



Foundations of linear mixed models

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The (very) basics about mixed models

Mixed models consist of:

Fixed terms

- Interest in **specific levels**
- Parameters of interest are **individual treatment effects**

Random terms

- Treatment levels are chosen to represent a **population of possible treatments**
- Parameters of interest **variance** parameters

Analysis of large experiments.

Analysis steps:

1. Fit a model with **genotypes as random**

- ↳ to calculate trial heritability
- ↳ genotype means are called 'BLUPs'

2. Fit a model with **genotypes as fixed**

- ↳ to extract genotype adjusted means (BLUEs) to be used for variety selection, GxE analysis or QTL detection.

Heritability (broad sense)

$$H^2 = \frac{\sigma_g^2}{\sigma_g^2 + \frac{\sigma_\epsilon^2}{n}}$$

σ_g^2 = genotypic variance

σ_ϵ^2 = error variance (residual)

n = number of replicates

$$H^2 = 0$$

variation is not
explained by
the genotypes

$$H^2 = 1$$

variation is fully
explained by the
genotypes

Analysis of large experiments

Example: each experiment has:

- 150 barley genotypes
- 2 replicates
- The only difference is the design that was used:

- a) RCBD
- b) Alpha
- c) Row-column
- d) Resolvable row-column

Uniformity trial (barley)

- **15 rows x 48 columns grid**
- **Variation across experiment due to uncontrolled conditions (=“noise”)**◦Minimum 1.490 ton/ha
- **Maximum 2.290 ton/ha**
- **Variance = 0.084 (ton/ha)²**
- **Std = 0.289 ton/ha**

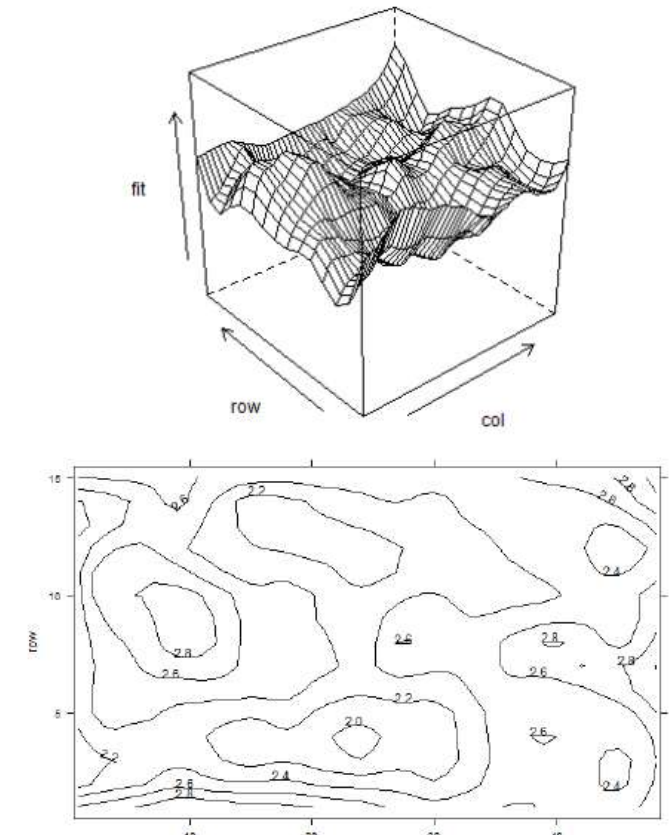
The use of uniformity data in the design and analysis of cotton and barley variety trials

ER Williams and DJ Lockett

Australian Journal of Agricultural Research 39(3) 339 - 350

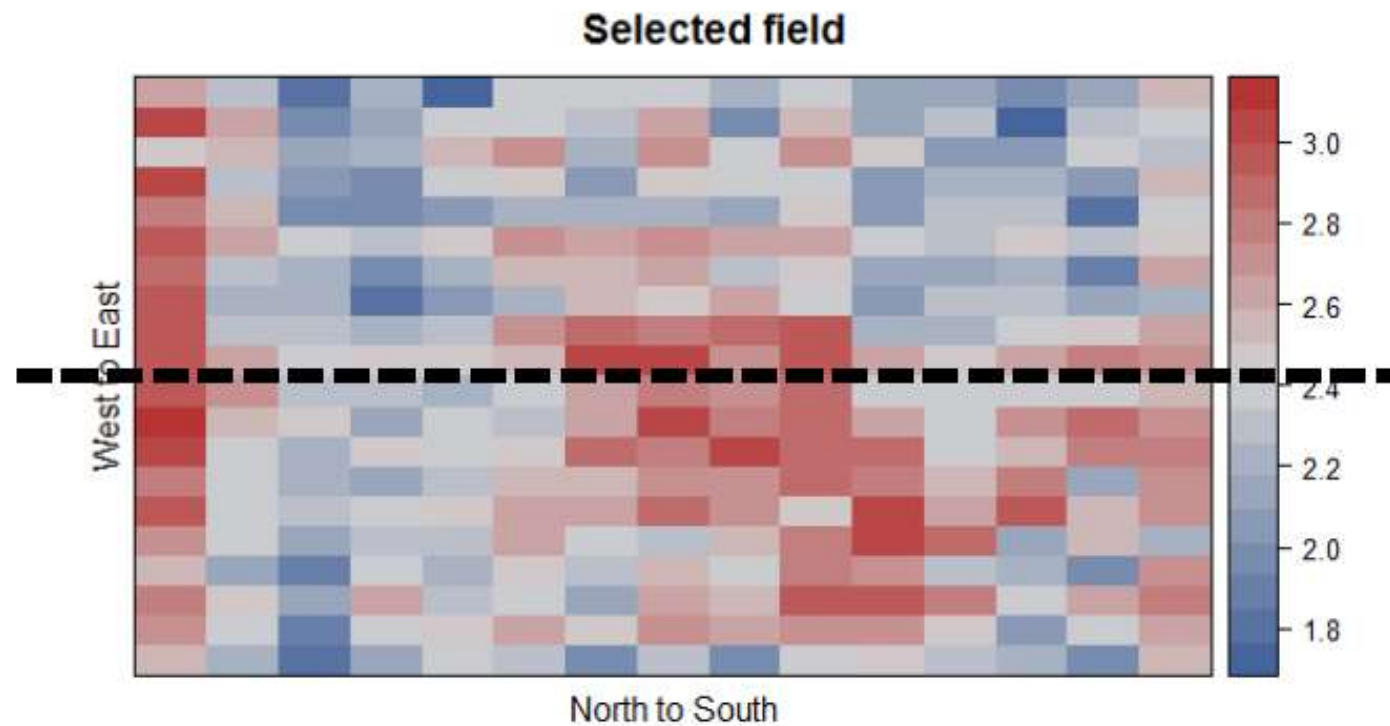
Published: 1988

<https://doi.org/10.1071/AR9880339>



Suppose we plan a trial in this field ...

