



# (HOS 6932)– Survey of Breeding Tools and Methods

## An introduction to Quantitative Genetics

**Felipe Ferrão**

Research Assistant Scientist

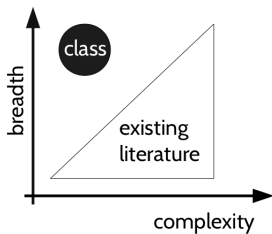
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January, 2023

# Introduction

## Main objectives

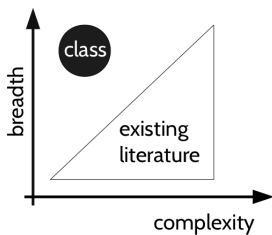


- This module address the practical implementation of genomic selection .
- What you can expect:
  - ▷ Understand general principles;
  - ▷ Keep mathematics to a minimum and focus instead on the intuition;
  - ▷ Fit a RR-BLUP predictive model

- What I expect from you:
  - ▷ Basic knowledge of statistics, genetics and breeding;
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# Introduction

## General Structure

- Genomic Selection can be presented under different perspectives
- Multiple courses at UF
- Quantitative Genetics: population genetics, resemblance between relatives, pedigree analysis (BLUP), GBLUP
- Statistical Learning: normal distribution, regression (linear model), regularization

# Introduction

*Let's grab some coffee and discuss methods and techniques used in plant breeding!!*



University of Florida

## Survey of Breeding Tools and Methods

(Genomic Selection)



<https://lfelipe-ferrao.github.io/teaching/>

# Quantitative Variation

## Definition















Quantitative genetics provides means for estimating the genetic architecture and predicting the evolutionary potential of complex traits.

- What is genetic architecture?
- Why we have complex and simple traits?
- How to study a complex trait?

# Quantitative Variation

## Background

- All traits measured by Mendel are very simplistic.
- Phenotypes were assumed to be completely determined by the genotype
- Discrete classes, with variation corresponding to single locus with two alleles

Pea trait	Dominant trait		Recessive trait		Numbers in second generation (F <sub>2</sub> )	Ratio
<b>Seeds</b>						
Seed shape	Round		Wrinkled		5474:1850	2.96:1
Seed colour	Yellow		Green		6002:2001	2.99:1
<b>Whole plants</b>						
Flower colour	Purple		White		705:224	3.15:1
Flower position	Axial		Terminal		651:207	3.14:1
Plant height	Tall		Short		787:277	2.84:1
Pod shape	Inflated		Constricted		882:299	2.95:1
Pod colour	Green		Yellow		428:152	2.82:1

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# Quantitative Variation

## Question

- How many traits in your crop do you know that follow this discrete pattern?
- Is this type of genetic architecture a general rule or an exception?



# Quantitative Variation

## Qualitative Traits

- Mendelian trait
- Fall into discrete categories
- One or few genes
- Example: Mendel's garden peas, color, insect and disease resistance

## Quantitative Genetics

- Continuous phenotype
- Greatly influenced by environment
- Joint action of multiple genes (or QTL) – Infinitesimal Model
- Example: yield, height and weight

Nature of quantitative traits has two important aspects

- Phenotype is a function of genotype and the environment
- Continuous traits usually follow a normal distribution, that can be fully described with only two parameters: mean and variance.

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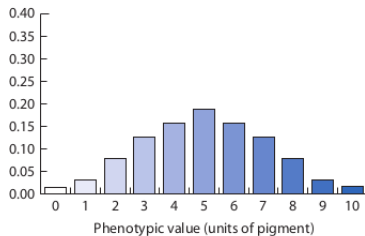
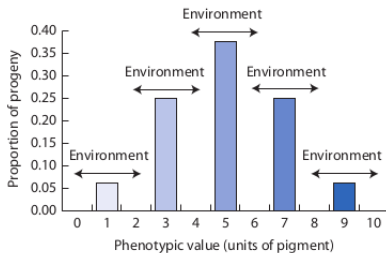
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# Quantitative Variation

## The environmental factor

- Ex: phenotypic distribution determined by two independent Mendelian loci.
- Environment can "increase" or "decrease" the phenotypic expression
- Even if there is only a single genotype, the phenotype expressed will change depending on the environmental conditions

