Appendix — Not Intended for Publication

A.1 Data sources and construction

Per capita income and debt

In the empirical analysis I employ per capita GDP in 1990 constant international dollars (in logarithms) and the percentage debt-to-GDP ratio (also in logarithms), where debt is generally gross central (external and domestic) government debt. The raw data for the analysis of four OECD countries is taken from the Excel spreadsheets available on the following websites: for data on debt www.reinhartandrogoff.com — the Reinhart and Rogoff (2009) companion website; and for data on per capita income (as well as population data) www.ggdc.net/maddison — the Maddison Project website at the University of Groningen.

A number of comments regarding these data as well as changes and/or adjustments are provided in the following:

- For Great Britain the debt series refer to net rather than gross central (external and domestic) government debt.
- The 'New' Maddison data provides two values for Great Britain's per capita GDP in 1851 — 2,330 and 2,718 — since this is where two data series come together: up to 1851 the estimates are taken from Van Zanden (2001), from 1851 onwards from the original Maddison (2010) estimates. I pick the arithmetic mean of the two values.
- Data on the debt-to-GDP ratio for Japan between 1941 and 1951 are computed from outstanding debt reported in Statistics Japan's *Historical Statistics of Japan*, available at www.stat.go.jp/english/data/chouki/05.htm. The 1952 and 1953 figures are computed from debt and GDP numbers in the excel spreadsheets marked 'RR' provided by Herndon, Ash and Pollin (2014). These are available at www.peri.umass.edu/236/hash/31e2ff374b6377b2ddec04deaa6388b1/publication/566/. Japanese GDP figures for 1941-51 are taken from Mitchell (2007*a*).
- I interpolated the debt-GDP ratio for Japan in 1882 where only a single observation was

missing in the Reinhart and Rogoff data. Similarly for the GDP series in 1945.

The analysis of time series of debt and per capita income in 27 economies is based on the same data sources. Below I discuss any changes made to variables in that sample:

– I interpolated the debt-to-GDP ratio in 3 cases where only a single observation was missing or (in case of India) where the recorded value was not credible (zero debt): Argentina 1866, India 1947, Peru 1960.

– For Brazil I chose debt data starting from 1889 since prior to this date the series covered only external debt. For Argentina, Italy, the Netherlands and New Zealand the debt series represent general rather than central government debt.

Additional Variables

Inflation I use inflation data from the Clio Infra project at the *International Institute of Social History* which reports the annual percentage change. Excel spreadsheets are available for download at www.clio-infra.eu. Downloads automatically include very detailed information on the original sources of the data. The following adjustments were made: for *Chile* in 1914 (two values provided), I took the arithmetic mean. Data for *Austria* in 1915 was missing so I linearly interpolated, similarly for *Australia* in 1911 and 1912, and *New Zealand* in 2010 (extrapolated).

Population The original Maddison (2010) database provides data on mid-year population (in thousands). My analysis employs the population growth rate. For all population series I extrapolate the value for 2010 from the Maddison data to maintain the integrity of the data series (alternative sources, e.g. the Penn World Table version 8.0, gave marginally different figures for the last years leading up to 2010 than the Maddison data).

– Population data for *Sweden* during 1800-1819 is taken from Schön and Krantz (2012). Excel spreadsheets are available from www.ekh.lu.se/en/research/economic_history_data/shna1560-2010.

– Population data for the *United States* during 1800–1819 is taken from the US Bureau of the Census, *Historical Statistics of the United States, Colonial Times to 1970*, Bicentennial Edi-

tion, Part 2, Washington, D.C., 1975. A pdf (Series A 6-8) is available from www.census.gov/prod/www/statistical_abstract.html.

– Population data for the *United Kingdom* 1800–1819 is taken from Mitchell (1971), page 8.

– Population data for *Argentina* in 1869 and 1895 is taken from Argentina's Instituto Nacional de Estadistica y Censos (indec), www.indec.mecon.ar/. Data for 1870–1894 and 1896–1899 are linearly interpolated.

Investment This is the investment-to-GDP ratio in percent (in the empirical analysis further in logarithms). The main source for these data are the tables available on the GGDC Maddison website, which report Maddison's (1992) domestic capital formation in percent of GDP. Below I indicate additional and alternative sources for these data ('Maddison' here refers to the 1992 *SJE* data):

– *Argentina* 1900–2010 is taken from the Montevideo-Oxford Latin American Economic History Data Base (MOxLAD) available at www.lac.ox.ac.uk/moxlad-database.

– *Australia* 1861–1869 computed from Mitchell (2007*a*); 1870–1989 from Maddison; for the remainder of the series I use the World Bank World Development Indicators (WDI) gross fixed capital formation (GFCF) series.

– *Brazil* 1900–2010 from MOxLAD.

– *Canada* 1870–1988 from Maddison; remainder from WDI.

– *Chile* 1900–2010 from MOxLAD.

– *Colombia* 1900–2010 from MOxLAD.

– *Denmark* 1850–1945 from Jones and Obstfeld (1997); 1946–1969 from Mitchell (2007*b*); remainder from WDI.

– *France* 1870–1988 from Maddison; 1850–1869, 1919–21, and 1939 from Jones and Obstfeld (1997); post-1939 from WDI.

– *Germany* 1925–1988 from Maddison; 1850–1913 from Jones and Obstfeld (1997); post-1988 from WDI.

– *India* 1870–1987 from Maddison; remainder from WDI.

– *Japan* 1885–1988 from Maddison, where I linearly interpolated for 1945; post-1988 from WDI.

– *Netherlands* 1921–1988 from Maddison (with gaps), for 1807–1913 I compute the investment/GDP ratio from GDP at current prices and total current gross fixed capital formation from the *National Accounts of the Netherlands 1800–1913*, provided by the Netherlands Research Institute and Graduate School on Economic and Social History at Data Archiving and Networked Services (DANS), available online at nationalaccounts.niwi.knaw.nl/start.htm; post-1988 from WDI.

– *Norway* 1830–2010 from Norges Bank *Historical Monetary Statistics – GDP and its components from 1830 to 2010* available at www.norges-bank.no/en/Statistics/.

– *Sweden* 1800–2000 is taken from the *Historical National Accounts for Sweden 1800–2000* available at www.ekh.lu.se/en/research/economic_history_data/shna1560-2010; post-2000 from WDI.

– *United Kingdom*: 1850–1945 from Jones and Obstfeld (1997); thereafter from Maddison (to 1988); post-1945 from WDI.

– *United States*: 1870–1986 from Maddison; post-1986 from WDI.

Human Capital Average years of education in the population is taken from the Excel spreadsheets provided by the Clio Infra project at www.clio-infra.eu.

A.2 Descriptives and Data Properties

Descriptive statistics for the four OECD countries analysed in the maintext are presented in Table A-1. Table A-2 reports results from unit root tests for all linear processes. This indicates that growth rates of per capita GDP or debt-to-GDP are stationary, while the levels of these variables are nonstationary. Figure A-1 charts the log levels and growth rates of per capita GDP (left column) and the debt-to-GDP ratio (right column) in the four sample countries.

Table A-1: Descriptive Statistics

		Start	End	Obs	Mean	Median	St.Dev.	Min	Max
USA	ln(GDP pc)	1800	2010	211	8.510	8.425	0.996	7.159	10.363
	Δln(GDP pc)			210	0.015	0.017	0.047	-0.241	0.171
	ln(Debt/GDP)			211	2.667	2.958	1.629	-5.878	4.798
	ln(Debt/GDP)^2			211	9.754	9.090	6.397	0.010	34.552
	ln(Debt/GDP)^3			211	31.026	25.870	36.127	-203.104	110.445
GBR	ln(GDP pc)	1800	2010	211	8.527	8.414	0.727	7.574	10.127
	Δln(GDP pc)			210	0.012	0.016	0.031	-0.114	0.091
	ln(Debt/GDP)			211	4.468	4.598	0.685	3.201	5.563
	ln(Debt/GDP)^2			211	20.432	21.139	6.057	10.247	30.946
	ln(Debt/GDP)^3			211	95.416	97.193	40.799	32.802	172.153
SWE	ln(GDP pc)	1800	2010	211	8.025	7.722	1.115	6.641	10.142
	Δln(GDP pc)			210	0.016	0.020	0.034	-0.094	0.120
	ln(Debt/GDP)			211	2.748	2.881	0.915	0.485	4.387
	ln(Debt/GDP)^2			211	8.385	8.297	4.605	0.235	19.246
	ln(Debt/GDP)^3			211	27.133	23.901	20.076	0.114	84.431
JPN	ln(GDP pc)	1872	2010	139	8.111	7.660	1.158	6.615	10.017
	Δln(GDP pc)			138	0.025	0.023	0.077	-0.681	0.162
	ln(Debt/GDP)			139	3.528	3.628	0.870	1.519	5.318
	ln(Debt/GDP)^2			139	13.197	13.164	6.066	2.307	28.280
	ln(Debt/GDP)^3			139	51.737	47.761	34.045	3.504	150.390