- 150 barley genotypes
- 2 reps each



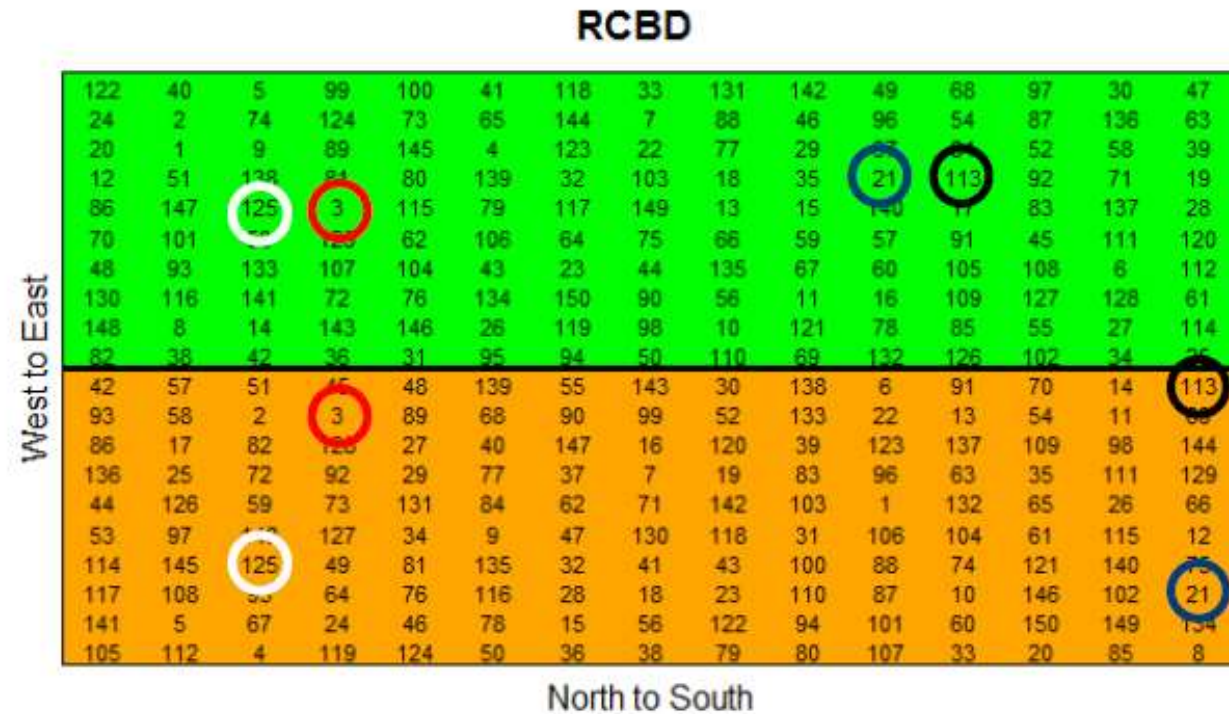
Two complete blocks = RCBD

Estimates: surrogates of true value

Phenotype	Phenotypic mean	BLUE	BLUP	PBLUP	GBLUP																		
<p>The phenotypic value of a single observation (e.g. plant or plot) is a surrogate of the observation's true genetic value.</p> <p>However, the phenotype deviates from the true genetic value due to non-genetic effects like environment and random chance. How much phenotypes deviate from the true genetic value can be estimated by measuring heritability.</p> <p>If phenotypes have high heritabilities, they are relatively accurate estimates of true value. If not, they are relatively inaccurate, and other surrogates can increase accuracy.</p>	<p>Phenotypic means of multiple observations are generally more accurate than single phenotypic values.</p> <p>This is because the error component of phenotypes is random, so the mean of the phenotypes is closer to the true genetic value than a single phenotype.</p> <p>Phenotypic means derived from phenotypes with high heritabilities still have higher accuracies than phenotypic means derived from phenotypes with low heritabilities.</p> <p>The increase in accuracy from using the phenotypic mean vs. phenotype is higher if the phenotypes have low heritabilities.</p>	<p>Best linear unbiased estimates can be obtained from linear regression of phenotypes on fixed effects, e.g.:</p> <p>Pheno ~ mean + block + genotype + error</p> <p>BLUEs allow more complex evaluation strategies which can increase accuracy (e.g. blocking) and accounting for non-genetic effects.</p> <p>In absence of non-genetic effects besides error and with equal replication, BLUEs of genetic value are equal to phenotypic means.</p> <p>BLUEs can increase accuracy compared to using phenotypic means indirectly by enabling use of experimental designs which increase accuracy.</p>	<p>Best linear unbiased predictions can also be obtained from linear regression of phenotypes, but the regression is on random effects or a mix of fixed and random effects, e.g.:</p> <p>Pheno ~ mean + block + genotype + error</p> <p>The random genotype effects, which estimate genetic value, are shrunk to the mean by an estimate of heritability for the genotype.</p> <p>Genotypes with fewer replications (as in unbalanced data) have lower heritabilities, so estimates of their genetic value are shrunk more to anticipate their regression to the mean. This can increase accuracy.</p>	<p>Pedigree best linear unbiased predictions are similar to BLUPs, but the random genetic effects are not assumed to be independent (since genotypes are related):</p> <p>Pheno ~ mean + block + genotype + error</p> <table><tr><td></td><td>A</td><td>B</td></tr><tr><td>A</td><td>1</td><td>0.50</td></tr><tr><td>B</td><td>0.5</td><td>1</td></tr></table> <p>The genetic effects are not only shrunk by heritability, but also weighted by the pedigree expectation of their relatedness to other observations. This borrowing of information from relatives can increase accuracy in some scenarios.</p>		A	B	A	1	0.50	B	0.5	1	<p>Genomic best linear unbiased predictions are quite similar to PBLUPs, except that genotype relatedness is estimated from markers rather than pedigree expectations.</p> <p>Pheno ~ mean + block + genotype + error</p> <table><tr><td></td><td>A</td><td>B</td></tr><tr><td>A</td><td>1</td><td>0.54</td></tr><tr><td>B</td><td>0.54</td><td>1</td></tr></table> <p>Genotype relatedness deviates from pedigree expectations because of recombination (Mendelian sampling), so accounting for this deviation can increase accuracy compared to PBLUP in some scenarios.</p>		A	B	A	1	0.54	B	0.54	1
	A	B																					
A	1	0.50																					
B	0.5	1																					
	A	B																					
A	1	0.54																					
B	0.54	1																					

