8
Rules for Differentiation

Evaluating the derivative using the definition can be time consuming. Although we have some techniques to help us evaluate limits, if the only way to take the derivative of a function was to evaluate $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ algebraically, calculus would not be as popular as it is. In this chapter we will discuss some short cuts for taking the derivative of a function. Two important rules that we will discuss are the product rule and the quotient rule.

Lesson 8-1: Sums and Differences

There are a few functions whose derivative is relatively easy to determine just by using the *interpretation* of the derivative. The derivative of a function represents the slope of the line tangent to the graph of the function. We will first consider the constant function: $f(x) = a$. The graph of this function is the horizontal line $y = a$. Lines that are tangent to this line will also be horizontal lines, and the slope of a horizontal line is 0. Therefore, the derivative of a constant is 0. This is true for *any* constant.

The next function to analyze is $f(x) = mx$. The graph of this function is a line with slope m. Lines tangent to this function will also have slope m, and the *derivative* of the function $f(x) = mx$ is m.

The rule for taking the derivative of the sum of two functions is based on the fact that the limit of a sum is the sum of the limits:

The derivative of the sum is the sum of the derivatives.

This means that if $f(x) = g(x) + h(x)$, finding $f'(x)$ can be done by finding $g'(x)$ and $h'(x)$ individually, and then adding the results. A similar relationship holds for the derivative of the difference between two functions: $(f(x) - g(x))' = f'(x) - g'(x)$.

We can use these two results and the first property of the derivative (the derivative of the sum is the sum of the derivative) to find the derivative of any *linear* function.

Example 1

Find the derivative of the following functions:

a. $f(x) = 3x + 4$

b. $f(x) = \pi x + e^2$

c. $f(x) = \left(\sin\frac{\pi}{4}\right)x + \cosh 2$

Solution: All of these functions are linear, with slope equal to the coefficient in front of x. The derivative of these functions will be their respective slopes:

a. $f'(x) = 3$

b. $f'(x) = \pi$

c. $f'(x) = \left(\sin\frac{\pi}{4}\right)$

There is another property of limits that will help us find the derivative of more complicated functions. The second property of limits states that $\lim_{x\to c}\left[k \cdot f(x)\right] = k \cdot \lim_{x\to c} f(x)$. In other words, the constant can be brought outside of the limit, because a constant does not care what x approaches. When taking the derivative of the product of a constant and a function, the constant waits patiently while the function is being differentiated. The constant does not go away, but it also does not change. This rule can be written as $(kf(x))' = kf'(x)$.

Lesson 8-1 Review

Find the derivative of the following functions:

1. $f(x) = 4x + \ln 5$
2. $f(x) = -2x - 4$
3. $f(x) = \frac{1}{3}x + 5$

Lesson 8-2: The Power Rule

The power rule enables us to find the derivative of any power function. Using the fact that the derivative of the sum is the sum of the derivative, you will then be able to find the derivative of any polynomial. The power rule will also play a role in establishing the chain rule. The power rule is as follows:

If $f(x) = x^n$ where a and n are constants, then $f'(x) = nx^{n-1}$.

We can practice using this first short cut for taking the derivative.

Example 1

Differentiate the following functions:

a. $f(x) = x^4$

b. $g(x) = \sqrt{x}$

c. $h(x) = x^{-3}$

d. $f(x) = \frac{1}{x^2}$

Solution: Apply the power rule: If $f(x) = x^n$, then $f'(x) = nx^{n-1}$. If the function is not explicitly written in the form $f(x) = x^n$, rewrite the function so that it is in this form.

a. $f(x) = x^4$: $f'(x) = 4x^{4-1} = 4x^3$

b. $g(x) = \sqrt{x}$: Rewrite this function so that the exponent is a fraction. Then use the power rule: $g(x) = x^{1/2}$, and $g'(x) = \frac{1}{2}x^{1/2-1} = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

c. $h(x) = x^{-3}$: $h'(x) = -3x^{-3-1} = -3x^{-4}$

d. $f(x) = \frac{1}{x^2}$: Rewrite this function so that the exponent is a negative integer. Then use the power rule: $f(x) = x^{-2}$ and $f'(x) = -2x^{-2-1} = -2x^{-3}$.

