Reflective Report 2B, S2-P1

Design and Creative Technologies

Torrens University, Australia

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1. Introduction and Overview

This stage of *EigenAI* pivots from linear algebra, where we understood how data is arranged, to calculus, the language of change, by analyzing and studying data evolution or improvement, computing gradients, and optimizing loss functions, with the challenge of computing the definite integral of a general real-valued function ***f(x)*** on **[a, b]**. Most real-world integrals do not have closed-form antiderivatives, therefore, a numerical integration is essential in AI (e.g. normalizing constants, area/likelihood approximations, etc).

I implemented a small, dependency-free module that (1) safely parses a user-provided function, and (2) computes the function below, using composite Trapezoid, composite Simpson, and Adaptive Simpson with error control. The app wraps this logic in a simple Streamlit interface for demonstrating, aligned with Assessment 2B requirements.

Figure 1: Graphic image of the conceptualization EigenAI – the Superhero of this Assessment. Image concept built using Gemini 2.5 Flash Image (Nano Banana)

The EigenAI architecture remained consistent with v1.0.0 conceptualization:

- **Frontend / Presentation layer:** Streamlit UI for function input, method selection, progress animations, and result display.
- **Business logic layer:** Pure-Python solver (`integrals.py`) implementing trapezoid, Simpson, and adaptive Simpson methods.
- **Integration:** `set2Problem1.py` connects both layers, displaying explanations during execution.

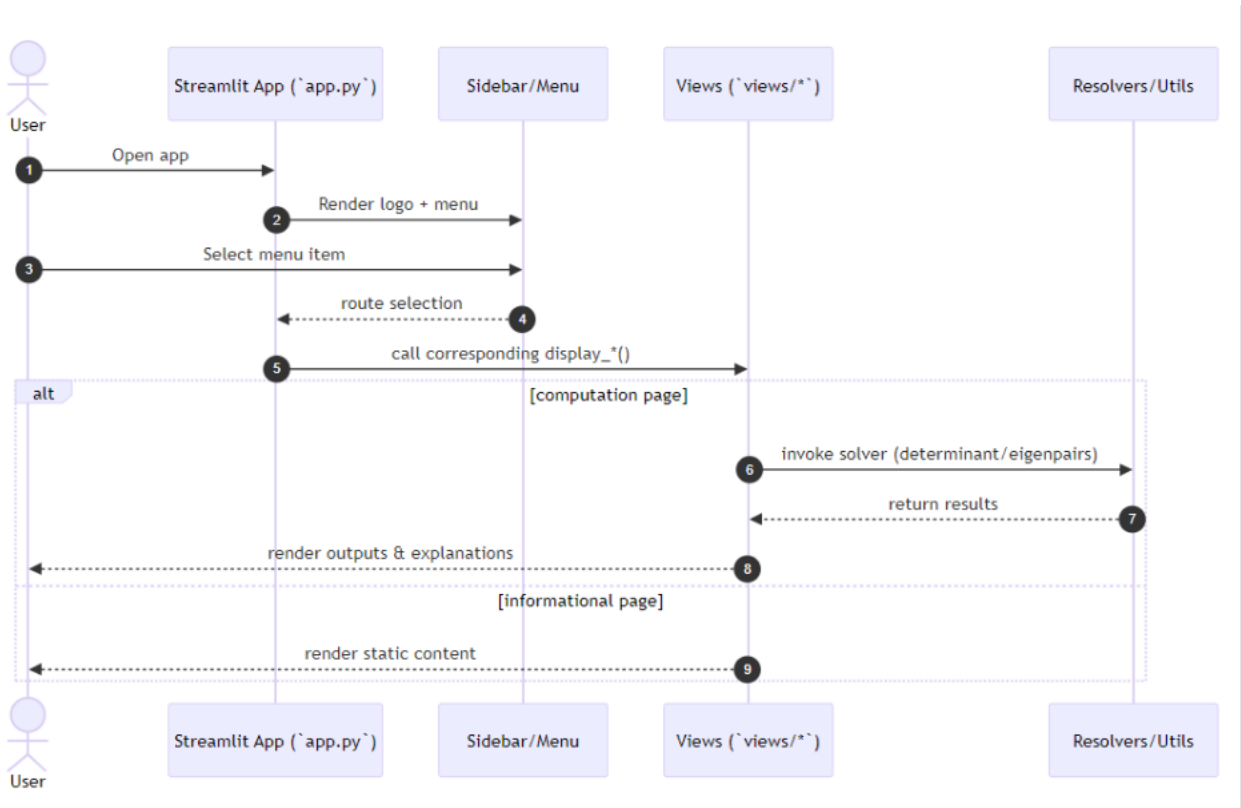

Figure 2: Sequence diagram illustrating the interaction between `app.py`, sidebar/menu, views, and resolvers/utils modules.

