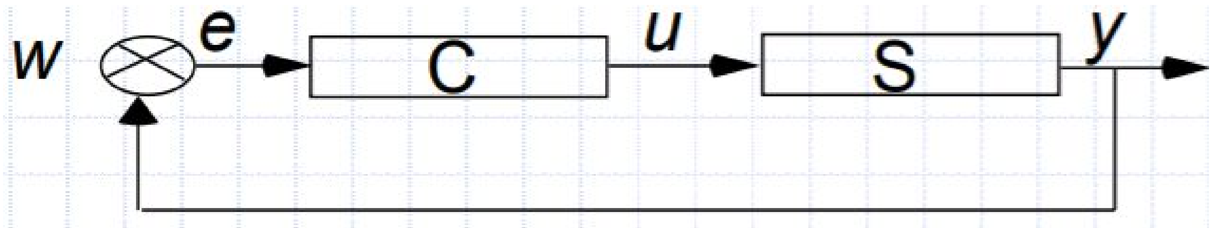
Control Theory - mechanism to guide or regulate the operation of a machine, often linked with **feedback:** a process of returning a fraction of the output signal to the input. Feedback can be negative (i.e. opposes the input), which dampens the input, or positive where it reinforces the input

Feedback Control - mechanism for guiding or regulating the operation of a system by returning to the input of the system a fraction of the output

Notation: system to guide (S), input (W), output (y), feedback controller (C), input to the controller - error signal (e)
Purpose of the controller is to guarantee the desired response of the output (y).



Control signal, u, can be said to be: proportional to the error, proportional to the magnitude of the error and the duration of the error, and proportional to relative changes in error values over time.
In classic control theory, would have to solve a non-linear equation for u to find 3 constants, but **in fuzzy logic, the non-linear function, u, is represented by a fuzzy mapping generally acquired from human beings.**

Traditional control system creates a mathematical model of the process, and specifications of the desired closed-loop behaviour to design a controller. This doesn't work if the model is:
- difficult to obtain
- partly unknown
- highly nonlinear

Use logical rules with vague predicates to derive inference from vague formulated data - linguistic control

A fuzzy controller is a device that is intended to modelise some vaguely known or vaguely described process.

Controller Type	**Typical Operation**
Mamdani (linguistic) controller with either fuzzy or singleton consequents	Direct closed-loop controller
Takagi-Sugeno (TS) or Takagi-Sugeno-Kang controller	Supervisory controller - as a self tuning device

Two modes of fuzzy controllers:

- **Feedback:** the fuzzy controller will act as a control device
- **Feedforward:** the controller is used as a prediction device

All inputs to, and outputs from, the controller are in the form of linguistic variables - **in a sense the fuzzy controller maps the input variables to a set of output linguistic variables.**

Regulation is the process of keeping the output, y, close to the setpoint (reference input), w, despite the presence of disturbances, fluctuations, and noise.
Error e = w-y

Principal message in fuzzy control literature: control algorithm is a knowledge-based algorithm, described by the methods of fuzzy logic

Typical Fuzzy Controller: **described by the relationship between change in control (u(k)) and error (e(k)), and the change in error**

$$\Delta e(k) = e(k) - e(k-1)$$

$$\Delta u(k) \ = \ F(e(k), \Delta e(k))$$

Heuristics for feedback control loop:

- **System responsiveness: IF** error is positive (negative) **AND** the change in error is approximately zero **THEN** the change in control is positive (negative)
- **Reduction in Overshooting: IF** error is approximately zero **AND** the change in error is positive (negative) **THEN** the change in control is positive (negative)
- **Steady State Control: IF** the error and change in error are approximately zero **THEN** the change in control is approximately zero

Balancing the cartpole

- Control system to output some force, F, to apply in order to balance the cartpole - engineering approach calculates two second order differential equations
- Can approach with a linguistic rule
- **IF** θ is approximately_zero **AND** $\frac{d\theta}{dt}$ is approximately_zero **THEN** F is approximately_zero
- So we assign the variable angle (θ) the linguistic variables: negative, approximately_zero, and positive
- Apply these terms to each variable (angular velocity, and force), note: they are all distinct sets
- We get a decision matrix using these terms

		Angular velocity	
Angle	$n_{\dot\theta}$	$az_{\dot\theta}$	$p_{\dot\theta}$
n_θ	n_F	n_F	az_F
az_θ	n_F	az_F	p_F
p_θ	az_F	p_F	p_F

- This coarse rule base is a starting point - need more specific rules: so negative big, negative medium, negative small, and the same for positive

	n_b	n_m	n_s	a_z	p_s	p_m	p_b
n_b			p_s	p_b			
n_m				p_m			
n_s	n_m		n_s	p_s			
a_z	n_b	n_m	n_s	a_z	p_s	p_m	p_b
p_s				n_s	p_s		p_m
p_m				n_m			
p_b				n_b	n_s		

  - 19 rule KB

Example of inference

- Find the rules which will fire for the supplied values
- Calculate the membership value of the values to the linguistic terms causing the rules to fire
- Rewrite their membership functions with an alpha level cut i.e. take the maximum output from the membership function (i.e. 1), and set it to the value's membership value e.g. 36 has membership of 0.6 to term positive_small, cap positive small membership function at 0.6 output
- Create a fuzzy set/membership function using the membership functions from last step - i.e. **construct the Union between the fuzzy sets**
- Choose approach to get a single crisp value
  - **Maximum Criterion Model:** take any value where the output of the rule achieves a maximum - from his example we would get [4,6]
  - **Mean of Maximum Model:** obtain the mean of the maximum output values - divide (the value times its membership to the set) by the sum of memberships

- **COG:** obtain the mean of the output of the fuzzy set over the whole fuzzy set - divide (every value times its membership to the set) by the sum of all the memberships