Module 4: Dynamic Modeling

Lesson 1: Kinematic Modeling in 2D

https://www.coursera.org/learn/intro-self-driving-cars/lecture/pScZH/lesson-1-kinematic-modeling-in-2d

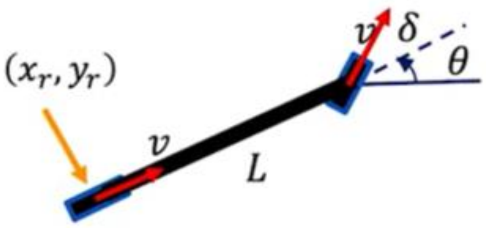
Overview of Module 4

- Basics of kinematic and coordinates
- Kinematic model development of a bicycle
- Basics of dynamic modeling
- Vehicle longitudinal dynamics and modeling
- Vehicle lateral dynamics and modeling
- Vehicle actuation system
- Tire slips and modeling

Kinematic Vs Dynamic Modeling

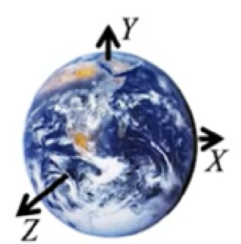
- At low speeds, it is often sufficient to look only at kinematic models of vehicles
  - Examples: Two wheeled robot, Bicycle model

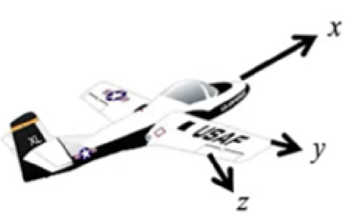
- Dynamic modeling is more involved, but captures vehicle behavior more precisely over a wide operating range
  - Examples: Dynamic vehicle model

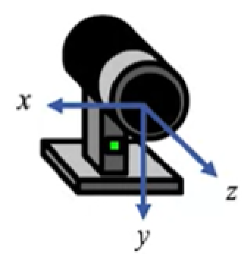


Coordinate Frames

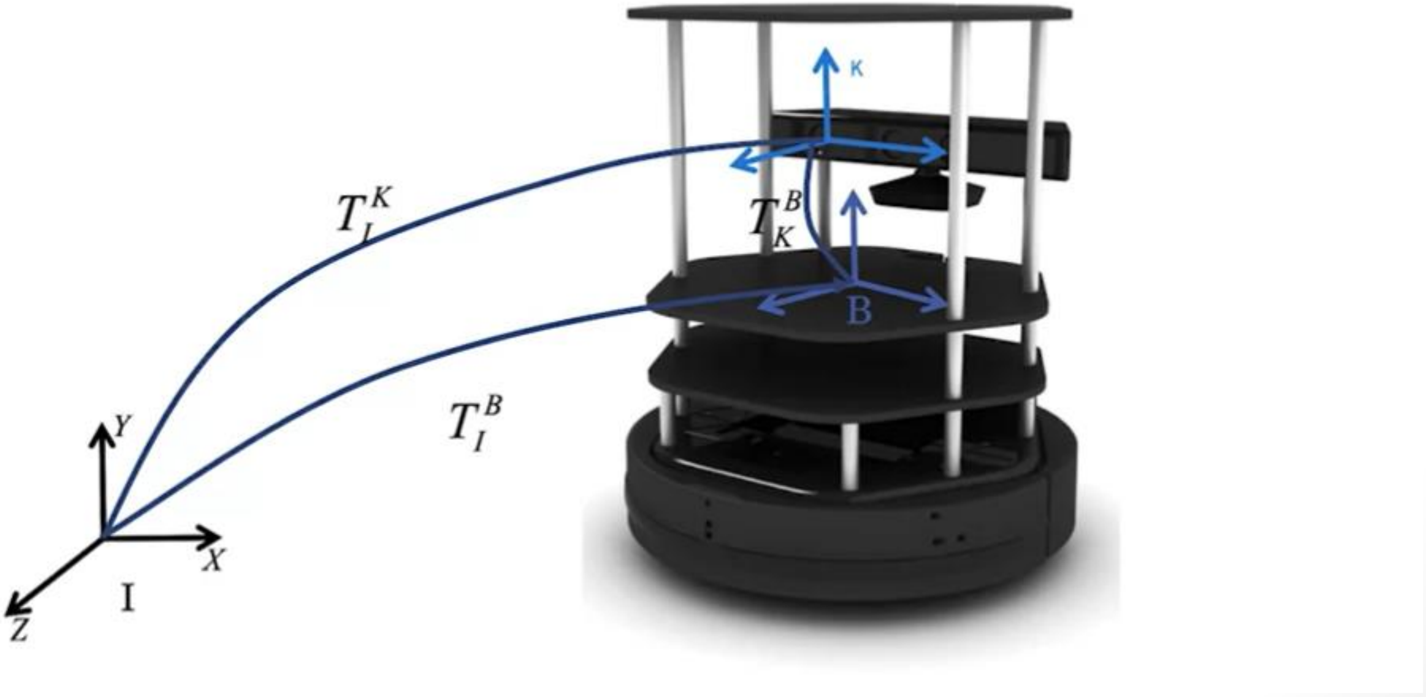
- Right handed by convention
- Inertial frame
  - Fixed, usually relative to earth
- Body frame
  - Attached to vehicle, origin at vehicle center of gravity, or center of rotation
- Sensor frame
  - Attached to sensor, convenient for expressing sensor measurements



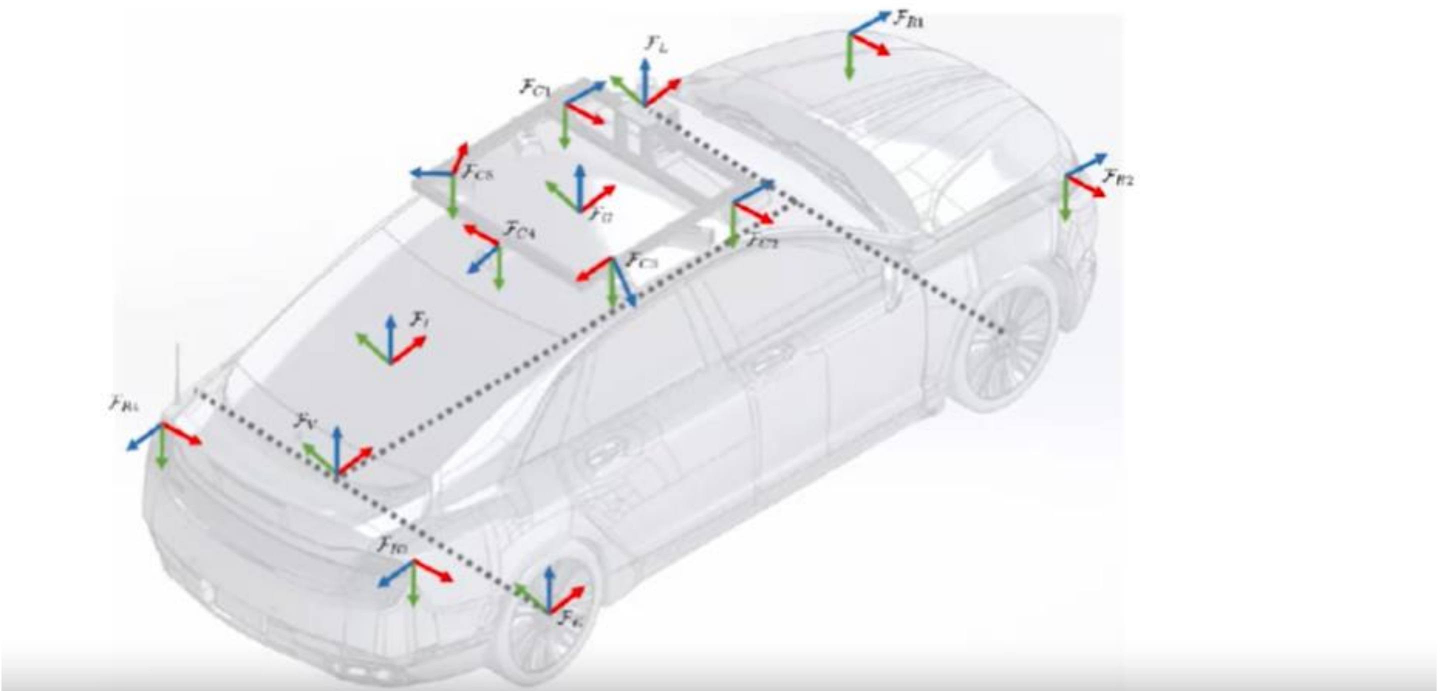




Why We Need Coordinate Transformation

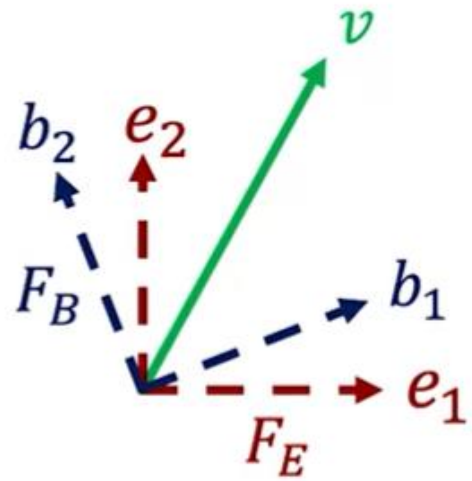


Why We Need Coordinate Transformation



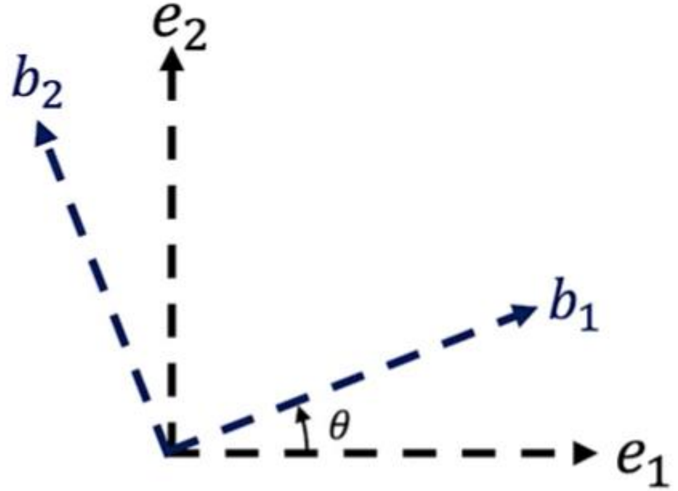
Vectors

- Vectors are variables with both magnitude and direction
- In this figure, v is a vector
- The vectors $\{b_1, b_2\}$, $\{e_1, e_2\}$ define two different coordinate frames, F_B and F_E



Rotation Matrices in 2D

$$C_{EB} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \qquad C_{BE} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$



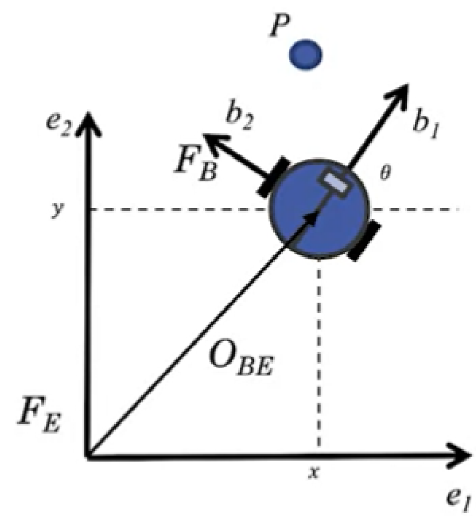
Coordinate Transformation

- Conversion between Inertial frame and Body coordinates is done with a translation vector and a rotation matrix
  - Location of point (P) in Body Frame (B)

$$P_B = C_{EB}(\theta)\, P_E + O_{EB} \longleftarrow$$ Translation term, expressed in body frame

  - Location of point (P) in Inertial Frame (E)

$$P_E = C_{BE}(\theta)\, P_B + O_{BE} \longleftarrow$$ Translation term, expressed in inertial frame



Homogeneous Coordinate Form

- A 2D vector in homogeneous form

$$P = \begin{bmatrix} x \\ y \end{bmatrix} \longrightarrow \bar{P} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Transforming a point from body to inertial coordinates with homogeneous coordinates

$$\overline{P_E} = [C_{EB}(\theta) \quad | \, O_{EB}]\overline{P_B}$$

2D Kinematic Modeling

- The kinematic constraint is nonholonomic
  - A constraint on rate of change of degrees of freedom
  - Vehicle velocity always tangent to current path

$$\frac{dy}{dx} = \tan\theta = \frac{\sin\theta}{\cos\theta}$$

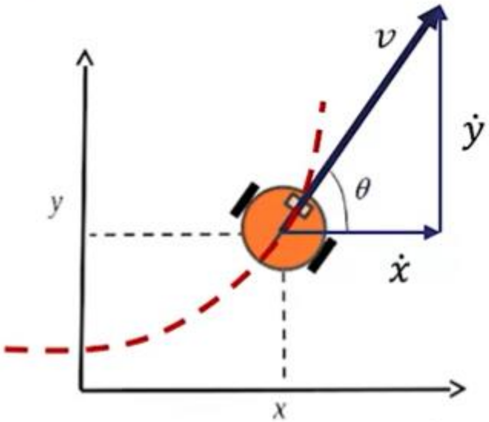
- Nonholonomic constraint

$$\dot{y}\cos\theta - \dot{x}\sin\theta = 0$$

- Velocity components

$$\dot{x} = v\cos\theta$$
$$\dot{y} = v\sin\theta$$

Velocity tangent to the path



Simple Robot Motion Kinematics

$[v, \omega]$

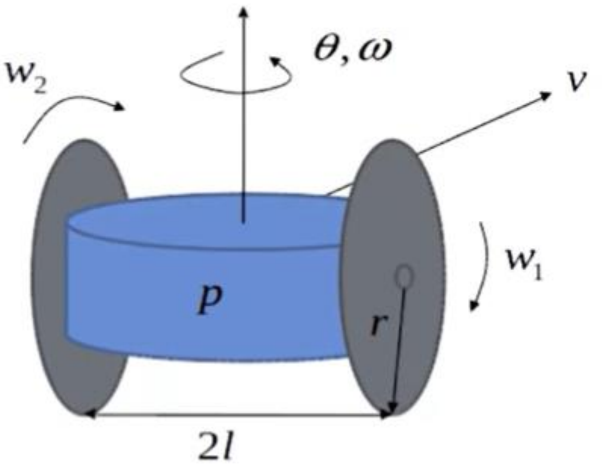
$$\dot{x} = v \cos\theta$$

$$\dot{y} = v \sin\theta$$

$$\dot{\theta} = \omega$$

Two-Wheeled Robot Kinematic Model

- Assume control inputs are wheel speeds
  - Center: p
  - Wheel to center: l
  - Wheel radius: r
  - Wheel rotation rates: w1, w2