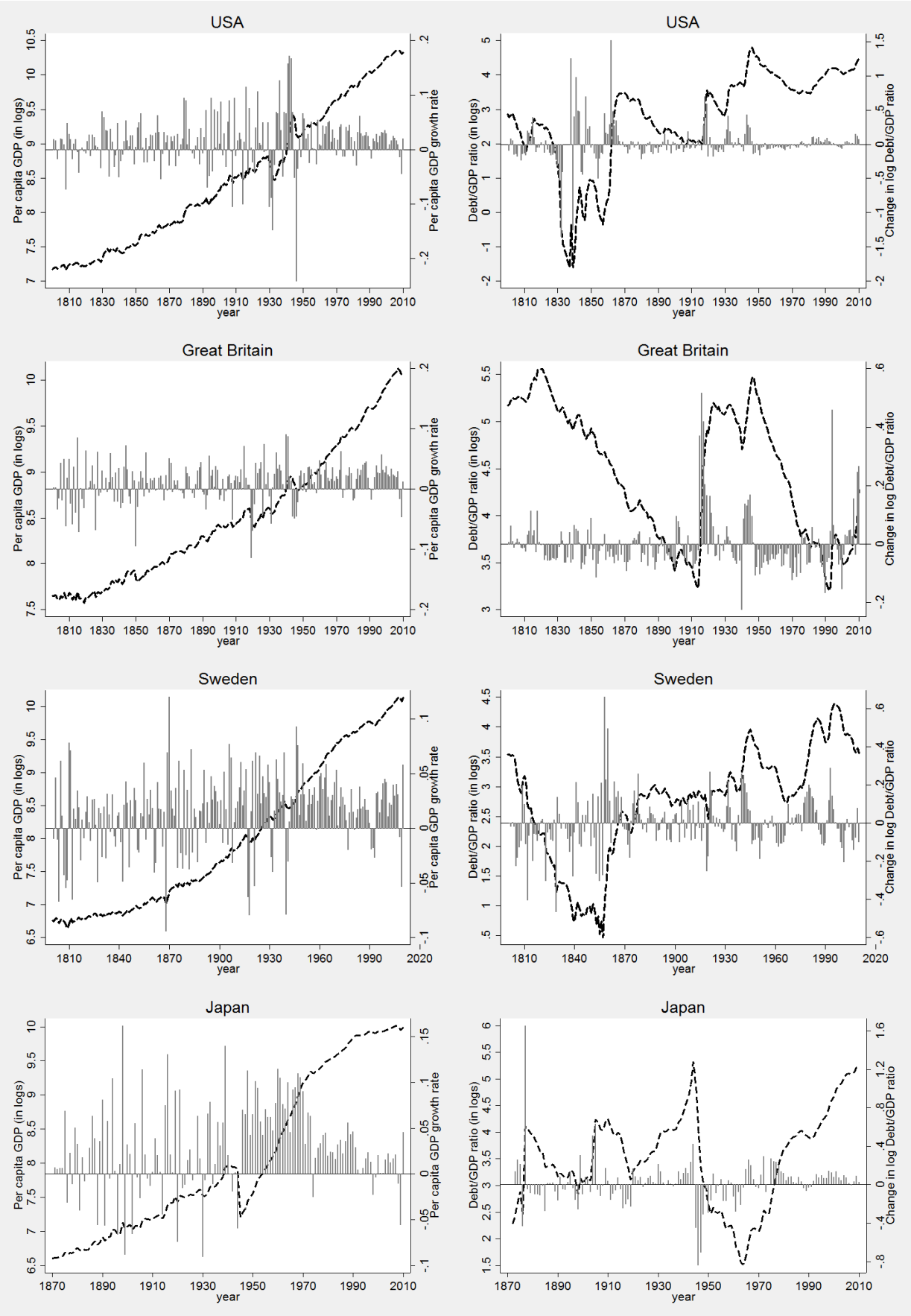
Notes: I provide the descriptive statistics for the levels variables included in our analysis (all in logarithms as indicated). Gap reports the number of missing observations. Δ is the difference operator.

Table A-2: Unit root/stationarity tests

		Start	End	Lags	DF-GLS	PP Z(t)	KPSS(i)	KPSS(ii)
USA	ln(GDP pc)	1800	2010	9	-0.88	0.65	0.47 ***	2.27 ***
	(Debt/GDP)			13	-2.55	-1.03	0.11	1.66 ***
	ln(Debt/GDP)			6	-2.38	-2.27	0.08	1.06 ***
	Δln(GDP pc)	1801	2010	8	-6.17***	-11.29***	0.03	0.14
	Δln(Debt/GDP)			5	-6.31***	-10.54***	0.03	0.04
GBR	ln(GDP pc)	1800	2010	6	0.09	2.26	0.66***	3.07***
	(Debt/GDP)			6	-1.89	-1.64	0.34***	0.94***
	ln(Debt/GDP)			1	-2.12	-1.66	0.93***	2.64***
	Δln(GDP pc)	1801	2010	8	-6.41***	-12.70***	0.03	0.52***
	Δln(Debt/GDP)			14	-2.08	-7.00***	0.08	0.10
SWE	ln(GDP pc)	1800	2010	10	-0.48	2.71	0.55***	1.99***
	(Debt/GDP)			10	-2.16	-1.95	0.12*	1.37***
	ln(Debt/GDP)			10	-1.66	-1.34	0.10	1.44***
	Δln(GDP pc)	1801	2010	9	-3.29**	-13.59***	0.14*	0.83***
	Δln(Debt/GDP)			9	-3.35**	-11.25***	0.10	0.12
JPN	ln(GDP pc)	1872	2010	0	-1.25	0.18	2.34***	13.30***
	(Debt/GDP)			2	-1.48	-0.88	0.39***	0.90***
	ln(Debt/GDP)			3	-1.76	-1.72	0.33***	0.39*
	Δln(GDP pc)	1873	2010	0	-10.77***	-10.76***	0.11	0.21
	Δln(Debt/GDP)			2	-4.54***	-9.78***	0.11	0.12

Notes: I use the Elliott, Rothenberg and Stock (1996) DF-GLS, Phillips and Perron (1988) PP and Kwiatkowski et al. (1992) KPSS tests. Lag length based on Ng and Perron (1995) criterion (DF-GLS and KPSS). KPSS(i) and (ii) have the null of trend and levels stationarity, respectively; DF-GLS and PP have the null of nonstationarity. Δ is the difference operator.

Figure A-1: Income and debt evolution



Notes: These plots chart the evolution of per capita GDP (in logs, left column) and the debt/GDP ratio (in logs, right column) for our four OECD countries. In each plot the levels variable (left axis, dashed line) is graphed alongside the variable in first differences (right axis, gray bars). Note that the per capita GDP growth series for Japan (-68%) excludes 1945 for ease of illustration.

Table A-3: Country Coverage (Extended Analysis)

Country	Argentina	Australia	Austria♯	Belgium♯	Brazil	Canada
Isocode	ARG	AUS	AUT	BEL	BRA	CAN
Start	1875	1861	1880	1846	1889	1870
End	2010	2010	2010	2010	2010	2010
Gaps	0	0	20	12	0	0
Obs	136	150	111	153	122	141

Country	Chile	Colombia	Denmark	France	Germany‡	Greece♯
Isocode	CHL	COL	DNK	FRA	DEU	GRC
Start	1870	1899	1880	1880	1880	1848
End	2010	2010	2010	2010	2010	2010
Gaps	0	0	0	23	37	15
Obs	141	112	131	108	94	163

Country	India	Italy	Japan	Netherlands	New Zealand♯	Norway
Isocode	IND	ITA	JPN	NLD	NZL	NOR
Start	1884	1861	1872	1815	1870	1880
End	2010	2010	2010	2010	2010	2010
Gaps	0	0	0	6	0	6
Obs	127	150	139	190	151	131

Country	Peru♯	Portugal♯	Spain♯	Sri Lanka⊗	Sweden	Switzerland♯
Isocode	PER	PRT	ESP	LKA	SWE	CHE
Start	1883	1865	1850	1870	1800	1880
End	2010	2010	2010	2009	2010	2010
Gaps	14	0	4	35	0	16
Obs	114	146	157	105	211	115

Country	Great Britain	USA	Uruguay♯
Isocode	GBR	USA	URY
Start	1800	1800	1871
End	2010	2010	2009
Gaps	0	0	23
Obs	211	211	116

Notes: I present start and end years of per capita GDP and debt-ratio time series for the set of 27 countries for which I report the summability, balance and co-summability results below. Data for all of these countries, with the exception of Sri Lanka (LKA) ⊗ are also included in the 'debt only' and 'debt and inflation' panel models (Table 5), although data coverage differs across models and countries. Excel files available at my personal website in due course will provide information on the country-time coverage by model. ♯ These countries are excluded from the 'Solow Model with Debt' and the 'Augmented Solow Model with Debt' due to limited time series data for one of the variable series. ‡ Germany is excluded from the 'Augmented Solow Model with Debt' for the same reason.

A.3 Additional Empirical Results

Table A-4
Balance (and implicitly Co-Summability) — Δln(GDP pc) specifications

						Balance			
	Start	End	Gaps	Obs	Nonlinearity	CI low	$\hat{\delta}_y - \hat{\delta}_z$	CI up	Verdict
USA	1801	2010	-	210	-	-2.199	-1.403	-0.606	$S(\delta_y) \neq S(\delta_z)$
				b =15	ln(Debt/GDP)^2	-2.642	-1.622	-0.603	$S(\delta_y) \neq S(\delta_z)$
				M =196	ln(Debt/GDP)^3	-3.237	-2.037	-0.836	$S(\delta_y) \neq S(\delta_z)$
					Threshold	-3.024	-1.822	-0.620	$S(\delta_y) \neq S(\delta_z)$
GBR	1801	2010	-	210	-	-1.781	-0.944	-0.106	$S(\delta_y) \neq S(\delta_z)$
				b =15	ln(Debt/GDP)^2	-2.573	-1.458	-0.343	$S(\delta_y) \neq S(\delta_z)$
				M =196	ln(Debt/GDP)^3	-3.252	-1.903	-0.554	$S(\delta_y) \neq S(\delta_z)$
					Thresholds	-3.416	-2.007	-0.597	$S(\delta_y) \neq S(\delta_z)$
SWE	1801	2010	-	210	-	-2.201	-1.457	-0.713	$S(\delta_y) \neq S(\delta_z)$
				b =15	ln(Debt/GDP)^2	-2.806	-1.884	-0.963	$S(\delta_y) \neq S(\delta_z)$
				M =196	ln(Debt/GDP)^3	-3.301	-2.218	-1.135	$S(\delta_y) \neq S(\delta_z)$
JPN	1873	2010		136	-	-1.925	-1.212	-0.499	$S(\delta_y) \neq S(\delta_z)$
				13	ln(Debt/GDP)^2	-2.680	-1.674	-0.667	$S(\delta_y) \neq S(\delta_z)$
				124	ln(Debt/GDP)^3	-3.293	-2.042	-0.792	$S(\delta_y) \neq S(\delta_z)$
					Threshold	-3.149	-1.908	-0.668	$S(\delta_y) \neq S(\delta_z)$

Notes: In all models I take the per capita GDP growth rate, Δln(GDP pc), as the dependent variable. See Table 2 for all other details. Since no model satisfies the balance test I do not carry out co-summability testing.

