Week 11:

Prepositional Logic

Today's Outline

I. Reasoning & Logic

II. Propositional Logic

III. Compound Propositions

IV. Operators

V. Combination of Three or more propositions

VI. Tutorial

Reasoning

- Capacity for **consciously** *making sense of things*.
- It is associated with thinking, cognition, and intellect.
- A primitive instinct based on *millions of years of evolution*.
- Reasoning: Can be Right / Wrong

Reasoning - Mythos vs Logos

- **Observation**: A thunderstorm lights up the sky
- Reasoning:

  1) Thor's hammer

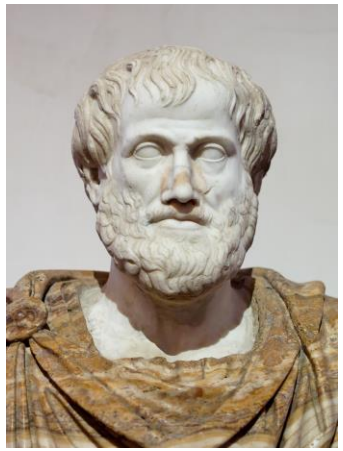
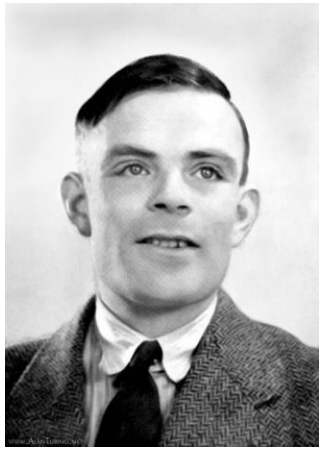
  2) A natural phenomenon due to static electricity discharge.

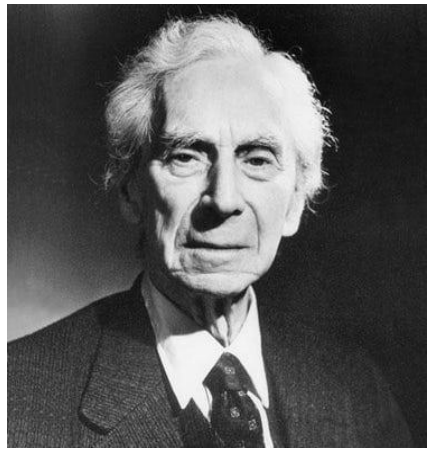
How do we know which is correct? →

Logic

Logic

- Origin: Greek
- logos (In contrast to *Mythos*)
- Aristotle began analysing logic systematically.
- Famous Logicians: *Aristotle*, *Boole*, *Turing*, *Gödel*, *Russel*, *Zadeh*, …




What is Logic?

- The study of the principles of correct reasoning.
- Logic is concerned with the truthfulness of a *chain of statements*.
- There are several different types of logic
- Each type → Particular limits in reasoning
- A statement can be represented in *different logics*
- Usually, a statement can be represented differently in *the same logic*.

