Ficha   Lei de Hartley-Shannon / Ritmo de Nyquist

1.

$\eta = 10^{-8}$ watt / Hz

$S = 100\ \mu$ watt $= 100 \times 10^{-6}$ watt

a)

$C = ?$

i) $B_T = 1$ KHz

$$C = 1 \times 10^3 \log_2 \left(1 + \frac{100 \times 10^{-6}}{0{,}00001} \right) \simeq 3459{,}43 \text{ bits/s}$$

$N = \eta \cdot B_T = 10^{-8} \times 1 \times 10^3 = 0{,}00001$

ii) $B_T = 10$ KHz

$$C = 10 \times 10^3 \log_2 \left(1 + \frac{100 \times 10^{-6}}{10000 \times 10^{-8}} \right) \simeq 10000 \text{ bits/s}$$

$N = 10^{-8} \times 10 \times 10^3 = 10000 \times 10^{-8}$

iii) $B_T = 100$ KHz

$$C = 100 \times 10^3 \log_2 \left(1 + \frac{100 \times 10^{-6}}{100000 \times 10^{-8}} \right) \simeq 13750{,}35 \text{ bits/s}$$

$N = 10^{-8} \times 100 \times 10^3 = 100000 \times 10^{-8}$

b)

$B_T \geq \frac{r_s}{2} \Leftrightarrow r_s \leq 2 B_T$

i) $r_s \leq 2 \times 1000$
≤ 2000

ii) $r_s \leq 2 \times 10000$
≤ 20000

iii) $r_s \leq 2 \times 100000$
≤ 200000

São símbolos por seg.

No caso de i) teríamos de usar outra base. (*)

Nos casos ii) e iii) não seria necessário alterar a base.

(*) Base 4 → por exemplo 1500 símbolos quaternários

$1500 < 2000$

$1500 \times 2 = 3000$ bits < 3459

2. $B_T = 4 \text{ KHz}$ $\qquad N = 4000 \times 10^{-13}$

$\eta = 10^{-13}$ watt/Hz

$S = ?$

$C = 64$ Kbit/s

$$64000 = 4000 \log_2 \left(1 + \frac{S}{4000 \times 10^{-13}}\right)$$

$\Rightarrow S = 2,62 \times 10^{-5}$ watt

$C = 128$ Kbits/s

$$128000 = 4000 \log_2 \left(1 + \frac{S}{4000 \times 10^{-13}}\right)$$

$\Rightarrow S = 0,72$ watt

$C = 256$ Kbits/s

$$256000 = 4000 \log_2 \left(1 + \frac{S}{4000 \times 10^{-13}}\right) \Rightarrow S = 7\,378\,697\,629 \text{ watt}$$

3.

