Module 4

Eigenvalues and Eigenvectors

Today's Outline

I. Eigenvectors & Eigenvalues- Conceptual Analysis

II. Eigenvectors & Eigenvalues- Definition

III. Eigenvectors & Eigenvalues- Attributes

IV. Eigenvectors & Eigenvalues- Computation general case

V. Eigenvectors & Eigenvalues- Computation for n= 3

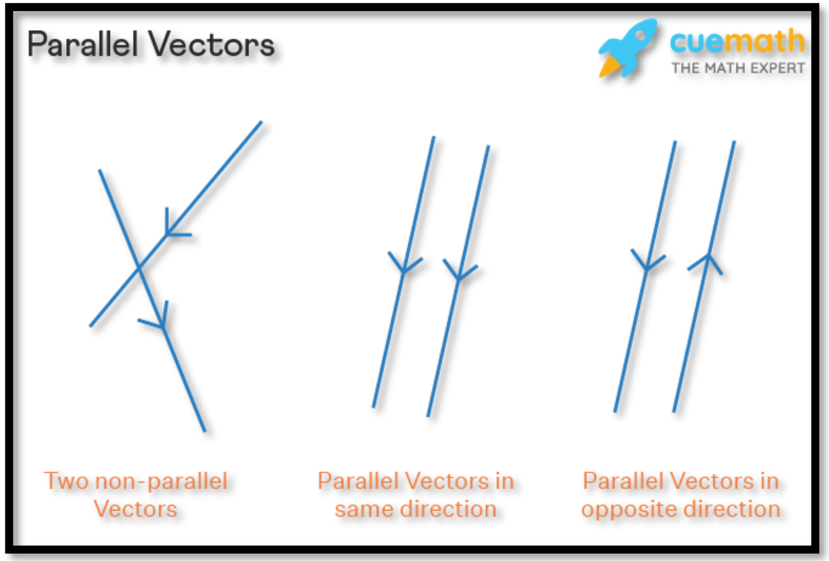
VI. Eigenspace

VII. Tutorials

Parallel Vectors

When are two vectors V & W parallel?

1. Two vectors V and W are called parallel if and only if the angle they form between them is 0°.
2. If two vectors V and W are parallel → $\vec{V} = \lambda \vec{u}$ $(where\ \lambda = Cte\ .)$



Activity

1. Are U [2,1,5] and W [3,8,0] Parallel?
2. Are U [2,1,5] and W [2,1,5] Parallel?
3. Are U [2,1,5] and W [-2,-1,-5] Parallel?
4. Are U [2,1,5] and W [10,5,25] Parallel?

Conceptual Example- General Case

$$A = \begin{pmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & -3 \end{pmatrix}$$

A is a 3X3 matrix

$$V0= \begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix}$$

V is a 3X1 Column matrix (Vector)

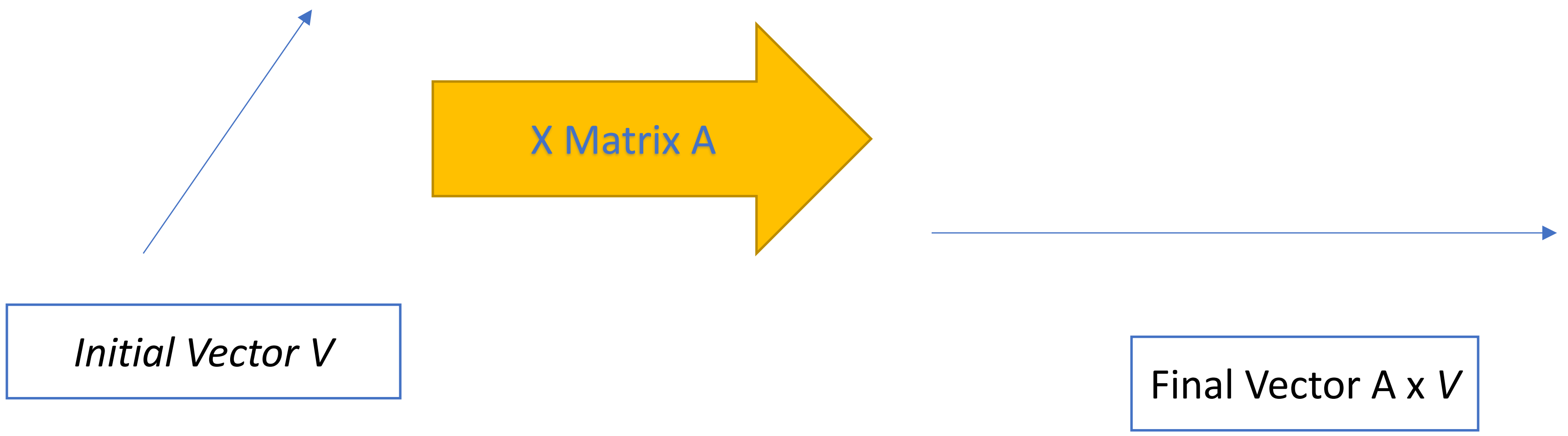
It is obvious:

$$\underbrace{\begin{pmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & -3 \end{pmatrix}}_{A} \times \underbrace{\boxed{\begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix}}}_{V0} = \underbrace{\boxed{\begin{bmatrix} 8 \\ 7 \\ -6 \end{bmatrix}}}_{V}$$

- Matrix A has been multiplied in Vector V0
- The magnitude of V0 has changed
- The Direction of V0 has changes

Eigenvectors & Eigenvalues

- In general, a matrix multiplied to a vector, changes **both** its
  I. Magnitude
  II. Direction.