RCBD

150 barley genotypes
2 reps each



- All 150 present once within each replicate (green and orange area).

Analysis RCBD trial (genotypes random = BLUPs)

$$y_{ik} = \mu + b_k + G_i + \epsilon_{ik}$$

$$G_i \sim N(0, \sigma_g^2)$$
$$\epsilon_{ik} \sim N(0, \sigma_\epsilon^2)$$

```
m1 <- lmer(yield ~ block + (1|geno), data = mytrial_RCBD$book)
summary(m1)
```

Random effects:

Groups	Name	Variance	Std.Dev.
geno	(Intercept)	0.01946	0.1395
Residual		0.08233	0.2869

Number of obs: 300, groups: geno, 150

Fixed effects:

	Estimate	Std. Error	df	t value	Pr(> t)
(Intercept)	2.50360	0.02605	287.49000	96.110	< 2e-16 ***
block2	-0.12587	0.03313	149.00000	-3.799	0.000211 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

$$H^2 = \frac{\sigma_g^2}{\sigma_g^2 + \frac{\sigma_\epsilon^2}{n}}$$
$$H^2 = \frac{0.01946}{0.01946 + \frac{0.08233}{2}} = 0.32$$

Analysis RCBD trial (genotypes fixed = BLUEs)

$$y_{ik} = \mu + b_k + G_i + \epsilon_{ik} \quad \epsilon_{ik} \sim N(0, \sigma_\epsilon^2)$$

```
m1f <- lm(yield ~ block + geno, data = mytrial_RCBD$book)
anova(m1f)
lsmeans(m1f, "geno")
```

Analysis of Variance Table

Response: yield

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
block	1	1.1882	1.18818	14.4326	0.0002111 ***
geno	149	18.0652	0.12124	1.4727	0.0093520 **
Residuals	149	12.2666	0.08233		

geno	lsmean	SE	df	lower.CL	upper.CL
1	2.654493	0.202887	149	2.253585	3.055400
2	2.498493	0.202887	149	2.097585	2.899400
3	2.086625	0.202887	149	1.685717	2.487532
4	1.928961	0.202887	149	1.528053	2.329868
5	2.140785	0.202887	149	1.739877	2.541692
6	2.303412	0.202887	149	1.902504	2.704319
7	2.808384	0.202887	149	2.407476	3.209291
8	2.400185	0.202887	149	1.999278	2.801093
9	2.282892	0.202887	149	1.881985	2.683800
10	2.834490	0.202887	149	2.433583	3.235398

- Adjusted means (= simple average over two blocks).
- Compare means using sed

$$sed = \sqrt{2 \times MSE / n} = \sqrt{2 \times 0.08233 / 2} = 0.287$$

(k=1...n), n= 2 replicates

RCBD Precision experiment → Least Significant Difference

$$LSD = 2 * sed = 2 * 0.287 = 0.574 \text{ ton/ha}$$

Standard error of the difference RCBD – the same

Because all genotypes are tested in all blocks, the s.e. is the same for all genotype comparisons

```
diff_rcbd[1:20,1:4]
```

contrast	estimate	SE
1 - 2	-0.466782955	0.2744649
1 - 3	-0.013491128	0.2744649
1 - 4	-0.107449108	0.2744649
1 - 5	-0.150717265	0.2744649
1 - 6	0.108819636	0.2744649
1 - 7	0.165882834	0.2744649
1 - 8	0.001472315	0.2744649
1 - 9	-0.289928923	0.2744649
1 - 10	-0.061383326	0.2744649
1 - 11	-0.157999909	0.2744649
1 - 12	-0.066398252	0.2744649
1 - 13	0.067532540	0.2744649
1 - 14	0.226920294	0.2744649
1 - 15	-0.450909076	0.2744649
1 - 16	-0.030708723	0.2744649
1 - 17	-0.017941970	0.2744649
1 - 18	0.161718042	0.2744649
1 - 19	0.126490129	0.2744649
1 - 20	0.703500385	0.2744649
1 - 21	0.015052154	0.2744649

Alpha design

Block 1

alpha design

2	126	90	147	08	99	81	1	87	133	160	28	26	123	07
82	48	129	122	19	27	57	53	18	98	116	125	45	32	47
14	141	04	46	5	108	82	4	130	52	90	17	115	01	06
117	09	124	138	13	11	88	110	111	116	51	24	20	114	09
146	58	90	99	12	106	15	128	23	49	7	21	63	10	100
6	88	121	113	18	105	143	83	85	43	72	97	120	132	16
5	9	132	139	140	00	42	114	56	31	25	01	50	38	134
24	09	3	76	22	11	79	77	112	91	60	39	65	30	54
127	79	142	80	55	103	145	09	47	40	41	93	36	131	05
71	54	02	11	55	85	21	54	88	85	04	05	78	08	104
31	134	47	12	76	25	127	74	131	64	133	11	141	85	30
65	90	113	120	115	135	80	52	72	121	111	42	129	139	7
122	5	107	41	00	09	54	83	86	26	21	46	04	84	112
20	145	7	3	46	57	58	02	57	66	5	96	52	136	37
1	147	122	53	08	40	99	10	64	91	24	114	78	88	72
33	52	118	125	143	59	61	71	07	110	40	35	84	81	82
14	142	35	16	73	83	27	12	146	123	05	07	42	58	137
10	90	39	144	29	9	98	89	27	140	44	11	138	38	8
38	55	123	132	116	67	13	98	23	148	68	1	61	150	86
75	79	128	80	5	05	17	138	70	89	15	1	43	119	45