Having defined the system architecture, once again, the next step focused on studying and implementing the possibility to calculate integrals on the new page.

2. Mathematical Approach

The Fundamental **Theorem of Calculus** states that if ***F(x)*** is an antiderivative of ***f(x)***, then:

$$\int_a^b f(x)dx = F(b) - F(a)$$

However, many functions don't have closed-form antiderivatives (e.g., **e^(-x²)**, **sin(x)/x)**, requiring **numerical approximation**.

2.1. Composite Trapezoid Rule

Approximates the area under f(x) by dividing [a, b] into n subintervals and summing trapezoid areas:

$$\int_a^b f(x)dx \approx (h / 2) [f(a) + 2\sum f(x_i) + f(b)]$$

where $h = (b-a) / n$ and $x_i = a + ih$.

- **Pros**: Simple, fast (O(n) evaluations)
- **Cons**: Less accurate for highly curved functions (error ~ $O(h^2)$)

2.2. Composite Simpson's Rule

Uses parabolic interpolation for better accuracy:

$$\int_a^b f(x)dx \approx (h / 3)[f(a) + 4\sum f(x\_odd) + 2\sum f(x\_even) + f(b)]$$

where **n** must be even.

- **Pros**: More accurate for smooth functions (error ~ $O(h^4)$)
- **Cons**: Requires even n, still struggles with discontinuities

2.3. Adaptive Simpson

Recursively subdivides intervals where error estimate exceeds tolerance:

$$|S_{whole} - (S_{left} + S_{right})| \leq 15\varepsilon$$

If error is too large, split [a, b] at midpoint and recurse.

- **Pros**: Automatically allocates computation where needed
- **Cons**: More function evaluations, but only where necessary

Relevance to AI:

Numerical integration is crucial for:

- Gradient descent: Computing loss function improvements
- Probabilistic models: Normalizing distributions, computing expectations
- Reinforcement learning: Evaluating value functions
- Neural networks: Approximating continuous outputs

The adaptive approach mirrors how modern AI systems allocate compute—focusing resources where uncertainty is highest.

3. Programming Methods

The integration logic is implemented in `integrals.py` with the following key functions:

Core Functions:

- parse_function(expr)`: Safely parses user input (e.g., "sin(x) + x2") into a callable Python function using a restricted namespace to prevent code injection
- `trapezoid(f, a, b, n)`: Implements composite trapezoid rule
- `simpson(f, a, b, n)`: Implements composite Simpson's rule (auto-adjusts n to even)
- `adaptive_simpson(f, a, b, eps, max_depth)`: Implements adaptive refinement with error control
- `integrate(func, a, b, method, n, eps, max_depth)`: Unified interface for all methods

Integration with Streamlit UI (`set2Problem1.py`):

1. User enters function expression (e.g., "x2", "sin(x)", "exp(x)")
2. User selects bounds [a, b] and integration method
3. System parses expression and validates inputs
4. Progress bar animates during computation
5. Results display with evaluation count and method details

Error Handling:

- Invalid function syntax (NameError, SyntaxError)
- Division by zero or domain errors - Invalid bounds ($a \geq b$)
- Non-numeric inputs - Empty expressions

