

Public Debt and Growth: Heterogeneity and Non-Linearity*

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Abstract: We study the long-run relationship between public debt and growth in a large panel of countries. Our analysis builds on theoretical arguments and data considerations in modelling the debt-growth relationship as heterogeneous across countries. We investigate the debt-growth nexus adopting linear and non-linear specifications, employing novel methods and diagnostics from the time-series literature adapted for use in the panel. We find some support for a negative relationship between public debt and long-run growth across countries, but no evidence for a similar, let alone common, debt threshold within countries.

Keywords: economic growth; public debt; common factor model; non-linearity

JEL classification: E62, F34, C23, O11

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1 Introduction

The relationship between public debt and economic growth has recently emerged once again as a hotly debated topic in academia and among policymakers. Starting from the seminal contribution of Reinhart & Rogoff (2010a,b) a large strand of literature has investigated this relationship, attempting to identify possible non-linearities and discussing to what extent debt accumulation has a detrimental and causal effect on GDP growth (for a recent review see Panizza & Presbitero, 2013).

This paper asks whether the relationship between public debt and economic growth is significantly negative and further investigates the presence of common or country-specific thresholds beyond which it changes in magnitude. The originality of our analysis arises from the adoption of recently developed methods from the panel time series literature which have significant bearings on how we can empirically model the debt-growth nexus: first, we can ask whether a negative long-run relationship between public debt and growth exists and whether this relationship differs substantially *across countries*. If the impact of debt on growth differed across countries then a focus on the *average* relation may be misleading for policy adoption in individual countries. Second, moving away from a strictly linear relationship for the debt-growth nexus we can investigate whether *within individual countries* there is any evidence for thresholds or ‘high vulnerability regions’ (Reinhart & Rogoff, 2010a, p. 7) where this relationship may change from a positive significant or insignificant to a significant negative one. Third, and incorporated in both these sets of analysis, we allow for a very flexible way to account for unobserved heterogeneity (and thus endogeneity) in our models, which could arise from omitted variables and/or the impact of global shocks which differ in their impact across countries.

We analyse the empirics of the debt-growth nexus within a standard neoclassical growth model. Given the recent interest in this topic, cross-country empirical papers that are closely related to our work include Cordella, Ricci & Ruiz-Arranz (2010); Checherita-Westphal & Rother (2012); Kourtellis, Stengos & Tan (2013); Panizza & Presbitero (2014), among others. We provide a synthetic review of this literature in a Technical Appendix.

Using total public debt data from 118 developing, emerging and advanced economies over the period 1960 to 2012 we find that long-run debt coefficients differ across countries and provide some evidence that countries with higher average debt-to-GDP ratios are more likely to see a negative effect on their long-run growth performance. This result is consistent with higher debt ratios being associated, on average, with lower GDP growth rates (Reinhart & Rogoff, 2010a,b). However, the debt-growth nexus differs significantly across countries and modelling non-linearities within-countries does not show the emergence of a common pattern in our sample. Viewed from this perspective, our results lend support to the view that debt overhang effects cannot be related to a specific debt thresholds, as one cannot “argue that growth will be normal at 89% and subpar (about 1% lower) at 91% debt/GDP any more than a car crash is unlikely at 54mph and near certain at 56mph” (Reinhart & Rogoff, 2010a). By contrast,

our evidence is suggestive of the fact that the relationship between public debt and growth is complex and the identification of a specific threshold which triggers a growth slowdown should take into account debt composition and a variety of country characteristics which could constrain government choices and affect the economy's vulnerability to crises.

Our analysis is based on total government debt, measured at face value, as this definition is broadly comparable across countries and makes it possible to use a large and sufficiently long panel dataset. However, this choice does not come without costs. First, the exclusion of private debt may be problematic as private debt is a potential source of financial instability and crisis (Gourinchas & Obstfeld, 2012; Schularick & Taylor, 2012). Second, our measure of public debt does not consider that a high proportion of foreign currency-denominated debt could increase financial fragility and lead to sub-optimal macroeconomic policies, as pointed out by the vast literature on the 'original sin' (Hausmann & Panizza, 2011). Third, we consider gross public debt, although net debt would seem to be a better measure of government indebtedness (Panizza & Presbitero, 2013). Finally, considering the face value of debt could be misleading given that countries can borrow at different maturities and contractual forms (Dias, Richmond & Wright, 2014). While data availability prevents us from dealing with some of these issues, we employ alternative definitions of the present value of public external debt for a large number of developing countries in order to focus on foreign-currency denominated debt and to have a better measure of indebtedness for developing countries. These results are qualitatively very similar to those for the total public (face value) debt data presented below and are therefore confined to a Technical Appendix.

The remainder of this study is organised as follows: Section 2 motivates our empirical approach from the existing theoretical and empirical literature. Section 3 considers how the complexities of the economic theory and data realities should inform our empirical analysis. Section 4 describes our data and provides an overview of the econometric methods we apply. In Section 5 we present our empirical results and detailed analysis of heterogeneity and non-linearity in the debt-growth relationship across and within countries. Section 6 concludes.

2 Related Literature

The first element of our analysis concerns the presence of a negative long-run relationship between public debt and growth. There are a number of theoretical arguments which can motivate such a long-run relationship between public debt and growth (Elmendorf & Mankiw, 1999). In standard overlapping generations models of growth public debt reduces savings and capital accumulation (via higher interest rates), thus weakening economic growth (Modigliani, 1961; Diamond, 1965; Blanchard, 1985). In endogenous growth models public debt has generally a negative effect on long-run growth (Barro, 1990; Saint-Paul, 1992). Alternatively, one could simply argue that debt has to be paid off by future reduction in public spending or distortionary taxation, with negative effects on growth. Consistent with this line of argument, Bohn

(1998), Mendoza & Ostry (2008) and Lo & Rogoff (2015) show that governments react to a rising public debt by increasing the primary surplus or running smaller deficits. Moreover, high public debt limits the effectiveness of productive public expenditures on long-run growth (Teles & Cesar Mussolini, 2014), creates uncertainty or expectations of future financial repression (Cochrane, 2011), and could be associated with higher sovereign yield spreads (Codogno, Favero & Missale, 2003) leading to higher real interest rates and lower private investment (Laubach, 2009).

The second element of our analysis establishes whether the long-run relationship studied is broadly the same in each country, or whether there are significant differences in the debt-growth nexus across countries. There are a number of reasons to assume the equilibrium relationship between public debt and growth may differ across countries. First, in line with the ‘new growth’ literature (see Temple, 1999) production technology may differ across countries, and thus also the relationship between debt and growth. In this vein some recent work (Reinhart, Reinhart & Rogoff, 2012; International Monetary Fund, 2012) has preferred to analyze single episodes of debt overhang in individual countries adopting qualitative methods in order to develop a typology of episodes. Second, the capacity to tolerate high levels of debt depends on a number of country-specific characteristics, related to past crises and the macro and institutional framework (Reinhart, Rogoff & Savastano, 2003; Kraay & Nehru, 2006; Manasse & Roubini, 2009), many of which are either unobserved or difficult to capture in the empirical setup. Third, vulnerability to public debt depends not only on debt levels, but also on debt composition – domestic versus external, foreign or domestic currency denominated, long-term versus short term public debt (Reinhart, Reinhart & Rogoff, 2012; Dell’Erba, Hausmann & Panizza, 2013) – which differs significantly across countries.

The final element of our analysis is the issue of non-linearity in the debt-growth relationship, which we approach with a number of alternative empirical strategies, enabling us to investigate a *country-specific* non-linearity or threshold.¹ This differs somewhat from the standard empirical approach to and interpretation of non-linearities in the debt-growth empirical literature adopting pooled models: in the latter, if country A has a higher debt-to-GDP ratio and worse growth performance than country B, then in interpreting the empirical results it is implicitly assumed that if country B were to reach the same level of indebtedness it would be subject to the same debt-growth effect as country A. By contrast, our investigation of country-specific non-linearities ties in closely with our concern over heterogeneity outlined above and our analysis of non-linearity focuses on country-specific thresholds or vulnerability regions. We refer to a well-established literature on the asymmetric effects of fiscal policy which could motivate a non-linear effect of public debt on output growth in advanced economies (Sutherland, 1997; Perotti, 1999). Non-linearities in the debt-growth nexus may also arise if there is a tipping point

¹We do not address the issue of *time-varying* thresholds (i.e. time-varying parameters in a linear or non-linear debt-growth model). One could imagine that if a country-specific threshold exists, it could change over time, depending on the evolution of macroeconomic and institutional variables. However, our empirical framework is not well-suited to tackle this issue in a very satisfactory fashion due to the limited time series available for a comparison of results over time. At worst if the debt-growth relationship changes within countries over time the estimates presented can econometrically be argued to represent averages over time.

of fiscal sustainability: when debt is too high debt overhang could directly distort investment, as investors believe that the proceeds of any new project will be taxed away to service the pre-existing debt (Krugman, 1988; Aguiar, Amador & Gopinath, 2009); alternatively, as debt levels rise with respect to GDP, creditors would ask for higher interest rates to compensate the risk of default and this effect would increase the cost of financing, constraining investment (Greenlaw et al., 2013). Consistent with these arguments, parts of the empirical literature lend support to the presence of a common debt threshold across (similar) countries (e.g. Kumar & Woo, 2010; Cecchetti, Mohanty & Zampolli, 2011; Checherita-Westphal & Rother, 2012; Greenlaw et al., 2013). However, the presence of a tipping point does not mean that it has to be common across countries. For instance, Ghosh et al. (2013) define ‘debt limit’ as the level of debt beyond which fiscal solvency fails and show that this debt limit is a function of countries’ structural characteristics and GDP growth. This argument resembles the idea of country-specific debt ‘vulnerability regions,’ which would be consistent with country-specific non-linearities (Reinhart, Rogoff & Savastano, 2003).

