Velocity Triangles

- You need to draw velocity triangles over a turbine rotor and compressor rotor.
- Be careful with signs and indices of stator and rotor.

Nomenclature

Stator exit angle	α_1
Rotor inlet angle	β_1
Rotor outlet angle	β_2
Rel. rotor inlet velocity	W_1
Rel. rotor outlet velocity	W_2
Absolute frame of reference:	
Velocity	C
Angle	α
Relative frame of reference:	
Velocity	W
Angle	β

Calculations

Rotational speed	$U = \omega \cdot r = 2\pi \cdot f \cdot r$
Mass flow	$\dot{m} = \rho \cdot A \cdot C_x$
Geometry	$C_{\theta 1} = C_x \tan(\alpha_1)$
	$C_{\theta 1} = r\omega - C_x \tan(\beta_2)$
	$\Delta h_T = U \cdot \Delta C_\theta = U (C_{\theta 2} - C_{\theta 1})$
Euler Work	$\Delta W = \frac{\text{Power}}{\dot{m}} = \Delta h_T = U \cdot \Delta C_\theta$

Thermodynamics

1st Law of Thermodynamics	$du = dq - dw$
	$\dot{W} - \dot{Q} = \dot{m} \left(h_1 - h_2 + \frac{v_1^2 - v_2^2}{\rho} + g(z_1 - z_2) \right)$
TDS Equation 1	$Tds = du + pdv$
TDS Equation 2	$Tds = dh - vdp$
Enthalpy	$h = u + pv$
	$dh = du + pdv + vdp$

Ideal Gas

Ideal Gas Equation	$pv = RT$
	$p = \rho RT$
Constant Volume	$c_v = (\partial u / \partial T)_v$
	$du = c_v(T)dT$
Constant Pressure	$c_p = (\partial h / \partial T)_p$
	$dh = c_p(T)dT$
Thermal coefficients	$R = c_p - c_v$
	$\gamma = k = \frac{c_p}{c_v}$
Specific Enthalpy of Gases	$h = u + pv = u(T) + RT = h(T)$
	$\Rightarrow$ does not depend on pressure

Ideal Gas – Isentropic Conditions

$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$ $\qquad$ $\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1}$

$\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{\gamma}$

Personally, I struggle with the above formulae. They are, however very easy to derive. You only need to know two things:
$\mathbf{pv} = \mathbf{RT}$ and $\mathbf{p_1 v_1^\gamma} = \mathbf{p_2 v_2^\gamma}$, the first is the ideal gas law and the second follows from the isentropic condition $p \cdot v^\gamma = \text{const}$.
From this we get:

$$\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{\gamma} \Rightarrow \frac{v_1}{v_2} = \left(\frac{p_2}{p_1}\right)^{1/\gamma}$$

$$\frac{T_2}{T_1} = \underbrace{\frac{p_2 v_2}{p_1 v_1}}_{\text{ideal gas law}} = \underbrace{\left(\frac{v_1}{v_2}\right)^{\gamma}}_{\text{from above}} \cdot \frac{v_2}{v_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1}$$

$$\frac{T_2}{T_1} = \underbrace{\frac{p_2 v_2}{p_1 v_1}}_{\text{ideal gas law}} = \frac{p_2}{p_1} \cdot \underbrace{\left(\frac{p_2}{p_1}\right)^{-1/\gamma}}_{\text{from above}} = \left(\frac{p_2}{p_1}\right)^{1-\frac{1}{\gamma}} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

Turbines, Compressors, Refrigerator and Heat Pump

Isentr. turbine efficiency	$\eta_{\text{turbine}} = \frac{h_{in}-h_{out}}{h_{in}-h_{out,s}} = \frac{\text{real work}}{\text{ideal work}}$
Isentr. compressor efficiency	$\eta_{\text{compressor}} = \frac{h_{out,s}-h_{in}}{h_{out}-h_{in}} = \frac{\text{ideal work}}{\text{real work}}$
Thermal efficiency	$\eta_{\text{thermal}} = \frac{W_{\text{out, net}}}{Q_{\text{in}}}$
Ideal pump work	$\Delta h = v \cdot \Delta p$
	Careful: p in Pa, not bar!
Isentropic Compression	$h_2 = h_1 + v_1 (p_2 - p_1)$
Heat pump "Leistungsziffer"	$\varepsilon_{HP} = \frac{\dot{Q}_{out}}{W_{komp}} \left(= \frac{h_2-h_3}{h_2-h_1}\right)$
Refrigerator "Leistungsziffer"	$\varepsilon_{KM} = \frac{\dot{Q}_{in}}{W_{komp}} \left(= \frac{h_1-h_4}{h_2-h_1}\right)$

General

You should know:

- The t-s diagrams by heart and be able to draw it.
- How to draw the p-v diagrams of the Otto and Diesel Cycle.
- How the Brayton Cycle works.
- How the heat pump and refrigerator cycles work.
- Ottoprozess: Gleichraumprozess (isochor), Diesel: Gleichdruckprozess (isobar).