Logical Reasoning

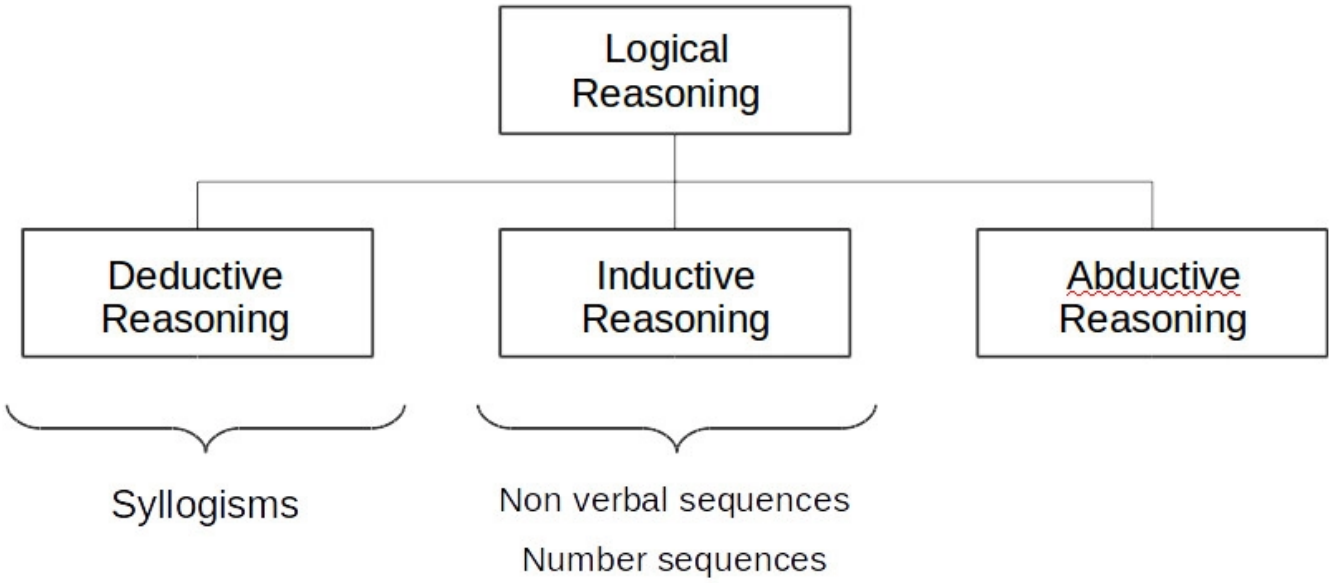
Logical Reasoning - Definition

- **Logical Reasoning = Logic + Reason**
- Form of Critical Thinking
- **Consciously** *making sense of things*.
- Using Logic → The conclusion is true

Logical Reasoning cont

Three types of **logical Reasoning**

1. Deductive Reasoning:
2. Inductive Reasoning
3. Abductive Reasoning



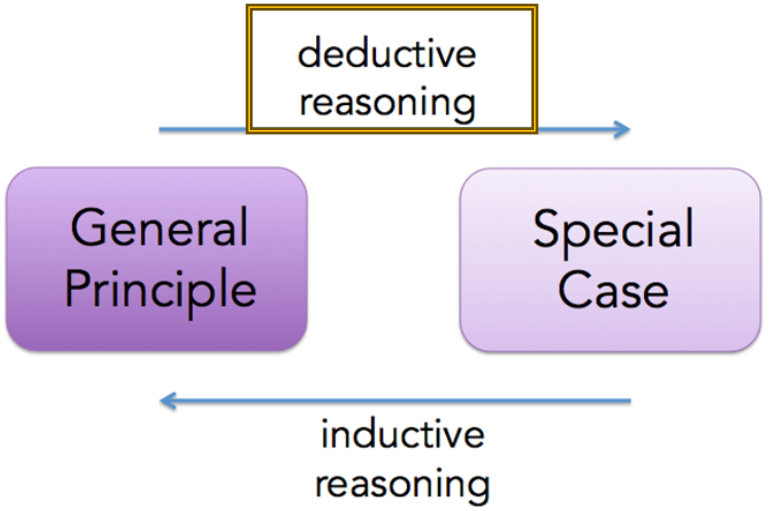
Logical Reasoning cont

Deductive Reasoning:

- The *process of reasoning* from one / more statements → to reach a *logical conclusion*.
- General Statement → Specific Statements

Example:

- P: All men are mortal
- Q: John is a man
- Therefore → R: John is mortal



Logical Reasoning cont

Inductive Reasoning:

- Inductive reasoning is the opposite of deductive reasoning.
- Specific observations → broad generalizations
- Data → Conclusions

