

# Online Appendix

## A.1 Data sources and construction

### Per capita income and debt

In the empirical analysis I employ per capita GDP in 1990 constant international dollars (in logarithms) and the percentage debt-to-GDP ratio (also in logarithms), where debt is generally gross central (external and domestic) government debt. The raw data for the analysis of four OECD countries is taken from the Excel spreadsheets available on the following websites: for data on debt [www.reinhartandrogoff.com](http://www.reinhartandrogoff.com) – the Reinhart and Rogoff (2009) companion website; and for data on per capita income (as well as population data) [www.ggdgc.net/maddison](http://www.ggdgc.net/maddison) – the Maddison Project website at the University of Groningen.

A number of comments regarding these data as well as changes and/or adjustments are provided in the following:

- For Great Britain the debt series refer to net rather than gross central (external and domestic) government debt.
- The ‘New’ Maddison data provides two values for Great Britain’s per capita GDP in 1851 – 2,330 and 2,718 – since this is where two data series come together: up to 1851 the estimates are taken from Van Zanden (2001), from 1851 onwards from the original Maddison (2010) estimates. I pick the arithmetic mean of the two values.
- Data on the debt-to-GDP ratio for Japan between 1941 and 1951 are computed from outstanding debt reported in Statistics Japan’s *Historical Statistics of Japan*, available at [www.stat.go.jp/english/data/chouki/05.htm](http://www.stat.go.jp/english/data/chouki/05.htm). The 1952 and 1953 figures are computed from debt and GDP numbers in the excel spreadsheets marked ‘RR’ provided by Hershdon, Ash and Pollin (2014). These are available at [www.peri.umass.edu/236/hash/31e2ff374b6377b2ddec04deaa6388b1/publication/566/](http://www.peri.umass.edu/236/hash/31e2ff374b6377b2ddec04deaa6388b1/publication/566/). Japanese GDP figures for 1941-51 are taken from Mitchell (2007a).
- I interpolated the debt-GDP ratio for Japan in 1882 where only a single observation was

missing in the Reinhart and Rogoff data. Similarly for the GDP series in 1945.

The analysis of time series of debt and per capita income in 27 economies is based on the same data sources. Below I discuss any changes made to variables in that sample:

- I interpolated the debt-to-GDP ratio in 3 cases where only a single observation was missing or (in case of India) where the recorded value was not credible (zero debt): Argentina 1866, India 1947, Peru 1960.
- For Brazil I chose debt data starting from 1889 since prior to this date the series covered only external debt. For Argentina, Italy, the Netherlands and New Zealand the debt series represent general rather than central government debt.

### **Additional Variables**

**Inflation** I use inflation data from the Clio Infra project at the *International Institute of Social History* which reports the annual percentage change. Excel spreadsheets are available for download at [www.clio-infra.eu](http://www.clio-infra.eu). Downloads automatically include very detailed information on the original sources of the data. The following adjustments were made: for *Chile* in 1914 (two values provided), I took the arithmetic mean. Data for *Austria* in 1915 was missing so I linearly interpolated, similarly for *Australia* in 1911 and 1912, and *New Zealand* in 2010 (extrapolated).

**Population** The original Maddison (2010) database provides data on mid-year population (in thousands). My analysis employs the population growth rate. For all population series I extrapolate the value for 2010 from the Maddison data to maintain the integrity of the data series (alternative sources, e.g. the Penn World Table version 8.0, gave marginally different figures for the last years leading up to 2010 than the Maddison data).

- Population data for *Sweden* during 1800-1819 is taken from Schön and Krantz (2012). Excel spreadsheets are available from [www.ekh.lu.se/en/research/economic\\_history\\_data/shna1560-2010](http://www.ekh.lu.se/en/research/economic_history_data/shna1560-2010).
- Population data for the *United States* during 1800–1819 is taken from the US Bureau of the Census, *Historical Statistics of the United States, Colonial Times to 1970*, Bicentennial Edi-

tion, Part 2, Washington, D.C., 1975. A pdf (Series A 6-8) is available from [www.census.gov/prod/www/statistical\\_abstract.html](http://www.census.gov/prod/www/statistical_abstract.html).

- Population data for the *United Kingdom* 1800–1819 is taken from Mitchell (1971), page 8.
- Population data for *Argentina* in 1869 and 1895 is taken from Argentina’s Instituto Nacional de Estadística y Censos (indec), [www.indec.mecon.ar/](http://www.indec.mecon.ar/). Data for 1870–1894 and 1896–1899 are linearly interpolated.

**Investment** This is the investment-to-GDP ratio in percent (in the empirical analysis further in logarithms). The main source for these data are the tables available on the GGDC Maddison website, which report Maddison’s (1992) domestic capital formation in percent of GDP. Below I indicate additional and alternative sources for these data (‘Maddison’ here refers to the 1992 *SJE* data):

- *Argentina* 1900–2010 is taken from the Montevideo-Oxford Latin American Economic History Data Base (MOxLAD) available at [www.lac.ox.ac.uk/moxlad-database](http://www.lac.ox.ac.uk/moxlad-database).
- *Australia* 1861–1869 computed from Mitchell (2007a); 1870–1989 from Maddison; for the remainder of the series I use the World Bank World Development Indicators (WDI) gross fixed capital formation (GFCF) series.
- *Brazil* 1900–2010 from MOxLAD.
- *Canada* 1870–1988 from Maddison; remainder from WDI.
- *Chile* 1900–2010 from MOxLAD.
- *Colombia* 1900–2010 from MOxLAD.
- *Denmark* 1850–1945 from Jones and Obstfeld (1997); 1946–1969 from Mitchell (2007b); remainder from WDI.
- *France* 1870–1988 from Maddison; 1850–1869, 1919–21, and 1939 from Jones and Obstfeld (1997); post-1939 from WDI.
- *Germany* 1925–1988 from Maddison; 1850–1913 from Jones and Obstfeld (1997); post-1988 from WDI.

- *India* 1870–1987 from Maddison; remainder from WDI.
- *Japan* 1885–1988 from Maddison, where I linearly interpolated for 1945; post-1988 from WDI.
- *Netherlands* 1921–1988 from Maddison (with gaps), for 1807–1913 I compute the investment/GDP ratio from GDP at current prices and total current gross fixed capital formation from the *National Accounts of the Netherlands 1800–1913*, provided by the Netherlands Research Institute and Graduate School on Economic and Social History at Data Archiving and Networked Services (DANS), available online at [nationalaccounts.niwi.knaw.nl/start.htm](http://nationalaccounts.niwi.knaw.nl/start.htm); post-1988 from WDI.
- *Norway* 1830–2010 from Norges Bank *Historical Monetary Statistics – GDP and its components from 1830 to 2010* available at [www.norges-bank.no/en/Statistics/](http://www.norges-bank.no/en/Statistics/).
- *Sweden* 1800–2000 is taken from the *Historical National Accounts for Sweden 1800–2000* available at [www.ekh.lu.se/en/research/economic\\_history\\_data/shna1560-2010](http://www.ekh.lu.se/en/research/economic_history_data/shna1560-2010); post-2000 from WDI.
- *United Kingdom*: 1850–1945 from Jones and Obstfeld (1997); thereafter from Maddison (to 1988); post-1945 from WDI.
- *United States*: 1870–1986 from Maddison; post-1986 from WDI.

