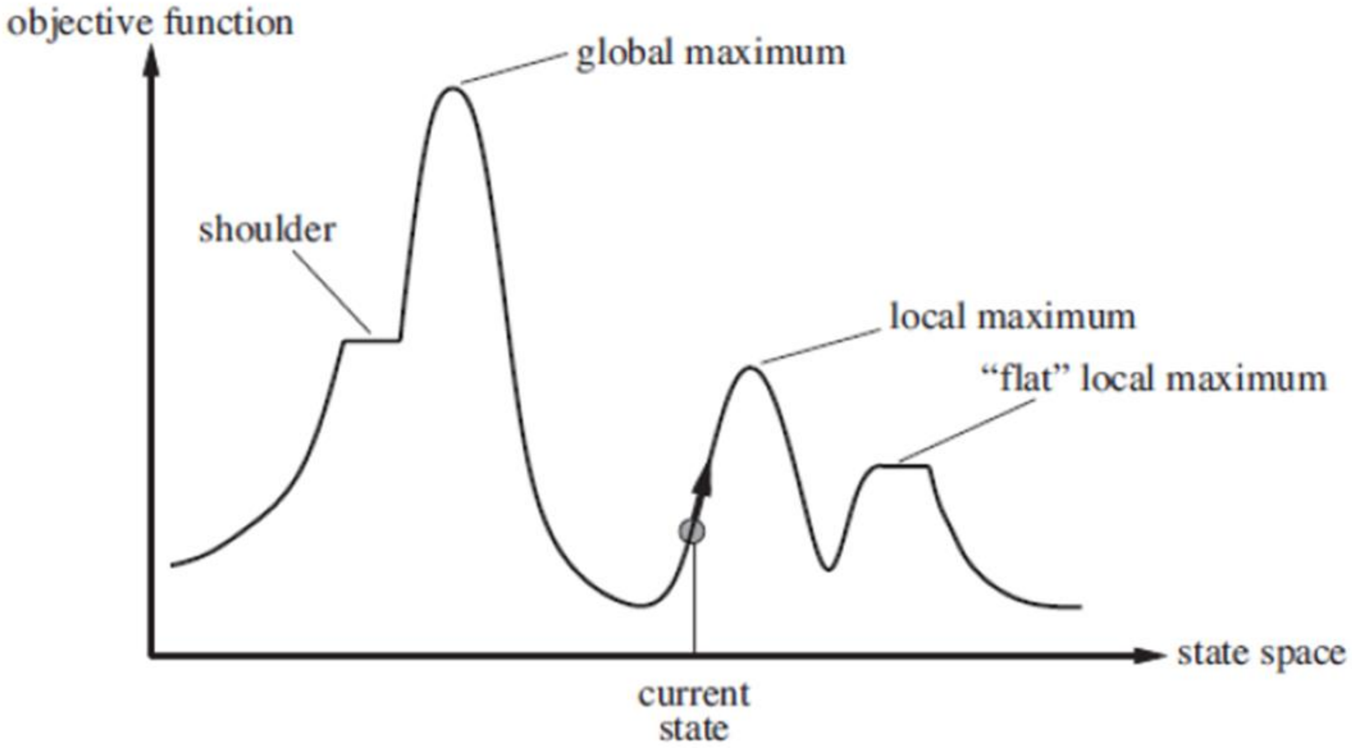
Module 6: Optimisation (Hill Climbing, Simulated Annealing and GA)

Source for the slides:

https://www.xpowerpoint.com/hill-climbing-search-main--PPT.html#

Local vs Global Search Algorithms

- Hill Climbing
- SA
- S.T.

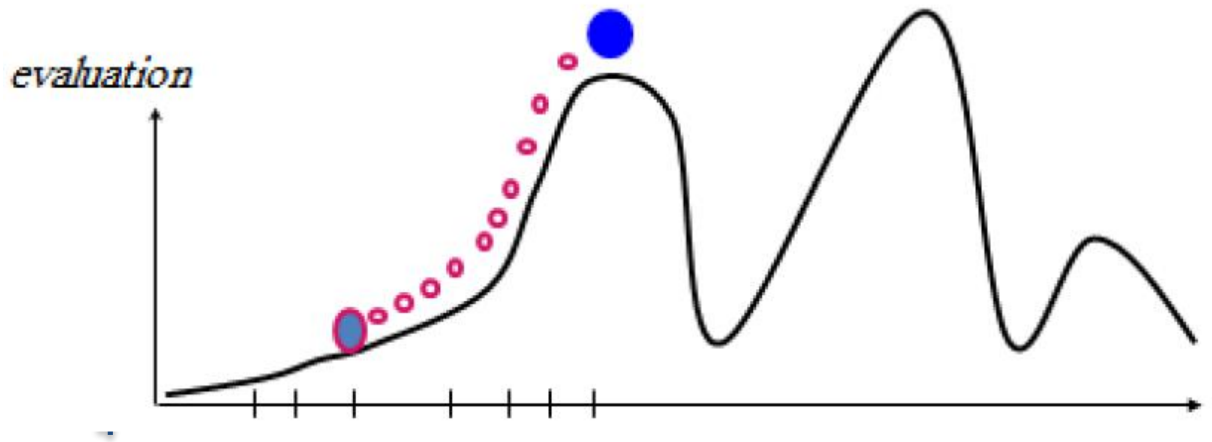


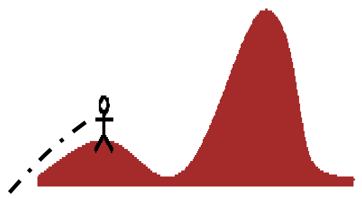
- GA

Hill-Climbing Search

Hill-Climbing Search

- **Main Idea**: Keep a single current node and move to a neighboring state to improve it.
- Uses a loop that continuously moves in the direction of increasing value (**uphill**):
- Choose the best successor, choose randomly if there is more than one.
- Terminate when a peak reached where no neighbor has a higher value.
- It also called **greedy local search**, steepest ascent/descent.



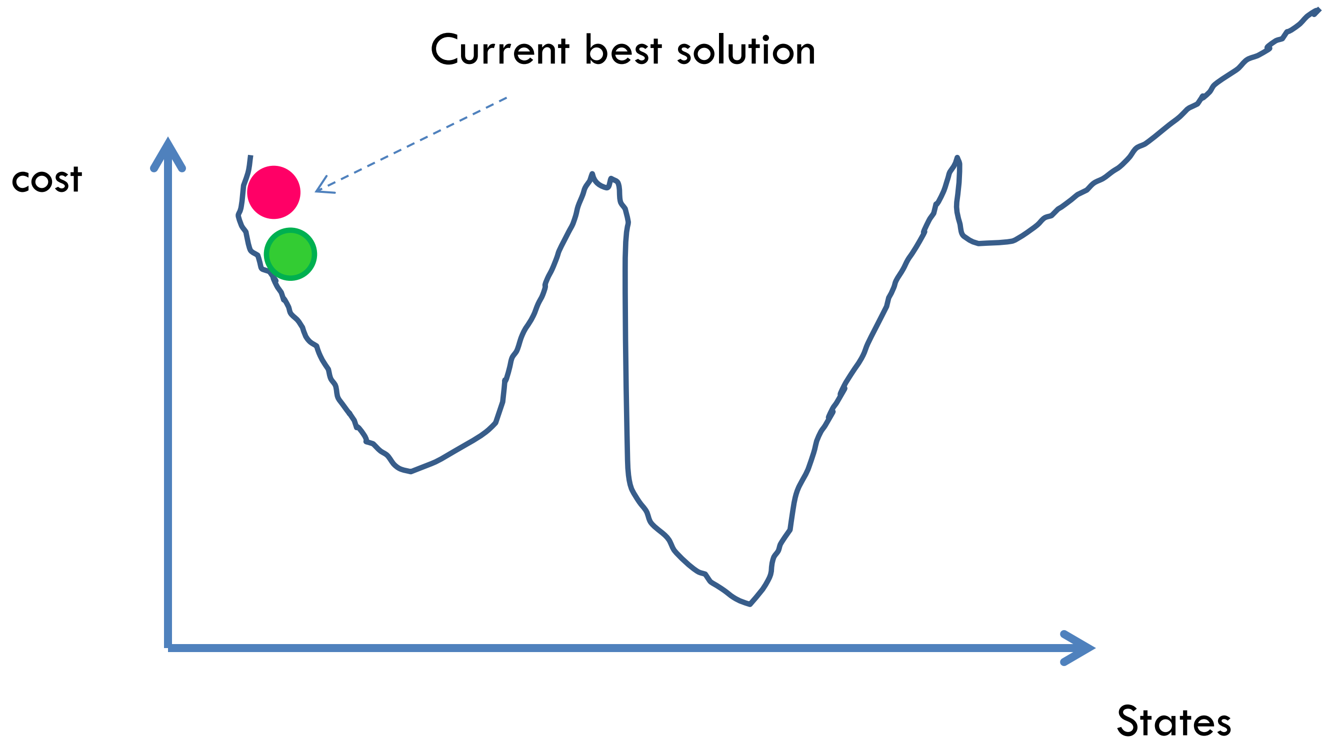


Hill-Climbing Search

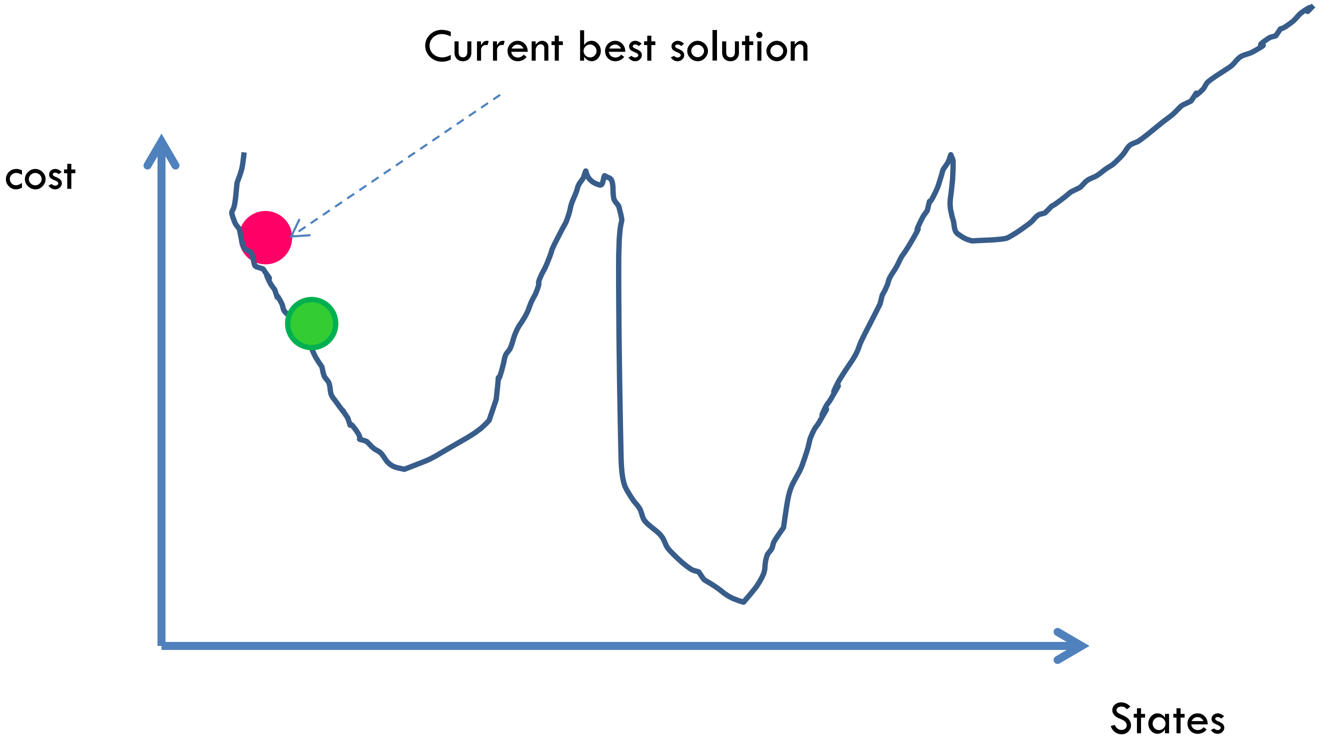
```
function HILL-CLIMBING(problem) returns a state that is a local maximum
   inputs: problem, a problem
   local variables: current, a node
                    neighbor, a node

   current ← MAKE-NODE(INITIAL-STATE[problem])
   loop do
      neighbor ← a highest-valued successor of current
      if VALUE[neighbor] ≤ VALUE[current] then return STATE[current]
      current ← neighbor
```

