Problem 1

a)

$$f(x \pm \delta) = f(x) \pm \delta f'(x) + \frac{1}{2}\delta^2 f''(x) \pm \frac{1}{6}\delta^3 f'''(x) + \frac{1}{24}\delta^4 f^{(4)}(x)$$

$$f'(x) = \frac{f(x+\delta) - f(x-\delta)}{2\delta} - \frac{1}{6}\delta^2 f'''(x)$$

$$f(x \pm 2\delta) = f(x) \pm 2\delta f'(x) + 2\delta^2 f''(x) + \frac{8}{6}\delta^3 f'''(x) + \frac{16}{24}\delta^4 f^{(4)}(x)$$

$$f'(x) = \frac{f(x+2\delta) - f(x-2\delta)}{4\delta} - \frac{2}{3}\delta^2 f'''(x)$$

$$\frac{3}{4}f'(x) = f'(x) - \frac{1}{4}f'(x)$$

$$= \left[\frac{f(x+\delta) - f(x-\delta)}{2\delta} - \frac{1}{6}\delta^2 f'''(x)\right] - \frac{1}{4}\left[\frac{f(x+2\delta) - f(x-2\delta)}{4\delta} - \frac{2}{3}\delta^2 f'''(x)\right]$$

$$\frac{3}{4}f'(x) = \left[\frac{f(x+\delta) - f(x-\delta)}{2\delta}\right] - \left[\frac{f(x+2\delta) - f(x-2\delta)}{16\delta}\right]$$

$$f'(x) = \frac{1}{12\delta}\left[8\left(f(x+\delta) - f(x-\delta)\right) - f(x+2\delta) + f(x-2\delta)\right]$$

b) Given that the leading order error term is f^5

$$err = \frac{f\varepsilon}{\delta} + f^{(5)}\delta^4$$

$$0 = -\frac{f\varepsilon}{\delta^2} + 4f^{(5)}\delta^3$$

$$\delta = \left(\frac{f\varepsilon}{4f^{(5)}}\right)^{1/5}$$

$f(x) = e^x$

$$\delta = \left(\frac{e^x\varepsilon}{4e^x}\right)^{1/5}$$

$$\delta = \left(\frac{\varepsilon}{4}\right)^{1/5}$$

$f(x) = e^{0.01x}$

$$\delta = \left(\frac{e^{0.01x}\varepsilon}{4(0.01)^5 e^{0.01x}}\right)^{1/5}$$

$$\delta = \left(\frac{\varepsilon}{4\times10^{-10}}\right)^{1/5}$$