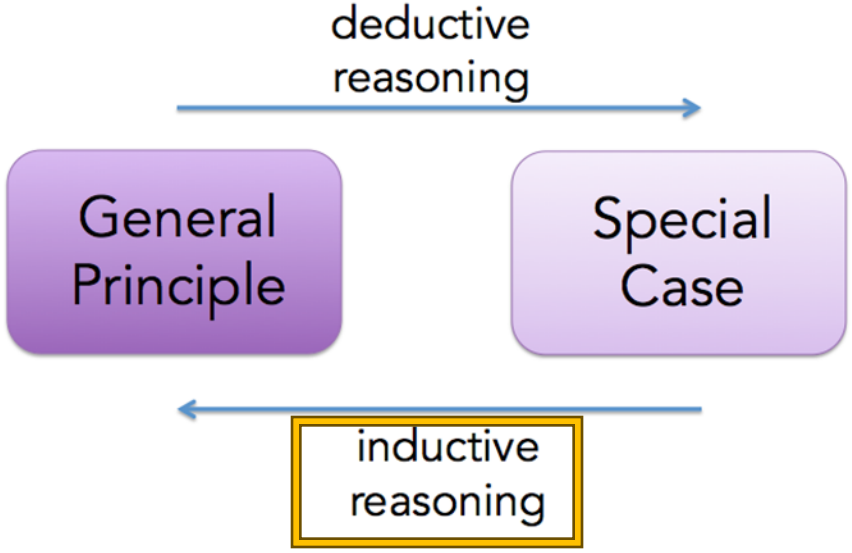
Example:

a) The coin I pulled from the bag is a penny.

b) A second coin I pulled from the bag is a penny.

c) A third coin I pulled from the bag is a penny.

d) Therefore → all the coins in the bag are pennies.



Logical Reasoning cont

Abductive Reasoning:

- Formulated by American philosopher Charles Sanders Peirce - 19th century.
- It consist of:

1) It starts with an observation / set of observations

2) Then seeks to find the simplest & most likely conclusion from the observations

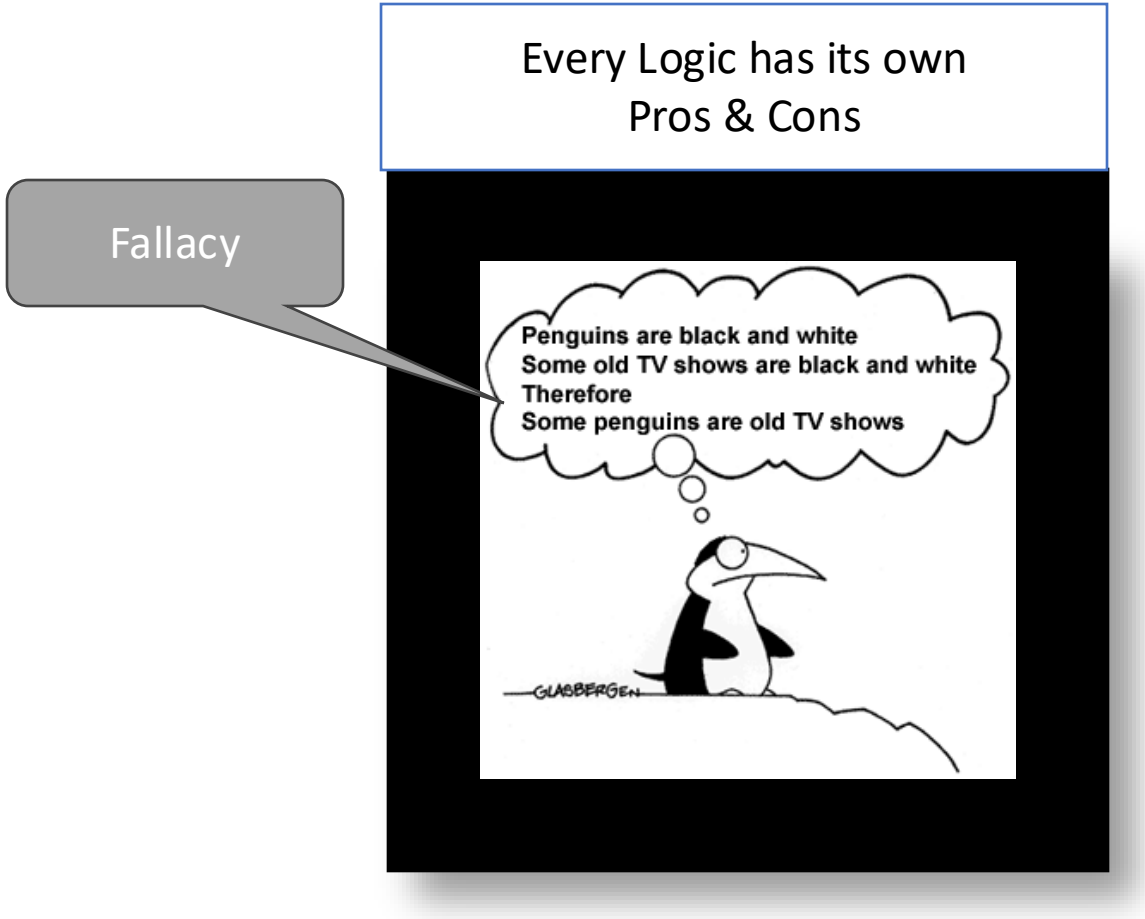
Example:

- Jury duty decisions : Observation → Conclusion
- Hot bowl of soup on a table → someone is eating the soup and will return soon

Logic- Different Types

Such As

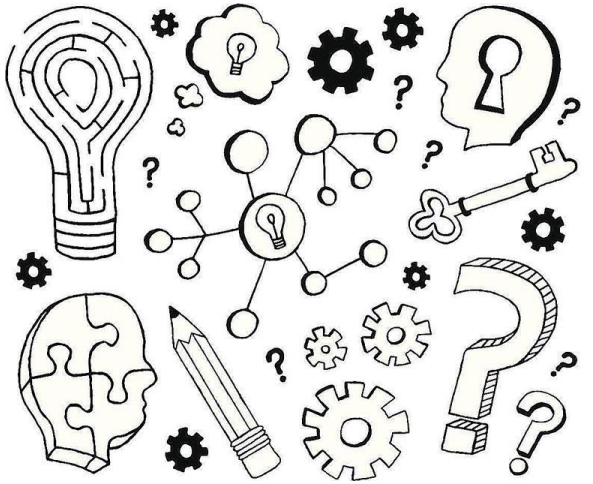
1. Propositional logic
2. Predicate logic
3. Syllogistic logic
4. Modal logic
5. Informal reasoning and dialectic
6. Mathematical logic
7. Philosophical logic
8. *More & More …*



Logic in computer science

- Computational logic : Oldest form of knowledge representation in computer.
- A branch of mathematical logic
- Intersection between mathematical **logic** & computer science.
- Application of Logic in Computer Science
- Its mainly based on:
    1. Propositional Logic: 0^{th} Order Logic
    2. Predicate Logic: 1^{st} Order Logic

Activity 1 (Individual, 10’)



1. What is reasoning? Is reasoning always, right?
2. What is logic?
3. What is the difference between reasoning & logic?
4. What are the different types of logical reasoning? Give an example for each.
5. What are the different types of logic?
6. What is the application of Logic in AI?
7. What is a fallacy? Give an examples.

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Propositional Logic

- A branch of logic.
- It deals with **propositions** and **relations** between them.
- Construction of *arguments* based on propositions.
- Can be traced back to Aristotle.
- AKA: *Zeroth-order logic*.

Proposition

- A *proposition* is a statement that is either true (T) or false (F).

Propositions- Examples:

- p: Two plus two is four.
- q: Toronto is the capital of Canada.
- r: There is an infinite number of primes.

Not propositions- Examples:

- s: What time is it?
- t: Have a nice day!

Propositional variables

- We can use propositional variables to represent propositions.
- propositional variables: p , q, …

- Examples:
- p: January has 31 days.
- q: February has 33 days.

Propositional Value

Proposition's truth value:

- A value indicating
- Whether the proposition is true/ false.
- Logic : Everyone should agree upon the structure, meaning and the value of the proposition