Hill-Climbing in Action …



Hill-Climbing in Action …



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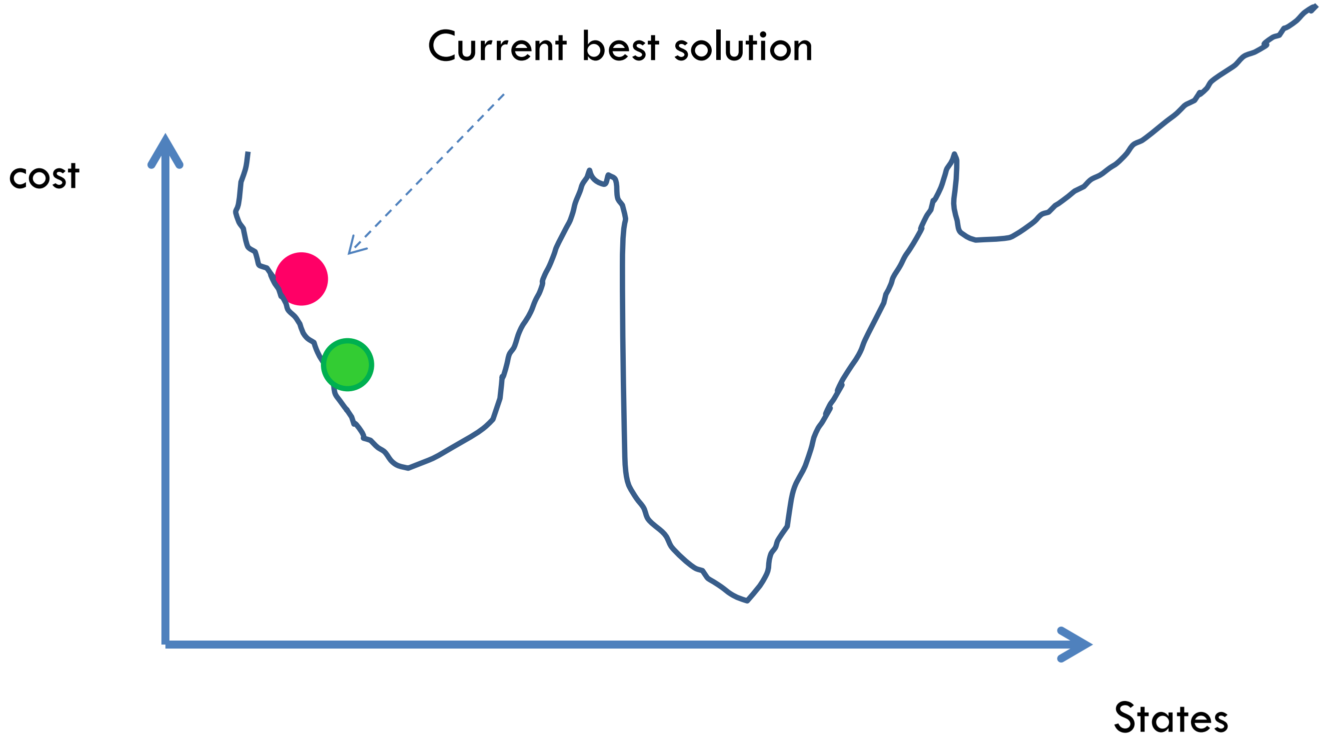
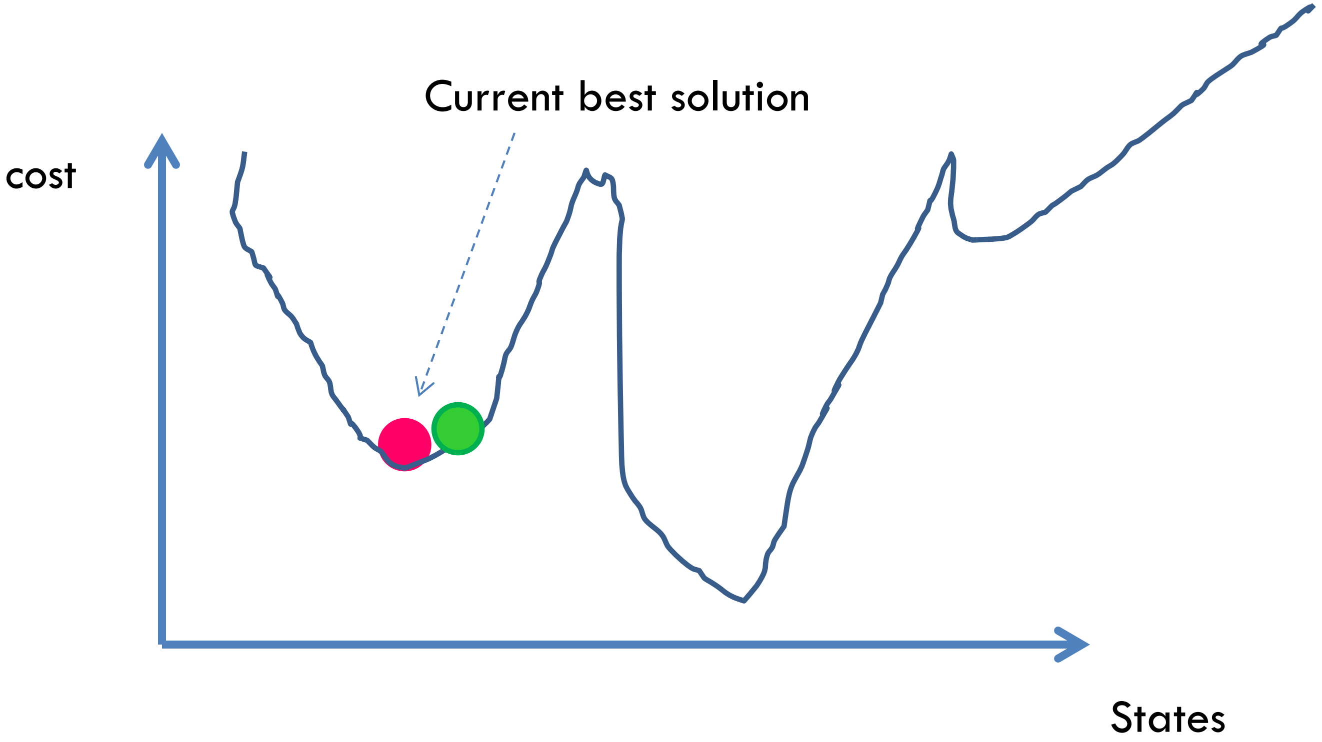
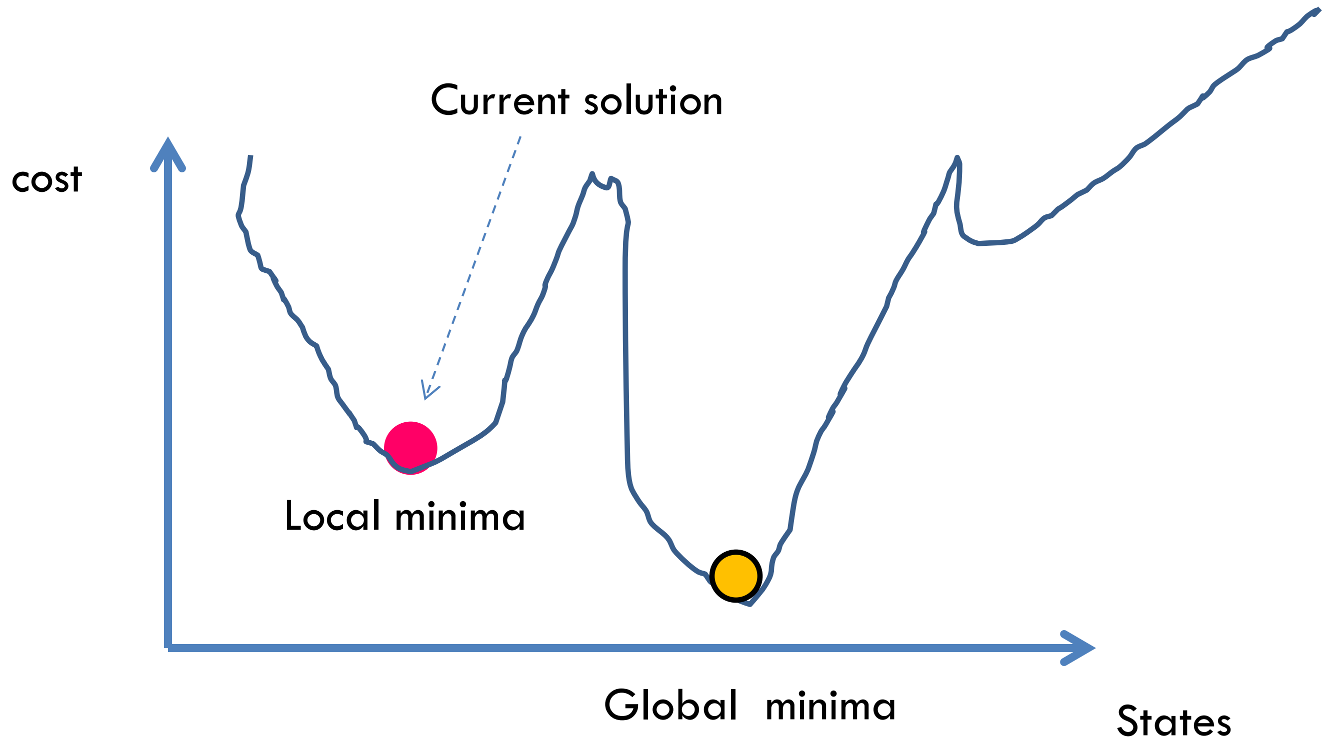


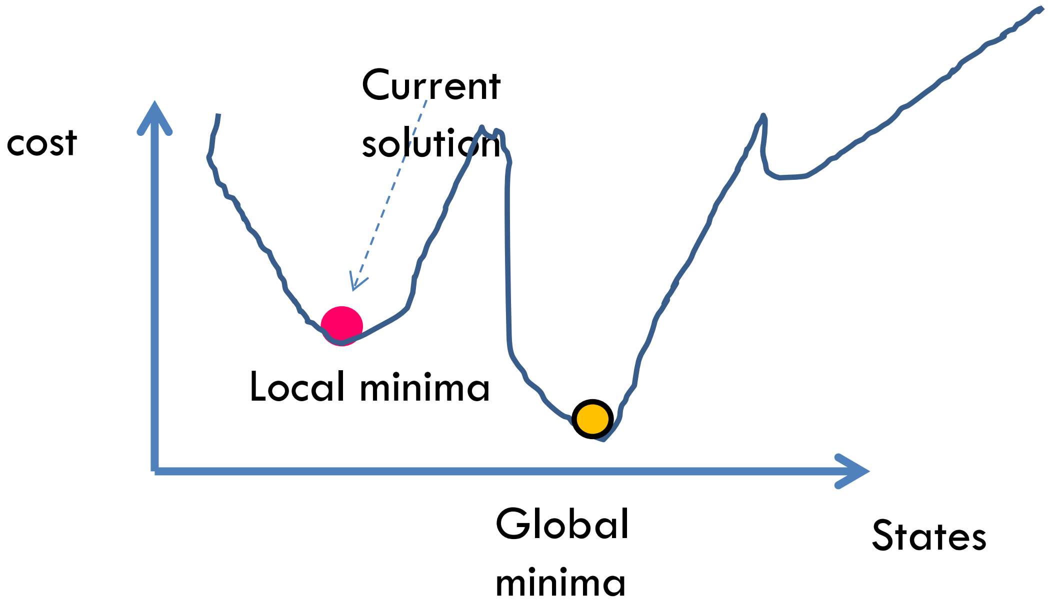
Hill-Climbing in Action …



Hill-Climbing in Action …



Hill-Climbing in Action …

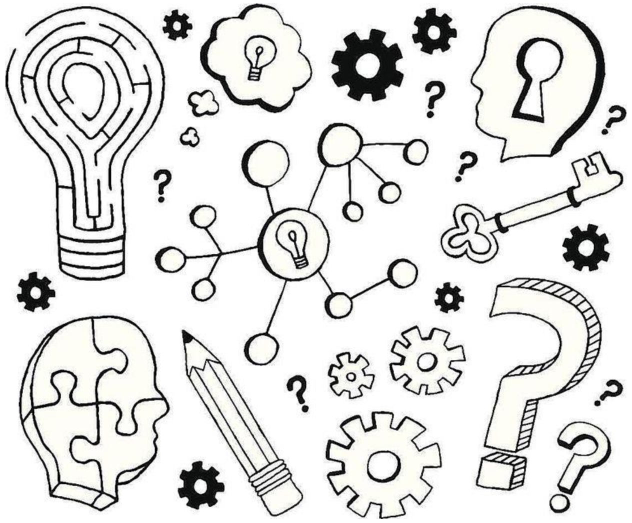


Drawback: Depending on initial state, it can get stuck in local maxima/minimum or flat local maximum and not find the solution.

Cure: Random restart.

Activity (Reflection, 20’)

Reflect on what you have studied in **Hill-Climbing Search.**



Simulated Annealing Search

The Problem

- Most minimization strategies find the *nearest* local minimum
- Standard strategy
  - Generate trial point based on current estimates
  - Evaluate function at proposed location
  - Accept new value if it improves solution

The Solution

- We need a strategy to find other minima
- This means, we must sometimes select new points that do not improve solution
- How?

Annealing

- One manner in which crystals are formed
- Gradual cooling of liquid …
    - ✓ At high temperatures, molecules move freely
    - ✓ At low temperatures, molecules are "stuck"
    - ✓ If cooling is slow

Low energy, organized crystal lattice formed

Simulated annealing Search

- **Main Idea**: escape local maxima by allowing some "bad" moves but gradually **decrease their frequency**.

- Instead of picking the **best** move, it picks a **random** move..

Simulated annealing Search

function SIMULATED-ANNEALING(*problem*, *schedule*) **returns** a solution state
  inputs: *problem*, a problem
    schedule, a mapping from time to "temperature"
  local variables: *current*, a node
    next, a node
    T, a "temperature" controlling prob. of downward steps

  current ← MAKE-NODE(INITIAL-STATE[*problem*])
  for $t \leftarrow 1$ **to** ∞ **do**
    $T \leftarrow schedule[t]$
    if $T = 0$ **then return** *current*
    next ← a randomly selected successor of *current*
    $\Delta E \leftarrow$ VALUE[*next*] – VALUE[*current*]
    if $\Delta E > 0$ **then** *current* ← *next*
    else *current* ← *next* only with probability $e^{\Delta E/T}$

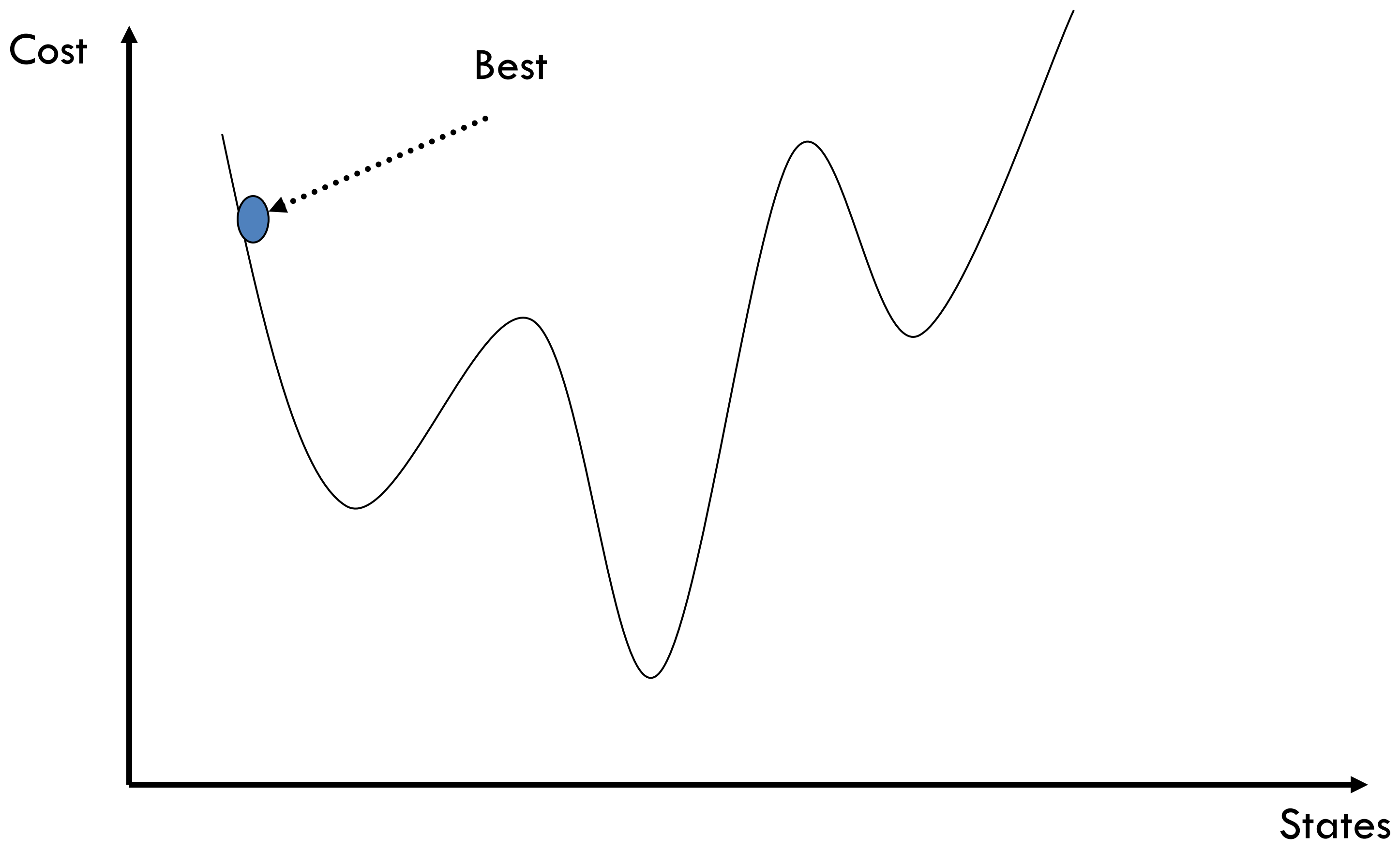
Similar to hill climbing, but a random move instead of best move.

Case of improvement, make the move.

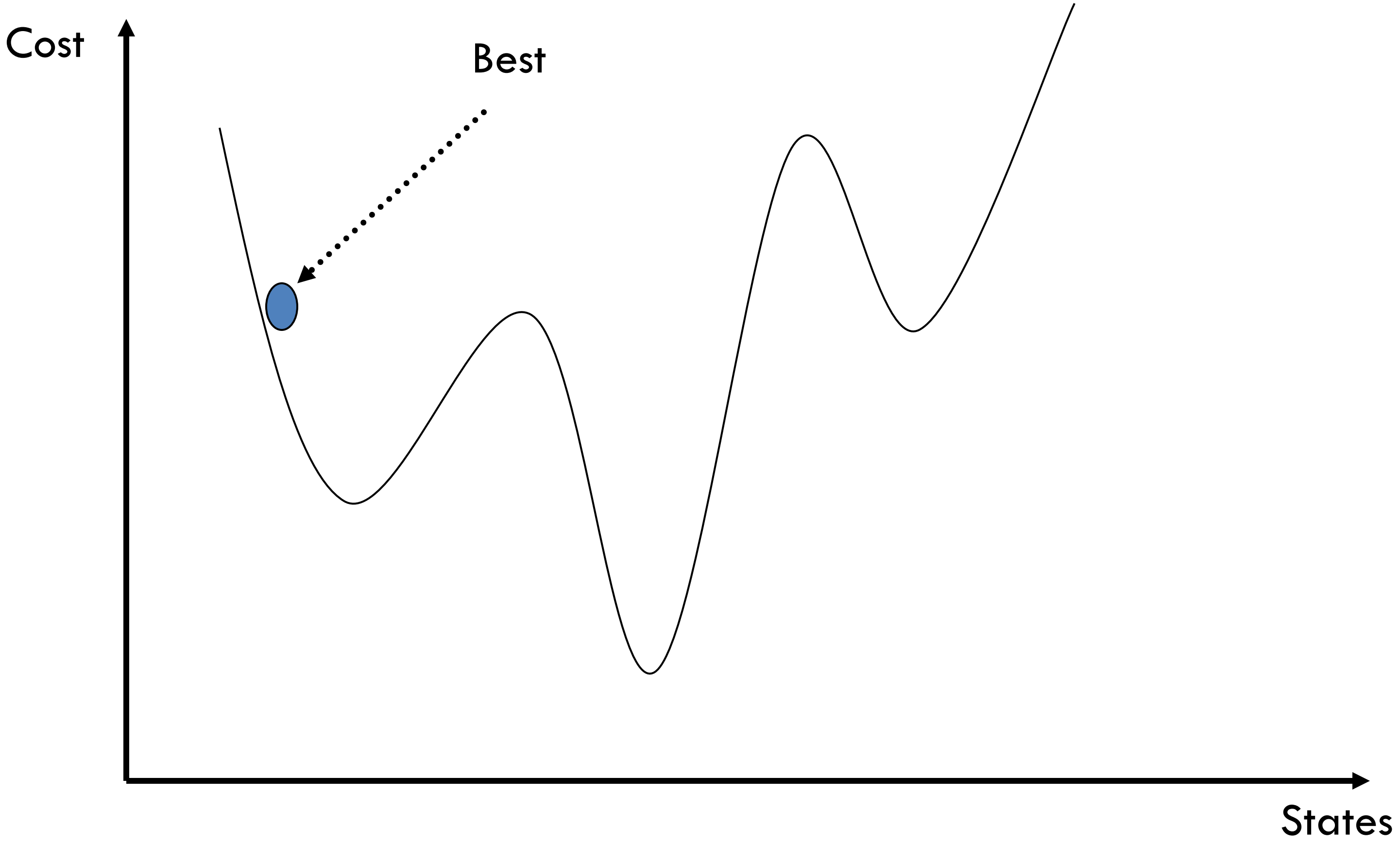
Otherwise, choose the move with probability that decreases exponentially with the "badness" of the move.