Conceptual Example- Special Case

$$A = \begin{pmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & -3 \end{pmatrix}$$

A is a 3X3 matrix

$$V = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

V is a 3X1 matrix (Vector)

It is obvious:

$$\underbrace{\begin{pmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & -3 \end{pmatrix}}_{A} \mathrm{X} \underbrace{\boxed{\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}}}_{V} = \underbrace{3}_{\lambda} \mathrm{X} \underbrace{\boxed{\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}}}_{V}$$

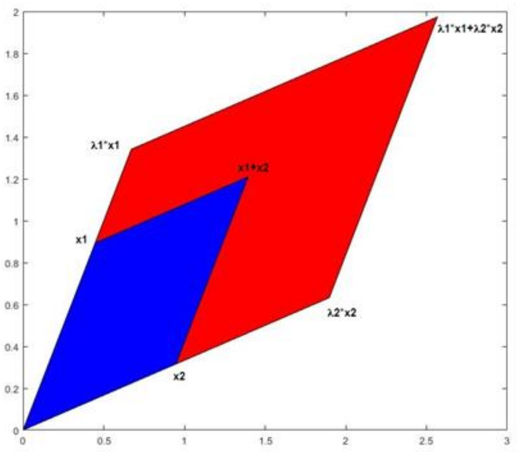
$$A\,\vec{V} = 3\,\vec{V}$$

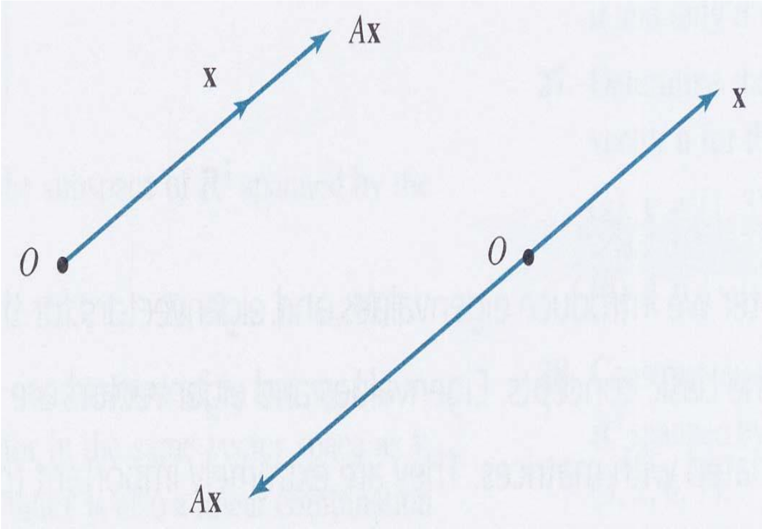
Should this equation be valid then:

- λ → eigenvalue of A
- **V** → eigenvector

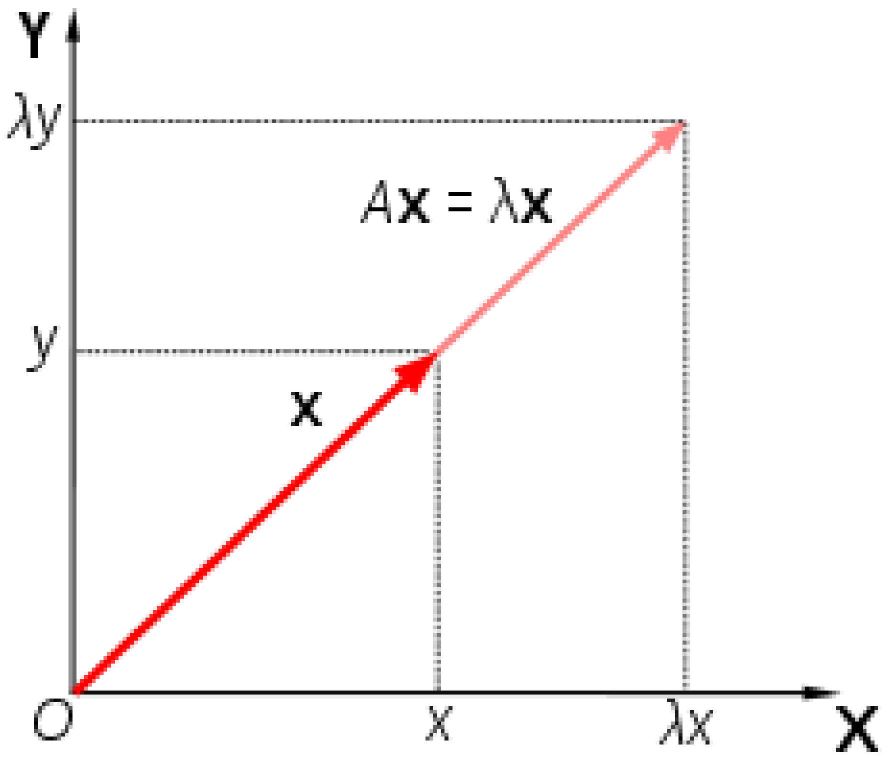
Eigenvectors & Eigenvalues

- In general, a matrix multiplied to a vector, changes **both** its
    I. Magnitude
    II. Direction.
- However, a matrix may operate on certain vectors by *changing only their magnitude.*
- The *new transformed vector* is just scaled *of the original vector*





Eigenvectors & Eigenvalues cont



Matrix A has transformed vector X → AX

- The direction of AX is **along** X
- Vector X is scaled to λX.

$A\mathbf{X} = \lambda\mathbf{X}$

- Number λ: *an eigenvalue of the matrix A.*
- Vector x: *an eigenvector corresponding to λ.*

- Eigen is German for "own" / "typical"

Today's Outline

I. Eigenvectors & Eigenvalues- Conceptual Analysis

II. Eigenvectors & Eigenvalues- Definition

III. Eigenvectors & Eigenvalues- Attributes

IV. Eigenvectors & Eigenvalues- Computation general case

V. Eigenvectors & Eigenvalues- Computation for n= 3

VI. Eigenspace

VII. Tutorials

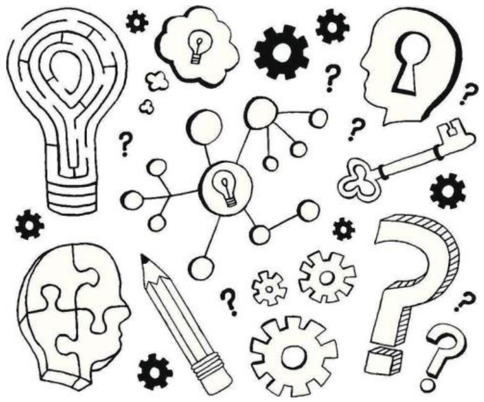
Eigenvectors & Eigenvalues- Definition

- Let A be an $n \times n$ matrix → $A_{n \times n}$
- Let λ be a nonzero scalar *(constant number)*
- If there exists a nonzero vector **X** in $\mathbf{R}^n$ such that

$$A\mathbf{X} = \lambda\mathbf{X}$$

- λ : an eigenvalue *of matrix A.*
- vector **x** : an eigenvector *corresponding to λ*.