3 Linking Theory and Empirics

Two aspects of our approach are related to the modelling of economic relationships as common or different across countries: first, we are concerned about common shocks (examples include the 1970s oil crises or the recent global financial crisis) and their distorting impact on identifying the debt-growth nexus in the data (cross-section correlation); second, we are interested in analysing the debt-growth relationship once we depart from the assumption of common parameters across all countries. Econometrically, we know that ignoring the impact of cross-section correlation, arising from global shocks or local spillover effects, yields seriously biased estimates for our parameters of interest (Phillips & Sul, 2003; Andrews, 2005), while non-linearities may spuriously appear if heterogeneous relationships are erroneously modeled as common across countries (Haque, Pesaran & Sharma, 1999). In the following we provide some simple descriptive analysis highlighting the cross-sectional correlation of debt accumulation across countries, but also the cross-country heterogeneity in the relationship between debt and growth — data and sources are described in detail in Section 4.4 and the Data Appendix.

We begin with the issue of correlation across countries, analysing the years in which countries in our sample reach their peak debt-to-GDP ratio: although there is some heterogeneity as to the sample coverage over the entire period, it is notable that in over one-third of countries these peaks occurred in only three years, namely 1985, 1994 and 2012. Given that the data stretches over fifty years, it is a remarkable indication of *common* effects across countries that the debt-to-GDP ratio peaks are clustered around a much smaller number of dates.

Our illustration in Figure 1 links countries’ debt-to-GDP ratio peaks to the deviation of per capita GDP growth rate during the ‘peak years’ (*ad hoc* defined as running from two years prior

to two years after the debt-to-GDP maximum) from that of the full time horizon (excluding the five peak years).² We highlight observations for the three years 1985, 1994 and 2012, as well as a small number of outliers. We can make a number of observations regarding this crude depiction of our empirical relationship of interest: first, there seems to be a negative correlation between the maximum debt level and relative growth performance between peak debt and other years (linear regression result reported in the figure footnote). Second, the figure highlights considerable heterogeneity across countries: for instance, among the countries for which debt-to-GDP peaked in 1985 (red triangles), one country experienced growth at around 2% *above* its average growth rate in all other years, while another country experienced a ‘peak years’ average growth rate which was 2% below its average in other years.³ Third, we note the dashed vertical line marking a debt-to-GDP ratio of 90%: a considerable number of countries to the right of this threshold had *better* growth performance in their peak debt years than at any other point since the 1960s.

[Figure 1 about here]

Figure 2 illustrates the potential for heterogeneity misspecification in the debt-growth relationship. In the first panel we plot a fractional polynomial regression line (as well as a 95% confidence interval) for per capita GDP against the debt-to-GDP ratio (both variables in logs) – the former is taken in deviation from the country-specific means (‘within’ transformation) to take account of different income levels across countries and thus focuses our analysis on changes relative to the country mean.⁴ As can be seen there is clearly a non-linear relationship between these two variables, in line with the standard arguments advanced in the literature, with a ‘threshold’ of 4.5 log points (equivalent to 90% debt-to-GDP) a distinct turning point: higher debt is associated with lower relative per capita GDP.⁵ In a second plot in the same figure we provide country-specific fractional polynomial regression lines for all countries in our sample, while a third plot randomly selects thirty countries from the previous plot. The latter two graphs illustrate that the seeming non-linearity assuming a pooled empirical model (black regression line and shaded confidence intervals) is far from obvious once we assume an empirical model which allows the relationship to differ across countries.

[Figure 2 about here]

Our descriptive analysis thus suggests that the raw data (adopting levels variables to elicit

²For peaks at the start (end) of our sample we limit these averages to the peak year and the two years after (before).

³Interestingly the gray diamonds indicating 2012 show that with the exception of SGP all countries in which debt peaked in that year (these are all High-Income countries except for GRD and LCA) had *worse* growth performance in 2010-2012 than in all other years (average growth rates, respectively).

⁴The same pattern emerges when we use untransformed per capita GDP. In order to aid presentation in Figure 2 we exclude ‘extreme’ values (in total 5% of observations) from this descriptive graph: in 1991 NIC had a debt/GDP ratio in excess of 2,000%; we further exclude all observations for which the within-transformed relative income exceeds 60%, which amounts to 239 observations (primarily fast-growing Middle- and (as a result of fast growth now) High-Income Countries such as KOR, SGP, MYS, THA, CHN, BWA, IRL).

⁵We further carried out the same descriptive exercise as in Reinhart & Rogoff (2010a, Figure 2) by analysing median and mean growth rates for different groupings based on level of indebtedness. Our results (contained in a Technical Appendix) are remarkably similar to those in the Reinhart and Rogoff study for this much larger and diverse set of countries.

the long-run relationship) show a clear non-linearity or threshold between the debt-to-GDP ratio and income at around 90% debt burden provided we assume that all countries in the sample have the same equilibrium relationship. However, relaxing this assumption seriously challenges this conclusion.

Of course this form of descriptive analysis is highly stylised, not to mention that there are other determinants of economic development and that such plots cannot provide any insights into any potentially causal relationship, be it from debt to growth or vice versa. Although our discussion is by no means conclusive, we feel that these illustrations cast some doubt over the stringent *implicit* assumptions adopted in most of the existing literature: first, that we can carry out empirical analysis assuming that correlation across countries does not matter when running standard panel regressions. Second, the assumption that all countries, regardless of their level of economic development, their industrial structure or institutional environment, follow the same equilibrium relationship between debt and growth. Third, the notion that all countries are subject to the same debt threshold, beyond which growth is affected detrimentally, which is econometrically implemented by use of exogenous or endogenous debt thresholds or by adopting a polynomial specification for debt in a pooled empirical model.

4 Empirical Strategy and Data

Our empirical analysis of the debt-growth nexus begins by considering differences in the relationship *across countries*. We adopt standard linear regression models, albeit of a fashion which accounts for both observed and unobserved heterogeneity. Identification of the long-run and short-run coefficients on debt is achieved by use of the Pesaran (2006) common correlated effects (CCE) estimator, which accounts for the presence of unobserved heterogeneity through a simple augmentation of the regression equation. Due to the dynamic setup and thus the presence of a lagged dependent variable it is necessary to adjust this augmentation following the suggestions in Chudik & Pesaran (2014). We then analyse the relationship between the estimated long-run coefficients and country-specific averages of debt levels, of debt-to-GDP ratios as well as peak debt-to-GDP ratios.

Next, we consider non-linearity in the debt-growth nexus *at the country-level* using two alternative approaches: first we employ the non-linear dynamic model by Shin, Yu & Greenwood-Nimmo (2013), where upon selecting an exogenously given threshold (we focus on 60%, the sample mean, and the popular 90% debt-to-GDP ratio) we are able to investigate heterogeneous growth regimes (below and above the threshold) whilst accounting for cross-section dependence. As a robustness exercise, our second approach will employ the familiar practice of including a squared term of the debt stock variable in a static regression model whilst accounting for cross-section dependence.

We discuss these specifications and identification strategies in detail below. Section 4.4 then introduces the data employed in the analysis and robustness checks.

4.1 Empirical Specification: Linear Dynamic Model

The basic equation of interest for our analysis of the debt-growth nexus is a log-linearised Cobb-Douglas production function augmented with a debt stock term:

$$y_{it} = \beta_i^K \text{cap}_{it} + \beta_i^D \text{debt}_{it} + u_{it} \quad u_{it} = \alpha_i + \lambda_i' f_t + \varepsilon_{it} \quad (1)$$

where y is aggregate GDP, cap is capital stock and debt is the total debt stock — all variables are in logarithms of *per capita* terms.⁶ These variables constitute the observable part of our model, with their parameter coefficients β_i^j (for $j=K, D$) allowed to differ across countries⁷ — this heterogeneity is a central feature of our empirical setup as motivated above.

Equation (1) also includes country-specific intercepts (α_i) and a set of unobserved common factors f_t with country-specific ‘factor loadings’ λ_i to account for the levels and evolution of unobserved Total Factor Productivity (TFP), respectively.⁸ The flexibility of this setup and how it encompasses existing approaches to modelling TFP is laid out in detail in the following paragraph. Allowing the common factors to be nonstationary has important implications for empirical analysis, since all observable and unobservable processes in the model are now integrated and standard inference is invalid (Kao, 1999). These common factors not only drive output, but also the capital and debt stocks, in line with the standard assumption of endogenous inputs to production.⁹ The parameters β_i^K and β_i^D on these endogenous variables are therefore not identified unless we find (i) some way to account for the unobservable factors in the error term u , or (ii) a valid and informative set of instruments. We return to the identification strategy in our discussion of the empirical implementation below. Suffice to highlight that standard instrumentation in a *pooled* empirical framework (e.g. Arellano & Bond, 1991; Blundell & Bond, 1998) is not appropriate in the present setup since we cannot obtain instruments which are both valid and informative due to the omnipresence of unobserved factors, and the underlying equilibrium relationship differing across countries.

The common factor framework encompasses a number of specifications in the existing cross-country growth literature. If, for instance, we believe that knowledge is a free public good and accordingly assume that TFP evolves in an identical fashion across all countries, but from

⁶Using per capita variables imposes constant returns to scale on the production process. Our specification of endogenous TFP in the form of common factors however allows for externalities such as knowledge spillovers at the local and global level (see Eberhardt, Helmers & Strauss, 2013). Note that our regressions use per capita total debt *stock* while in some of the graphs we employ the debt-to-GDP ratio for comparison with the existing literature.

⁷In line with the literature we assume these parameter coefficients are fixed (Pesaran & Smith, 1995, footnote 2). This means their magnitudes matter and do not just differ randomly across countries.

