

## Velocity Triangles

- You need to draw velocity triangles over a turbine rotor and compressor rotor.
- Be careful with signs and indices of stator and rotor.

## Nomenclature

Stator exit angle	$\alpha_1$
Rotor inlet angle	$\beta_1$
Rotor outlet angle	$\beta_2$
Rel. rotor inlet velocity	$W_1$
Rel. rotor outlet velocity	$W_2$
<i>Absolute frame of reference:</i>	
Velocity	$C$
Angle	$\alpha$
<i>Relative frame of reference:</i>	
Velocity	$W$
Angle	$\beta$

## Calculations

Rotational speed	$U = \omega \cdot r = 2\pi \cdot f \cdot r$
Mass flow	$\dot{m} = \rho \cdot A \cdot C_x$
Geometry	$C_{\theta 1} = C_x \tan(\alpha_1)$
	$C_{\theta 1} = r\omega - C_x \tan(\beta_2)$
	$\Delta h_T = U \cdot \Delta C_\theta = U (C_{\theta 2} - C_{\theta 1})$
Euler Work	$\Delta W = \frac{\text{Power}}{\dot{m}} = \Delta h_T = \dot{m} \cdot U \cdot \Delta C_\theta$

## Thermodynamics

1st Law of Thermodynamics

$$du = dq - dw$$

TDS Equation 1

$$\dot{W} - \dot{Q} = \dot{m} \left( h_1 - h_2 + \frac{v_1^2 - v_2^2}{2} + g(z_1 - z_2) \right)$$

TDS Equation 2

$$Tds = du + pdv$$

Enthalpy

$$Tds = dh - vdp$$

$$h = u + pv$$

$$dh = du + pdv + vdp$$

## Ideal Gas

Ideal Gas Equation

$$pv = RT$$

Constant Volume

$$p = \rho RT$$

$$c_v = (\partial u / \partial T)_v$$

$$du = c_v(T) dT$$

Constant Pressure

$$c_p = (\partial h / \partial T)_p$$

$$dh = c_p(T) dT$$

Thermal coefficients

$$R = c_p - c_v$$

$$\gamma = k = \frac{c_p}{c_v}$$

Specific Enthalpy of Gases

$$h = u + pv = u(T) + RT = h(T)$$

$\Rightarrow$  does not depend on pressure

## Ideal Gas – Isentropic Conditions

$$\frac{T_2}{T_1} = \left( \frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{p_2}{p_1} = \left( \frac{v_1}{v_2} \right)^{\gamma}$$

$$\frac{T_2}{T_1} = \left( \frac{v_1}{v_2} \right)^{\gamma-1}$$

Personally, I struggle with the above formulae. They are, however very easy to derive. You only need to know two things:

$\mathbf{p}\mathbf{v} = \mathbf{R}\mathbf{T}$  and  $\mathbf{p}_1\mathbf{v}_1^\gamma = \mathbf{p}_2\mathbf{v}_2^\gamma$ , the first is the ideal gas law and the second follows from the isentropic condition  $p \cdot v^\gamma = \text{const.}$

From this we get:

$$\frac{p_2}{p_1} = \left( \frac{v_1}{v_2} \right)^{\gamma} \Rightarrow \frac{v_1}{v_2} = \left( \frac{p_2}{p_1} \right)^{1/\gamma}$$

$$\frac{T_2}{T_1} = \underbrace{\frac{p_2 v_2}{p_1 v_1}}_{\text{ideal gas law}} = \underbrace{\left(\frac{v_1}{v_2}\right)^\gamma}_{\text{from above}} \cdot \frac{v_2}{v_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1}$$

$$\frac{T_2}{T_1} = \underbrace{\frac{p_2 v_2}{p_1 v_1}}_{\text{ideal gas law}} = \frac{p_2}{p_1} \cdot \underbrace{\left(\frac{p_2}{p_1}\right)^{-1/\gamma}}_{\text{from above}} = \left(\frac{p_2}{p_1}\right)^{1-\frac{1}{\gamma}} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

## Turbines, Compressors, Refrigerator and Heat Pump

Isentr. turbine efficiency	$\eta_{\text{turbine}} = \frac{h_{in}-h_{out}}{h_{in}-h_{out,s}} = \frac{\text{real work}}{\text{ideal work}}$
Isentr. compressor efficiency	$\eta_{\text{compressor}} = \frac{h_{out,s}-h_{in}}{h_{out}-h_{in}} = \frac{\text{ideal work}}{\text{real work}}$
Thermal efficiency	$\eta_{\text{thermal}} = \frac{W_{\text{out, net}}}{Q_{\text{in}}}$
Ideal pump work	$\Delta h = v \cdot \Delta p$
	<b>Careful:</b> $p$ in Pa, not bar!
Isentropic Compression	$h_2 = h_1 + v_1 (p_2 - p_1)$
Heat pump "Leistungsziffer"	$\varepsilon_{HP} = \frac{\dot{Q}_{out}}{W_{komp}} \left( = \frac{h_2-h_3}{h_2-h_1} \right)$
Refrigerator "Leistungsziffer"	$\varepsilon_{KM} = \frac{\dot{Q}_{in}}{W_{komp}} \left( = \frac{h_1-h_4}{h_2-h_1} \right)$

## General

You should know:

- The t-s diagrams by heart and be able to draw it.
- How to draw the p-v diagrams of the Otto and Diesel Cycle.
- How the Brayton Cycle works.
- How the heat pump and refrigerator cycles work.
- Ottoprozess: Gleichraumprozess (isochor), Diesel: Gleichdruckprozess (isobar).