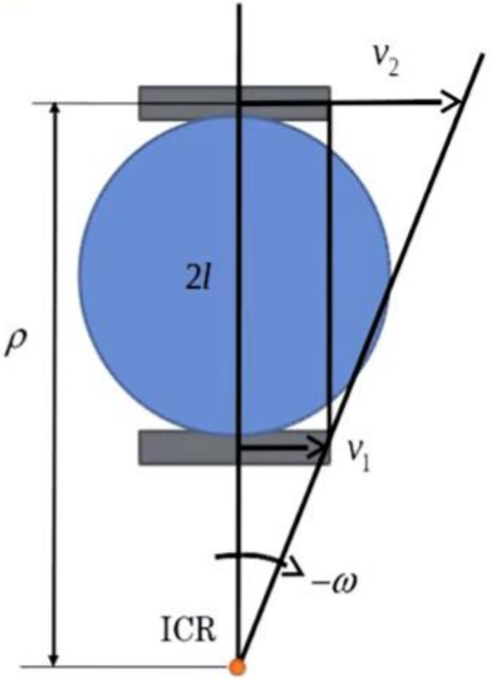


Two-wheeled Kinematic Model

- Use the instantaneous center of rotation (ICR)
- Equivalent triangles give the angular rate of rotation

$$\omega = \frac{-v_2}{\rho} = \frac{-(v_2 - v_1)}{2l}$$

$$\omega = \frac{rw_1 - rw_2}{2l}$$



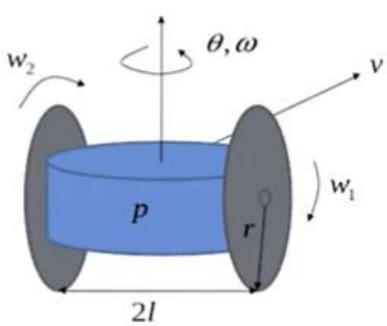
Kinematic Model of a Simple 2D Robot

- Continuous time model:

$$\dot{x} = \left[\left(\frac{rw_1 + rw_2}{2}\right)\cos\theta\right]$$

$$\dot{y} = \left[\left(\frac{rw_1 + rw_2}{2}\right)\sin\theta\right]$$

$$\dot{\theta} = \left(\frac{rw_1 - rw_2}{2l}\right)$$



- Discrete time model:

$$x_{k+1} = x_k + \left[\left(\frac{rw_{1,k} + rw_{2,k}}{2}\right)\cos\theta_k\right]\Delta t$$

$$y_{k+1} = y_k + \left[\left(\frac{rw_{1,k}+rw_{2,k}}{2}\right)\sin\theta_k\right]\Delta t$$

$$\theta_{k+1} = \theta_k + \left(\frac{rw_{1,k} - rw_{2,k}}{2l}\right)\Delta t$$

Summary

What we have learned from this lesson:

- Basics of 2D kinematics
- Coordinate frames and transformations
- Continuous and discrete kinematic model of a two wheeled robot

- **Supplementary Reading for Module 4, Lesson 1: Kinematic Modeling in 2D**
- Read more about 2D plan motion and coordinate frames in the PDF below:
- Chapter 2, "Models of Mobile Robots in the Plane" B.A. Francis and M. Maggiore, *Flocking and Rendezvous in Distributed Robotics*, SpringerBriefs in Control, Automation and Robotics (2016), https://www.springer.com/cda/content/document/cda_downloaddocument/9783319247274-c2.pdf?SGWID=0-0-45-1532785-p177708750.

Lesson 2: The Kinematic Bicycle Model

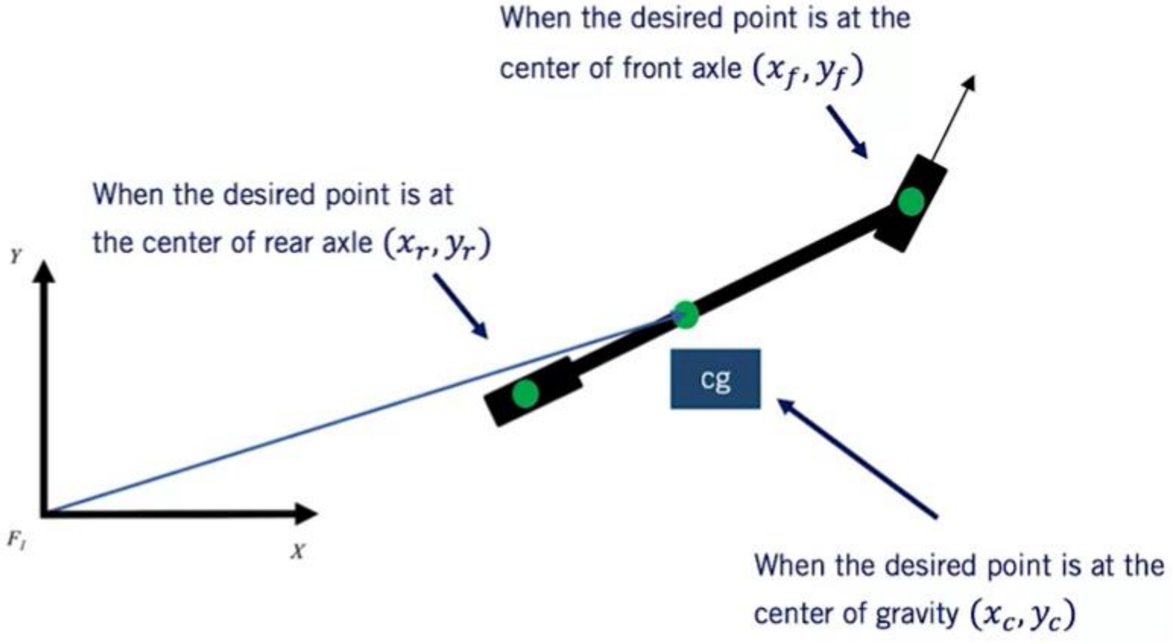
https://www.coursera.org/learn/intro-self-driving-cars/lecture/Bi8yE/lesson-2-the-kinematic-bicycle-model

In this video…

- Learn about slip angle
- Develop the kinematic bicycle model

Bicycle Kinematic Model

- 2D bicycle model (simplified car model)
- Front wheel steering



Two-Wheeled Robot Kinematic Model

- Rear Wheel Reference Point
  - Apply Instantaneous Center of Rotation (ICR)

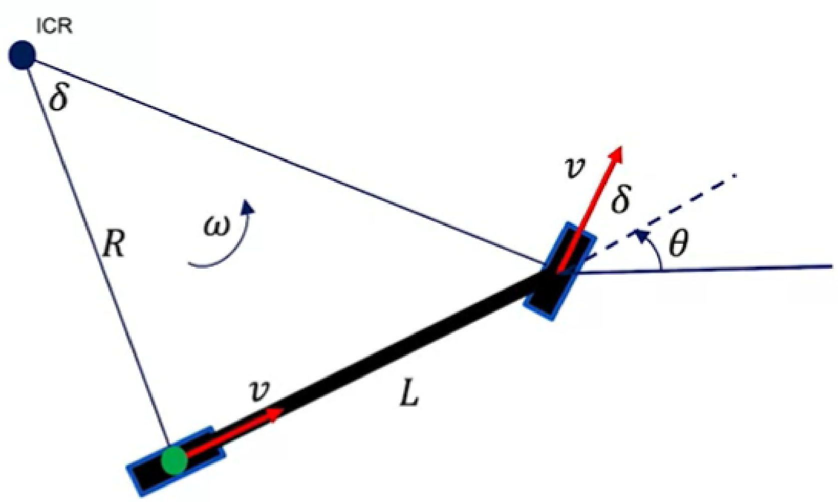
$$\dot{\theta} = \omega = \frac{v}{R}$$

  - Similar triangles

$$\tan \delta = \frac{L}{R}$$

  - Rotation rate equation

$$\dot{\theta} = \omega = \frac{v}{R} = \frac{v \tan \delta}{L}$$



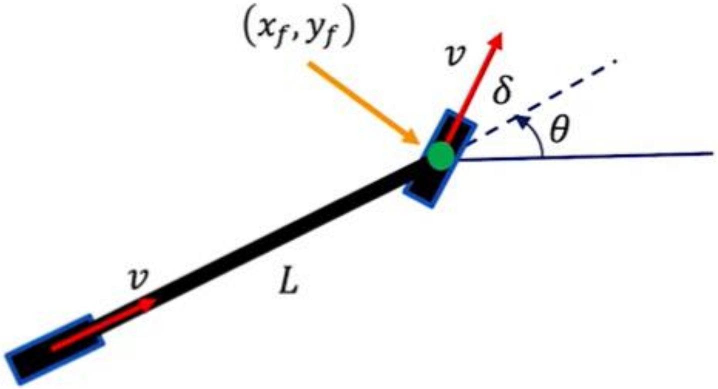
Bicycle Kinematic Model

- If the desired point is at the center of the front axle

$$\dot{x}_f = v\cos(\theta + \delta)$$

$$\dot{y}_f = v\sin(\theta + \delta)$$

$$\dot{\theta} = \frac{v\sin\delta}{L}$$



Bicycle Kinematic Model

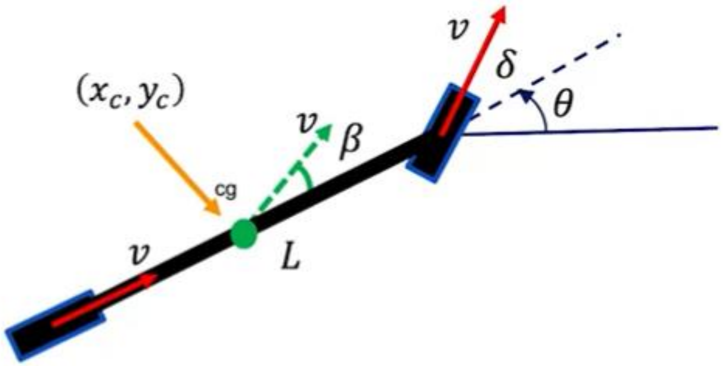
- If the desired point is at the center of the gravity (cg)

$$\dot{x}_c = v\cos(\theta + \beta)$$

$$\dot{y}_c = v\sin(\theta + \beta)$$

$$\dot{\theta} = \frac{v\cos\beta\tan\delta}{L}$$

$$\beta = \tan^{-1}\left(\frac{l_r\tan\delta}{L}\right)$$



State-space Representation

- Modify CG kinematic bicycle model to use steering rate input
  - State: $[x, y, \theta, \delta]^T$ Inputs: $[v, \varphi]^T$

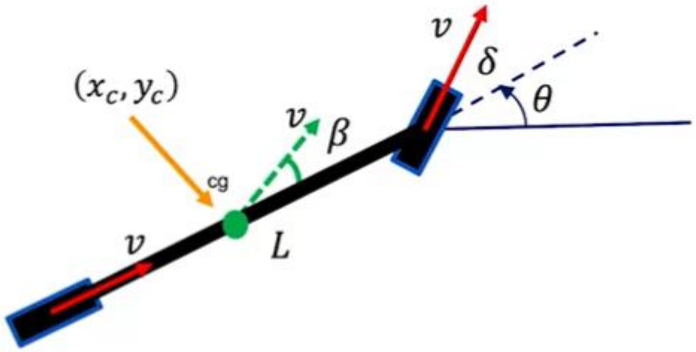
$$\dot{x}_c = v\cos(\theta + \beta)$$

$$\dot{y}_c = v\sin(\theta + \beta)$$

$$\dot{\theta} = \frac{v\cos\beta\tan\delta}{L}$$

$$\dot{\delta} = \varphi$$

Modified Input: rate of change of steering angle



Summary

- What we have learned from this lesson:
  - Formulation of a kinematic bicycle model
  - State-space representation.

