Module 3

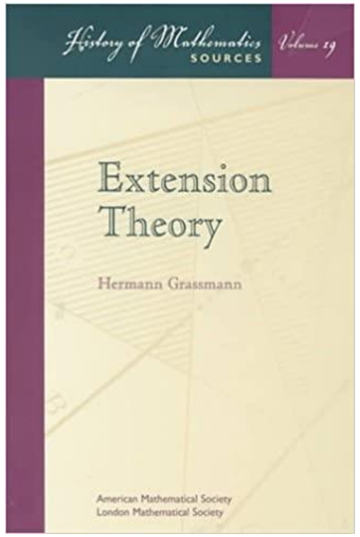
Matrices

Today's Outline

I. An Introduction to Linear Algebra - Revisited
II. Introduction to Matrices
III. Few Important Matrices
IV. Fundamental Operations on Matrices
V. Determinants
VI. Inverse Matrix

Linear Algebra- Matrix Theory

- A branch of Abstract Mathematics
- Hermann Grassmann (1844): published his "**Theory of Extension**"
- Discussing a foundational new topics of what is known as **Modern linear algebra**.
- Linear Algebra → deeper understanding of machine learning.
- The most important concept in linear algebra → Matrix Theory

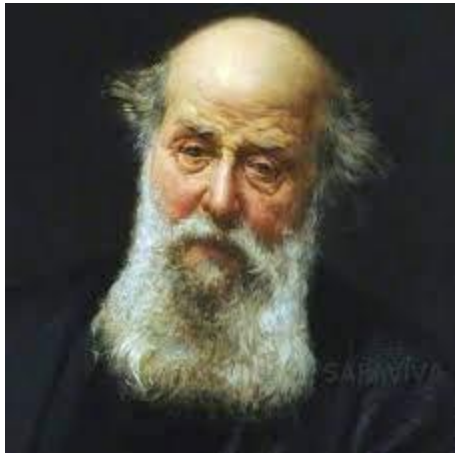


Grassmann

Matrix Theory

- In 1848, James Joseph Sylvester coined the term **matrix**, (*Latin for womb*).
- Basic elements of matrix theory → computational intelligence.
- Matrix Theory studies:

    1. The general theory of matrices
    2. The associated algebraic operations.



James Joseph Sylvester

- Linear algebra → **mathematics of data.**
- Matrices → **language of data.**

Today's Outline

~~I. An Introduction to Linear Algebra~~

II. Introduction to Matrices

III. Few Important Matrices

IV. Fundamental Operations on Matrices

V. Determinants

VI. Inverse Matrix

Definition

Matrix

- Plural: matrices
- A rectangular array
- Of numbers / functions
- Written within brackets.

$$\begin{pmatrix} 2 & 5 & 7 & 8 \\ 1 & 2 & 3 & 1 \\ 4 & 5 & 0 & 1 \end{pmatrix}$$

$$A = \begin{bmatrix} 3t - 3t^2 & \sqrt{t} - t \\ 0 & t^3 - t \\ |t| - t & 0 \\ \dfrac{t}{3} - \dfrac{t^3}{3} & 0 \end{bmatrix}.$$

Importance of Matrices

Matrices

- Provides a **systematic approach** for arranging large arrays of values / Functions.
- **Simplifies** analysis of large amount of data.
- **Simplifies** the analysis of large arrays of equations and their solutions.
- **Simplifies** computer algorithms for manipulating large arrays of data.

Size of a Matrix

- The number of horizontal rows → m
- The number of vertical columns → n

Example: State the dimensions of each matrix.

A. $\begin{bmatrix} 4 & 6 & 5 \\ 2 & -3 & -7 \\ 1 & 0 & 9 \end{bmatrix}$

B. $\begin{bmatrix} -4 & \frac{1}{3} & -3 \end{bmatrix}$

C. $\begin{bmatrix} 1 \\ 2 \\ 0 \\ 0.5 \end{bmatrix}$

Size of a Matrix cont

- m Rows
- n Columns
- Notation: A_{mxn}

$$\mathbf{A} = \begin{bmatrix} a_{11} \; a_{12} \; a_{13} \ldots . a_{1n} \\ a_{21} \; a_{22} \; a_{23} \ldots . a_{2n} \\ \vdots \qquad\qquad\qquad \vdots \\ a_{m1} \; a_{m2} \; a_{m3} \ldots a_{mn} \end{bmatrix}$$

Example: Determine the size of the matrix shown below.

$$\mathbf{A} = \begin{bmatrix} 2 & -3 & 5 \\ -1 & 4 & 6 \end{bmatrix}$$

- The matrix has 2 rows and 3 columns. Its size is 2 x 3. → $\mathbf{A}_{2\times3}$

$$\mathbf{B} = \begin{bmatrix} 2 & 1 \\ 7 & -4 \\ 3 & 1 \end{bmatrix}$$

- The matrix has 3 rows and 2 columns. Its size is 3 x 2. → $\mathbf{B}_{3\times2}$

Element of a Matrix

- Numbers /functions **of a matrix** are called **elements** of the **matrix.:**
- Each element a of a matrix, is addressed by two indices i & j: $\boldsymbol{a_{ij}}$
    - i: number of the row (where the element is located in the matrix)
    - j: number of the column (where the element is located in the matrix)