A1. Falso. r_s não depende de $\frac{S}{N}$

$r_s \leq 2 B_T$

B2. Verdadeiro $r_s \leq 1000$ símb/s $\rightarrow B_T = 500$

$\eta = 10^{-2}$ watt/Hz

$C = 1000$ bit/s

$S = 15$ watts

$$1000 = 500 \log_2 \left(1 + \frac{15}{500 \times 10^{-2}}\right)$$

$1000 = 1000$

$N = \eta \cdot B_T$
$= 10^{-2} \times 500$

C3. Verdadeiro $\frac{S}{N} = 7$ $\qquad C = 3 B_T$

$C = B_T \log_2 (1 + 7)$

$= B_T \cdot 3$

D4. Falso. Exercício 1

Ficha: Análise de Sinais

4

A1. V

B2. F. Componente constante = 0,7

$$T = \frac{1}{f} = \frac{1}{200} = 0{,}005\,s = 5\,ms$$

$$f = 200$$

$$\cos(2\pi \times 1 \times 200 \times t)$$

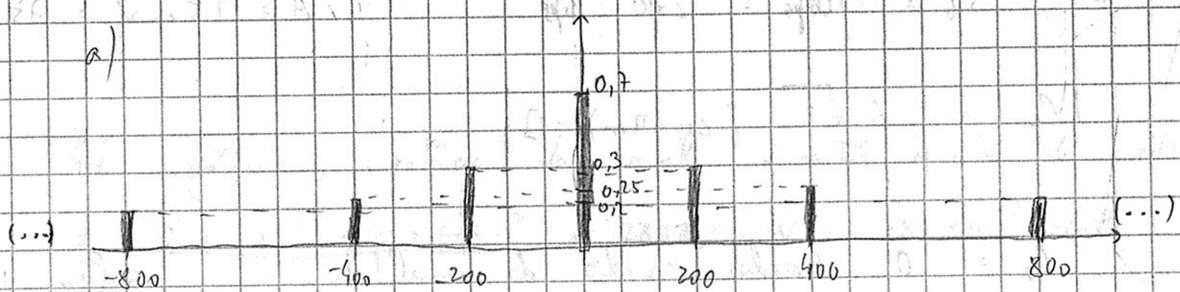
$\downarrow$ n, $\downarrow$ f_0

C3. F $f_0 = 200\ Hz$

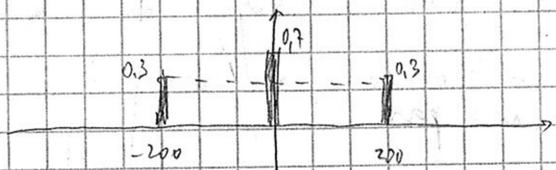
D4. F. Por B2.

5.

a)

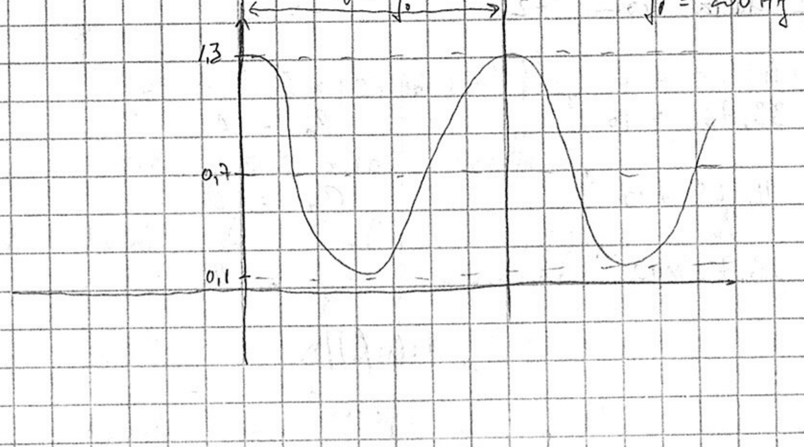


b)



$$x(t) = 0{,}7 + 0{,}6\cos(400\pi t)$$

$$T_0 = \frac{1}{f_0} \qquad f_0 = 200\,Hz$$



Ficha Análise Sinais II

1. $C_m = \dfrac{1}{T_0} \displaystyle\int x(t) \cdot e^{-j 2\pi f_0 t} \, dt$

$$C_m = \frac{1}{T_0} \left[\int_{-\frac{T_0}{2}}^{0} -A \cdot e^{-j 2\pi f_0 t} \, dt + \int_{0}^{\frac{T_0}{2}} A \cdot e^{-j 2\pi f_0 t} \, dt \right]$$

(⋮)

$|C_m| = \_\_$

$\hookrightarrow$
$$\left.\begin{array}{l} C_0 = \ldots \\ C_1 = \ldots \\ C_{-1} = \ldots \\ C_2 = \ldots \\ C_{-2} = \ldots \end{array}\right\} \text{Espectro}$$

2. $r_b = 2 \text{ Mbps} = 2 \times 10^6 \text{ bps}$ $\qquad A = 75 \text{ mV} = 75 \times 10^{-3}$

$$|C_m| = \left| \frac{A \sqrt{2}}{2 \pi m} \left[\cos(\pi m) - 1 \right] \right|$$

$C_0 = 0$ (valor médio do sinal)