It is important that you avoid the traps set by disguised power functions. Rewrite the functions so that you can use the power rule directly. It may be tempting to save time by trying to work with functions such as $f(x) = \frac{1}{x^2}$ or

$g(x)=\sqrt{x}$ without rewriting the function. If you try to skip steps, the result could be costly. By taking a little extra time, you can be sure that you are taking the derivative correctly.

Now we are ready to take the derivative of more complicated functions. We can differentiate any polynomial, any sum of power functions, or any constant times a power function.

Example 2

Differentiate the following functions:

a. $g(x)=\sqrt{x}$

b. $h(x) = 5x^{-3} - 3x^2 + 2$

c. $f(x)=4x^3+6\sqrt{x}$

Solution: Apply the power rule and the properties of the derivative. Rewrite any power functions that are not in their proper format.

a. $f(x)=4x^3+6\sqrt{x}=4x^3+6x^{1/2}$:

$f'(x)=4\left(3x^{3-1}\right)+6\left(\frac{1}{2}x^{1/2-1}\right)=12x^2+3x^{-1/2}$

b. $h(x) = 5x^{-3} - 3x^2 + 2$: $h'(x) = 5(-3x^{-3-1}) - 3(2x^{2-1}) + 0 = -15x^{-4} - 6x$

c. $f(x)=\frac{2}{x^2}-\frac{3}{x^5}=2x^{-2}-3x^{-5}$:

$f'(x) = 2(-2x^{-2-1}) - 3(-5x^{-5-1}) = -4x^{-3} + 15x^{-6}$

Now that we can find the derivative of a variety of functions, we can practice using the derivative to find the equations of tangent lines. To find the equation of a tangent line, we need a point and a slope. The point can be found by evaluating the *function* at the specific x–coordinate. The slope can be found by evaluating the *derivative* of the function at the specific *x*-coordinate.

Example 3

Find the equation of the line tangent to the following functions at the point indicated. Write your equations in slope-intercept form.

a. $f(x)=3x^2+\sqrt{x}$ at $x = 1$

b. $g(x)=\frac{4}{x}$ at $x=2$

Solution: Find the point and the slope, and then use the point-slope formula:

a. $f(x)=3x^2+\sqrt{x}$: at $x = 1$: The point is (1,4). To find the slope, evaluate the derivative at $x = 1$: $f'(x)=6x+\frac{1}{2}x^{-½}$, and $f'(1)=\frac{13}{2}$. Use the point-slope formula:

$$y-4=\frac{13}{2}(x-1)$$
$$y=\frac{13}{2}x-\frac{5}{2}$$

The graphs of $f(x)=3x^2+\sqrt{x}$ and of $y=\frac{13}{2}x-\frac{5}{2}$ are shown in Figure 8.1.

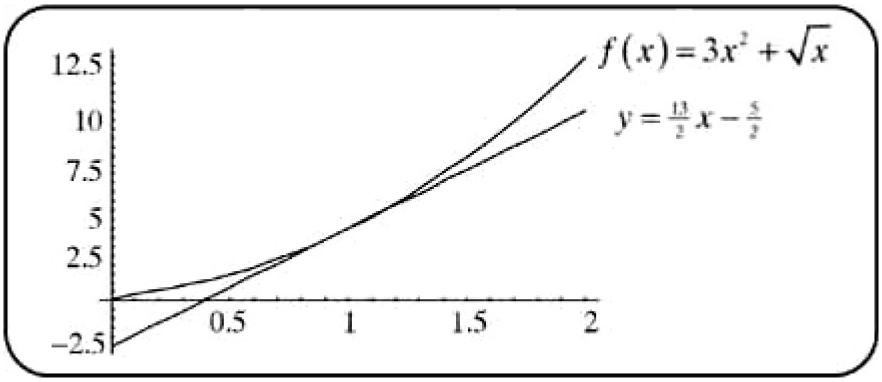

Figure 8.1.

b. $g(x)=\frac{4}{x}$ at $x=2$: The point is (2, 2). To find the slope, evaluate the derivative at $x = 2$: $g'(x) = -4x^{-2}$, and $g'(2) = -1$. Use the point-slope formula:

$$y-2=-1(x-2)$$
$$y=-x+4$$

The graphs of $g(x)=\frac{4}{x}$ and $y = -x + 4$ are shown in Figure 8.2.