- say the change in objective function is δ
- if δ is positive, then move to that state
- **otherwise**:
  - move to this state with probability proportional to δ
  - thus: worse moves (very large negative δ) are executed less often

Simulated Annealing



Simulated Annealing



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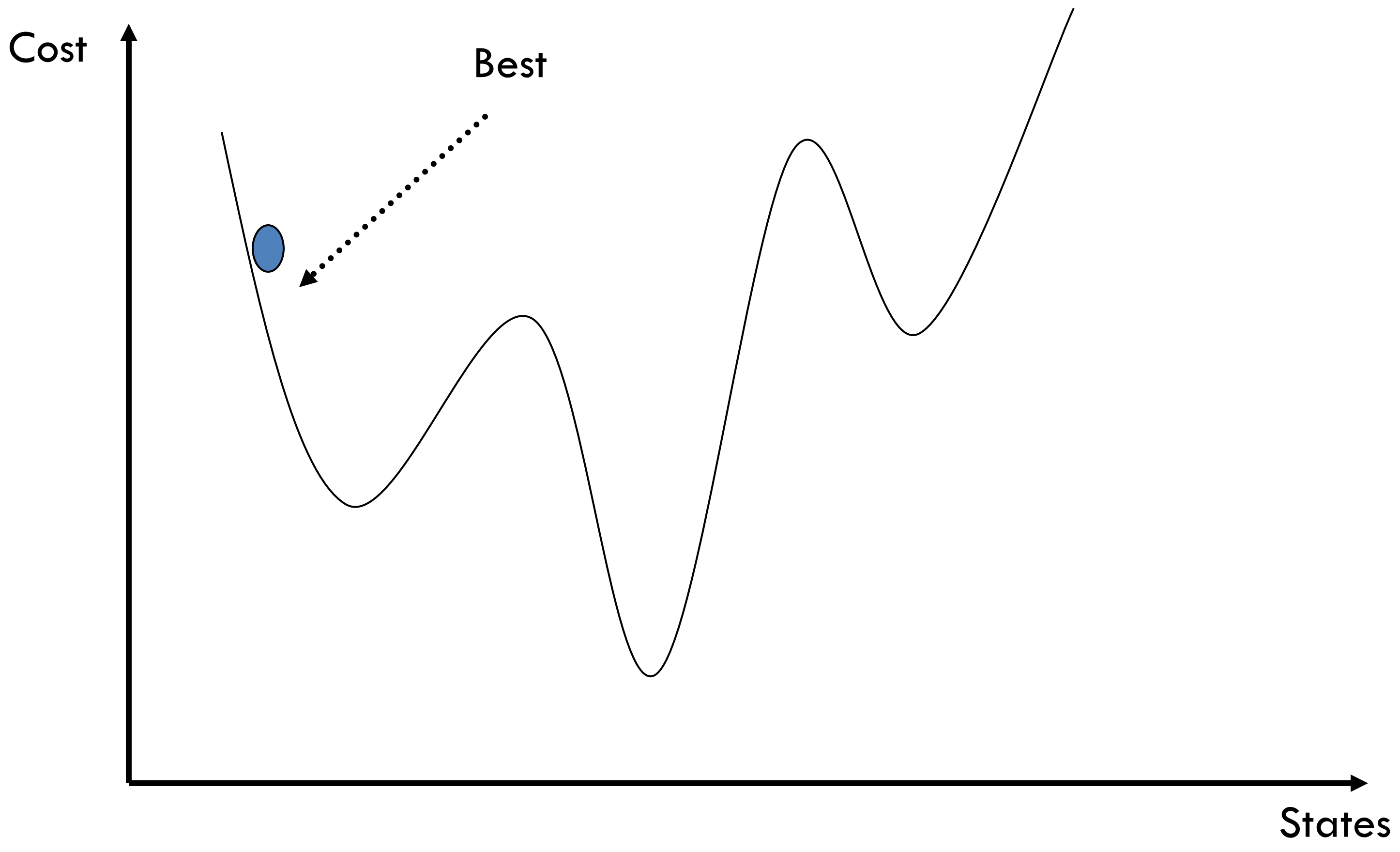

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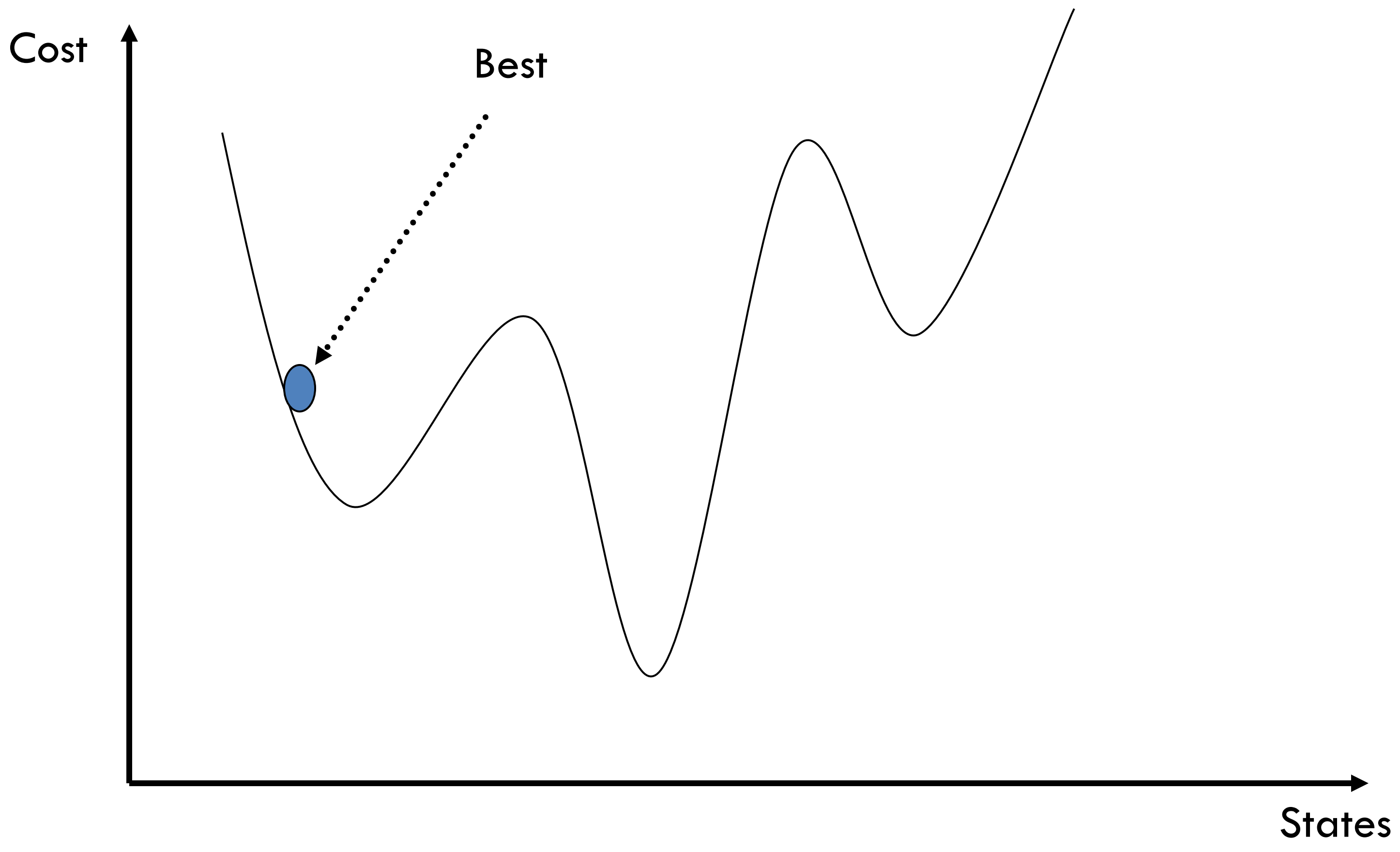
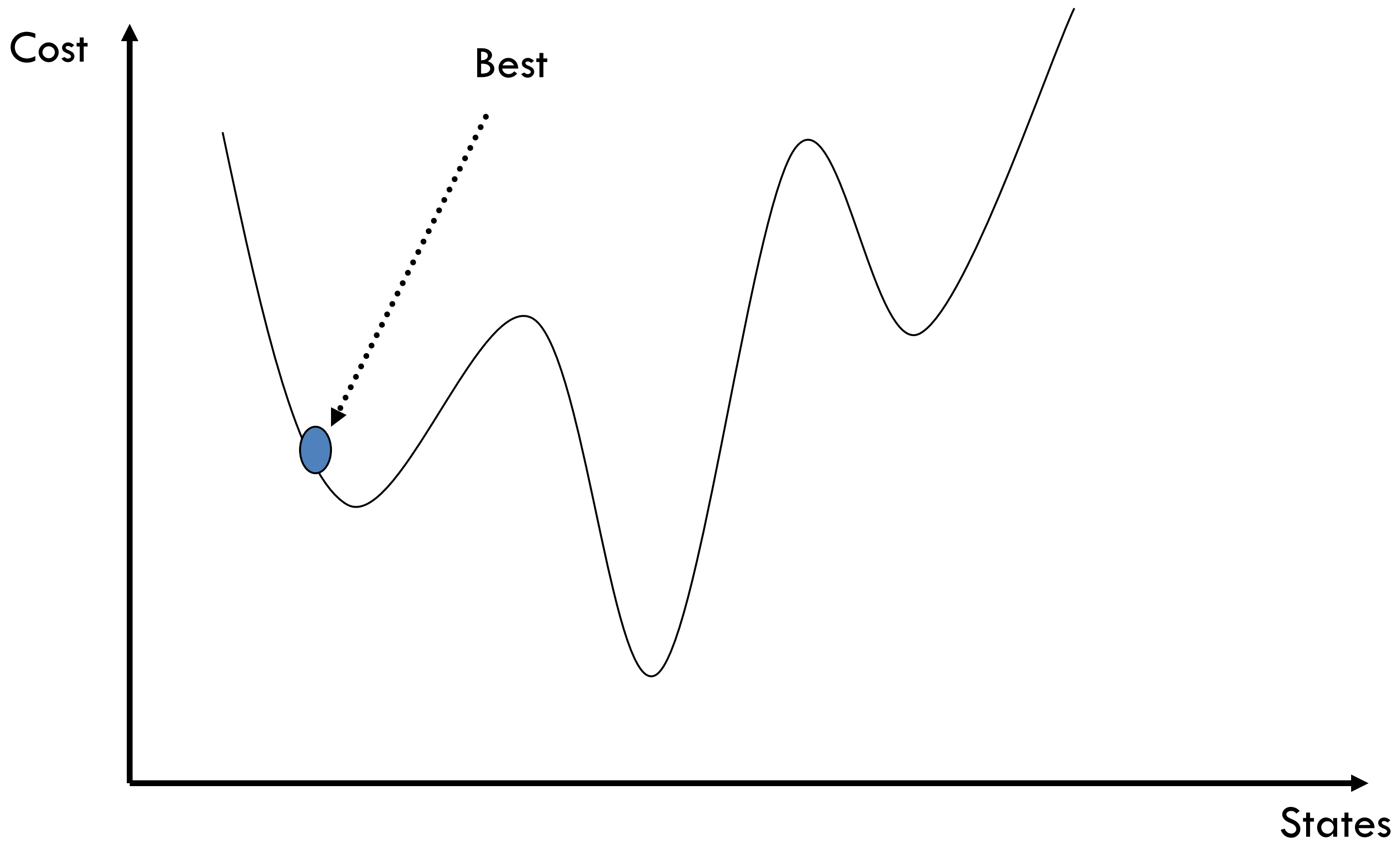


Simulated Annealing



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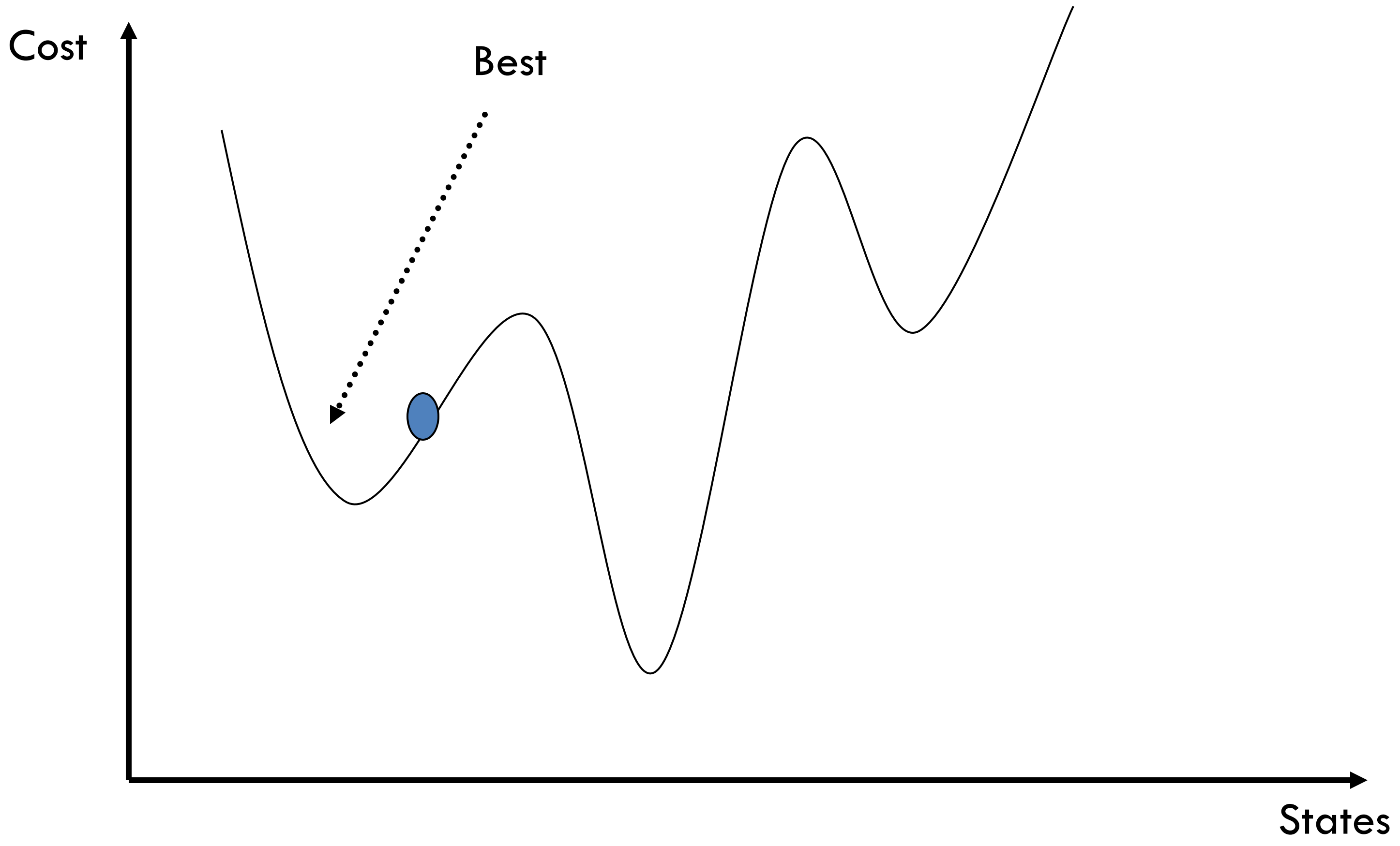