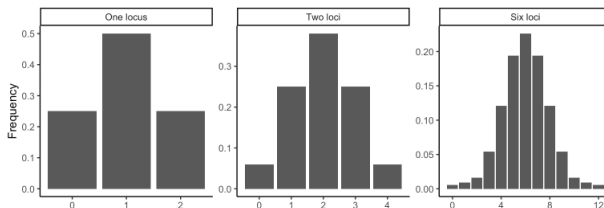


# Quantitative Variation

## Normal Distribution

### Infinitesimal Model

Fisher in 1918 showed that a large number of Mendelian factors (genes) influencing a trait would cause a nearly continuous distribution of trait values. Therefore, mendelian genetics can lead to an approximately normal distribution



# Quantitative Variation

- For a trait controlled by many genes and influenced by the environment, the measured character for any trait on an individual is called **phenotypic value**
- Formally, we can divide the phenotypic value

$$P = G + E$$

## Environment (E)

- Include all non-genetic effects (systematic and non-systematic)
- In plant breeding:  $G \times E$  is also important

## Genotype (G)

- The particular set of genes in a given individuals
- Can be decomposed in additional terms

$$V_P = V_G + V_E$$

$$V_P = V_A + V_D + V_I + V_E$$

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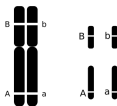
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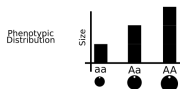
# Quantitative Variation

## Gene action

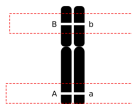
### Additive



- \* Cumulative phenotypic effects of alleles
- \* Phenotypic effect of each allele can be added

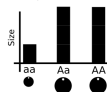


### Dominance

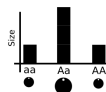


- \* Depends on the combination of alleles within a locus
- \* Very important in hybrids
- \* Different levels

#### Complete Dominance



#### Overdominance



### Epistasis

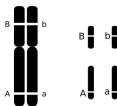


- \* Combination of genotypes at two or more loci
- \* can be thought of as the "leftover" part of genotypic variance
- \* Different levels:
  - \*\* add-by-add
  - \*\* add-by-dom
  - \*\* dom-by-dom

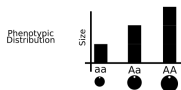
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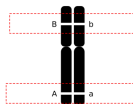
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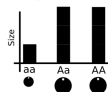


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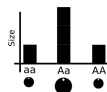


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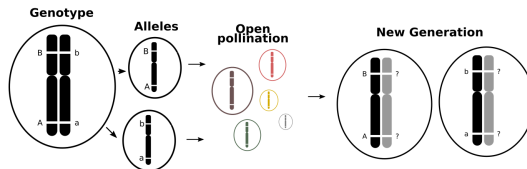


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**Why are these concepts important?**





## Connecting the dots



- Differences between quantitative and qualitative traits
- Environment has an important impact on the phenotype expression
- Our unity of study is a population, with a normal distribution and mean and variance

What are we missing?

We need to define the means to study a complex trait

## Connecting the dots



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What are we missing?

We need to define the means to study a complex trait

# Regression

CAN WE PREDICT A STUDENT'S WEIGHT  $y$  FROM HIS OR HER HEIGHT  $x$ ?

## Regression analysis

FITS A STRAIGHT LINE TO THIS MESSY SCATTERPLOT.  $x$  IS CALLED THE INDEPENDENT OR PREDICTOR VARIABLE, AND  $y$  IS THE DEPENDENT OR RESPONSE VARIABLE. THE REGRESSION OR PREDICTION LINE HAS THE FORM

$$y = a + bx$$



# Regression

## Background

- Depending on the causal connections between two variables, their true relationship may be linear and can be described using a linear regression
- Examples
  - ▷ How gender ( $x$ ) is associated with salary ( $y$ ) income?
  - ▷ How fertilization ( $x$ ) is associated to yield ( $y$ ) in corn?
  - ▷ How the phenotypic value ( $x$ ) is associated to the gene content ( $y$ )?

# Regression

**We can write such questions using a model**

$$Y_i = \beta_0 + \beta_1 X_i + e$$

- The terms  $\beta_0$  and  $\beta_1$  are the intercept and slope of the model, respectively.
- Intercept is the point at which the line crosses the y axis at  $x = 0$ .
- The slope expresses the relationship between y and x.