$\cos(\pi m - 1)$

m ímpar $= -2$ $\qquad$ m par $= 0$

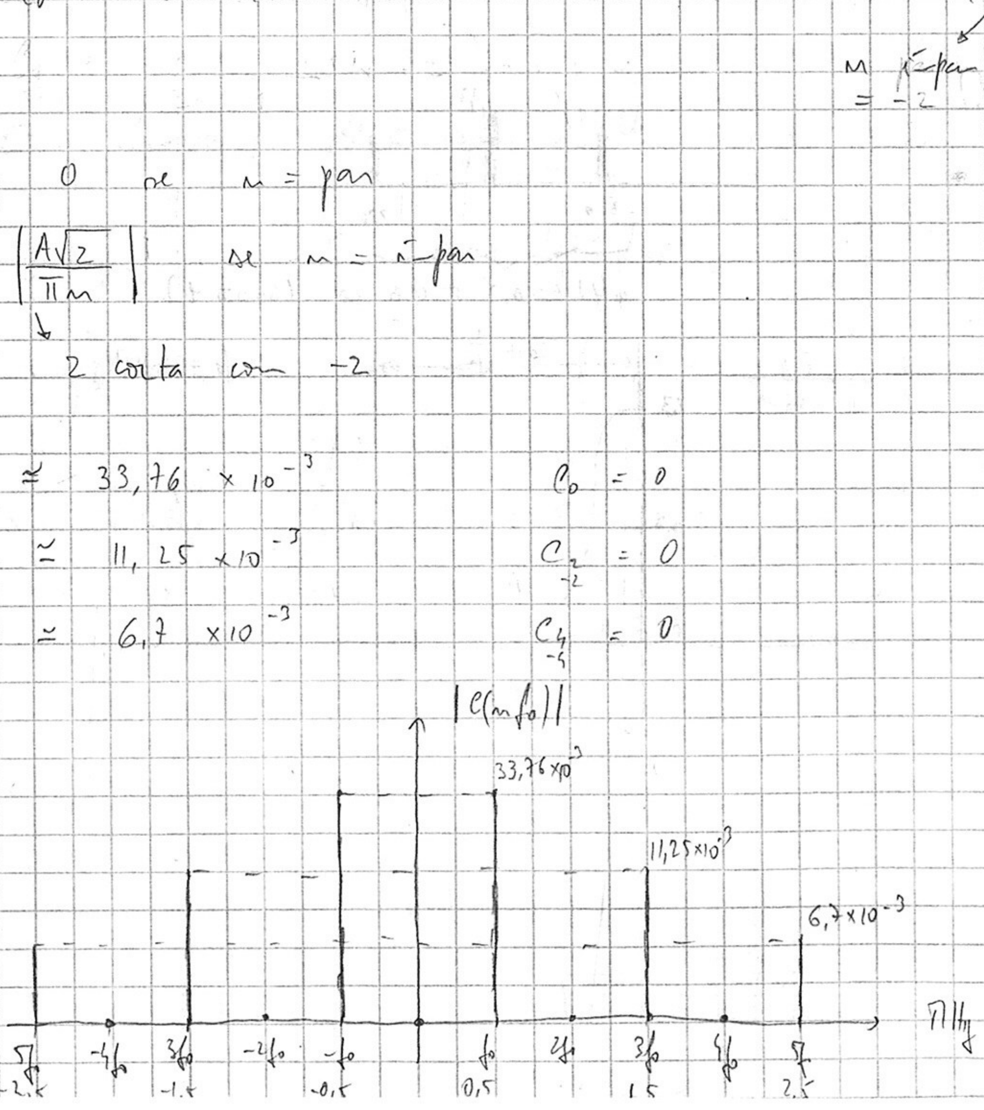
$$|C_m| \begin{cases} 0 & \text{se } m = \text{par} \\ \left| \dfrac{A\sqrt{2}}{\pi m} \right| & \text{se } m = \text{ímpar} \end{cases}$$

2 corta com -2

$C_{1, -1} \simeq 33{,}76 \times 10^{-3}$ $\qquad$ $C_0 = 0$

$C_{3, -3} \simeq 11{,}25 \times 10^{-3}$ $\qquad$ $C_{2, -2} = 0$

$C_{5, -5} \simeq 6{,}7 \times 10^{-3}$ $\qquad$ $C_{4, -4} = 0$



Período do bit

$$T_b = \frac{1}{2\times10^6}$$

Período:

$$T_0 = 4\,T_b = \frac{4}{2\times10^6}$$

$$f_0 = \frac{1}{T_0} \Leftrightarrow f_0 = \frac{1}{\frac{4}{2\times10^6}} = 0{,}5\ \text{MHz}$$

b) Potência do sinal:

$$S = \langle |v(t)|^2 \rangle = \frac{1}{T_0}\int_{T_0} |v(t)|^2\,dt$$

$$=$$

Pelo gráfico, potência do sinal é a média de $v(t)^2$

$$= \frac{0 + 75^2 + 0 + (-75)^2}{4} \simeq 2812{,}5\,\text{mW} = 2{,}81\times10^{-3}\ \text{Watt}$$