⁸These common factors can be a combination of ‘strong’ factors, representing global shocks such as the recent financial crisis, the 1970s oil crises or the emergence of China as a major economic power; and ‘weak’ factors, capturing local spillover effects along channels determined by shared culture heritage, geographic proximity, economic or social interaction (Chudik, Pesaran & Tosetti, 2011). They should not be regarded as merely omitted variables, but a set of latent drivers of the macro economy. We should certainly view them as artificial constructs, in the same spirit as when they are employed to capture the evolution of hundreds of macro variables in macro forecasting models (e.g. Stock & Watson, 2002).

⁹A formal motivation for this setup from economic theory can be found in Mundlak, Butzer & Larson (2012) and Eberhardt & Teal (2013b). Note that covariates are not assumed to be only driven by common factors also contained in the estimation equation (f_t), but can have additional factors exclusive to their evolution.

differential starting points (TFP levels), then we could specify $u_{it} = \alpha_i + \lambda_t f_t + \varepsilon_{it}^a$, where f_t now represents a set of time fixed effects and λ_t their common parameter (identical across countries). This type of specification was adopted in the seminal studies by Islam (1995), Caselli, Esquivel & Lefort (1996) and Bond, Hoeffler & Temple (2001). An alternative would be to allow TFP growth rates to differ across countries, but to assume constant rates over time, which would be specified as $u_{it} = \alpha_i + \lambda_i t + \varepsilon_{it}^b$, as in the empirical model of Pedroni (2007). There could be variations on these two specifications whereby elements of TFP evolution are common, while each country can deviate from world TFP evolution, but once again in a fashion which assumes constant (relative) TFP growth: $u_{it} = \alpha_i + \lambda_1 f_t + \lambda_{2,i} t + \varepsilon_{it}^c$, where f_t once again represents a set of year fixed effects. The empirical setup we specify in this study, which has previously been adopted by Eberhardt, Helmers & Strauss (2013) and Eberhardt & Teal (2013a) amongst others, allows for a more flexible evolution for TFP over time: $u_{it} = \alpha_i + \lambda_{it} + \varepsilon_{it}^d$, whereby TFP evolution differs in each country and over time but is not constrained to be linear or in linear deviations from world TFP. We can indicate the underlying assumptions we are making in adopting the factor model framework to capture λ_{it} using the following equation:

$$u_{it} = \alpha_i + \sum_{s=1}^M \lambda_{s,i}^s f_{s,t}^s + \sum_{k=1}^{\infty} \lambda_{w,i}^k f_{w,t}^k + \varepsilon_{it}^d \quad (2)$$

Our empirical implementation described below can capture this heterogeneity, provided there are only a limited number (M) of ‘strong’ factors $f_{s,t}$ (see footnote 8) which affect all countries in the world. There can however be an infinity of ‘weak’ factors, $f_{w,t}$, which only affect small subsets of countries (Chudik, Pesaran & Tosetti, 2011).¹⁰

Given the importance of time series properties and dynamics in macro panel analysis, we employ an error correction model (ECM) representation of the above equation of interest. This offers three advantages over static models and restricted dynamic specifications: (i) we can readily distinguish short-run from long-run behaviour;¹¹ (ii) we can investigate the error correction term and deduce the speed of adjustment for the economy to the long-run equilibrium; and (iii) we can test for cointegration in the ECM by closer investigation of the statistical significance of the error correction term. The ECM representation is as follows:

$$\Delta y_{it} = \alpha_i + \rho_i (y_{i,t-1} - \beta_i^K \text{cap}_{i,t-1} - \beta_i^D \text{debt}_{i,t-1} - \lambda_i' f_{t-1}) \quad (3)$$

¹⁰Finally, we have marked the error component for our various specifications ε_{it}^a to ε_{it}^d : this highlights that we do not expect these to be identical white noise processes. As an illustration, if TFP differed across countries but followed a more flexible factor structure, then the simple specification in Islam (1995) would be unable to capture this flexibility and as a result the residuals ε_{it}^a from this model would be correlated across countries (cross-sectionally dependent). In our empirical analysis below we put a lot of emphasis on testing the properties of the regression residuals in informing our choice of preferred empirical model.

¹¹Note that our use of the term ‘long-run’ is in line with an econometric rather than a macroeconomic definition: the former, including examples in the analysis of firm-level production in panels of 4 or 5 years (e.g. Blundell & Bond, 2000), specifies a dynamic model in order to allow for the notion that productivity evolution is persistent. Based on this dynamic model the short-run estimates can then be distinguished from the long-run implications. The macroeconomic literature attempts the same by adopting error correction models like that employed in our estimations. In both these econometric literatures, it could be argued that the ‘long-run’ refers to the range of years in the sample, rather than some macroeconomic principle which may extend beyond one or two generations.

$$\begin{aligned}
& +\gamma_i^K \Delta \text{cap}_{it} + \gamma_i^D \Delta \text{debt}_{it} + \gamma_i^{F'} \Delta \mathbf{f}_t + \varepsilon_{it} \\
\Leftrightarrow \Delta y_{it} = & \pi_{0i} + \pi_i^{EC} y_{i,t-1} + \pi_i^K \text{cap}_{i,t-1} + \pi_i^D \text{debt}_{i,t-1} + \pi_i^{F'} \mathbf{f}_{t-1} \\
& + \pi_i^k \Delta \text{cap}_{it} + \pi_i^d \Delta \text{debt}_{it} + \pi_i^{f'} \Delta \mathbf{f}_t + \varepsilon_{it}
\end{aligned} \tag{4}$$

where the β_i^j in equation (3) represent the long-run equilibrium relationship between GDP (y) and the measures for capital and debt in our model, while the γ_i^j represent the short-run relations. ρ_i indicates the speed of convergence of the economy to its long-run equilibrium. Taken together the terms in round brackets represent the candidate cointegrating relationship we seek to identify in our panel time series approach. We included the common factors f in our long-run equation, which implies that we seek to investigate an equilibrium relationship between output, capital, debt and TFP

In equation (4) we have relaxed the restrictions between the parameters ρ_i and β_i implicit in equation (3) and reparameterized the model. From the coefficients on the ‘levels’ terms (π_i^j for $j = K, D$) we can now back out the long-run parameters, $\beta_i^K = -\pi_i^K / \pi_i^{EC}$ and $\beta_i^D = -\pi_i^D / \pi_i^{EC}$, whereas from the coefficient on the terms in first difference (π_i^m for $m = k, d$, lowercase to distinguish from the long-run coefficients) we can read off the short-run parameters directly. π_i^{EC} relates to the speed at which the economy returns to the long-run equilibrium,¹² while inference on this π_i^{EC} parameter will provide insights into the presence of a long-run equilibrium relationship.¹³

Following Pesaran (2006) and Banerjee & Carrion-i-Silvestre (2011) we employ cross-section averages of all variables in the model to capture unobservables and omitted elements of the cointegration relationship.¹⁴ Recent work by Chudik & Pesaran (2014) has highlighted that in a dynamic panel this approach is subject to small sample bias, in particular for moderate time series dimensions. Furthermore, these authors relax the assumption of strict exogeneity for the observables and thus allow for feedback between (in our application) debt, capital stock and output, which provides a more serious challenge to consistency for the original Pesaran (2006) approach. As a remedy these authors suggest to include further lags of the cross-section averages *in addition* to the cross-section averages of all model variables. Our estimation equation is thus

$$\begin{aligned}
\Delta y_{it} = & \pi_{0i} + \pi_i^{EC} y_{i,t-1} + \pi_i^K \text{cap}_{i,t-1} + \pi_i^D \text{debt}_{i,t-1} + \pi_i^k \Delta \text{cap}_{it} + \pi_i^d \Delta \text{debt}_{it} \\
& + \pi_{1i}^{CA} \overline{\Delta y}_t + \pi_{2i}^{CA} \overline{y}_{t-1} + \pi_{3i}^{CA} \overline{\text{cap}}_{t-1} + \pi_{4i}^{CA} \overline{\text{debt}}_{t-1} + \pi_{5i}^{CA} \overline{\Delta \text{cap}}_t + \pi_{6i}^{CA} \overline{\Delta \text{debt}}_t
\end{aligned} \tag{5}$$

¹²The half-life (in our data: in years) is computable as $(\log(0.5)/\log(1 + \pi_i^{EC}))$.

¹³If $\pi_i^{EC} = \rho_i = 0$ we have no cointegration and the model reduces to a regression with variables in first differences (i.e. the levels terms in brackets in equation (3) drop out). If $\pi_i^{EC} = \rho_i \neq 0$ we observe ‘error correction’, i.e. following a shock the economy returns to the long-run equilibrium path, and thus there exists cointegration between the variables and processes in round brackets/levels.

¹⁴The simple algebraic mechanics of accounting for the unobservable factors f with cross-section averages of all variables is provided in Eberhardt & Teal (2013b), the asymptotic theory in Pesaran (2006) and Kapetanios, Pesaran & Yamagata (2011).

$$+ \sum_{\ell=2}^p \pi_{7i\ell}^{CA} \overline{\Delta y}_{t-\ell} + \sum_{\ell=1}^p \pi_{8i\ell}^{CA} \overline{\Delta cap}_{t-\ell} + \sum_{\ell=1}^p \pi_{9i\ell}^{CA} \overline{\Delta debt}_{t-\ell} + \varepsilon_{it}$$

The first line of equation (5) represents the specification for a standard Mean Group (MG) estimator (Pesaran & Smith, 1995), the addition of the second line yields the standard Common Correlated Effects (CCE) Mean Group estimator (Pesaran, 2006), while all three lines taken together represent the Chudik & Pesaran (2014) dynamic CCE Mean Group estimator. These authors show that once augmented with a sufficient number of lagged cross-section averages ($p = T^{1/3}$ is suggested as a rule of thumb) the CCE Mean Group estimator performs well even in a dynamic model with weakly exogenous regressors. We test for weak exogeneity in the various empirical models presented (see Technical Appendix) and conclude that evidence for a causal relationship from debt to growth is strongest in the heterogeneous parameter CCE models and weakest in the pooled specifications.