Replicate 1

Replicate 2

- Resolvable: 2 full replicates
- 15 Incomplete blocks within each replicate
 - Blocks occur along the columns direction
 - A subset of genotypes within a particular block
 - Partially-balanced: genotypes occur together in a block either **once** ($\lambda_2 = 1$) or **never** ($\lambda_1 = 0$) over the whole experiment $\rightarrow \alpha(0,1)$ design

Analysis α -design trial (genotypes random = BLUPs)

$$y_{ijk} = \mu + R_j + b_{k(j)} + G_i + \epsilon_{ijk}$$

$$\begin{aligned} G_i &\sim N(0, \sigma_g^2) \\ b_{k(j)} &\sim N(0, \sigma_b^2) \\ \epsilon_{ijk} &\sim N(0, \sigma_\epsilon^2) \end{aligned}$$

When analysing an alpha design, blocks MUST be random, because they are incomplete

```
m2 <- lmer(yield ~ replication + (1|block:replication) + (1|geno),
           data = mytrial_alpha$book)
summary(m2)
```

Random effects:

Groups	Name	Variance	Std.Dev.
geno	(Intercept)	0.02508	0.1584
block:replication	(Intercept)	0.03463	0.1861
Residual		0.03840	0.1960

Number of obs: 300, groups: geno, 150; block:replication, 30

Fixed effects:

	Estimate	Std. Error	df	t value	Pr(> t)
(Intercept)	2.50360	0.05227	28.41700	47.902	<2e-16 ***
replication2	-0.12587	0.07162	25.38300	-1.758	0.0909 .

signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

$$H^2 = \frac{\sigma_g^2}{\sigma_g^2 + \frac{\sigma_\epsilon^2}{n}}$$

$$H^2 \cong \frac{0.02508}{0.02508 + \frac{0.03840}{2}} = 0.56$$

Analysis α -design trial (genotypes fixed = BLUEs)

$$y_{ijk} = \mu + R_j + b_{k(j)} + G_i + \epsilon_{ijk}$$

$$b_{k(j)} \sim N(0, \sigma_b^2)$$

$$\epsilon_{ijk} \sim N(0, \sigma_\epsilon^2)$$

```
m2f <- lmer(yield ~ replication + (1|block:replication) + geno,
            data = mytrial_alpha$book)
```

Least Squares Means table:

	replication	geno	Estimate	standard Error	DF	t-value	Lower CI	Upper CI
geno 1	NA	1	2.315	0.151	142	15.3	2.02	2.61
geno 2	NA	63	2.289	0.151	142	15.2	1.99	2.59
geno 3	NA	74	2.379	0.151	142	15.8	2.08	2.68
geno 4	NA	85	2.257	0.151	142	14.9	1.96	2.56
geno 5	NA	96	2.507	0.151	142	16.6	2.21	2.81
geno 6	NA	107	2.621	0.151	142	17.4	2.32	2.92
geno 7	NA	118	2.596	0.151	142	17.2	2.30	2.89
geno 8	NA	129	2.686	0.151	142	17.8	2.39	2.98
geno 9	NA	140	2.203	0.151	142	14.6	1.90	2.50
geno 10	NA	2	2.381	0.151	142	15.8	2.08	2.68
..

- Adjusted means (\neq simple average over two reps).
- Lower standard errors! (was 0.202 and now 0.151!)
- Standard error of difference = 0.206

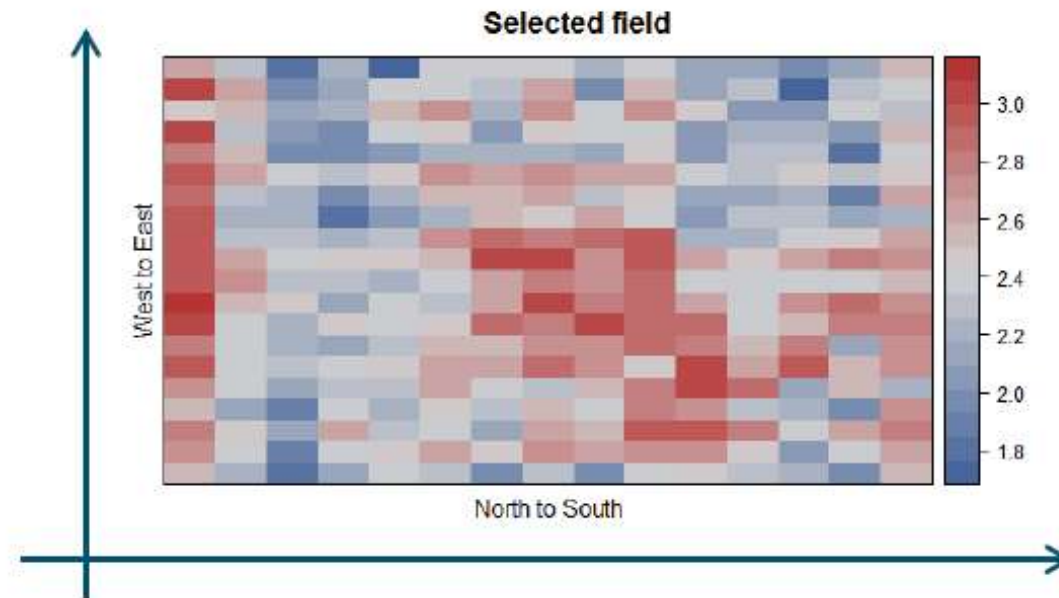
Higher precision than the RCBD $\rightarrow LSD = 2 \times 0.206 = 0.412 \text{ ton/ha}$

Standard error of the difference α -design

- It might differ between pairs
- The SE depends on the incomplete block and error variance and on with whom the genotypes occur together in incomplete blocks

