Knowledge Representation and Reasoning

- Knowledge represented in knowledge-base
    - **'Facts' are represented through linguistic variables and rules follow fuzzy logic**
    - In real world systems we actually have crisp inputs based on sensor data, so we need to map this to linguistic variables - **fuzzification**
    - We then invoke rules systematically to see what fires and to what degree - **inference**
        - Contrasts with traditional systems - rules fire or dont, fuzzy system - rules fire to a degree
    - We then calculate the influence of each rule with a form of 'averaging' procedure - **composition**
    - Finally, we convert the fuzzy values into a crisp number which can be used in a real world system - **defuzzification**
- **Four major tasks of a fuzzy system: fuzzification, inference, composition, defuzzification**
- **Air-Conditioner example (16 degrees):**
    - Fuzzification - 16 degrees relates to the linguistic variables cool and pleasant
    - Inference - rules containing cool and pleasant are rules 2 and 3
    - Composition - perform alpha level cut on cool and pleasant at the degree belonging and create new membership function
    - Defuzzification - examine the sets of slow and medium to obtain a crisp speed

Knowledge-based expert systems

- Embodies organised knowledge pertaining to specific area of human activity
    - Expected to perform to the standard of a human expert
- Human expertise consists of domain-specific knowledge, and experience based solution recognition
- Recognition based expertise then is the process of breaking the goal into many sub-goals, and once all sub-goals are satisfied, the key goal is deemed satisfied
- **Knowledge of a subject domain is encoded within its terminology**

Representation

- Production system based on production rules/productions: "if this condition then do this action"
    - Works on the assumption that rules are explicit, and rules cannot invoke each other
- **Fuzzy knowledge based system (KBS) is a KBS performing approximate reasoning**
    - Comprises vague facts and vague rules
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KB Entity	**Fuzzy KB**	**Crisp KB**
Fact	X is μ_x	X is TRUE or X is NOT TRUE
Rule	If X is μ_x Then Y is μ_y	If X Then Y

    - Two Challenges:

        - **How to interpret and represent vague rules**
        - **How to find an inference mechanism with well-defined semantics that permits approximate reasoning by means of a general system of vague rules, and case-specific vague facts?**
    - We describe the antecedent/premise/condition as an object/event/state in the form of a fuzzy specification of a measured value
    - The consequent/conclusion/action consists of an appropriate fuzzy value
    - **If age is a linguistic variable, then each term in T(age) is a label of a fuzzy subset of a universe of discourse, say U = [0,100]**
    - **Linguistic variables can be associated with two rules:**
        - **Syntactic Rule**
            - Defines well formed sentences in T()
        - **Semantic Rule**
            - By which the meaning of the terms of T() may be determined. If X is a term in T() then its meaning is a subset off U. That is, the primary terms of T() can be used to give meaning to the non primary terms
            - Example semantic rule: $\mu_{very\ young} = (\mu_{young})^2$
    - Knowledge representation aims to represent complex concepts that are rich in meaning
        - Simple concepts to us (height, heat or colour), are given meaning-rich and ambiguous linguistic assignments (instead of crisp but meaningless numbers) e.g. {short,medium,tall} ; {cold, warm, hot} ; {red, green, blue}
        - This mapping of data to linguistic description involves mapping raw data to a fuzzy set
        - **Fuzzy modifiers can be used to diffuse or focus the meaning of the variable**
    - Example: Air conditioner
        - The rules that are used to control can be expressed:
          $Control = Temp * Speed$
        - Can use our observations to formalise membership functions/rules
- **Composition post alpha level cut**
    - We get an alpha level set from the two membership functions
    - Need to get a single number from the curve below corresponding to the motor speed
    - **Defuzzification**
        - Two popular techniques: Centroid, and Maximum
        - **Centroid**
            - Compute Center of Gravity(COG) or Center of Area(COA) of the rule outputs
            - COG involves computing the weighted sum of the Speed and the corresponding membership function of the output fuzzy set and weighted sum of the membership function
            - The COG attempts to take the applicativeness of rules into account

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$$\eta = \frac{1}{\sum \mu^{output}_{x1,\ldots,xn}(y)} \sum y \quad \mu^{output}_{x1,\ldots,xn}(y)$$
    - Use the sum to approximate the integral
- **Mean of Maxima**
    - Weighted sum and weighted membership are calculated, except the membership function is given another alpha level cut corresponding to the maximum value of the output fuzzy set
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$$\eta = \frac{1}{\left| Max(\mu^{output}_{x1,\ldots,xn}) \right|} \sum_{y \in Max(\mu^{output}_{x1\ldots.xn})} y$$

where $Max(\mu^{output}_{x1\ldots.xn})$ denotes the set of all outputs y, with

$$\mu^{output}_{x1\ldots.xn}(y) = Max(\mu^{output}_{x1\ldots.xn})$$

    - These two calculations will differ

- **Fuzzification**
    - Membership function defined on input variables are applied to their actual values to determine the degree of truth for each rule premise
    - The degree of truth for a rule's premise is referred to as an alpha value
    - If a rule's premise has a non-zero alpha value, it is said to fire
- **Inference**
    - The truth value is computed and the conclusion is applied to each part of the rule - results in one fuzzy subset assigned to each output variable for each rule.
    - **Two methods**
    - **Min**
        - The output membership function is clipped off at a height corresponding to the computed degree of truth of a rule's premise - based on the AND operation of fuzzy logic
    - **Product**
        - Output membership function is scaled by the premise's degree of truth
- **Composition**
    - **Max**
        - Combined fuzzy subset is constructed by taking the pointwise maximum over all fuzzy subsets assigned to the output variable by the inference rule
    - **Sum**

- Combined output fuzzy subset is constructed by taking the pointwise sum over all fuzzy subsets assigned to output variable by their inference rule (**can result in truth values > 1**)

This is all mamdani calculus where membership functions of antecedent and consequent variables are considered at composition.

- For real-time systems mamdani calculus isnt feasible
- And not necessarily always desirable due to Occam's logic (simplify where possible)