An important characteristic of the implementation adopted is that all models are estimated by OLS: features such as nonstationarity, cross-section correlation, heterogeneity in the equilibrium relationship across countries and non-linearity/asymmetry in the long- and/or short-run relationship are captured by the empirical specification and the use of additional terms in the regression equation.

4.2 Empirical Specification: Asymmetric Dynamic Model

We follow the discussion in Shin, Yu & Greenwood-Nimmo (2013) and define the asymmetric long-run regression model

$$y_{it} = \alpha_i + \beta_i^K cap_{it} + \beta_i^{D+} debt_{it}^+ + \beta_i^{D-} debt_{it}^- + \lambda_i' f_t + \varepsilon_{it} \quad (6)$$

where we again assume that observable and unobservable processes are nonstationary and where debt stock has been decomposed into $debt_{it} = debt_{i0} + debt_{it}^+ + debt_{it}^-$. The latter two terms are partial sums of values above and below a specific threshold, $debt_{i0}$ has been subsumed into the constant term. For instance, if we assume a threshold of zero then these debt terms define positive and negative changes in debt accumulation for each country i

$$\begin{aligned} debt_{it}^+ &= \sum_{j=1}^t \Delta debt_{ij}^+ = \sum_{j=1}^t \max(\Delta debt_{ij}, 0) \\ debt_{it}^- &= \sum_{j=1}^t \Delta debt_{ij}^- = \sum_{j=1}^t \min(\Delta debt_{ij}, 0) \end{aligned} \quad (7)$$

This setup would suit the analysis of an asymmetric response to debt accumulation and relief, whereby the hypothesised substantial growth benefits of debt relief could be empirically investigated for a differential relationship between debt accumulation and growth on the one hand and debt reduction and growth on the other. In the present study we instead create partial

sums for debt stock below and above a number of (exogenously determined) debt-to-GDP ratio thresholds, namely 60% (sample mean) and the ‘canonical’ 90%. Thus the partial sums are constructed from the per capita *debt stock variable*, while the assignment to one or the other regime is determined by the *debt-to-GDP ratio* — we follow this practice in order to be able to compare our results with those in the literature adopting the debt-to-GDP ratio as the primary variable of interest.

The ECM version of our asymmetric dynamic model is then

$$\begin{aligned} \Delta y_{it} = & \pi_{0i} + \pi_i^{EC} y_{i,t-1} + \pi_i^K \text{cap}_{i,t-1} + \pi_i^{D+} \text{debt}_{i,t-1}^+ + \pi_i^{D-} \text{debt}_{i,t-1}^- \\ & + \pi_i^{F'} f_{t-1} + \pi_i^k \Delta \text{cap}_{it} + \pi_i^{d+} \Delta \text{debt}_{it}^+ + \pi_i^{d-} \Delta \text{debt}_{it}^- + \pi_i^f \Delta f_t + \varepsilon_{it} \end{aligned} \quad (8)$$

The dynamic asymmetry can be included in the long-run relationship (lagged levels terms), in the short-run behaviour (first difference terms) or both. As before we allow for cross-country heterogeneity in *all* long-run and short-run parameters and account for the presence of unobserved time-varying heterogeneity by augmenting the country regressions with cross-section averages of the dependent and independent variables. While in the original Shin, Yu & Greenwood-Nimmo (2013) time series approach the parameter estimates are identified by augmentation of the empirical equation with additional lagged differences, our panel approach relies on the common factor framework for identification. As motivated above we augment the estimation equation with additional lags of the cross-section averages (Chudik & Pesaran, 2014).

This implementation raises a number of problems in the case where the debt threshold is relatively high: if only a very small number of observations for a specific country are above the threshold, then the estimated coefficient may be very imprecise. In order to guard against this we present results of the estimated long-run debt parameters in the low and high debt regimes only for those countries where at least 25% of all time series observations are in one regime. This amounts to a total of 27 countries for the 90% debt/GDP threshold, and 54 countries in case of the 60% threshold.

4.3 Empirical Specification: Static Non-Linear Model

As a robustness exercise to the threshold approach in the previous section, we estimate different static models with a polynomial in debt: our analysis is limited to the linear and squared debt stock terms most popular in the empirical literature.¹⁵ We focus exclusively on the following *static* model, given that reconciling non-linearities with cross-section dependence, parameter heterogeneity and a dynamic specification within a panel of moderate time series dimension

¹⁵We also added the cubed debt term, to allow for more complex non-linearities (as in Ghosh et al., 2013), though results (available on request) are difficult to interpret given the level of flexibility we allow across countries.

represents a complexity beyond the scope of this study:

$$y_{it} = \alpha_i + \beta_i^K \text{cap}_{it} + \beta_i^{DL} \text{debt}_{it} + \beta_i^{DS} \text{debt}_{it}^2 + \lambda_i' f_t + \varepsilon_{it} \quad (9)$$

One main concern for our analysis here is the most appropriate specification with regards to the time-series properties of the data: reliable inference on a relationship between variable series which are nonstationary involves establishing that these variables are cointegrated. Crucially, cointegration defines a *linear* combination of variables integrated of order one (in our case) which is stationary (i.e. integrated of order zero). However, when modelling potentially non-linear relationships, such as that between debt and growth, the order of integration of the square (or cube) of an integrated variable is not defined within the linear integration and cointegration framework.¹⁶

We apply novel methods on the order of summability, model balance and the concept of co-summability from the time series econometric literature (Berenguer-Rico & Gonzalo, 2013a,b) to provide pre-estimation testing as to the validity of our empirical equation incorporating country-specific non-linearities.¹⁷ The summability test can be seen in analogy to a test for unit root behaviour, while the balance test investigates whether the left- and right-hand side of the empirical equation are of the same order of summability — like in a linear model all variables are required to be of *the same order* of integration/summability. Co-summability testing can then be viewed in analogy to cointegration testing in the linear case. Further details on these tests of long-run co-movement as well as the results are confined to a Technical Appendix.

4.4 Data

Our main variables are GDP, population, capital stock (constructed from gross fixed capital formation using the standard perpetual inventory method and assuming a common and constant 5% depreciation rate) and total public debt stock (all in logarithms of real US\$ values). Data are taken from the World Bank World Development Indicators (WDI) database with the exception of the debt data, which are taken from the IMF historical public debt database (Abbas et al., 2011, Fall 2013 vintage). Debt refers to gross general government debt; in many cases, especially for the period before 1980, only central government data was available and this is what is reported in the database. The final sample contains 4,588 observations (dynamic specification) from 118 countries (22 Low-Income, 27 Lower Middle-Income, 33 Upper Middle-Income and 36 High Income countries based on current World Bank classification), thus on average 38.9 years per country (range of 21 to 52 country observations) from 1961 to 2012.

¹⁶Integration and cointegration are linear concepts: if x is nonstationary and integrated of order 1, $I(1)$, then the order of integration of x^2 is not defined.

¹⁷To the best of our knowledge our study is the first to adopt these methods in the panel context, further addressing the concerns over cross-section dependence.

Our empirical analysis will focus on total public debt, given that the literature has generally identified an association between episodes of public debt overhangs and lower growth (Reinhart, Reinhart & Rogoff, 2012). A potential limitation of this choice – driven by data availability – is the exclusion of private debt, which has been shown to play a significant role for financial instability and crises (Reinhart & Rogoff, 2009; Gourinchas & Obstfeld, 2012; Schularick & Taylor, 2012). Limited data availability prevents us from undertaking a comprehensive analysis of the relationship between debt and growth using more granular definitions of debt, separating between gross and net debt¹⁸ and between foreign- and domestic-currency denominated total debt. The issue of foreign currency denomination is likely to be particularly relevant in developing countries: the presence of foreign currency debt increases financial fragility and leads to sub-optimal macroeconomic policies, as pointed out by the literature on the ‘original sin’ (Hausmann & Panizza, 2011; Dell’Erba, Hausmann & Panizza, 2013). By contrast, the issue of foreign currency denomination issue is likely to have a less relevance in advanced economies: Panizza & Presbitero (2014) show that in a sample of 17 OECD countries between 1980 and 2010 the share of foreign currency denominated debt is merely 9%. Given the lack of comprehensive data on currency denomination of *total* public debt, we resort to a robustness check which considers *external* public debt in developing countries, where foreign currency-denominated debt is a more relevant issue.¹⁹ Our analysis here further takes into consideration that “the comparison of debt stocks *at face value* over time and across countries can generate misleading inferences as a result of significant differences in the contractual structure of debt portfolios over time and across countries” (Dias, Richmond & Wright, 2014, p. 1), given that the contractual face values of debt could over-estimate the indebtedness of low-income countries compared to middle-income countries. Thus, in analysis presented in a Technical Appendix we adopt a present value measure of public external debt, as calculated by Dias, Richmond & Wright (2014) for a sample of 89 developing countries. Results for the dynamic linear models support the robustness of our findings.²⁰

5 Empirical Results

We carried out panel unit root tests following Pesaran (2007) and investigated the cross-section correlation properties of the raw data including formal CD tests following Pesaran (2004). Results are provided in a Technical Appendix and indicate that the levels variable series are

¹⁸The distinction between net and gross debt is available for a sample of OECD economies since 1996 (OECD Economic Outlook). In that sample, the correlation between the two series was above 0.8 in 2011-2012. In addition, the difference between gross and net debt in OECD economies (averaged between 2002-2009, to maximize sample size) is about 33% of GDP, with just a few countries for which this difference is significantly larger (Finland, Iceland, Japan, Korea, Luxembourg, Norway and Sweden). For details see Panizza & Presbitero (2013, pp. 194-5). As a robustness check, we have run our baseline empirical model excluding these countries and results are robust (available on request).