Element of a Matrix cont

- For a Matrix: A mxn
- General notation of elements

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \dots . a_{1n} \\ a_{21} & a_{22} & a_{23} \dots . a_{2n} \\ \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} \dots a_{mn} \end{bmatrix}$$

Example: Identifying a Matrix Element

$$A = \begin{bmatrix} \sin x & \cos x \\ \operatorname{tg} x & \sin x \end{bmatrix}$$

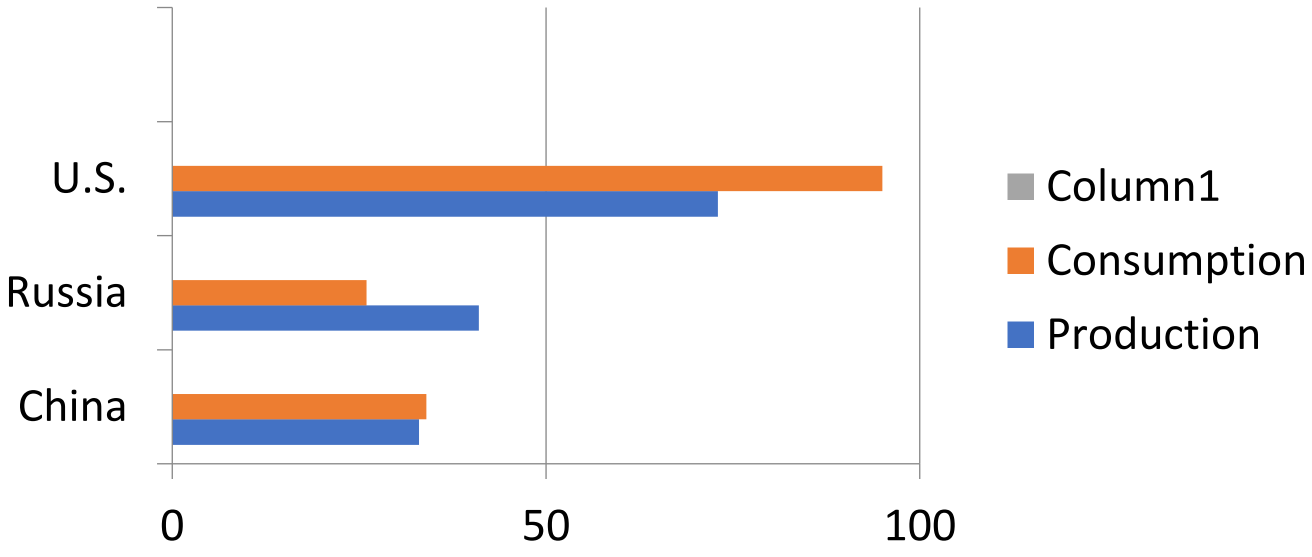
Find each of the following elements:

$\boldsymbol{a_{31}}$ $\boldsymbol{a_{22}}$ $\boldsymbol{a_{21}}$ $\boldsymbol{a_{11}}$

Example. Organizing data 1

Energy is often measured in British Thermal Units (Btu).

1. We can write a 3 x 2 matrix to represent the bellow data.
2. And rewrite the information as a 2 x 3 matrix.



	US	Russia	China
Consumption (Btu)	92	28	36
Production (Btu)	73	41	34

$$\begin{pmatrix} 92 & 28 & 36 \\ 73 & 41 & 34 \end{pmatrix}$$

Example. Organizing data 2

For the bellow table:

a. Write a matrix to represent the information.

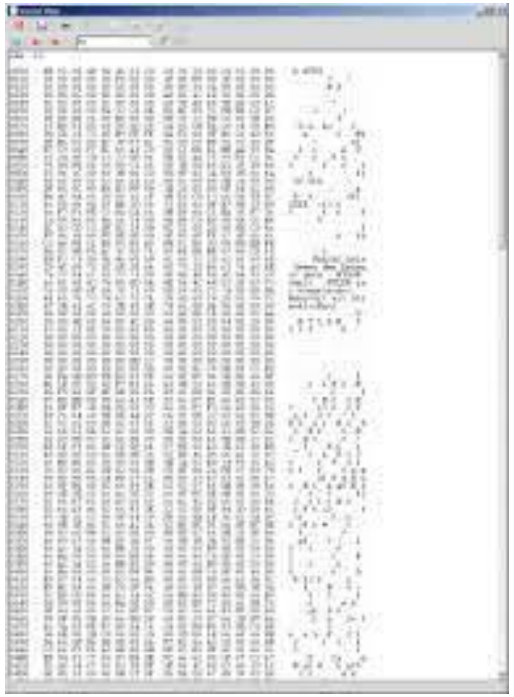
b. What are the dimensions of the matrix?