Exercise (Individual, 10')



Which of the following is an Eigen value for vector X ?

$$A=\begin{bmatrix}-6 & 3\\ 4 & 5\end{bmatrix}$$

$$\mathbf{x}=\begin{bmatrix}1\\ 4\end{bmatrix}$$

a) $\lambda=6$

b) $\lambda=-6$

c) $\lambda=0$

d) $\lambda=3$

Example) Show that for matrix A, V is an eigenvector & 6 is an eigenvalue .

$$A=\begin{bmatrix}-6 & 3\\ 4 & 5\end{bmatrix}$$

$$\mathbf{x}=\begin{bmatrix}1\\ 4\end{bmatrix}$$

We should Show that: $Ax=\lambda x$

- $Ax=\begin{bmatrix}-6 & 3\\ 4 & 5\end{bmatrix}\begin{bmatrix}1\\ 4\end{bmatrix}=\begin{bmatrix}6\\ 24\end{bmatrix}$
- $\lambda x=\begin{bmatrix}6\\ 24\end{bmatrix}=6\begin{bmatrix}1\\ 4\end{bmatrix}$

$$\begin{bmatrix}-6 & 3\\ 4 & 5\end{bmatrix}\begin{bmatrix}1\\ 4\end{bmatrix}=6\begin{bmatrix}1\\ 4\end{bmatrix}$$

$$Ax=\lambda x$$

$$\lambda=6$$

Hence, for the Matrix A:

- $\lambda=6$ is an eigenvalue
- X is an eigenvector

Today’s Outline

I. Eigenvectors & Eigenvalues- Conceptual Analysis

II. Eigenvectors & Eigenvalues- Definition

III. Eigenvectors & Eigenvalues- Attributes

IV. Eigenvectors & Eigenvalues- Computation general case

V. Eigenvectors & Eigenvalues- Computation for n= 3

VI. Eigenspace

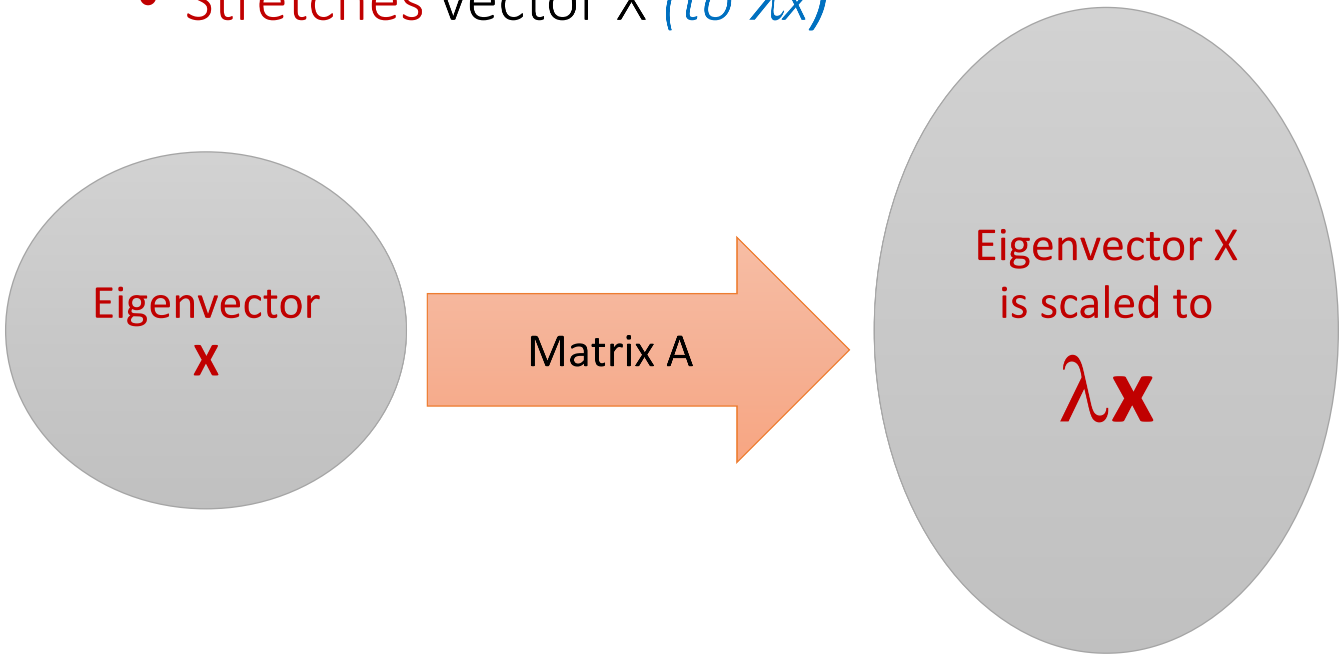
VII. Tutorials

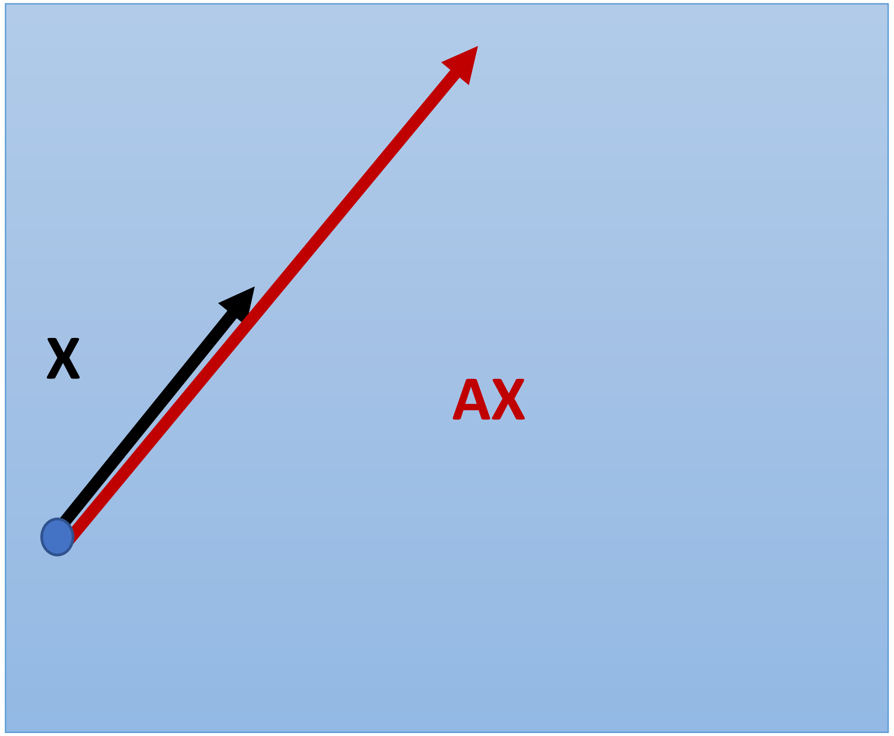
Geometrical Interpretation

A is a **transformation matrix** acting on a vector X:

- Does not change *the direction* of the vector X
- Stretches vector X *(to λx)*

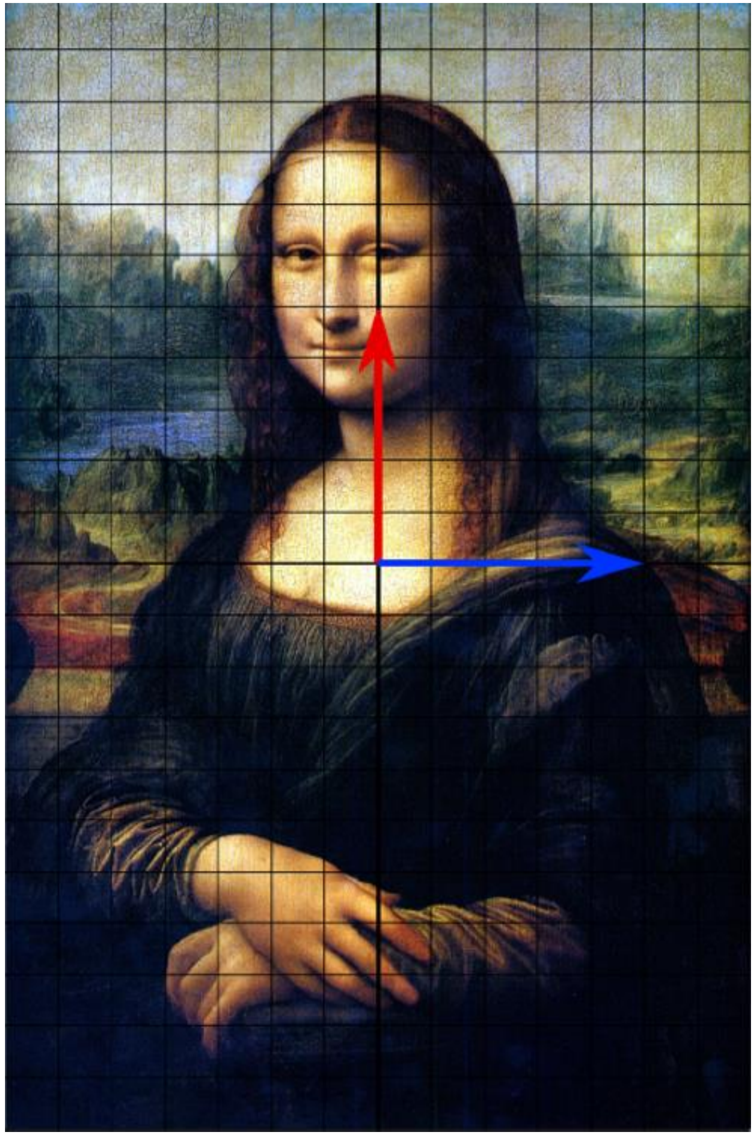
$$\boldsymbol{A}\mathbf{x} = \lambda \mathbf{x}$$

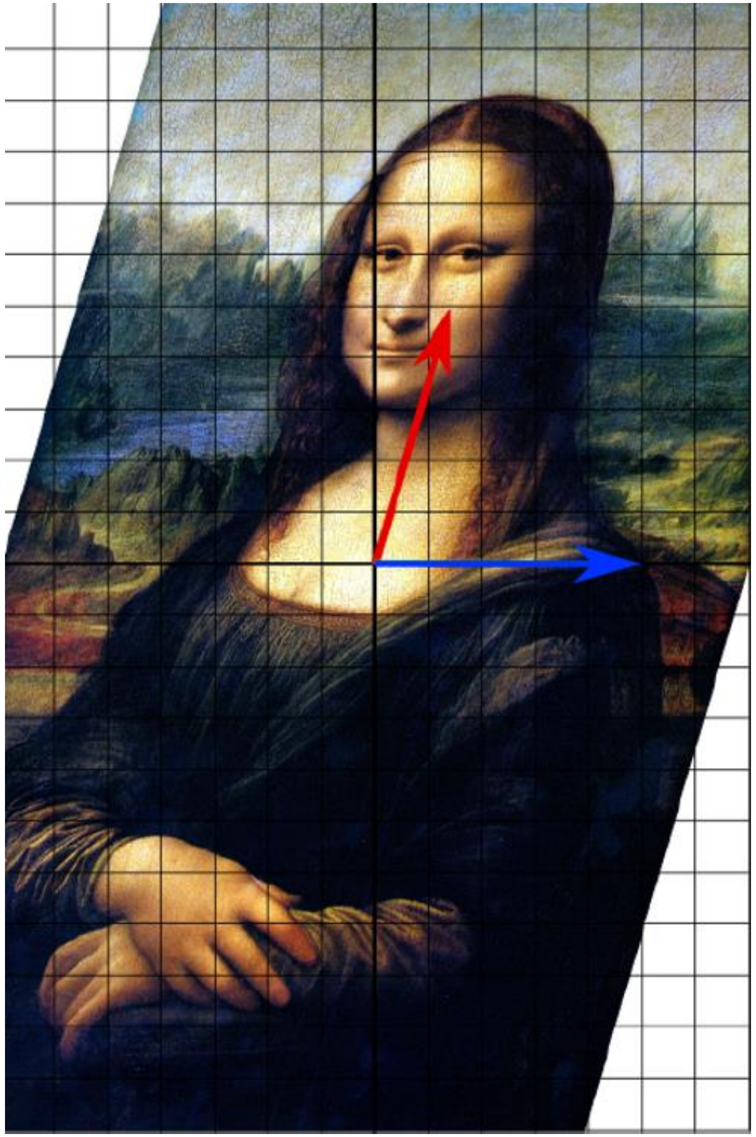




Vector space U

Vector space V





In this shear mapping:

- Red arrow changes direction
- Blue arrow does not.