¹⁹According to World Bank data for a sample of low- and middle-income countries since 1980, on average half of all public and publicly guaranteed (PPG) external debt is US\$-denominated.

²⁰An additional analysis for 96 countries using an alternative measure of the present value of public external debt compiled by the World Bank also yields qualitatively identical results (available upon request).

integrated of order 1 and subject to considerable cross-section dependence.

5.1 Linear Dynamic Models

Table 1 presents results derived from an ECM specification, with estimates for a standard two-way fixed effects and pooled CCE (CCEP) in columns [1] and [2] imposing parameter homogeneity across countries and all other models in columns [3]-[6] allowing for differential relationships. The model in column [3] is the standard Mean Group estimator which ignores any unobserved common factors, while that in column [4] represents the standard CCE estimator in the Mean Group version. The remainder specifications in columns [5] and [6] add further lags of the cross-section averages as suggested in Chudik & Pesaran (2014).²¹ These *average* results in [3] to [6] are of interest due to their comparability with those arising from pooled empirical models; further we use this table to report on the diagnostic tests we have carried out. Subsequently we move on to analyse patterns in long-run debt coefficients across countries.

[Table 1 about here]

In each model we focus on the long-run estimates as well as the coefficient on the lagged level of GDP to investigate error correction and thus evidence for a long-run relationship — full ECM results are available on request. In the heterogeneous models we present results for two concepts of average long-run estimates, since the panel aspect enables the alternatives to (i) compute the long-run coefficient in each country first (ALR), which is then averaged, and (ii) to average ECM coefficients first and then compute the long-run (LRA).²² For all heterogeneous models which address concerns over cross-section dependence there is strong evidence of error correction²³ – the lagged GDP per capita levels variable is highly statistically significant – and the long-run coefficients on debt appear statistically significant and positive throughout, whereas short-run coefficients are insignificant. The latter does not imply the absence of any significant effects, but rather highlights the heterogeneity across countries with dynamics on average cancelling out. The MG estimator in contrast yields statistically insignificant debt coefficients, similarly to the pooled fixed effects (2FE) estimator.

Diagnostic tests highlight that the use of cross-section averages considerably reduces residual cross-section dependence — the CD statistic drops from 23 in the MG to between -0.5 and 0.5 in the CMG models. The null of cross-sectionally independent residuals cannot be rejected in the three CMG models; recall that the presence of cross-section dependence indicates that we

²¹We use their recommended rule of thumb ($p = T^{1/3} \approx 3$), with relevance for an ARDL. In our ECM this equates to adding up to 2 lagged differences.

²²We follow standard practice in this literature and employ robust regression (see Hamilton, 1992) to weigh down outliers in the computation of the averages in models [3]-[6]. LRA standard errors are computed via the Delta method. In the ALR case standard errors are constructed following Pesaran & Smith (1995). For more details on these concepts see Smith (2001).

²³The reported half-life indicates “the length of time after a shock before the deviation in output shrinks to half of its impact value” (Chari, Kehoe & McGrattan, 2000, p. 1161).

have misspecified the TFP process which may indicate our estimates are biased. This seems to be the case in the 2FE, CCEP and MG models.

Once we move from a pooled to a heterogeneous parameter specification, statistically significant positive *average* long-run coefficients as we find in our sample only provide insights regarding the *central tendency* of the panel. This result is consistent with the positive correlation between debt and growth found by Dreger & Reimers (2013) and Baum, Checherita-Westphal & Rother (2013) when debt is sufficiently low, and it indicates that, on average, the countries in our sample are on the ‘right’ (actually, left) side of an hypothetical Debt-Laffer curve.²⁴ In Figure 3 we provide a number of plots indicating the cross-section *dispersion* of the long-run debt coefficients, primarily focusing on the estimates in the dynamic CCE model with two additional lags (column [6] of Table 1). With the exception of panel (b) all plots capture the country-specific average debt-to-GDP ratio over the entire sample period (in logs) on the *x*-axis and estimated debt-coefficients on the *y*-axis. Panel (a) suggests that there is a mildly non-linear relationship between the debt-to-GDP ratio and the long-run impact of debt, which has a turning point around 90% debt-to-GDP. Panel (c) makes the same point grouping countries into quintiles based on average debt/GDP ratio and providing distributional plots for each of them (group #5 represents debt burden over 90% of GDP).

Panel (b) however cautions against this conclusion: instead of average debt-to-GDP ratio we plot the debt-to-GDP ratio *peak* for each country. It is notable that many countries still have positive coefficients despite peak debt-to-GDP ratios in excess of 90%. Panel (d) splits the data into the 25% richest countries and the rest — the non-linearity between debt burden and the long-run debt coefficient across countries seems to be driven primarily by the richer countries in the sample,²⁵ for which the relationship turns negative at around 90% debt-to-GDP, consistent with the evidence on advanced economies discussed by Reinhart & Rogoff (2010b) and Checherita-Westphal & Rother (2012), among others. Panels (e) and (f) provide fitted fractional polynomial regression lines for all the CMG models in Table 1. With regard to long-run results for average debt in panel (e) or peak debt in panel (f) the graphs consistently suggest an inverted-U shaped relationship.

[Figure 3 about here]

We thus find some tentative evidence for a non-linearity in the long-run relationship between debt and growth *across countries*. We can be reasonably certain that these empirical models represent cointegrating relationships between debt, income and TFP, but this does not rule out the possibility of feedback from income to debt, which would question the validity of our empirical results. Our weak exogeneity tests, reported in the Technical Appendix, suggest that our augmented production function model is valid, and rejects the notion that this represents a misspecified investment demand or fiscal policy equation.

²⁴This is hardly surprising as the mean debt-to-GDP ratio is around 60% (Table TA10), a value below the ‘tipping points’ typically reported by the literature on developing and advanced economies (see Table TA1).

²⁵This subsample constitutes those OECD member states which joined the organisation in the 1960s, plus The Bahamas, Bahrain, Barbados, Israel, New Zealand (latter two joined OCED after 1969), Saudi-Arabia and Singapore but excluding Turkey.

The purpose of the analysis up to this point was to investigate the possibility of a non-linear relationship between the debt burden and the long-run debt coefficient in the cross-country dimension. A number of empirical models were evaluated and we can conclude that on balance there is evidence for heterogeneity in the long-run coefficients across countries. We now turn to empirical models which allow for heterogeneous long-run relations across countries while at the same time allowing for thresholds in the relationship within countries, which represents a departure from the apparent consensus of a common threshold in large parts of the existing empirical literature (see our review in the Technical Appendix).

5.2 Asymmetric Dynamic Models

In Figure 4 we present results from the asymmetric (heterogeneous) dynamic regression models where we account for unobserved common factors by inclusion of cross-section averages of all covariates (in the left column of plots) as well as two further lags of the cross-section averages (in the right column).²⁶ For each specification the two plots correspond to subsamples for an adopted threshold of 90% (top) and 60% (bottom) for the debt-to-GDP ratio – in each case we only include countries which have at least 25% of their observations in one of the two regimes (below/above threshold).

[Figure 4 about here]

The x-axis in each plot represents the average debt-to-GDP ratio (in logs) over the entire time horizon — the left tip of each arrow represents the average value for the ‘low debt’ regime where debt is below 60% or 90% of GDP, while the right arrow tip marks the average value for the ‘high debt’ regime above the threshold. The y-axis in each plot captures the estimated long-run debt coefficient which by construction is allowed to differ across regimes (and countries). Under the working hypothesis that a shift to the ‘high debt’ regime would have an additional negative impact on long-run growth, we would expect most arrows to run from NW to SE, i.e. to indicate a negative relationship. As can be seen, this hypothesis is not borne out by the empirical results: there is no evidence for any *systematic* change in the relationship between debt and growth when countries shift from a ‘low’ to ‘high’ debt regime, with only around half of all countries experiencing a drop in the debt coefficient.²⁷ Average coefficient *changes* in each of the threshold cases are statistically insignificant (based on standard or robust means).

Thus our test of within-country threshold effects in the debt-growth relationship suggests that the consensus in much of the empirical literature of a *common* debt threshold does not hold up for the cutoffs tested if we allow for observed and unobserved heterogeneity across countries. In the following section we investigate a popular alternative empirical representation for debt

²⁶Empirical results on which these graphs are based can be found in a Technical Appendix (Table TA6, models [2] and [6] with no or two additional lags, respectively. In both cases we allow for asymmetry in the long- and short-run specification).

²⁷This simple count does not take statistical significance into account.

thresholds adopting polynomial specifications.

5.3 Non-Linear Static Model

As a robustness check on the results for the asymmetric dynamic model we present estimates from a static non-linear model, where the non-linearity is specified by simple inclusion of a squared debt stock term as is common practice in many existing empirical papers in this literature. Estimates from the non-linear models are presented in Table 2 – we indicate that the balance and co-summability analysis (see Technical Appendix for discussion and detailed results) suggests that only the CMG specification augmented with 2 additional lags of the cross-section averages offers strong evidence for a long-run equilibrium relationship. We present results for all 118 countries in Panel A, while in Panel B we adopt the same subsamples as in the previous section, focusing on those countries with at least 25% of their observations in either regime beyond a certain (60%, 90% debt-to-GDP) threshold.

Consistent with our previous results, our static non-linear model estimates highlight the heterogeneity in the country-specific results and do not support the presence of a common debt threshold. For instance, in the models with linear and squared debt there is more evidence for concave relations – in line with the debt threshold story – but it would be difficult to claim that that this result is *uniform across all countries*, as a non-negligible number of countries show a U-shaped debt-growth relationship.²⁸

On the whole, the investigation of heterogeneous nonlinear models confirm our previous findings: once we relax the assumption of common parameters *across countries* results do not lend support to the notion that countries possess identical or even similar non-linearities in the debt-growth relationship *over time*.