- Examples:
- p: January has 31 days = T
- q: February has 33 days = F

p
T
F

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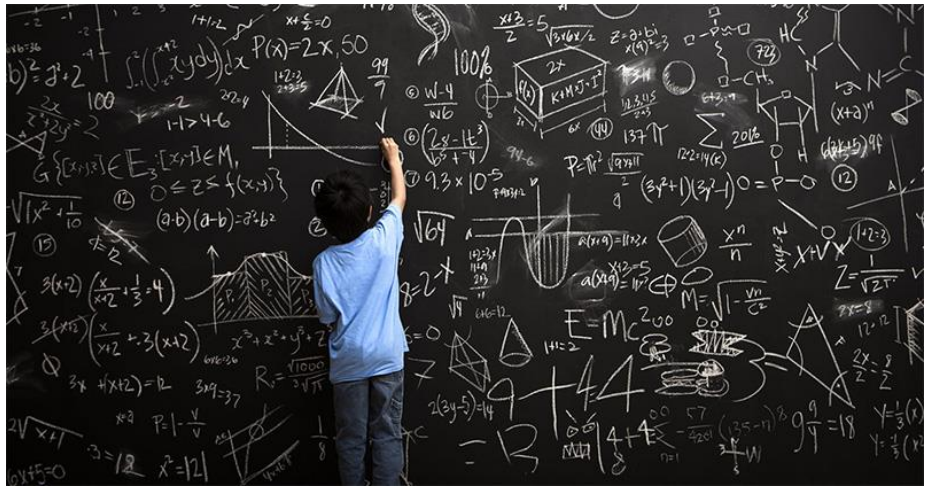
Compound Proposition

- **Compound Proposition**: A **proposition** that involves *assembly of multiple propositions*.
- Compound Propositions may be made thorough **different operators**:
    - Negations
    - Conjunctions
    - Disjunctions
    - Implications
    - Biconditionals
    - etc…

Propositions + Operators → Compound Propositions

Compound Propositions cont

- Language: A **compound proposition** have different meanings
- Logic: A **compound proposition** → has a **Specific value proposition: (T / F)**
- This value proposition is *evident to everyone*



Truth Table

- A truth table is a table used in logic → determines every possible combination of the *variables contained in it*.
- We can define an operator using its *truth table.*
- *Number of possible combinations: 2^n*

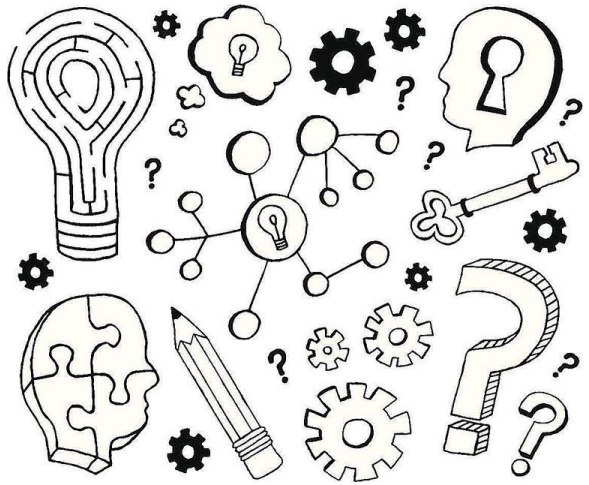
p
T
F

p	q
T	T
T	F
F	T
F	F

Activity 2 (Individual, 10’)

1. What is a proposition? Give an example
2. What are values of a proposition?
3. What is Truth table?
4. How can we make compound propositions? Give an example
5. Why the values of a proposition may change in language?

   Can this happen in a program? Give reason for your answer.



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Negation

- The negation of *p* is denoted by ¬*p*
- In English, this is the statement "not *p*"
- Negation of a proposition
  - T → F
  - F → T

p	¬p
T	F
F	T

The Conjunction Operator

- *Conjunction of p and q* : p ∧ q
- In English, this is the statement "p and q"

p	q	p ∧ q
T	T	T
T	F	F
F	T	F
F	F	F

Example:

- p: January has 31 days.
- q: February has 33 days.
- **p ∧ q**: January has 31 days & February has 33 days.

The Inclusive Disjunction Operator

- The *inclusive disjunction* operation: $p \vee q$
- The proposition is read "p or q" .

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Example:

- p: January has 31 days.
- q: February has 33 days.
- **$p \vee q$**: January has 31 days or February has 33 days.

The Exclusive Disjunction Operator

- The exclusive disjunction Operator : $p \oplus q$
- The proposition $p \oplus q$ is read: "p xor q"
- It is true → (one is true & the other is false)

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Example: When a parent tells their child :

- P: you can have chocolate
- q: you can have a lollypop
- $p \oplus q$: *child can have one of the two but NOT both.*

The Implication Operator

- $p \Rightarrow q$: The Implication operation
- P implies q *(If p then q)*
- $(p \Rightarrow q) \equiv (\neg p \vee q)$ [Equivalent*]

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

* Propositions are equivalent,

if their truth tables are identical.