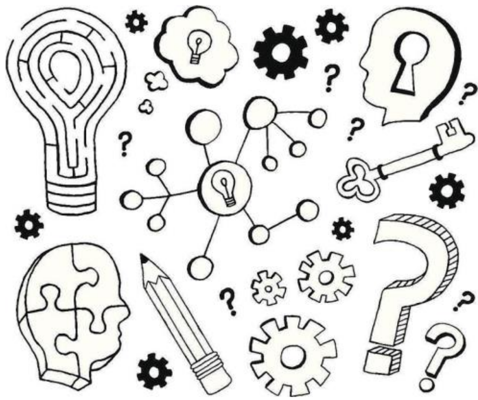
Blue arrow → eigenvector

1. It is a transformation matrix
2. It does not change direction
3. Its length is unchanged → *its eigenvalue is 1*

Application in Machine Learning

- Matrices represent a large set of data & information.
- Dealing with large-scale datasets might be problematic due to the need for a huge amount of memory and slow computational speed.
- Using eigenvalues & eigenvectors, one value& one vector can **represent** a large matrix , therefore alleviate the problem.
- You may think of eigenvalues and eigenvectors as **providing** summary of a large matrix.

Reflection (Individual, 10’)



1. What is an eigenvector? What is an eigenvalue?
2. What is the application of eigenvalues & eigenvectors in AI & ML?
3. What is the characteristic that distinguishes between eigenvectors and other matrix transformations?
4. What is the geometrical interpretation of eigenvectors & eigenvalues?
5. Have you ever used this mathematical concept in your coding?

Today's Outline

I. Eigenvectors & Eigenvalues- Conceptual Analysis

II. Eigenvectors & Eigenvalues- Definition

III. Eigenvectors & Eigenvalues- Attributes

IV. Eigenvectors & Eigenvalues- Computation general case

V. Eigenvectors & Eigenvalues- Computation for n= 3

VI. Eigenspace

VII. Tutorials

Computation Eigenvalues and Eigenvectors

- Let A be an $n \times n$ matrix → $A_{n \times n}$
- With eigenvalue λ
- And corresponding eigenvector $\mathbf{x}_{nx1}$.
- Thus;

$$A\mathbf{x} = \lambda\mathbf{x}$$

- Which can be re-written:

$$A\mathbf{x} - \lambda\mathbf{x} = \mathbf{0}$$

- Given that: $\lambda = \lambda I_n$ (I_n is an $n \times n$ identity matrix), we can write:

$$(A - \lambda I_n)\mathbf{x} = \mathbf{0}$$

- λ is an eigenvalue of matrix A if and only, if the equation $(A - \lambda I_n)\mathbf{x} = \mathbf{0}$, has a nontrivial solution →

$|A - \lambda I_n| = 0$

Computation Eigenvalues and Eigenvectors cont

- The equation $|A - \lambda I_n| = 0$ is called the characteristic equation of A.
- On expending the determinant $|A - \lambda I_n|$, we get a **polynomial in λ**.
- This polynomial is called the characteristic polynomial of A.
- The roots of characteristic polynomial are the eigenvalues of A.

Procedure for Computation

Matrix $A_{n \times n}$ is given. To compute it's eigenvalue & eigenvector:

1) The eigenvector for $A_{n \times n}$ will be of the form: $\mathbf{x}_{n \times 1}$

2) And I_n will be of the form: $I_{n \times n}$

3) Hence, $A\mathbf{x} = \lambda I_n \mathbf{x} \rightarrow (A - \lambda I_n)\mathbf{x} = \mathbf{0}$

4) Drive the characteristic equation $|A - \lambda I_n|$ (determinant of equation 3)

5) Expand the characteristic equation $\rightarrow$ characteristic Polynomial

6) Find the roots of the characteristic Polynomial $\rightarrow |A - \lambda I_n| = 0 \rightarrow$ Find λ_1, λ_2, etc.

7) Replace each λ in equation 3. For example, for $\lambda_1 \rightarrow A_{n \times n} - (\lambda_1 I_{n \times n})] \mathbf{x}_{n \times 1} = \mathbf{0}_{n \times 1}$

8) For each λ, calculate the elements of $\mathbf{x}_{n \times 1}$

Conceptual Example) Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} -4 & -6 \\ 3 & 5 \end{bmatrix}$$

Let’s do this together !

Procedure- Compute Eigenvalues

Step1) Finding Eigenvalues

1) $A_{n \times n} \rightarrow$ what is $I_{n \times n}$

2) Derive $A - \lambda I_n$

3) Compute $| A - \lambda I_n |$

Characteristic Polynomial

4) For what values of $\lambda \rightarrow | A - \lambda I_n | = 0$

Eigenvalue of Matrix A

Step1) Finding Eigenvalues

- $A = \begin{bmatrix} -4 & -6 \\ 3 & 5 \end{bmatrix} \Rightarrow$ n=2 $\Rightarrow I = I_2$

- $A - \lambda I_2 = \boxed{\begin{bmatrix} -4 & -6 \\ 3 & 5 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}} = \begin{bmatrix} -4-\lambda & -6 \\ 3 & 5-\lambda \end{bmatrix} \Rightarrow$

  $|A - \lambda I_2| = (-4-\lambda)(5-\lambda) + 18 = \lambda^2 - \lambda - 2$ Characteristic Polynomial

- We now solve the characteristic polynomial of *A*.

  $\lambda^2 - \lambda - 2 = 0 \Rightarrow (\lambda - 2)(\lambda + 1) = 0 \Rightarrow \lambda = 2$ or $\lambda = -1$ Eigenvalue of Matrix A

- **The eigenvalues of *A* are 2 and –1** → The corresponding eigenvectors are found by using these values of λ in the equation$(A - \lambda I_2)\mathbf{x} = \mathbf{0}$. There are many eigenvectors corresponding to each eigenvalue.

Procedure- Compute First Eigenvector

Step 2- Compute eigenvectors corresponding to $\lambda = 2$

1) What would be $(A - 2I_2)$?

2) What would be $(A - 2I_2)X$ → Expands to a set of two equations ?