Largura de banda do sinal: 90% S

$$0{,}9\times S = 0{,}9\times 2{,}81\times10^{-3} \simeq 2{,}531\times10^{-3}\ \text{watts}$$

$$S = \sum_{-\infty}^{+\infty} |C_n|^2 = C_0 + 2|C_1|^2 + 2|C_2|^2 + 2|C_3|^2 + (\dots)$$

Somar até igualar ou passar $\underbrace{0{,}9S}$

$$= 0 + 2\times|33{,}76\times10^{-3}|^2 + 2\times0 + 2\times|11{,}25\times10^{-3}|^2$$ largura de banda

$$\simeq 2{,}532\times10^{-3}\ \text{watts}$$

Largura de banda do sinal: amplitude do menor intervalo espectral positivo que contém 90%

Largura de banda $= 1$ MHz $(1{,}5 - 0{,}5)$

Se fosse banda base $= 1{,}5$ MHz (começava do 0)

3.

A1. F. É periódico

B2. V $T = \frac{1}{f_0} = \frac{1}{50} = 0{,}02\ s = 20\ ms$

C3. F. $f_0 = 50\ Hz$

D4. V $S = 400 \times 10^{-3}$

$90\%\ S = 360 \times 10^{-3}$ (0,360)

$$S = \sum_{-\infty}^{+\infty} |C_n|^2 = C_0^2 + 2|C_1|^2 + 2|C_2|^2 + 2|C_3|^2 + \dots$$

(*) Não tem $C_2, C_3, C_5, \dots$ $= C_0^2 + 2|C_1|^2 + 2|C_4|^2 + 2|C_8|^2$ (*)

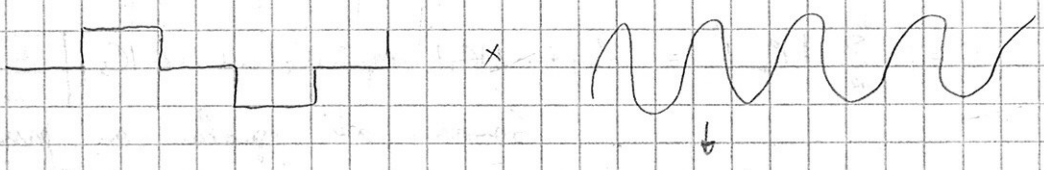
$= 0{,}5^2 + 2 \times 0{,}2^2 + 2 \times 0{,}15^2$

$= 0{,}375$

$0{,}375 > 0{,}360$ Foi dos 0 aos 200 Hz

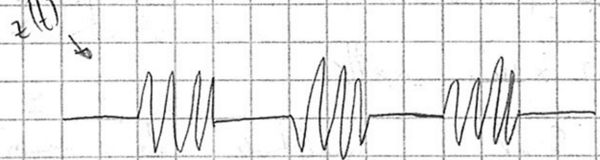
Largura de Banda = 200 Hz

4. $z(t) = v(t) \cdot \cos(2\pi f_p t)$



10 MHz

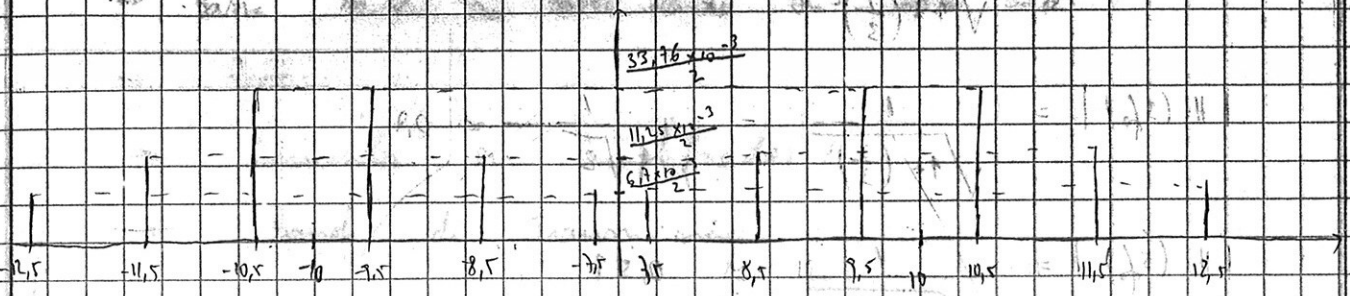
=



(Adequação ao meio de comunicação)