6 Concluding Remarks

This article investigates the relationship between public debt and long-run growth and provides important insights for the current debate on threshold effects in the debt-growth nexus sparked by the work of Reinhart and Rogoff (2009; 2010a; 2010b; 2011).

Our paper makes three contributions to this empirical literature: first, we investigated the long-run relationship by means of a dynamic empirical model and adopted time series arguments to establish the presence of a long-run equilibrium, taking into account possible endogeneity issues. Since estimation results are likely to be spurious and seriously biased if these well-known data properties are not recognised and addressed in the empirical analysis our approach signals a significant departure from the standard empirical modelling in this literature.

²⁸We do not compute sample average ‘turning point’ estimates since our models are based on debt *stocks* rather than debt-to-GDP *ratios*.

Second, we adopted empirical specifications which allow for heterogeneity in the long-run relationship across countries, thus reflecting a host of theoretical and empirical arguments. This heterogeneity in the specification extends to the unobservable determinants of growth and public debt, which we addressed by means of a flexible common factor model framework. Ours is the first panel study on debt and growth to address parameter heterogeneity and cross-section dependence, allowing for a closer match between economic theory and data restrictions on the one hand and empirical modelling on the other.

Third, we used a number of empirical estimators and testing procedures to shed light on the potential non-linearity in the debt-growth relationship, focusing on both the possibility of a debt-growth non-linearity *across* and *within* countries. It bears emphasising that no empirical study modelling the debt-growth relationship in a pooled panel model can claim to be able to distinguish these two types of non-linearity.

Our empirical analysis provided some evidence for systematic differences in the debt-growth relationship *across countries*, but no evidence for systematic *within-country* non-linearities in the debt-growth relationship for all countries in our sample. With regard to the first result we observed that long-run debt coefficients appeared to be lower in countries with higher average public debt burdens. Regarding the second result, in piecewise linear specifications with various pre-specified thresholds we found that the change in the debt coefficient at the threshold was just as likely to be positive as negative. Alternative specifications using polynomials of the debt stock term came to the same conclusion. These findings imply that whatever the shape and form of the debt-growth relationship, it differs across countries, so that appropriate policies for *one* country may be seriously misguided in *another*.

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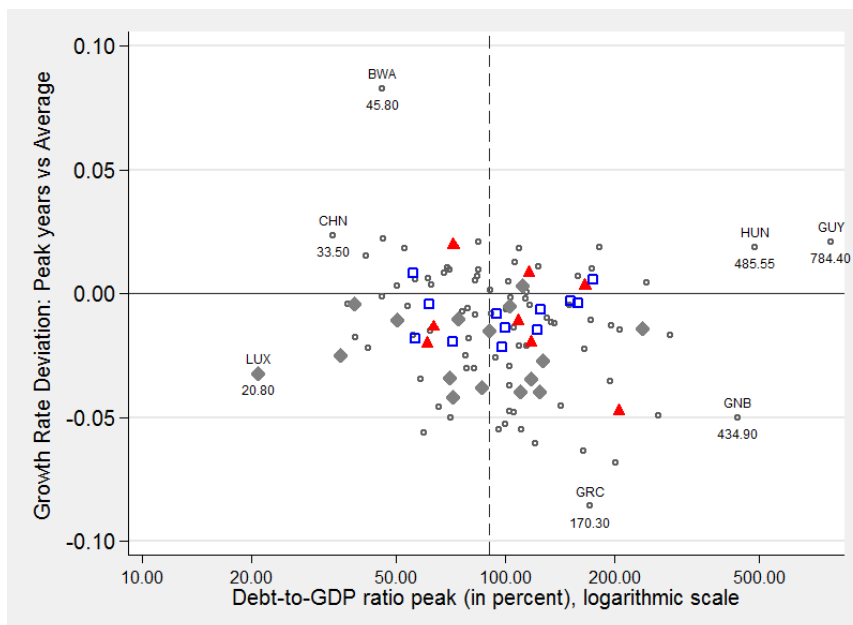
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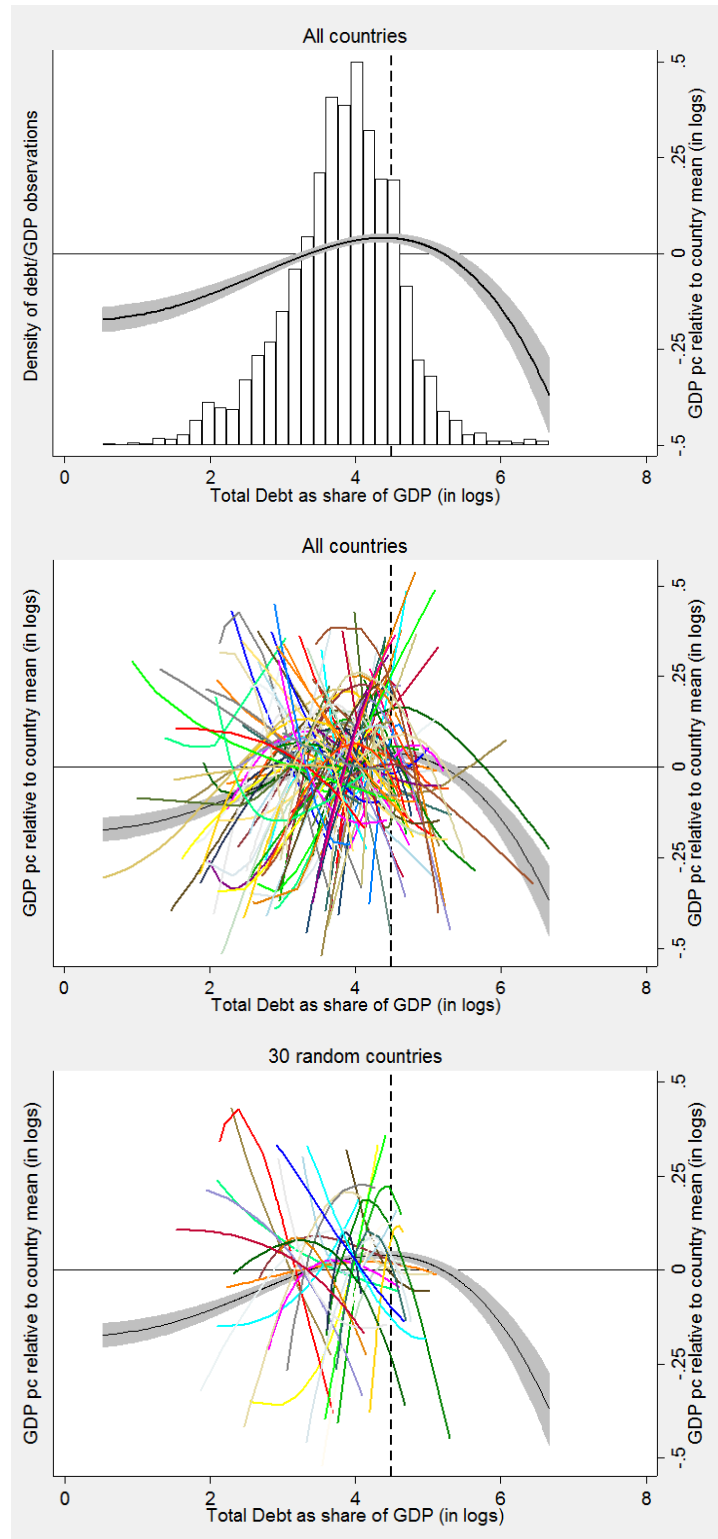
FIGURES AND TABLES