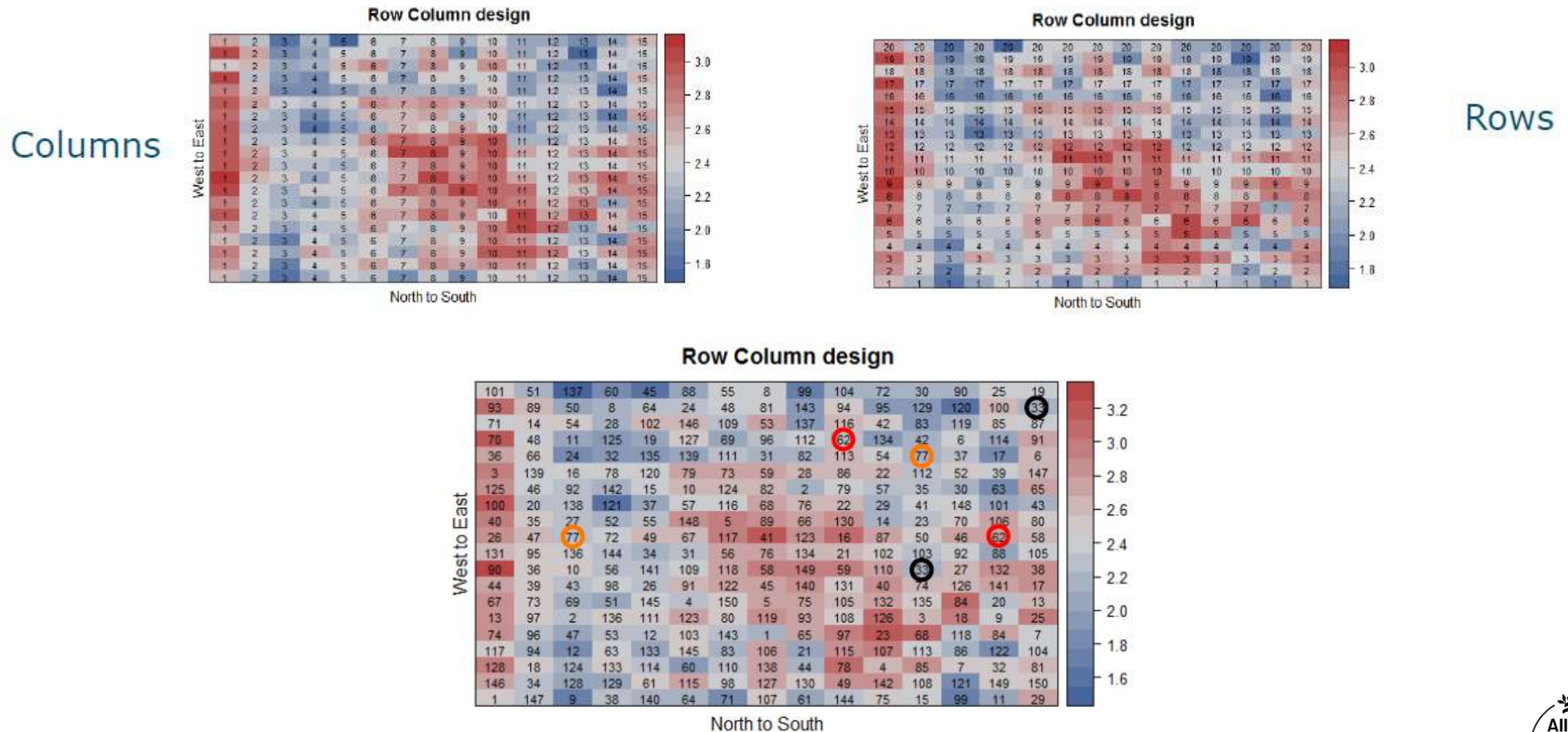
```
> diff_alpha$contrasts
contrast      estimate      SE
1 - 2        -7.44e-01 0.235
1 - 3        -2.69e-01 0.234
1 - 4        -3.78e-01 0.236
1 - 5        -4.12e-01 0.235
1 - 6        -3.49e-01 0.235
1 - 7        -6.15e-01 0.235
1 - 8        -1.87e-01 0.235
1 - 9        -4.43e-01 0.236
1 - 10       -8.44e-02 0.236
1 - 11       -1.46e-01 0.234
1 - 12       -1.89e-01 0.234
1 - 13       -2.50e-01 0.235
1 - 14       -3.81e-01 0.226
1 - 15       -4.66e-01 0.234
1 - 16       -2.51e-01 0.235
1 - 17       -3.27e-01 0.234
1 - 18       -1.77e-01 0.234
1 - 19       -5.75e-01 0.234
1 - 20       -2.36e-01 0.226
```

Double blocking: row-column designs



- Regard the field as a full $(r \times c)$ grid.
- Rows and columns \rightarrow incomplete blocks over both directions.

Double direction blocking



Analysis of a row-column design (genotypes random = BLUPs)

$$y_{ijk} = \mu + R_j + C_k + G_i + \epsilon_{ijk}$$

$$\begin{aligned} G_i &\sim N(0, \sigma_g^2) \\ R_j &\sim N(0, \sigma_r^2) \\ C_k &\sim N(0, \sigma_c^2) \\ \epsilon_{ijk} &\sim N(0, \sigma_\epsilon^2) \end{aligned}$$

```
m3 <- lmer(yield ~ (1|Row) + (1|Col) + (1|Geno),
           data = rowcol1$Book)
```

Random effects:

Groups	Name	Variance	Std.Dev.
Geno	(Intercept)	0.02096	0.1448
Col	(Intercept)	0.02456	0.1567
Row	(Intercept)	0.03075	0.1754
Residual		0.03217	0.1794

Number of obs: 300, groups: Geno, 150; Col, 20; Row, 15

Fixed effects:

	Estimate	Std. Error	t value
(Intercept)	2.44067	0.05937	41.11

$$H^2 = \frac{\sigma_g^2}{\sigma_g^2 + \frac{\sigma_\epsilon^2}{n}}$$

$$H^2 \cong \frac{0.02096}{0.02096 + \frac{0.03217}{2}} = 0.57$$

- Note that the model consists of rows and columns
 - Non-resolvable design because subsets of rows/cols do not form complete replicates
- Similar H^2 as with the α -design (and better than RCBD).

