Advanced Algorithms, Fall 2014

Prof. Bernard Moret

Homework Assignment #2

due Sunday night, Oct. 5

Write your solutions in LaTeX using the template provided on the Moodle and web sites and upload your PDF file through Moodle by 4:00 am Monday morning, Oct. 6.

Question 1.
Suppose you are going to start skiing but you do not have skis yet. Now there are these options for you:

1. rent skis, which costs 1 CHF each time;
2. buy cheap skis, which costs N CHF, and these skis only last for $2N$ times of skiing;
3. buy expensive skis, which costs $2N$ CHF, and these skis will last forever.

You want to minimize the total money that you spend on renting/buying skis. The problem is that you do not know T, which is the total number of times that you will go skiing, while an off-line OPT knows T. Design an on-line strategy, which gives the smallest possible worst-case competitive ratio with OPT.

Question 2.
Consider a variation of the move-to-front heuristic in which each item has an associated bit. Initially every bit is 0. When we access an item, if its bit is 0, we set it to 1, but do not move the item; if its bit is 1, we reset it to 0 and move the item to the front. Thus we move the item to the front on every second access only. What is the competitive ratio of this strategy? What would happen if we moved the item every kth access, for $k > 2$?

Question 3.
A dog with no sense of smell knows that there is one bone somewhere on the road on which he stands. The road is of infinite length in each direction, and the bone could be in either direction from where the dog stands. The dog has to walk along the road to search for the bone. Let x denote the distance between the bone and the original position of the dog. If the dog walks in the right direction, the cost to find the bone is x, but if he walks in the wrong direction, the cost would be infinite. Note that the dog does not know x.

1. Describe a competitive algorithm for the dog to find the bone in at most kx cost where k is the competitive ratio. Try to make k as small as possible.
2. Now assume the dog is standing at a crossroads of $m > 2$ roads (m was 2 in the previous case). Show that the competitive ratio is $\Theta(m)$. That is, find a $O(m)$-competitive algorithm, and show that, for any m, there exists an input with distance x such that the cost to the algorithm is at least $\Omega(mx)$.