Figure 1: Peak Debt/GDP Ratio and Relative Growth



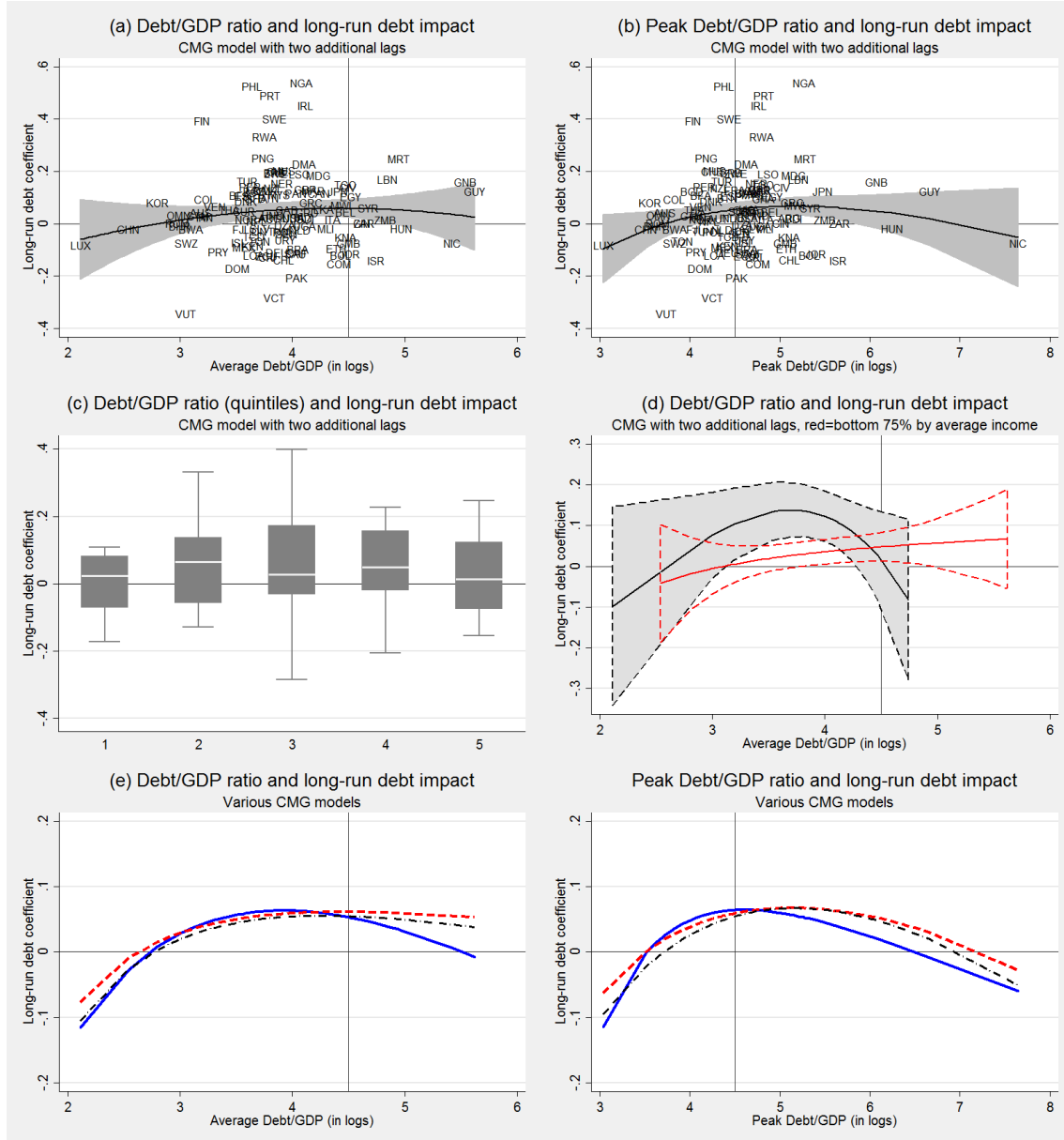
Notes: Along the x-axis we arrange countries by the value of the maximum debt-to-GDP ratio (in logarithms), highlighting a number of outliers as well as three years in particular: 1985 (Triangles), 1994 (Squares), and 2012 (Diamonds). Along the y-axis we plot the deviation of countries' (i) average per capita growth rate in the five years around their peak debt year (i.e. peak debt occurs in year 3) from (ii) their average per capita growth rate over the entire time horizon 1960-2012 excluding the five 'peak debt years.' These averages are adjusted to the peak year and two previous years if the peak year occurred in 2012. A simple (outlier-robust) linear regression of average per capita growth rates on debt-to-GDP peaks (in logarithms) yields (absolute t -ratios in brackets): $.019 [1.16] - .007 [2.00] \log (\text{debt/GDP})_i^{\max}$

Figure 2: Non-Linearities in the Country-Specific Debt-Income Nexus



Notes: We plot the unconditional relation between debt/GDP ratio and within-transformed per capita GDP (both in logs) employing fractional polynomial regression (solid regression line; shaded 95% confidence intervals) – see Footnote 4 for details on sample restriction. In the top panel we add a histogram for the debt/GDP ratio (in logs) to indicate that the bulk of observations (93%) are in the [2,5] log point range (\approx [7%,150%] debt/GDP). In the middle panel we instead add country-specific polynomial plots, in the bottom panel we do the same but chose a random subset of 30 countries.

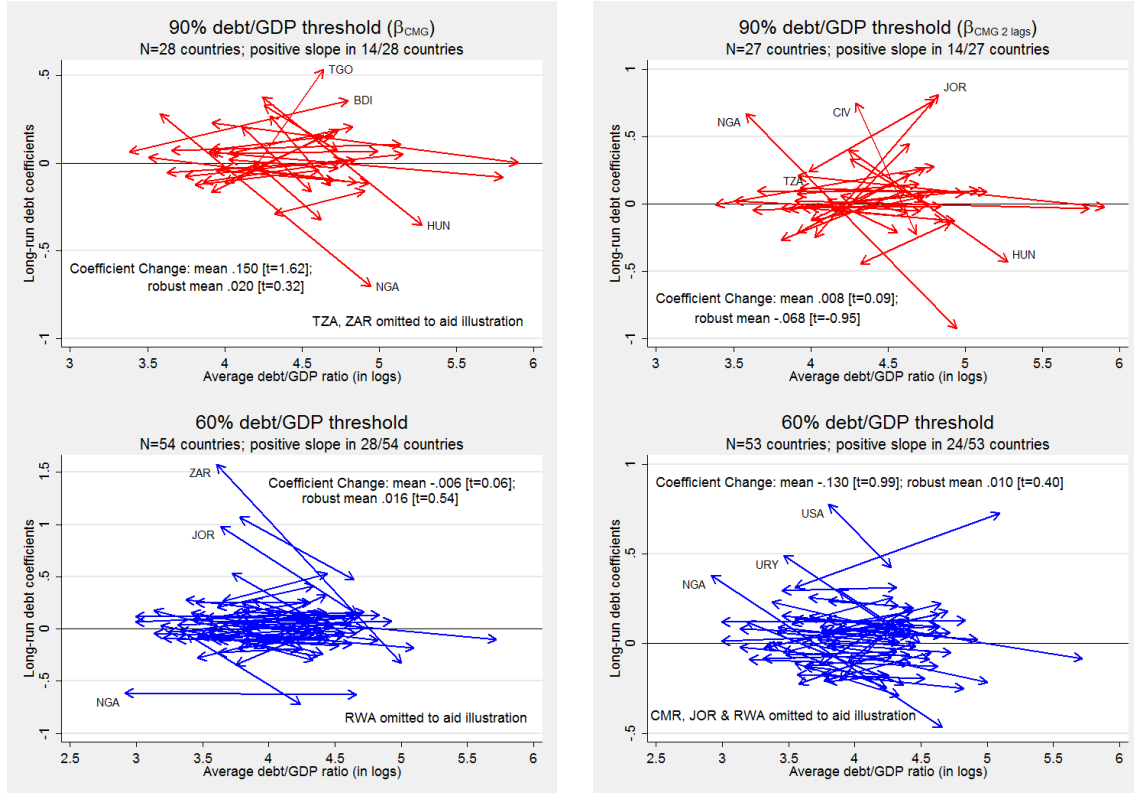
Figure 3: Patterns for CMG Debt Coefficients



Notes: We plot the country specific long-run coefficients for debt in each country, taken from the dynamic CMG model with one additional lag (in column [6] of Table 1) against (a) the country-specific average debt/GDP ratio (in logs), and (b) the country-specific peak value for debt/GDP (in logs) — for both plots we reduce the number of countries as detailed below to improve illustration. In both cases we added fitted fractional polynomial regression lines along with 5% and 95% confidence bands (shaded area). We further provide (c) box plots for all 118 country-estimates divided into quintiles of the average country debt/GDP ratio distribution — outliers are omitted from these box plots. In (d) we split the sample into the top 25% and bottom 75% by average income and fit fractional polynomial regression lines alongside 5% and 95% confidence bands for each grouping (reduced sample in the plot for illustration). The final set of plots in (e) and (f) presents fitted fractional polynomial regression lines of long-run debt coefficients against average debt/GDP ratio and peak debt/GDP for all CMG models (columns [4]-[6]), respectively.

In each case (as in the first two scatter plots) we omit those countries (based on the estimated long-run debt coefficient) which the robust regression method (Hamilton, 1992) indicates as outliers, resulting in 112 [4], 112 [5] and 113 [6] countries out of a possible 118. This practice excludes the following country estimates: BHS (not excluded in [4]), OMN (not in [6]), SGP, TTO, TZA, ZAF, ZAR (excluded in [4] only). In all plots we add a horizontal line to mark zero, in most plots we also add a vertical line at 4.5 log points ($\approx 90\%$) of the debt/GDP ratio.

Figure 4: Debt Coefficient Comparison: Debt-to-GDP Thresholds



Notes: We plot the long-run debt coefficients in the low and high debt regime for (top) 90% and (bottom) 60% debt/GDP thresholds. In each case the left plot uses the CMG results and the right plot the results for CMG with two additional lags of cross-section averages (models [2] and [5] in Table TA6) for 54 and 28 countries, respectively — countries are only included if they have at least 25% of their observations in one of the two regimes (below/above threshold). The values on the x-axis represent the average debt/GDP ratio (in logarithms) for the lower and higher regimes (average over all years in each regime). A positive (negative) slope indicates the debt coefficient increased (decreased), i.e. had a positive or less negative impact on growth, in the higher debt/GDP regime. As indicated small number of country estimates is omitted to aid illustration.

We carried out empirical tests for statistical significance of average coefficient changes at each threshold and report the mean and robust mean estimates together with respective t -ratios. Results for other threshold values are available on request. Parameter shifts at the two thresholds are almost exactly split into positive and negative ones, as is indicated in each plot.

Table 1: Linear Dynamic Models

	[1] 2FE	[2] CCEP	[3] MG [†]	[4] CMG	[5] CMG [‡] 2nd lag	[6] CMG [‡] 3rd lag
additional lagged CA						
<i>Debt coefficients</i>						
LRA	-0.031 [0.027]	0.050 [0.014]***	-0.010 [0.011]	0.041 [0.011]***	0.030 [0.012]***	0.027 [0.013]**
ALR			-0.016 [0.011]	0.044 [0.011]***	0.039 [0.011]***	0.035 [0.013]***
SR	-0.007 [0.006]	0.001 [0.005]	-0.012 [0.006]**	0.008 [0.007]	0.003 [0.007]	0.004 [0.006]
<i>Capital coefficients</i>						
LRA	0.549 [0.063]***	0.607 [0.034]***	0.314 [0.063]***	0.587 [0.070]***	0.570 [0.072]***	0.583 [0.071]***
ALR			0.330 [0.050]***	0.594 [0.051]***	0.590 [0.050]***	0.593 [0.048]***
SR	0.631 [0.069]***	0.894 [0.065]***	1.393 [0.089]***	1.489 [0.092]***	1.420 [0.093]***	1.417 [0.099]***
<i>EC coefficient</i>						
$y_{i,t-1}$	-0.069 [0.011]***	-0.254 [0.014]***	-0.412 [0.023]***	-0.482 [0.026]***	-0.507 [0.028]***	-0.539 [0.029]***
t -statistic ^b	-6.42	-18.29	-17.76	-18.53	-17.99	-18.88
\bar{t} -statistic			-3.21	-3.30	-3.19	-2.99
Implied half-life (years)	9.69	2.37	1.31	1.05	0.98	0.90
<i>Diagnostics[‡]</i>						
RMSE	0.039	0.037	0.030	0.026	0.023	0.022
CD test	-2.05	4.93	23.07	-0.52	0.31	0.59
Observations	4,588	4,588	4,588	4,588	4,562	4,536

Notes: Results for full sample of $N = 118$ countries, based on an error correction model with the first difference of log real GDP per capita as dependent variable. We report the robust mean of coefficients across countries in the heterogeneous parameter models in [3]-[6] (Hamilton, 1992); standard errors in these models are constructed non-parametrically following Pesaran & Smith (1995).