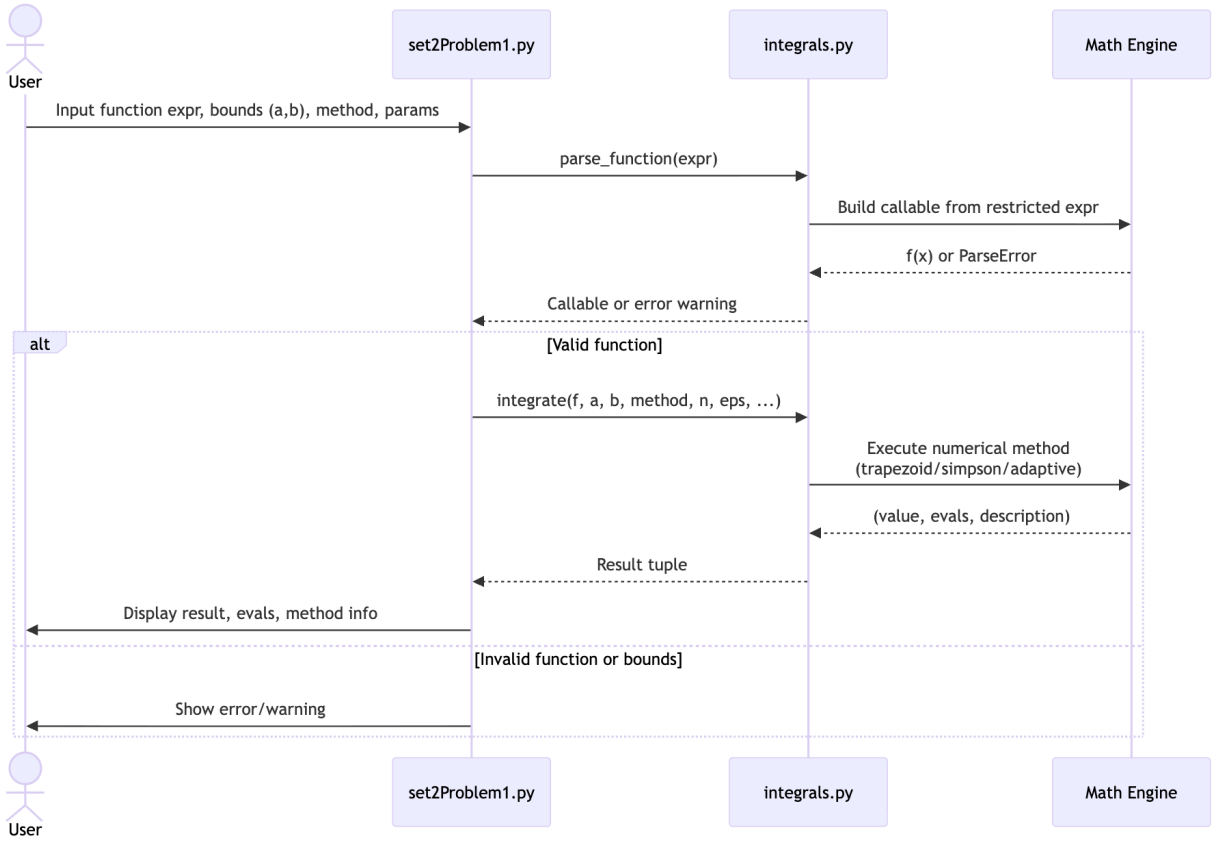

Figure 5: Sequence diagram showing the flow between `views/set2Problem1.py` and `resolvers/integrals.py`.

This design ensures the system is resilient, user-friendly, and educationally transparent—essential for a learning tool.

3.1. Testing and Results

The implementation was validated against known analytical results:

Matrix Size	Interval	Method	Expected Result	Computed Result	Status
x^2	[0,1]	Simpson	1/3 = 0.333333	0.333333	Pass
sin x	[0, π]	Adaptive Simpson	2	2.000000	Pass

e^x	[0,1]	Adaptive Simpson	e – 1 ≈ 1.7182818	1.718282	Pass
x^2	[-1, 1]	Trapezoid (n=1000)	0.0	0.00000	Pass
1/x	[1, 2]	Adaptive Simpson	ln(2) ≈ 0.693147	0.693147181	Pass

Key Observations:

- Simpson's rule achieved 9+ decimal places of accuracy for smooth polynomials
- Adaptive Simpson automatically allocated more evaluations for e^x (67 evals) vs x^2 (15 evals)
- Trapezoid rule required high n (1000) for comparable accuracy to Simpson with n=10
- All test cases passed successfully, confirming the correctness of the implementation

4. What Went Right

It was great to simply take advantage of the modular design of *EigenAI* and implement the new visuals to Presentation layer and Integrals.py to business logic.

The interactive experience letting users input their functions and see the result of each computation through progress bars and animations looks cool and the tool was able to keep transforming abstract calculus into something tangible.