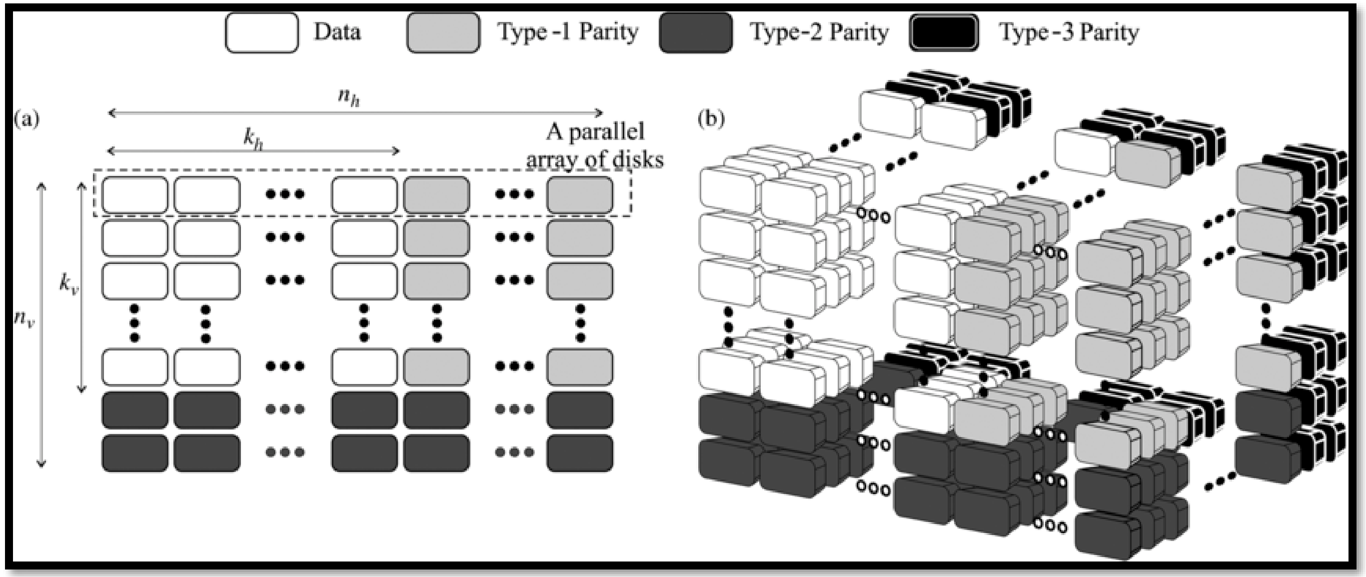
c. Which element represents Kristin Maloney's score on the vault?

Gymnast	Floor Exercise	Vault	Balance Beam	Uneven Bars
Amy Chow	9.525	9.468	9.625	9.400
Dominique Dawes	9.087	9.393	8.600	9.675
Kristin Maloney	9.525	9.225	9.312	9.575
Elise Ray	9.225	9.468	9.687	9.687

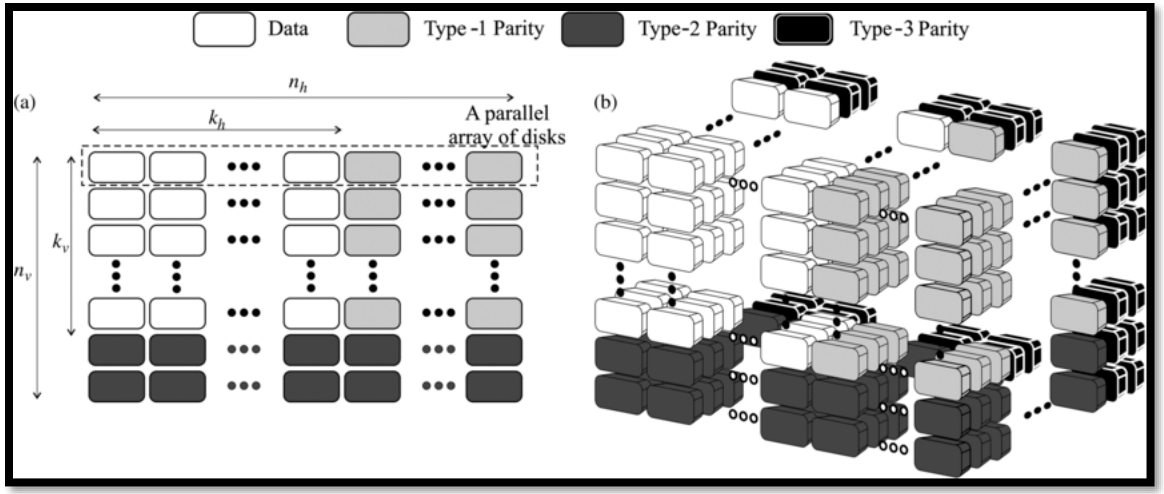
Matrices and Hard Disk

- Using matrices → we can correlate a unique address to every data point in memory of a hard disk