**Human Capital** Average years of education in the population is taken from the Excel spreadsheets provided by the Clio Infra project at [www.clio-infra.eu](http://www.clio-infra.eu).

## A.2 Descriptives and Data Properties

Descriptive statistics for the four OECD countries analysed in the maintext are presented in Table A-1. Table A-2 reports results from unit root tests for all linear processes. This indicates that growth rates of per capita GDP or debt-to-GDP are stationary, while the levels of these variables are nonstationary. Figure A-1 charts the log levels and growth rates of per capita GDP (left column) and the debt-to-GDP ratio (right column) in the four sample countries.

Table A-1: Descriptive Statistics

		Start	End	Obs	Mean	Median	St.Dev.	Min	Max
USA	ln(GDP pc)	1800	2010	211	8.510	8.425	0.996	7.159	10.363
	$\Delta \ln(\text{GDP pc})$			210	0.015	0.017	0.047	-0.241	0.171
	ln(Debt/GDP)			211	2.667	2.958	1.629	-5.878	4.798
	$\ln(\text{Debt/GDP})^2$			211	9.754	9.090	6.397	0.010	34.552
	$\ln(\text{Debt/GDP})^3$			211	31.026	25.870	36.127	-203.104	110.445
GBR	ln(GDP pc)	1800	2010	211	8.527	8.414	0.727	7.574	10.127
	$\Delta \ln(\text{GDP pc})$			210	0.012	0.016	0.031	-0.114	0.091
	ln(Debt/GDP)			211	4.468	4.598	0.685	3.201	5.563
	$\ln(\text{Debt/GDP})^2$			211	20.432	21.139	6.057	10.247	30.946
	$\ln(\text{Debt/GDP})^3$			211	95.416	97.193	40.799	32.802	172.153
SWE	ln(GDP pc)	1800	2010	211	8.025	7.722	1.115	6.641	10.142
	$\Delta \ln(\text{GDP pc})$			210	0.016	0.020	0.034	-0.094	0.120
	ln(Debt/GDP)			211	2.748	2.881	0.915	0.485	4.387
	$\ln(\text{Debt/GDP})^2$			211	8.385	8.297	4.605	0.235	19.246
	$\ln(\text{Debt/GDP})^3$			211	27.133	23.901	20.076	0.114	84.431
JPN	ln(GDP pc)	1872	2010	139	8.111	7.660	1.158	6.615	10.017
	$\Delta \ln(\text{GDP pc})$			138	0.025	0.023	0.077	-0.681	0.162
	ln(Debt/GDP)			139	3.528	3.628	0.870	1.519	5.318
	$\ln(\text{Debt/GDP})^2$			139	13.197	13.164	6.066	2.307	28.280
	$\ln(\text{Debt/GDP})^3$			139	51.737	47.761	34.045	3.504	150.390

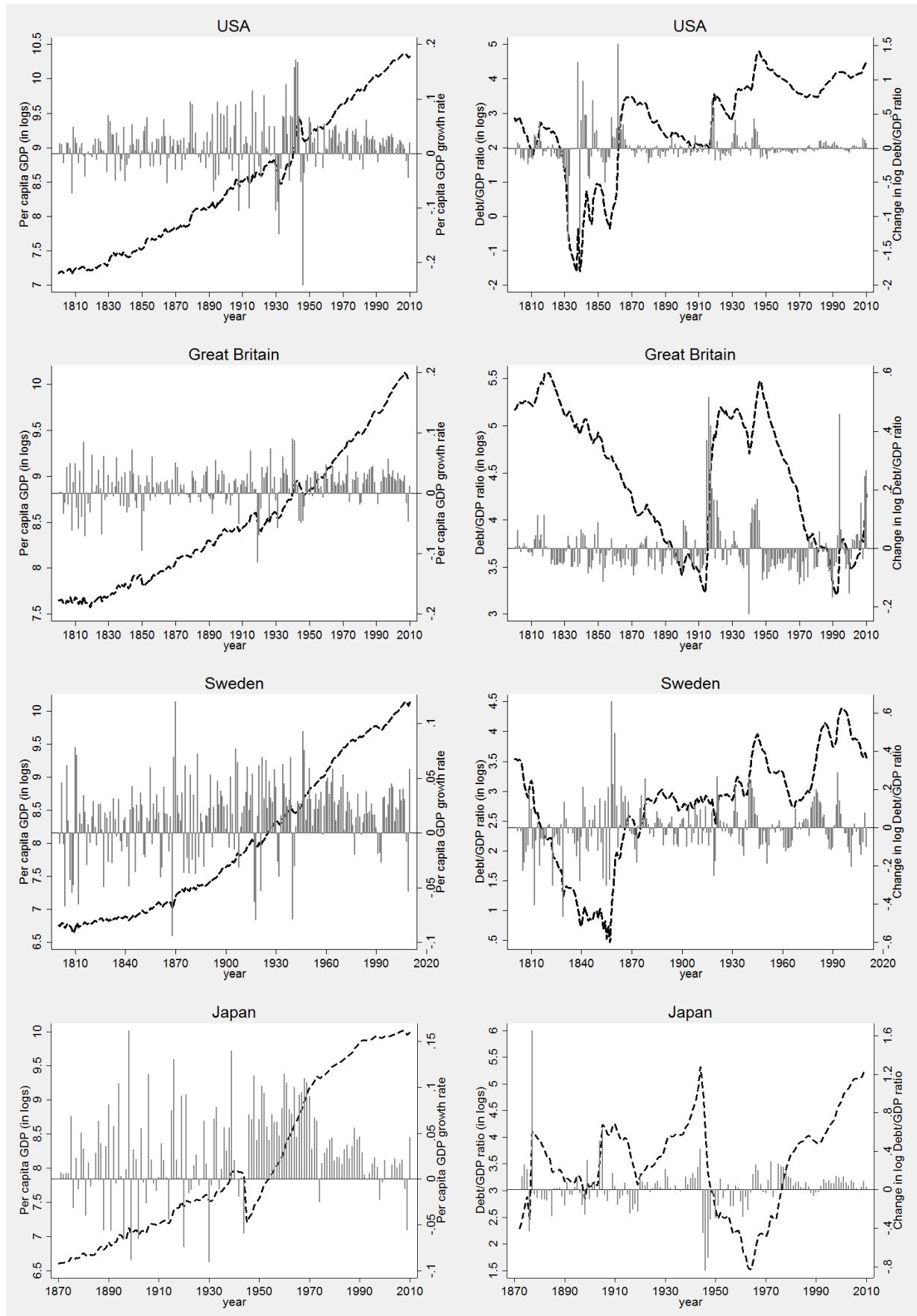
**Notes:** I provide the descriptive statistics for the levels variables included in our analysis (all in logarithms as indicated). Gap reports the number of missing observations.  $\Delta$  is the difference operator.