3) Show that the solution to above equations are: $x_1 = -r$ & $x_2 = r$

4) Eigenvectors of A corresponding to $\lambda = 2$ is the nonzero vectors below. Why?

$$r\begin{bmatrix}-1\\1\end{bmatrix}$$

$\lambda = 2$

- We solve the equation $(A - 2I_2)\mathbf{x} = \mathbf{0}$ for $\mathbf{x}$. The matrix $(A - 2I_2)$ is obtained by subtracting 2 from the diagonal elements of A. We get

$$\begin{bmatrix} -6 & -6 \\ 3 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \mathbf{0}$$

- This leads to the system of equations

$$\begin{cases} -6x_1 - 6x_2 = 0 \\ 3x_1 + 3x_2 = 0 \end{cases}$$

- Giving $x_1 = -x_2$. The solutions to this system of equations are $x_1 = -r$, $x_2 = r$, where r is a scalar. Thus, the **eigenvectors of A corresponding to $\lambda = 2$** are nonzero vectors of the form

eigenvectors of A corresponding to $\lambda = 2$

$$r\begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

- $\lambda = 2$ is an eigenvalue of the matrix A
- vector X_1 is an eigenvector of the matrix A corresponding to $\lambda = 2$

$$X_1 = r\begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -4 & -6 \\ 3 & 5 \end{bmatrix} * \; r\begin{bmatrix} -1 \\ 1 \end{bmatrix} = 2\left\{ r\begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\}$$

Procedure- Compute Second Eigenvector

Step 3- Compute eigenvectors corresponding to $\lambda = -1$

1) What would be $(A - \lambda I_2)$

2) What would be $(A - \lambda I_2)$ X → Expands to a set of two equations

3) Show that the solution to above equations are: $x_1 = -2s$ & $x_2 = s$

4) Eigenvectors of A corresponding to $\lambda = -1$ is the nonzero vectors below. Why?

$$s\begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$\lambda = -1$

- We solve the equation $(A + 1I_2)x = 0$ for x. The matrix $(A + 1I_2)$ is obtained by adding 1 to the diagonal elements of A. We get

$$\begin{bmatrix} -3 & -6 \\ 3 & 6 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \mathbf{0}$$

- This leads to the system of equations

$$\begin{cases} -3x_1 - 6x_2 = 0 \\ 3x_1 + 6x_2 = 0 \end{cases}$$

- Thus $x_1 = -2x_2$. The solutions to this system of equations are $x_1 = -2s$ and $x_2 = s$, where s is a scalar. Thus the eigenvectors of A corresponding to $\lambda = -1$ are nonzero vectors of the form

eigenvectors of A corresponding to λ = -1

$$s\begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

- $\lambda = -1$ is an eigenvalue of the matrix A
- vector X_2 is an eigenvector of the matrix A corresponding to $\lambda = -1$

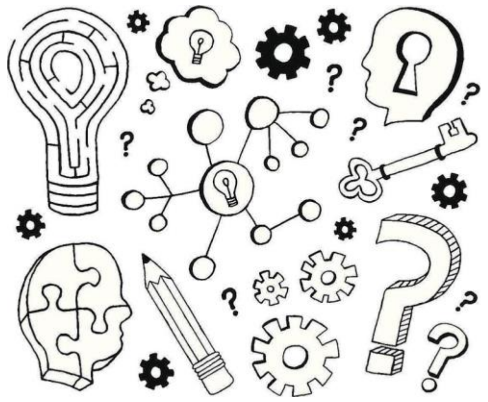
$$X_2 = s\begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -4 & -6 \\ 3 & 5 \end{bmatrix} * s\begin{bmatrix} -2 \\ 1 \end{bmatrix} = (-1)\left\{ s\begin{bmatrix} -2 \\ 1 \end{bmatrix} \right\}$$

Activity (Individual, 20’)

- What is the eigenvalues & eigen vectors of of A?

$$A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$$



Check your Answer

$$r\begin{pmatrix} 1 \\ 1 \end{pmatrix}, s\begin{pmatrix} -6/5 \\ 1 \end{pmatrix}$$

Break- 20’

Today's Outline

I. Eigenvectors & Eigenvalues- Conceptual Analysis

II. Eigenvectors & Eigenvalues- Definition

III. Eigenvectors & Eigenvalues- Attributes

IV. Eigenvectors & Eigenvalues- Computation general case

V. Eigenvectors & Eigenvalues- Computation for n= 3

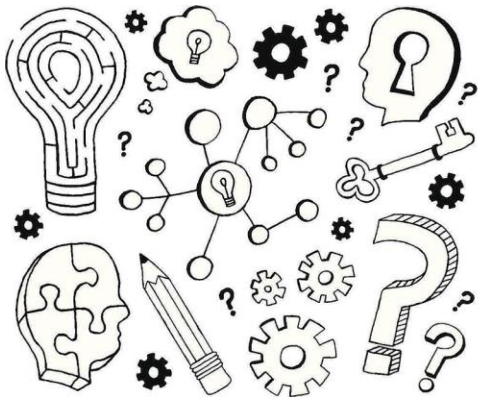
VI. Eigenspace

VII. Tutorials

Determinants

- The process of calculating the determinants of matrix with $n > 2$ → Hectic
- We use different tricks, including:
  1. Interchange Property
  2. Sign Property
  3. Zero Property
  4. Multiplication Property
  5. Sum Property
  6. Property Of Invariance
  7. Triangular Property

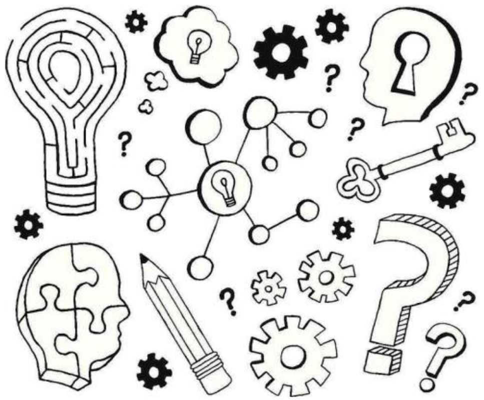
Reading(Individual, 15’)



- Bellow article discuss the properties of a determinant. Read and investigate each property.