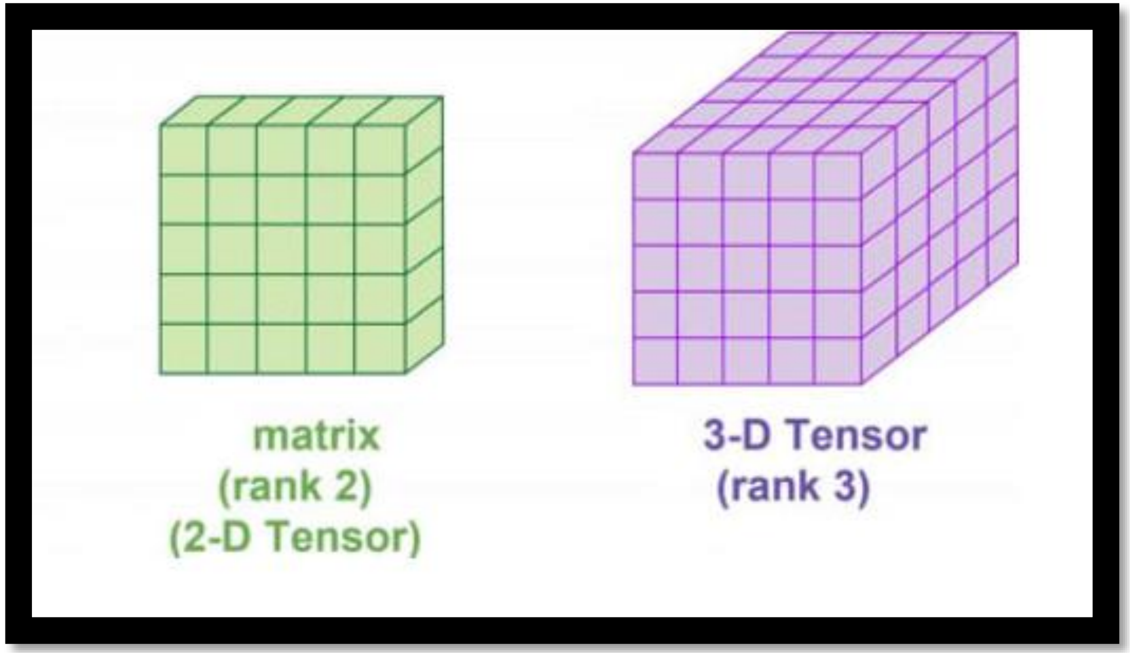


Matrices and Hard Disk



Matrix

Tensor



Today's Outline

I. ~~An Introduction to Linear Algebra~~

II. ~~Introduction to Matrices~~

III. Few Important Matrices in machine learning

IV. Fundamental Operations on Matrices

V. Determinants

VI. Inverse Matrix

Scalar

- A 1 x 1 matrix is called a *scalar*
- Such as: [1],[∏],[e]

Row & Column Matrix

- A matrix having only one row is called a *row matrix*
- A matrix having only one column is called a *column matrix*
- A matrix of *either form* is called a *vector*.

$$\mathbf{t} = [0\ 1\ 2\ 3\ 4\ 5]$$

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Square Matrix

- In a Matrix A_{mxn}
- If $n = m$ → Square Matrix.

$$\mathbf{A}_{\mathbf{m,m}}$$

$$\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}_{2\times 2}$$

$$\begin{bmatrix} 9 & 5 & 2 \\ 1 & 8 & 5 \\ 3 & 1 & 6 \end{bmatrix}_{3\times 3}$$

$$\begin{bmatrix} x & y & z & 1 \\ a & b & c & 1 \\ p & q & r & 1 \\ m & n & o & 1 \end{bmatrix}_{4\times 4}$$

Main Diagonal of A Matrix

- In a Square Matrix, the elements $a_{11} \rightarrow a_{ii} \rightarrow a_{nn}$,
- Are the **main diagonal** of the matrix.

$$\mathbf{C} = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 6 & 1 \\ -5 & 2 & 1 \end{bmatrix}$$

The main diagonal of C:

$c_{11}=2$

$c_{22}=6$

$C_{33}=1$

Diagonal Matrix

- A diagonal matrix is a Square Matrix, *if the elements*

  a) If the main diagonal is **non-zero**: (*for i, 1≤ i ≤ n*: $a_{ii} \neq 0$)

  b) Other elements are 0.

Example: Diagonal Matrix

$$A = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

$$\begin{bmatrix} a_1 & 0 & 0 & \cdots & 0 \\ 0 & a_2 & 0 & \cdots & 0 \\ 0 & 0 & a_3 & & \vdots \\ \vdots & \vdots & & \ddots & 0 \\ 0 & 0 & \cdots & 0 & a_n \end{bmatrix}$$

Identity / unit Matrix

- The identity matrix is a *square matrix* that:

  a) Has 1 along the *main diagonal*

  b) Has 0 for all other entries.

- This matrix is often written simply as $\mathbb{I}$.

- It *acts like 1* in matrix multiplication $[\ A_{mxn} \times I_n = A_{mxn}\ ]$

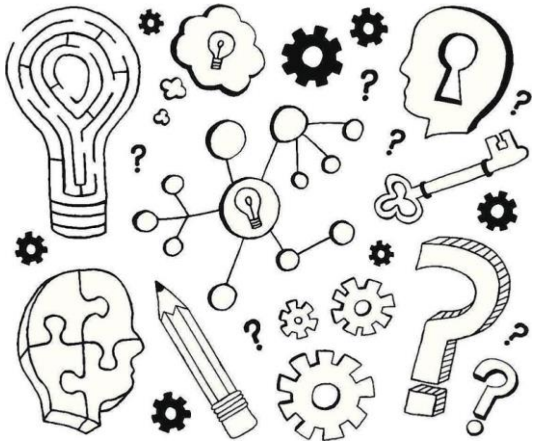
- AKA *Unit Matrix*

$$I_1 = [1],\ I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},\ I_n = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}$$