Table A-5: Estimated Order of Summability — 27 countries

Country	Start Year	End Year	Gaps	Obs	Variable	CI low	$\hat{\delta}$	CI up
ARG	1875	2010	-	136	ln(GDP pc)	0.279	0.851	1.423
					Δln(GDP pc)	-0.855	-0.249	0.357
					ln(Debt/GDP)	0.117	0.661	1.205
					ln(Debt/GDP) squared	0.075	0.691	1.308
					ln(Debt/GDP) cubed	0.117	0.727	1.337
AUS	1861	2010	-	150	ln(GDP pc)	0.262	0.847	1.432
					Δln(GDP pc)	-0.222	0.464	1.151
					ln(Debt/GDP)	0.402	0.981	1.560
					ln(Debt/GDP) squared	0.300	0.978	1.656
					ln(Debt/GDP) cubed	0.308	1.011	1.714
AUT	1880	2010	20	111	ln(GDP pc)	0.056	0.775	1.495
					Δln(GDP pc)	-0.552	0.104	0.760
					ln(Debt/GDP)	0.546	1.225	1.904
					ln(Debt/GDP) squared	0.473	1.190	1.907
					ln(Debt/GDP) cubed	0.476	1.158	1.839
BEL	1846	2010	12	153	ln(GDP pc)	0.347	0.730	1.113
					Δln(GDP pc)	-0.484	0.163	0.810
					ln(Debt/GDP)	0.147	0.680	1.213
					ln(Debt/GDP) squared	0.131	0.675	1.220
					ln(Debt/GDP) cubed	0.106	0.673	1.240
BRA	1889	2010	-	122	ln(GDP pc)	0.650	1.157	1.664
					Δln(GDP pc)	0.094	1.064	2.035
					ln(Debt/GDP)	-0.385	0.376	1.137
					ln(Debt/GDP) squared	-0.510	0.321	1.152
					ln(Debt/GDP) cubed	-0.596	0.244	1.084
CAN	1870	2010	-	141	ln(GDP pc)	0.101	0.552	1.003
					Δln(GDP pc)	-1.235	-0.354	0.527
					ln(Debt/GDP)	-0.005	0.627	1.258
					ln(Debt/GDP) squared	0.024	0.657	1.291
					ln(Debt/GDP) cubed	0.136	0.690	1.244
CHL	1870	2010	-	141	ln(GDP pc)	0.145	0.717	1.289
					Δln(GDP pc)	-0.584	0.022	0.628
					ln(Debt/GDP)	0.135	0.818	1.500
					ln(Debt/GDP) squared	0.176	0.828	1.480
					ln(Debt/GDP) cubed	0.216	0.859	1.501
COL	1899	2010	-	112	ln(GDP pc)	0.817	1.537	2.257
					Δln(GDP pc)	0.374	1.157	1.940
					ln(Debt/GDP)	0.673	1.312	1.950
					ln(Debt/GDP) squared	0.745	1.243	1.740
					ln(Debt/GDP) cubed	0.766	1.179	1.593
DNK	1880	2010	-	131	ln(GDP pc)	0.141	0.677	1.213
					Δln(GDP pc)	-0.573	0.155	0.882
					ln(Debt/GDP)	0.389	1.182	1.976
					ln(Debt/GDP) squared	0.484	1.228	1.972
					ln(Debt/GDP) cubed	0.514	1.240	1.967

Table continued on the following page

Table A-5: Estimated Order of Summability — 27 countries (continued)

Country	Start Year	End Year	Gaps	Obs	Variable	CI low	$\hat{\delta}$	CI up
FRA	1880	2010	23	108	ln(GDP pc)	0.463	1.307	2.150
					Δln(GDP pc)	-0.490	0.226	0.942
					ln(Debt/GDP)	0.773	1.564	2.356
					ln(Debt/GDP) squared	1.153	2.336	3.519
					ln(Debt/GDP) cubed	1.557	3.156	4.755
DEU	1880	2010	37	94	ln(GDP pc)	0.549	1.239	1.929
					Δln(GDP pc)	-0.567	0.121	0.808
					ln(Debt/GDP)	0.514	0.940	1.367
					ln(Debt/GDP) squared	0.509	0.935	1.360
					ln(Debt/GDP) cubed	0.531	0.937	1.343
GRC	1848	2010	15	148	ln(GDP pc)	0.431	1.169	1.907
					Δln(GDP pc)	-0.534	0.138	0.809
					ln(Debt/GDP)	0.324	0.856	1.388
					ln(Debt/GDP) squared	0.406	0.960	1.515
					ln(Debt/GDP) cubed	0.367	0.990	1.613
IND	1884	2010	-	127	ln(GDP pc)	0.031	0.507	0.982
					Δln(GDP pc)	-0.925	-0.130	0.666
					ln(Debt/GDP)	0.279	0.991	1.702
					ln(Debt/GDP) squared	0.393	1.045	1.697
					ln(Debt/GDP) cubed	0.325	1.098	1.872
ITA	1861	2010	-	150	ln(GDP pc)	0.495	1.162	1.829
					Δln(GDP pc)	-0.210	0.364	0.939
					ln(Debt/GDP)	0.380	0.921	1.462
					ln(Debt/GDP) squared	0.384	0.937	1.491
					ln(Debt/GDP) cubed	0.390	0.953	1.517
JPN	1872	2010	-	139	ln(GDP pc)	0.987	2.390	3.792
					Δln(GDP pc)	-0.692	-0.004	0.683
					ln(Debt/GDP)	0.427	1.091	1.755
					ln(Debt/GDP) squared	0.433	1.101	1.769
					ln(Debt/GDP) cubed	0.410	1.114	1.819
NLD	1815	2010	6	190	ln(GDP pc)	0.055	0.569	1.083
					Δln(GDP pc)	-0.353	0.304	0.961
					ln(Debt/GDP)	0.462	1.084	1.705
					ln(Debt/GDP) squared	0.477	1.089	1.702
					ln(Debt/GDP) cubed	0.528	1.097	1.666
NZL	1870	2010	-	141	ln(GDP pc)	0.098	0.503	0.909
					Δln(GDP pc)	-0.323	0.299	0.920
					ln(Debt/GDP)	0.479	0.960	1.441
					ln(Debt/GDP) squared	0.419	0.986	1.553
					ln(Debt/GDP) cubed	0.486	1.009	1.533
NOR	1880	2010	6	125	ln(GDP pc)	0.656	1.349	2.042
					Δln(GDP pc)	-0.179	0.579	1.337
					ln(Debt/GDP)	0.398	1.073	1.749
					ln(Debt/GDP) squared	0.394	1.086	1.778
					ln(Debt/GDP) cubed	0.394	1.101	1.808

Table continued on the following page

Table A-5: Estimated Order of Summability — 27 countries (continued)

Country	Start Year	End Year	Gaps	Obs	Variable	CI low	$\hat{\delta}$	CI up
PER	1883	2010	14	114	ln(GDP pc)	0.284	0.820	1.357
					Δln(GDP pc)	-0.073	0.665	1.404
					ln(Debt/GDP)	0.677	1.122	1.566
					ln(Debt/GDP) squared	0.683	1.063	1.444
					ln(Debt/GDP) cubed	0.676	1.009	1.342
PRT	1865	2010	-	146	ln(GDP pc)	0.464	1.087	1.709
					Δln(GDP pc)	0.010	0.802	1.594
					ln(Debt/GDP)	0.397	0.933	1.470
					ln(Debt/GDP) squared	0.347	0.940	1.533
					ln(Debt/GDP) cubed	0.381	0.945	1.510
ESP	1850	2010	4	157	ln(GDP pc)	0.212	0.767	1.322
					Δln(GDP pc)	-0.499	0.067	0.633
					ln(Debt/GDP)	0.394	0.994	1.595
					ln(Debt/GDP) squared	0.350	0.979	1.609
					ln(Debt/GDP) cubed	0.380	0.966	1.551
LKA	1870	2009	35	105	ln(GDP pc)	0.411	0.816	1.220
					Δln(GDP pc)	-0.319	0.379	1.078
					ln(Debt/GDP)	0.210	0.771	1.332
					ln(Debt/GDP) squared	0.240	0.797	1.354
					ln(Debt/GDP) cubed	0.224	0.822	1.420
SWE	1800	2010	-	211	ln(GDP pc)	0.361	0.904	1.334
					Δln(GDP pc)	-0.359	0.030	0.357
					ln(Debt/GDP)	0.637	1.624	2.603
					ln(Debt/GDP) squared	0.614	1.577	2.451
					ln(Debt/GDP) cubed	0.473	1.538	2.399
CHE	1880	2010	16	115	ln(GDP pc)	0.159	0.669	1.179
					Δln(GDP pc)	-0.690	-0.097	0.497
					ln(Debt/GDP)	0.506	1.265	2.023
					ln(Debt/GDP) squared	0.508	1.254	2.001
					ln(Debt/GDP) cubed	0.477	1.255	2.033
GBR	1800	2010	-	211	ln(GDP pc)	0.731	1.696	2.662
					Δln(GDP pc)	-0.444	0.126	0.695
					ln(Debt/GDP)	0.540	0.967	1.393
					ln(Debt/GDP) squared	0.509	0.948	1.386
					ln(Debt/GDP) cubed	0.475	0.931	1.387
USA	1800	2010	-	211	ln(GDP pc)	0.686	1.561	2.436
					Δln(GDP pc)	-0.522	0.052	0.627
					ln(Debt/GDP)	0.551	1.082	1.613
					ln(Debt/GDP) squared	0.383	0.860	1.336
					ln(Debt/GDP) cubed	0.404	0.993	1.582
URY	1871	2009	23	116	ln(GDP pc)	-0.128	0.539	1.206
					Δln(GDP pc)	-0.195	0.545	1.285
					ln(Debt/GDP)	0.419	1.061	1.704
					ln(Debt/GDP) squared	0.401	1.089	1.777
					ln(Debt/GDP) cubed	0.440	1.127	1.815

Notes: I report full sample order of summability estimates, CI low and up indicate the 95% confidence interval for the summability estimate $S(\hat{\delta})$ — shaded cells indicate variable series where the summability confidence interval includes zero. In all tests conducted I allow for deterministic terms (constant and trend). Country codes are detailed in the Data Appendix.