[†] This model is augmented with country-specific linear trend terms; we also augmented the various CMG models but this resulted in CD test statistics (see below) above 1.96, indicating empirical misspecification (result available on request).

[‡] The CMG estimator (Pesaran, 2006; Chudik & Pesaran, 2014) is implemented using further cross-section averages (CA) of additional lags as indicated – see main text for details.

‘LRA’ refers to the long-run average coefficient, which is calculated directly from the pooled model ECM results in [1] and [2] and the robust mean estimates of the heterogeneous model ECM results (standard errors computed via the Delta method) in [3]-[6].

‘ALR’ refers to the average long-run coefficient in the heterogeneous models, whereby the long-run coefficients are computed from the ECM results in each country first and then averaged across the panel. ‘SR’ refers to the short-run coefficients.

^b The first set of t -statistics are non-parametric statistics derived from the country-specific coefficients following Pesaran & Smith (1995). The second set represent averages across country-specific t -statistics.

*, ** and *** indicate significance at 10%, 5% and 1% level respectively.

[‡] RMSE is the root mean squared error, CD test reports the Pesaran (2004) test, which under the null of cross-section independence is distributed standard normal.

Table 2: Static Linear and Non-Linear Models

PANEL A: Full sample analysis				
	No CA augmentation		CA augmentation	
	[1]	[2]	[3]	[4]
Estimator	MG	MG	CMG	CMG
Lags of add. CA			2	2
Bal & Co-Sum			×	×
cap_{it}	0.566 [0.046]***	0.556 [0.044]***	0.786 [0.038]***	0.785 [0.038]***
$debt_{it}$	-0.057 [0.011]***	0.272 [0.117]**	-0.004 [0.010]	0.441 [0.093]***
$debt_{it}^2$		-0.027 [0.009]***		-0.031 [0.007]***
non-linearity †				
# of Countries				36 \cup , 82 \cap
Observations	4,676	4,676	4,612	4,612
Countries	118	118	118	118
Diagnostics ‡				
RMSE	0.062	0.055	0.042	0.035
$I(\cdot) \hat{e}_{it}$	I(0)/I(1)	I(0)	I(0)	I(0)
CD Test	26.97	22.58	7.22	5.92

PANEL B: Subsample analysis				
	60% threshold sample		90% threshold sample	
	[3a]	[4a]	[3b]	[4b]
Estimator	CMG	CMG	CMG	CMG
Lags of add. CA	2	2	2	2
Bal & Co-Sum	×	×	×	×
cap_{it}	0.778 [0.055]***	0.793 [0.054]***	0.698 [0.106]***	0.804 [0.089]***
$debt_{it}$	0.001 [0.016]	0.461 [0.124]***	0.021 [0.022]	0.555 [0.208]***
$debt_{it}^2$		-0.031 [0.010]***		-0.037 [0.016]**
non-linearity †				
# of Countries		13 \cup , 41 \cap		8 \cup , 20 \cap
Countries	54	54	28	28
Diagnostics ‡				
RMSE	0.046	0.038	0.045	0.037
$I(\cdot) \hat{e}_{it}$	I(0)	I(0)	I(0)	I(0)
CD Test	2.85	3.28	2.73	0.56

Notes: We report the estimates and diagnostic tests for static production functions with linear and polynomial debt terms. All estimates are robust means (see notes to Table 1). The MG models further include country-specific trend terms, we also omitted to report the averaged constant terms in all models (available on request). ‘Bal & Co-Sum’ indicates those specification which were found to be balanced and co-summable (Tables TA8 and TA9 in a Technical Appendix). In models (1)-(2) there is no augmentation with cross-section averages; in (3)-(4) we add the cross-section averages of all model variables in the standard Pesaran (2006) fashion and include two further lags of these — alternative specifications (one lag, no lag) yield similar results (available on request). † We report the number of countries with convex and concave debt-growth relationships using \cup and \cap , respectively. ‡ All residual series were found to be stationary.

DATA APPENDIX

A-I Data Construction

The principle data sources for our empirical analysis are the World Bank World Development Indicators (WDI) and an update to the dataset provided by Abbas et al. (2011, Fall 2013 vintage). From the former we take real GDP in year 2000 US\$ values, the per capita series of the same variable, population as well as gross fixed capital formation (investment) as a share of GDP. The latter data source provides total debt series, comprising the sum of domestic and external debt (not reported separately), in face value terms as a percentage of GDP, enabling us to construct the real debt stock series.

With the WDI investment series we can construct real capital stock by adopting the standard perpetual inventory method with an annual depreciation rate of 5%. If country investment series contained gaps of less than three years' length we used cubic spline interpolation to fill these gaps for a small number of countries. Note that this interpolation does *not* affect the overall sample size, since the observations in question are also missing for GDP and other variables; a fairly 'continuous' investment series does however aid the construction of the capital stock series.

In the process of constructing the capital stock series we investigated a number of basic magnitudes, including the investment-to-GDP ratio in 1960 (found to be between 10 and 50% – Equatorial Guinea was omitted for a values in excess of 98%) and the capital-output ratio in 1960 (found to be between 1.5 and 4.7 – Ukraine's K/Y ratio was 6.1 and the country thus omitted), which other than for those countries highlighted were all within reasonable bounds. We did however limit our analysis to countries with at least 21 years of data, which effectively excluded transition economies as well as a small number of African and Latin American countries. The final sample contains 4,588 observations (dynamic specification) from 118 countries (22 Low-Income, 27 Lower Middle-Income, 33 Upper Middle-Income and 36 High Income countries based on current World Bank income classification), thus on average 38.9 years per country (range of 21 to 52 country observations) from 1961 to 2012.

We present descriptive statistics for our sample in Table A1. Detailed information about the sample make-up is confined to a Technical Appendix.

A-II Descriptive Statistics

Table A1: Descriptive Statistics

PANEL A: RAW VARIABLES AND TRANSFORMATIONS						
variable	type	mean	median	sd	min	max
GDP	level	2.89E+11	1.86E+10	1.04E+12	1.56E+08	1.42E+13
GDP growth	%age growth rate	3.589	3.773	4.636	-69.812	35.354
GDP per capita	level	9,075	2,802	12,962	112	87,717
GDP pc growth	%age growth rate	1.831	2.105	4.610	-64.082	35.077
Population	level	3.98E+07	9.22E+06	1.32E+08	4.08E+04	1.35E+09
Population growth	%age growth rate	1.757	1.793	1.186	-7.597	11.181
Investment/GDP ratio	%age share of GDP	21.595	20.978	7.416	-0.906	74.821
Capital Stock	level	8.21E+11	4.16E+10	2.97E+12	5.01E+08	4.13E+13
Capital Stock growth	%age growth rate	3.815	3.469	2.953	-5.591	28.413
Capital Stock per capita	level	2.63E+04	7.15E+03	3.90E+04	1.81E+02	2.06E+05
Capital Stock pc growth	%age growth rate	2.057	1.944	2.935	-8.966	24.555
Debt (total)	level	1.81E+11	9.04E+09	8.32E+11	5.70E+06	1.46E+13
Debt growth	%age growth rate	5.273	4.418	18.275	-206.854	147.786
Debt (total) per capita	level	4.79E+03	1.17E+03	8.38E+03	7.49E+00	8.75E+04
Debt pc growth	%age growth rate	3.516	2.908	18.288	-209.393	145.533
Debt/GDP ratio	%age share of GDP	59.904	47.000	64.184	1.700	2,092.900
PANEL B: REGRESSION VARIABLES (IN LOGS OR FIRST DIFFERENCES OF LOGS)						
variable		mean	median	sd	min	max
Δy_{it}		0.018	0.021	0.046	-0.641	0.351
$y_{i,t-1}$		7.994	7.916	1.603	4.717	11.382
$cap_{i,t-1}$		8.911	8.856	1.743	5.198	12.236
$debt_{i,t-1}$		7.161	7.037	1.700	1.056	11.332
Δcap_{it}		0.021	0.019	0.029	-0.090	0.246
$\Delta debt_{it}$		0.035	0.029	0.183	-2.094	1.455

Notes: We present descriptive statistics for the full sample of 4,588 observations from $N = 118$ countries (average $T=38.9$). In Panel A we added a number of standard transformations of the data applied, e.g. the debt/GDP ratio and the investment/GDP ratio as well as per capita GDP and its growth rate. Some of these variables are used in the post-estimation analysis. In Panel B we present descriptives for the error correction model regression variables, namely Δy_{it} — GDP per capita growth rate, $y_{i,t-1}$ — lagged level of GDP per capita (in logs), $cap_{i,t-1}$ — lagged level of capital stock per capita (in logs), $debt_{i,t-1}$ — lagged level of debt stock per capita (in logs), Δcap_{it} — growth rate of capital stock per capita, $\Delta debt_{it}$ — growth rate of debt stock per capita.