Analysis of a row-column design (genotypes fixed>BLUEs)

$$y_{ijk} = \mu + R_j + C_k + G_i + \epsilon_{ijk}$$

```
m3f <- lmer(yield ~ (1|Row) + (1|Col) + Geno,  
            data = rowcol1$Book)
```

$$R_j \sim N(0, \sigma_r^2)$$
$$C_k \sim N(0, \sigma_c^2)$$
$$\epsilon_{ijk} \sim N(0, \sigma_\epsilon^2)$$

Least Squares Means table:

	Geno	Estimate	Standard Error	DF	t-value	Lower CI	Upper CI	p-value	
Geno 1	1	2.146	0.148	147	14.530	1.85	2.44	<2e-16	***
Geno 2	63	2.360	0.149	147	15.860	2.07	2.65	<2e-16	***
Geno 3	74	2.503	0.147	147	16.990	2.21	2.79	<2e-16	***
Geno 4	85	2.353	0.148	147	15.930	2.06	2.64	<2e-16	***
Geno 5	96	2.712	0.148	147	18.310	2.42	3.00	<2e-16	***
Geno 6	107	2.645	0.148	147	17.930	2.35	2.94	<2e-16	***
Geno 7	118	2.367	0.148	147	16.040	2.08	2.66	<2e-16	***
Geno 8	129	2.457	0.148	147	16.600	2.16	2.75	<2e-16	***
Geno 9	140	2.249	0.148	147	15.250	1.96	2.54	<2e-16	***
Geno 10	2	2.604	0.148	147	17.580	2.31	2.90	<2e-16	***

When analysing a row-column design, rows and columns MUST be random, because they are incomplete

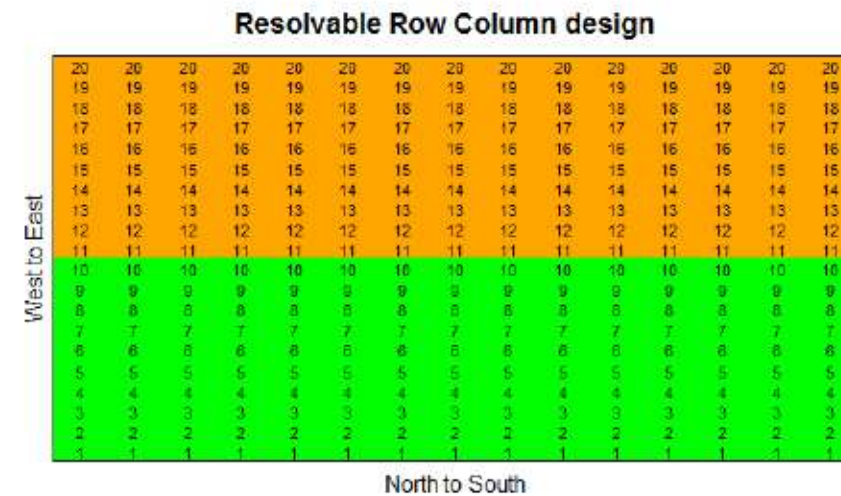
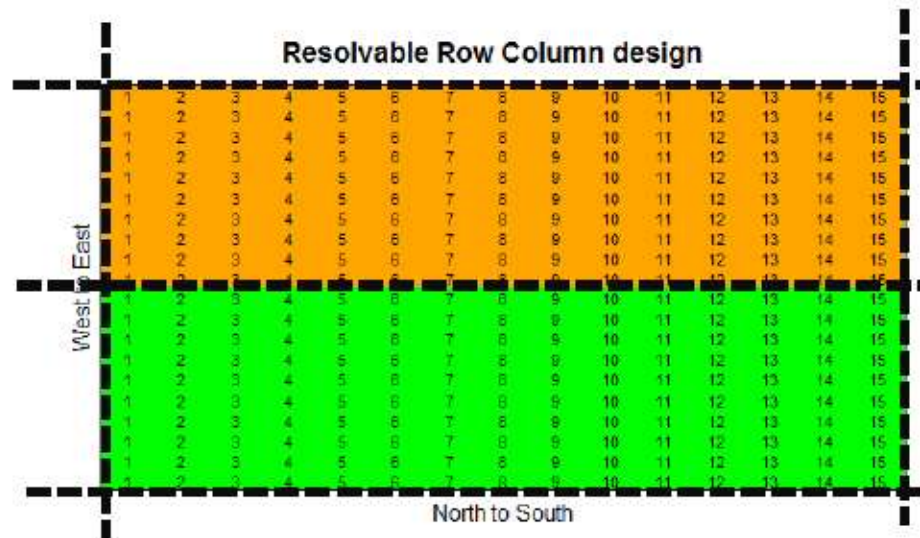
- ❑ Adjusted means (≠ simple average over two reps).
- ❑ Standard errors slightly lower than with alpha design
- ❑ Average standard error of difference = 0.192

Standard error of the difference row column design

- It might differ between pairs
- SE depends on row, column, and error variance and which genotypes occur together in rows and columns.