Table A-6: Balance and Co-Summability — ln(GDP pc) specifications, 27 countries

						Balance				Co-Summability			
	Start	End	Gaps	obs	Nonlinearity	CI low	$\hat{\delta}_y - \hat{\delta}_z$	CI up	Verdict	CI low	$\hat{\delta}_{\hat{e}_t}$	CI up	Verdict
ARG	1875	2010	-	136	-	-0.766	-0.109	0.549	$\hat{\delta}_y = \hat{\delta}_z$	1.483	2.654	3.826	$S(\delta_{\hat{e}_t}) \neq 0$
				b =13	ln(Debt/GDP)^2	-1.506	-0.624	0.258	$\hat{\delta}_y = \hat{\delta}_z$	0.947	1.605	2.264	$S(\delta_{\hat{e}_t}) \neq 0$
				M =124	ln(Debt/GDP)^3	-2.151	-1.055	0.042	$\hat{\delta}_y = \hat{\delta}_z$	0.859	1.536	2.212	$S(\delta_{\hat{e}_t}) \neq 0$
AUS	1861	2010	-	150	-	-1.081	-0.421	0.239	$\hat{\delta}_y = \hat{\delta}_z$	0.894	1.378	1.861	$S(\delta_{\hat{e}_t}) \neq 0$
				b =13	ln(Debt/GDP)^2	-1.504	-0.801	-0.098	$\hat{\delta}_y \neq \hat{\delta}_z$				
				138	ln(Debt/GDP)^3	-2.074	-1.145	-0.217	$\hat{\delta}_y \neq \hat{\delta}_z$				
AUT	1880	2010	20	111	-	-1.247	-0.398	0.452	$\hat{\delta}_y = \hat{\delta}_z$	0.451	1.038	1.626	$S(\delta_{\hat{e}_t}) \neq 0$
				b =12	ln(Debt/GDP)^2	-2.083	-0.936	0.210	$\hat{\delta}_y = \hat{\delta}_z$	0.153	0.772	1.392	$S(\delta_{\hat{e}_t}) \neq 0$
				M =100	ln(Debt/GDP)^3	-2.818	-1.388	0.043	$\hat{\delta}_y = \hat{\delta}_z$	0.074	0.588	1.102	$S(\delta_{\hat{e}_t}) \neq 0$
BEL	1846	2010	12	153	-	-1.066	-0.397	0.273	$\hat{\delta}_y = \hat{\delta}_z$	0.103	0.531	0.960	$S(\delta_{\hat{e}_t}) \neq 0$
				b =13	ln(Debt/GDP)^2	-1.693	-0.876	-0.060	$\hat{\delta}_y \neq \hat{\delta}_z$				
				M =141	ln(Debt/GDP)^3	-2.352	-1.273	-0.194	$\hat{\delta}_y \neq \hat{\delta}_z$				
BRA	1889	2010	-	122	-	-0.706	0.020	0.747	$\hat{\delta}_y = \hat{\delta}_z$	1.106	2.038	2.970	$S(\delta_{\hat{e}_t}) \neq 0$
				b =12	ln(Debt/GDP)^2	-1.440	-0.486	0.469	$\hat{\delta}_y = \hat{\delta}_z$	0.658	1.274	1.891	$S(\delta_{\hat{e}_t}) \neq 0$
				M =111	ln(Debt/GDP)^3	-2.111	-0.854	0.403	$\hat{\delta}_y = \hat{\delta}_z$	0.724	1.144	1.563	$S(\delta_{\hat{e}_t}) \neq 0$
CAN	1870	2010	-	141	-	-0.774	-0.246	0.282	$\hat{\delta}_y = \hat{\delta}_z$	0.198	0.770	1.342	$S(\delta_{\hat{e}_t}) \neq 0$
				b =13	ln(Debt/GDP)^2	-1.541	-0.763	0.016	$\hat{\delta}_y = \hat{\delta}_z$	0.172	0.745	1.317	$S(\delta_{\hat{e}_t}) \neq 0$
				M =129	ln(Debt/GDP)^3	-2.203	-1.197	-0.190	$\hat{\delta}_y \neq \hat{\delta}_z$				

Table continued on the following page

Table A-6: Balance and Co-Summability — ln(GDP pc) specifications, 27 countries (continued)

						Balance				Co-Summability			
	Start	End	Gaps	obs	Nonlinearity	CI low	$\hat{\delta}_y - \hat{\delta}_z$	CI up	Verdict	CI low	$\hat{\delta}_{\hat{e}_t}$	CI up	Verdict
CHL	1870	2010	-	141	-	-1.193	-0.095	1.004	$\hat{\delta}_y = \hat{\delta}_z$	0.626	1.120	1.614	$S(\delta_{\hat{e}_t}) \neq 0$
				b =13	ln(Debt/GDP)^2	-1.919	-0.570	0.779	$\hat{\delta}_y = \hat{\delta}_z$	0.484	1.115	1.746	$S(\delta_{\hat{e}_t}) \neq 0$
				M =129	ln(Debt/GDP)^3	-2.529	-0.977	0.575	$\hat{\delta}_y = \hat{\delta}_z$	0.550	1.149	1.748	$S(\delta_{\hat{e}_t}) \neq 0$
COL	1899	2010	-	112	-	-1.513	-0.814	-0.116	$\hat{\delta}_y \neq \hat{\delta}_z$				
				b =12	ln(Debt/GDP)^2	-2.262	-1.329	-0.396	$\hat{\delta}_y \neq \hat{\delta}_z$				
				M =101	ln(Debt/GDP)^3	-2.896	-1.757	-0.617	$\hat{\delta}_y \neq \hat{\delta}_z$				
DNK	1880	2010	-	131	-	-1.076	-0.372	0.332	$\hat{\delta}_y = \hat{\delta}_z$	1.041	1.793	2.545	$S(\delta_{\hat{e}_t}) \neq 0$
				b =12	ln(Debt/GDP)^2	-1.841	-0.899	0.043	$\hat{\delta}_y = \hat{\delta}_z$	0.600	1.648	2.696	$S(\delta_{\hat{e}_t}) \neq 0$
				M =120	ln(Debt/GDP)^3	-2.443	-1.309	-0.175	$\hat{\delta}_y \neq \hat{\delta}_z$				
FRA	1880	2010	23	108	-	-2.224	-0.770	0.684	$\hat{\delta}_y = \hat{\delta}_z$	0.818	1.175	1.531	$S(\delta_{\hat{e}_t}) \neq 0$
				b =11	ln(Debt/GDP)^2	-4.627	-2.110	0.406	$\hat{\delta}_y = \hat{\delta}_z$	0.442	1.081	1.719	$S(\delta_{\hat{e}_t}) \neq 0$
				M =98	ln(Debt/GDP)^3	-7.041	-3.419	0.203	$\hat{\delta}_y = \hat{\delta}_z$	0.393	1.117	1.840	$S(\delta_{\hat{e}_t}) \neq 0$
DEU	1880	2010	37	94	-	-1.173	-0.304	0.566	$\hat{\delta}_y = \hat{\delta}_z$	0.927	1.375	1.824	$S(\delta_{\hat{e}_t}) \neq 0$
				11	ln(Debt/GDP)^2	-2.015	-0.849	0.316	$\hat{\delta}_y = \hat{\delta}_z$	0.547	1.136	1.726	$S(\delta_{\hat{e}_t}) \neq 0$
				84	ln(Debt/GDP)^3	-2.738	-1.319	0.100	$\hat{\delta}_y = \hat{\delta}_z$	0.395	0.795	1.195	$S(\delta_{\hat{e}_t}) \neq 0$
GRC	1848	2010	15	148	-	-0.766	-0.084	0.598	$\hat{\delta}_y = \hat{\delta}_z$	0.270	0.930	1.589	$S(\delta_{\hat{e}_t}) \neq 0$
				b =13	ln(Debt/GDP)^2	-1.680	-0.786	0.108	$\hat{\delta}_y = \hat{\delta}_z$	0.391	1.065	1.739	$S(\delta_{\hat{e}_t}) \neq 0$
				M =136	ln(Debt/GDP)^3	-2.463	-1.343	-0.223	$\hat{\delta}_y \neq \hat{\delta}_z$				

Table continued on the following page

Table A-6: Balance and Co-Summability — ln(GDP pc) specifications, 27 countries (continued)

						Balance				Co-Summability			
	Start	End	Gaps	obs	Nonlinearity	CI low	$\hat{\delta}_y - \hat{\delta}_z$	CI up	Verdict	CI low	$\hat{\delta}_{\hat{e}_t}$	CI up	Verdict
IND	1884	2010	-	127	-	-0.899	-0.254	0.392	$\hat{\delta}_y = \hat{\delta}_z$	0.163	0.709	1.256	$S(\delta_{\hat{e}_t}) \neq 0$
				b =12	ln(Debt/GDP)^2	-1.605	-0.725	0.155	$\hat{\delta}_y = \hat{\delta}_z$	0.212	0.710	1.209	$S(\delta_{\hat{e}_t}) \neq 0$
				M =116	ln(Debt/GDP)^3	-2.177	-1.120	-0.064	$\hat{\delta}_y \neq \hat{\delta}_z$				
ITA	1861	2010	-	150	-	-1.121	-0.435	0.251	$\hat{\delta}_y = \hat{\delta}_z$	0.713	1.203	1.693	$S(\delta_{\hat{e}_t}) \neq 0$
				b =13	ln(Debt/GDP)^2	-1.778	-0.956	-0.134	$\hat{\delta}_y \neq \hat{\delta}_z$				
				M =138	ln(Debt/GDP)^3	-2.446	-1.393	-0.341	$\hat{\delta}_y \neq \hat{\delta}_z$				
JPN	1872	2010		139	-	-1.430	-0.597	0.236	$\hat{\delta}_y = \hat{\delta}_z$	0.580	1.128	1.676	$S(\delta_{\hat{e}_t}) \neq 0$
				b =13	Debt/GDP squared	-2.171	-1.057	0.058	$\hat{\delta}_y = \hat{\delta}_z$	0.236	0.873	1.511	$S(\delta_{\hat{e}_t}) \neq 0$
				M =127	Debt/GDP cubed	-2.781	-1.429	-0.077	$\hat{\delta}_y \neq \hat{\delta}_z$				
NLD	1815	2010	6	190	-	-1.127	-0.411	0.305	$\hat{\delta}_y = \hat{\delta}_z$	0.254	0.254	0.839	$S(\delta_{\hat{e}_t}) \neq 0$
				b =15	ln(Debt/GDP)^2	-1.952	-0.948	0.057	$\hat{\delta}_y = \hat{\delta}_z$	0.146	0.560	0.974	$S(\delta_{\hat{e}_t}) \neq 0$
				M =176	ln(Debt/GDP)^3	-2.668	-1.409	-0.151	$\hat{\delta}_y \neq \hat{\delta}_z$				
NZL	1870	2010	-	141	-	-1.188	-0.527	0.134	$\hat{\delta}_y = \hat{\delta}_z$	0.575	0.869	1.163	$S(\delta_{\hat{e}_t}) \neq 0$
				b =13	ln(Debt/GDP)^2	-1.817	-1.067	-0.317	$\hat{\delta}_y \neq \hat{\delta}_z$				
				M =129	ln(Debt/GDP)^3	-2.529	-1.521	-0.513	$\hat{\delta}_y \neq \hat{\delta}_z$				
NOR	1880	2010	6	125	-	-0.670	0.001	0.672	$\hat{\delta}_y = \hat{\delta}_z$	0.723	1.318	1.913	$S(\delta_{\hat{e}_t}) \neq 0$
				b =12	ln(Debt/GDP)^2	-1.408	-0.467	0.475	$\hat{\delta}_y = \hat{\delta}_z$	0.529	1.161	1.792	$S(\delta_{\hat{e}_t}) \neq 0$
				M =114	ln(Debt/GDP)^3	-2.004	-0.842	0.319	$\hat{\delta}_y = \hat{\delta}_z$	0.504	1.209	1.914	$S(\delta_{\hat{e}_t}) \neq 0$

