⑤ a) Let $\alpha = e^{-2\pi i k/N}$

$$\sum_{x=0}^{N-1} e^{-2\pi i k x/N} = \sum_{x=0}^{N-1} \alpha^x$$

Using solution to geometric power series

$$\sum_{k=0}^{n} ar^k = a\left(\frac{1-r^{n+1}}{1-r}\right)$$ where $x=k$, $N-1=n$, $a=1$, $r=\alpha$

$$\sum_{x=0}^{N-1} \alpha^x = \left(\frac{1-\alpha^N}{1-\alpha}\right) = \frac{1-\exp(-2\pi i k)}{1-\exp(-2\pi i k/N)}$$

b) Using l'hopital's rule

$$\lim_{k\to 0} \frac{f(k)}{g(k)} \to \frac{0}{0} \qquad f(k) = 1-e^{-2\pi i k}$$
$$g(k) = 1-e^{-2\pi i k/N}$$

$$\lim_{k\to 0} \frac{f'(k)}{g'(k)} = \frac{2\pi i e^{-2\pi i k}}{\frac{2\pi i}{N} e^{-2\pi i k/N}} \to \frac{2\pi i}{\frac{2\pi i}{N}} \to N$$

$e^{-2\pi i k} = 1$ for $k =$ any integer, therefore

$$\frac{f(k)}{g(k)} = \frac{1-1}{g(k)} = 0$$

If k is a multiple of N, then $g(k) = 1-e^{-2\pi i t}$ where t is integer

then $\frac{f(k)}{g(k)} = \frac{0}{0}$ undefined.