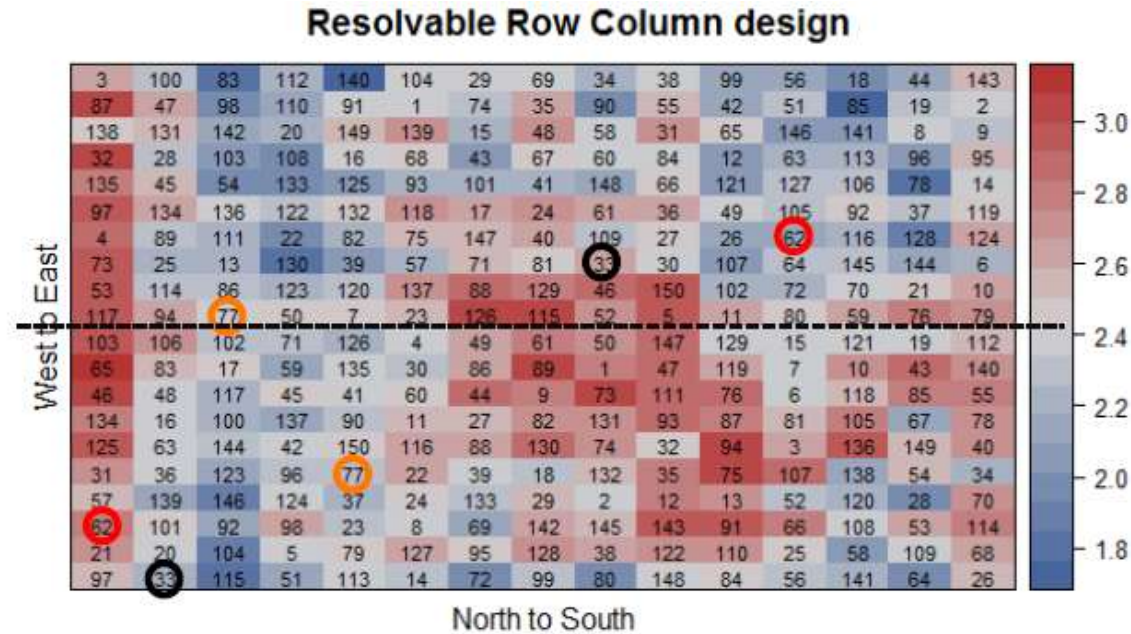
```
> diff_rowcol$contrasts
contrast      estimate      SE
1 - 2        -2.14e-01 0.195
1 - 3        -3.57e-01 0.190
1 - 4        -2.07e-01 0.195
1 - 5        -5.66e-01 0.191
1 - 6        -4.99e-01 0.194
1 - 7        -2.21e-01 0.189
1 - 8        -3.11e-01 0.190
1 - 9        -1.04e-01 0.189
1 - 10       -4.58e-01 0.195
1 - 11       -2.79e-01 0.189
1 - 12       -8.41e-02 0.189
1 - 13       -2.35e-01 0.190
1 - 14       -1.83e-01 0.195
1 - 15       -5.42e-01 0.189
1 - 16       -4.26e-01 0.197
1 - 17       -1.82e-01 0.195
1 - 18       -5.85e-01 0.194
1 - 19       -2.47e-01 0.194
1 - 20       -2.54e-01 0.194
```

Resolvable Row-Column design



- Define two full replicates → make it resolvable
- Within Replicates incomplete rows and columns (as before).

Randomization: resolvable row-column



- ❑ All 150 in each half of the field.
- ❑ Genotypes within same row (column) in rep 1 will not be in the same row (column) in rep 2.

Analysis Resolvable Row-Column design (genotypes random = BLUPs)

$$y_{ijklm} = \mu + \text{Rep}_m + R_{j(m)} + C_{k(m)} + G_i + \epsilon_{ijklm}$$

$$\begin{aligned} G_i &\sim N(0, \sigma_g^2) \\ R_j &\sim N(0, \sigma_r^2) \\ C_k &\sim N(0, \sigma_c^2) \\ \epsilon_{ij} &\sim N(0, \sigma_\epsilon^2) \end{aligned}$$

```
m4 <- lmer(yield ~ Rep + (1|Rep:Row) + (1|Rep:Col) + (1|Geno),
  data = rowcol2$Book)
```

Random effects:

Groups	Name	Variance	Std.Dev.
Geno	(Intercept)	0.02652	0.1629
Rep:Row	(Intercept)	0.03792	0.1947
Rep:Col	(Intercept)	0.01986	0.1409
Residual		0.02464	0.1570

Number of obs: 300, groups: Geno, 150; Rep:Row, 30; Rep:Col, 20

Fixed effects:

	Estimate	Std. Error	df	t value	Pr(> t)
(Intercept)	2.37774	0.06968	42.04000	34.123	<2e-16 ***
Rep1	0.12587	0.09673	39.15000	1.301	0.201

 signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

$$H^2 = \frac{\sigma_g^2}{\sigma_g^2 + \frac{\sigma_\epsilon^2}{n}}$$

$$H^2 \cong \frac{0.02652}{0.02652 + \frac{0.02464}{2}} = 0.68$$

- Note that rows and columns are **nested** within replicates (the : operator used in R).
- Heritability is the highest among all the designs.

Analysis resolvable row-column (genotypes fixed = BLUEs)

$$y_{ijkm} = \mu + Rep_m + R_{j(m)} + C_{k(m)} + G_i + \epsilon_{ijkm}$$

$R_j \sim N(0, \sigma_r^2)$
 $C_k \sim N(0, \sigma_c^2)$
 $\epsilon_{ijkm} \sim N(0, \sigma_e^2)$

```
m4f <- lmer(yield ~ Rep + (1|Rep:Row) + (1|Rep:Col) + Geno,
            data = rowcol2$Book)
```

Least squares Means table:

	Rep	Geno	Estimate	Standard Error	DF	t-value	Lower CI	Upper CI	p-value	
Geno 1	NA	1	2.379	0.136	140	17.6	2.11	2.65	<2e-16	***
Geno 2	NA	63	2.379	0.135	140	17.6	2.11	2.65	<2e-16	***
Geno 3	NA	74	2.355	0.134	139	17.5	2.09	2.62	<2e-16	***
Geno 4	NA	85	2.241	0.135	139	16.6	1.97	2.51	<2e-16	***
Geno 5	NA	96	2.632	0.135	140	19.5	2.36	2.90	<2e-16	***
Geno 6	NA	107	2.312	0.136	140	17.0	2.04	2.58	<2e-16	***
Geno 7	NA	118	2.347	0.135	140	17.4	2.08	2.61	<2e-16	***
Geno 8	NA	129	2.382	0.133	139	17.9	2.12	2.65	<2e-16	***
Geno 9	NA	140	2.218	0.135	139	16.5	1.95	2.48	<2e-16	***
Geno 10	NA	2	2.484	0.135	139	18.4	2.22	2.75	<2e-16	***