Por $v(t) \cdot \cos(2\pi f_p t) \leftrightarrow \frac{1}{2}\left[V(f - f_p) + V(f + f_p)\right]$

Espectro $z(t)$:



Ficha Sistemas de Transmissão (I + II)

1.

a) Característica de amplitude: $|H(f)|$

Característica de potência: $|H(f)|^2$

$$H(f) = \frac{1}{1 + j\frac{f}{B_T}} \qquad |H(f)| = \frac{1}{\sqrt{1+\left(\frac{f}{B_T}\right)^2}}$$

$$|H(f)|^2 = \frac{1}{1+\left(\frac{f}{B_T}\right)^2}$$

Gráfico $|H(f)|$:

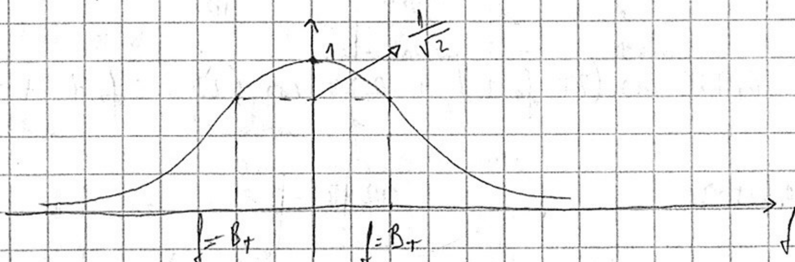
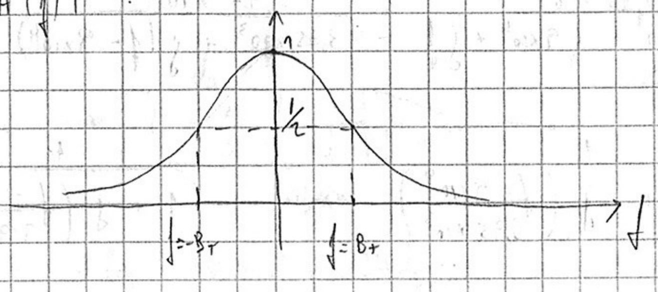


Gráfico $|H(f)|^2$:



b)

$$H(f) = \frac{1}{1 + j\frac{f}{3f_0}}$$

180° para fazer positivo

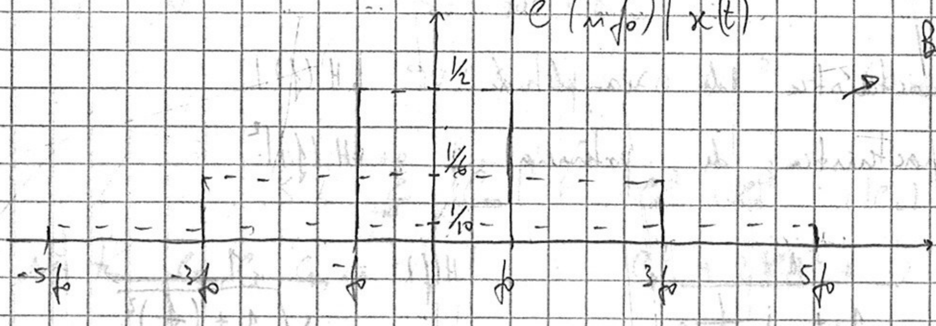
$$x(t) = \cos(2\pi f_0 t) + \frac{1}{3}\cos(6\pi f_0 t + \pi) + \frac{1}{5}\cos(10\pi f_0 t)$$

$$= \cos(2\pi f_0 t) + \frac{1}{3}\cos(3\times2\pi f_0 t + \pi) + \frac{1}{5}\cos(5\times2\pi f_0 t)$$

