COMS 572: Homework #5

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Problem 1

(48 pts.) Represent the following sentences in first-order logic, using a consistent vocabulary (which you must define):

Definitions: **Takes(x,y,z)**: student x takes course y in semester z; **Passes(x,y,z)**: student x passes course y in semester z; **Score(x,y)**: The score of student x in course y; **Buys(x,y,z)**: person x buys y from z; **Sells(x,y,z)**: person x sells y to z; **Shaves(x,y)**: x shaves y;**Citizen(x,y,z)**: person x is a citizen of country y for reason z; **Resident(x,y)**: person x is a resident of country y; **Fools(x,y,z)**: x fools y for time z.

a. Some students took French in spring 2001.

   Answer: $\exists x \; Student(x) \land Takes(x, French, Spring2011)$

b. Every student who takes French passes it.

   Answer: $\forall x \; Student(x) \land Takes(x, French, Spring2011) \Rightarrow Passes(x, French, Spring2011)$

c. Only one student took Greek in spring 2001.

   Answer:

   $\exists x, Student(x) \land Takes(x, Greek, Spring2001) \land [\forall y \; y \neq x, Student(y)] \Rightarrow \neg Takes(y, Greek, Spring2001)$

d. The best score in Greek is always higher than the best score in French.

   Answer: $\exists x \; \forall y, \; Score(x, Greek) > Score(y, French)$

e. Every person who buys a policy is smart.

   Answer: $\forall x \; Person(x) \land [\exists y, z \; Policy(y) \land Buys(x, y, z)] \Rightarrow Smart(x)$

f. No person buys an expensive policy.

   Answer: $\forall x, y, z \; Person(x) \land Policy(y) \land Expensive(y) \Rightarrow \neg Buys(x, y, z)$

g. There is an agent who sells policies only to people who are not insured.

   Answer: $\exists x \; Agent(x) \land \{\forall y \; Policy(y) \land [\forall z \; Sells(x, y, z)]\} \Rightarrow Person(z) \land \neg Insured(z)$

h. There is a barber who shaves all men in town who do not shave themselves.

   Answer: $\exists b \; \forall m \; Barber(x) \land Man(m) \land \neg Shaves(b, b) \Rightarrow Shaves(b, m)$

i. A person born in the UK, each of whose parents is a UK citizen or a UK resident, is a UK citizen by birth.

   Answer:

   $\forall x \; Person(x) \land Born(x, UK) \land [\forall y \; Parent(y, x) \Rightarrow Citizen(y, UK, r) \lor Resident(y, UK)] \Rightarrow Citizen(x, UK, birth)$

j. A person born outside the UK, one of whose parents is a UK citizen by birth, is a UK citizen by descent.

   Answer: $\forall x \; Person(x) \land \neg Born(x, UK) \land [\exists y \; Parent(y, x) \land Citizen(y, UK, birth)] \Rightarrow Citizen(x, UK, descent)$

k. Politicians can fool some of the people all of the time, and they can fool all of the people some of the time, but they cant fool all of the people all of the time.

   Answer: $\forall x \; Politician(x) \Rightarrow [\exists y \; \forall t \; Person(y) \Rightarrow Fools(x, y, t)] \land [\forall y \; \exists t \; Person(y) \Rightarrow Fools(x, y, t)] \land \neg[\forall y \; \forall t \; Person(y) \Rightarrow Fools(x, y, t)]$

l. All Greeks speak the same language. (Use $Speaks(x, l)$ to mean that person x speaks language l.)

   Answer: $\forall x, y, l \; Greek(x) \land Greek(y) \land Speaks(x, l) \Rightarrow Speaks(y, l)$

Problem 2

(8 pts.) For each pair of atomic sentences, give the most general unifier if it exists:

A. $P(A, B, B), P(x, y, z)$.

Answer: $\{x/A, y/B, z/B\}$

Unification:

$P(A, B, B), P(x, y, z) : \{x/A\}$

$P(A, B, B), P(A, y, z) : \{x/A, y/B\}$

$P(A, B, B), P(A, B, z) : \{x/A, y/B, z/B\}$

$P(A, B, B), P(A, B, B)$

B. $Q(y, G(A, B)), Q(G(x, x), y)$.

Answer: Do not exist

Unification:

$Q(y, G(A, B)), Q(G(x, x), y) : \{y/G(x, x)\}$

$Q(G(x, x), G(A, B)), Q(G(x, x), G(x, x)) : \{y/G(x, x), x/A\}$

$Q(G(A, A), G(A, B)), Q(G(A, A), G(A, A)) : \{y/G(x, x), x/A\}$ Cannot unify A/B

C. $Older(Father(y), y), Older(Father(x), John)$.

Answer: $\{x/John, y/John\}$

Unification:

$Older(Father(y), y), Older(Father(x), John)$: need recursion

$Older(Father(y), y), Older(Father(x), John) : \{x/y\}$

$Older(Father(y), y), Older(Father(y), John) : \{x/y, y/John\} = \{x/John, y/John\}$

$Older(Father(John), John), Older(Father(John), John)$

D. $Knows(Father(y), y), Knows(x, x)$.

Answer: Do not exist

Unification:

$Knows(Father(y), y), Knows(x, x) : \{x/Father(y)\}$

$Knows(Father(y), y), Knows(Father(y), Father(y)) : \{x/Father(y)\}$ Cannot unify $y/Father(y)$