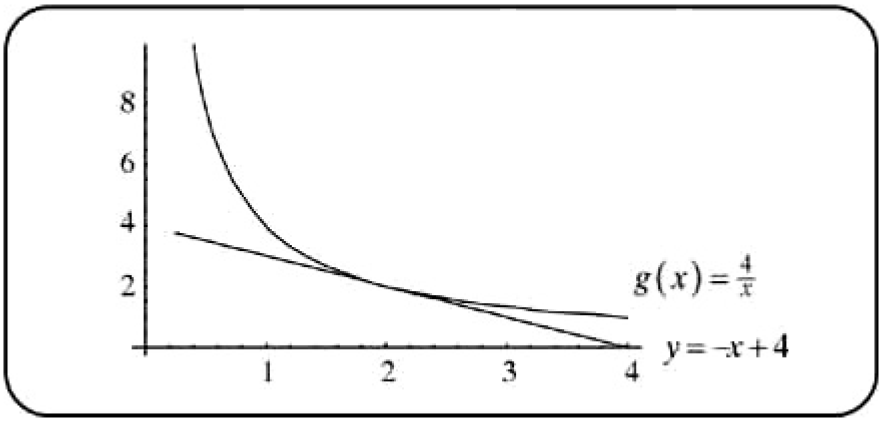

Figure 8.2.

Lesson 8-2 Review

1. Differentiate the following functions:

   a. $f(x) = \sqrt[3]{x} + \frac{2}{x}$

   b. $g(x) = 8x - \frac{3}{x^2}$

2. Find the equation of the line tangent to the following functions at the point indicated. Write your equations in slope-intercept form.

   a. $f(x) = x^4 + 2x + 1$ at $x = 1$

   b. $g(x) = \frac{4}{x} - 2x$ at $x = 2$

Lesson 8-3: The Product Rule

We can now turn our attention to differentiating more complicated functions. The derivative of a function measures how a change in the independent variable affects, or changes, the dependent variable. The difference quotient is the ratio of these changes: $\frac{\Delta f}{\Delta x}$, and the derivative is $\lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x}$. In this lesson, we will examine how a product of functions changes as the independent variable changes.

As the independent variable of a product of two functions changes, *both* of the functions involved in the product will change. Too much change can result in a sensory overload, so we only allow one function to change at a time. The product rule is as follows:

$$(f(x)\cdot g(x))' = f'(x)\cdot g(x) + f(x)\cdot g'(x)$$

In other words, when taking the derivative of the product of two functions, take the derivative of the first function (changed) and multiply it by the second function (unchanged). Then take the derivative of the second function (changed) and multiply it by the first function (unchanged). Add the two results together. In effect, the derivative of the product of two functions is the derivative of the first times the second plus the first one left alone times the derivative of the second. Not only is it important that you learn what the product rule says, but how to use it correctly.

Example 1

Differentiate the following functions. Do not simplify.

a. $f(x) = (4x^2 + 1)(6x - 2)$

b. $g(x) = \left(\sqrt{x} + \frac{1}{x}\right)(x - 2)$

Solution: Apply the product rule:

a. $f(x) = (4x^2 + 1)(6x - 2)$:

$f'(x) = (4x^2 + 1)'(6x - 2) + (4x^2 + 1)(6x - 2)'$

$f'(x) = (8x)(6x - 2) + (4x^2 + 1)(6)$

b. $g(x) = \left(\sqrt{x} + \frac{1}{x}\right)(x - 2)$:

$$g'(x) = \left(\sqrt{x} + \frac{1}{x}\right)'(x - 2) + \left(\sqrt{x} + \frac{1}{x}\right)(x - 2)'$$

$$g'(x) = \left(x^{1/2} + x^{-1}\right)'(x - 2) + \left(\sqrt{x} + \frac{1}{x}\right)(x - 2)'$$

$$g'(x) = \left(\tfrac{1}{2}x^{-1/2} - x^{-2}\right)(x - 2) + \left(\sqrt{x} + \frac{1}{x}\right)(1)$$