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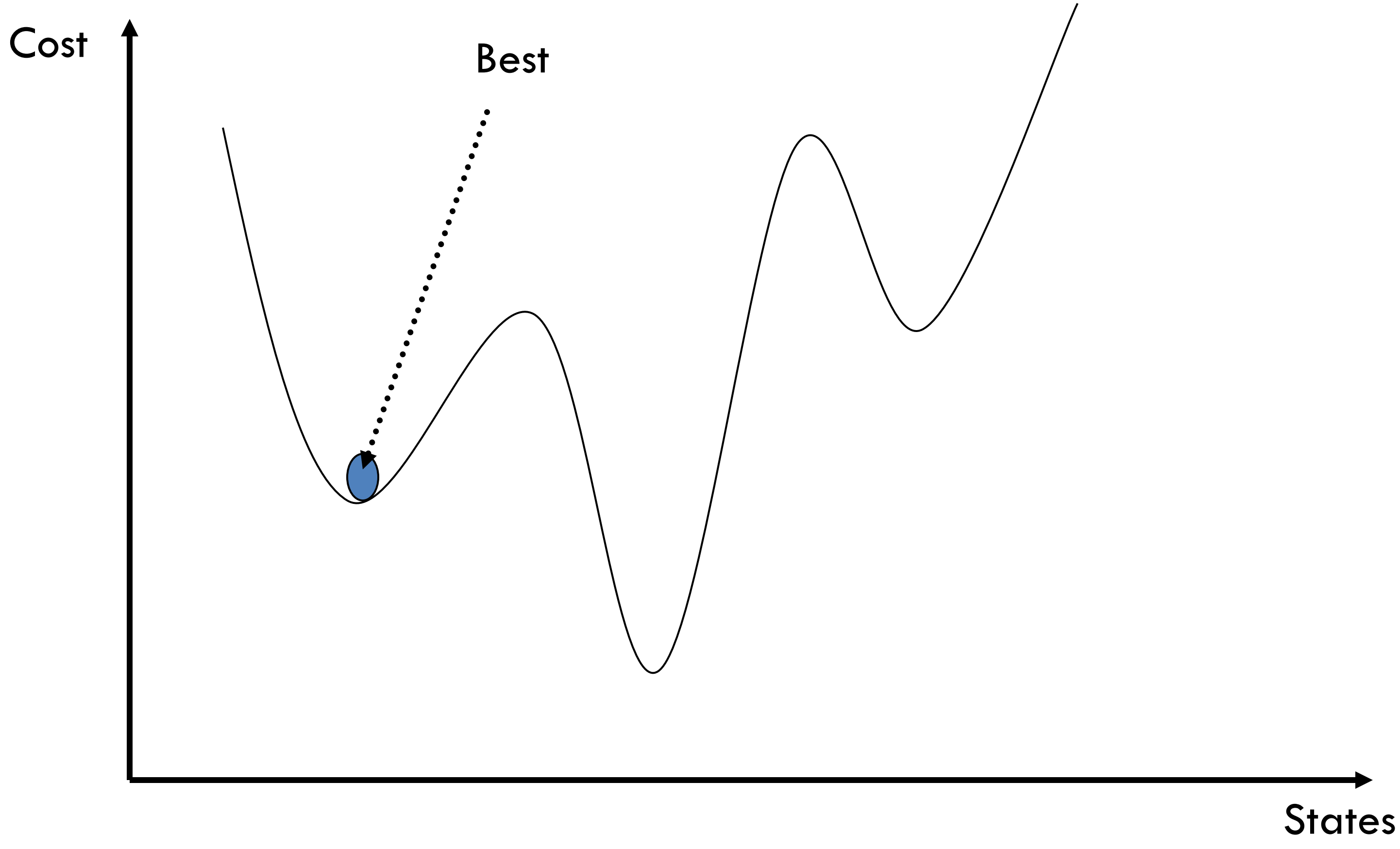

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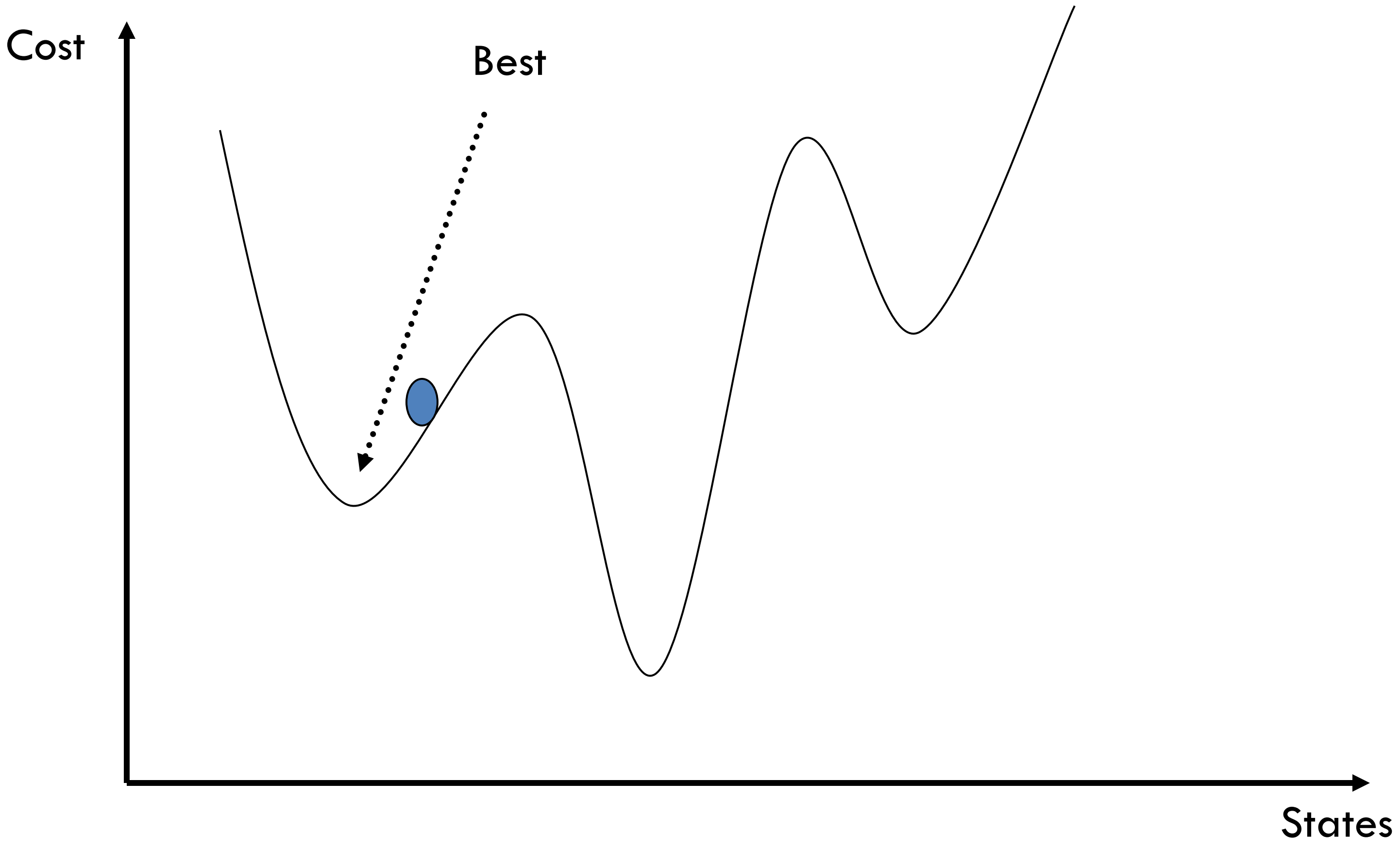

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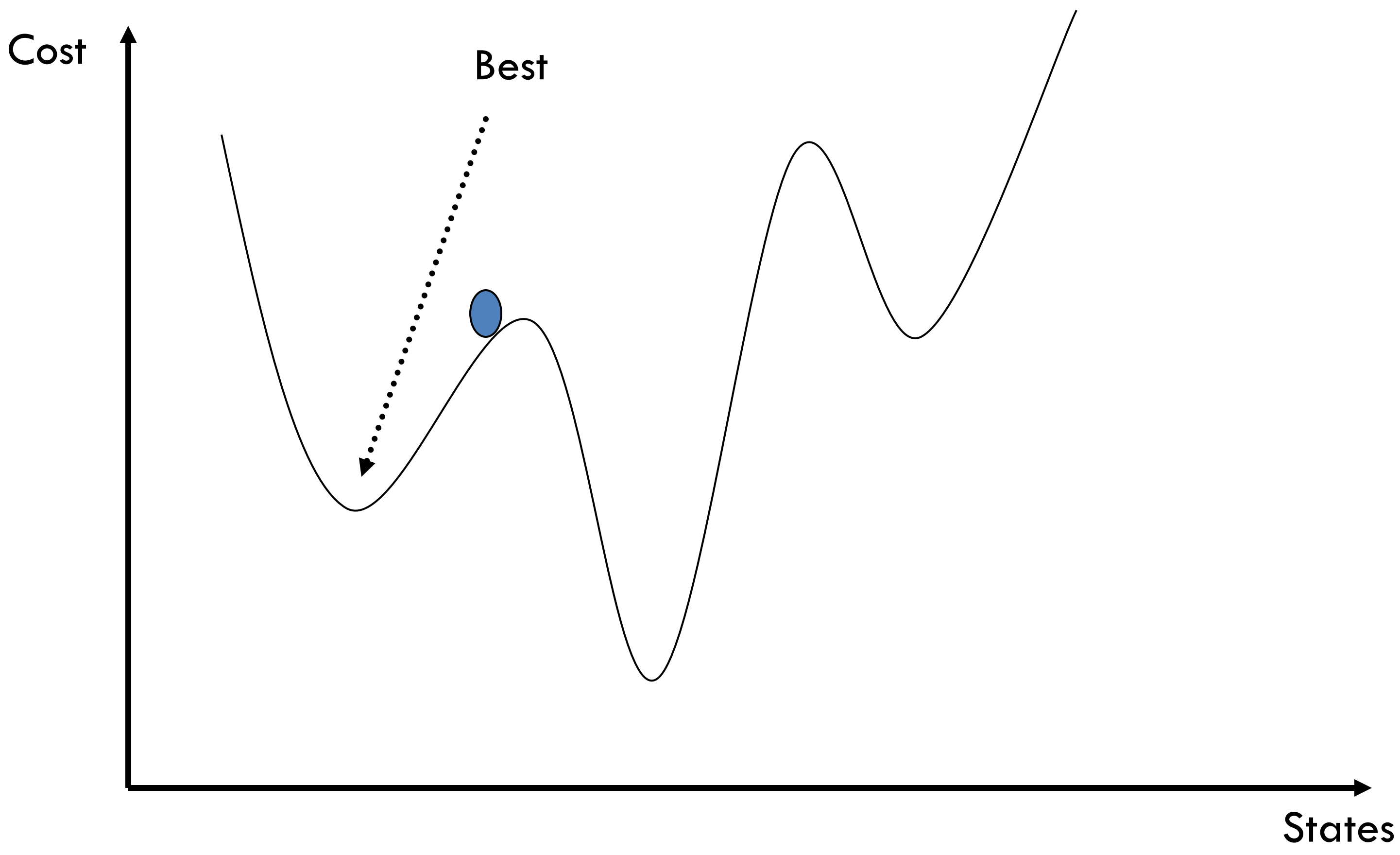

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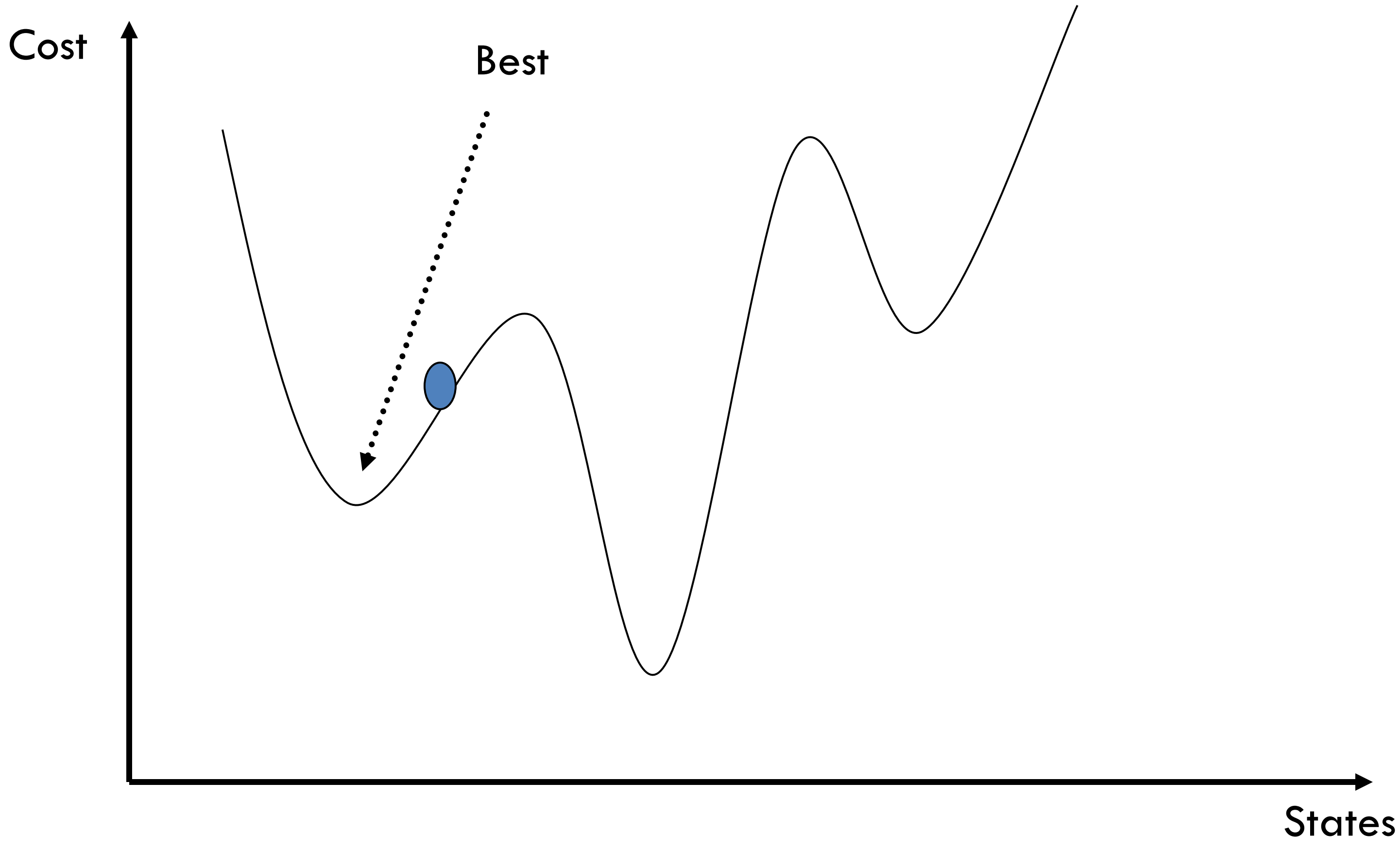