Table A-2: Unit root/stationarity tests

		Start	End	Lags	DF-GLS	PP Z(t)	KPSS(i)	KPSS(ii)
USA	ln(GDP pc)	1800	2010	9	-0.88	0.65	0.47 ***	2.27 ***
	(Debt/GDP)			13	-2.55	-1.03	0.11	1.66 ***
	ln(Debt/GDP)			6	-2.38	-2.27	0.08	1.06 ***
	$\Delta$ ln(GDP pc)	1801	2010	8	-6.17***	-11.29***	0.03	0.14
	$\Delta$ ln(Debt/GDP)			5	-6.31***	-10.54***	0.03	0.04
GBR	ln(GDP pc)	1800	2010	6	0.09	2.26	0.66***	3.07***
	(Debt/GDP)			6	-1.89	-1.64	0.34***	0.94***
	ln(Debt/GDP)			1	-2.12	-1.66	0.93***	2.64***
	$\Delta$ ln(GDP pc)	1801	2010	8	-6.41***	-12.70***	0.03	0.52***
	$\Delta$ ln(Debt/GDP)			14	-2.08	-7.00***	0.08	0.10
SWE	ln(GDP pc)	1800	2010	10	-0.48	2.71	0.55***	1.99***
	(Debt/GDP)			10	-2.16	-1.95	0.12*	1.37***
	ln(Debt/GDP)			10	-1.66	-1.34	0.10	1.44***
	$\Delta$ ln(GDP pc)	1801	2010	9	-3.29**	-13.59***	0.14*	0.83***
	$\Delta$ ln(Debt/GDP)			9	-3.35**	-11.25***	0.10	0.12
JPN	ln(GDP pc)	1872	2010	0	-1.25	0.18	2.34***	13.30***
	(Debt/GDP)			2	-1.48	-0.88	0.39***	0.90***
	ln(Debt/GDP)			3	-1.76	-1.72	0.33***	0.39*
	$\Delta$ ln(GDP pc)	1873	2010	0	-10.77***	-10.76***	0.11	0.21
	$\Delta$ ln(Debt/GDP)			2	-4.54***	-9.78***	0.11	0.12

**Notes:** I use the Elliott, Rothenberg and Stock (1996) DF-GLS, Phillips and Perron (1988) PP and Kwiatkowski et al. (1992) KPSS tests. Lag length based on Ng and Perron (1995) criterion (DF-GLS and KPSS). KPSS(i) and (ii) have the null of trend and levels stationarity, respectively; DF-GLS and PP have the null of nonstationarity.  $\Delta$  is the difference operator.

Figure A-1: Income and debt evolution



**Notes:** These plots chart the evolution of per capita GDP (in logs, left column) and the debt/GDP ratio (in logs, right column) for our four OECD countries. In each plot the levels variable (left axis, dashed line) is graphed alongside the variable in first differences (right axis, gray bars). Note that the per capita GDP growth series for Japan (-68%) excludes 1945 for ease of illustration.

Table A-3: Country Coverage (Extended Analysis)

Country	Argentina	Australia	Austria‡	Belgium‡	Brazil	Canada
Isocode	ARG	AUS	AUT	BEL	BRA	CAN
Start	1875	1861	1880	1846	1889	1870
End	2010	2010	2010	2010	2010	2010
Gaps	0	0	20	12	0	0
Obs	136	150	111	153	122	141

Country	Chile	Colombia	Denmark	France	Germany‡	Greece‡
Isocode	CHL	COL	DNK	FRA	DEU	GRC
Start	1870	1899	1880	1880	1880	1848
End	2010	2010	2010	2010	2010	2010
Gaps	0	0	0	23	37	15
Obs	141	112	131	108	94	163

Country	India	Italy	<b>Japan</b>	Netherlands	New Zealand‡	Norway
Isocode	IND	ITA	JPN	NLD	NZL	NOR
Start	1884	1861	1872	1815	1870	1880
End	2010	2010	2010	2010	2010	2010
Gaps	0	0	0	6	0	6
Obs	127	150	139	190	151	131

Country	Peru‡	Portugal‡	Spain‡	Sri Lanka	<b>Sweden</b>	Switzerland‡
Isocode	PER	PRT	ESP	LKA	SWE	CHE
Start	1883	1865	1850	1870	1800	1880
End	2010	2010	2010	2009	2010	2010
Gaps	14	0	4	35	0	16
Obs	114	146	157	105	211	115

Country	<b>Great Britain</b>	<b>United States</b>	Uruguay‡
Isocode	GBR	USA	URY
Start	1800	1800	1871
End	2010	2010	2009
Gaps	0	0	23
Obs	211	211	116

**Notes:** I present start and end years of per capita GDP and debt-ratio time series for the set of 27 countries for which I report the summability and co-summability results below. The countries in bold are studied in the main section of the paper. Excel files available at my personal website in due course will provide information on the country-time coverage by model.



### A.3 Additional Empirical Results

I investigate the time series properties in a more diverse set of 23 additional economies (including some developing countries) in order to establish whether the patterns of results are fairly consistent across all countries investigated. The patterns observed in the summability analysis presented above are confirmed by results for the larger set of countries in Table A-4: in 23 out of 27 countries there is a pattern whereby we cannot reject the null that the per capita GDP *growth rate* is  $S(0)$  but reject this null in the equivalent *levels* series.<sup>17</sup> In 25 out of 27 countries all three debt variables reject summability of order zero.<sup>18</sup> Investigation of the co-summability results for the larger set of countries in Table A-5 again confirms that the patterns of results in the four OECD countries are qualitatively identical to those in the additional 23 countries investigated – only a single case (polynomial specification with linear, squared and cubed debt terms for Uruguay) satisfies co-summability. These results provide strong evidence against any nonlinear – or, for that matter, linear – long-run equilibrium relationship in all countries investigated.

In cointegration analysis the choice of the dependent variable has crucial bearings on the empirical results. I therefore also consider whether the same might be the case for my investigation of a nonlinear long-run relationship between debt and growth: I employ the debt-to-GDP ratio (in logs) as the dependent variable for models including the (log of) per capita GDP, and its squared and cubed polynomial terms as regressors in my analysis of co-summability. Adopting the debt-to-GDP ratio as dependent variable in balance and co-summability analysis in combination with polynomials of per capita GDP (results in Table A-6) finds only a single model – the nonlinear model with squared and cubed per capita income for Japan – co-summable. This confirms that my findings above are not the outcome of the (arbitrary) choice of the debt-to-GDP ratio as dependent variable.

The focus on empirical models limited to measures of income and debt may be overly simplistic and subject to serious misspecification. This aside these simply models do not have any solid justification from an economic theory point of view. In a further set of robustness checks I therefore

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<sup>17</sup>For URY both series cannot reject  $S(0)$ , for BRA, COL and PRT both series reject the  $S(0)$  null – see Table A-3 for country codes.

<sup>18</sup>For BRA all three polynomials cannot reject  $S(0)$ , while (marginally) the same holds for the linear debt-to-GDP series for CAN.

extend the empirical model to include a number of determinants of growth as favoured by the cross-country growth literature. The following specifications are tested (obviously debt terms are always included): (i) I add inflation to the model, motivated by theoretical considerations of an investment-enhancing ‘Tobin effect’ of inflation as well as its reverse, and the strong negative impact found in cross-country empirical work (e.g. Barro, 1991); (ii) I analyse a ‘Solow Model with Debt,’ which includes the investment-to-GDP ratio (in logs) as well as the population growth rate and is motivated by the empirical equilibrium analysis of a standard Solow growth model in Mankiw, Romer and Weil (1992, equation (7) and Table I); (iii) I extend this to the ‘Augmented Solow Model with Debt’ by adding a measure of human capital (schooling) which was shown by the same authors to reconcile empirical estimates with observed income shares of capital and labour (equation (12) and Table II of that paper).