Example:

- p:It is a rainy day
- q: I should take an umbrella
- If "it is a rainy day", then "I should take an umbrella"

The Biconditional Operator

- $p \Leftrightarrow q$: The Biconditional Operator
- p if & only if q
- $p \Leftrightarrow q \equiv (p \Rightarrow q) \wedge (q \Rightarrow p)$

p	q	$p \Leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Example:

- The polygon has only four sides ⇒ the polygon is a quadrilateral.
- The polygon is a quadrilateral ⇒ the polygon has only four sides.
  - The polygon is a quadrilateral if and only if the polygon has only four sides. $(p \Leftrightarrow q)$

Summary of operator truth tables

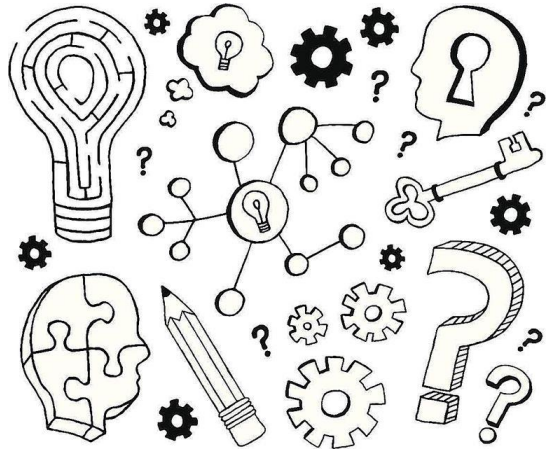
p	q	$p \wedge q$	$p \vee q$	$p \oplus q$	$\neg p$
T	T	T	T	F	F
T	F	F	T	T	F
F	T	F	T	T	T
F	F	F	F	F	T

Truth Tables (in Python)

a	b	a and b	a or b	a^b	not a
True	True	True	True	False	False
True	False	False	True	True	False
False	True	False	True	True	True
False	False	False	False	False	True

Notice the different notation for the operators.
Here a and b are Boolean (`bool`) variables.

Activity 3 (Individual, 15')



Using the truth table, show:

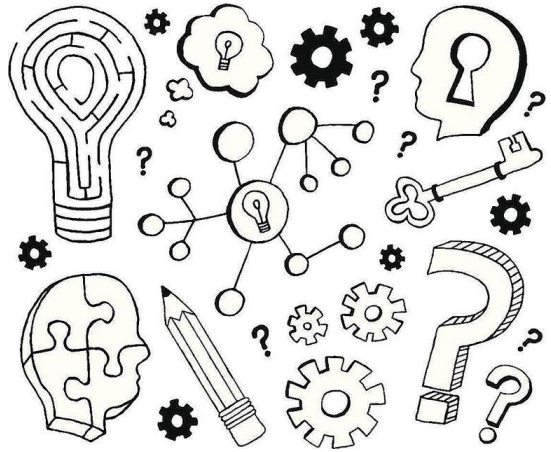
1. $(p \Rightarrow q) \equiv (\neg p \vee q)$
2. $(p \Leftrightarrow q) \equiv (q \Leftrightarrow p)$
3. What is the truth table for: $(p \vee \neg q) \vee (p \wedge q)$
4. Complete the following truth table :

$\neg p$	$\neg q$	$\neg p \wedge \neg q$	$\neg p \vee \neg q$	$\neg p \oplus q$	$p \oplus \neg q$	$\neg p \oplus \neg q$

Exercise 3.3- Answer

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \vee (p \wedge q)$
T	T	F	T	T	T
T	F	T	T	F	T
F	T	F	F	F	F
F	F	T	T	F	T

Exercise 3.4- Answer



Complete the following truth table :

$\neg p$	$\neg q$	$\neg p \wedge \neg q$	$\neg p \vee \neg q$	$\neg p \oplus q$	$p \oplus \neg q$	$\neg p \oplus \neg q$
F	F	F	F	T	T	F
F	T	F	T	F	F	T
T	F	F	T	F	F	T
T	T	T	T	T	T	F