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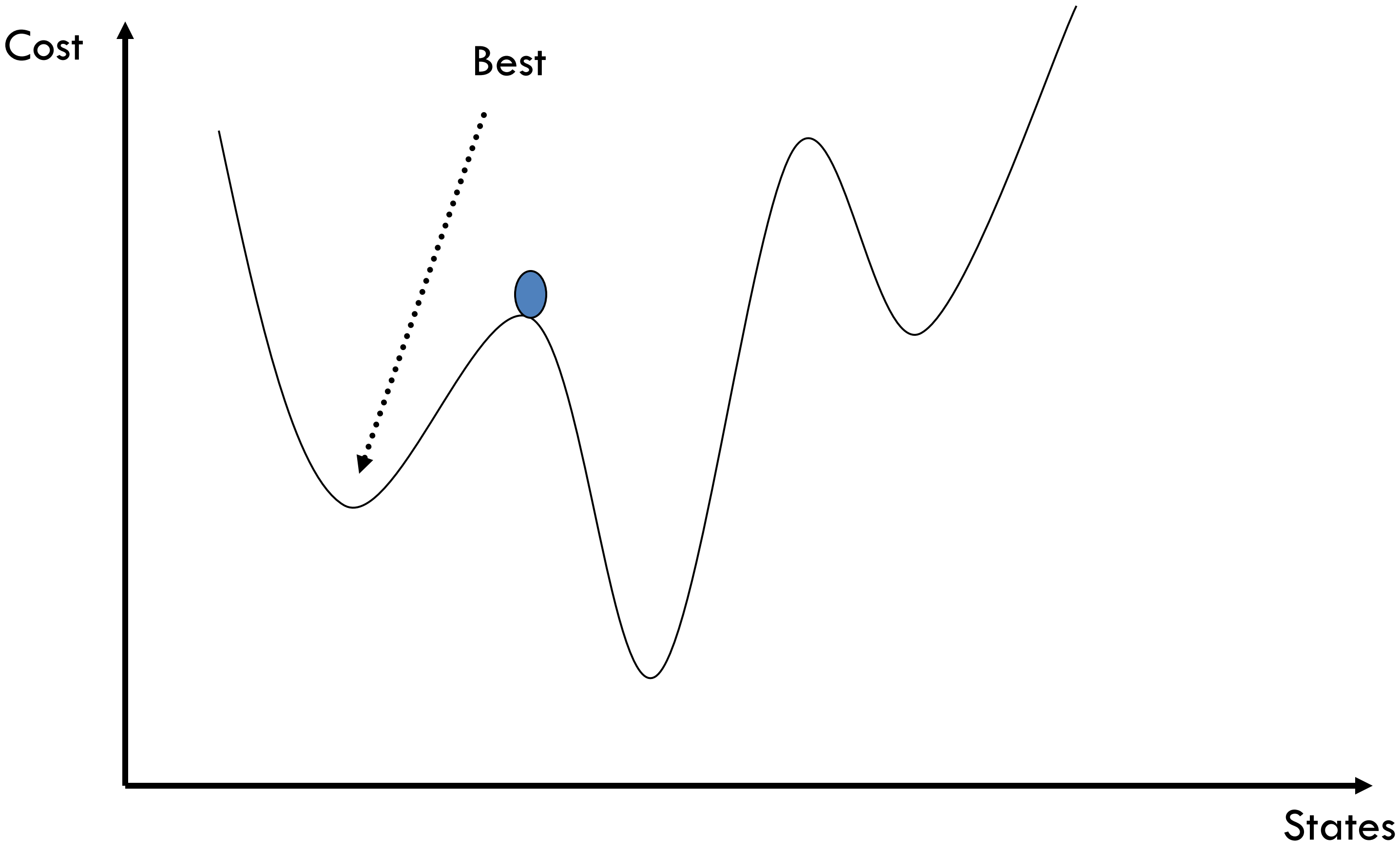

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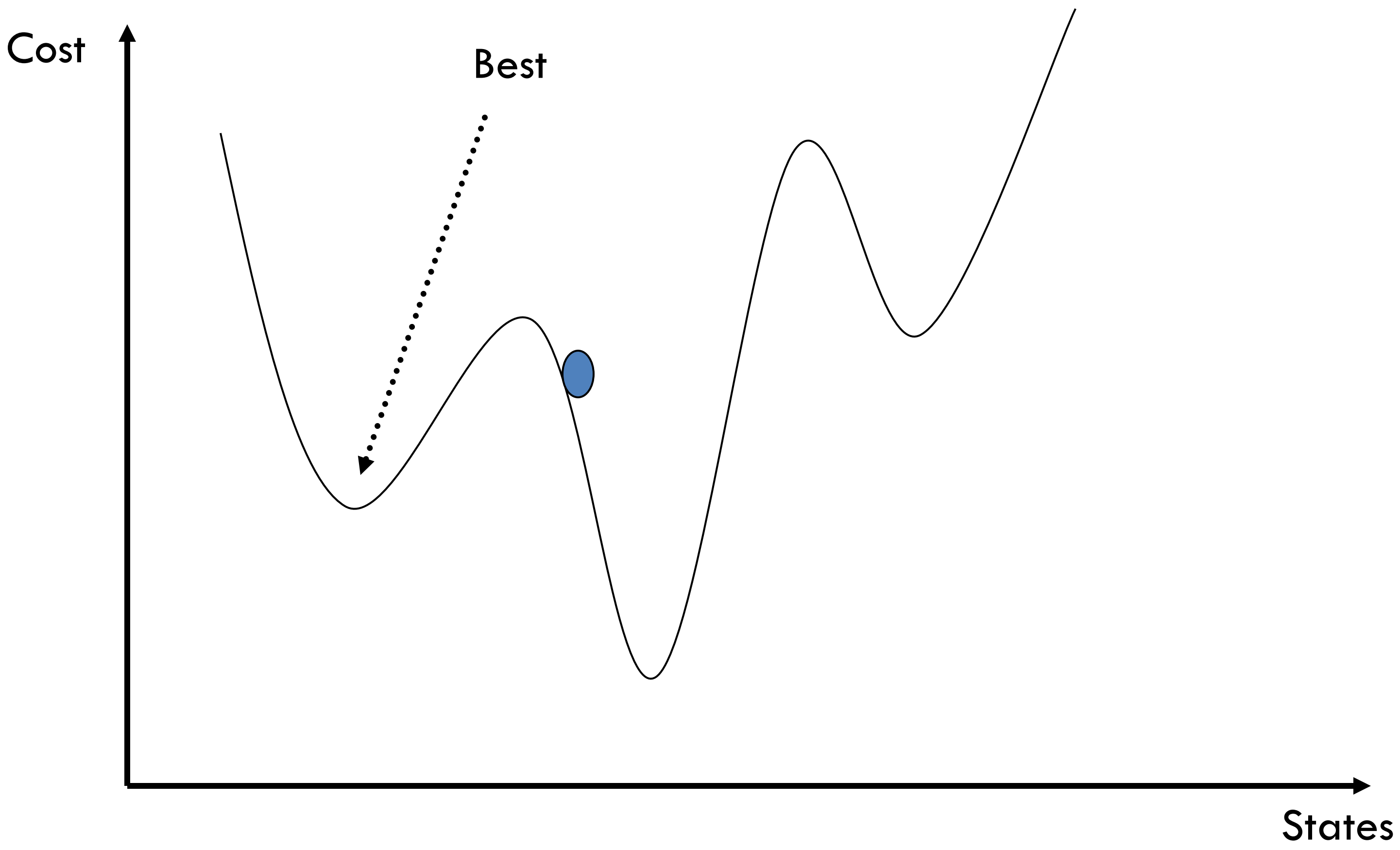

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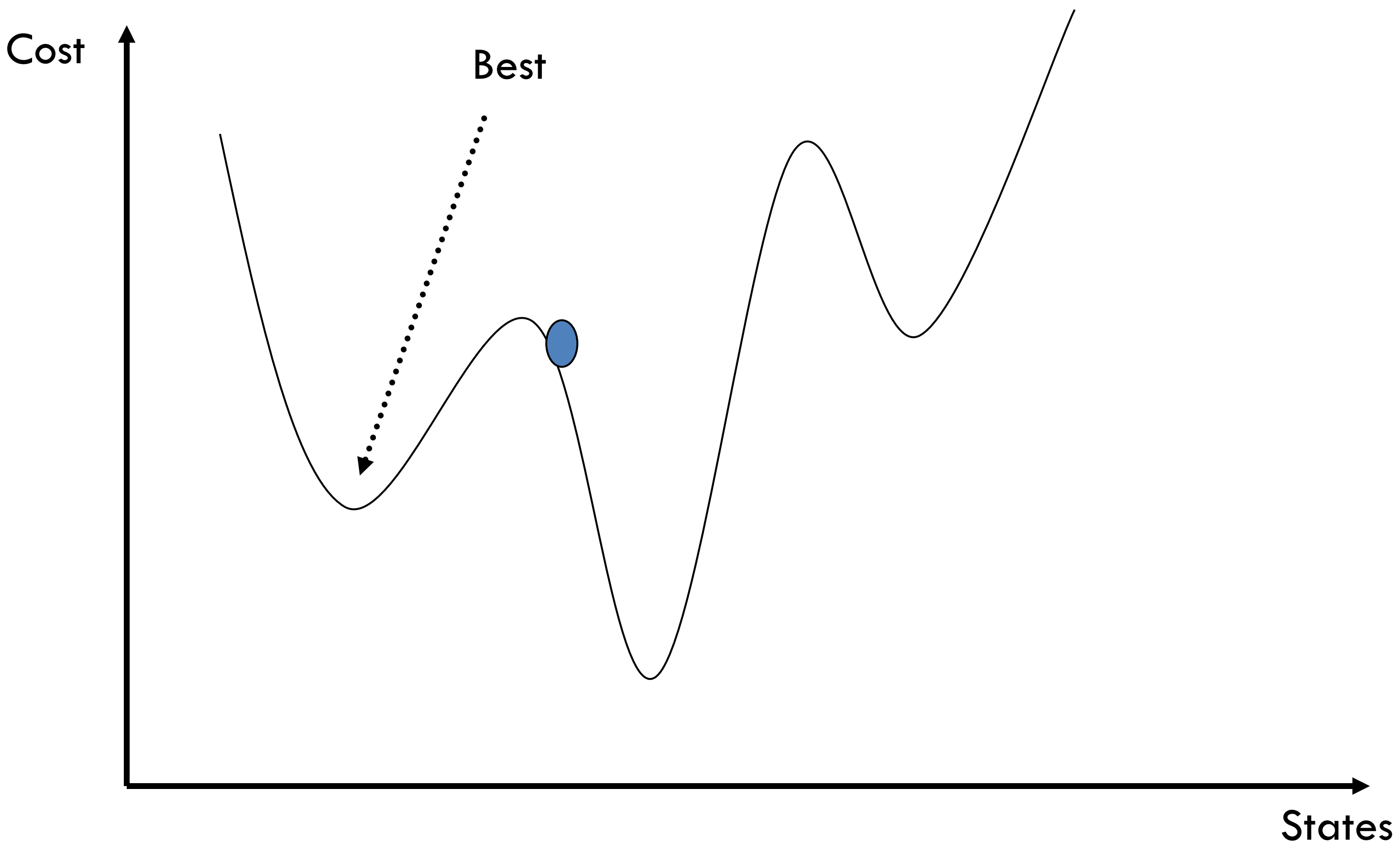

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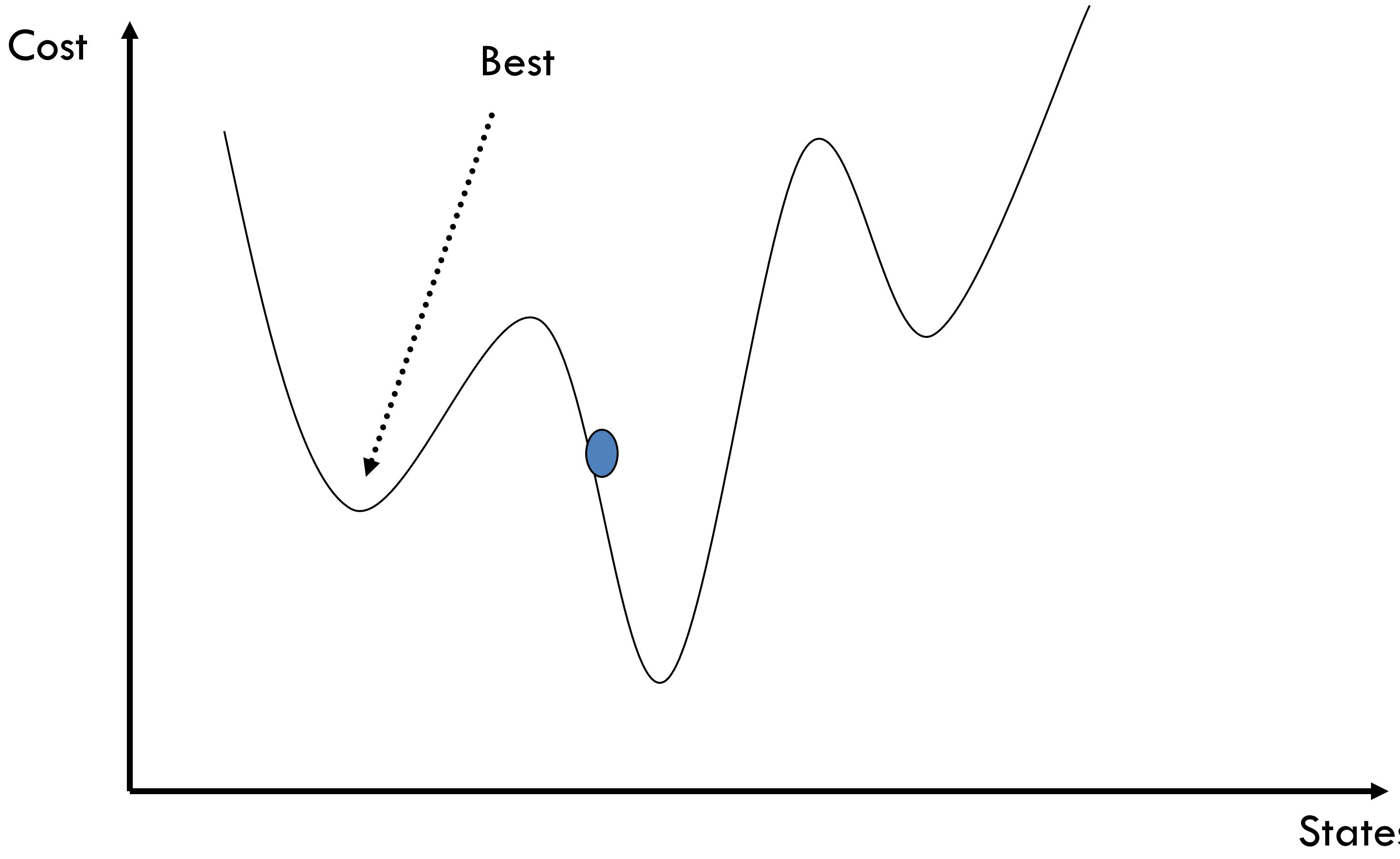