Summability test results for the new variables included are provided in Table A-7. Results here are not necessarily consistent across countries, in particular with regard to the population variable: the slow pace of demographic transition in advanced economies such as those studied here typically translates into the time series of this variable (in logarithms) appearing to be integrated of order two,  $I(2)$ , with the growth rate thus  $I(1)$ . This is the case for Sweden and Great Britain, where  $S(0)$  and thus  $I(0)$  is rejected, however the population growth rates for the United States and Japan cannot reject this property. There are also minor disagreements across countries with regards to the inflation and investment share (in logs). Table A-8 reports co-summability results from a model where inflation is added as additional regressor to the debt term(s). This setup is found to be co-summable across all models for Great Britain but not in any other country bar the threshold specification for the United States. The ‘Solow Model with Debt’ then finds all models for Sweden and Japan co-summable but none for the United States (again with the same exception of the 50% debt/GDP threshold) and Great Britain – see Table A-9. Finally, the ‘Augmented Solow Model with Debt’ in Table A-10 again finds all models for Sweden co-summable, yet none for the other countries, with the exception of the model with linear and squared debt terms for the United States and the two polynomial specifications for Great Britain.

Table A-4: Estimated Order of Summability – 27 countries

Country	Start Year	End Year	Gaps	Obs	Variable	CI low	$\hat{\delta}$	CI up
ARG	1875	2010	-	136	ln(GDP pc)	0.279	0.851	1.423
					$\Delta$ ln(GDP pc)	-0.855	-0.249	0.357
					ln(Debt/GDP)	0.117	0.661	1.205
					ln(Debt/GDP) squared	0.075	0.691	1.308
					ln(Debt/GDP) cubed	0.117	0.727	1.337
AUS	1861	2010	-	150	ln(GDP pc)	0.262	0.847	1.432
					$\Delta$ ln(GDP pc)	-0.222	0.464	1.151
					ln(Debt/GDP)	0.402	0.981	1.560
					ln(Debt/GDP) squared	0.300	0.978	1.656
					ln(Debt/GDP) cubed	0.308	1.011	1.714
AUT	1880	2010	20	111	ln(GDP pc)	0.056	0.775	1.495
					$\Delta$ ln(GDP pc)	-0.552	0.104	0.760
					ln(Debt/GDP)	0.546	1.225	1.904
					ln(Debt/GDP) squared	0.473	1.190	1.907
					ln(Debt/GDP) cubed	0.476	1.158	1.839
BEL	1846	2010	12	153	ln(GDP pc)	0.347	0.730	1.113
					$\Delta$ ln(GDP pc)	-0.484	0.163	0.810
					ln(Debt/GDP)	0.147	0.680	1.213
					ln(Debt/GDP) squared	0.131	0.675	1.220
					ln(Debt/GDP) cubed	0.106	0.673	1.240
BRA	1889	2010	-	122	ln(GDP pc)	0.650	1.157	1.664
					$\Delta$ ln(GDP pc)	0.094	1.064	2.035
					ln(Debt/GDP)	-0.385	0.376	1.137
					ln(Debt/GDP) squared	-0.510	0.321	1.152
					ln(Debt/GDP) cubed	-0.596	0.244	1.084
CAN	1870	2010	-	141	ln(GDP pc)	0.101	0.552	1.003
					$\Delta$ ln(GDP pc)	-1.235	-0.354	0.527
					ln(Debt/GDP)	-0.005	0.627	1.258
					ln(Debt/GDP) squared	0.024	0.657	1.291
					ln(Debt/GDP) cubed	0.136	0.690	1.244
CHL	1870	2010	-	141	ln(GDP pc)	0.145	0.717	1.289
					$\Delta$ ln(GDP pc)	-0.584	0.022	0.628
					ln(Debt/GDP)	0.135	0.818	1.500
					ln(Debt/GDP) squared	0.176	0.828	1.480
					ln(Debt/GDP) cubed	0.216	0.859	1.501
COL	1899	2010	-	112	ln(GDP pc)	0.817	1.537	2.257
					$\Delta$ ln(GDP pc)	0.374	1.157	1.940
					ln(Debt/GDP)	0.673	1.312	1.950
					ln(Debt/GDP) squared	0.745	1.243	1.740
					ln(Debt/GDP) cubed	0.766	1.179	1.593
DNK	1880	2010	-	131	ln(GDP pc)	0.141	0.677	1.213
					$\Delta$ ln(GDP pc)	-0.573	0.155	0.882
					ln(Debt/GDP)	0.389	1.182	1.976
					ln(Debt/GDP) squared	0.484	1.228	1.972
					ln(Debt/GDP) cubed	0.514	1.240	1.967

Table continued on the following page

Table A-4: Estimated Order of Summability – 27 countries (continued)

Country	Start Year	End Year	Gaps	Obs	Variable	CI low	$\hat{\delta}$	CI up
FRA	1880	2010	23	108	ln(GDP pc)	0.463	1.307	2.150
					$\Delta$ ln(GDP pc)	-0.490	0.226	0.942
					ln(Debt/GDP)	0.773	1.564	2.356
					ln(Debt/GDP) squared	1.153	2.336	3.519
					ln(Debt/GDP) cubed	1.557	3.156	4.755
DEU	1880	2010	37	94	ln(GDP pc)	0.549	1.239	1.929
					$\Delta$ ln(GDP pc)	-0.567	0.121	0.808
					ln(Debt/GDP)	0.514	0.940	1.367
					ln(Debt/GDP) squared	0.509	0.935	1.360
					ln(Debt/GDP) cubed	0.531	0.937	1.343
GRC	1848	2010	15	148	ln(GDP pc)	0.431	1.169	1.907
					$\Delta$ ln(GDP pc)	-0.534	0.138	0.809
					ln(Debt/GDP)	0.324	0.856	1.388
					ln(Debt/GDP) squared	0.406	0.960	1.515
					ln(Debt/GDP) cubed	0.367	0.990	1.613
IND	1884	2010	-	127	ln(GDP pc)	0.031	0.507	0.982
					$\Delta$ ln(GDP pc)	-0.925	-0.130	0.666
					ln(Debt/GDP)	0.279	0.991	1.702
					ln(Debt/GDP) squared	0.393	1.045	1.697
					ln(Debt/GDP) cubed	0.325	1.098	1.872
ITA	1861	2010	-	150	ln(GDP pc)	0.495	1.162	1.829
					$\Delta$ ln(GDP pc)	-0.210	0.364	0.939
					ln(Debt/GDP)	0.380	0.921	1.462
					ln(Debt/GDP) squared	0.384	0.937	1.491
					ln(Debt/GDP) cubed	0.390	0.953	1.517
JPN	1872	2010	-	139	ln(GDP pc)	0.987	2.390	3.792
					$\Delta$ ln(GDP pc)	-0.692	-0.004	0.683
					ln(Debt/GDP)	0.427	1.091	1.755
					ln(Debt/GDP) squared	0.433	1.101	1.769
					ln(Debt/GDP) cubed	0.410	1.114	1.819
NLD	1815	2010	6	190	ln(GDP pc)	0.055	0.569	1.083
					$\Delta$ ln(GDP pc)	-0.353	0.304	0.961
					ln(Debt/GDP)	0.462	1.084	1.705
					ln(Debt/GDP) squared	0.477	1.089	1.702
					ln(Debt/GDP) cubed	0.528	1.097	1.666
NZL	1870	2010	-	141	ln(GDP pc)	0.098	0.503	0.909
					$\Delta$ ln(GDP pc)	-0.323	0.299	0.920
					ln(Debt/GDP)	0.479	0.960	1.441
					ln(Debt/GDP) squared	0.419	0.986	1.553
					ln(Debt/GDP) cubed	0.486	1.009	1.533
NOR	1880	2010	6	125	ln(GDP pc)	0.656	1.349	2.042
					$\Delta$ ln(GDP pc)	-0.179	0.579	1.337
					ln(Debt/GDP)	0.398	1.073	1.749
					ln(Debt/GDP) squared	0.394	1.086	1.778
					ln(Debt/GDP) cubed	0.394	1.101	1.808