## Estimation

- $\beta_0$  and  $\beta_1$  are two parameters estimated from the data
- Ordinary Least Squares (OLS) to estimate  $\hat{\beta}_0$  and  $\hat{\beta}_1$

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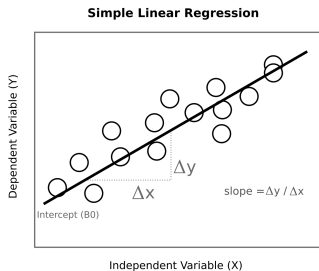
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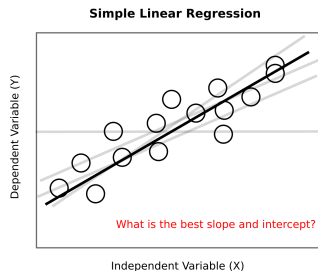
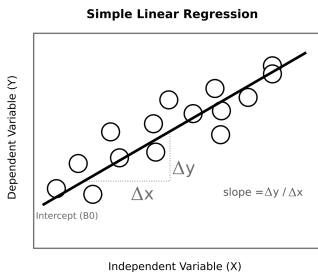
# Regression

## Geometric Representation



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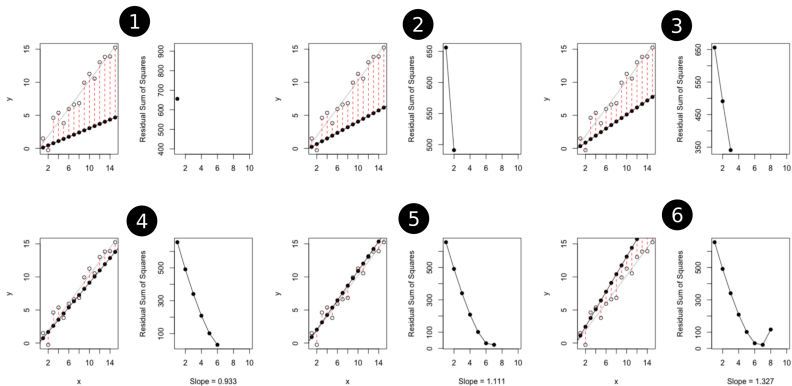




# Regression

## OLS in action

- Straight lines can be drawn in multiple ways – different slopes and intercepts.
- What is the best fit? Minimizes the error between the original and predicted values



# Regression

## Least-Squares Linear Regression

- We seek estimators for the intercept and slope that minimize the residual
- First, we need to define the errors
- Differentiate with respect to  $\beta_0$  and  $\beta_1$  and set the results equal to zero

$$y_i = \beta_0 + \beta_1 x_i + e$$

$$\hat{e}'\hat{e} = y_i - \hat{y}$$

$$\hat{e} = \sum \hat{e}_i^2 = \sum (y_i - \hat{y}_i)^2 = \sum (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)^2$$

$$\frac{\partial \hat{e}'\hat{e}}{\partial \hat{\beta}_0} = -2 \sum (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i) = 0$$

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### Simple Linear Regression Estimators

- Slope:  $\hat{\beta}_1 = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$
- Intercept:  $\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$

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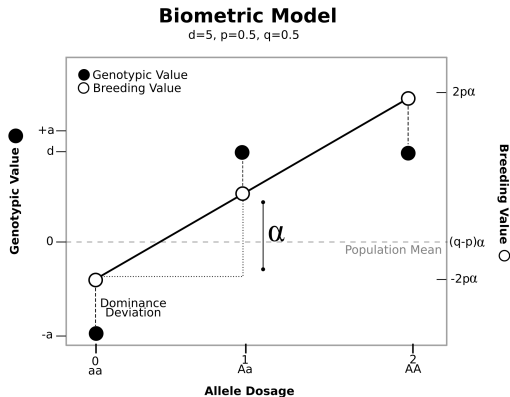
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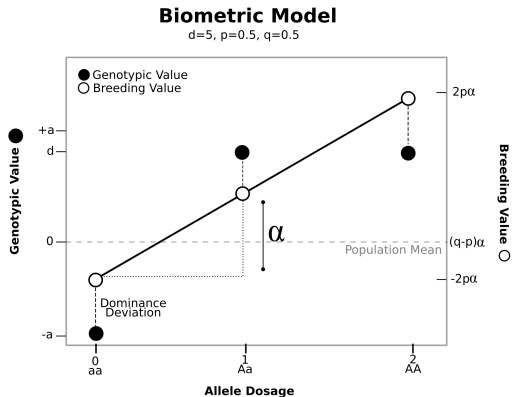
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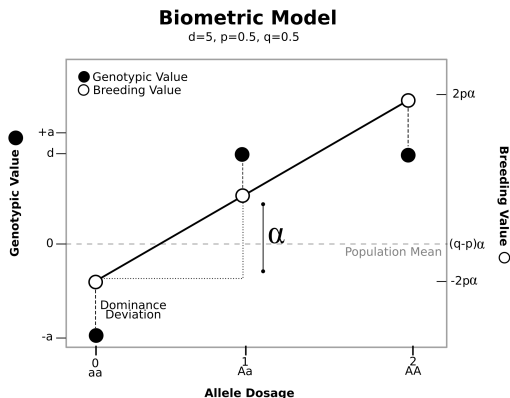
- $\hat{\beta}_1 = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sum(x_i - \bar{x})^2} = \alpha$  (average effect of allelic substitution)
- $\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} = -2p\alpha$  (breeding value for aa)

