Week 12-

First Order Logic

Today's Outline

- Introduction
- Predicate Logic
- Elements of Predicate Logic
- Quantifiers
- Boolean Algebra
- Epilogue

What is Logic?

- The study of the principles of correct reasoning.
- Logic is concerned with the truthfulness of a *chain of statements*.
- There are several different types of logic
- Each type → Particular limits in reasoning

Different Types of Logic- Logic in computer science

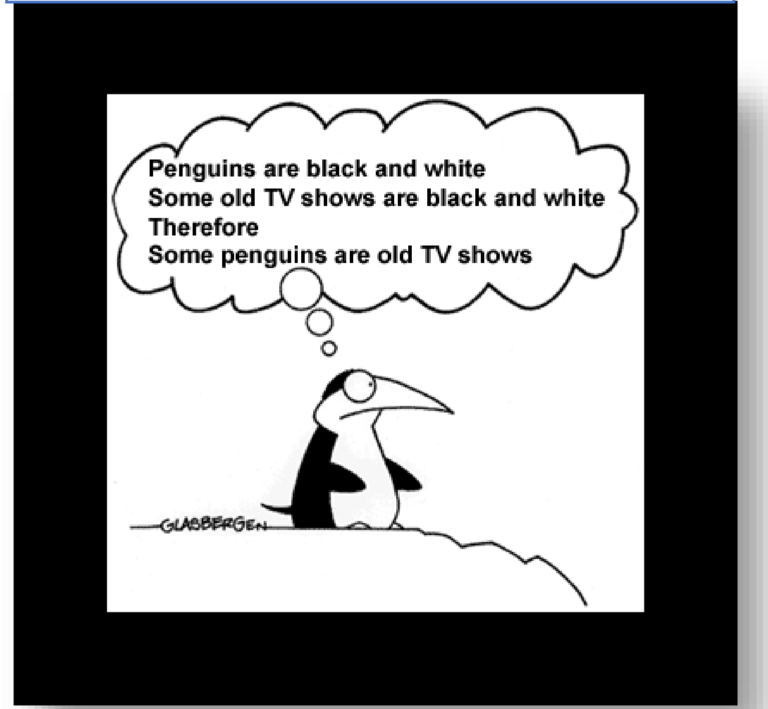
- Computational logic : Oldest form of knowledge representation in computer.
- A branch of mathematical logic
- Intersection between mathematical logic & computer science.
- Application of Logic in Computer Science

Different Types of Logic

Such As

1. Propositional logic
2. Predicate logic
3. Boolean Logic
4. Syllogistic logic
5. Modal logic
6. Informal reasoning and dialectic
7. Mathematical logic
8. Philosophical logic
9. *More & More …*

Every Logic has its own
Pros & Cons



Propositional Logic- Pros

1. Propositional logic is declarative → True or False

2. Propositional logic facilitates compound propositions

3. Propositional logic is compositional→ The meaning of $p \wedge q$ is derived from the meaning of p and the meaning of q

4. Meaning in propositional logic is context-independent *(unlike natural language, where meaning depends on context)*

Propositional Logic- Cons

1. Propositional Logic can not represent all statements.

   Example) All men are mortals. Some dogs like cats

2. Expressing a simple context may need thousands of pages

   Example To Express Rules of chess: 100,000 pages in propositional logic

Thus, we need a more general form of logic

capable of representing the details.

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Predicate Logic (FOL)

- Developed **independently** by Gottlob Frege and Charles Sanders Peirce (late 1800s)
- It is a more expressive logic than propositional
- AKA FOL
- Enhances processing by allowing the *use of **variables, functions, Relations, Predicates, …***
- Example: Rules of chess:

  100,000 pages in propositional logic → Few pages in first-order logic


FOL vs ZOL

Language	What exists in the world	What an agent believes about facts
Propositional logic	Facts	true / false / unknown
Predicate logic	Facts, objects, relations, Functions	true / false / unknown

Predicate Logic (FOL)- Elements

- Propositional logic assumes the world contains facts → FOL assumes the world also contains:

1. Objects: (Constants, Variables): people, houses, numbers, theories, Ronald McDonald, colors, baseball games, wars, centuries, *etc.*
2. Relations: (return **true/false**): red, round, prime, brother of, bigger than, inside, part of, has color, occurred after, owns, *etc.*
3. Functions: (return an **object**): father of..., best friend..., one more than..., end of..., *etc.*

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Elements of FOL

Objects- Constant

- Specific objects.
- Example: King John, 2, e, TUA, elephant

Elements of FOL cont

Objects- Variables

- Represent general classes of objects
- Represent general classes of properties
- Example: x, y, z, a

Elements of FOL cont

Predicates

- Predicates → Express a relation
- Return True / False
- Example: James teaches MFA501 → teach (James, MFA501)
    - Teach → predicate → Denoting the relationship between James & MFA501.

Elements of FOL cont

Functions

- A function denotes a mapping from entities of a set to a unique element of another set : f: A→ B
- Returns an object
- Example:
  - father(Alexander) = Philip
  - mother(Alexander) = Olympias.

Elements of FOL cont

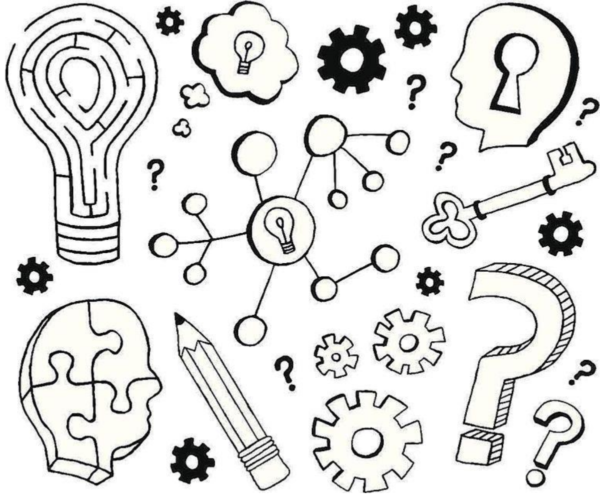
Operations

- Connectivity & Equality
- FOL uses the same operators found in P.L.
- Example: $\wedge$ $\vee$ $\neg$ $\Rightarrow$ $\Leftrightarrow$

Logics in General

Language	What exists in the world	What an agent believes about facts
Propositional logic	Facts	true / false / unknown
Predicate logic	Facts, objects, relations, Functions	true / false / unknown
Probability theory	Facts	degree of belief
Fuzzy logic	Facts + degree of truth	known interval value

Reflection (Individual, 10’)



Research and answer the following:

1. What are the applications of logic in computer science?

2. What types of logic are utilized in computer science? Why?

3. What types of logic have you used so far in your coding?

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Quantifiers

Quantifiers

- Quantifier: is a word that *goes before a noun* to express the quantity of the object;
- Example: little milk, all students, no man, some kids, ...
- FOL introduces different symbols called variable quantifiers.

1. $\forall$: Universal quantifier: **for all.**
2. $\exists$: Existential quantifier: **there exist**

Universal Quantifiers

- $\forall$ Indicates the expression is TRUE for all values of designated variable.

Example:

- $\forall$ X likes (X,mary)
- means for all values of X, the statement is true → everybody likes Mary