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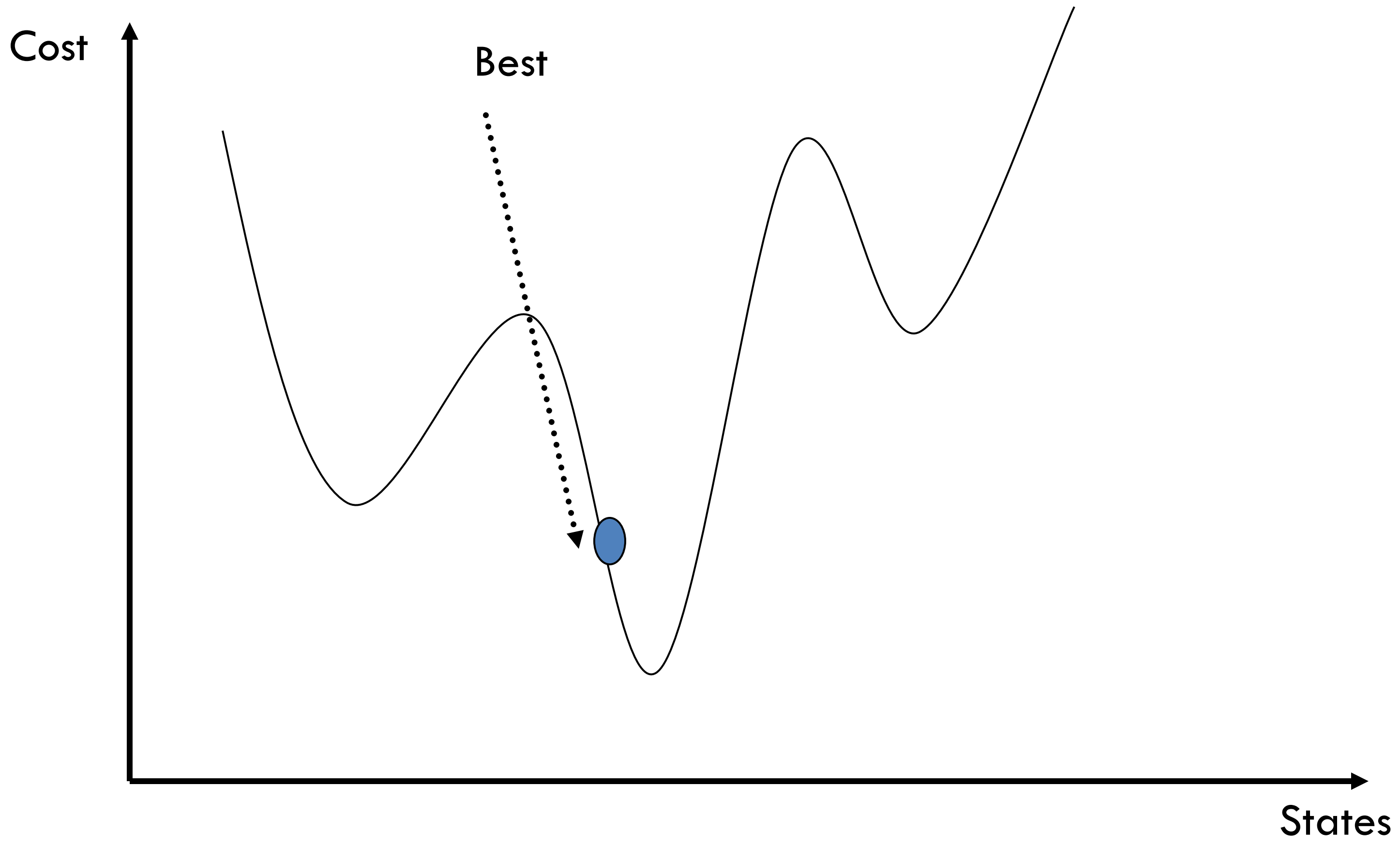

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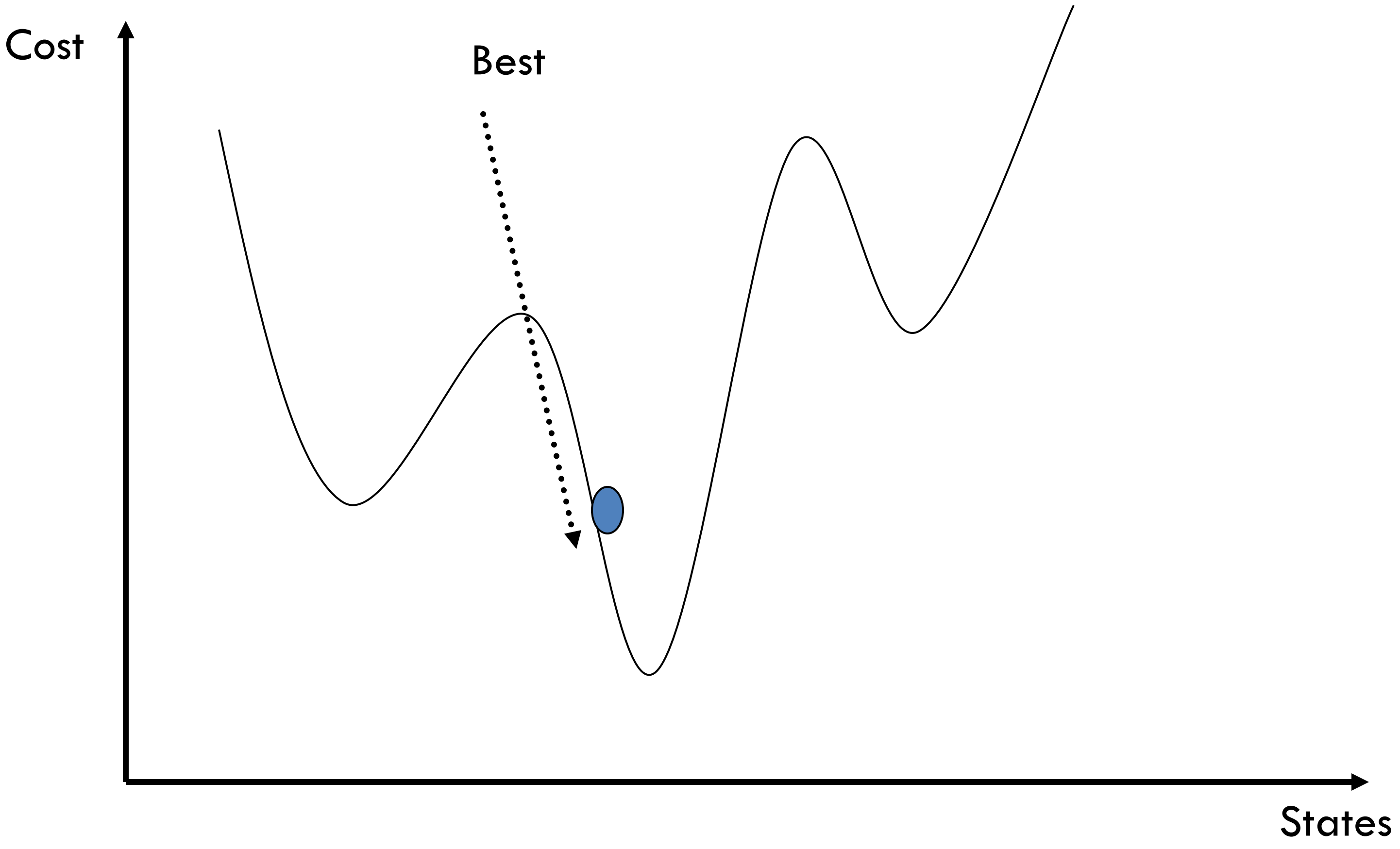

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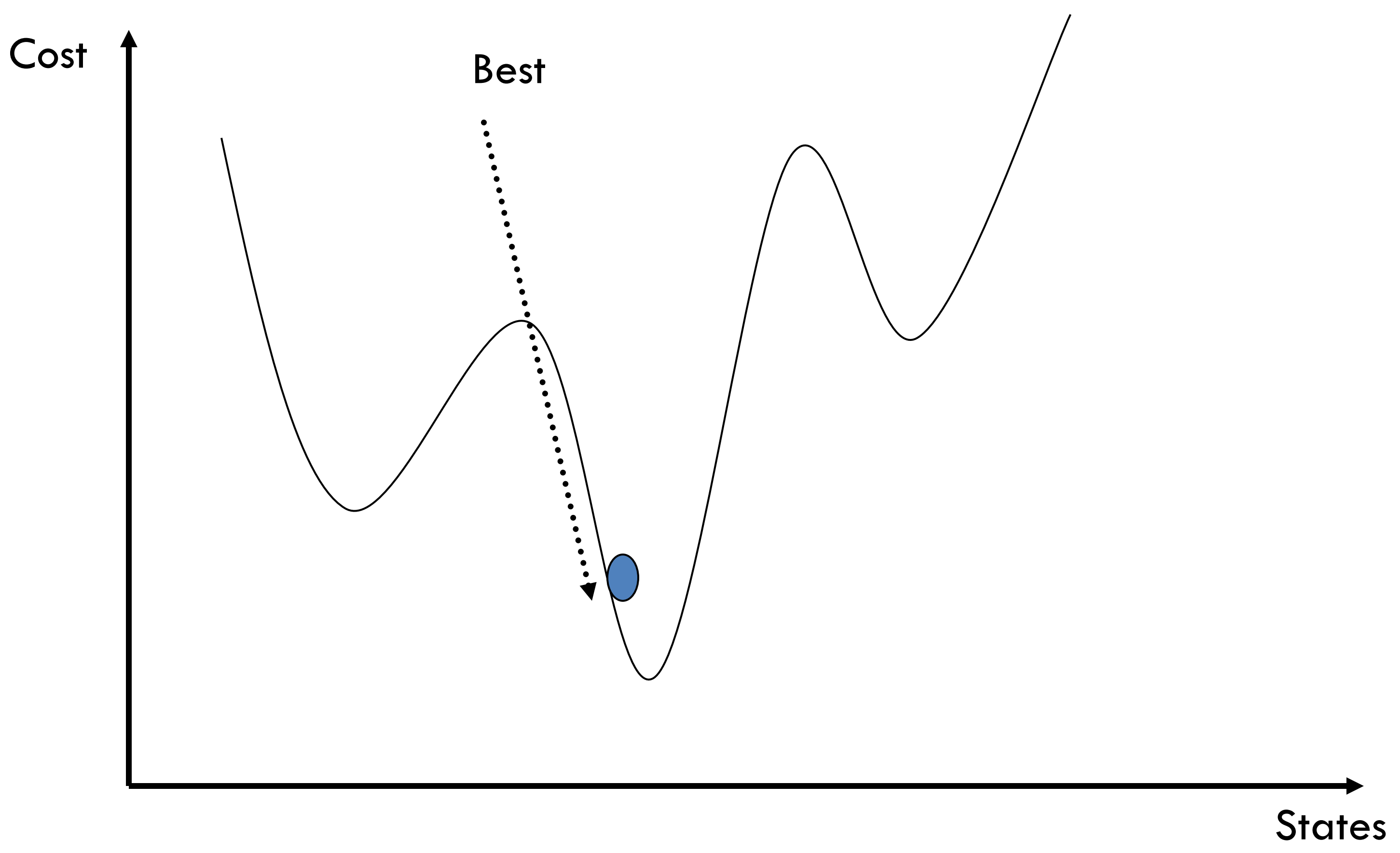

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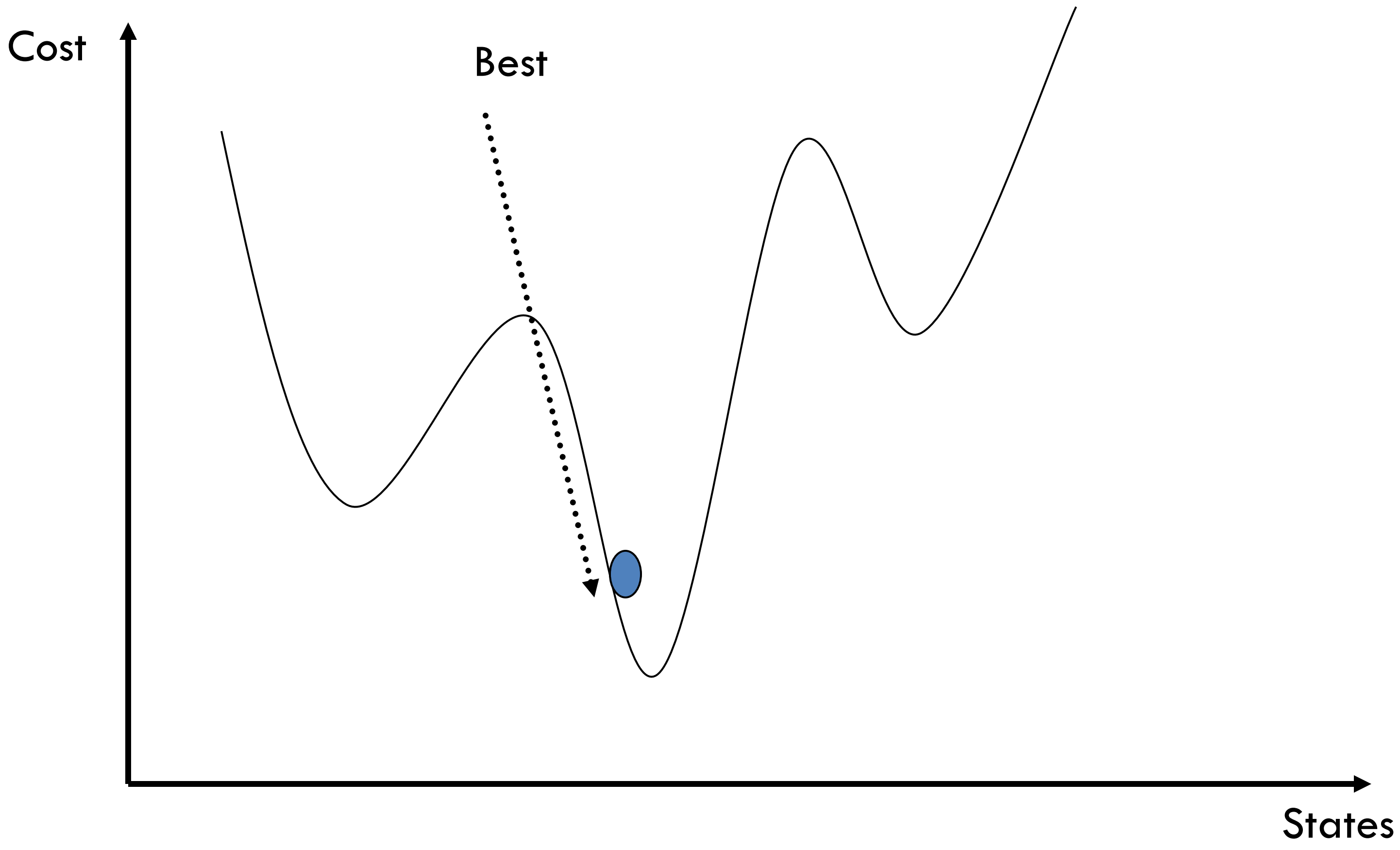

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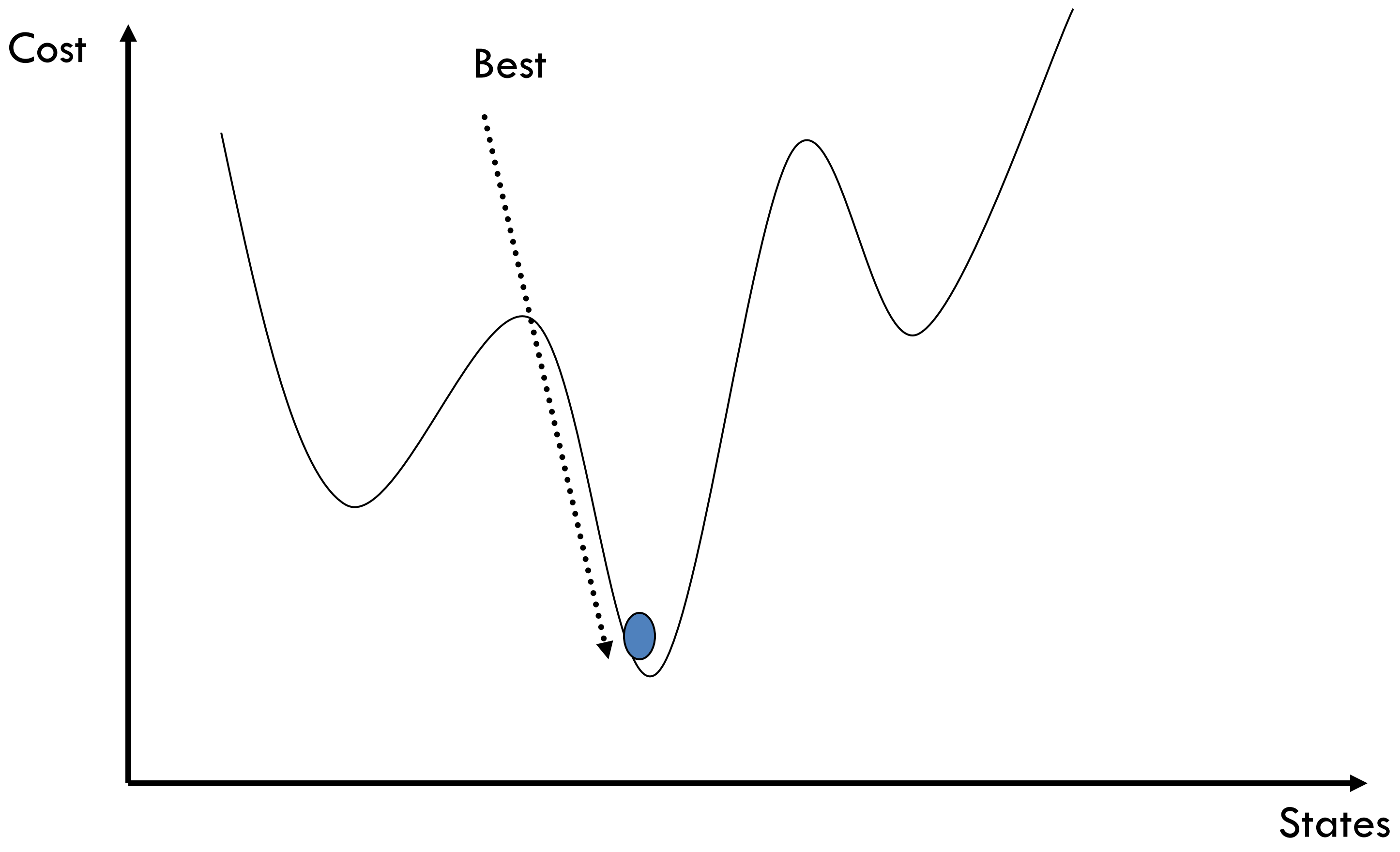