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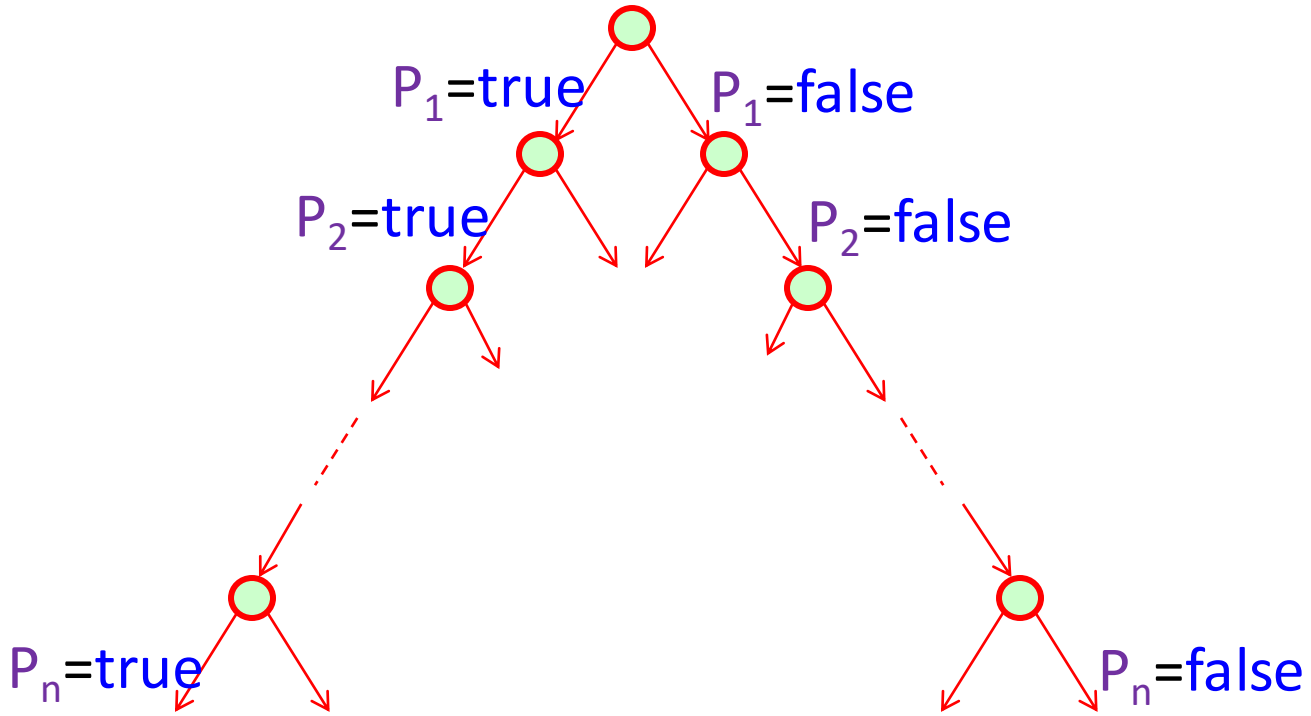
Combination of 3 & More Propositions

- Let p, q & r be three propositions.
- We can make compound propositions using p, q, & r
- To create the "Truth Table" → use the following strategy



Combination of 3 & More Propositions cont

- Number of rows in the truth table of n variables → 2^n

p	q	r
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

Different Combinations of Propositions

- Having 2 or more variables → different compound propositions using different operators
- Example: p, q & r can form different combinations; such as:

I. $(p \wedge (q)) \wedge r$

II. $(p \vee (q)) \wedge r$

III. $(p \wedge (q)) \vee r$

IV. $(p \vee (q)) \vee r$

V. More & more

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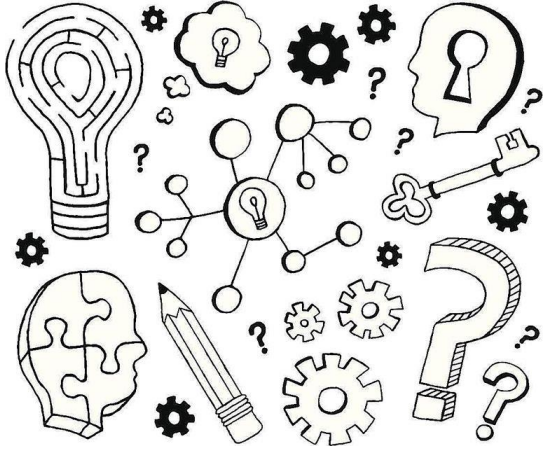
VI. Tutorials