Pay attention to the notation. When I apply the product rule, I put *primes* on the pieces that need to be differentiated. The product rule says to take the derivative of the first times the second plus the first one left alone times the derivative of the second. Notice that the first step in differentiating the product of two functions is to figure out which piece needs to be differentiated at each stage. Take an extra step to apply the product rule correctly and then take the derivatives of each of the individual functions. Use a systematic approach with these problems. If you establish a routine early and are careful at each step along the way, differentiating products of functions should be no problem.

In the previous examples, I specifically indicated that you should not simplify. Right now, the focus is on the mechanics of taking the derivative. Multiplying polynomials together and collecting terms is a skill that has been practiced enough in algebra; you should be able to simplify these expressions when necessary.

We can now find equations of lines tangent to more complicated functions at a given point. Tangent lines are important in calculus, and we will discuss them in more detail in Chapter 11. For now, we will focus on the *mechanics* of finding equations of tangent lines.

Example 2

Find the equation of the line tangent to the given functions at the indicated point.

a. $f(x)=\left(3x^3+\frac{2}{\sqrt{x}}\right)\left(6x^2-2x\right)$ at $x=1$

b. $h(x)=\left(\frac{2}{x^3}-\frac{3}{x^4}\right)\left(2x^4-7x^2+5\right)$ at $x=1$

Solution: Evaluate the function at the given *x*-coordinate to find the point, and find the slope by evaluating the *derivative* of the function at the given *x*-coordinate. Then use the point-slope formula. To verify our answers, both the function and the tangent line will be graphed. The tangent lines should just touch the graph of the functions at the given point.

a. $f(x)=\left(3x^3+\frac{2}{\sqrt{x}}\right)\left(6x^2-2x\right)$ at $x = 1$: The point is (1, 20).

The derivative is:

$$f'(x)=\left(3x^3+\frac{2}{\sqrt{x}}\right)'\left(6x^2-2x\right)+\left(3x^3+\frac{2}{\sqrt{x}}\right)\left(6x^2-2x\right)'$$

$$f'(x)=\left(9x^2+2\left(-\frac{1}{2}x^{-3/2}\right)\right)\left(6x^2-2x\right)+\left(3x^3+\frac{2}{\sqrt{x}}\right)\left(12x-2\right)$$

$f'(1) = 82$

Finally, use the point-slope formula:

$y - 20 = 82(x - 1)$

$y = 82x - 62$

The graphs of $f(x) = \left(3x^3 + \frac{2}{\sqrt{x}}\right)\left(6x^2 - 2x\right)$ at $x = 1$ and $y = 82x - 62$ are shown in Figure 8.3.

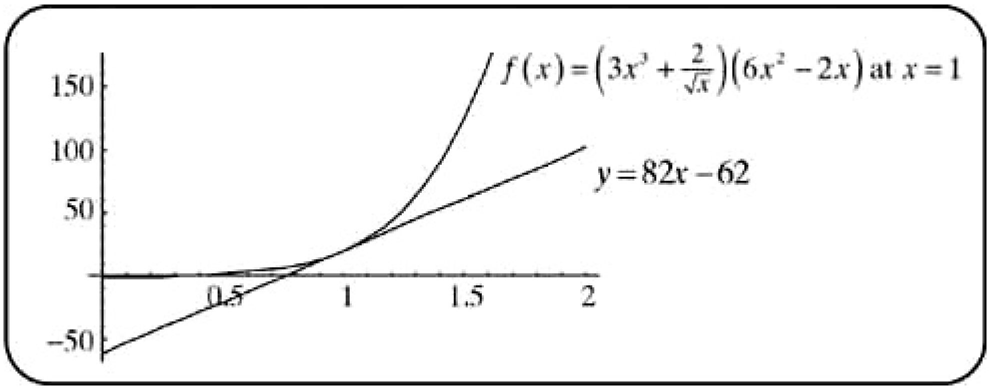

Figure 8.3

b. $h(x) = \left(\frac{2}{x^3} - \frac{3}{x^4}\right)\left(2x^4 - 7x^2 + 5\right)$ at $x = 1$: The point is (1, 0). To find the slope of the tangent line, evaluate $h'(1)$:

$$h'(x) = \left(\frac{2}{x^3} - \frac{3}{x^4}\right)'\left(2x^4 - 7x^2 + 5\right) + \left(\frac{2}{x^3} - \frac{3}{x^4}\right)\left(2x^4 - 7x^2 + 5\right)'$$

$$h'(x) = \left(-6x^{-4} + 12x^{-5}\right)\left(2x^4 - 7x^2 + 5\right) + \left(\frac{2}{x^3} - \frac{3}{x^4}\right)\left(8x^3 - 14x\right)$$

$$h'(1) = (-6 + 12)(2 - 7 + 5) + (2 - 3)(8 - 14) = 6$$

Finally, use the point-slope formula:

$y - 0 = 6(x - 1)$

$y = 6x - 6$

The graphs of $h(x) = \left(\frac{2}{x^3} - \frac{3}{x^4}\right)\left(2x^4 - 7x^2 + 5\right)$ and $y = 6x - 6$ are shown in Figure 8.4.

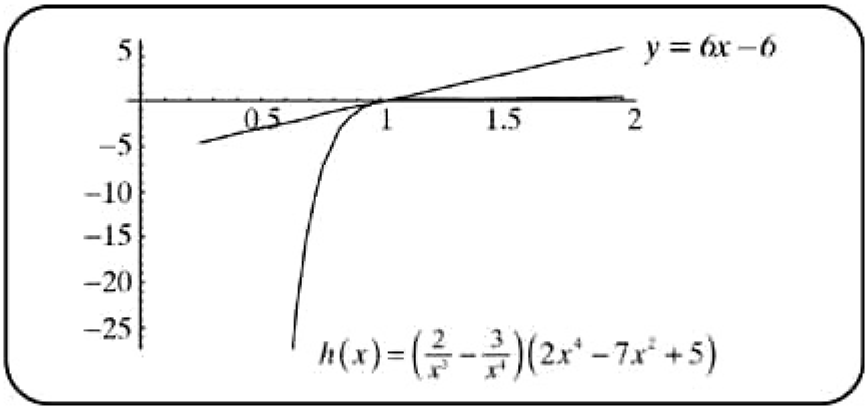

Figure 8.4

Lesson 8-3 Review

1. Differentiate the following functions:

   a. $f(x) = (4x^6 - 3x^2 + 2)(x^9 - 8x + 1)$

   b. $g(x)=\left(\sqrt{x}-5x\right)\left(2x^4-7x^2+5\right)$

2. Find the equation of the line tangent to the graph of the following functions at the indicated point:

   a. $f(x)=\left(x+\frac{3}{x}\right)\left(x^4-2x+1\right)$ at $x = 1$

   b. $g(x)=\left(\sqrt{x}-2\right)\left(\sqrt{x}+5\right)$ at $x = 1$

Lesson 8-4: The Quotient Rule

Another important differentiating rule is the quotient rule. The quotient rule will enable us to evaluate the derivative of the quotient, or ratio, of two functions. It is similar to the product rule, but there are enough differences that you need to be careful when applying it. The derivative of the quotient of two functions is given by the following formula:

$$\left(\frac{f(x)}{g(x)}\right)'=\frac{f'(x)g(x)-f(x)g'(x)}{(g(x))^2}$$

Notice the similarities between the quotient rule and the product rule. The numerator of the derivative involves taking the derivative of the numerator and the derivative of the denominator of the function one at a time, just as we did using the product rule. Instead of the results being added, they are subtracted. This rule actually comes from the product rule, though we can also derive it using the chain rule, which we will discuss in Chapter 10. Right now, we will focus on applying the quotient rule and reinforcing our tangent line skills.