- **Supplementary Reading: The Kinematic Bicycle Model**
- Read more about the Kinematic Bicycle Model (pages 15-26) in the PDF below:
- "Chapter 2, Lateral Vehicle Dynamics", R. Rajamani, *Vehicle Dynamics and Control*, Mechanical Engineering Series, https://www.springer.com/cda/content/document/cda_downloaddocument/9781461414322-c1.pdf?SGWID=0-0-45-1265143-p174267791. (2012)

Lesson 3: Dynamic Modeling in 2D

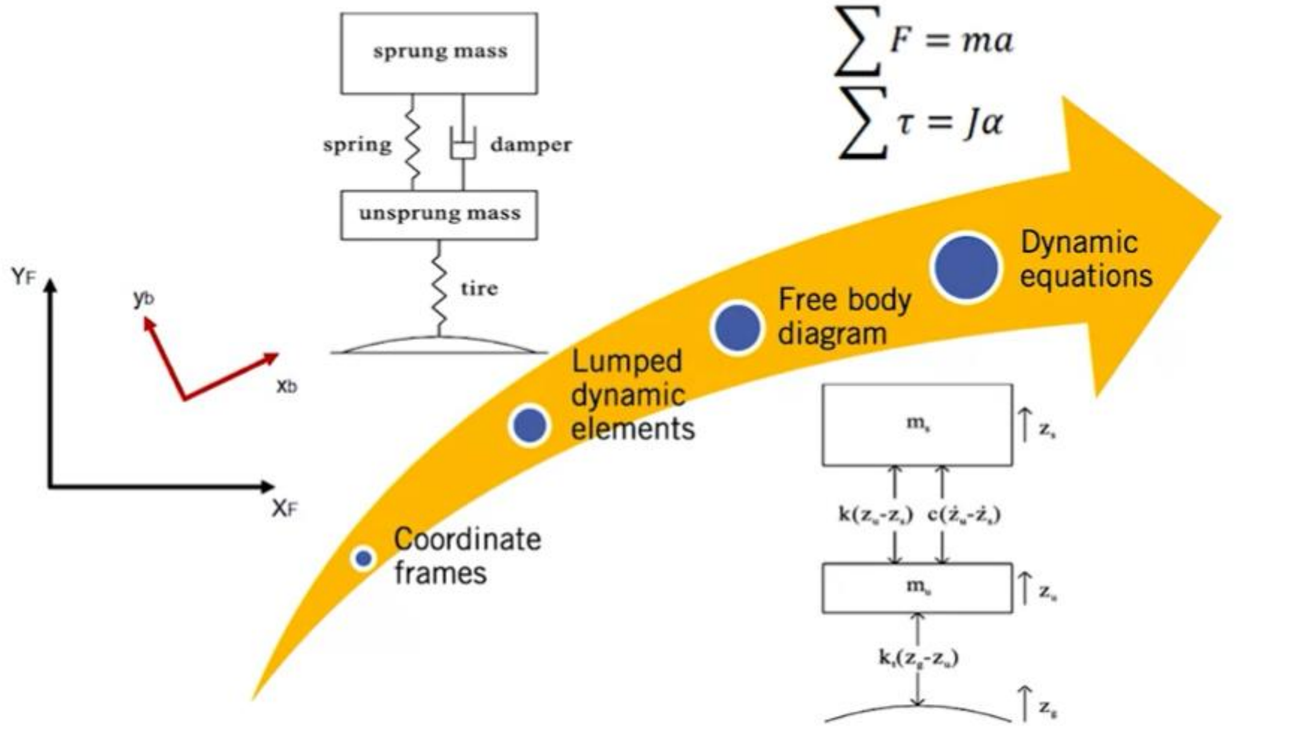
https://www.coursera.org/learn/intro-self-driving-cars/lecture/yNa0v/lesson-3-dynamic-modeling-in-2d

Dynamic Modeling

- Why Dynamic Modeling is Important?
  - At higher speed and slippery roads, vehicles do not satisfy no slip condition
  - Forces such as drag, road friction govern required throttle inputs

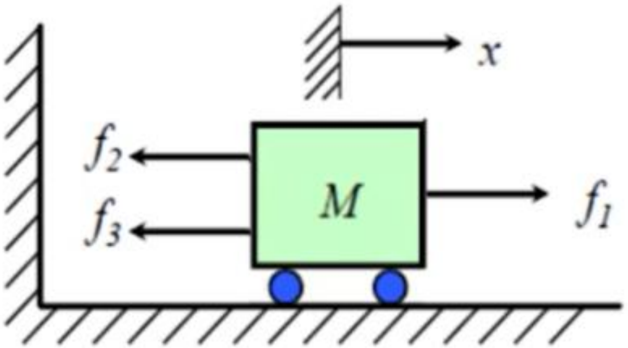
Dynamic Modeling

- Steps to build a typical dynamic model:



Dynamic Modeling - Translational System

- Deals with forces and torques
- Roughly, need to equate all forces
- Governed by Newton's second law

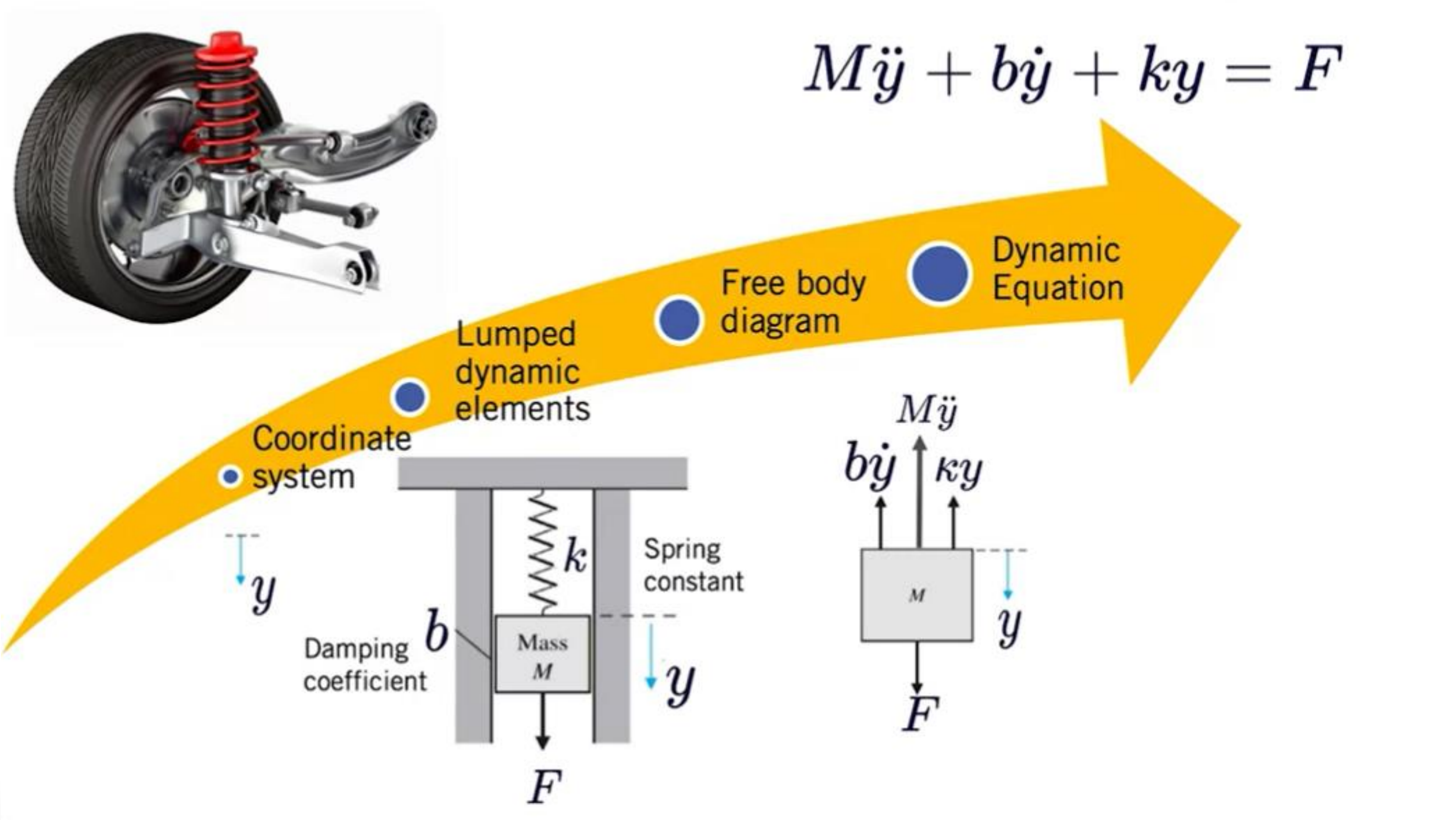


$$Ma = \sum F$$

$$M\ddot{x} = f_1 - f_2 - f_3$$

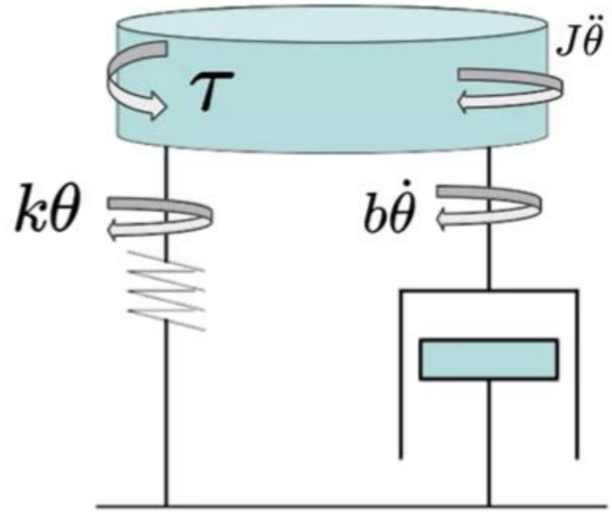
Example - Vehicle Shock Absorber (Suspension)

$$M\ddot{y} + b\dot{y} + ky = F$$

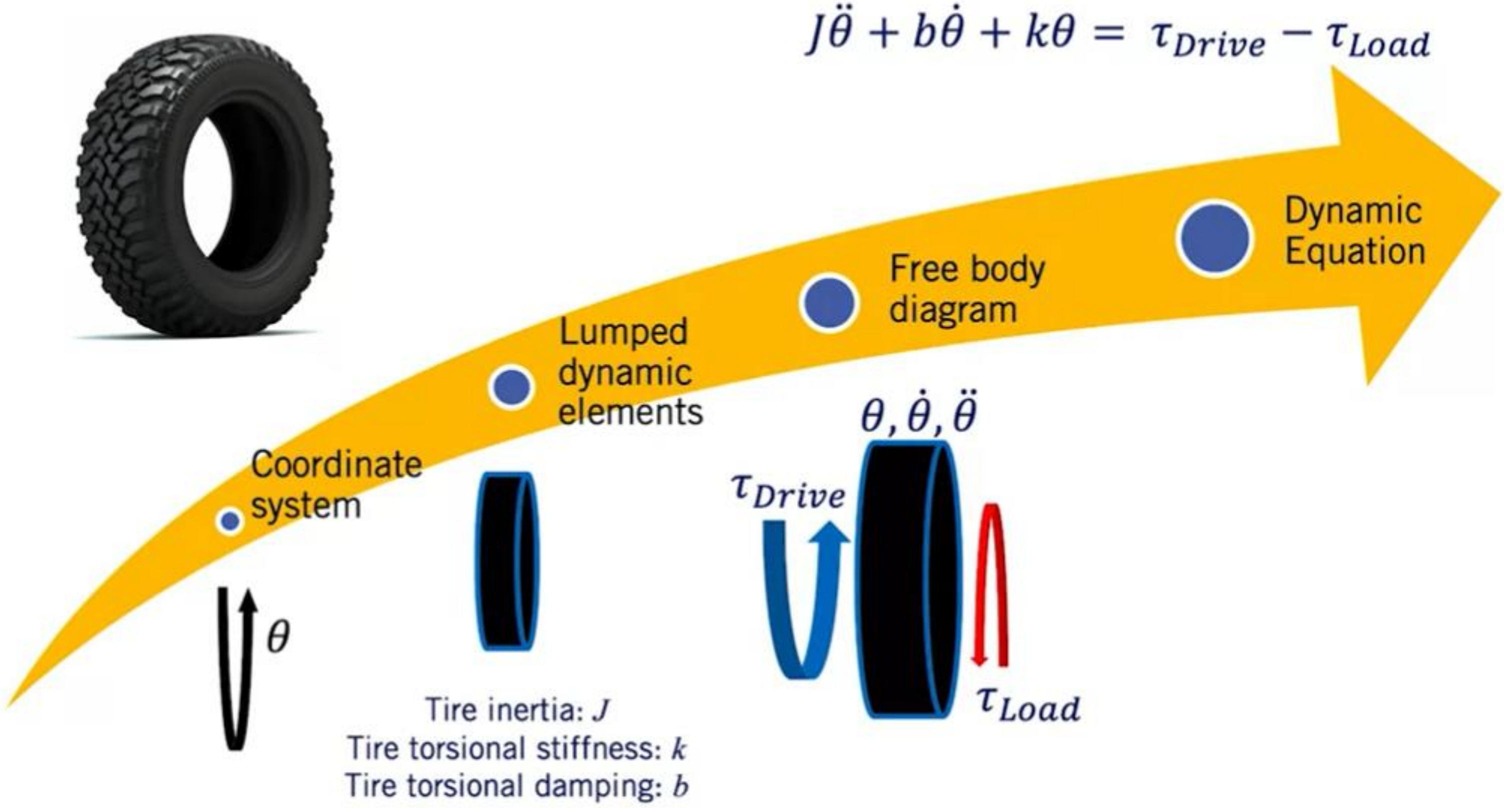


Dynamic Modeling - Rotational Systems

- Inertia, J
- Torsional force, τ
- Forces resisting that torsional force
  - Spring force
  - Damping force
  - Inertia force