Table continued on the following page

Table A-4: Estimated Order of Summability – 27 countries (continued)

Country	Start Year	End Year	Gaps	Obs	Variable	CI low	$\hat{\delta}$	CI up
PER	1883	2010	14	114	ln(GDP pc)	0.284	0.820	1.357
					$\Delta$ ln(GDP pc)	-0.073	0.665	1.404
					ln(Debt/GDP)	0.677	1.122	1.566
					ln(Debt/GDP) squared	0.683	1.063	1.444
					ln(Debt/GDP) cubed	0.676	1.009	1.342
PRT	1865	2010	-	146	ln(GDP pc)	0.464	1.087	1.709
					$\Delta$ ln(GDP pc)	0.010	0.802	1.594
					ln(Debt/GDP)	0.397	0.933	1.470
					ln(Debt/GDP) squared	0.347	0.940	1.533
					ln(Debt/GDP) cubed	0.381	0.945	1.510
ESP	1850	2010	4	157	ln(GDP pc)	0.212	0.767	1.322
					$\Delta$ ln(GDP pc)	-0.499	0.067	0.633
					ln(Debt/GDP)	0.394	0.994	1.595
					ln(Debt/GDP) squared	0.350	0.979	1.609
					ln(Debt/GDP) cubed	0.380	0.966	1.551
LKA	1870	2009	35	105	ln(GDP pc)	0.411	0.816	1.220
					$\Delta$ ln(GDP pc)	-0.319	0.379	1.078
					ln(Debt/GDP)	0.210	0.771	1.332
					ln(Debt/GDP) squared	0.240	0.797	1.354
					ln(Debt/GDP) cubed	0.224	0.822	1.420
SWE	1800	2010	-	211	ln(GDP pc)	0.361	0.904	1.334
					$\Delta$ ln(GDP pc)	-0.359	0.030	0.357
					ln(Debt/GDP)	0.637	1.624	2.603
					ln(Debt/GDP) squared	0.614	1.577	2.451
					ln(Debt/GDP) cubed	0.473	1.538	2.399
CHE	1880	2010	16	115	ln(GDP pc)	0.159	0.669	1.179
					$\Delta$ ln(GDP pc)	-0.690	-0.097	0.497
					ln(Debt/GDP)	0.506	1.265	2.023
					ln(Debt/GDP) squared	0.508	1.254	2.001
					ln(Debt/GDP) cubed	0.477	1.255	2.033
GBR	1800	2010	-	211	ln(GDP pc)	0.731	1.696	2.662
					$\Delta$ ln(GDP pc)	-0.444	0.126	0.695
					ln(Debt/GDP)	0.540	0.967	1.393
					ln(Debt/GDP) squared	0.509	0.948	1.386
					ln(Debt/GDP) cubed	0.475	0.931	1.387
USA	1800	2010	-	211	ln(GDP pc)	0.686	1.561	2.436
					$\Delta$ ln(GDP pc)	-0.522	0.052	0.627
					ln(Debt/GDP)	0.551	1.082	1.613
					ln(Debt/GDP) squared	0.383	0.860	1.336
					ln(Debt/GDP) cubed	0.404	0.993	1.582
URY	1871	2009	23	116	ln(GDP pc)	-0.128	0.539	1.206
					$\Delta$ ln(GDP pc)	-0.195	0.545	1.285
					ln(Debt/GDP)	0.419	1.061	1.704
					ln(Debt/GDP) squared	0.401	1.089	1.777
					ln(Debt/GDP) cubed	0.440	1.127	1.815

**Notes:** I report full sample order of summability estimates, CI low and up indicate the 95% confidence interval for the summability estimate  $S(\hat{\delta})$  – shaded cells indicate variable series where the summability confidence interval includes zero. In all tests conducted I allow for deterministic terms (constant and trend).

Table A-5: Co-Summability – ln(GDP pc) specifications, 27 countries

	Start	End	Gaps	obs	Nonlinearity	Co-Summability			
						CI low	$\hat{\delta}_{\hat{e}_t}$	CI up	Verdict
ARG	1900	2008	-	109	-	0.543	0.990	1.438	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$b = 11$	$\ln(\text{Debt/GDP})^2$	0.343	0.928	1.512	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$M = 99$	$\ln(\text{Debt/GDP})^3$	0.423	0.936	1.449	$\hat{\delta}_{\hat{e}_t} \neq 0$
AUS	1861	2008	-	148	-	0.882	1.356	1.830	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$b = 13$	$\ln(\text{Debt/GDP})^2$	0.883	1.430	1.977	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$M = 136$	$\ln(\text{Debt/GDP})^3$	0.859	1.376	1.894	$\hat{\delta}_{\hat{e}_t} \neq 0$
AUT	1880	2008	2	109	-	0.444	1.012	1.580	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$b = 11$	$\ln(\text{Debt/GDP})^2$	0.218	0.760	1.301	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$M = 99$	$\ln(\text{Debt/GDP})^3$	0.101	0.579	1.057	$\hat{\delta}_{\hat{e}_t} \neq 0$
BEL	1846	2008	2	151	-	0.096	0.537	0.978	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$b = 13$	$\ln(\text{Debt/GDP})^2$	0.017	0.559	1.100	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$M = 139$	$\ln(\text{Debt/GDP})^3$	0.039	0.550	1.062	$\hat{\delta}_{\hat{e}_t} \neq 0$
BRA	1889	2008	-	120	-	1.022	1.832	2.643	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$b = 12$	$\ln(\text{Debt/GDP})^2$	0.687	1.298	1.908	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$M = 109$	$\ln(\text{Debt/GDP})^3$	0.684	1.056	1.428	$\hat{\delta}_{\hat{e}_t} \neq 0$
CAN	1870	2008	-	139	-	0.223	0.770	1.318	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$b = 13$	$\ln(\text{Debt/GDP})^2$	0.233	0.751	1.269	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$M = 127$	$\ln(\text{Debt/GDP})^3$	0.242	0.751	1.259	$\hat{\delta}_{\hat{e}_t} \neq 0$
CHL	1827	2008	-	182	-	0.870	1.424	1.978	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$b = 14$	$\ln(\text{Debt/GDP})^2$	0.871	1.418	1.965	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$M = 169$	$\ln(\text{Debt/GDP})^3$	0.877	1.488	2.099	$\hat{\delta}_{\hat{e}_t} \neq 0$
COL	1900	2008	-	109	-	0.810	1.265	1.720	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$b = 11$	$\ln(\text{Debt/GDP})^2$	0.424	0.946	1.467	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$M = 99$	$\ln(\text{Debt/GDP})^3$	0.054	0.639	1.224	$\hat{\delta}_{\hat{e}_t} \neq 0$
DNK	1880	2008	-	129	-	1.049	1.801	2.553	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$b = 12$	$\ln(\text{Debt/GDP})^2$	0.589	1.654	2.719	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$M = 118$	$\ln(\text{Debt/GDP})^3$	0.695	1.604	2.513	$\hat{\delta}_{\hat{e}_t} \neq 0$
FRA	1880	2008	2	106	-	0.393	1.073	1.753	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$b = 11$	$\ln(\text{Debt/GDP})^2$	0.329	1.108	1.887	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$M = 96$	$\ln(\text{Debt/GDP})^3$	0.392	1.040	1.689	$\hat{\delta}_{\hat{e}_t} \neq 0$
DEU	1880	2008	2	106	-	0.330	0.757	1.185	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$b = 11$	$\ln(\text{Debt/GDP})^2$	0.353	0.959	1.564	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$M = 96$	$\ln(\text{Debt/GDP})^3$	0.326	0.893	1.459	$\hat{\delta}_{\hat{e}_t} \neq 0$
IND	1884	2008	-	125	-	0.164	0.691	1.218	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$b = 12$	$\ln(\text{Debt/GDP})^2$	0.163	0.691	1.218	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$M = 114$	$\ln(\text{Debt/GDP})^3$	0.335	0.751	1.167	$\hat{\delta}_{\hat{e}_t} \neq 0$
ITA	1861	2008	-	148	-	0.704	1.124	1.544	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$b = 13$	$\ln(\text{Debt/GDP})^2$	0.473	0.975	1.477	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$M = 136$	$\ln(\text{Debt/GDP})^3$	0.526	1.058	1.591	$\hat{\delta}_{\hat{e}_t} \neq 0$
JPN	1872	2008	2	123	-	0.462	1.135	1.809	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$b = 12$	$\ln(\text{Debt/GDP})^2$	0.194	0.848	1.503	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$M = 112$	$\ln(\text{Debt/GDP})^3$	0.200	0.859	1.518	$\hat{\delta}_{\hat{e}_t} \neq 0$

