Prob. 1	Prob. 2	Prob. 3	Prob. 4

Problem 1.

Let $n \times m$ be the size of the matrix. We call c_i the integer sum of the column i of the matrix, and r_i the integer sum of the row i of the matrix. We are going to show a way to compute a right rounding using max-flow algorithm.

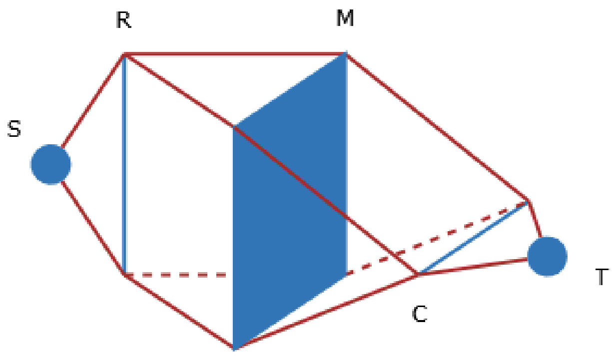

Figure 1: Blue = Vertices; Red = Edges

We construct the following graph (see Fig. 1) :

Vertices

- a source S
- a sink T
- n vertices called $R_1, \ldots, R_n$
- m vertices called $C_1, \ldots, C_m$
- n x m vertices called $M_{1,1}, \ldots M_{n,m}$

Edges

- n edges from S to R_i with capacity r_i
- m edges from C_i to T with capacity c_i
- n x m edges from R_i to $M_{i,j}$ with capacity 1
- n x m edges from $M_{i,j}$ to C_j with capacity 1

Our graph is made so that the not-rounded matrix gives us a possible flow of capacity $r_1 + ... + r_n$ which is maximal because it saturates both the source and the sink. However, since every capacity of an edge of the graph is an integer, the solution given by the max-flow algorithm will have integer values. That means we get our rounding with the solution of max-flow algorithm.

Furthermore our algorithm is polynomial, because the size of the graph is a linear function of the size of the input matrix and Max-Flow is polynomial.

Problem 2.

We are going to use min-cut algorithm. For this we construct the following graph (see Fig. 2) :

Vertices

- a source S
- a sink T
- n vertices called $F_1, ..., F_n$

Edges

- n edges from S to F_i with capacity b_i
- n edges from F_i to T with capacity a_i
- n x (n-1) / 2 edges from F_i to F_j with capacity c_{ij}

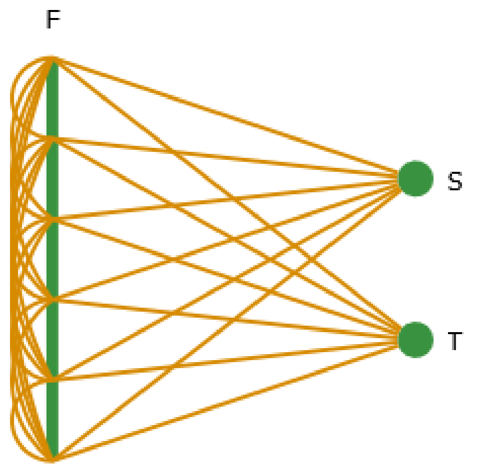

Figure 2: Green = Vertices; Yellow = Edges

We then realize that the capacity of a cut in this graph corresponds to the yearly cost of building firms linked to the source in town A and firms linked to the sink in town B. We conclude that min-cut algorithm gives an optimal repartition of the firms

Furthermore our algorithm is polynomial, because the size of the graph is a linear function of the size of the input and Min-Cut is polynomial.

Problem 3.

Problem 4.