Example - Tire Model

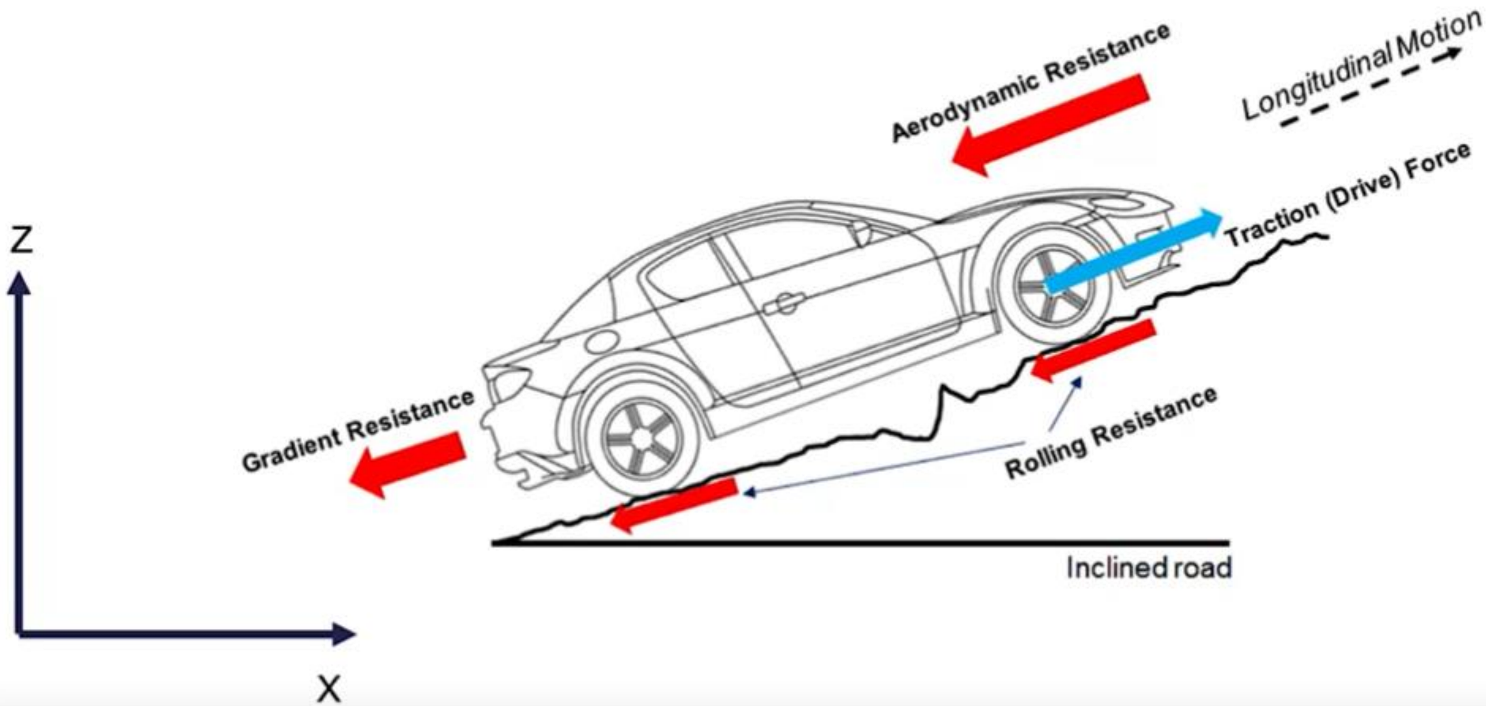


Full Vehicle Modeling

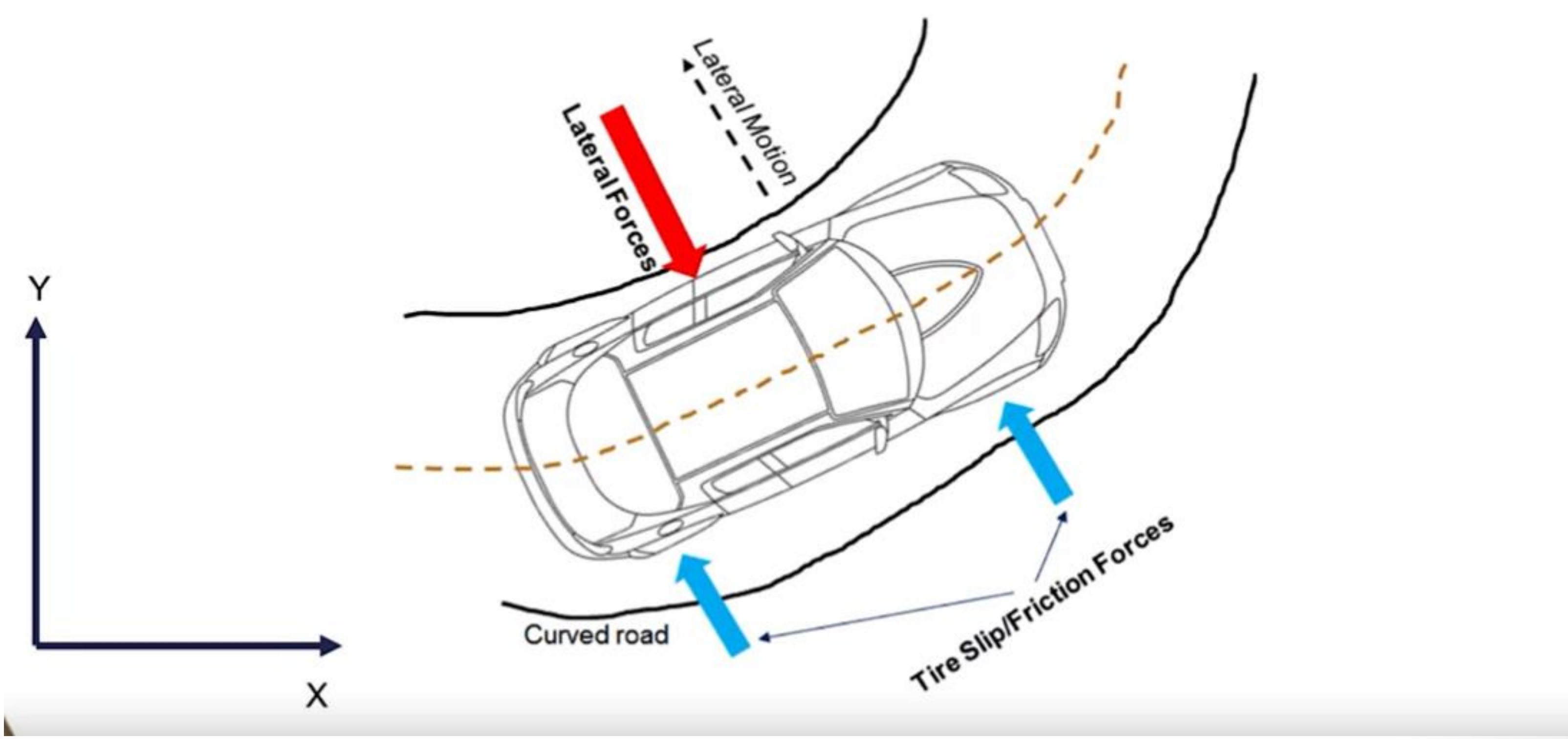
- All components, forces and moments in 3D
  - Pitch, roll, normal forces
  - Suspension, drivetrain, component models

Project Chrono

2D Dynamics - Vehicle Longitudinal Motion



2D Dynamics - Vehicle Lateral Motion



Summary

- What we have learned from this lesson?
  - Basics of 2D dynamic and how to start modeling a dynamical system, along with some application in automotive.

- **Supplementary Reading: Dynamic Modeling in 3D**
- Read more about the fundamentals of dynamics and vehicle dynamics in the textbooks and links below:
- General Dynamics:
- Ardema, Mark D. *Newton-Euler Dynamics*, Springer: Santa Clara University, Santa Clara (2005).
- Tong, David. *Classical Dynamics* University of Cambridge Course Notes (2004)
- Vehicle Modeling:
- Rajamani, Rajesh. *Vehicle dynamics and control,* Springer Science & Business Media (2011).
- Jacobson, Bengt, et al. *Vehicle Dynamics,* Vehicle Dynamics Group, Division of Vehicle and Autonomous Systems, Department of Applied Mechanics, Chalmers University of Technology (2016)
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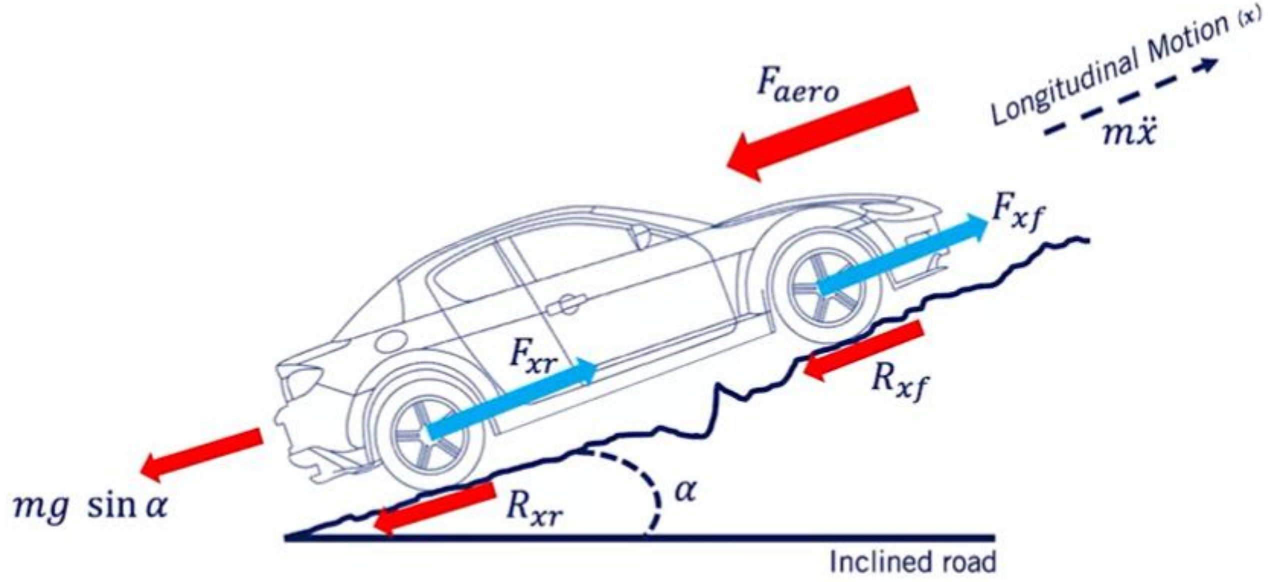
Lesson 4: Longitudinal Vehicle Modeling

https://www.coursera.org/learn/intro-self-driving-cars/lecture/V8htX/lesson-4-longitudinal-vehicle-modeling

Learning Objectives

- Define dynamic force balance on a vehicle
- Describe powertrain component models
- Connect models to create a full longitudinal motion model

Longitudinal Vehicle Model



Vehicle acceleration	Front & rear tire forces	Aerodynamic forces	Front & rear road rolling resistance	Gravitational force due to the road inclination

$$m\ddot{x} = F_{xf} + F_{xr} - F_{aero} - R_{xf} - R_{xr} - mg\sin\alpha$$

Simplified Longitudinal Dynamics

- The full longitudinal dynamics

$$m\ddot{x} = F_{xf} + F_{xr} - F_{aero} - R_{xf} - R_{xr} - mg\sin\alpha$$

- Let F_x - total longitudinal force: $F_x = F_{xf} + F_{xr}$
- Let R_x - total rolling resistance: $R_x = R_{xf} + R_{xr}$
- Assume α is a small angle: $\sin\alpha = \alpha$

- Then the simplified longitudinal dynamics become

$$\underbrace{m\ddot{x}}_{\text{Inertial Term}} = \underbrace{F_x}_{\text{Traction Force}} - \underbrace{\boxed{F_{aero} - R_x - mg\alpha}}_{\text{Total Resistant Forces } (F_{Load})}$$

Simple Resistance Force Models

- Total resistance load:

$$F_{load} = F_{aero} + R_x + mg\alpha$$

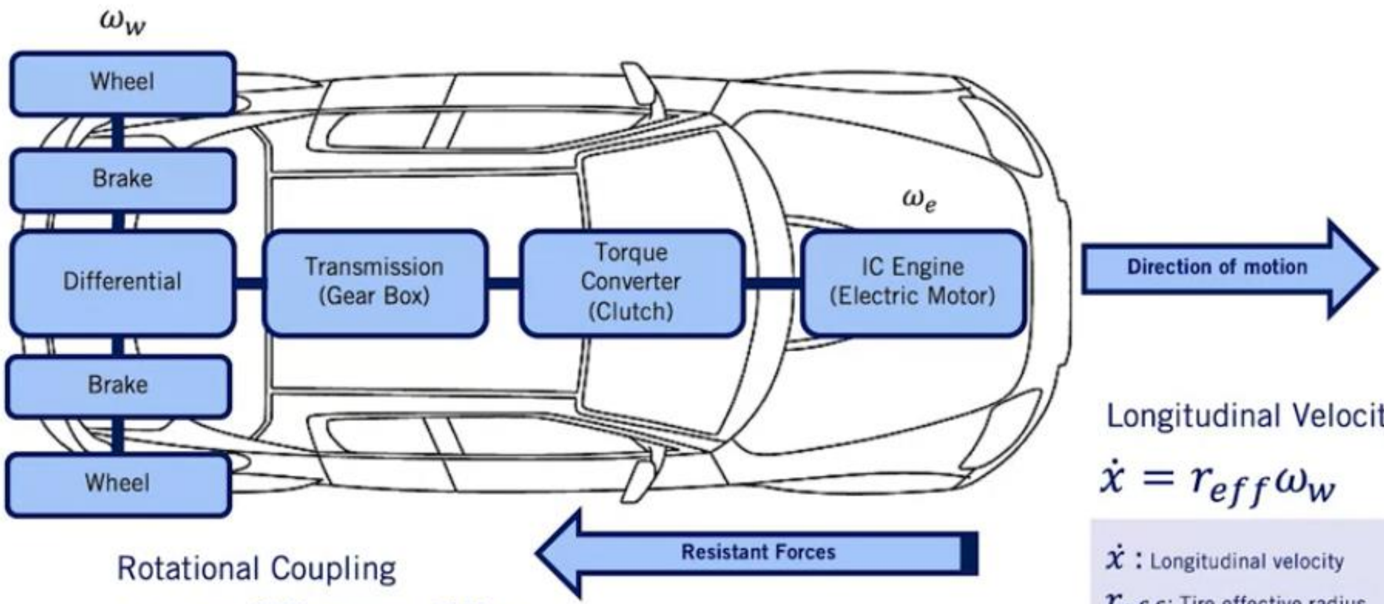
- The aerodynamic force can depend on air density, frontal area, on the speed of the vehicle

$$F_{aero} = {}^{1}/_{2}\, C_a \rho A \dot{x}^2 = c_a \dot{x}^2$$

- The rolling resistance can depend on the tire normal force, tire pressures and vehicle speed

$$R_x = N(\hat{c}_{r,0} + \hat{c}_{r,1}|\dot{x}| + \hat{c}_{r,2}\dot{x}^2) \approx c_{r,1}|\dot{x}|$$

Powertrain Modeling



Rotational Coupling

$$\omega_w = GR\omega_t = GR\omega_e$$

ω_w: wheel angular speed
ω_t: turbine angular speed
ω_e: engine angular speed
GR: Combined gear ratios