Table continued on the following page

Table A-6: Balance and Co-Summability — ln(GDP pc) specifications, 27 countries (continued)

						Balance				Co-Summability			
	Start	End	Gaps	obs	Nonlinearity	CI low	$\hat{\delta}_y - \hat{\delta}_z$	CI up	Verdict	CI low	$\hat{\delta}_{\hat{e}_t}$	CI up	Verdict
PER	1883	2010	14	114	-	-1.477	-0.730	0.018	$\hat{\delta}_y = \hat{\delta}_z$	0.635	1.186	1.736	$S(\delta_{\hat{e}_t}) \neq 0$
				b =12	ln(Debt/GDP)^2	-2.314	-1.301	-0.287	$\hat{\delta}_y \neq \hat{\delta}_z$				
				M =103	ln(Debt/GDP)^3	-3.037	-1.788	-0.539	$\hat{\delta}_y \neq \hat{\delta}_z$				
PRT	1865	2010	-	146	-	-1.117	-0.486	0.144	$\hat{\delta}_y = \hat{\delta}_z$	0.402	0.822	1.242	$S(\delta_{\hat{e}_t}) \neq 0$
				b =13	ln(Debt/GDP)^2	-1.781	-0.981	-0.181	$\hat{\delta}_y \neq \hat{\delta}_z$				
				M =134	ln(Debt/GDP)^3	-2.421	-1.390	-0.360	$\hat{\delta}_y \neq \hat{\delta}_z$				
ESP	1850	2010	4	157	-	-1.055	-0.384	0.287	$\hat{\delta}_y = \hat{\delta}_z$	0.253	0.911	1.570	$S(\delta_{\hat{e}_t}) \neq 0$
				b =14	ln(Debt/GDP)^2	-1.855	-0.907	0.040	$\hat{\delta}_y = \hat{\delta}_z$	0.232	0.840	1.449	$S(\delta_{\hat{e}_t}) \neq 0$
				M =144	ln(Debt/GDP)^3	-2.562	-1.351	-0.141	$\hat{\delta}_y \neq \hat{\delta}_z$				
LKA	1870	2010	35	106	-	-1.047	-0.452	0.142	$\hat{\delta}_y = \hat{\delta}_z$	0.253	0.696	1.139	$S(\delta_{\hat{e}_t}) \neq 0$
				b =11	ln(Debt/GDP)^2	-1.900	-1.028	-0.155	$\hat{\delta}_y \neq \hat{\delta}_z$				
				M =96	ln(Debt/GDP)^3	-2.619	-1.516	-0.414	$\hat{\delta}_y \neq \hat{\delta}_z$				
SWE	1800	2010	-	211	-	-1.054	-0.350	0.354	$\hat{\delta}_y = \hat{\delta}_z$	0.777	1.546	2.314	$S(\delta_{\hat{e}_t}) \neq 0$
				b =16	ln(Debt/GDP)^2	-1.660	-0.767	0.126	$\hat{\delta}_y = \hat{\delta}_z$	0.658	1.602	2.546	$S(\delta_{\hat{e}_t}) \neq 0$
				M =196	ln(Debt/GDP)^3	-2.245	-1.108	0.028	$\hat{\delta}_y = \hat{\delta}_z$	0.697	1.598	2.499	$S(\delta_{\hat{e}_t}) \neq 0$
CHE	1880	2010	18	115	-	-1.254	-0.315	0.624	$\hat{\delta}_y = \hat{\delta}_z$	0.588	1.033	1.479	$S(\delta_{\hat{e}_t}) \neq 0$
				b =12	ln(Debt/GDP)^2	-2.062	-0.814	0.434	$\hat{\delta}_y = \hat{\delta}_z$	0.639	1.036	1.434	$S(\delta_{\hat{e}_t}) \neq 0$
				M =104	ln(Debt/GDP)^3	-2.720	-1.229	0.263	$\hat{\delta}_y = \hat{\delta}_z$	0.445	0.974	1.503	$S(\delta_{\hat{e}_t}) \neq 0$

Table continued on the following page

Table A-6: Balance and Co-Summability — ln(GDP pc) specifications, 27 countries (continued)

						Balance				Co-Summability			
	Start	End	Gaps	obs	Nonlinearity	CI low	$\hat{\delta}_y - \hat{\delta}_z$	CI up	Verdict	CI low	$\hat{\delta}_{\hat{e}_t}$	CI up	Verdict
GBR	1800	2010	-	211	-	-0.913	-0.178	0.558	$\hat{\delta}_y = \hat{\delta}_z$	0.660	1.194	1.728	$S(\delta_{\hat{e}_t}) \neq 0$
				b =16	ln(Debt/GDP)^2	-1.705	-0.694	0.317	$\hat{\delta}_y = \hat{\delta}_z$	0.699	1.196	1.693	$S(\delta_{\hat{e}_t}) \neq 0$
				M =196	ln(Debt/GDP)^3	-2.383	-1.137	0.109	$\hat{\delta}_y = \hat{\delta}_z$	0.702	1.196	1.689	$S(\delta_{\hat{e}_t}) \neq 0$
USA	1800	2010	-	211	-	-1.265	-0.507	0.252	$\hat{\delta}_y = \hat{\delta}_z$	0.467	1.049	1.631	$S(\delta_{\hat{e}_t}) \neq 0$
				16	ln(Debt/GDP)^2	-1.611	-0.726	0.160	$\hat{\delta}_y = \hat{\delta}_z$	0.277	0.943	1.609	$S(\delta_{\hat{e}_t}) \neq 0$
				196	ln(Debt/GDP)^3	-2.158	-1.145	-0.132	$\hat{\delta}_y \neq \hat{\delta}_z$				
URY	1871	2010	23	117	-	-1.233	-0.354	0.525	$\hat{\delta}_y = \hat{\delta}_z$	0.224	0.602	0.981	$S(\delta_{\hat{e}_t}) \neq 0$
				12	ln(Debt/GDP)^2	-2.091	-0.896	0.298	$\hat{\delta}_y = \hat{\delta}_z$	0.118	0.602	1.087	$S(\delta_{\hat{e}_t}) \neq 0$
				106	ln(Debt/GDP)^3	-2.814	-1.359	0.097	$\hat{\delta}_y = \hat{\delta}_z$	-0.156	0.477	1.111	$S(\delta_{\hat{e}_t}) = 0$

Notes: In all models I take per capita GDP (in logarithms) as the dependent variable. CI low and up indicate the 95% confidence interval for the balance and co-summability estimates. In all tests conducted I allow for deterministic terms (constant and trend). $\hat{\delta}_y \neq (=)\hat{\delta}_z$ implies that balance is (not) rejected, $\hat{\delta}_{\hat{e}_t} \neq (=)0$ that co-summability is (not) rejected. $b = int\sqrt{T} + 1$ refers to the time series length of the subsample, $M = T - b + 1$ to the number of subsamples used in the analysis. Regarding the 'Nonlinearity,' the model with ln(Debt/GDP)^2 also includes ln(Debt/GDP), while the model with ln(Debt/GDP)^3 also includes ln(Debt/GDP)^2 and ln(Debt/GDP).

Figure A-2: Balance Testing (Sub-Samples)

