HW#4

[redacted]

6.4 Give precise formulations for each of the following as constraint satisfaction problems:

a. Rectilinear floor-planning: find non-overlapping places in a large rectangle for a number of smaller rectangles.

b. Class scheduling: There is a fixed number of professors and classrooms, a list of classes to be offered, and a list of possible time slots for classes. Each professor has a set of classes that he or she can teach.

Solution:

6.4 **a.** For rectilinear floor-planning, one possibility is to have a variable for each of the small rectangles, with the value of each variable being a 4-tuple consisting of the x and y coordinates of the upper left and lower right corners of the place where the rectangle will be located. The domain of each variable is the set of 4-tuples that are the right size for the corresponding small rectangle and that fit within the large rectangle. Constraints say that no two rectangles can overlap; for example if the value of variable R_1 is $[0, 0, 5, 8]$, then no other variable can take on a value that overlaps with the $0, 0$ to $5, 8$ rectangle.

b. For class scheduling, one possibility is to have three variables for each class, one with times for values (e.g. MWF8:00, TuTh8:00, MWF9:00, ...), one with classrooms for values (e.g. Wheeler110, Evans330, ...) and one with instructors for values (e.g. Abelson, Bibel, Canny, ...). Constraints say that only one class can be in the same classroom at the same time, and an instructor can only teach one class at a time. There may be other constraints as well (e.g. an instructor should not have two consecutive classes).

6.7 Consider the following logic puzzle: In five houses, each with a different color, live five persons of different nationalities, each of whom prefers a different brand of candy, a different drink, and a different pet. Given the following facts, the questions to answer are "Where does the zebra live, and in which house do they drink water?"

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The Englishman lives in the red house.
The Spaniard owns the dog.
The Norwegian lives in the first house on the left.
The green house is immediately to the right of the ivory house.
The man who eats Hershey bars lives in the house next to the man with the fox.
Kit Kats are eaten in the yellow house.
The Norwegian lives next to the blue house.
The Smarties eater owns snails.
The Snickers eater drinks orange juice.
The Ukrainian drinks tea.
The Japanese eats Milky Ways.
Kit Kats are eaten in a house next to the house where the horse is kept.
Coffee is drunk in the green house.
Milk is drunk in the middle house.

Discuss different representations of this problem as a CSP. Why would one prefer one representation over another?

Solution:

Two possible alternatives using finite domain constraints:

1. Variables: 25. The first 5 variables could represent the 5 colors (i.e. red, yellow, blue, green and ivory), the second 5 group the 5 animals, the next 5 the nationalities, the next 5 the brands of candy and the last 5 the drinks.

   Domain: Each variable will store a value in the range $\{1, ..., 5\}$. The value represent the house number, so if for example, the first variable has a value of 3 it will represent that the red house is the third one.

   Examples of constraints: Englishman = Red (The Englishman lives in the red house), Spaniard = Dog (The Spaniard owns the dog), Green = Ivory + 1 (The green house is immediately to the right of the ivory house), and |Norwegian - Blue | = 1 (The Norwegian lives next to the blue house).

   Number of possible solutions: $O(5!^5)$

   Advantages: It is easy for modeling and programming.

2. Variables: 25. Each group of 5 variables represent the attributes of a house. For example, the variable 1 could be the color of the first house, the second variable the nationality of the person living in the first house, and so on.

   Domain: Each variable will store a value in the range 1..5. The problem with this representation is that any value has a different semantic meaning for each variable of the 5-element set. For example, if in the first variable we have a 1, it could mean that the color of the first house is red, but if we have a 1 in the second variable it means that the Englishman is in the first house.

   Examples of constraints: Var2 = 3 (The Norwegian lives in the first house). With this representation it is difficult to implement constraints as *The English lives in the red house*, and we require additional variables.

   Number of possible solutions: $O(5!^5)$

   Another disadvantages: Given that we need additional variables, the size of the search space is greater than the size of the search space in the first alternative. In addition, it is difficult for programming and even for understanding the code. For those reasons, I prefer the first alternative.

In general, one important aspect to have into account for selecting between different CSP specifications is the size of the search space. Another aspect to consider is that a representation could need less constraints than another.