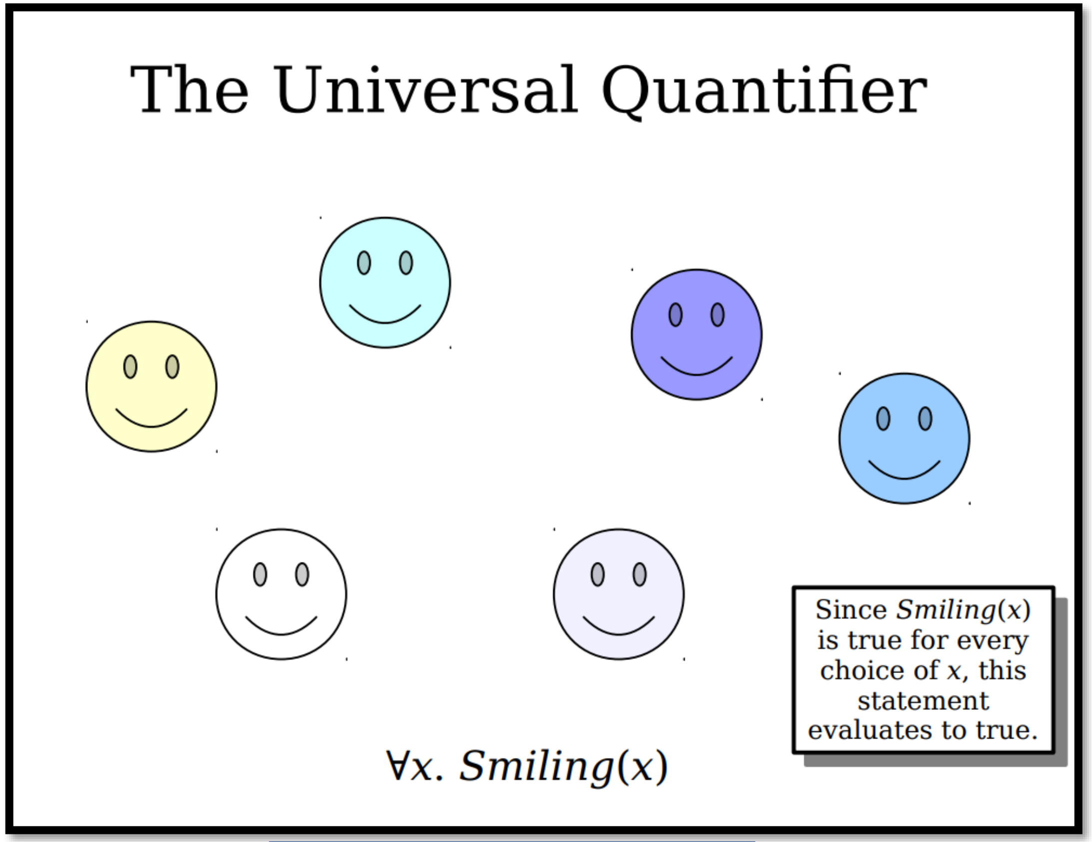
Existential Quantifiers

- ∃ indicates the expression is TRUE for some values of the variable;
- At least one value exist that makes the statement true.