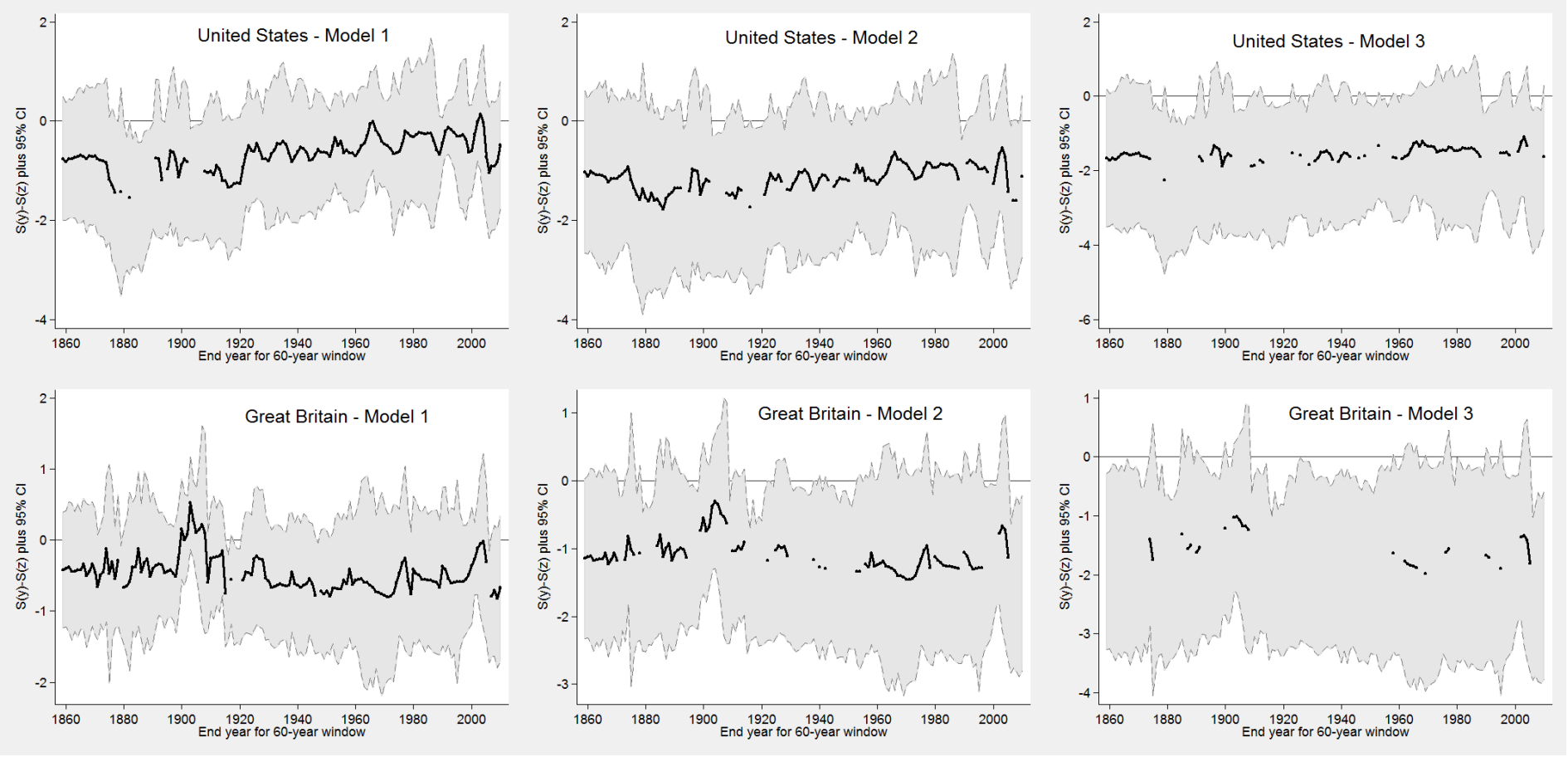
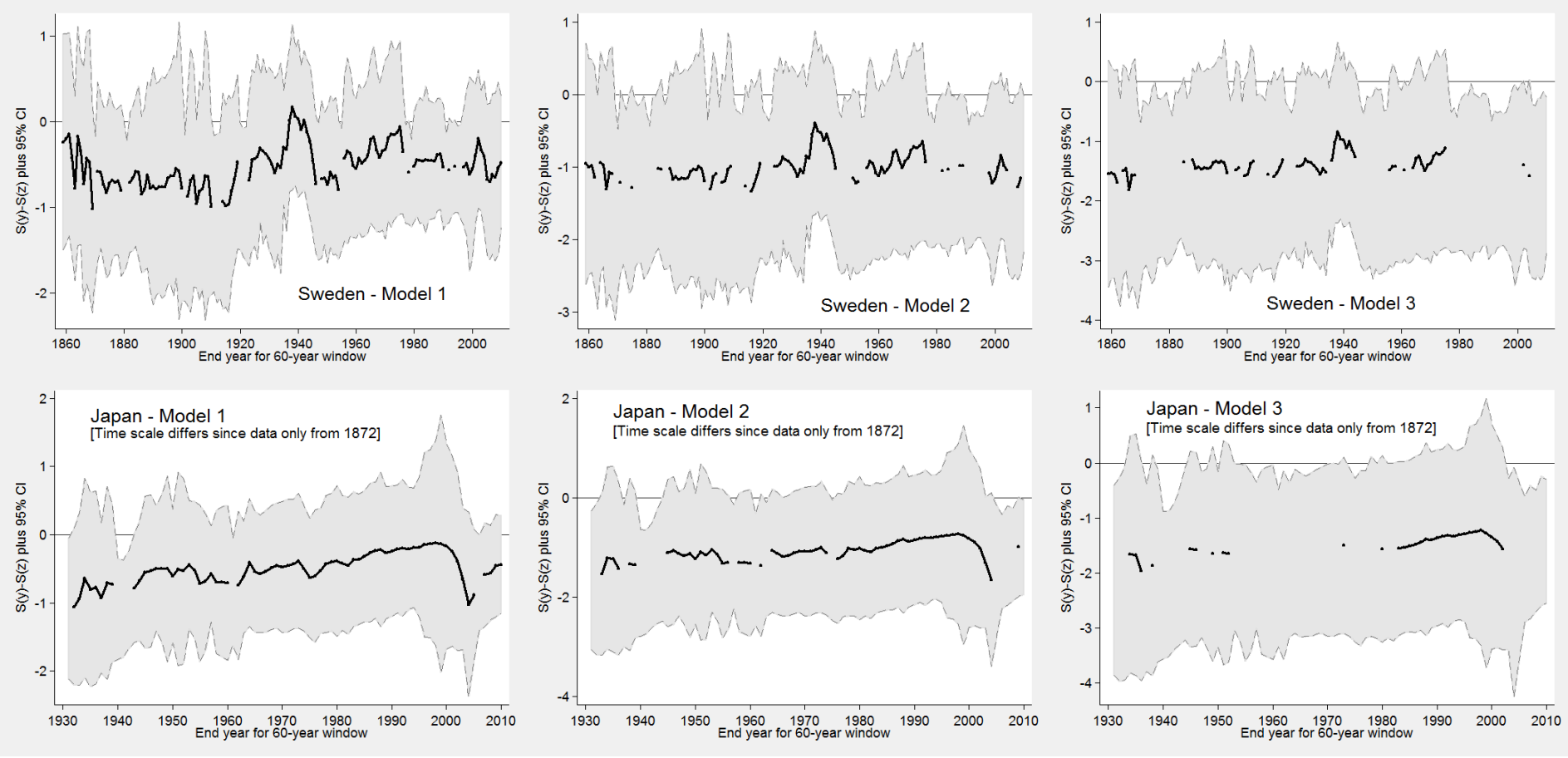


Figure continued on the following page

Figure A-2: Balance Testing (Sub-Samples) continued



Notes: The shaded areas represent the Bonferroni-corrected 95% Confidence Intervals for the Balance statistic computed in a moving window of 60-year time periods; the solid line represents the balance estimate for consecutive windows: I only plot this when balance cannot be rejected. The coverage of the data differs across countries: for the US, Great Britain and Sweden I have data from 1800-2010 (152 subsamples), for Japan from 1872-2010 (80 subsamples). Model 1 refers to a specification with linear debt terms only, Model 2 to a specification with linear and quadratic debt terms, Model 3 further includes a cubed debt term. The graphs capture both subsequent end years in which subsamples were balanced (lines) as well as 'isolated' years (dots).

Figure A-3: Co-Summability Testing (Sub-Samples)

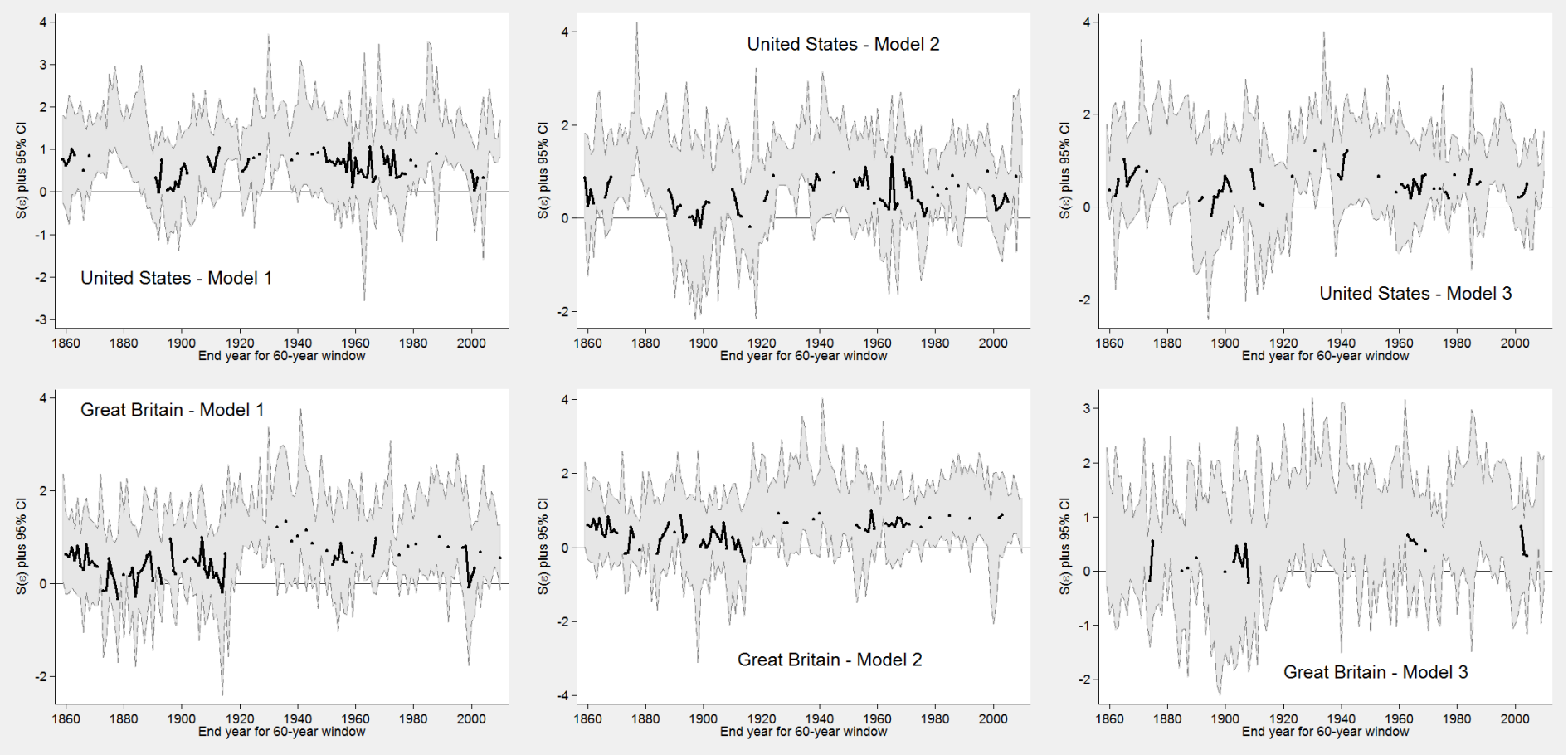
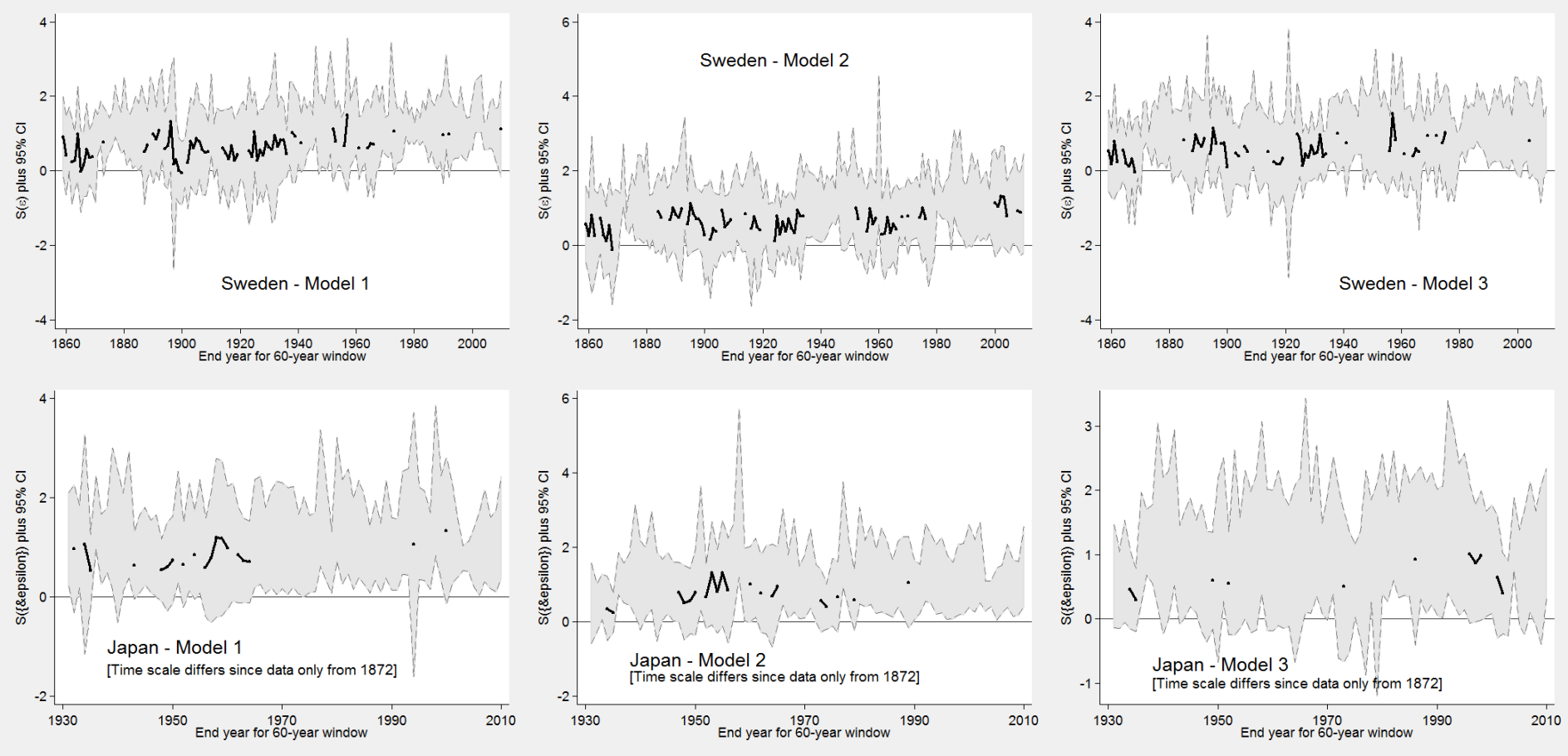