Identity/Unit Matrix cont

$$\mathbf{I} = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & & & & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix}$$

Activity (Individual, 10’)



Write an example of each below matrices:

i. Scalar
ii. Vectors
iii. Matrices
iv. Row Matrix
v. Column Matrix
vi. Zero Matrix
vii. Square Matrix
viii. Diagonal Matrix
ix. Scalar Matrix
x. Unit Matrix
xi. Upper Triangular Matrix
xii. Lower Triangular Matrix

Types of Matrices

Row Matrix

$$\begin{pmatrix} a & b & c \end{pmatrix}$$

Column Matrix
Vector Matrix

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

Zero Matrix
Null Matrix

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Diagonal Matrix

$$\begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}$$

Scalar Matrix

$$\begin{pmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{pmatrix}$$

Unit Matrix

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Upper Triangular Matrix

$$\begin{pmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{pmatrix}$$

Lower Triangular Matrix

$$\begin{pmatrix} a & 0 & 0 \\ b & c & 0 \\ d & e & f \end{pmatrix}$$

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IV. Fundamental Operations on Matrices
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Fundamental Operations on Matrices

- Transpose of a Matrix
- Equality of Matrices
- Addition
- Subtraction
- Multiplication
- Division?

Transpose of a Matrix

- The transpose of a matrix A is denoted as A'. $A \xrightarrow{T} A'$
- A'=T(A),
- It is obtained by interchanging the rows and columns.
- Thus, if A has a size of *m* x *n* → A' will have a size of *n* x *m*.
- If the *transpose operation is applied twice*, the original matrix is restored.

$$\boxed{T\,(T(A)) = A}$$

Example: Determine the transpose of the matrix **A** below.

$$\mathbf{A} = \begin{bmatrix} \boxed{2 \ \ -3 \ \ 5} \\ -1 \ \ 4 \ \ 6 \end{bmatrix} \Rightarrow \mathbf{A}' = \begin{bmatrix} \boxed{\begin{matrix} 2 \\ -3 \\ 5 \end{matrix}} & \begin{matrix} -1 \\ 4 \\ 6 \end{matrix} \end{bmatrix}$$

Equality of Matrices

Two matrices A & B are equal, if & only if:

i. Both matrices have the same size

ii. Corresponding elements of both matrices are equal.

Thus if $A = (a_{ij})_{m,n}$ and $B = (b_{ij})_{m,n}$ then **A = B**

if and only if $a_{ij} = b_{ij}$ *(for $1 \leq i \leq m$ & $1 \leq j \leq n$)*

Addition and Subtraction of Matrices

- Matrices can be added together or subtracted from each other if and only if *they are of the same size*.
- Corresponding elements are *added or subtracted*.

$$\mathbf{C}_{\mathbf{m,n}} = \mathbf{A}_{\mathbf{m,n}} \pm \mathbf{B}_{\mathbf{m,n}}$$

Addition and Subtraction of Matrices cont

ADD

$$A + B = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} + \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix}$$

$$= \begin{bmatrix} a_1 + b_1 & a_2 + b_2 \\ a_3 + b_3 & a_4 + b_4 \end{bmatrix}$$

SUBTRACT

$$A - B = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} - \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix}$$

$$= \begin{bmatrix} a_1 - b_1 & a_2 - b_2 \\ a_3 - b_3 & a_4 - b_4 \end{bmatrix}$$

Example: Determine the matrices:

i) C = A+B

ii) D = B-A

$$\mathbf{A} = \begin{bmatrix} 3 & 2 & 1 \\ -4 & 5 & 6 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 2 & -3 & 5 \\ -1 & 4 & 6 \end{bmatrix}$$

Multiplication of Two Matrices

Two matrices can be multiplied if & only if:

- The *number of columns* → of the **first matrix**
- Is Equal To
- The *number of rows* → of the **second matrix**.

$$\begin{array}{cc} & \text{3 columns} \\ \text{2 rows} & \begin{vmatrix} 1 & 0 & 0 \\ 1 & .5 & 1 \end{vmatrix} \end{array}$$

$2 \times \boxed{3}$ Matrix

$$\begin{array}{cc} & \text{2 columns} \\ \text{3 rows} & \begin{vmatrix} 2 & 0 \\ 1 & 3 \\ 3 & .5 \end{vmatrix} \end{array}$$

$\boxed{3} \times 2$ Matrix

Multiplication of Two Matrices cont

The size of the result matrix:

- The number of rows is equal to the number of rows in the first matrix
- The number of columns is equal to the number of columns in the second matrix.

$$\mathbf{A}_{\boxed{\mathbf{m}},\cancel{\mathbf{n}}}\mathbf{B}_{\cancel{\mathbf{n}},\boxed{\mathbf{k}}} = \mathbf{C}_{\mathbf{m},\mathbf{k}}$$

Multiplication of Two Matrices cont

$$\overset{4\times 2\ \text{matrix}}{\begin{bmatrix} a_{11} & a_{12} \\ \cdot & \cdot \\ a_{31} & a_{32} \\ \cdot & \cdot \end{bmatrix}} \overset{2\times 3\ \text{matrix}}{\begin{bmatrix} \cdot & b_{12} & b_{13} \\ \cdot & b_{22} & b_{23} \end{bmatrix}} = \overset{4\times 3\ \text{matrix}}{\begin{bmatrix} \cdot & c_{12} & c_{13} \\ \cdot & \cdot & \cdot \\ \cdot & c_{32} & c_{33} \\ \cdot & \cdot & \cdot \end{bmatrix}}$$