3) $$|H(f_0)| = \frac{1}{\sqrt{1+\left(\frac{1}{3}\right)^2}} \simeq 0{,}95$$

$$|H(3f_0)| = \frac{1}{\sqrt{1+\left(\frac{3}{3}\right)^2}} = \frac{1}{\sqrt{2}} \simeq 0{,}7$$

$$|H(5f_0)| = \frac{1}{\sqrt{1+\left(\frac{5}{3}\right)^2}} \simeq 0{,}51$$



→ Bilateral, por isso divide por 2

$$|C_{\underset{-1}{1}}|\,y(t) = |C_{\underset{-1}{1}}|\,x(t) \times |H(f_0)| = \frac{1}{2} \times 0{,}95 \simeq 0{,}475$$

$$|C_{\underset{-3}{3}}|\,y(t) = |C_{\underset{-1}{1}}|\,x(t) \times |H(3f_0)| = \frac{1}{6} \times 0{,}7 \simeq 0{,}12$$

$$|C_{\underset{-5}{5}}|\,y(t) = |C_{\underset{-5}{5}}|\,x(t) \times |H(5f_0)| = \frac{1}{10} \times 0{,}5 \simeq 0{,}051$$

valores bilaterais

$$y(t) = \underset{0{,}475\times 2}{0{,}95} \cos(2\pi f_0 t) + \underset{0{,}12\times 2}{0{,}24} \cos(2\pi \cdot 3 f_0 t + \pi) + \underset{0{,}051\times 2}{0{,}102} \cos(2\pi \cdot 5 f_0 t)$$

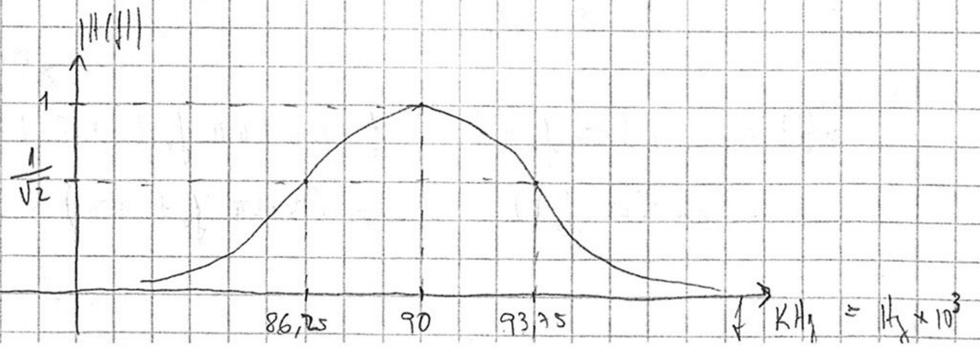
4.

$$H(f) = \frac{3{,}75\times 10^3}{3{,}75\times 10^3 - j\,9\times 10^4 + jf} = \frac{3{,}75\times 10^3}{3{,}75\times 10^3 + j(f - 9\times 10^4)}$$

$$= \frac{1}{1 + j\left(\frac{f - 9\times 10^4}{3{,}75\times 10^3}\right)} = \frac{1}{1 + j\left(\frac{f - 90\times 10^3}{3{,}75\times 10^3}\right)}$$

a)

$$|H(f)| = \frac{1}{\sqrt{1 + \left(\frac{f - 90\times 10^3}{3{,}75\times 10^3}\right)^2}}$$



Características de um sistema:
→ Passa banda ou Passa baixo ou Passa alto

→ Ordem

→ Atenuador ou amplificador

→ Banda de Transmissão

Características do sistema de 4:

→ Passa banda (não está centrado em 0)

→ 1ª ordem

→ Atenuador (ganho máximo 1, o resto é tudo abaixo)

