COMS 572: Homework #4

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Problem 1

(10 pts.) Give precise formulations for each of the following as constraint satisfaction problems:

b. Class scheduling: There is a fixed number of professors and classrooms, a list of classes to be offered, and a list of possible time slots for classes. Each professor has a set of classes that he or she can teach.

Answer:

Variables:

- P_i for each professor. $P_i.class$ is the set of classes a certain professor can teach.
- R_i for each classroom
- C_i for each class
- T_i for each time slot
- Class assignment A_i, with each assignment being a 4 tuple: $(A_i.prof, A_i.room, A_i.class, A_i.time)$.

Domain:

- $A_i.prof \in P_j$
- $A_i.room \in R_j$
- $A_i.class \in C_j$
- $A_i.time \in T_j$

Constraint:

- $A_i.class \in A_i.prof.class$ for all i.
- $\neg(A_i.time = A_j.time \wedge A_i.prof = A_j.prof)$ for all $i \neq j$
- $\neg(A_i.time = A_j.time \wedge A_i.room = A_j.room)$ for all $i \neq j$

Problem 2

(20 pts.) Consider the following logic puzzle: In five houses, each with a different color, live five persons of different nationalities, each of whom prefers a different brand of candy, a different drink, and a different pet. Given the following facts, the questions to answer are Where does the zebra live, and in which house do they drink water?

A. The Englishman lives in the red house.

B. The Spaniard owns the dog.

C. The Norwegian lives in the first house on the left.

D. The green house is immediately to the right of the ivory house.

E. The man who eats Hershey bars lives in the house next to the man with the fox.

F. Kit Kats are eaten in the yellow house.

G. The Norwegian lives next to the blue house.

H. The Smarties eater owns snails.

I. The Snickers eater drinks orange juice.

J. The Ukrainian drinks tea.

K. The Japanese eats Milky Ways.

L. Kit Kats are eaten in a house next to the house where the horse is kept.

M. Coffee is drunk in the green house.

N. Milk is drunk in the middle house.

Discuss different representations of this problem as a CSP.
Formulate the puzzle as a CSP. (Note: Discussion of different representations not required, solution of puzzle not required.)
Answer:
For this problem, we have 5 different variables each with 5 different values:

Color	**Nationality**	**Drink**	**Candy**	**Pet**
red	English	orange juice	Hershey bar	dog
green	Spanish	tea	Kit Kat	fox
ivory	Norwegian	coffee	Smarties	snail
yellow	Ukrainian	milk	Snickers	horse
blue	Japanese	water	Milky Way	zebra

We can assign a number from 1 to 5 to each house, left to right respectively. We have constraints from A to N given by the problem. Each letter of the constraint should have exact one corresponding house number from 1 to 5. We can now generate a 5x5 table and insert each constraint to its corresponding slot. In this table, for each column, each number 1 to 5 should appear exactly once.

If we use $\underline{W}$ to represent water, and $\underline{Z}$ to represent Zebra, We will have:

Color	**Nationality**	**Drink**	**Candy**	**Pet**
A	A	I	E	B
D=M	B	J	F=L	E±1
D-1	C=1=G±1	M	H	H
F	J	N=3	I	L±1
G	K	$\underline{W}$	K	$\underline{Z}$

Then, we can cancel out some duplication(like D=M, C=G±1, F=L, etc.), and **deduce F=1**, we have:

Color	**Nationality**	**Drink**	**Candy**	**Pet**
A	A	I	E	B
D	B	J	F=1	E±1
D-1	1	D	H	H
F=1	J	3	I	2
2	K	$\underline{W}$	K	$\underline{Z}$

Thus, we can have following constraints with unavailable values crossed out:

	1	**2**	**3**	**4**	**5**
A	X	X			
B	X	X			
D	X	X	X		
E	X				
F		X	X	X	X
H	X	X			
I	X		X		
J	X		X		
K	X				

Because I, J, D cannot be 1, if we look at column Drink, we know that **W=1**.

If D=4, then from column Color and Drink, we know that A=5, I=5, J=2. So H, B, K must chose from $2, 3$ which means one of column Nationality, Candy and Pet must have duplicate numbers, which is not allowed. Thus, **D has to be 5 and A=3**.