Multiplication of Two Matrices cont

- This means that

$$\mathbf{AB} \neq \mathbf{BA}$$

Example. For these matrices, determine possible orders of multiplication.

$$\mathbf{A} = \begin{bmatrix} 2 & -3 & 5 \\ -1 & 4 & 6 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 2 & 1 \\ 7 & -4 \\ 3 & 1 \end{bmatrix}$$

- $\mathbf{AB} = \mathbf{A}_{2,3}\mathbf{B}_{3,2} = \mathbf{C}_{2,2}$
- $\mathbf{BA} = \mathbf{B}_{3,2}\mathbf{A}_{2,3} = \mathbf{D}_{3,3}$

Multiplication of Two Matrices cont

- An **element** in the product matrix is obtained by summing successive products of elements in the row of the first with elements of the column of the second.

$$c_{ij} = \sum_{r=1}^{n} a_{ir} b_{rj}$$

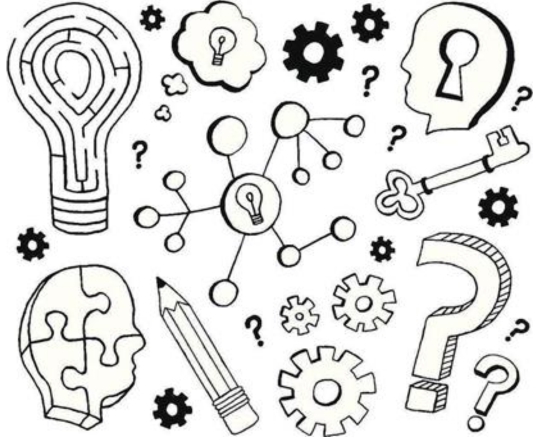
Multiplication of Two Matrices cont

$$\mathbf{A}_{2x2} \times \mathbf{B}_{2x2} = \mathbf{C}_{2x2}$$

$$AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 1(5) + 2(7) & 1(6) + 2(8) \\ 3(5) + 4(7) & 3(6) + 4(8) \end{bmatrix}$$

$$= \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$$



Activity (Individual, 10’):

Matrices A & B are given (below). Find C = AB.

$$\mathbf{A} = \mathbf{A}_{2,3} = \begin{bmatrix} 2 & -3 & 5 \\ -1 & 4 & 6 \end{bmatrix}$$

$$\mathbf{B} = \mathbf{B}_{3,2} = \begin{bmatrix} 2 & 1 \\ 7 & -4 \\ 3 & 1 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 2 & -3 & 5 \\ -1 & 4 & 6 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 7 & -4 \\ 3 & 1 \end{bmatrix}$$

Example. Continuation.

$$c_{11} = (2)(2) + (-3)(7) + (5)(3) = 4 - 21 + 15 = -2$$

$$c_{12} = (2)(1) + (-3)(-4) + (5)(1) = 2 + 12 + 5 = 19$$

$$c_{21} = (-1)(2) + (4)(7) + (6)(3) = -2 + 28 + 18 = 44$$

$$c_{22} = (-1)(1) + (4)(-4) + (6)(1) = -1 - 16 + 6 = -11$$

$$\mathbf{C} = \begin{bmatrix} -2 & 19 \\ 44 & -11 \end{bmatrix}$$

Division of Matrices ?

- There is no such thing as division of matrices.
- However, *matrix inversion* can be viewed in some sense as a procedure similar to division.

Some Important Properties of Matrices

$$AB \neq BA$$

$$(AB)C = A(BC)$$

$$A(B + C) = AB + AC$$

$$(B + C)A = BA + CA$$

Break- 20’

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5. Determinants
6. Inverse Matrix

Determinants

The determinant of a matrix **A:**

- Is defined only for a *square* matrix.
- It is a *scalar* value.
- Various representations are shown as follows:

$$\det(\mathbf{A}) \qquad |\mathbf{A}| \qquad \Delta$$

Determinant of 2 x 2 Matrix

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$\det(\mathbf{A}) = a_{11}a_{22} - a_{12}a_{21}$$

Example. Determine the determinant of the matrix shown below.

$$\mathbf{A} = \begin{bmatrix} 3 & 2 \\ -4 & 5 \end{bmatrix}$$

$$\det(\mathbf{A}) = \begin{vmatrix} 3 & 2 \\ -4 & 5 \end{vmatrix} = (3)(5) - (2)(-4)$$

$$= 15 + 8 = 23$$

Determinant of 3 x 3 Matrix

- For determinants of matrices of higher order than 2 x 2, the *process can become tedious*.
- There are many "tricks", but some are useful only when the matrix has simple numbers.
- We will investigate this, for a 3X3 matrix.