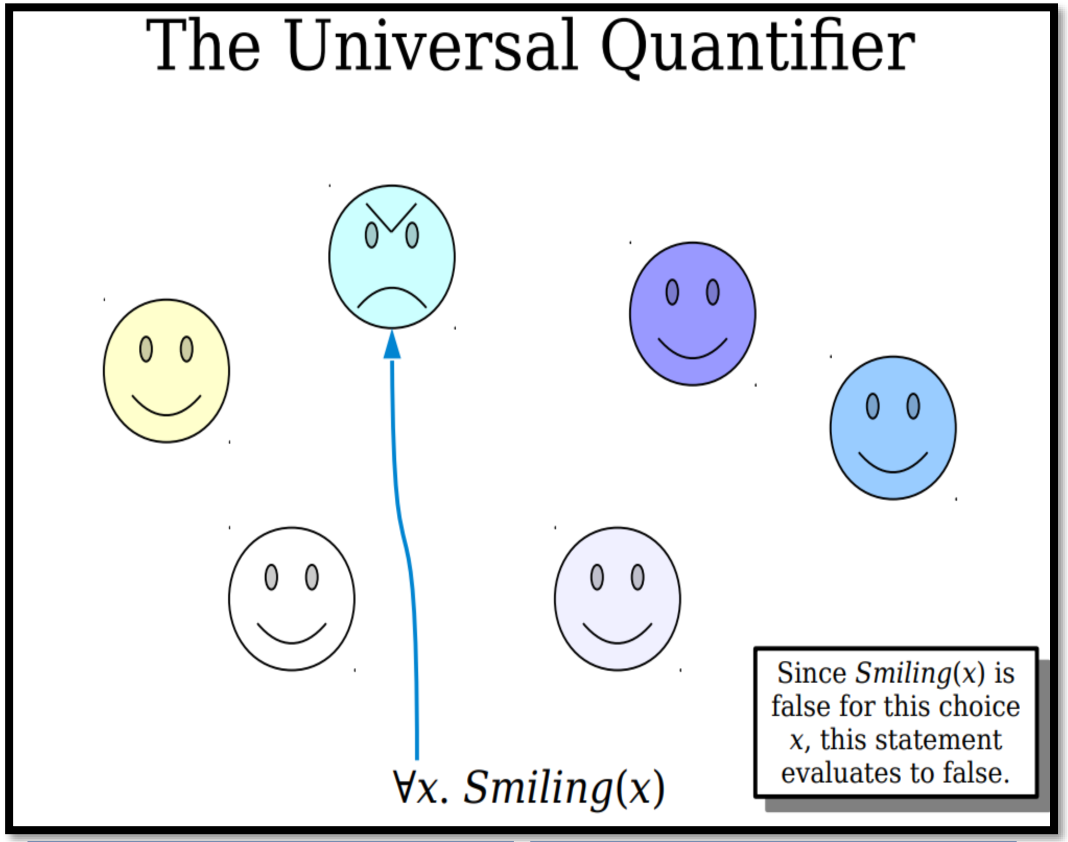
Example

- ∃ X likes (X, Mary)
- At least one person likes Mary

Examples of Quantifiers



∀ X. Smiling (x) = T



∀ X. Smiling (x) = F

∃ X. Smiling (x) = T

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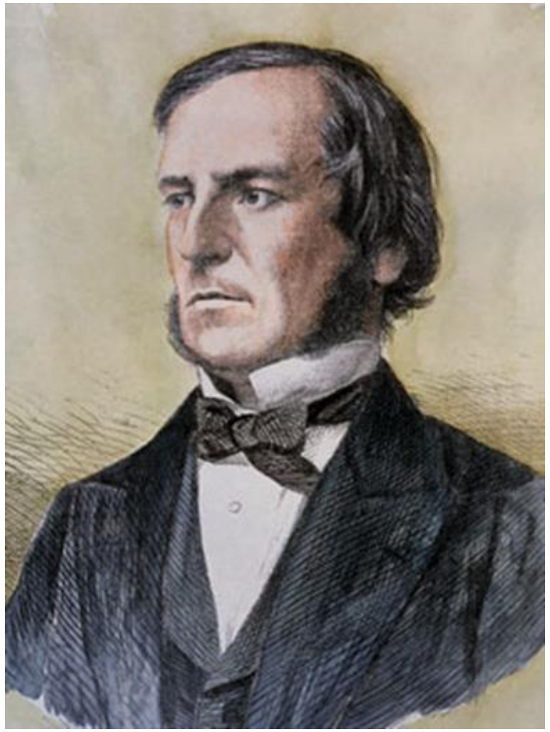
Boolean Logic

- Boolean Logic was introduced by George Boole in 1847
- In his book "*The Mathematical Analysis of Logic*"
- It is deeply connected to propositional & Predicate logic
- Applications:
  - AI & Machine Learning
  - Computer science
  - Digital electronic circuits
  - Microprocessors
  - More & More…



Boolean Algebra

- **Boolean Variables:** A variable → has two possible values
- **Boolean's value:** Just like the Propositional variable is either True or False
- Binary Codes represent the Boolean's value (T→ 1 and F→ 0)