→ Banda de transmissão [86,25 , 93,75] KHz

b) 93,75 − 86,25 = 7,5 KHz

2. $P_e = 10\,W = 10\,000\,mW$ $P_e\,dB_m = 10 \log_{10} 10000 = 40\,dB_m$

Ganho (g) = 100 dB

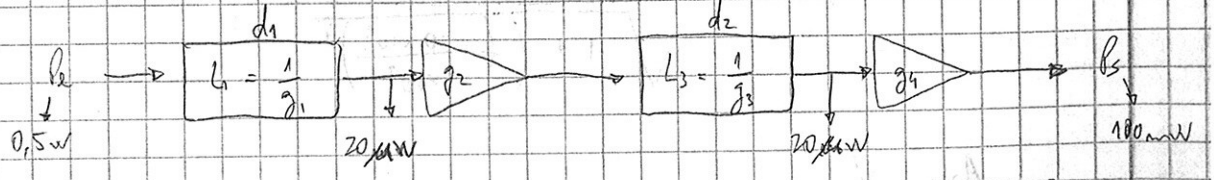
Atenuação (L) $= \underbrace{10 \log_{10}}_{\text{calcular } \alpha} \underset{\downarrow\, L_{Km}}{10} \times \underset{\downarrow\, 11Km}{11} = 110\,dB$

$L = \alpha . d$

$P_s\,dB_m = P_e\,dB_m + g - L$
$= 40 + 100 - 110$
$= 30\,dB_m$

$P_s = \left(\frac{30}{10}\right)^{10} mW = 3^{10}\,mW$

3.



$P_e = 0,5\,W = 10 \log_{10} 0,5 \times 10^3 = 27\,dB_m$
$\alpha = 2\,dB/Km$
$d_1 + d_2 = d = 40\,Km$
$P_s = 100\,mW = 10 \log_{10} 100 = 20\,dB_m$

$P_{e\,g_2} = 20\,\mu W = P_{e\,g_4} =$
$= 10 \log_{10} 20 \times 10^{-6} \times 10^3 =$
$= 10 \log_{10} 20 \times 10^{-3} = -17\,dB_m$

$P_s\,dB_m = P_e\,dB_m + g - L$
$-17 = 27 + 0 - 2 \times d_1$
$44 = 2d_1$
$d_1 = 22\,Km$

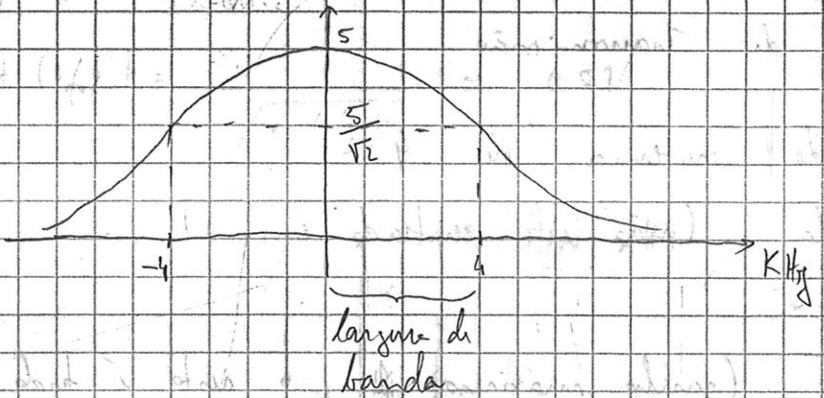
$d_1 + d_2 = 40 \Leftrightarrow d_2 = 18\,Km$

$-17 = -17 + g_2 + g - d_2 \times 2$
$g_2 = 2d_2$
$g_2 = 2 \times 18$
$g_2 = 36\,dB$

$20 = -17 + g_4 - L$
$37 = g_4 - 0$
$g_4 = 37\,dB$

5.

$$|H(f)| = \frac{5}{\sqrt{1 + \left(\frac{f}{4\times10^3}\right)^4}}$$



a) · sistema / filtro 2ª ordem (n=2)
passa baixo (está centrado no 0)
· Amplificador (K ≥ 1)

b) · $B_T = 4$ KHz

6.

$$H(f) = \frac{1}{25 + j\left(\frac{f - 10\times10^3}{10^3}\right)^2}$$

$$= \frac{1/25}{1 + j\left(\frac{f - 10\times10^3}{5\times10^3}\right)^2}$$

$$|H(f)| = \frac{1/25}{\sqrt{1 + \left(\frac{f - 10\times10^3}{5\times10^3}\right)^4}}$$

A_1 . V
B_2 . F . $1/625$ → Atenua 625 vezes
C_3 . V
D_4 . V