Determinant of 3 x 3 Matrix, cont

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$\begin{aligned} \det(\mathbf{A}) = a_{11}&(a_{22}a_{33} - a_{23}a_{32}) \\ &+a_{12}(-a_{21}a_{33} + a_{23}a_{31}) \\ &+a_{13}(a_{21}a_{32} - a_{22}a_{31}) \end{aligned}$$

Determinant of 3 x 3 Matrix, cont

2x2 Matrix = $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

Determinant = $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$

$\boxed{= ad - bc}$

3x3 Matrix M = $\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$

$$|M| = a * \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b * \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c * \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

$\boxed{= \quad a(ei-fh) - b(di-gf) + c(dh-ge)}$

Singular Matrix

- If det(A) = 0 → the matrix is said to be *singular*.

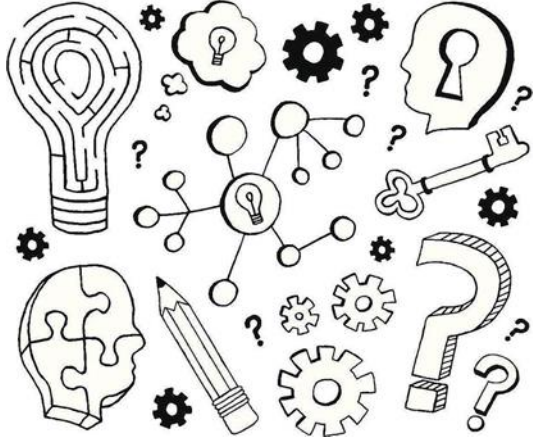
$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \Rightarrow |A| = 0 \Rightarrow$$

A is a singular Matrix

Activity (Individual, 10’):

1) Show that Matrix A is singular.

$$A = \begin{bmatrix} 3 & 8 & 1 \\ -4 & 1 & 1 \\ -4 & 1 & 1 \end{bmatrix}$$



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Inverse Matrix

- The inverse of a matrix **A** is denoted by A^{-1}.
- A matrix is inversible → **det(A)** $\neq$ 0
- If A is inversible, then:

$$\mathbf{A}\,\mathbf{A}^{-1} = \mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$$

Inverse of a 2 x 2 Matrix

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \Rightarrow \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}^{-1} = \frac{\begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}}{\det(\mathbf{A})} = \begin{bmatrix} \dfrac{a_{22}}{\det(\mathbf{A})} & \dfrac{-a_{12}}{\det(\mathbf{A})} \\ \dfrac{-a_{21}}{\det(\mathbf{A})} & \dfrac{a_{11}}{\det(\mathbf{A})} \end{bmatrix}$$

1) Change the location of elements on the main diagonal
2) Multiply the other two elements in (-1)
3) Divide the matrix by det(A)

Example. Determine the inverse of the matrix **A** below.

$$\mathbf{A} = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$$

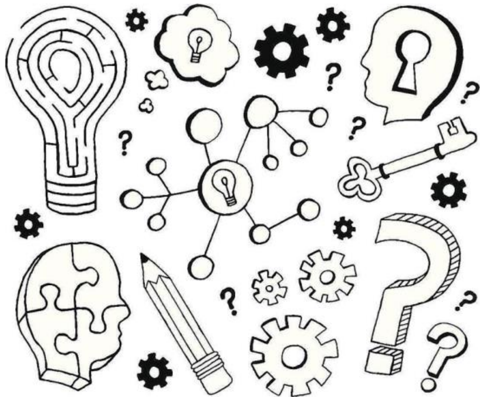
$$\det(\mathbf{A}) = (2)(5) - (3)(4) = -2$$

$$\mathbf{A}^{-1} = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}^{-1} = \frac{\begin{bmatrix} 5 & -3 \\ -4 & 2 \end{bmatrix}}{-2} = \begin{bmatrix} -2.5 & 1.5 \\ 2 & -1 \end{bmatrix}$$

Reading,10’

1. Read the bellow article and summaries your findings?

https://www.linkedin.com/pulse/linear-algebra-fuels-artificial-intelligence-kayode-odeyemi/



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VII. Tutorial

Exercise 1) Show that C = A + B

$$\mathbf{A} = \begin{bmatrix} 3 & 2 & 1 \\ -4 & 5 & 6 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 4 & 9 & x \\ 6 & -3 & y \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 7 & 11 & 1+x \\ 2 & 2 & 6+y \end{bmatrix}$$

Exercise 2) Show that D = A - B

$$\mathbf{A} = \begin{bmatrix} 3 & 2 & 1 \\ -4 & 5 & 6 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 4 & 9 & x \\ 6 & -3 & y \end{bmatrix}$$

$$\mathbf{D} = \begin{bmatrix} -1 & -7 & 1-x \\ -10 & 8 & 6-y \end{bmatrix}$$

Exercise 3) Proof D = BA.

$$\mathbf{A} = \mathbf{A}_{2,3} = \begin{bmatrix} 2 & -3 & 5 \\ -1 & 4 & 6 \end{bmatrix} \qquad \mathbf{B} = \mathbf{B}_{3,2} = \begin{bmatrix} 2 & 1 \\ 7 & -4 \\ 3 & 1 \end{bmatrix}$$

$$\mathbf{D} = \mathbf{BA} = \begin{bmatrix} 2 & 1 \\ 7 & -4 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 2 & -3 & 5 \\ -1 & 4 & 6 \end{bmatrix} = \begin{bmatrix} 3 & -2 & 16 \\ 18 & -37 & 11 \\ 5 & -5 & 21 \end{bmatrix}$$