$T \rightarrow 1$

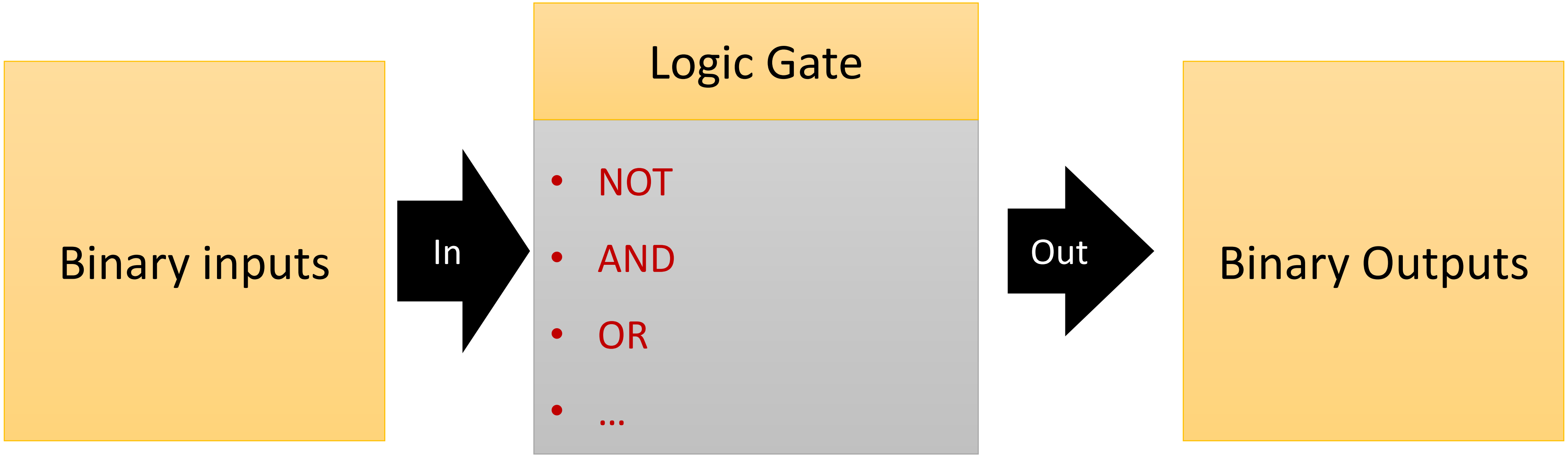
$F \rightarrow 0$

$\wedge \rightarrow \times$

$\vee \rightarrow +$

Logic Gate

- A logic gate is an idealized / physical electronic device.
- Works based upon Boolean Logic.