Example 1

Differentiate the following functions. Do not simplify.

a. $f(x)=\frac{(4x^2+1)}{(6x-2)}$

b. $g(x)=\frac{\left(\sqrt{x}+\frac{1}{x}\right)}{(x^2+1)}$

Solution: Apply the quotient rule.

a.
$$f(x)=\frac{(4x^2+1)}{(6x-2)}: \quad f'(x)=\frac{(4x^2+1)'(6x-2)-(4x^2+1)(6x-2)'}{(6x-2)^2}$$

$$f'(x)=\frac{(8x)(6x-2)-(4x^2+1)(6)}{(6x-2)^2}$$

b.
$$g(x)=\frac{\left(\sqrt{x}+\frac{1}{x}\right)}{(x^2+1)}: \quad g'(x)=\frac{\left(\sqrt{x}+\frac{1}{x}\right)'(x^2+1)-\left(\sqrt{x}+\frac{1}{x}\right)(x^2+1)'}{(x^2+1)^2}$$

$$g'(x)=\frac{\left(\frac{1}{2}x^{-1/2}-x^{-2}\right)(x^2+1)-\left(\sqrt{x}+\frac{1}{x}\right)(2x)}{(x^2+1)^2}$$

We can find equations of tangent lines and work out other types of applications.

Example 2

Where does the graph of the function $f(x)=\frac{x}{x^2+1}$ have a horizontal tangent line?

Solution: The slope of the tangent line can be found by taking the derivative of the function. The slope of a horizontal line is 0. In order to answer this question, we need to find where the derivative is equal to 0. First, find the derivative using the quotient rule:

$$f'(x)=\frac{(x)'\left(x^2+1\right)-(x)\left(x^2+1\right)'}{\left(x^2+1\right)^2}$$

$$f'(x)=\frac{(1)\left(x^2+1\right)-(x)(2x)}{\left(x^2+1\right)^2}$$

Now, set the derivative equal to 0 and solve for *x*:

$$\frac{(1)\left(x^2+1\right)-(x)(2x)}{\left(x^2+1\right)^2}=0$$

$$\frac{\left(x^2+1\right)-2x^2}{\left(x^2+1\right)^2}=0$$

$$\frac{-x^2+1}{\left(x^2+1\right)^2}=0$$

Now, $f'(x) = 0$ when the numerator is equal to 0:

$$-x^2 + 1 = 0$$

$$x^2 = 1$$

$$x^2 = \pm 1$$

The graph of the function $f(x)=\frac{x}{x^2+1}$ has horizontal tangent lines at $x = \pm 1$.

The graph of $f(x)=\frac{x}{x^2+1}$ is shown in Figure 8.5. This function is an odd function. We can see this algebraically:

$$f(-x)=\frac{(-x)}{(-x)^2+1}=-\frac{x}{x^2+1}=-f(x)$$

We can also see this because the numerator is an odd function and the denominator is an even function, and an odd function divided by an even function will be an odd function. Either way, we expect the graph to be symmetric about the origin, and we expect the tangent lines to be horizontal at $x = \pm 1$.

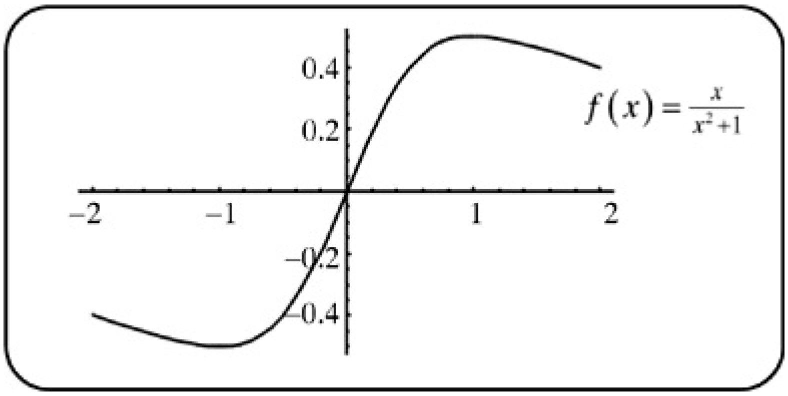

Figure 8.5

We can differentiate some functions by using either the power rule or the quotient rule. Let's differentiate the function $f(x)=\frac{4}{x}$ using both techniques:

Power Rule	Quotient Rule
$f(x)=4x^{-1}$	$f(x)=\frac{4}{x}$
$f'(x)=-4x^{-2}$	$f'(x)=\frac{(4)'(x)-(4)(x)'}{(x)^2}$
	$f'(x)=\frac{(0)(x)-(4)(1)}{(x)^2}$
	$f'(x)=-\frac{4}{x^2}=-4x^{-2}$

Either way, we get the same result. It is natural to treat $f(x)=\frac{4}{x}$ as a quotient and differentiate it using the quotient rule, but using the power rule involves fewer steps and less simplification. Regardless of the rule you use to take a derivative, if you apply the rule correctly, and carefully differentiate each piece of the function at the right time, you cannot go wrong.

Lesson 8-4 Review

1. Differentiate the following functions:

a. $f(x)=\dfrac{\left(\frac{2}{x}-4x\right)}{\left(2x^4-7x^2+5\right)}$

b. $g(x)=\dfrac{x^4+7x+1}{\sqrt{x}+10}$

2. Find the equation of the line tangent to the following functions at the indicated point:

a. $f(x)=\dfrac{4x+2}{x^2+1}$ at $x=0$

b. $g(x)=\dfrac{2x-1}{3x+2}$ at $x=-1$

3. Where does the graph of the function $f(x)=\dfrac{(2x+1)}{(x^2+4)}$ have a horizontal tangent line?

4. Where does the graph of the function $f(x)=\dfrac{(2x+1)}{(x+4)}$ have a tangent line with slope equal to 1?

Answer Key

Lesson 8-1 Review

1. $f'(x) = 4$

2. $f'(x) = -2$

3. $f'(x) = \frac{1}{3}$

Lesson 8-2 Review

1. a. $f(x) = \sqrt[3]{x} + \frac{2}{x} = x^{1/3} + 2x^{-1}$, $f'(x) = \frac{1}{3}x^{-2/3} - 2x^{-2}$

   b. $g(x) = 8x - \frac{3}{x^2} = 8x - 3x^{-2}$, $g'(x) = 8 + 6x^{-3}$

2. a. $f(x) = x^4 + 2x + 1$ at $x = 1$:

   The point is (1, 4), $f'(x) = 4x^3 + 2$, so the slope is $f'(1) = 6$.

   The equation of the tangent line is $y - 4 = 6(x - 1)$, or $y = 6x - 2$.

   b. $g(x) = \frac{4}{x} - 2x$ at $x = 2$:

   The point is (2, −2), $g'(x) = -4x^{-2} - 2$, so the slope is $g'(2) = -3$.

   The equation of the tangent line is $y + 2 = -3(x - 2)$, or $y = -3x + 4$.

Lesson 8-3 Review

1. a. $f(x) = (4x^6 - 3x^2 + 2)(x^9 - 8x + 1)$

   $f'(x) = (4x^6 - 3x^2 + 2)'(x^9 - 8x + 1) + (4x^6 - 3x^2 + 2)(x^9 - 8x + 1)'$

   $f'(x) = (24x^5 - 6x)(x^9 - 8x + 1) + (4x^6 - 3x^2 + 2)(9x^8 - 8)$

   $g(x) = (\sqrt{x} - 5x)(2x^4 - 7x^2 + 5)$

   $g'(x) = (\sqrt{x} - 5x)'(2x^4 - 7x^2 + 5) + (\sqrt{x} - 5x)(2x^4 - 7x^2 + 5)'$

   b. $g'(x) = (\frac{1}{2}x^{-1/2} - 5)(2x^4 - 7x^2 + 5) + (\sqrt{x} - 5x)(8x^3 - 14x)$

2. a. $f(x) = (x + \frac{3}{x})(x^4 - 2x + 1)$ at $x = 1$:

   The point is (1,0), and $f'(x) = (1 - 3x^{-2})(x^4 - 2x + 1) + (x + \frac{3}{x})(4x^3 - 2)$, so the slope is f'(1) = 8.