If J=4, from column Nationality, and $B \neq 2$, B=5 and K=2; from column Drink, I=2. However, it violates column Candy because I=K=2 is not allowed here.

Therefore, **J=2, I=4, K=5, B=4, E=3, H=3, Z=5**.

Now, we have the solution to this puzzle:

House	1	2	3	4	5
Color	yellow	blue	red	ivory	green
Nationality	Norwegian	Ukrainian	English	Spainish	Japanese
Drink	**WATER**	tea	milk	orange juice	coffee
Candy	Kit Kat	Hershey bar	Smarties	Snickers	Milky Way
Pet	fox	horse	snails	dog	**ZEBRA**

So **Zebra lives in green house**, and **in yellow house they drink water**.

Problem 3

(20 pts.) (Adapted from Barwise and Etchemendy (1993).) Given the following, can you prove that the unicorn is mythical? How about magical? Horned?

> If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal.
>
> If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

Answer:
From the desctiptions we have:

1. $Mythical \implies Immortal$
2. $\neg Mythical \implies \neg Immortal \wedge Mammal$
3. $Immortal \vee Mammal \implies Horned$
4. $Horned \implies Magical$

From 2, we get:

5. $\neg Mythical \implies Mammal$

From 1 and 5, we get:

6. $Mythical \vee \neg Mythical \implies Immortal \vee Mammal$

Then, with 3 and 6, we have:

7. $Horned$

With 4 and 7, we have:

8. $Magical$

Therefore, the unicorn is horned and magical.
However, we cannot prove it is mythical.

Problem 4

(30 pts.) Consider the following sentence:

$$[(Food \implies Party) \lor (Drinks \implies Party)] \implies [(Food \land Drinks) \implies Party]$$

a. Determine, using enumeration, whether this sentence is valid, satisfiable (but not valid), or unsatisfiable.

Answer:

This sentence is valid because it is true for all cases.

F	P	D	$F \Rightarrow P$	$D \Rightarrow P$	$(F \Rightarrow P)$ $\lor(D \Rightarrow P)$	$F \land D$	$F \land D$ $\Rightarrow P$	$[(F \Rightarrow P) \lor (D \Rightarrow P)]$ $\Rightarrow [(F \land D) \Rightarrow P]$
T	T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T	T
T	F	T	F	F	F	T	F	T
T	F	F	F	T	T	F	T	T
F	T	T	T	T	T	F	T	T
F	T	F	T	T	T	F	T	T
F	F	T	T	F	T	F	T	T
F	F	F	T	T	T	F	T	T

b. Convert the left-hand and right-hand sides of the main implication into CNF, showing each step, and explain how the results confirm your answer to (a).

Proof.

$$[(Food \implies Party) \lor (Drinks \implies Party)] \implies [(Food \land Drinks) \implies Party]$$

$$[(\neg Food \lor Party) \lor (\neg Drinks \lor Party)] \implies [\neg(Food \land Drinks) \lor Party]$$

$$[(\neg Food \lor Party) \lor (\neg Drinks \lor Party)] \implies [(\neg Food \lor \neg Drinks) \lor Party]$$

$$(\neg Food \lor \neg Drinks \lor Party) \implies (\neg Food \lor \neg Drinks \lor Party)$$

After the conversion, we can see that we have the exactly same form on both sides which implies that the original sentence is valid. Therefore, the answer to (a) is correct. □

c. Prove your answer to (a) using resolution.

Proof. Assume we represent the left hand side and right hand side with LHS and RHS respectively. To prove the answer to (a), we need to show that $LHS \land \neg RHS$ cannot be satisfied.

Initially, we have:

$$LHS \land \neg RHS \equiv [(Food \implies Party) \lor (Drinks \implies Party)] \land \neg[(Food \land Drinks) \implies Party]$$

Then, we have:

$$LHS \land \neg RHS \equiv (\neg Food \lor \neg Drinks \lor Party) \land (Food) \land (Drinks) \land (\neg Party)$$

As we can see, all the elements will be canceled so $LHS \land \neg RHS$ resolves to empty clause. Thus, we have proved that the original sentense is valid. □