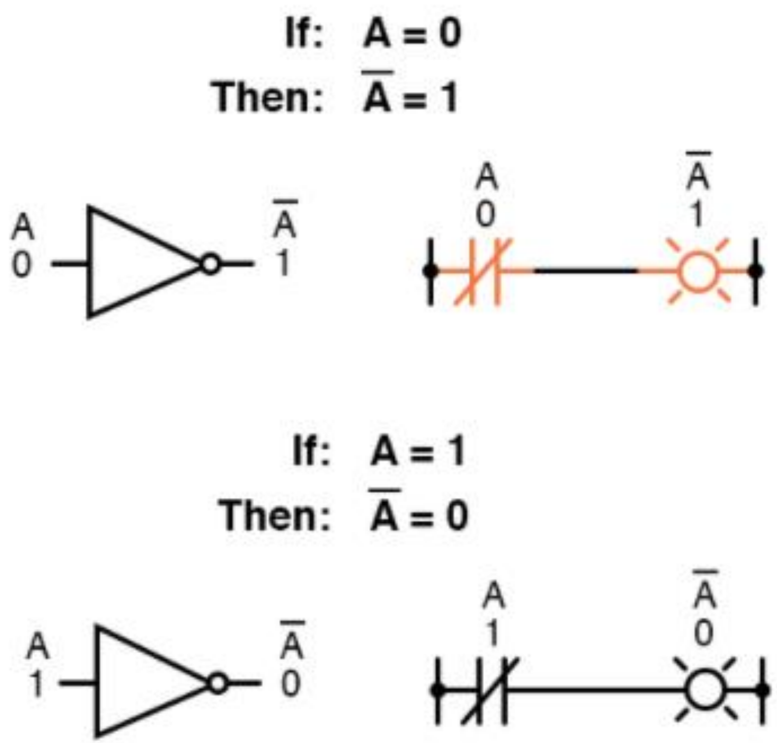
NOT Gate

Negation of P

p	¬p
T	F
F	T

Not Gate

p	$\bar{p}$
1	0
0	1



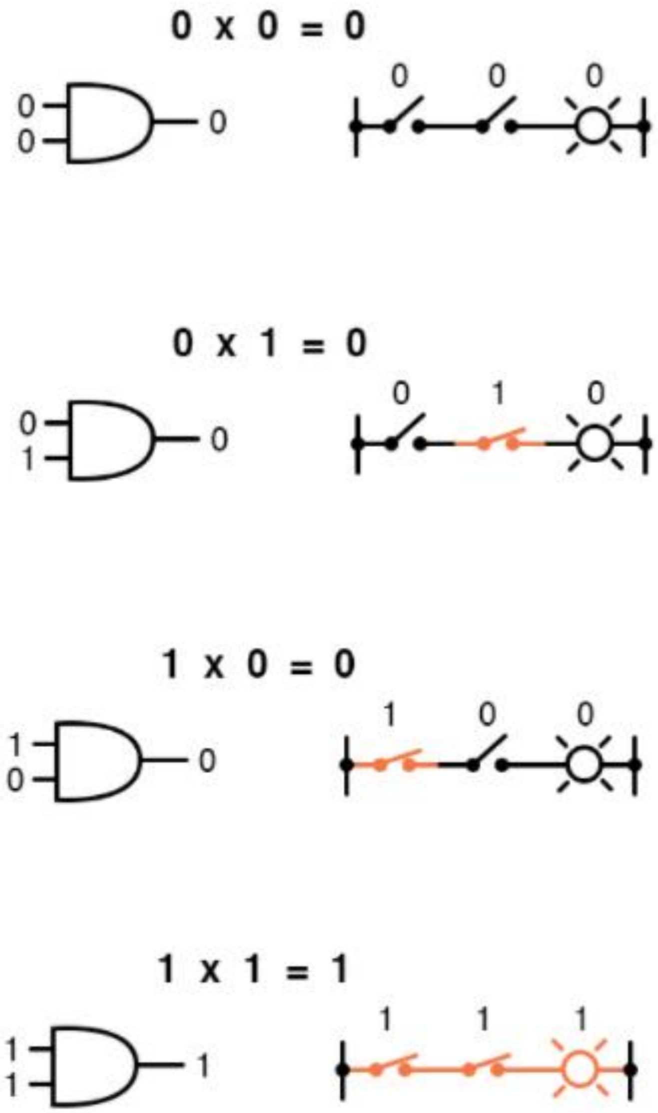
AND Gate

p	q	p ∧ q
T	T	T
T	F	F
F	T	F
F	F	F

p	q	p x q
1	1	1
1	0	0
0	1	0
0	0	0

Two switches- connected in Series

0 x 0 = 0
0 x 1 = 0
1 x 0 = 0
1 x 1 = 1

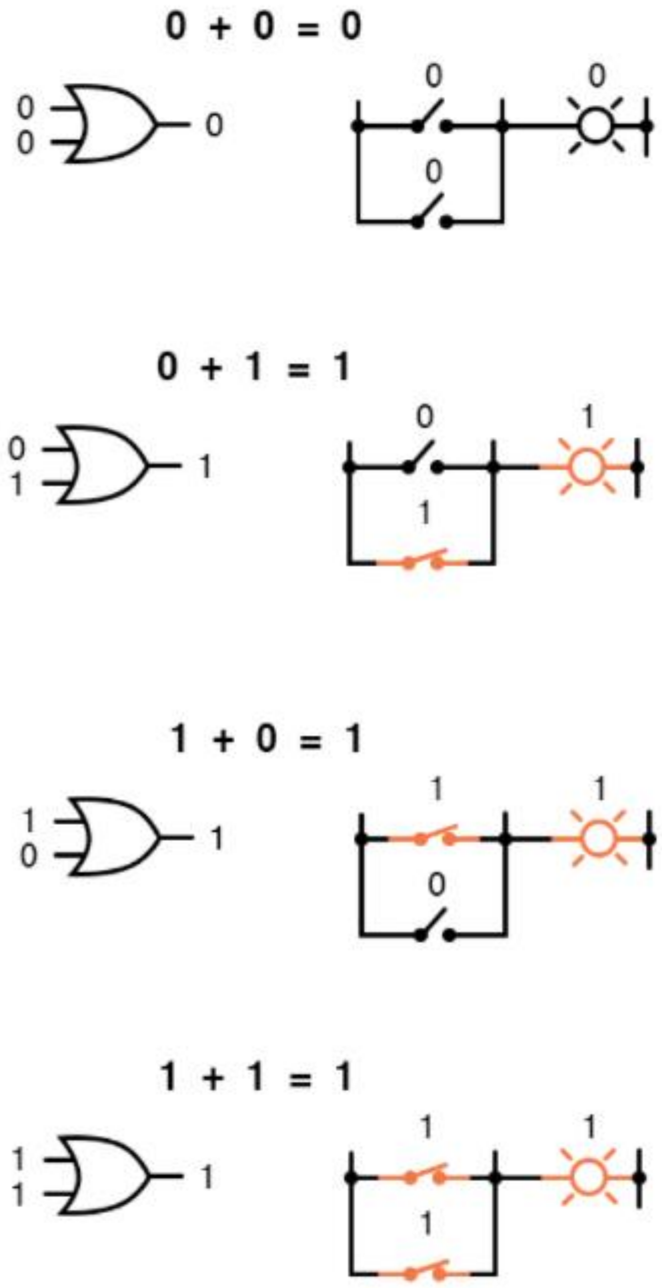


OR Gate

p	q	p ∨ q	p	q	p + q
T	T	T	1	1	1
T	F	T	1	0	1
F	T	T	0	1	1
F	F	F	0	0	0

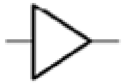
Two switches- connected in Parallel

0 + 0 = 0
0 + 1 = 1
1 + 0 = 1
1 + 1 = 1



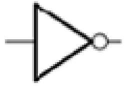
