

## ASSIGNMENTS

### Introduction to Finite Volume method

1. Consider the computational domain  $\Omega = [0, 1]^2$ . The source term is  $\mathbf{f} = \mathbf{0}$ . The exact solution of such a problem is:  $\mathbf{u}_{ex}(x, y) = (y, -x)$  and  $p_{ex}(x, y) = 0.5(x^2 + y^2) - 1/3$ . Perform a mesh convergence analysis based on Cartesian uniform orthogonal grids for  $\nu = 10$ .
2. Consider the lid driven cavity benchmark. Employ the following initial condition and source term for the temperature transport equation:

$$T_0(x, y) = 1 \quad \text{for} \quad \sqrt{(x - 0.05)^2 + (y - 0.05)^2} \leq 0.01 \quad \text{otherwise} \quad T_0(x, y) = 0,$$
$$f_T(x, y) = \alpha T \quad \text{with} \quad \alpha \in R.$$

Note that the source term needs to be treated in an implicit way.

### Navier Stokes equations in stream function vorticity formulation

1. Perform the vortex merger simulation by using icoFoam and carry out a comparison with the results obtained by the stream-function vorticity formulation in terms of all the variables involved (velocity, pressure, stream function and vorticity). Concerning the computation of the pressure for the stream-function vorticity formulation, you can refer, e.g., to <https://www.iist.ac.in/sites/default/files/people/psi-omega.pdf>
2. Perform the lid driven cavity simulation by using the stream-function vorticity formulation and carry out a comparison with the results obtained by icoFoam in terms of all the variables involved (velocity, pressure, stream function and vorticity). Concerning the boundary conditions to be enforced for the vorticity, you can refer, e.g., to <https://www.iist.ac.in/sites/default/files/people/psi-omega.pdf>

### Quasi Geostrophic equations for ocean flows

1. Perform the double-gyre wind forcing simulation for  $Re = 200$  and  $Ro = 0.0016$  on meshes  $4 \times 8$ ,  $8 \times 16$  and  $16 \times 32$  and carry out a comparison (efficiency and accuracy) between QGE and BV- $\alpha$  models.
2. Employ the BV-Bardina model (<https://www.lume.ufrgs.br/bitstream/handle/10183/128048/000974186.pdf?sequence=1>) and carry out a comparison (efficiency and accuracy) between it and the BV- $\alpha$  model for the double-gyre wind forcing simulation for  $Re = 450$  and  $Ro = 0.0036$  on meshes  $4 \times 8$  and  $8 \times 16$ .

### Cardiovascular flows

1. Extend to a three-dimensional configuration the idealized blood vessel benchmark. Then, consider a more complex Windkessel model based on a RCR circuit providing the outlet boundary condition for the pressure.