Figure continued on the following page

Figure A-3: Co-Summability Testing (Sub-Samples) continued



Notes: The shaded areas represent the Bonferroni-corrected 95% Confidence Intervals for the Co-Summability statistic computed in a moving window of 60-year time periods. The solid black line represents the computed Co-Summability statistic — this line is only shown if the prerequisite balance test could not reject this feature for the specific subsample. I allow for an intercept in the co-summability analysis. For further details see Figure A-2. The graphs capture both subsequent end years in which subsamples were balanced (line) as well as 'isolated' years (dots).

Table A-7
Balance and Co-Summability — ln(debt) as dependent variable

					Balance				Co-Summability			
	Start	End	Obs	Nonlinearity	CI low	$\hat{\delta}_y - \hat{\delta}_z$	CI up	Verdict	CI low	$\hat{\delta}_{\hat{e}_t}$	CI up	Verdict
USA	1800	2010	211	-	-0.252	0.507	1.265	$S(\delta_y) = S(\delta_z)$	0.340	0.820	1.300	$S(\delta_{\hat{e}_t}) \neq 0$
			b =16	ln(GDP pc)^2	-1.267	-0.128	1.010	$S(\delta_y) = S(\delta_z)$	0.246	0.886	1.527	$S(\delta_{\hat{e}_t}) \neq 0$
			M =196	ln(GDP pc)^3	-2.198	-0.689	0.821	$S(\delta_y) = S(\delta_z)$	0.284	0.885	1.485	$S(\delta_{\hat{e}_t}) \neq 0$
GBR	1800	2010	211	-	-0.557	0.178	0.913	$S(\delta_y) = S(\delta_z)$	0.560	1.263	1.965	$S(\delta_{\hat{e}_t}) \neq 0$
			16	ln(GDP pc)^2	-1.158	-0.453	0.251	$S(\delta_y) = S(\delta_z)$	0.501	1.173	1.846	$S(\delta_{\hat{e}_t}) \neq 0$
			196	ln(GDP pc)^3	-1.984	-1.013	-0.042	$S(\delta_y) \neq S(\delta_z)$				
SWE	1800	2010	211	-	-0.360	0.350	1.059	$S(\delta_y) = S(\delta_z)$	0.709	1.623	2.538	$S(\delta_{\hat{e}_t}) \neq 0$
			16	ln(GDP pc)^2	-1.190	-0.262	0.666	$S(\delta_y) = S(\delta_z)$	0.601	1.534	2.467	$S(\delta_{\hat{e}_t}) \neq 0$
			196	ln(GDP pc)^3	-2.079	-0.802	0.476	$S(\delta_y) = S(\delta_z)$	0.590	1.515	2.439	$S(\delta_{\hat{e}_t}) \neq 0$
JPN	1872	2010	139	-	-0.236	0.597	1.430	$S(\delta_y) = S(\delta_z)$	0.145	0.669	1.193	$S(\delta_{\hat{e}_t}) \neq 0$
			13	ln(GDP pc)^2	-0.928	-0.098	0.732	$S(\delta_y) = S(\delta_z)$	0.204	0.806	1.408	$S(\delta_{\hat{e}_t}) \neq 0$
			127	ln(GDP pc)^3	-1.904	-0.713	0.479	$S(\delta_y) = S(\delta_z)$	-0.160	0.338	0.837	$S(\delta_{\hat{e}_t}) = 0$

Notes: In all models I take the debt-to-GDP ratio (in logarithms) as the dependent variable. CI low and up indicate the 95% confidence interval for the balance and co-summability estimates. In all tests conducted I allow for deterministic terms (constant and trend). $\hat{\delta}_y \neq (=)\hat{\delta}_z$ implies that balance is (not) rejected, $\hat{\delta}_{\hat{e}_t} \neq (=)0$ that co-summability is (not) rejected. Obs reports the number of observations, $b = int\sqrt{T} + 1$ refers to the time series length of each subsample, $M = T - b + 1$ to the number of subsamples used in the analysis. Regarding the 'Nonlinearity,' the model with ln(GDP pc)^2 also includes ln(GDP pc), while the model with ln(GDP pc)^3 also includes ln(GDP pc)^2 and ln(GDP pc). ‡ Results for balance do not differ across different threshold values since $X_t\mathbb{1}(X_t < \text{threshold}) + X_t\mathbb{1}(X_t \geq \text{threshold}) = X_t$.

Table A-8: Estimated Order of Summability

Country	Start & End Year		n	Variable	CI low	$\hat{\delta}$	CI up
USA	1800	2010	211	Inflation	-0.572	0.035	0.642
	1870	2010	141	Δln(Population)	-0.130	0.417	0.964
				ln(Invest/GDP)	0.520	1.407	2.294
	1880	2010	131	Schooling	1.234	2.471	3.708
GBR	1800	2010	211	Inflation	0.457	1.210	1.963
	1850	2010	161	Δln(Population)	0.103	0.590	1.078
				ln(Invest/GDP)	-0.178	0.386	0.950
	1870	2010	141	Schooling	0.918	1.696	2.474
SWE	1800	2010	211	Inflation	0.492	1.376	2.260
	1801	2010	210	Δln(Population)	0.131	0.644	1.158
				ln(Invest/GDP)	0.211	0.919	1.627
	1870	2010	141	Schooling	1.094	2.153	3.212
JPN	1800	2010	211	Inflation	0.030	0.608	1.185
	1850	2010	161	Δln(Population)	-0.269	0.540	1.350
				ln(Invest/GDP)	0.027	0.529	1.031
	1870	2010	141	Schooling	0.368	1.058	1.749

Notes: CI low and up indicate the 95% confidence interval for the summability estimate $S(\delta)$ constructed from subsampling — shaded cells indicate variable series where the summability confidence interval includes zero. In all tests conducted I allow for deterministic terms (constant and trend).

Table A-9
Balance and Co-Summability — Debt and Inflation Model

					Balance				Co-Summability			
	Start	End	Obs	Nonlinearity	CI low	$\hat{\delta}_y - \hat{\delta}_z$	CI up	Verdict	CI low	$\hat{\delta}_{\hat{e}_t}$	CI up	Verdict
USA	1800	2010	211	-	-1.624	-0.666	0.292	$S(\delta_y) = S(\delta_z)$	0.417	0.951	1.486	$S(\delta_{\hat{e}_t}) \neq 0$
			b =16	ln(Debt/GDP)^2	-1.592	-0.758	0.077	$S(\delta_y) = S(\delta_z)$	0.312	0.892	1.472	$S(\delta_{\hat{e}_t}) \neq 0$
			M =196	ln(Debt/GDP)^3	-2.121	-1.147	-0.172	$S(\delta_y) \neq S(\delta_z)$				
				Threshold 50%	-1.809	-0.944	-0.079	$S(\delta_y) \neq S(\delta_z)$				
GBR	1800	2010	211	-	-2.017	-0.995	0.028	$S(\delta_y) = S(\delta_z)$	-0.118	0.248	0.614	$S(\delta_{\hat{e}_t}) = 0$
			b =16	ln(Debt/GDP)^2	-2.151	-0.980	0.192	$S(\delta_y) = S(\delta_z)$	-0.146	0.246	0.639	$S(\delta_{\hat{e}_t}) = 0$
			M =196	ln(Debt/GDP)^3	-2.415	-1.143	0.129	$S(\delta_y) = S(\delta_z)$	-0.178	0.245	0.669	$S(\delta_{\hat{e}_t}) = 0$
				Threshold 50%	-2.513	-1.250	0.013	$S(\delta_y) = S(\delta_z)$	-0.169	0.268	0.706	$S(\delta_{\hat{e}_t}) = 0$
				Threshold 70%	-2.513	-1.250	0.013	$S(\delta_y) = S(\delta_z)$	-0.146	0.302	-0.751	$S(\delta_{\hat{e}_t}) = 0$
				Threshold 90%	-2.513	-1.250	0.013	$S(\delta_y) = S(\delta_z)$	-0.107	0.304	0.715	$S(\delta_{\hat{e}_t}) = 0$
SWE	1800	2010	211	-	-1.905	-0.875	0.155	$S(\delta_y) = S(\delta_z)$	0.386	0.924	1.463	$S(\delta_{\hat{e}_t}) \neq 0$
			b =16	ln(Debt/GDP)^2	-2.068	-0.986	0.097	$S(\delta_y) = S(\delta_z)$	0.354	0.949	1.544	$S(\delta_{\hat{e}_t}) \neq 0$
			M =196	ln(Debt/GDP)^3	-2.346	-1.185	-0.023	$S(\delta_y) \neq S(\delta_z)$	0.337	0.989	1.641	$S(\delta_{\hat{e}_t}) \neq 0$
JPN	1872	2010	125	-	-2.589	-1.261	0.068	$S(\delta_y) = S(\delta_z)$	0.770	1.589	2.409	$S(\delta_{\hat{e}_t}) \neq 0$
			b =12	ln(Debt/GDP)^2	-2.679	-1.323	0.034	$S(\delta_y) = S(\delta_z)$	0.228	0.783	1.338	$S(\delta_{\hat{e}_t}) \neq 0$
			M =114	ln(Debt/GDP)^3	-2.878	-1.495	-0.112	$S(\delta_y) \neq S(\delta_z)$				
				Threshold 50%	-2.781	-1.431	-0.080	$S(\delta_y) \neq S(\delta_z)$				