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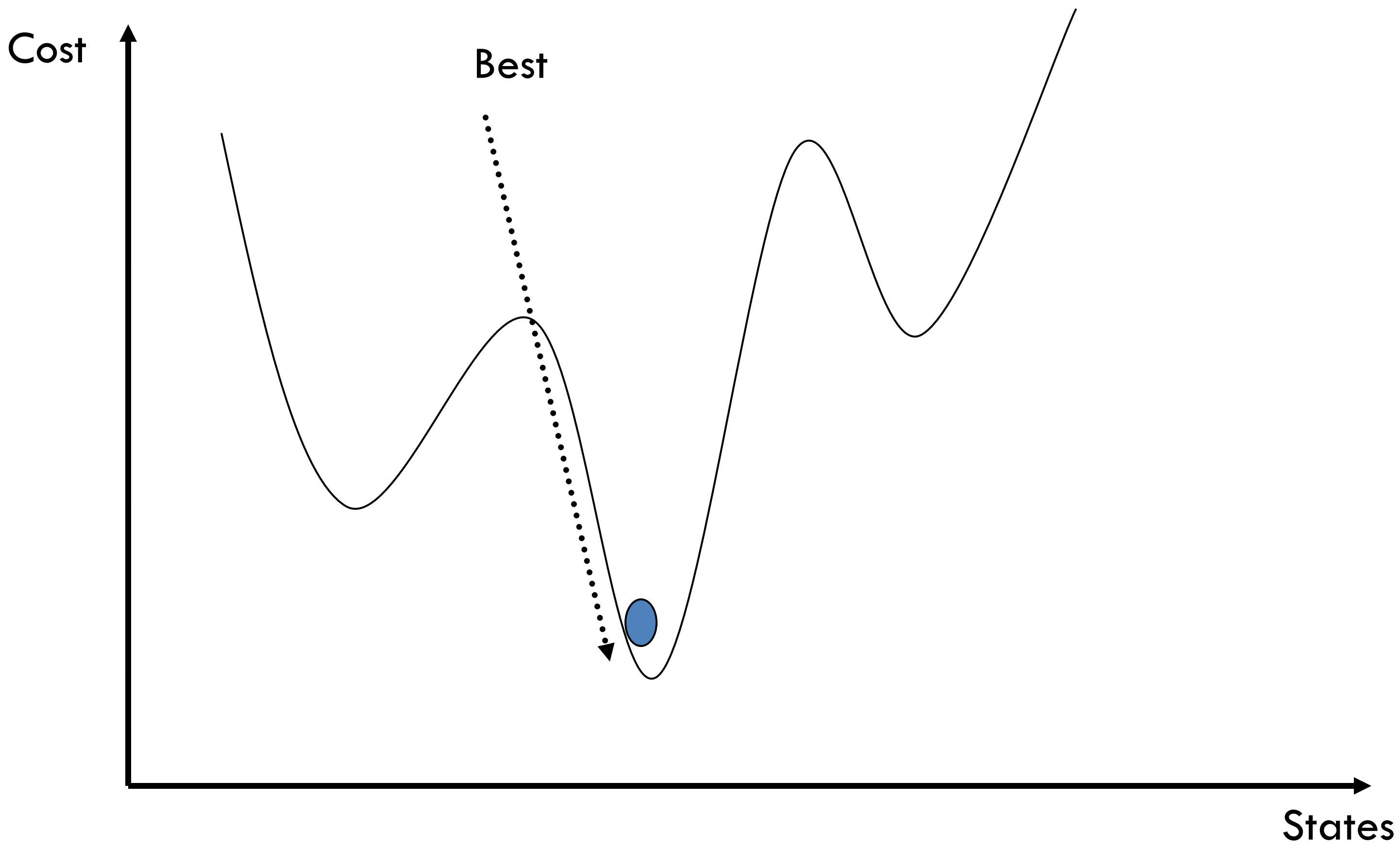

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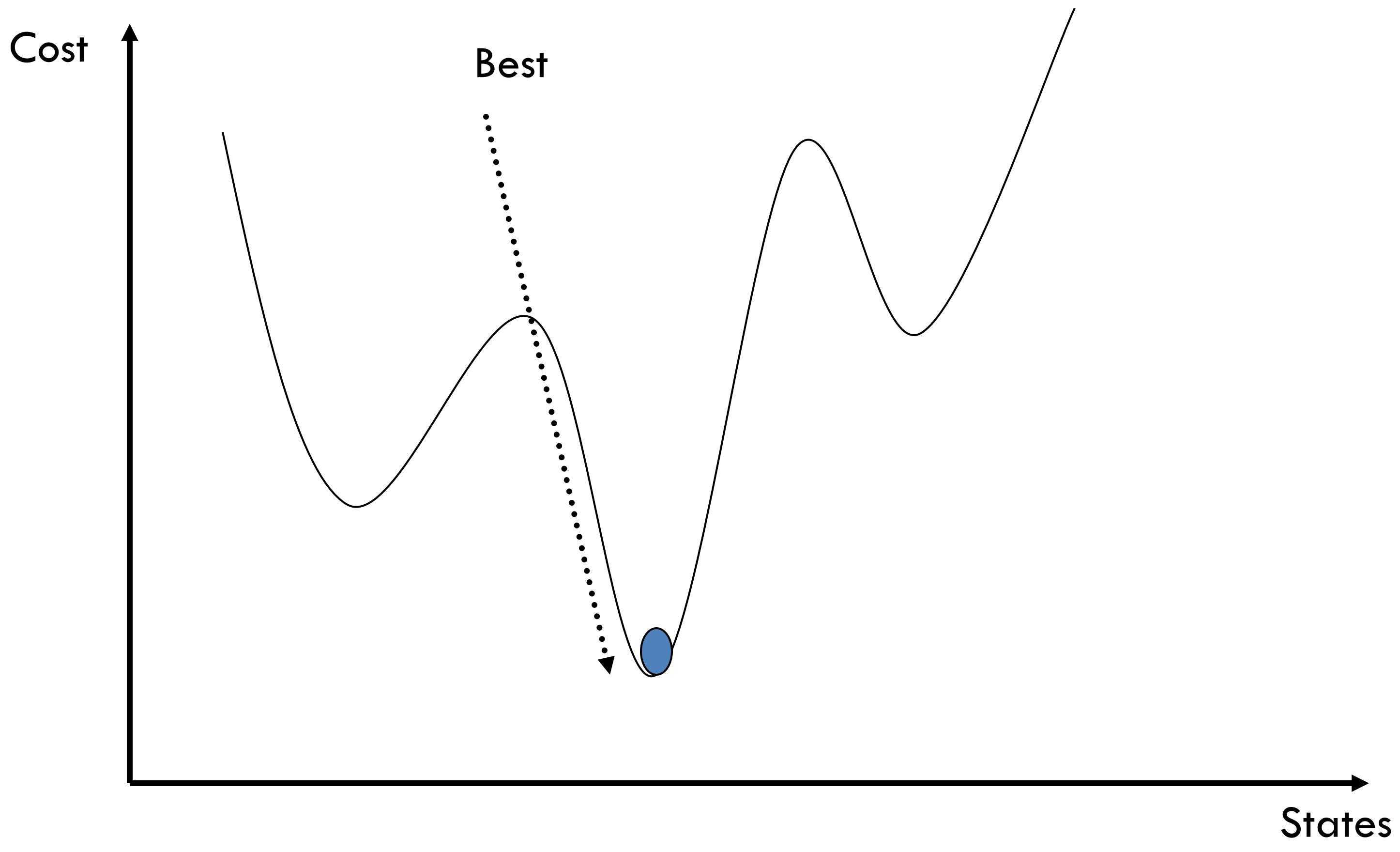

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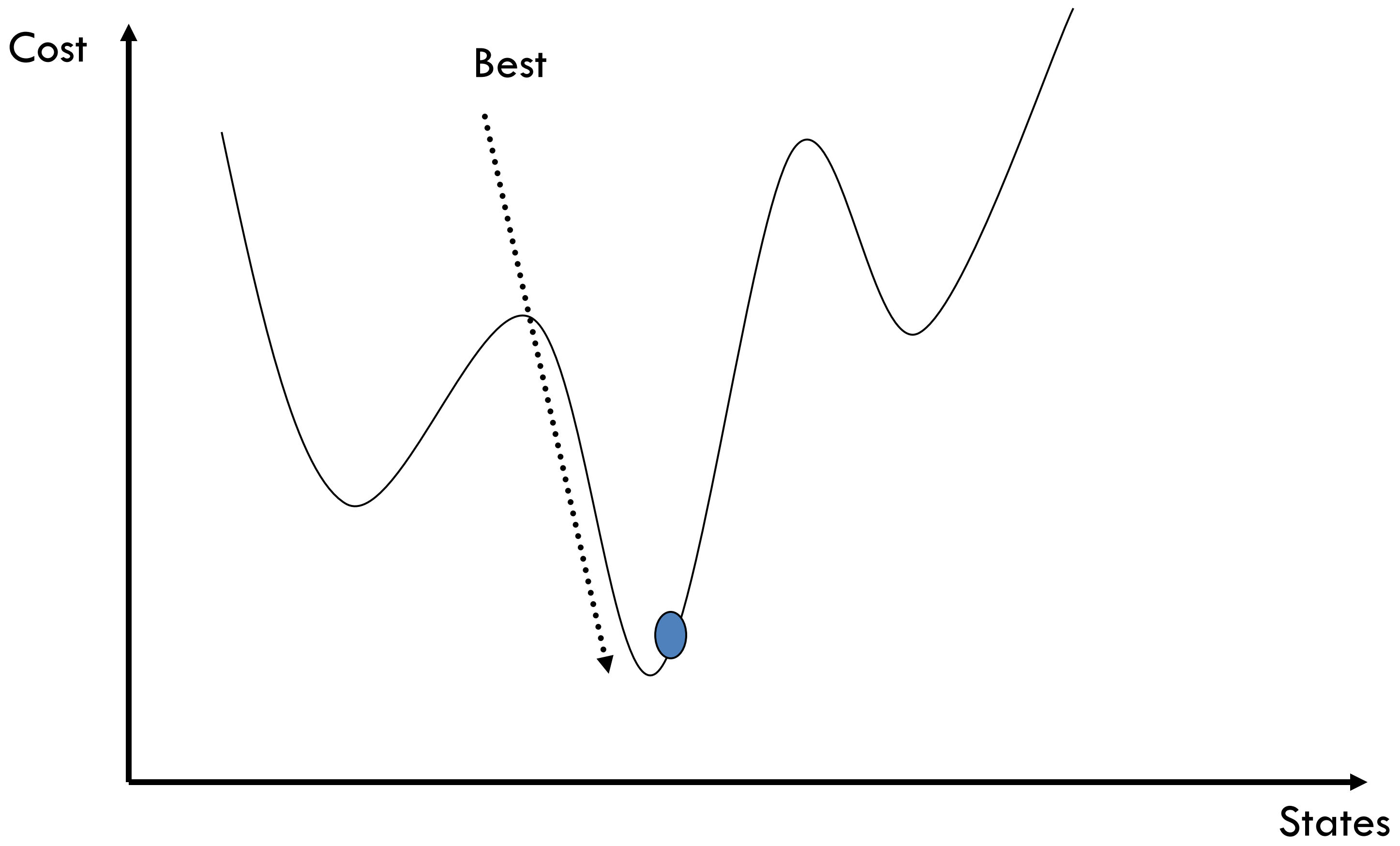

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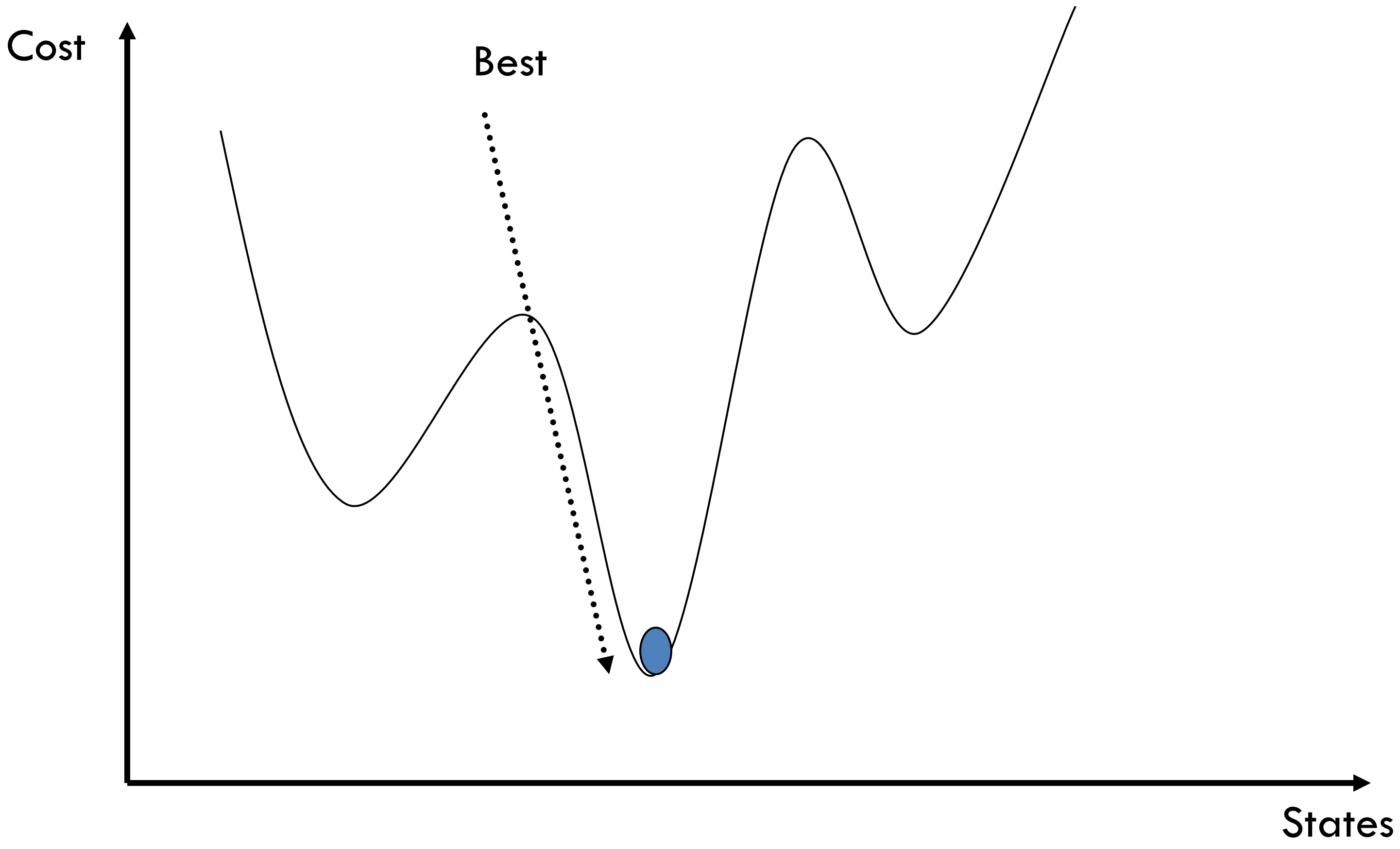