Table continued on the following page

Table A-5: Co-Summability – ln(GDP pc) specifications, 27 countries (cont'd)

	Start	End	Gaps	obs	Nonlinearity	Co-Summability			
						CI low	$\hat{\delta}_{\hat{e}_t}$	CI up	Verdict
NLD	1820	2008	1	183	-	0.210	0.595	0.980	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$b = 15$	$\ln(\text{Debt/GDP})^2$	0.238	0.640	1.042	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$M = 169$	$\ln(\text{Debt/GDP})^3$	0.291	0.708	1.124	$\hat{\delta}_{\hat{e}_t} \neq 0$
NZL	1870	2008	-	139	-	0.587	0.872	1.158	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$b = 13$	$\ln(\text{Debt/GDP})^2$	0.545	0.832	1.119	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$M = 127$	$\ln(\text{Debt/GDP})^3$	0.470	0.809	1.149	$\hat{\delta}_{\hat{e}_t} \neq 0$
NOR	1880	2008	1	123	-	0.721	1.325	1.930	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$b = 12$	$\ln(\text{Debt/GDP})^2$	0.476	1.161	1.846	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$M = 112$	$\ln(\text{Debt/GDP})^3$	0.540	1.221	1.902	$\hat{\delta}_{\hat{e}_t} \neq 0$
PER	1896	2008	4	99	-	0.327	0.922	1.517	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$b = 11$	$\ln(\text{Debt/GDP})^2$	0.319	0.890	1.460	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$M = 89$	$\ln(\text{Debt/GDP})^3$	0.261	0.877	1.494	$\hat{\delta}_{\hat{e}_t} \neq 0$
PRT	1865	2008	-	144	-	0.304	0.790	1.275	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$b = 13$	$\ln(\text{Debt/GDP})^2$	0.423	0.782	1.140	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$M = 132$	$\ln(\text{Debt/GDP})^3$	0.132	0.615	1.098	$\hat{\delta}_{\hat{e}_t} \neq 0$
SWE	1820	2008	-	189	-	0.331	0.928	1.524	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$b = 15$	$\ln(\text{Debt/GDP})^2$	0.281	0.854	1.426	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$M = 175$	$\ln(\text{Debt/GDP})^3$	0.280	0.957	1.634	$\hat{\delta}_{\hat{e}_t} \neq 0$
CHE	1880	2008	3	113	-	0.699	1.173	1.647	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$b = 12$	$\ln(\text{Debt/GDP})^2$	0.673	1.173	1.673	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$M = 102$	$\ln(\text{Debt/GDP})^3$	0.548	1.054	1.560	$\hat{\delta}_{\hat{e}_t} \neq 0$
ESP	1850	2008	1	155	-	0.251	0.911	1.571	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$b = 13$	$\ln(\text{Debt/GDP})^2$	0.219	0.849	1.478	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$M = 143$	$\ln(\text{Debt/GDP})^3$	0.300	0.862	1.423	$\hat{\delta}_{\hat{e}_t} \neq 0$
LKA	1870	2008	1	104	-	0.248	0.702	1.157	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$b = 11$	$\ln(\text{Debt/GDP})^2$	0.196	0.740	1.285	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$M = 94$	$\ln(\text{Debt/GDP})^3$	0.157	0.700	1.243	$\hat{\delta}_{\hat{e}_t} \neq 0$
GBR	1830	2008	-	179	-	0.600	0.995	1.390	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$b = 14$	$\ln(\text{Debt/GDP})^2$	0.580	1.014	1.448	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$M = 166$	$\ln(\text{Debt/GDP})^3$	0.425	0.951	1.477	$\hat{\delta}_{\hat{e}_t} \neq 0$
USA	1870	2008	-	139	-	0.483	1.116	1.748	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$b = 13$	$\ln(\text{Debt/GDP})^2$	0.415	1.090	1.764	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$M = 127$	$\ln(\text{Debt/GDP})^3$	0.344	0.943	1.541	$\hat{\delta}_{\hat{e}_t} \neq 0$
URY	1871	2008	1	115	-	0.218	0.602	0.986	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$b = 12$	$\ln(\text{Debt/GDP})^2$	0.156	0.602	1.048	$\hat{\delta}_{\hat{e}_t} \neq 0$
				$M = 104$	$\ln(\text{Debt/GDP})^3$	-0.119	0.477	1.074	$\hat{\delta}_{\hat{e}_t} = 0$

**Notes:** In all models I take per capita GDP (in logarithms) as the dependent variable. CI low and up indicate the 95% confidence interval for the co-summability estimates. In all tests conducted I allow for deterministic terms (constant and trend).  $\hat{\delta}_{\hat{e}_t} \neq (=)0$  implies that co-summability is (not) rejected.  $b = \text{int}\sqrt{T} + 1$  refers to the time series length of the subsample,  $M = T - b + 1$  to the number of subsamples used in the analysis. Regarding the ‘Nonlinearity,’ the model with  $\ln(\text{Debt/GDP})^2$  also includes  $\ln(\text{Debt/GDP})$ , while the model with  $\ln(\text{Debt/GDP})^3$  also includes  $\ln(\text{Debt/GDP})^2$  and  $\ln(\text{Debt/GDP})$ .