Exercise 4) Calculate all the element of the Matrix C.

$$A_{2\text{x}4} \; \text{X} \; B_{4\text{x}3} = C_{2\text{x}3}$$

$$c_{11} = a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} + a_{14}b_{41}$$

$$\underset{2 \times 4}{\begin{bmatrix} \boxed{a_{11} \; a_{12} \; a_{13} \; a_{14}} \\ a_{21} \; a_{22} \; a_{23} \; a_{24} \end{bmatrix}} \underset{4 \times 3}{\begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \\ b_{41} & b_{42} & b_{43} \end{bmatrix}} = \underset{2 \times 3}{\begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \end{bmatrix}}$$

$$c_{22} = a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} + a_{24}b_{42}$$

$$\begin{bmatrix} a_{11} \; a_{12} \; a_{13} \; a_{14} \\ \boxed{a_{21} \; a_{22} \; a_{23} \; a_{24}} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \\ b_{41} & b_{42} & b_{43} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \end{bmatrix}$$

Exercise 5) Calculate Cij

$$
\begin{array}{cc}
 & \\
 & \\
\text{row } i \hookrightarrow & \\
 & \\
 &
\end{array}
\begin{bmatrix}
a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
a_{i1} & a_{i2} & a_{i3} & \dots & a_{in} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn}
\end{bmatrix}
\cdot
\begin{array}{c}
\begin{array}{cccccc} & & & \text{column } j & & \\ & & & \downarrow & & \end{array} \\
\begin{bmatrix}
b_{11} & b_{12} & \dots & b_{1j} & \dots & b_{1n} \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
b_{i1} & b_{i2} & \dots & b_{ij} & \dots & b_{in} \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
b_{n1} & b_{n2} & \dots & b_{nj} & \dots & b_{nn}
\end{bmatrix}
\end{array}
=
$$

$$
=
\begin{bmatrix}
c_{11} & c_{12} & \dots & c_{1j} & \dots & c_{1n} \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
c_{i1} & c_{i2} & \dots & \boxed{\mathbf{c_{ij}}} & \dots & c_{in} \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
c_{n1} & c_{n2} & \dots & c_{nj} & \dots & c_{nn}
\end{bmatrix}
$$

entry on row i
column j

Exercise 6)

1. Derive the determinant of a 3x3 generic Matrix.

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

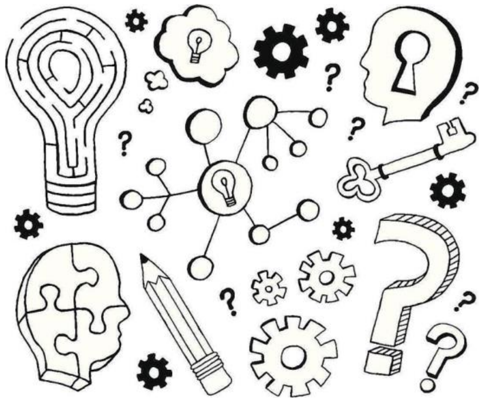
Activity 7)

1. For the Matrix $\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, investigate:

   a) $A^{-1}A = I_2$

   b) $AA^{-1} = I_2$

Research 1

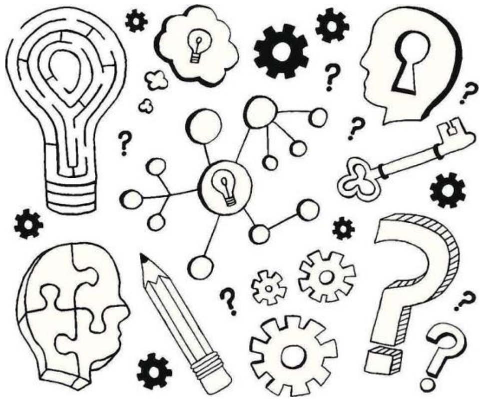


1. What is the procedure for driving the determinant of a 4x4 generic matrix?

$$|A| = \begin{vmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{vmatrix}$$

$$= A_{11} \begin{vmatrix} A_{22} & A_{23} & A_{24} \\ A_{32} & A_{33} & A_{34} \\ A_{42} & A_{43} & A_{44} \end{vmatrix} - A_{21} \begin{vmatrix} A_{12} & A_{13} & A_{14} \\ A_{32} & A_{33} & A_{34} \\ A_{42} & A_{43} & A_{44} \end{vmatrix}$$
$$+ A_{31} \begin{vmatrix} A_{12} & A_{13} & A_{14} \\ A_{22} & A_{23} & A_{24} \\ A_{42} & A_{43} & A_{44} \end{vmatrix} - A_{41} \begin{vmatrix} A_{12} & A_{13} & A_{14} \\ A_{22} & A_{23} & A_{24} \\ A_{32} & A_{33} & A_{34} \end{vmatrix}.$$

Research 2



1. What is the procedure for driving the inverse of a 3x3 matrix?

   (Try to solve this for a generic matrix A)

2. Show that $A\,A^{-1} = A^{-1}A = I_3$

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Next Class

(Module 4)

Eigenvalues & Eigenvectors

Come prepared!

Any Questions or Concerns?

Sources for the slides:

https://fdocuments.in/

And

https://www.xpowerpoint.com/