Longitudinal Velocity

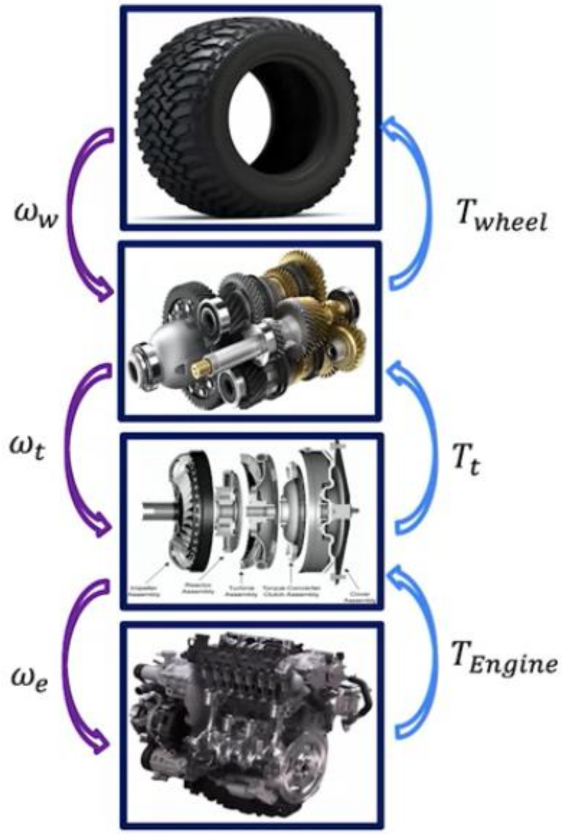
$$\dot{x} = r_{eff}\omega_w$$

$\dot{x}$: Longitudinal velocity
r_{eff}: Tire effective radius

Longitudinal acceleration

$$\ddot{x} = r_{eff}GR\dot{\omega}_e$$

Power Flow in Powertrain



Wheel

$$I_w \dot{\omega}_w = T_{wheel} - r_{eff} F_x$$

$$T_{wheel} = I_w \dot{\omega}_w + r_{eff} F_x$$

Transmission

$$I_t \dot{\omega}_t = T_t - (GR) T_{wheel}$$

$$I_t \dot{\omega}_t = T_t - GR(I_w \dot{\omega}_w + r_{eff} F_x)$$

Torque Converter

$$\omega_t = \omega_e$$

$$T_t = (I_t + I_w GR^2)\dot{\omega}_e + GR r_{eff} F_x$$

Engine

$$I_e \dot{\omega}_e = T_{Engine} - T_t$$

$$I_e \dot{\omega}_e = T_{Engine} - (I_t + I_w GR^2)\dot{\omega}_e - GR r_{eff} F_x$$

Engine Dynamics

- Tire force in terms of inertia and load force:

$$F_x = m\ddot{x} + F_{load} = mr_{eff}GR\dot{\omega}_e + F_{load}$$

- Combining with our engine dynamics model yields:

$$(\underbrace{I_e + I_t + I_w GR^2 + m(GR^2)r_{eff}^2}_{J_e})\dot{\omega}_e = T_{Engine} - (GR)(r_{eff}F_{Load})$$

- Finally, the engine dynamic model simplifies to

$$J_e\dot{\omega}_e = T_{Engine} - (GR)(r_{eff}F_{Load})$$

Total Load Torque (T_{Load})

Summary

What we have learned from this lesson?

- Vehicle longitudinal dynamics, resistance forces
- Powertrain components and component models
- Unified longitudinal dynamic model for speed control

- **Supplementary Reading: Longitudinal Vehicle Modeling**
- To learn more about longitudinal dynamics and vehicle drivetrain, read Chapter 4 in the textbook below:
- Rajamani R. (2012) "Longitudinal Vehicle Dynamics." In: *Vehicle Dynamics and Control*. Mechanical Engineering Series. Springer, Boston, MA. http://link.springer.com/content/pdf/10.1007%2F978-1-4614-1433-9_4.pdf.
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Lesson 5: Lateral Dynamics of Bicycle Model

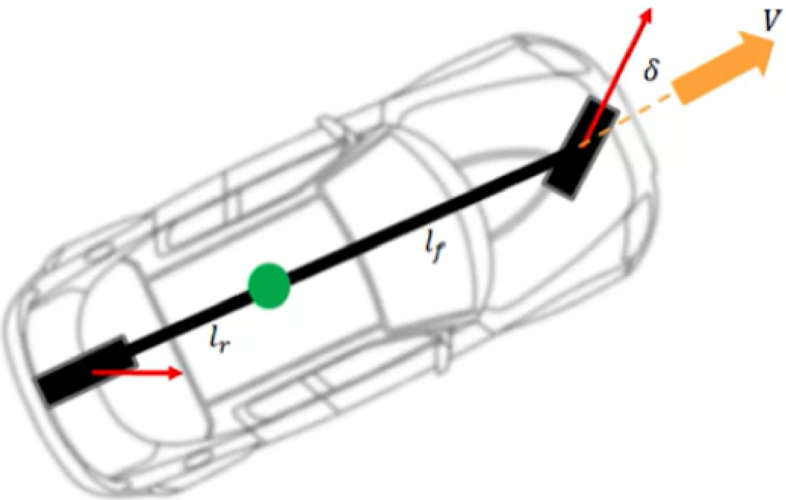
https://www.coursera.org/learn/intro-self-driving-cars/lecture/1Opvo/lesson-5-lateral-dynamics-of-bicycle-model

Learning Objectives

- Build a dynamic model of a car using the kinematic bicycle model as a starting point
- Convert to standard state space representation

Vehicle Model to Bicycle Model

- Assumptions
  - Longitudinal velocity is constant
  - Left and right axle are lumped into a single wheel (bicycle model)

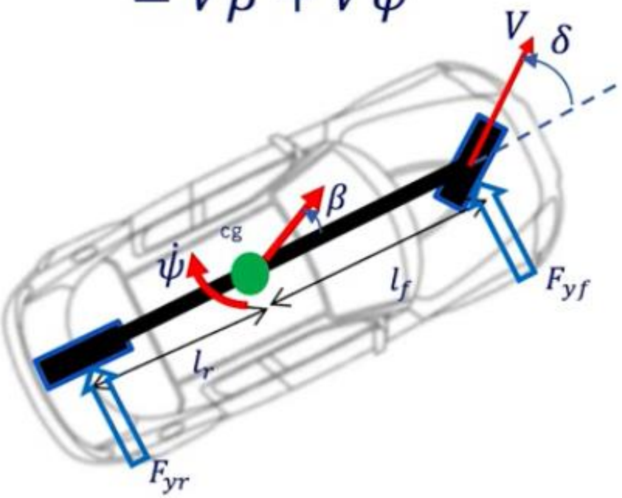


Lateral Dynamics

- Lateral dynamics can be written as

Lateral acceleration

$$a_y = \ddot{y} + \omega^2 R = V\dot{\beta} + V\dot{\psi}$$



vehicle mass

side slip rate

yaw rate

front and rear tire forces

$$mV(\dot{\beta} + \dot{\psi}) = F_{yf} + F_{yr}$$

vehicle velocity

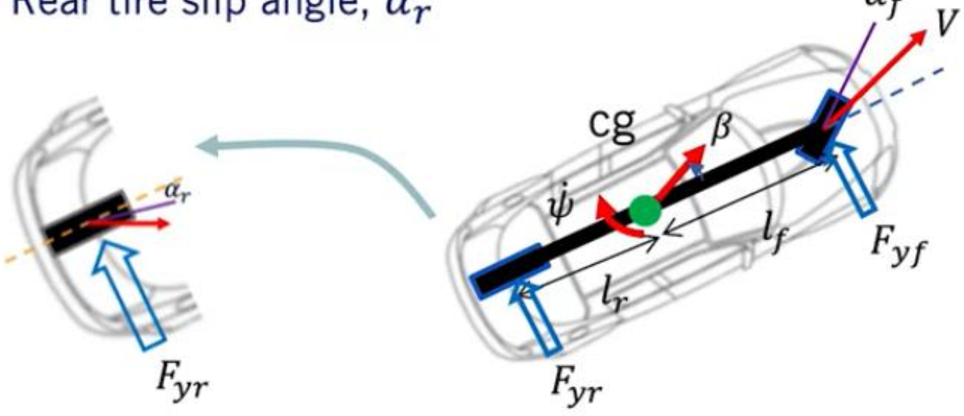
$$I_z\ddot{\psi} = l_f F_{yf} - l_r F_{yr}$$

vehicle inertia

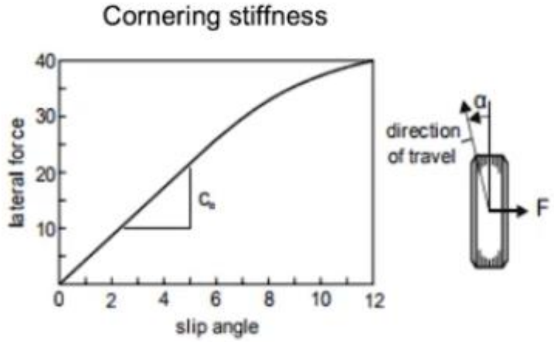
center of gravity distance from front and rear tires

Tire Slip Angles

- Many different tire slip models
- For small tire slip angles, the lateral tire forces are approximated as a linear function of tire slip angle
- Tire variables
  - Front tire slip angle, α_f
  - Rear tire slip angle, α_r



Front and Rear Tire Forces



- C_f: linearized cornering stiffness of the front wheel

$$F_{yf} = C_f \alpha_f = C_f \left(\delta - \beta - \frac{l_f \dot{\psi}}{V} \right)$$

- C_r : linearized cornering stiffness of the rear wheel

$$F_{yr} = C_r \alpha_r = C_r \left(-\beta + \frac{l_r \dot{\psi}}{V} \right)$$

Lateral and Yaw Dynamics

- From the previous slide formulations:

$$F_{yf} = C_f\alpha_f = C_f\left(\delta - \beta - \frac{l_f\dot{\psi}}{V}\right)$$

$$F_{yr} = C_r\alpha_r = C_r\left(-\beta + \frac{l_r\dot{\psi}}{V}\right)$$

Substitute the lateral forces

$$mV(\dot{\beta} + \dot{\psi}) = F_{yf} + F_{yr}$$

$$I_z\ddot{\psi} = l_f F_{yf} - l_r F_{yr}$$

Rearranging the equations

$$\dot{\beta} = \frac{-(C_r + C_f)}{mV}\beta + \left(\frac{C_r l_r - C_f l_f}{mV^2} - 1\right)\dot{\psi} + \frac{C_f}{mV}\delta$$

$$\ddot{\psi} = \frac{C_r l_r - C_f l_f}{I_z}\beta - \frac{C_r l_r^2 + C_f l_f^2}{I_z V}\dot{\psi} + \frac{C_f l_f}{I_z}\delta$$

Standard State Space Representation

- State Vector: $X_{lat} = [\underbrace{y}_{\text{lateral position}} \quad \underbrace{\beta}_{\text{side slip angle}} \quad \underbrace{\psi}_{\text{yaw angle}} \quad \underbrace{\dot{\psi}}_{\text{yaw rate}}]^T$

$$A_{lat} = \begin{bmatrix} 0 & V & V & 0 \\ 0 & -\dfrac{C_r + C_f}{mV} & 0 & \dfrac{C_r l_r - C_f\, l_f}{mV^2} - 1 \\ 0 & 0 & 0 & 1 \\ 0 & \dfrac{C_r l_r - C_f\, l_f}{I_z} & 0 & -\dfrac{C_r l_r^2 + C_f\, l_f^2}{I_z V} \end{bmatrix}$$

$$\boxed{\dot{X}_{lat} = A_{lat} X_{lat} + B_{lat}\delta}$$

$$B_{lat} = \begin{bmatrix} 0 \\ \dfrac{C_f}{mV} \\ 0 \\ \dfrac{C_f l_f}{I_z} \end{bmatrix}$$

Summary

What we have learned from this lesson?

- Formulated the lateral dynamics of a bicycle model
- Defined a state space representation of lateral model

- **Supplementary Reading: Lateral Dynamics of Bicycle Model**
- Read more about the Lateral Dynamics of Bicycle Model (pages 27-44) in the PDF below:
- R. Rajamani (2012), "Lateral Vehicle Dynamics" In: *Vehicle Dynamics and Control*, Mechanical Engineering Series, https://www.springer.com/cda/content/document/cda_downloaddocument/9781461414322-c1.pdf?SGWID=0-0-45-1265143-p174267791.
- **Lateral Vehicle Dynamics.pdf**Archivo PDF

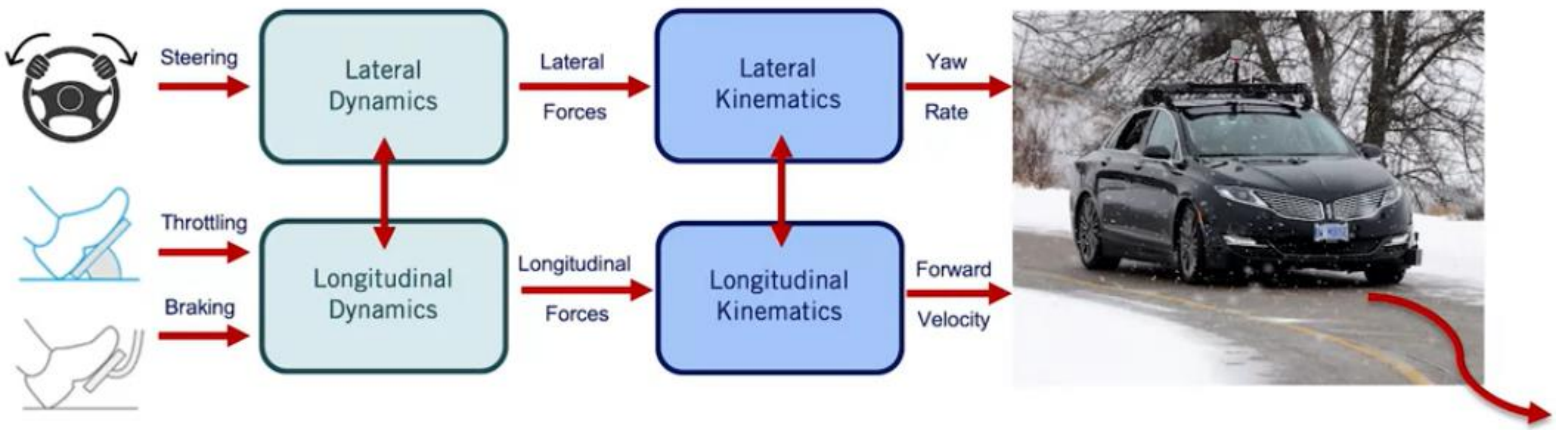
Lesson 6: Vehicle Actuation

https://www.coursera.org/learn/intro-self-driving-cars/lecture/fSAQG/lesson-6-vehicle-actuation