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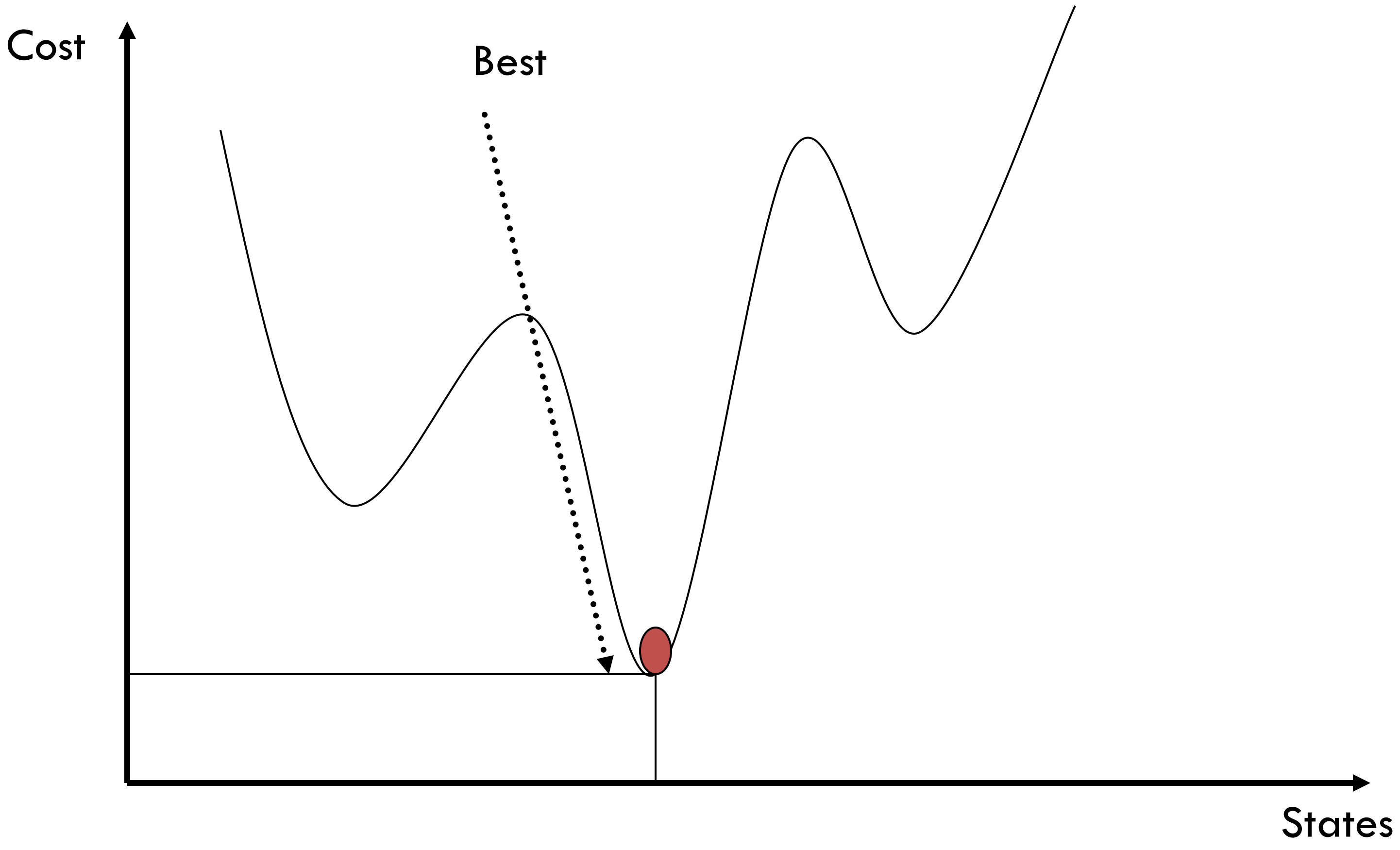
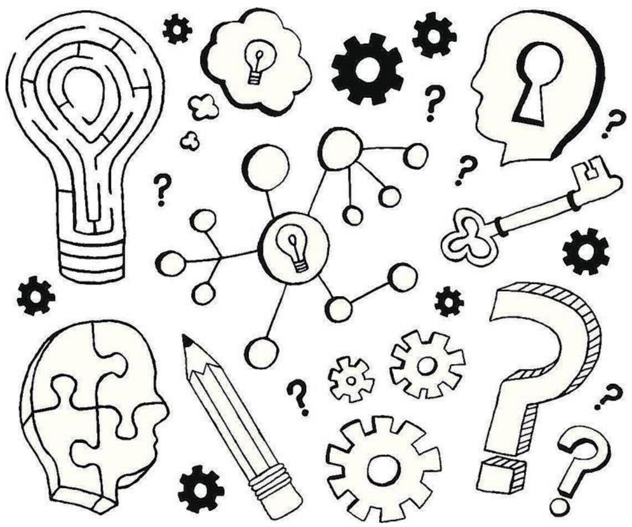


Note:

Finding Global Minimum can only happen if we traverse the required iterations otherwise this method finds the local minimum only.

Activity (Reflection, 20’)



Reflect on what you have studied in **Simulated Annealing Search**.

Time for a break – 20’

Genetic Algorithm (Search)

Genetic Algorithms

- Formally introduced in the US in the 70s by John Holland.
- GAs emulate ideas from genetics and natural selection and can search potentially large spaces.
- Before we can apply Genetic Algorithm to a problem, we need to answer:

  - How is an individual represented?
  - What is the fitness function?
  - How are individuals selected?
  - How do individuals reproduce?

- Note:

What is explained here is one of the approaches to GA. GA is a wide area to study

Genetic Algorithms: Representation of states (solutions)

Each state or individual is represented as a string over a finite alphabet. It is also called **chromosome** which Contains **genes**.

genes

Solution: 607 $\xrightarrow{\text{Encoding}}$ 1001011111

Chromosome:

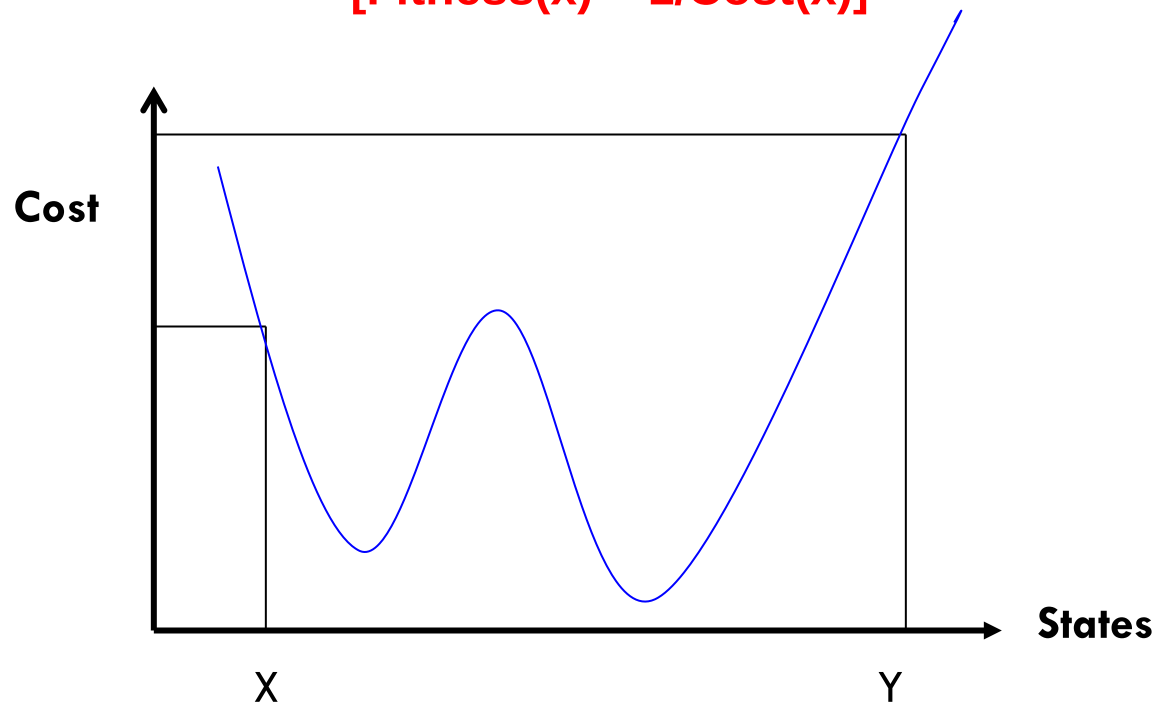
Binary String

Genetic Algorithms: Fitness Function

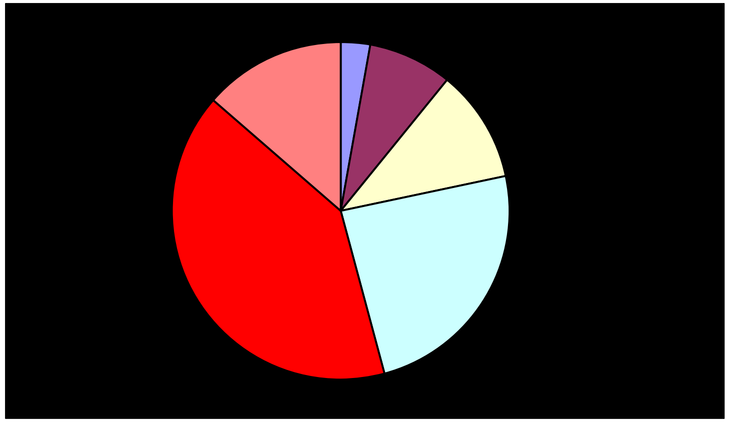
- Each state is rated by the evaluation function called **fitness function**. Fitness function should return higher values for better states:

Fitness(X) should be greater than Fitness(Y) !!

[Fitness(x) = 1/Cost(x)]



GA Parent Selection - Roulette Wheel



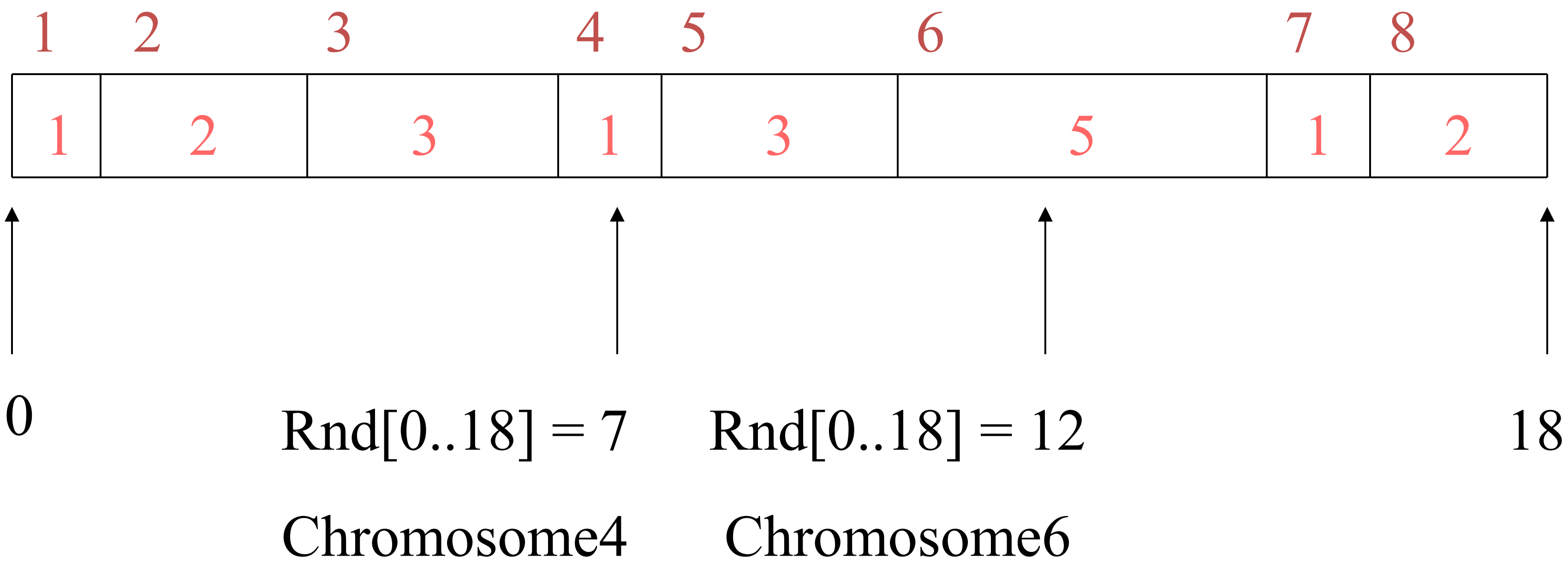
Roulette Wheel Selection

- Sum the fitnesses of all the population members, ***TF***
- Generate a random number, ***m***, between **0** and ***TF***
- Return the **first population member** whose fitness added to the preceding population members is greater than or equal to ***m***

Genetic Algorithms: Selection

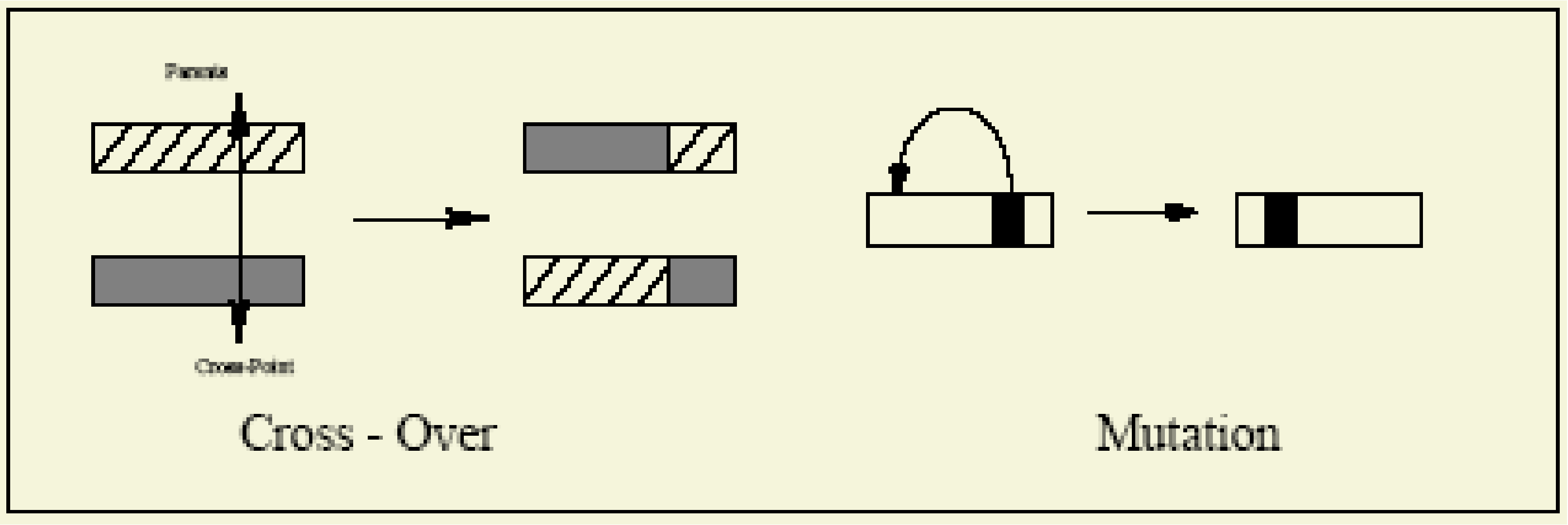
How are individuals selected ?

Roulette Wheel Selection



Genetic Algorithms: Cross-Over and Mutation

How do individuals reproduce ?



Genetic Algorithms
Crossover - Recombination

1010000000	Parent1	Offspring1	1011011111
1001011111	Parent2	Offspring2	1010000000

Crossover single point - random

With some high probability (*crossover rate*) apply crossover to the parents. (*typical values are 0.8 to 0.95*)

Stochastic Search: Genetic Algorithms Mutation

mutate

	Original offspring	Mutated offspring
Offspring1	1011011111	1011001111
Offspring2	1010000000	1000000000

Original offspring

Mutated offspring

With some small probability (the *mutation rate*) flip each bit in the offspring (*typical values between 0.1 and 0.001*)

Example

- If P_3 and P_2 are chosen as parents and we apply one point crossover show the resulting children, C_1 and C_2. Use a crossover point of 1 (first digit)
- Do the same using P_4 and P_2 with a crossover point of 2(two digit) and create C_3 and C_4
- Do multiple point crossover using parent P1 and P3 to give C5 and C6 on digts 1 and 4

Chromosome	Binary String
P_1	11100
P_2	01111
P_3	10111
P_4	00100

Chromosome	Binary String
C_1	11111
C_2	00111
C_3	00111
C_4	01100

C_5	10110
C_6	11101

Genetic Algorithms

Algorithm:

1. Initialize population with **p** Individuals at random
2. For each Individual **h** compute its fitness
3. While max fitness < threshold do

   Create a new generation Ps
4. Return the Individual with highest fitness

Activity (Reflection, 20’)

Reflect on what you have studied in **GA Search.**