Table A-6: Co-Summability – ln(debt) as dependent variable

	Start	End	Obs	Nonlinearity	Co-Summability			
					CI low	$\hat{\delta}_{\hat{e}_t}$	CI up	Verdict
USA	1800	2010	211	-	0.340	0.820	1.300	$S(\delta_{\hat{e}_t}) \neq 0$
			$b = 16$	$\ln(\text{GDP pc})^2$	0.246	0.886	1.527	$S(\delta_{\hat{e}_t}) \neq 0$
			$M = 196$	$\ln(\text{GDP pc})^3$	0.284	0.885	1.485	$S(\delta_{\hat{e}_t}) \neq 0$
GBR	1800	2010	211	-	0.560	1.263	1.965	$S(\delta_{\hat{e}_t}) \neq 0$
			$b = 16$	$\ln(\text{GDP pc})^2$	0.501	1.173	1.846	$S(\delta_{\hat{e}_t}) \neq 0$
			$M = 196$	$\ln(\text{GDP pc})^3$	0.369	0.917	1.466	$S(\delta_{\hat{e}_t}) \neq 0$
SWE	1800	2010	211	-	0.709	1.623	2.538	$S(\delta_{\hat{e}_t}) \neq 0$
			$b = 16$	$\ln(\text{GDP pc})^2$	0.601	1.534	2.467	$S(\delta_{\hat{e}_t}) \neq 0$
			$M = 196$	$\ln(\text{GDP pc})^3$	0.590	1.515	2.439	$S(\delta_{\hat{e}_t}) \neq 0$
JPN	1872	2010	139	-	0.145	0.669	1.193	$S(\delta_{\hat{e}_t}) \neq 0$
			$b = 13$	$\ln(\text{GDP pc})^2$	0.204	0.806	1.408	$S(\delta_{\hat{e}_t}) \neq 0$
			$M = 127$	$\ln(\text{GDP pc})^3$	-0.160	0.338	0.837	$S(\delta_{\hat{e}_t}) = 0$

**Notes:** In all models I take the debt-to-GDP ratio (in logarithms) as the dependent variable. CI low and up indicate the 95% confidence interval for the co-summability estimates. In all tests conducted I allow for deterministic terms (constant and trend).  $\hat{\delta}_{\hat{e}_t} \neq (=) 0$  implies that co-summability is (not) rejected. Obs reports the number of observations,  $b = \text{int}\sqrt{T} + 1$  refers to the time series length of each subsample,  $M = T - b + 1$  to the number of subsamples used in the analysis. Regarding the ‘Nonlinearity,’ the model with  $\ln(\text{GDP pc})^2$  also includes  $\ln(\text{GDP pc})$ , while the model with  $\ln(\text{GDP pc})^3$  also includes  $\ln(\text{GDP pc})^2$  and  $\ln(\text{GDP pc})$ .



Table A-7: Estimated Order of Summability – Additional Covariates

Country	Start & End Year		$n$	Variable	CI low	$\hat{\delta}$	CI up
USA	1800	2010	211	Inflation	-0.572	0.035	0.642
	1870	2010	141	$\Delta \ln(\text{Population})$	-0.130	0.417	0.964
				$\ln(\text{Invest/GDP})$	0.520	1.407	2.294
	1880	2010	131	Schooling	1.234	2.471	3.708
GBR	1800	2010	211	Inflation	0.457	1.210	1.963
	1850	2010	161	$\Delta \ln(\text{Population})$	0.103	0.590	1.078
				$\ln(\text{Invest/GDP})$	-0.178	0.386	0.950
	1870	2010	141	Schooling	0.918	1.696	2.474
SWE	1800	2010	211	Inflation	0.492	1.376	2.260
	1801	2010	210	$\Delta \ln(\text{Population})$	0.131	0.644	1.158
				$\ln(\text{Invest/GDP})$	0.211	0.919	1.627
	1870	2010	141	Schooling	1.094	2.153	3.212
JPN	1800	2010	211	Inflation	0.030	0.608	1.185
	1850	2010	161	$\Delta \ln(\text{Population})$	-0.269	0.540	1.350
				$\ln(\text{Invest/GDP})$	0.027	0.529	1.031
	1870	2010	141	Schooling	0.368	1.058	1.749

**Notes:** CI low and up indicate the 95% confidence interval for the summability estimate  $S(\delta)$  constructed from subsampling – shaded cells indicate variable series where the summability confidence interval includes zero. In all tests conducted I allow for deterministic terms (constant and trend).

Table A-8: Co-Summability – Debt and Inflation Model

	Start	End	Obs	Nonlinearity	Co-Summability			
					CI low	$\hat{\delta}_{\hat{e}_t}$	CI up	Verdict
USA	1800	2010	211	-	0.417	0.951	1.486	$S(\delta_{\hat{e}_t}) \neq 0$
			$b=16$	$\ln(\text{Debt/GDP})^2$	0.312	0.892	1.472	$S(\delta_{\hat{e}_t}) \neq 0$
			$M=196$	$\ln(\text{Debt/GDP})^3$	0.353	0.855	1.357	$S(\delta_{\hat{e}_t}) \neq 0$
				Threshold 50%	-0.169	0.268	0.706	$S(\delta_{\hat{e}_t}) = 0$
GBR	1800	2010	211	-	-0.118	0.248	0.614	$S(\delta_{\hat{e}_t}) = 0$
			$b=16$	$\ln(\text{Debt/GDP})^2$	-0.146	0.246	0.639	$S(\delta_{\hat{e}_t}) = 0$
			$M=196$	$\ln(\text{Debt/GDP})^3$	-0.178	0.245	0.669	$S(\delta_{\hat{e}_t}) = 0$
				Threshold 50%	-0.169	0.268	0.706	$S(\delta_{\hat{e}_t}) = 0$
				Threshold 70%	-0.146	0.302	-0.751	$S(\delta_{\hat{e}_t}) = 0$
				Threshold 90%	-0.107	0.304	0.715	$S(\delta_{\hat{e}_t}) = 0$
SWE	1800	2010	211	-	0.386	0.924	1.463	$S(\delta_{\hat{e}_t}) \neq 0$
			$b=16$	$\ln(\text{Debt/GDP})^2$	0.354	0.949	1.544	$S(\delta_{\hat{e}_t}) \neq 0$
			$M=196$	$\ln(\text{Debt/GDP})^3$	0.337	0.989	1.641	$S(\delta_{\hat{e}_t}) \neq 0$
JPN	1872	2010	125	-	0.770	1.589	2.409	$S(\delta_{\hat{e}_t}) \neq 0$
			$b=12$	$\ln(\text{Debt/GDP})^2$	0.228	0.783	1.338	$S(\delta_{\hat{e}_t}) \neq 0$
			$M=114$	$\ln(\text{Debt/GDP})^3$	0.109	0.739	1.369	$S(\delta_{\hat{e}_t}) \neq 0$
				Threshold 50%	0.388	1.029	1.670	$S(\delta_{\hat{e}_t}) \neq 0$

**Notes:** In all models I take per capita GDP (in logarithms) as the dependent variable. CI low and up indicate the 95% confidence interval for the balance and co-summability estimates. In all tests conducted I allow for deterministic terms (constant and trend).  $\hat{\delta}_y \neq (=) \hat{\delta}_z$  implies that balance is (not) rejected,  $\hat{\delta}_{\hat{e}_t} \neq (=) 0$  that co-summability is (not) rejected. Obs reports the number of observations,  $b = \text{int}\sqrt{T} + 1$  refers to the time series length of each subsample,  $M = T - b + 1$  to the number of subsamples used in the analysis. Regarding the ‘Nonlinearity,’ the model with  $\ln(\text{Debt/GDP})^2$  also includes  $\ln(\text{Debt/GDP})$ , while the model with  $\ln(\text{Debt/GDP})^3$  also includes  $\ln(\text{Debt/GDP})^2$  and  $\ln(\text{Debt/GDP})$ .