Summary of Basic Logical Gates

YES



INPUT	OUTPUT
A	
0	0
1	1

NOT



INPUT	OUTPUT
A	
0	1
1	0

AND

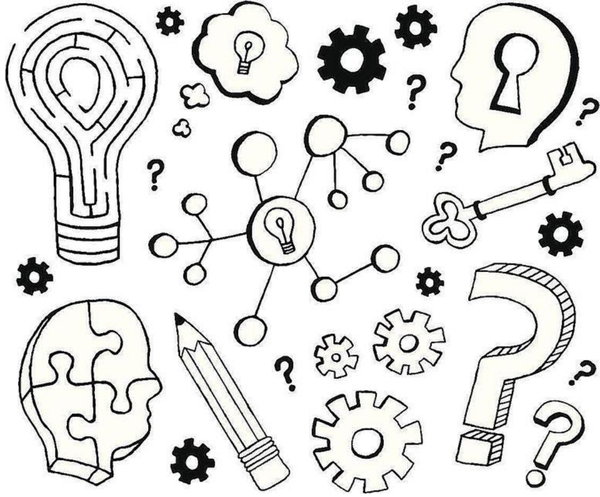
INPUT		OUTPUT
A	B	
0	0	0
1	0	0
0	1	0
1	1	1

OR

INPUT		OUTPUT
A	B	
0	0	0
1	0	1
0	1	1
1	1	1

Reflection (Individual, 10’)

Reflect on below table.



	Propositional Logic	Equivalents in Boolean Logic	Equivalents in Set Theory	Equivalents in Arithmetic
Characters	T , F	0 , 1	Sets	Numbers
Negation	$\neg$	NOT	~ (Complement of set)	-
Conjunction	$\wedge$	AND	$\cap$ (Intersection)	x
Disjunction	$\vee$	OR	$\cup$ (Union)	+