- ❑ Adjusted means (≠ simple average over two reps).
- ❑ Standard errors lower than with α design and row-column.
- ❑ Average standard error of difference = 0.178

$$\rightarrow LSD = 2 \times 0.178 = 0.356 \text{ ton/ha}$$

Standard error in a resolvable row column design

- It might differ between pairs
- SE depends on row, column, and error variance and which genotypes occur together in rows and columns.

```
> diff_resrowcol$contrasts
contrast      estimate      SE
1 - 2         0.000383 0.165
1 - 3         0.024864 0.181
1 - 4         0.138576 0.183
1 - 5        -0.252565 0.178
1 - 6         0.067122 0.181
1 - 7         0.032708 0.175
1 - 8        -0.002491 0.182
1 - 9         0.161730 0.181
1 - 10        -0.104665 0.176
1 - 11        -0.259703 0.180
1 - 12        -0.073505 0.180
1 - 13        -0.215201 0.177
1 - 14        -0.125324 0.183
1 - 15        -0.006636 0.182
1 - 16        -0.148138 0.180
1 - 17        -0.241101 0.176
1 - 18         0.178004 0.182
1 - 19         0.049960 0.175
1 - 20        -0.098286 0.183
```


Broad sense heritability

$$H^2 = \frac{\sigma_g^2}{\sigma_g^2 + \frac{\sigma_\epsilon^2}{n}}$$

<i>RCBD</i>	<i>Alpha design</i>
$H^2 \cong \frac{0.01946}{0.01946 + \frac{0.08233}{2}} = 0.32$	$H^2 \cong \frac{0.02508}{0.02508 + \frac{0.03840}{2}} = 0.56$
<i>Row-Col (non-resolvable)</i>	<i>Row-Col (resolvable)</i>
$H^2 \cong \frac{0.02096}{0.02096 + \frac{0.03217}{2}} = 0.57$	$H^2 \cong \frac{0.02652}{0.02652 + \frac{0.02464}{2}} = 0.68$

Variance components estimated for models with genotypes as random

Least Significant Difference (LSD) – precision experiments

RCBD $\rightarrow LSD = 2 * 0.287 = 0.574 \text{ ton/ha}$

Alpha design $\rightarrow LSD = 2 \times 0.206 = 0.412 \text{ ton/ha}$

Row-Col (non-resolvable) $\rightarrow LSD = 2 \times 0.192 = 0.384 \text{ ton/ha}$

Row-Col (resolvable) $\rightarrow LSD = 2 \times 0.178 = 0.356 \text{ ton/ha}$

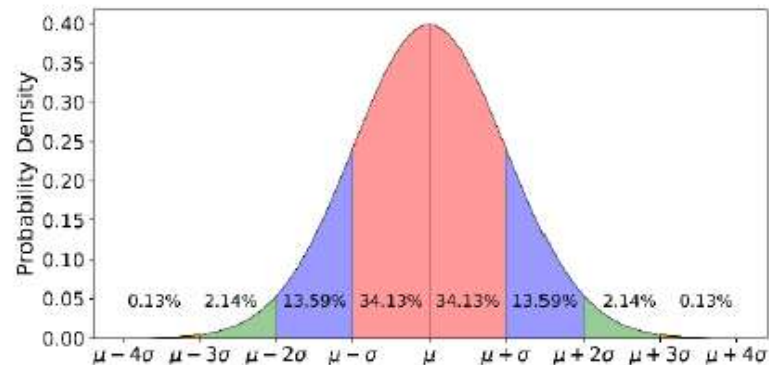
Estimated for models with genotypes fixed

Shrinkage (genotypes fixed or random?)

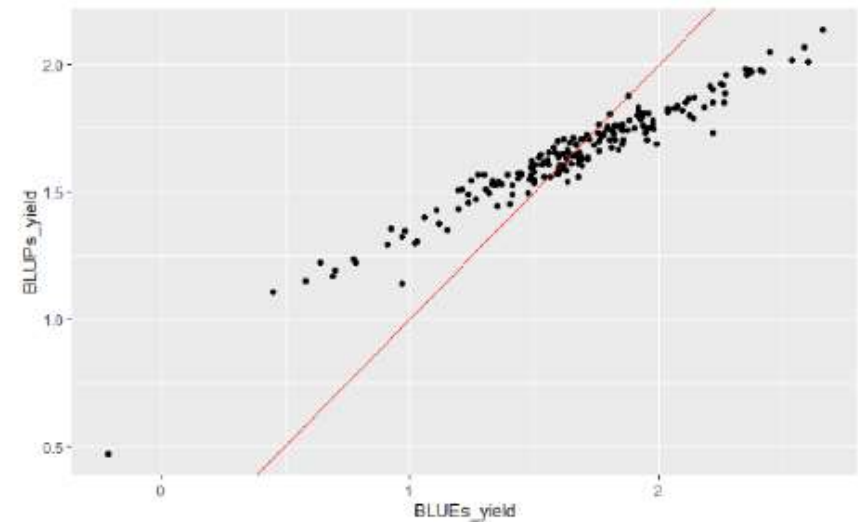
Shrinkage: Random effects are pulled towards the mean.

Extreme values are less likely to occur

$$\hat{g}_i = \frac{\sigma_g^2}{(\sigma_g^2 + \sigma_\epsilon^2/n)} (\bar{y}_i - \bar{y}_{..})$$



Example: wheat data (next practical)



```
> # calculate the range for BLUES
> range(WH09BBSBLUES_yield)
[1] -0.2129382 2.6567850
>
>
> # calculate the range for BLUPS
> range(WH09BBSBLUPS_yield)
[1] 0.4710533 2.1358297
```


Experimental design and analysis

- Good experimental design that helps to control extraneous variation.
 - Soil variability, operator/equipment effects, etc
- Precise data collection/phenotyping.
 - Trait definition (units, measurement procedure, etc)
- Advanced data analysis methods (simple ANOVA models far **too restrictive**)
 - Mixed models
 - Spatial models

Takeways

- With large trials we need to account for local variability
 - Smaller blocks
 - Row / column effects
- Also true in greenhouses and platforms!
- Clear design = clear analysis
- Useful designs are to separate spatial field variation from genotypic variation:
 - Alpha designs (genotypes assigned to incomplete blocks that form full replicates)
 - Row-column designs (blocking in the row and column directions)
- Design + advanced analyses leads to better/more accurate genotypic evaluation (e.g. higher Heritability)



Thank you.