# Regression



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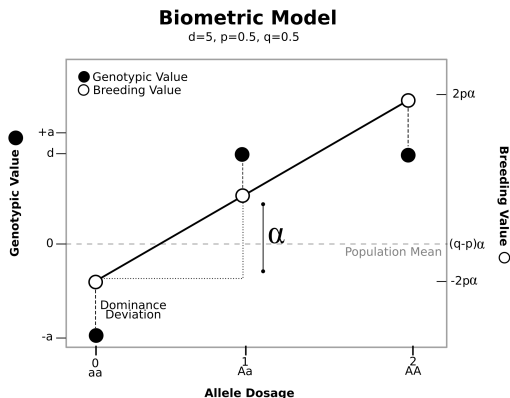
# Regression



## Partition the Genetic Variance

- $SS_{Total} = SS_{Regression} + SS_{deviation}$
- $SS_{Total} = \sum f_i y_i^2 = p^2(y_1)^2 + 2pq(y_2)^2 + q^2(y_3) = \sigma_g^2$
- $SS_{Reg} = \hat{\beta}_1 \sum f_i x_i y_i = 2pq\alpha^2 = \sigma_a^2$
- $SS_{De} = \sum f_i \hat{e}_i^2 = (2pqd)^2 = \sigma_d^2$

# Regression



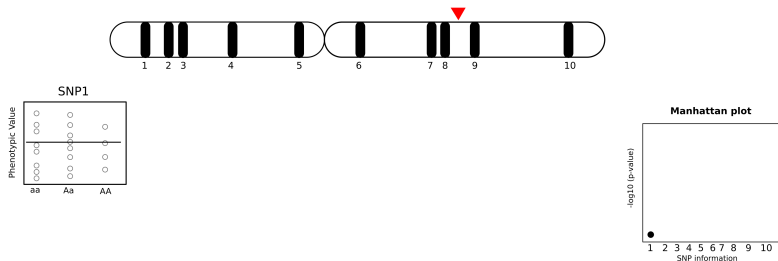
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# Regression

## Including Molecular Markers

- 10 markers (allele dosage) and a single QTL (red)
- 20 individuals measured for a given phenotypic trait (ex: yield)
- Regression Model:  $y \sim f(\text{marker})$  and testing  $H_0 : \beta_1 = 0$