Learning Objectives

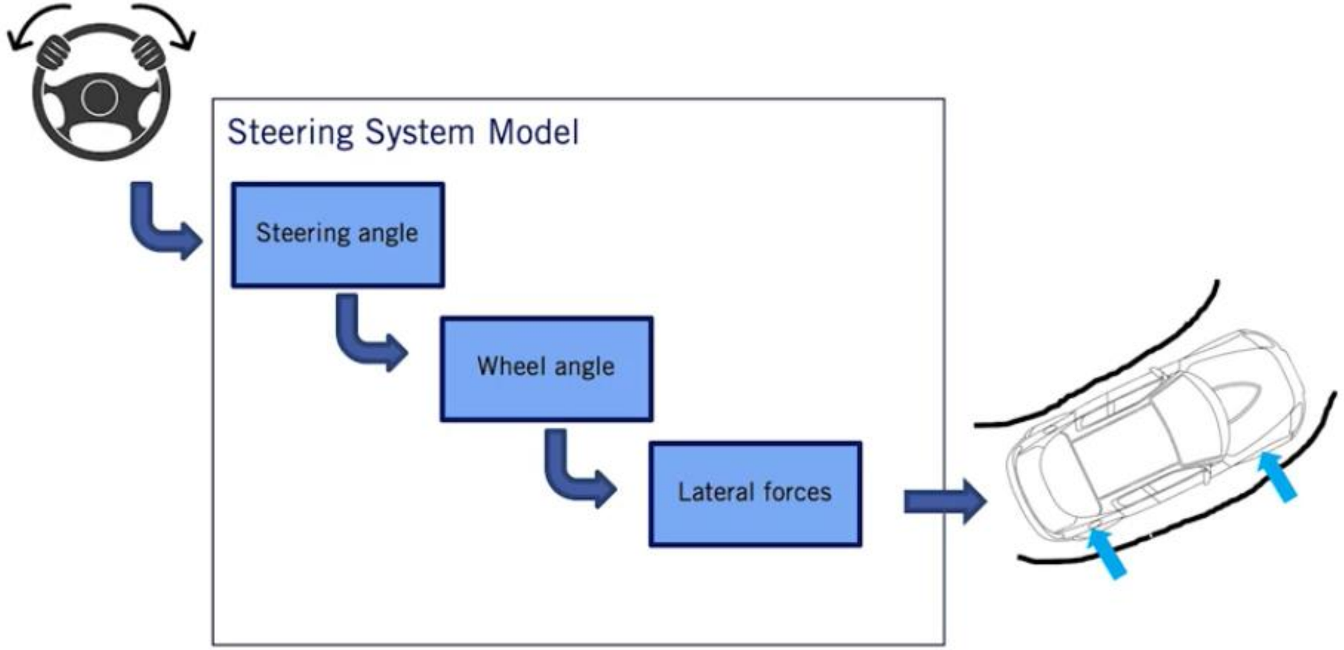
- Build models for the main vehicle actuation systems such as steering, throttling, and braking
- Connect these models to longitudinal and lateral vehicle dynamic models

Coupled Lateral & Longitudinal

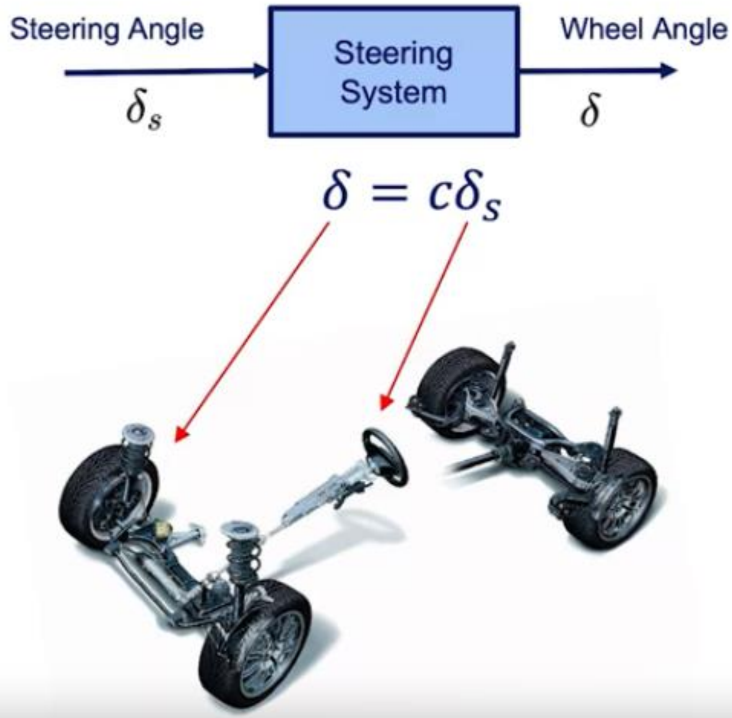


Main Control Task:
To keep the vehicle on the defined path at the desired velocity

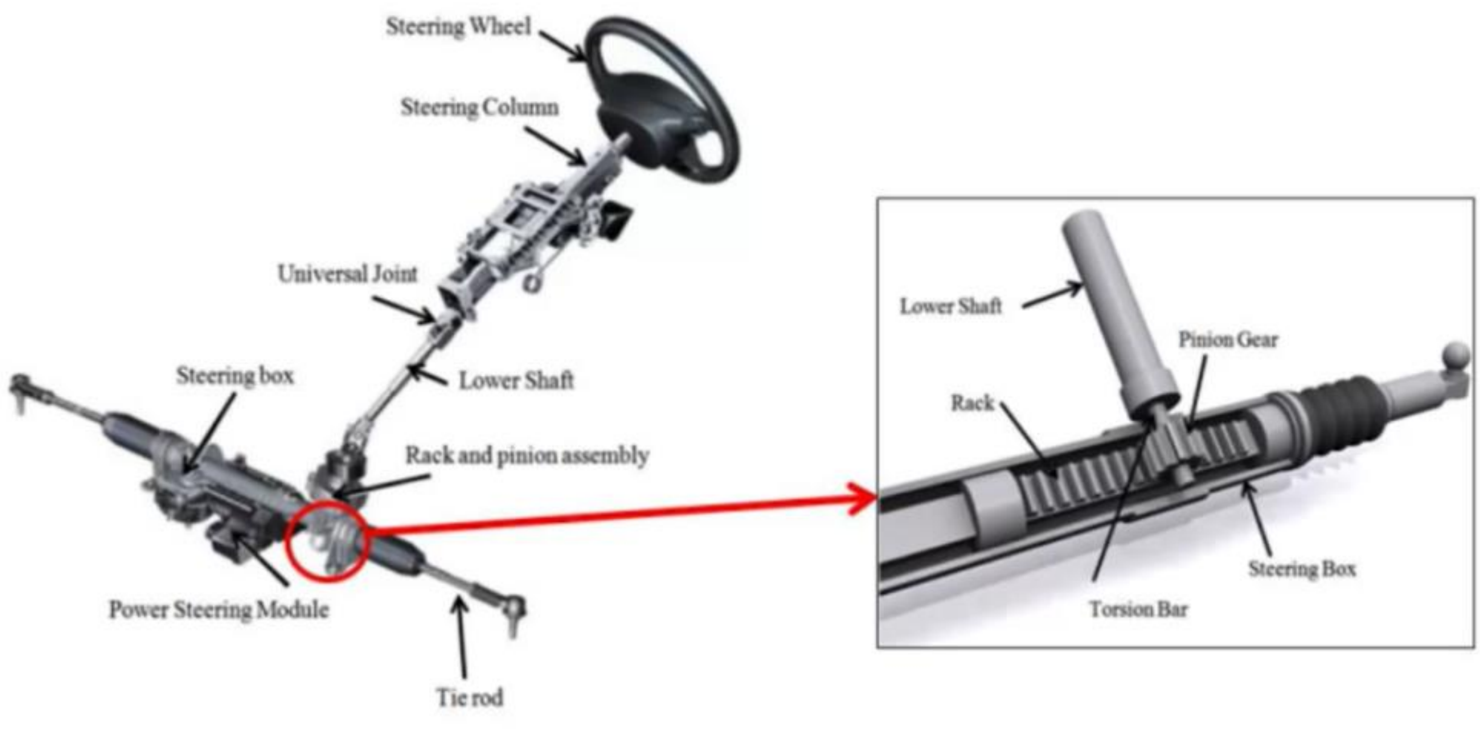
Steering


Steering System Model
Steering angle
Wheel angle
Lateral forces

Simple Steering Model

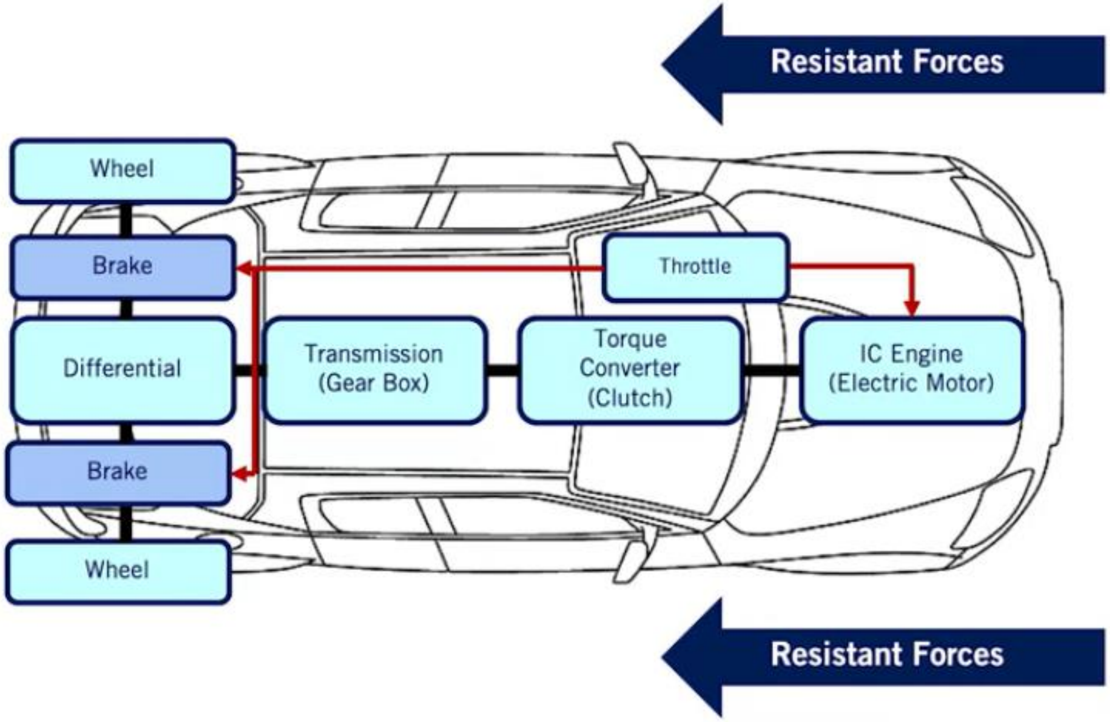


Actual Steering System

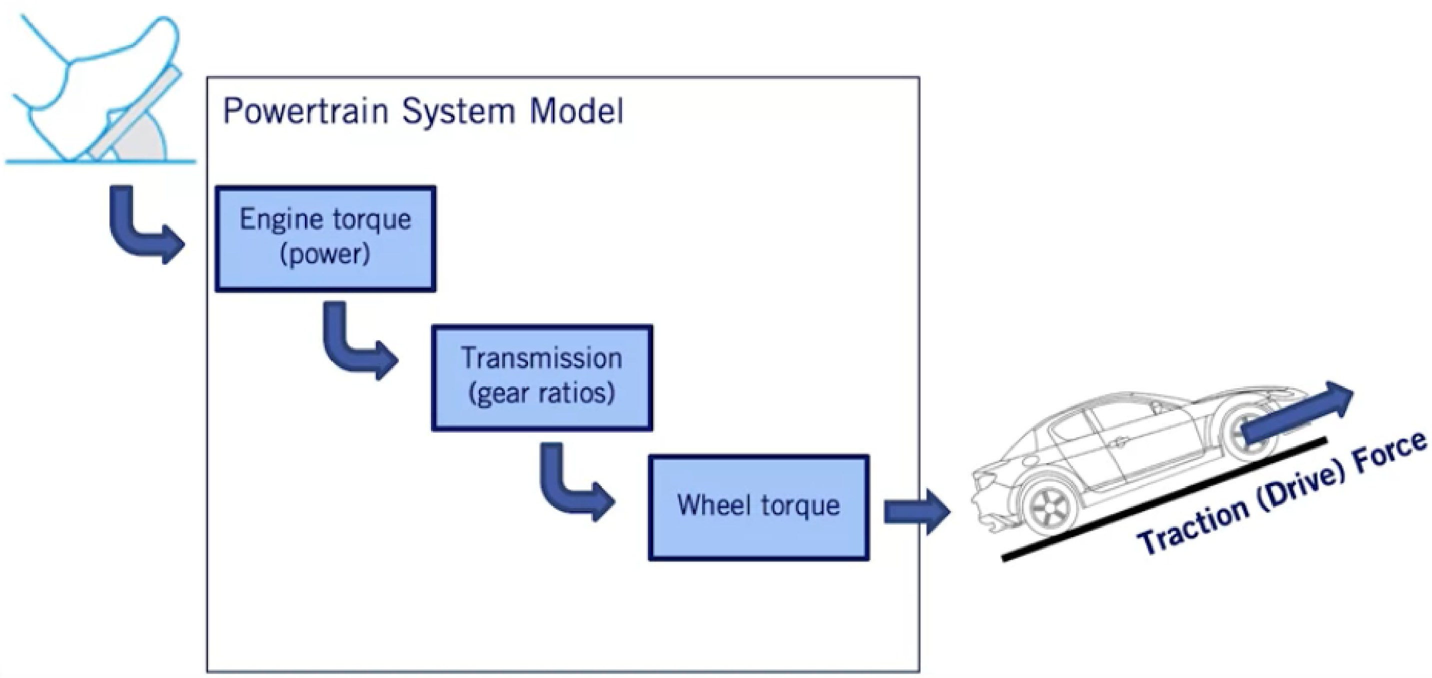


Powertrain System (Driveline)