Properties of Determinants - Properties, Formulas, Examples (cuemath.com)

Activity (Individual, 30’)



- What are the eigenvalues & eigenvectors for Matrix A?

$$A = \begin{bmatrix} 5 & 4 & 2 \\ 4 & 5 & 2 \\ 2 & 2 & 2 \end{bmatrix}$$

Procedure- Compute Eigenvalues

1) $A_{n \times n} \rightarrow$ what is $I_{n \times n}$

2) Derive $A - \lambda I_n$

3) Compute $| A - \lambda I_n |$ Characteristic Polynomial

4) For what values of $\lambda \rightarrow | A - \lambda I_n | = 0$ Eigenvalue of Matrix A

Solution

1) The matrix $A - \lambda I_3$ is obtained by subtracting λ from the diagonal elements of A.

$$A - \lambda I_3 = \begin{bmatrix} 5 & 4 & 2 \\ 4 & 5 & 2 \\ 2 & 2 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 5-\lambda & 4 & 2 \\ 4 & 5-\lambda & 2 \\ 2 & 2 & 2-\lambda \end{bmatrix}$$

- So, the characteristic polynomial of A is $|A - \lambda I_3| \rightarrow |A - \lambda I_3| = \begin{vmatrix} 5-\lambda & 4 & 2 \\ 4 & 5-\lambda & 2 \\ 2 & 2 & 2-\lambda \end{vmatrix}$

2) Using row and column operations to simplify determinants

[Row 1 - Row2 : : $\boxed{\{((5-\lambda)-4),\ (4-(5-\lambda)),\ (2-2)\}}$ → we get

Property of Invariance

$$|A - \lambda I_3| = \begin{vmatrix} 5-\lambda & 4 & 2 \\ 4 & 5-\lambda & 2 \\ 2 & 2 & 2-\lambda \end{vmatrix} = \begin{vmatrix} 1-\lambda & -1+\lambda & 0 \\ 4 & 5-\lambda & 2 \\ 2 & 2 & 2-\lambda \end{vmatrix}$$

3) $|A-\lambda I_3| = \begin{vmatrix} 1-\lambda & -1+\lambda & 0 \\ 4 & 5-\lambda & 2 \\ 2 & 2 & 2-\lambda \end{vmatrix}$

Using Row & Column operation → Simplify determinant →

Replace Column 2 with → Column 1 + Column 2

$$\boxed{|A-\lambda I_3| = \begin{vmatrix} 1-\lambda & 0 & 0 \\ 4 & 9-\lambda & 2 \\ 2 & 4 & 2-\lambda \end{vmatrix}}$$

Multiplying inside bracket

Algebraic Operation

4) $|A-\lambda I_3| = \begin{vmatrix} 1-\lambda & 0 & 0 \\ 4 & 9-\lambda & 2 \\ 2 & 4 & 2-\lambda \end{vmatrix} = (1-\lambda)[(9-\lambda)(2-\lambda)-8] = (1-\lambda)\,[\lambda^2 - 11\lambda + 10]$

$= (1-\lambda)\,[(\lambda-10)(\lambda-1)] = -(\lambda-10)(\lambda-1)^2$

5) solving the characteristic equation of A: $-(\lambda-10)(\lambda-1)^2 = 0 \rightarrow \lambda = 10$ or $\lambda = 1$

- The eigenvalues of A are 10 and 1.

6) The corresponding eigenvectors are found by using values of λ in the equation $(A - \lambda I_3)\mathbf{x} = \mathbf{0}$.

Procedure- Compute Eigenvector

Compute eigenvectors corresponding to $\lambda = 10$

1) What would be $(A - \lambda I_3)$

2) What would be $(A - \lambda I_3)$ X → Expands to a set of three equations → Find the solutions of X

$$A = \begin{bmatrix} 5 & 4 & 2 \\ 4 & 5 & 2 \\ 2 & 2 & 2 \end{bmatrix}$$

- $\lambda = 10$

  We get $(A - 10I_3)\mathbf{x} = \mathbf{0} \rightarrow \left[\begin{bmatrix} 5 & 4 & 2 \\ 4 & 5 & 2 \\ 2 & 2 & 2 \end{bmatrix} - 10\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right]\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \mathbf{0}$ ⇨ $\boxed{\begin{bmatrix} -5 & 4 & 2 \\ 4 & -5 & 2 \\ 2 & 2 & -8 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \mathbf{0}}$

The solution to this system of equations are $x_1 = 2r$, $x_2 = 2r$, and $x_3 = r$, where r is a scalar.

Thus, the eigenvector of $\lambda = 10$ is the one-dimensional space of vectors of the form.

$$\boxed{r\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}}$$

Procedure- Compute Eigenvector

Compute eigenvectors corresponding to $\lambda = 1$

1) What would be $(A - \lambda I_3)$

2) What would be $(A - \lambda I_3)$ X → Expands to a set of three equations → Find the solutions of X

- $\lambda = 1$

Let $\lambda = 1$ in $(A - \lambda I_3)\mathbf{x} = \mathbf{0}$. We get

Subtract 1 from the elements of the main diagonal of A

$$(A - 1I_3)\mathbf{x} = \mathbf{0}$$

$$\begin{bmatrix} 4 & 4 & 2 \\ 4 & 4 & 2 \\ 2 & 2 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \mathbf{0}$$

The solution to this system of equations can be shown to be $x_1 = -s - t$, $x_2 = s$, and $x_3 = 2t$, where s and t are scalars.

Thus, the eigenspace of $\lambda = 1$ is the space of vectors of the form:

$$\boxed{\begin{bmatrix} -s - t \\ s \\ 2t \end{bmatrix}}$$

Separating the parameters s and t, we can write

$$\begin{bmatrix} -s-t \\ s \\ 2t \end{bmatrix} = s\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$$

Thus, the eigenvector of $\lambda = 1$ is a two-dimensional subspace of $\mathbf{R}^2$ with basis

$$\boxed{\left\{ S\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, t\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \right\}}$$

Today's Outline

I. Eigenvectors & Eigenvalues- Conceptual Analysis

II. Eigenvectors & Eigenvalues- Definition

III. Eigenvectors & Eigenvalues- Attributes

IV. Eigenvectors & Eigenvalues- Computation general case

V. Eigenvectors & Eigenvalues- Computation for n= 3

VI. Eigenspace

VII. Tutorials

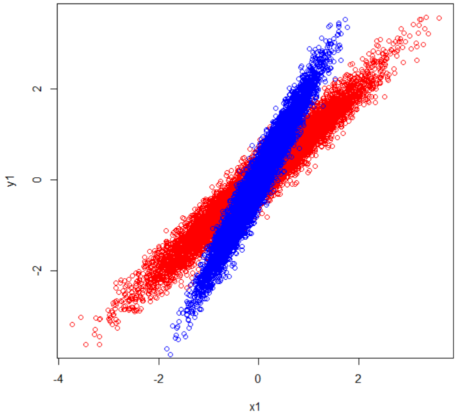
Eigenspace

- As evident , there are different eigenvalues for the matrix A.
- For every eigenvalue, there is a group of eigenvectors **corresponding** to *that eigenvalue*.
- Further, zero vector is the nontrivial solution to the equation $(A - \lambda I_n)\mathbf{x} = \mathbf{0}$
- All the eigenvectors corresponding **to** λ together with the zero vector forms a set.
- This set is known as eigenspace.