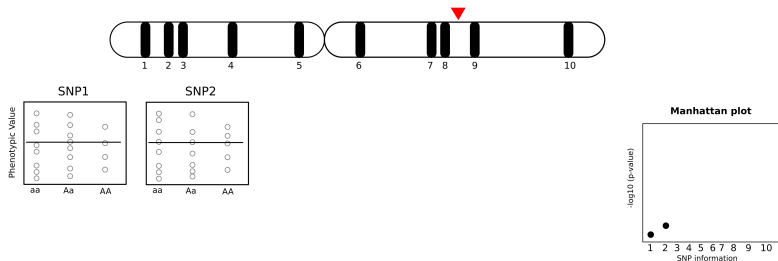




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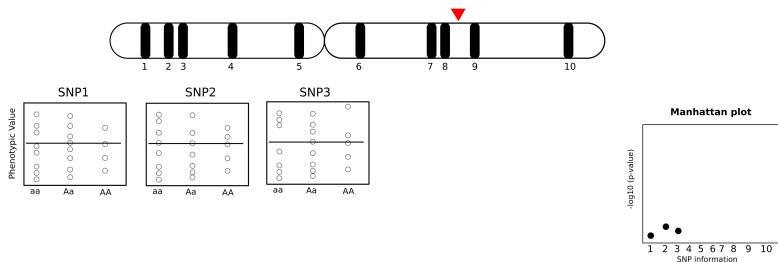
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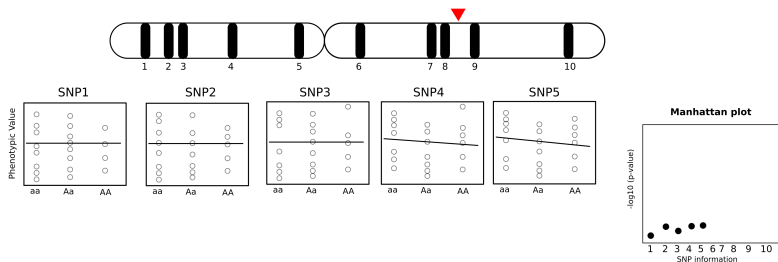
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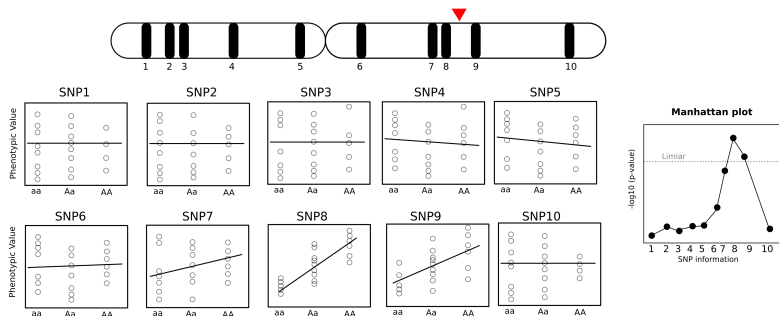
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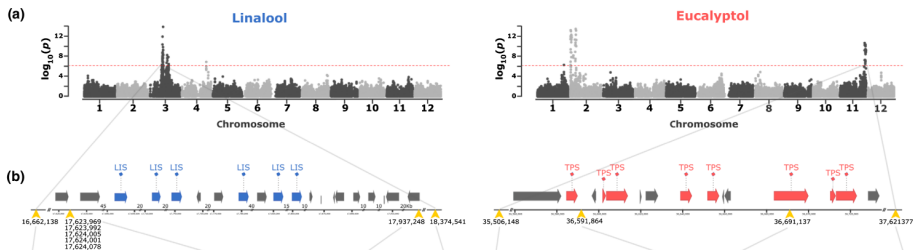
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# Regression

## Genome-wide association of volatiles reveals candidate loci for blueberry flavor



Ferrão et al., 2020. New Phytologist doi: 10.1111/nph.16459

# Regression

## Including Molecular Markers

- Single Marker Regression
- Version of the Biometric Model: we can compute genetic parameters
- Theoretical basis for GWAS models and QTL mapping
- Precursor of genomic selection methods
- Problems:
  - ▷ Testing millions of markers, one at a time, inflate type I error
  - ▷ Lack of power: small effect can rarely be detected
  - ▷ Beavis (or winner's curse) effect: noises will occur in analysis with many markers, and this biases the estimates, making it look much larger than real

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# Final Considerations

## Summary

- Differences between qualitative and quantitative traits
- How to compute means and variance using genetic information
- Key concepts: additive, dominance, epistasis, additive variance, dominance variance and average effect of allelic substitution
- First marker-assisted selection model !!

## Next class

- Why use one marker at a time in the regression analyses?
- What happens if we use all markers simultaneously?



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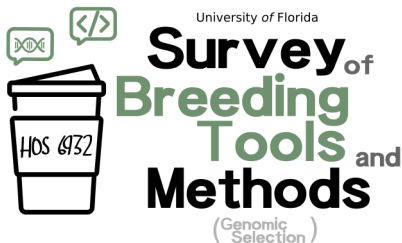
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## Hands-on 1



<https://lfelipe-ferrao.github.io/teaching/>

# References

## References



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Book – *Intermediate knowledge, more focuses in breeding*



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Falconer and Mackay, 1996. Introduction to Quantitative Genetics

Book – *Main reference in the field, advanced knowledge*



Cruz, 2005. Princípios de Genética Quantitativa.

Book – *Portuguese version*