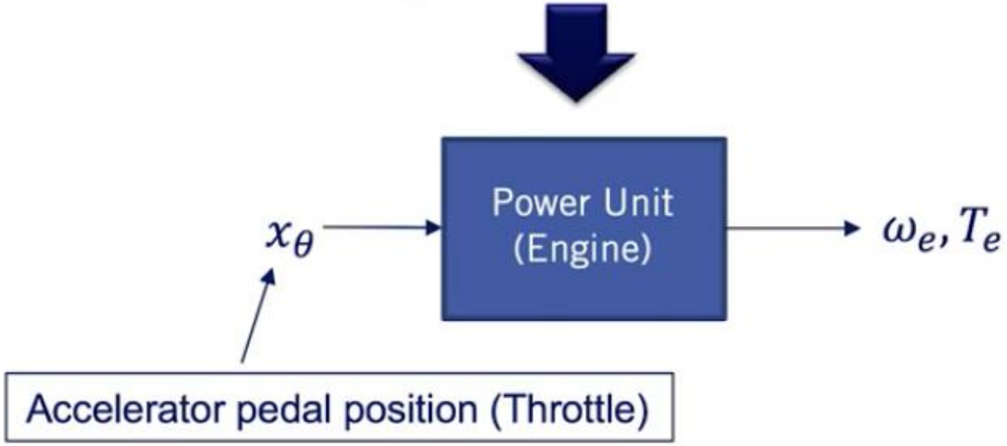
- Throttle and brake commands affect torque balance



Throttling (Accelerating)

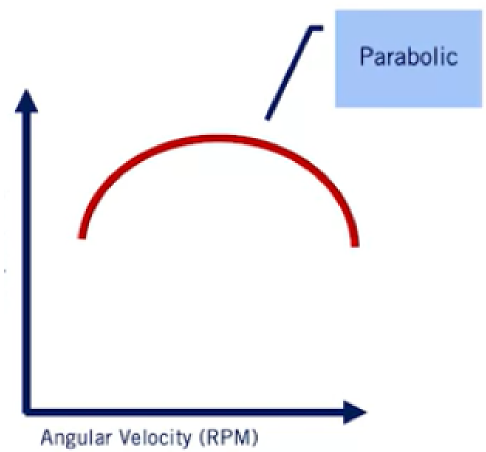


Accelerating Model

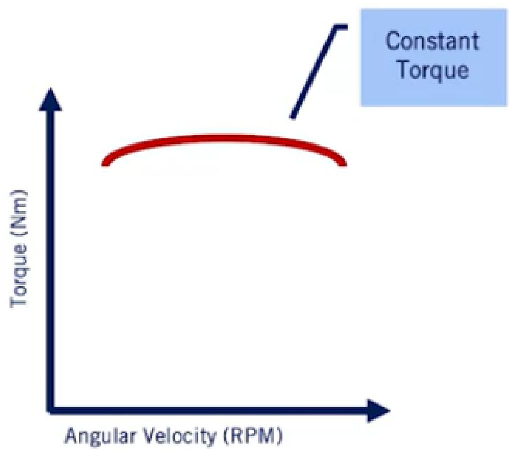


Characteristics Plots

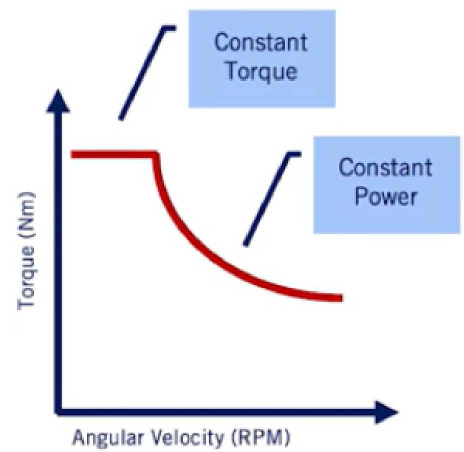
Gasoline Engines



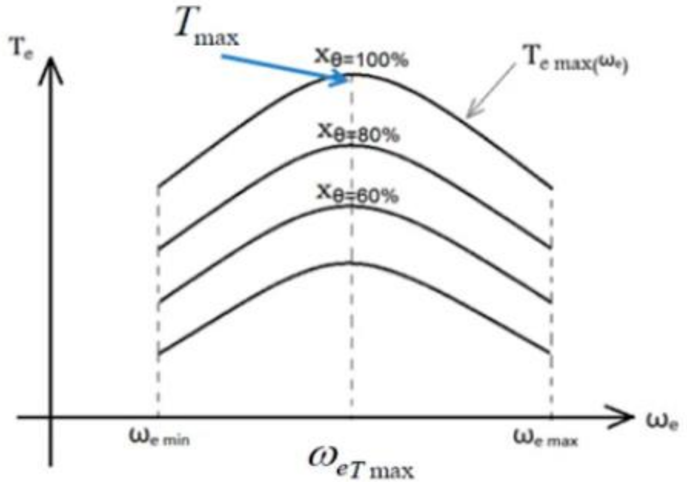
Diesel Engines

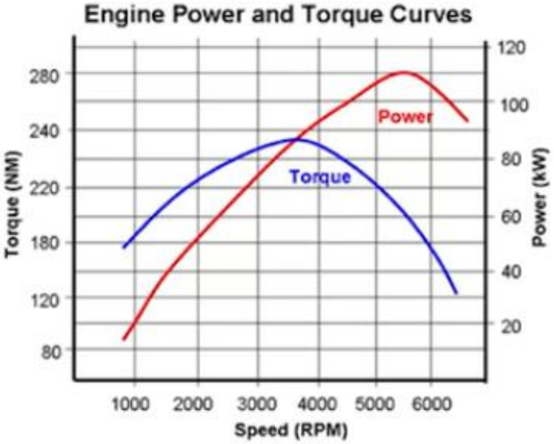


Electric Motors



Typical Torque Curves for Gasoline Engines



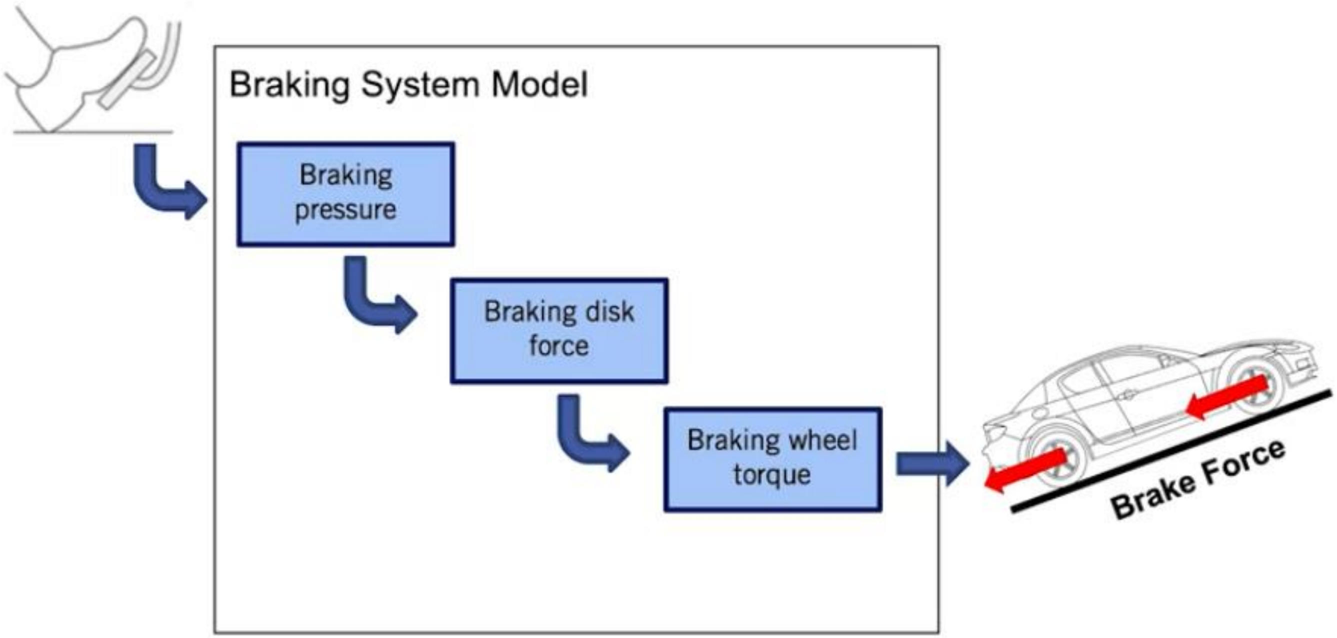


$$T_{e_{max}}(\omega_e) = A_0 + A_1\omega_e + A_2\omega_e^2$$

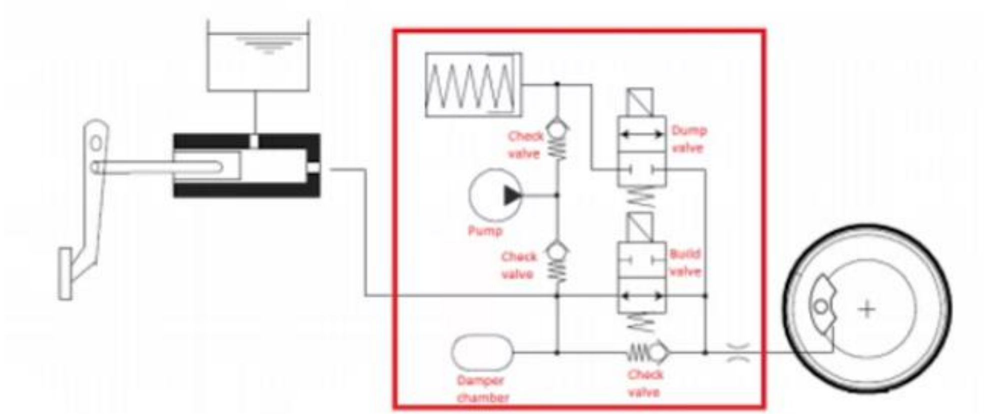
$$T_e(\omega_e, x_\theta) \approx x_\theta(A_0 + A_1\omega_e + A_2\omega_e^2)$$

Throttle position (percentage)

Braking (Decelerating)



Braking Model



$$T_{brake} = k\ \Delta P$$

Braking System

- Basic functionality of braking includes:
  - Shorten stopping distance
  - Steerability during braking through ABS systems
  - Stability during braking to avoid overturning

Summary

What we have learned from this lesson:

- The workings of the vehicle actuation systems such as steering, power generation, and braking
- How to convert steering, throttle and brake inputs to wheel angles and torques

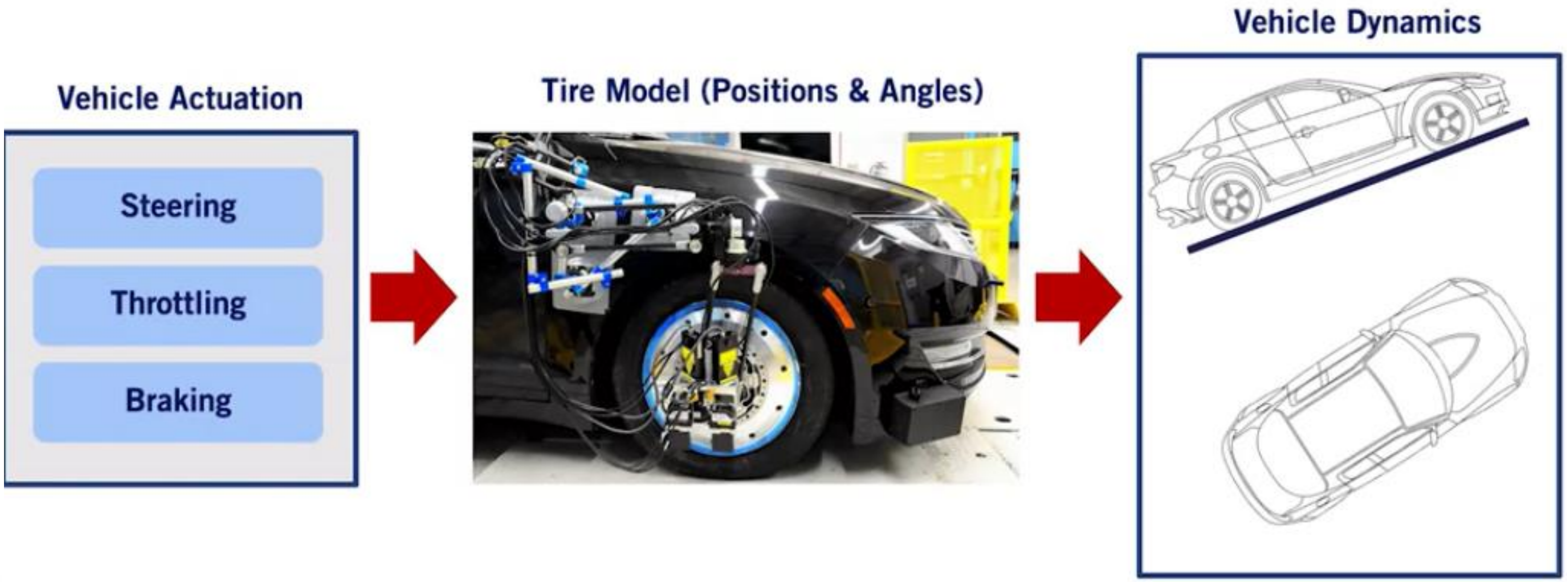
- **Supplementary Reading: Vehicle Actuation**
- Read more about vehicle steering system in the Journal article below:
- Reimann G., Brenner P., Büring H. (2015) "Steering Actuator Systems". In: Winner H., Hakuli S., Lotz F., Singer C. (eds) *Handbook of Driver Assistance Systems*. Springer, Cham
- Read more about vehicle driveline (throttling and braking system) in the textbook below:
- Mashadi, B., Crolla, D, *Vehicle Powertrain Systems*. Wiley (2012)