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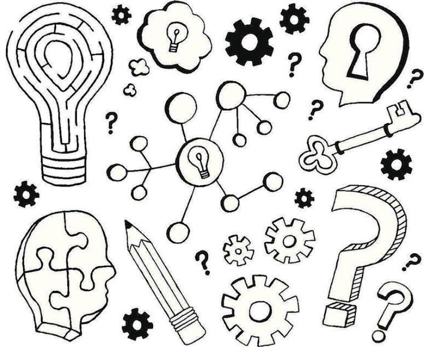
A Brief History of Mathematics in AI

- Ancient Greece → Geometry + Algebra
- Descartes: Geometry + Algebra → Geometric Algebra
- Geometric Algebra → Calculus + Linear Algebra → Clockwork Universe
- Clockwork Universe → Differential Equations
- Real Life Problems → Nonlinear DE → Not solvable through conventual methods
- Statistics & Probability Theory → Limited Accuracy
- Chaos Theory → Butterfly effect
- Numerical Analysis + Logic → Language of the Machines
- Mathematics for AI

Foundation of Mathematics for AI, Machine Learning & Depp Learning

Set Theory → Functions; Logic	W.1
Linear Algebra → Vectors; Matrices	W.2,3,4,5
Calculus (Differential Calculus; Integral Calculus)	W 6,7,8
Optimization	W.9
Statistics & Probability Theory	W.10
Logic (ZOL, FOL, Boolean)	W.11, 12

Workshop (Individual, 120’)

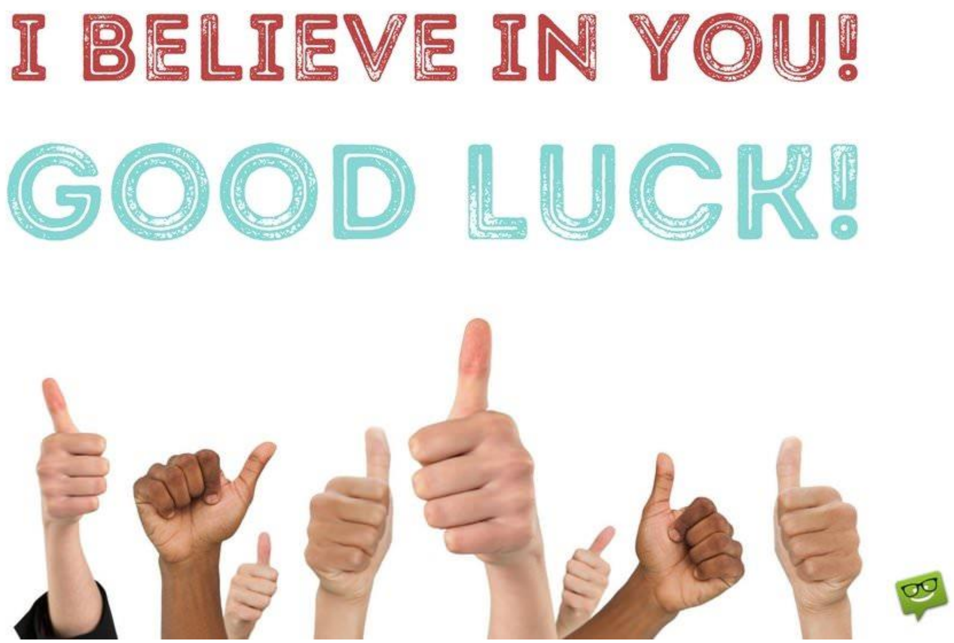


Write a paragraph of 500 words in approx. 5’ for each topic and summarize the following

1. What have you learned about this topic?
2. How is this topic related to computer science?
3. How can it assist you in your career?

i. Set Theory
ii. Undefined Concepts
iii. Linear Algebra
iv. Coordinate systems
v. Vectors, Matrices & Tensors
vi. Transformation
vii. Eigenvalue & Eigenvectors
viii. Relations & Functions
ix. Limits & Continuity
x. Differential calculus
xi. Integral Calculus
xii. Optimization
xiii. Different types of phenomena
xiv. Probability Theorem
xv. Statistics
xvi. Distribution
xvii. Reasoning & Logic
xviii. Propositional Logic
xix. Predicate Logic
xx. Stochastic Systems

This is the end of this Subject!



Any Questions or Concerns?