   The equation of the tangent line is $y - 0 = 8(x - 1)$, or $y = 8x - 8$.

b. $g(x)=\left(\sqrt{x}-2\right)\left(\sqrt{x}+5\right)$ at $x=1$:

The point is (1, −6), and $g'(x)=\left(\frac{1}{2}x^{-1/2}\right)\left(\sqrt{x}+5\right)+\left(\sqrt{x}-2\right)\left(\frac{1}{2}x^{-1/2}\right)$,

so the slope is $g'(1)=\frac{5}{2}$

The equation of the tangent line is $y+6=\frac{5}{2}(x-1)$, or $y=\frac{5}{2}x-\frac{17}{2}$.

Lesson 8-4 Review

1. a. $f(x)=\frac{\left(\frac{2}{x}-4x\right)}{\left(2x^4-7x^2+5\right)}$: $f'(x)=\frac{\left(\frac{2}{x}-4x\right)'\left(2x^4-7x^2+5\right)-\left(\frac{2}{x}-4x\right)\left(2x^4-7x^2+5\right)'}{\left(2x^4-7x^2+5\right)^2}$

$$f'(x)=\frac{\left(-2x^{-2}-4\right)\left(2x^4-7x^2+5\right)-\left(\frac{2}{x}-4x\right)\left(8x^3-14x\right)}{\left(2x^4-7x^2+5\right)^2}$$

$g(x)=\frac{x^4+7x+1}{\sqrt{x}+10}$: $g'(x)=\frac{\left(x^4+7x+1\right)'\left(\sqrt{x}+10\right)-\left(x^4+7x+1\right)\left(\sqrt{x}+10\right)'}{\left(\sqrt{x}+10\right)^2}$

b. $g'(x)=\frac{\left(4x^3+7\right)\left(\sqrt{x}+10\right)-\left(x^4+7x+1\right)\left(\frac{1}{2}x^{-1/2}\right)}{\left(\sqrt{x}+10\right)^2}$

2. a. $f(x)=\frac{4x+2}{x^2+1}$ at $x=0$:

The point is (0,2) and $f'(x)=\frac{(4)\left(x^2+1\right)-(4x+2)(2x)}{\left(x^2+1\right)^2}$, so the slope is $f'(0)=4$.

The equation of the tangent line is $y-2=4(x-0)$, or $y=4x+2$.

b. $g(x)=\frac{2x-1}{3x+2}$ at $x=-1$:

The point is (−1,3) and $g'(x)=\frac{(2)(3x+2)-(2x-1)(3)}{(3x+2)^2}$, so the slope is $g'(0)=7$

The equation of the tangent line is $y-3=7(x+1)$, or $y=7x+10$.

3. Take the derivative, set the numerator of the derivative equal to 0, and solve for *x:*

$$f'(x)=\frac{(2)(x^2+4)-(2x+1)(2x)}{(x^2+4)^2}=0$$

$$(2)(x^2+4)-(2x+1)(2x)=0$$
$$2x^2+8-4x^2-2x=0$$
$$-2x^2-2x+8=0$$

$$x=\frac{2\pm\sqrt{4-4(-2)(8)}}{2(-2)}$$

$$x=\frac{2\pm\sqrt{68}}{-4}=\frac{-1\pm\sqrt{17}}{2}$$

The graph of $f(x)=\frac{(2x+1)}{(x^2+4)}$ has a horizontal tangent line at $x=\frac{-1+\sqrt{17}}{2}$ and $x=\frac{-1-\sqrt{17}}{2}$.

4. Take the derivative, set it equal to 1, and solve for *x:*

$$f'(x)=\frac{(2)(x+4)-(2x+1)(1)}{(x+4)^2}=1$$

$$\frac{2x^2+8-4x^2-2x}{(x+4)^2}=1$$

$$-2x^2-2x+8=(x+4)^2$$

$$-2x^2-2x+8=x^2+8x+16$$

$$3x^2+10x+8=0$$

$$(3x+4)(x+2)=0$$

The graph of $f(x)=\frac{2x+1}{x+4}$ has a tangent line with slope equal to 1 at $x=-2$ and $x=-\frac{4}{3}$