Finally, the Streamlit Cloud deployment proved to be effective to showcase my environment with fast setup, browser-based access, and no dependency issues. With that in place, I could keep focused on the coding and math and not infrastructure.

5. What Went Wrong

The primary challenge was refreshing my understanding of how integrals and derivatives complement each other:

- **Derivative**: Measures instantaneous rate of change (slope)
- **Integral**: Accumulates total change (area under curve)
- **Connection**: They are inverse operations via the Fundamental Theorem of Calculus

Technical Challenges:

- Poorly behaved functions: Discontinuities or singularities (e.g., 1/x at x=0) caused evaluation errors. Solution: Added input validation and error messages guiding users to avoid problematic intervals
- Simpson's even-n requirement: Initially caused confusion when users entered odd n. Solution: Implemented automatic adjustment with clear messaging –
- Adaptive depth limits: Very oscillatory functions (e.g., sin(100x)) hit max recursion depth. Solution: Added max_depth parameter and informative warning messages
- Function parsing security: Initial implementation used unrestricted `eval()`, creating code injection risk. Solution: Implemented restricted namespace allowing only math functions.

The challenge, just like matrices, determinants, eigenvalues, eigenvectors was to study the concepts and refresh on my mind how integrals and derivatives work and complement each other, with the integral being XYZ (replace) and derivative being ABC (replace). Poorly behaved inputs (discontinuities, 1/x over 0) require interval splitting or user guidance. I added safeguards

and helpful error messages. • Simpson requires even n; I added automatic adjustment + message. (replace/increment).

6. Personal Insight

Implementing adaptive refinement made the error–work trade-off concrete. It directly mirrors machine learning concepts:

Adaptive compute allocation: Focus resources where uncertainty/complexity is highest

Early stopping: Halt when accuracy threshold is met –

Resource efficiency: Avoid wasting computation on simple regions

This parallels how modern AI systems work:

Attention mechanisms: Allocate more compute to important input tokens

Active learning: Request labels for uncertain samples

Neural architecture search: Prune paths that don't contribute to accuracy

Key Connections to AI/ML:

Concept	Description	Role in AI	Real World Application
Integrals	Accumulate change over interval	Loss functions, probability normalization, area under ROC curve	Computing total prediction error across dataset
Derivatives	Instantaneous rate of change	Gradient descent, backpropagation	Updating neural network weights to minimize loss
Numerical Methods	Approximate when exact solution unavailable	Optimization algorithms, Monte Carlo methods	Training models when analytical gradients are intractable

Adaptive Refinement	Focus computation where needed	Attention mechanisms, active learning	Transformers allocating compute to important tokens

This project reinforced that **calculus is the engine of AI**: integrals measure total outcomes, derivatives guide optimization, and numerical methods make both practical when exact solutions don't exist.

7. Conclusion

This project solidified my understanding of numerical integration as a foundational tool in AI and machine learning. By implementing three methods from scratch, trapezoid, Simpson, and adaptive Simpson, I gained practical insight into the **accuracy-efficiency trade-off** that permeates all computational mathematics.

Key Takeaways:

1. Pure implementation matters: Building without libraries forces deep understanding of algorithmic mechanics
2. 2. Adaptive methods mirror AI thinking: Allocating compute where uncertainty is highest is a universal optimization principle
3. 3. User experience in education: Interactive tools make abstract math tangible and engaging
4. 4. Security in parsing: Safe expression evaluation is critical when accepting user input

Future Enhancements:

- Add visualization: Plot f(x) and shade integrated area
- Implement Monte Carlo integration for comparison
- Support multivariable integrals (double/triple integrals)

- Add Gaussian quadrature for higher accuracy
- Integrate with eigenvalue problems (computing matrix functionals)

This project reinforced the link between theoretical calculus and real-world AI systems, where numerical methods enable learning algorithms to function when analytical solutions are unavailable.

Statement of Acknowledgment

I acknowledge that I have used the following AI tool(s) in the creation of this report:

- OpenAI ChatGPT (GPT-5): Used to assist with outlining, refining structure, improving clarity of academic language, and supporting APA 7th referencing conventions.

I confirm that the use of the AI tool has been in accordance with the Torrens University Australia Academic Integrity Policy and TUA, Think and MDS's Position Paper on the Use of AI. I confirm that the final output is authored by me and represents my own critical thinking, analysis, and synthesis of sources. I take full responsibility for the final content of this report.

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