Lesson 7: Tire Slip and Modeling

https://www.coursera.org/learn/intro-self-driving-cars/lecture/iSRpt/lesson-7-tire-slip-and-modeling

Importance of Tire Modeling

- The tire is the interface between the vehicle and road



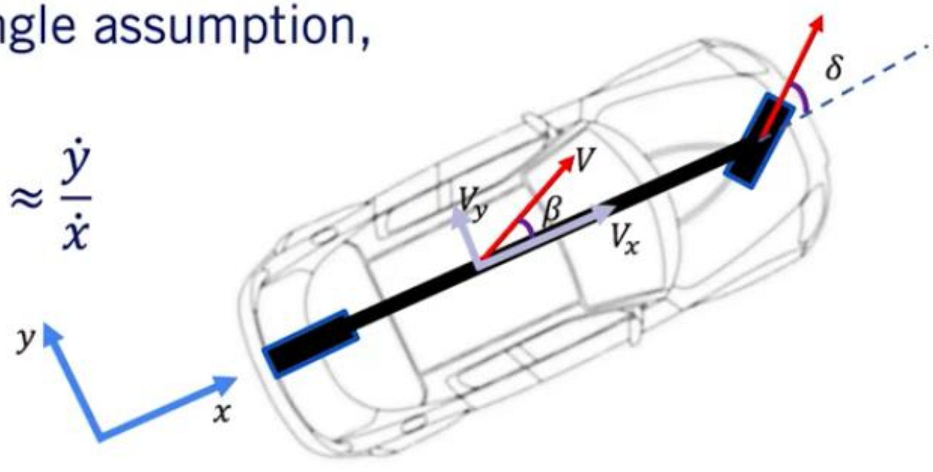
Vehicle (Bicycle) Slip Angle

- Slip angle

$$\beta = \tan^{-1}\frac{V_y}{V_x} = \tan^{-1}\frac{\dot{y}}{\dot{x}}$$

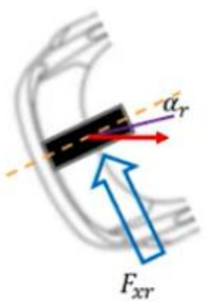
- Using small angle assumption,

$$\beta \approx \frac{\dot{y}}{\dot{x}}$$



Tire Slip Angles

- Tire slip angle is the angle between the direction in which a wheel is pointing and the direction in which it is actually travelling



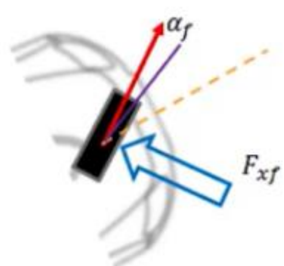
Rear tire slip angle

$$\alpha_r = -\beta + \frac{l_r \dot{\psi}}{V}$$

yaw rate

vehicle slip angle

forward velocity



Front tire slip angle

$$\alpha_f = \delta - \beta - \frac{l_f \dot{\psi}}{V}$$

steering angle

Slip Ratios

- Longitudinal slip (also called slip ratio)

wheel angular speed

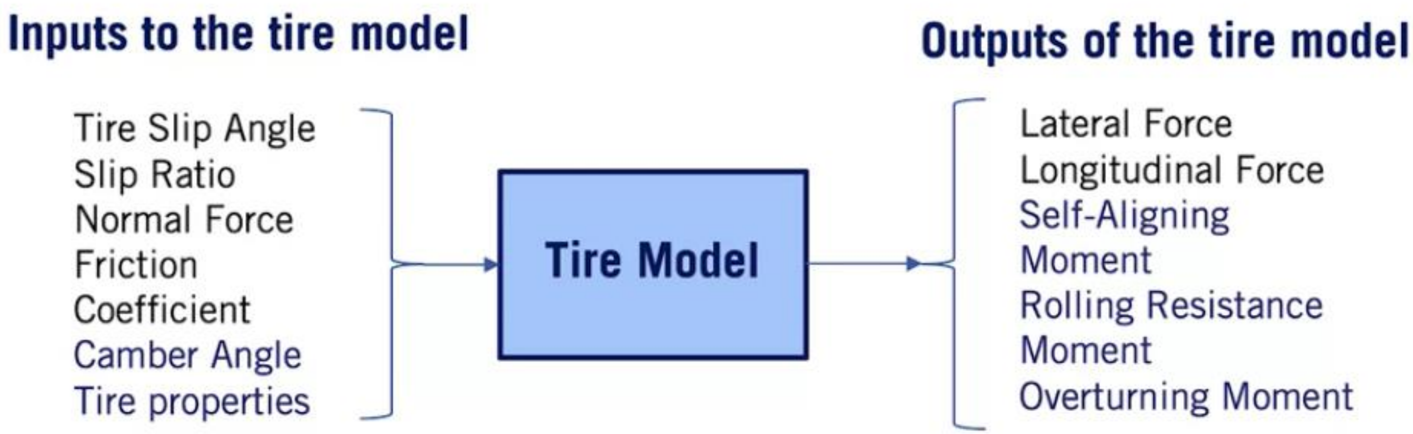
tire effective radius

$$s = \frac{w\,r_e - V}{V}$$

vehicle forward velocity

$wr_e < V$	Wheels are skidding, this happens during deceleration of the vehicle, during normal braking
$wr_e > V$	Wheels are spinning, this happens during acceleration, especially in low friction driving (icy road)
$wr_e = 0$	Wheels are locked, this happens during heavy or panic braking where the vehicle loses its desired traction

Tire Modeling

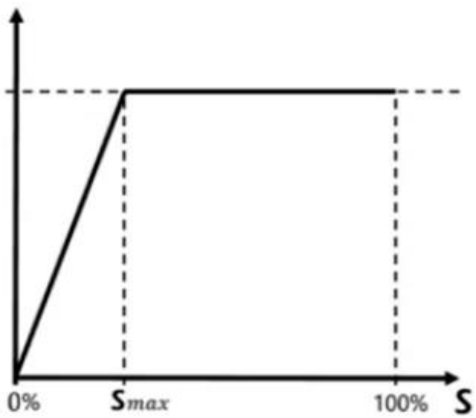


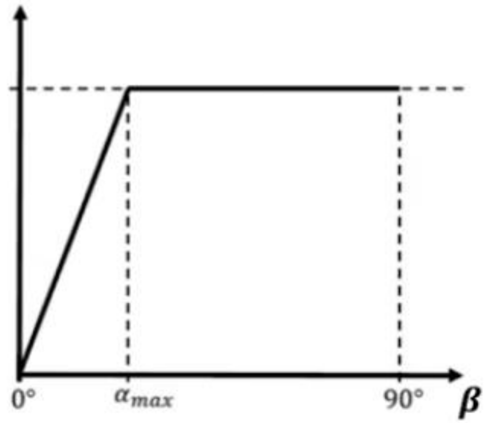
Tire Modeling

- Analytical - Brush, Fiala, Linear
  - Tire physical parameters are explicitly employed
  - Low precision, but simple
- Numerical
  - Look up tables instead of mathematical equations
  - No explicit mathematical form
  - Geometry and material property of tire are considered
- Parameterized - Linear, Pacejka, Dugoff
  - Need experiments for each specific tire
  - Formed by fitting model with experimental data
  - Match experimental data very well
  - Used widely for vehicle dynamics simulation studies and control design

Linear Tire Model

- Assumption: the relationship between slip angle and force is linear
  - Piecewise linear curves: $F(x) = \begin{cases} Cx & if\ |x| < x_{max} \\ F_{max} & if\ |x| \geq x_{max} \end{cases}$



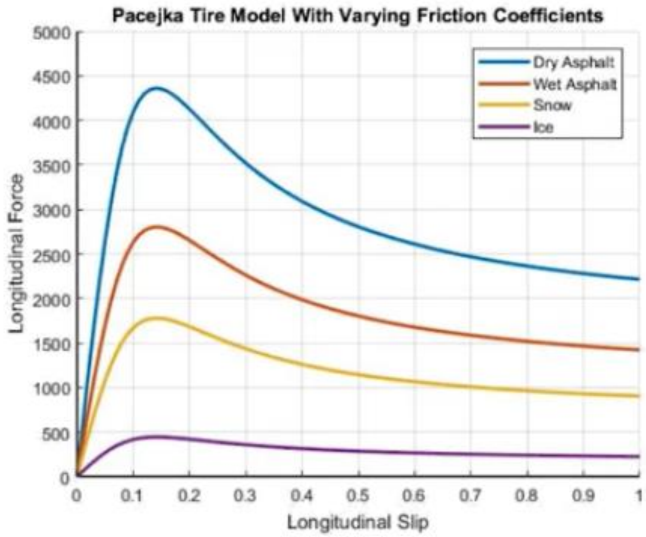


Pacejka Tire Model

- Also called Magic Formula tire model
  - Widely used in model-based control development.

tire vertical force

$$F(x, F_z) = D \sin(C \tan^{-1}(Bx - E(Bx - \tan^{-1}(Bx))))\mu F_z$$

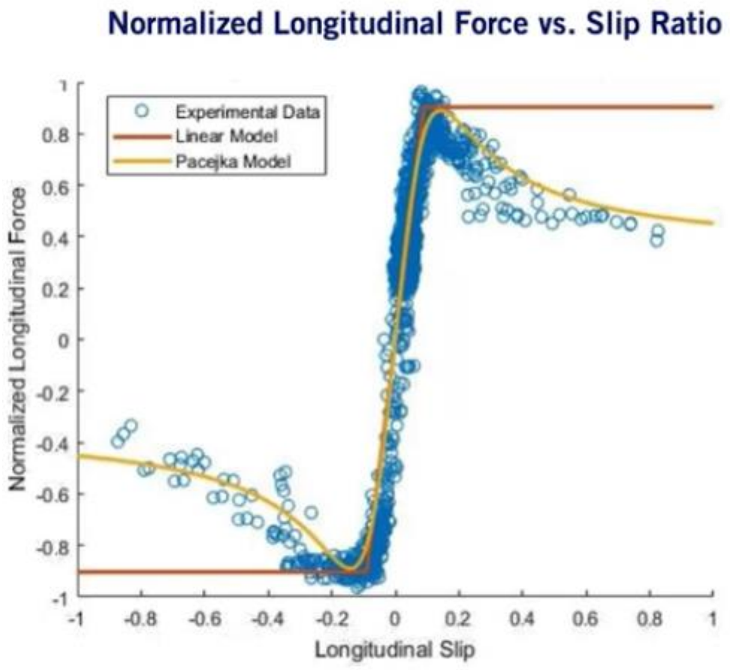


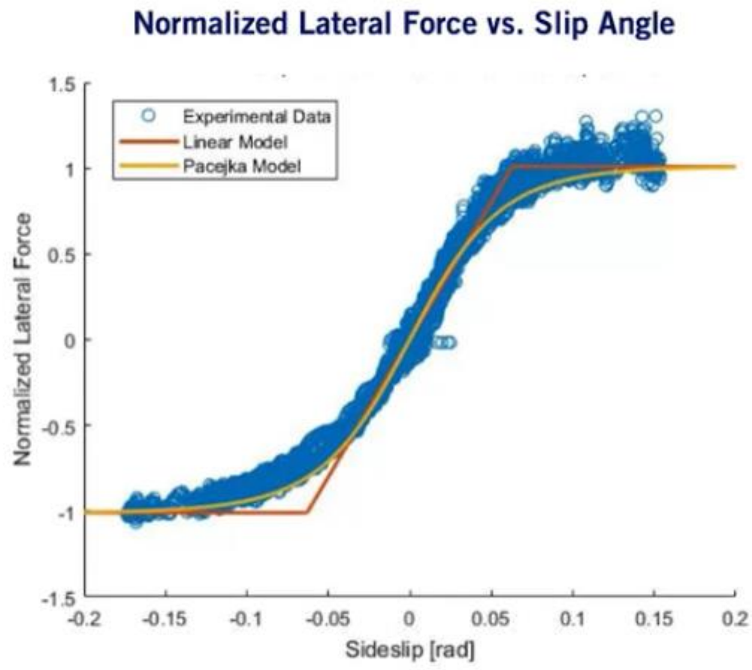
x could be either slip ratio or slip angle (in tire modeling)

road friction coefficient

B – Stiffness Factor
C – Shape Factor
D – Peak Factor
E – Curvature Factor

Forces vs Slips





Lesson Summary

What we have learned from this lesson:

- The role of tire in vehicle dynamics
- The terminology used in tire modelling such as slip angle and slip ratio
- The linear and Pacejka tire models

- **Supplementary Reading: Tire Slip and Modeling**
- Read more about different tire model and formulation in the PDF below:
- Moad Kissai, Bruno Monsuez, Adriana Tapus, Didier Martinez. "A new linear tire model with varying parameters". 2017 2nd IEEE International Conference on Intelligent Transportation Engineering (ICITE), Sep 2017, Singapore, Singapore. IEEE, Intelligent Transportation Engineering (ICITE), 2017 2nd IEEE International Conference on. <10.1109/ICITE.2017.8056891>. https://hal.archives-ouvertes.fr/hal-01690792/
- **Tire model.pdf**Archivo PDF

Challenges for the Industry

- https://www.coursera.org/learn/intro-self-driving-cars/lecture/BBJF6/challenges-for-the-industry