Notes: In all models I take per capita GDP (in logarithms) as the dependent variable. CI low and up indicate the 95% confidence interval for the balance and co-summability estimates. In all tests conducted I allow for deterministic terms (constant and trend). $\hat{\delta}_y \neq (=)\hat{\delta}_z$ implies that balance is (not) rejected, $\hat{\delta}_{\hat{e}_t} \neq (=)0$ that co-summability is (not) rejected. Obs reports the number of observations, $b = int\sqrt{T} + 1$ refers to the time series length of each subsample, $M = T - b + 1$ to the number of subsamples used in the analysis. Regarding the 'Nonlinearity,' the model with ln(Debt/GDP)^2 also includes ln(Debt/GDP), while the model with ln(Debt/GDP)^3 also includes ln(Debt/GDP)^2 and ln(Debt/GDP). ‡ Results for balance do not differ across different threshold values since $X_t\mathbb{1}(X_t <$ threshold$) + X_t\mathbb{1}(X_t \geq$ threshold$) = X_t$.

Table A-10
Balance and Co-Summability — Solow Model with Debt

					Balance				Co-Summability			
	Start	End	Obs	Nonlinearity	CI low	$\hat{\delta}_y - \hat{\delta}_z$	CI up	Verdict	CI low	$\hat{\delta}_{\hat{e}_t}$	CI up	Verdict
USA	1870	2010	141	-	-1.625	-0.792	0.040	$S(\delta_y) = S(\delta_z)$	0.212	0.888	1.565	$S(\delta_{\hat{e}_t}) \neq 0$
			b =13	ln(Debt/GDP)^2	-2.043	-1.234	-0.425	$S(\delta_y) \neq S(\delta_z)$				
			M =129	ln(Debt/GDP)^3	-2.613	-1.614	-0.614	$S(\delta_y) \neq S(\delta_z)$				
				Threshold 50%	-2.421	-1.495	-0.569	$S(\delta_y) \neq S(\delta_z)$				
GBR	1850	2010	161	-	-1.286	-0.628	0.029	$S(\delta_y) = S(\delta_z)$	0.155	1.003	1.851	$S(\delta_{\hat{e}_t}) \neq 0$
			b =14	ln(Debt/GDP)^2	-2.027	-1.142	-0.256	$S(\delta_y) \neq S(\delta_z)$				
			M =148	ln(Debt/GDP)^3	-2.747	-1.604	-0.460	$S(\delta_y) \neq S(\delta_z)$				
				Thresholds	-2.846	-1.686	-0.525	$S(\delta_y) \neq S(\delta_z)$				
SWE	1800	2010	211	-	-1.010	-0.338	0.334	$S(\delta_y) = S(\delta_z)$	-0.152	0.334	0.820	$S(\delta_{\hat{e}_t}) = 0$
			b =16	ln(Debt/GDP)^2	-1.709	-0.772	0.165	$S(\delta_y) = S(\delta_z)$	-0.372	0.284	0.940	$S(\delta_{\hat{e}_t}) = 0$
			M =196	ln(Debt/GDP)^3	-2.271	-1.107	0.057	$S(\delta_y) = S(\delta_z)$	-0.540	0.195	0.931	$S(\delta_{\hat{e}_t}) = 0$
JPN	1872	2010	125	-	-0.985	-0.258	0.469	$S(\delta_y) = S(\delta_z)$	-0.253	0.246	0.745	$S(\delta_{\hat{e}_t}) = 0$
			b =12	ln(Debt/GDP)^2	-1.729	-0.770	0.190	$S(\delta_y) = S(\delta_z)$	-0.277	0.239	0.754	$S(\delta_{\hat{e}_t}) = 0$
			M =114	ln(Debt/GDP)^3	-2.397	-1.186	0.025	$S(\delta_y) = S(\delta_z)$	-0.262	0.232	0.726	$S(\delta_{\hat{e}_t}) = 0$
				Threshold 50%	-2.319	-1.130	0.053	$S(\delta_y) = S(\delta_z)$	-0.014	0.517	1.048	$S(\delta_{\hat{e}_t}) = 0$

Notes: In all models I take per capita GDP (in logarithms) as the dependent variable. CI low and up indicate the 95% confidence interval for the balance and co-summability estimates. In all tests conducted I allow for deterministic terms (constant and trend). $\hat{\delta}_y \neq (=) \hat{\delta}_z$ implies that balance is (not) rejected, $\hat{\delta}_{\hat{e}_t} \neq (=) 0$ that co-summability is (not) rejected. Obs reports the number of observations, $b = int\sqrt{T} + 1$ refers to the time series length of each subsample, $M = T - b + 1$ to the number of subsamples used in the analysis. Regarding the 'Nonlinearity,' the model with ln(Debt/GDP)^2 also includes ln(Debt/GDP), while the model with ln(Debt/GDP)^3 also includes ln(Debt/GDP)^2 and ln(Debt/GDP). ‡ Results for balance do not differ across different threshold values since $X_t \mathbb{1}(X_t < \text{threshold}) + X_t \mathbb{1}(X_t \geq \text{threshold}) = X_t$.

Table A-11
Balance and Co-Summability — Augmented Solow Model with Debt

					Balance				Co-Summability			
	Start	End	Obs	Nonlinearity	CI low	$\hat{\delta}_y - \hat{\delta}_z$	CI up	Verdict	CI low	$\hat{\delta}_{\hat{e}_t}$	CI up	Verdict
USA	1880	2010	131	-	-1.523	-0.662	0.200	$S(\delta_y) = S(\delta_z)$	0.304	0.892	1.479	$S(\delta_{\hat{e}_t}) \neq 0$
			b =12	ln(Debt/GDP)^2	-2.165	-1.018	0.129	$S(\delta_y) = S(\delta_z)$	-0.011	0.619	1.250	$S(\delta_{\hat{e}_t}) = 0$
			M =120	ln(Debt/GDP)^3	-2.708	-1.365	-0.022	$S(\delta_y) \neq S(\delta_z)$				
				Threshold 50%	-2.485	-1.239	0.008	$S(\delta_y) = S(\delta_z)$	0.144	0.720	1.295	$S(\delta_{\hat{e}_t}) \neq 0$
GBR	1870	2010	141	-	-1.202	-0.475	0.253	$S(\delta_y) = S(\delta_z)$	0.275	0.915	1.554	$S(\delta_{\hat{e}_t}) \neq 0$
			b =13	ln(Debt/GDP)^2	-1.905	-0.941	0.023	$S(\delta_y) = S(\delta_z)$	0.286	0.801	1.317	$S(\delta_{\hat{e}_t}) \neq 0$
			M =129	ln(Debt/GDP)^3	-2.619	-1.395	-0.171	$S(\delta_y) \neq S(\delta_z)$				
				Thresholds‡	-2.681	-1.454	-0.227	$S(\delta_y) \neq S(\delta_z)$				
SWE	1870	2010	141	-	-1.177	-0.396	0.384	$S(\delta_y) = S(\delta_z)$	-0.078	0.437	0.952	$S(\delta_{\hat{e}_t}) = 0$
			b =13	ln(Debt/GDP)^2	-1.705	-0.448	0.809	$S(\delta_y) = S(\delta_z)$	-0.233	0.387	1.007	$S(\delta_{\hat{e}_t}) = 0$
			M =129	ln(Debt/GDP)^3	-2.339	-0.761	0.817	$S(\delta_y) = S(\delta_z)$	-0.185	0.340	0.866	$S(\delta_{\hat{e}_t}) = 0$
JPN	1890	2010	107	-	-1.191	-0.337	0.516	$S(\delta_y) = S(\delta_z)$	0.055	0.533	1.011	$S(\delta_{\hat{e}_t}) \neq 0$
			b =11	ln(Debt/GDP)^2	-1.898	-0.830	0.237	$S(\delta_y) = S(\delta_z)$	0.069	0.509	0.950	$S(\delta_{\hat{e}_t}) \neq 0$
			M =97	ln(Debt/GDP)^3	-2.569	-1.248	0.072	$S(\delta_y) = S(\delta_z)$	0.065	0.509	0.952	$S(\delta_{\hat{e}_t}) \neq 0$
				Thresholds‡	-2.505	-1.197	0.110	$S(\delta_y) = S(\delta_z)$	0.435	0.790	1.145	$S(\delta_{\hat{e}_t}) \neq 0$

Notes: In all models I take per capita GDP (in logarithms) as the dependent variable. CI low and up indicate the 95% confidence interval for the balance and co-summability estimates. In all tests conducted I allow for deterministic terms (constant and trend). $\hat{\delta}_y \neq (=)\hat{\delta}_z$ implies that balance is (not) rejected, $\hat{\delta}_{\hat{e}_t} \neq (=)0$ that co-summability is (not) rejected. Obs reports the number of observations, $b = int\sqrt{T} + 1$ refers to the time series length of each subsample, $M = T - b + 1$ to the number of subsamples used in the analysis. Regarding the 'Nonlinearity,' the model with ln(Debt/GDP)^2 also includes ln(Debt/GDP), while the model with ln(Debt/GDP)^3 also includes ln(Debt/GDP)^2 and ln(Debt/GDP). ‡ Results for balance do not differ across different threshold values since $X_t\mathbb{1}(X_t < \text{threshold}) + X_t\mathbb{1}(X_t \geq \text{threshold}) = X_t$.