Eigenspace cont.

Definition

- The set of all solutions of $(A - \lambda I)x = 0$
- Is called the **eigenspace** of A corresponding to λ .
- This is the set of all the eigenvectors corresponding to a λ



Eigenspace cont

- In Exercise 1, the eigenspace is:

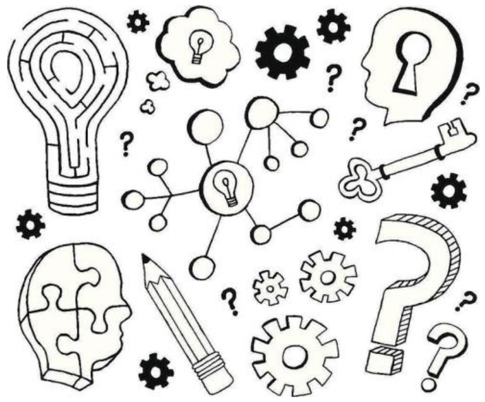
$$\left\{ r\begin{bmatrix}-1\\1\end{bmatrix}, s\begin{bmatrix}-2\\1\end{bmatrix}, t\begin{bmatrix}0\\0\end{bmatrix} \right\}$$

- *Where, r, s & t are three constants*

Exercise (Individual, 10’)

- What is the eigenspace of A?

$$A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$$



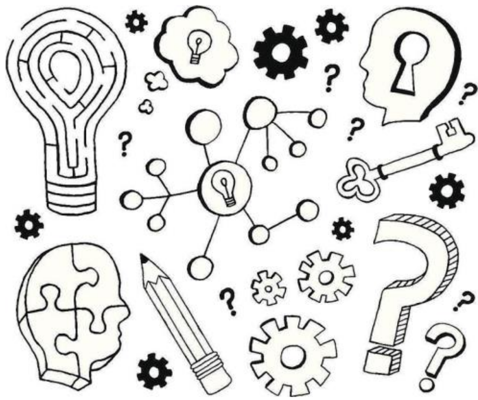
Check your Answer

- $\vec{v} = \{ t\begin{bmatrix}0\\0\end{bmatrix}, r\begin{pmatrix}1\\1\end{pmatrix}, s\begin{pmatrix}-6/5\\1\end{pmatrix}\}$

Today's Outline

I. Eigenvectors & Eigenvalues- Conceptual Analysis
II. Eigenvectors & Eigenvalues- Definition
III. Eigenvectors & Eigenvalues- Attributes
IV. Eigenvectors & Eigenvalues- Computation general case
V. Eigenvectors & Eigenvalues- Computation for n= 3
VI. Eigenspace
VII. Tutorials

Reflection (Individual, 20’)



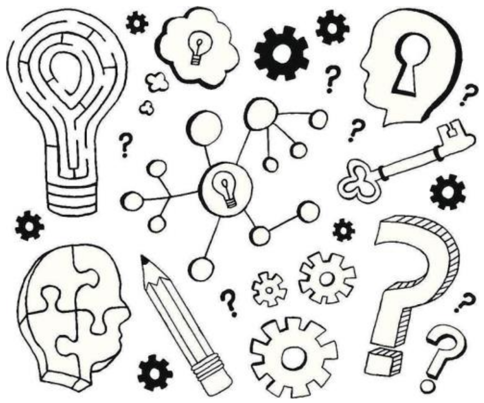
1. What is an eigenvector? What is an eigenvalue? What is an eigenspace?
2. What is the application of eigenvalues & eigenvectors in AI & ML?
3. What is the characteristic that distinguishes between eigenvectors and other matrix transformations?
4. What is the geometrical interpretation of eigenvectors & eigenvalues?
5. The eigenvalue for the matrix A (bellow)is λ=2. Determine if U or V are the eigenvectors corresponding to λ.

$$A = \begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix},$$

a) $u = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

b) $v = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

Exercise 5- Answer



The eigenvalue for the matrix A (bellow)is $\lambda = 2.$

Determine if U or V are the eigenvectors corresponding to λ.

$$A = \begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix},$$

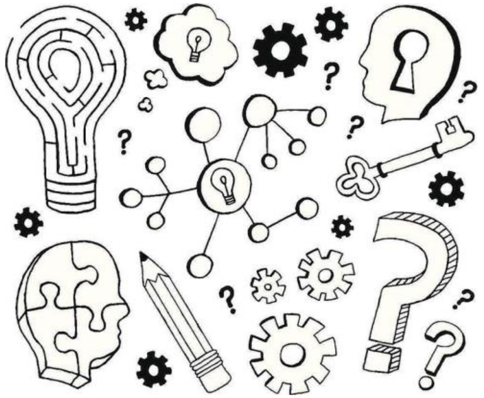
$$a.\ \ u = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$b.\ \ v = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

a. $AU \neq \lambda U \ \rightarrow U\ is\ not\ an\ eigenvector\ corresponding\ to \lambda$

b. $AV = \lambda V \ \rightarrow V\ is\ an\ eigenvector\ corresponding\ to\ this\ \lambda$

Reading (Individual, 20')



1. Read the bellow article and summaries the application of eigenvalues & eigenvectors in computer science.

https://www.linkedin.com/pulse/understanding-eigenvalues-eigenvectors-computer-vision-kanishka-gabel/?utm_source=rss&utm_campaign=articles_sitemaps

Any Questions or Concerns?

Sources for the slides:

https://fdocuments.in/

And

https://www.xpowerpoint.com/