Table A-9: Balance and Co-Summability – Solow Model with Debt

	Start	End	Obs	Nonlinearity	Co-Summability			
					CI low	$\hat{\delta}_{\hat{e}_t}$	CI up	Verdict
USA	1870	2010	141	-	0.212	0.888	1.565	$S(\delta_{\hat{e}_t}) \neq 0$
			$b=13$	$\ln(\text{Debt/GDP})^2$	0.158	0.745	1.332	$S(\delta_{\hat{e}_t}) \neq 0$
			$M=129$	$\ln(\text{Debt/GDP})^3$	0.125	0.939	1.753	$S(\delta_{\hat{e}_t}) \neq 0$
				Threshold 50%	-0.321	0.213	0.747	$S(\delta_{\hat{e}_t}) = 0$
GBR	1850	2010	161	-	0.155	1.003	1.851	$S(\delta_{\hat{e}_t}) \neq 0$
			$b=14$	$\ln(\text{Debt/GDP})^2$	0.148	0.820	1.492	$S(\delta_{\hat{e}_t}) \neq 0$
			$M=148$	$\ln(\text{Debt/GDP})^3$	0.117	0.754	1.392	$S(\delta_{\hat{e}_t}) \neq 0$
				Threshold 50%	0.209	0.884	1.559	$S(\delta_{\hat{e}_t}) \neq 0$
				Threshold 70%	0.343	0.972	1.602	$S(\delta_{\hat{e}_t}) \neq 0$
SWE	1800	2010	211	-	-0.152	0.334	0.820	$S(\delta_{\hat{e}_t}) = 0$
			$b=16$	$\ln(\text{Debt/GDP})^2$	-0.372	0.284	0.940	$S(\delta_{\hat{e}_t}) = 0$
			$M=196$	$\ln(\text{Debt/GDP})^3$	-0.540	0.195	0.931	$S(\delta_{\hat{e}_t}) = 0$
				Threshold 50%	-0.014	0.517	1.048	$S(\delta_{\hat{e}_t}) = 0$
JPN	1872	2010	125	-	-0.253	0.246	0.745	$S(\delta_{\hat{e}_t}) = 0$
			$b=12$	$\ln(\text{Debt/GDP})^2$	-0.277	0.239	0.754	$S(\delta_{\hat{e}_t}) = 0$
			$M=114$	$\ln(\text{Debt/GDP})^3$	-0.262	0.232	0.726	$S(\delta_{\hat{e}_t}) = 0$
				Threshold 50%	-0.014	0.517	1.048	$S(\delta_{\hat{e}_t}) = 0$

**Notes:** In all models I take per capita GDP (in logarithms) as the dependent variable. CI low and up indicate the 95% confidence interval for the balance and co-summability estimates. In all tests conducted I allow for deterministic terms (constant and trend).  $\hat{\delta}_y \neq (=)\hat{\delta}_z$  implies that balance is (not) rejected,  $\hat{\delta}_{\hat{e}_t} \neq (=)0$  that co-summability is (not) rejected. Obs reports the number of observations,  $b = \text{int}\sqrt{T} + 1$  refers to the time series length of each subsample,  $M = T - b + 1$  to the number of subsamples used in the analysis. Regarding the ‘Nonlinearity,’ the model with  $\ln(\text{Debt/GDP})^2$  also includes  $\ln(\text{Debt/GDP})$ , while the model with  $\ln(\text{Debt/GDP})^3$  also includes  $\ln(\text{Debt/GDP})^2$  and  $\ln(\text{Debt/GDP})$ .

Table A-10: Balance and Co-Summability – Augmented Solow Model with Debt

	Start	End	Obs	Nonlinearity	Co-Summability			
					CI low	$\hat{\delta}_{\hat{e}_t}$	CI up	Verdict
USA	1880	2010	131	-	0.304	0.892	1.479	$S(\delta_{\hat{e}_t}) \neq 0$
			$b=12$	$\ln(\text{Debt/GDP})^2$	-0.011	0.619	1.250	$S(\delta_{\hat{e}_t}) = 0$
			$M=120$	$\ln(\text{Debt/GDP})^3$	0.059	0.663	1.267	$S(\delta_{\hat{e}_t}) \neq 0$
				Threshold 50%	0.144	0.720	1.295	$S(\delta_{\hat{e}_t}) \neq 0$
GBR	1870	2010	141	-	0.275	0.915	1.554	$S(\delta_{\hat{e}_t}) \neq 0$
			$b=13$	$\ln(\text{Debt/GDP})^2$	-0.183	0.368	0.919	$S(\delta_{\hat{e}_t}) = 0$
			$M=129$	$\ln(\text{Debt/GDP})^3$	-0.010	0.634	1.278	$S(\delta_{\hat{e}_t}) = 0$
				Threshold 50%	0.230	0.853	1.475	$S(\delta_{\hat{e}_t}) \neq 0$
				Threshold 70%	0.343	0.972	1.602	$S(\delta_{\hat{e}_t}) \neq 0$
				Threshold 90%	0.345	1.059	1.773	$S(\delta_{\hat{e}_t}) \neq 0$
SWE	1870	2010	141	-	-0.078	0.437	0.952	$S(\delta_{\hat{e}_t}) = 0$
			$b=13$	$\ln(\text{Debt/GDP})^2$	-0.233	0.387	1.007	$S(\delta_{\hat{e}_t}) = 0$
			$M=129$	$\ln(\text{Debt/GDP})^3$	-0.185	0.340	0.866	$S(\delta_{\hat{e}_t}) = 0$
JPN	1890	2010	107	-	0.055	0.533	1.011	$S(\delta_{\hat{e}_t}) \neq 0$
			$b=11$	$\ln(\text{Debt/GDP})^2$	0.069	0.509	0.950	$S(\delta_{\hat{e}_t}) \neq 0$
			$M=97$	$\ln(\text{Debt/GDP})^3$	0.065	0.509	0.952	$S(\delta_{\hat{e}_t}) \neq 0$
				Threshold 50%	0.435	0.790	1.145	$S(\delta_{\hat{e}_t}) \neq 0$

**Notes:** In all models I take per capita GDP (in logarithms) as the dependent variable. CI low and up indicate the 95% confidence interval for the co-summability estimates. In all tests conducted I allow for deterministic terms (constant and trend).  $\hat{\delta}_{\hat{e}_t} \neq (=)0$  implies that co-summability is (not) rejected. Obs reports the number of observations,  $b = \text{int}\sqrt{T} + 1$  refers to the time series length of each subsample,  $M = T - b + 1$  to the number of subsamples used in the analysis. Regarding the ‘Nonlinearity,’ the model with  $\ln(\text{Debt/GDP})^2$  also includes  $\ln(\text{Debt/GDP})$ , while the model with  $\ln(\text{Debt/GDP})^3$  also includes  $\ln(\text{Debt/GDP})^2$  and  $\ln(\text{Debt/GDP})$ .