Reflection



- Read & summarize the below article regarding the application of propositional logic in AI.

  https://www.javatpoint.com/propositional-logic-in-artificial-intelligence

Exercise 1

- Evaluate proposition: $(p \vee (\neg q)) \wedge r$
- Where, the variables have the values: p:F, q:F, r:F

$(p \vee (\neg q)) \wedge r$

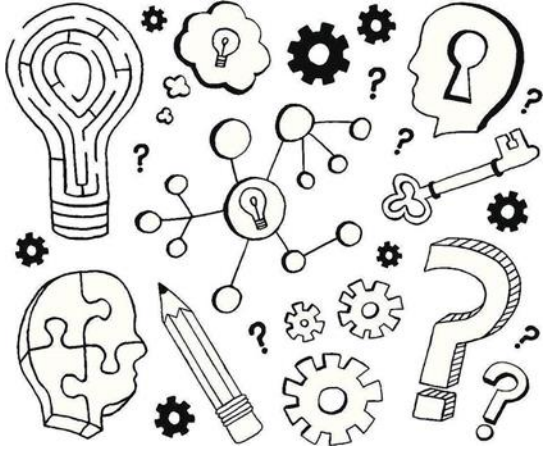
a. $(F \vee (\neg F)) \wedge F$

b. $(F \vee T) \wedge F$

c. $(T) \wedge F$

d. F

Exercise 2



- Evaluate proposition: $(p \vee (\neg q)) \wedge r$; Where, the variables have the values: p:F, q:F, r:F

$(p \vee (\neg q)) \wedge r$

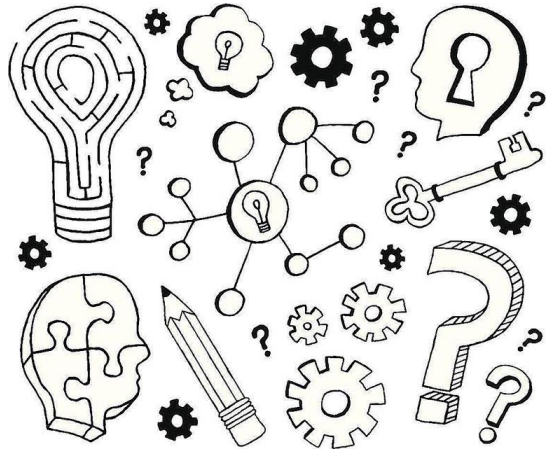
a. $(F \vee (\neg F)) \wedge F$

b. $(F \vee T) \wedge F$

c. $(T) \wedge F$

d. F

Exercise 3



1. What is the truth table for: **$\neg q \wedge (p \vee r)$**
2. What is the truth table for: **$(p \vee r) \wedge \neg q$**
3. Describe in words when the expression $p \vee q \vee r \vee s \vee t$ is true and when it is false.
4. Describe in words when the expression $p \wedge q \wedge r \wedge s \wedge t$ is true and when it is false.

Use intermediate columns,

Exercise 3.1- Answer

p	q	r	¬q	(p ∨ r)	¬q ∧ (p ∨ r)
T	T	T	F	T	F
T	T	F	F	T	F
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	T	F
F	T	F	F	F	F
F	F	T	T	T	T
F	F	F	T	F	F

Exercise 3.2-Answer

p	q	r	$(p \vee r) \wedge \neg q$
T	T	T	F
T	T	F	F
T	F	T	T
T	F	F	T
F	T	T	F
F	T	F	F
F	F	T	T
F	F	F	F

Next Week

We will continue studying about Logic:

- Boolean Logic
- Predicate Logic

Source of the slides:

Thomas Calculus – 11e

Stewart Calculus

https://www.slideserve.com/search/presentations/derivatives-and-integrals

https://www.xpowerpoint.com/ppt/first-order-logic-ai.html