laser_ode_entropy_analytic_approx

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Approximation of Entropy based on Gaussian Distribution

- author: Longfei Fan
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In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
import seaborn as sns

from scipy.stats import poisson
from scipy.special import erfc

from qutip import *
import laser, entropy_utils

%matplotlib inline
%reload_ext autoreload
%autoreload 1
%aimport laser, entropy_utils
```

In [2]:
```
from IPython.display import set_matplotlib_formats
set_matplotlib_formats('pdf', 'png')
```

helpers

In [3]:
```
def plot_n_vs_t(filename, xlim, ylim):
    n1_df = pd.read_csv(filename)
    entropy_utils.df_plot(n1_df, xlim=xlim, ylim=ylim, \
                          style = ['-', '-.', ':', '--'], \
                          xlabel=r'$gt$', ylabel=r'$\bar{n}$ (mean photon number)')
    plt.title(r'Evolution of $\bar{n}$ under Different $A/C$', fontsize=14);


def plot_entr_vs_t(filename, xlim, ylim):
    entr1_df = pd.read_csv(filename)
    entropy_utils.df_plot(entr1_df, xlim=xlim, ylim=ylim, \
```

```
                                  xlabel=r'$gt$', ylabel=r'$S$ (entropy)', \
                                  style = ['-', '-.', ':', '--'], \
                                  entr_cohe=ENTR_COHE, entr_thml=False)
            plt.title(r'Evolution of $S$ under Different $A/C$', fontsize=14)
```

```
In [4]: G = 0.001
        KAPPA = 0.0001

        NBAR = 200
        N_max = 1000
        n_list = np.arange(N_max)

        # vacuum
        vacu = fock(N_max, 0)

        init_psi = vacu
        solver = 'pn'
```

The entropy calculated given on the Poisson distribution with an average photon number of 200.

```
In [5]: pns_cohe = [poisson.pmf(n, NBAR) for n in n_list]
        ENTR_COHE = - sum([pn * np.log(pn) for pn in pns_cohe if pn > 0])
        print('ENTROPY COHERENT: {:.4f}'.format(ENTR_COHE))
```

```
ENTROPY COHERENT: 4.0677
```

0.0.1 Evolution of Photon Statistics

```
In [6]: def plot_photon_statistics_evolution(l, gts, title, x1, x2, y1, y2):
            """ plot photon statistics with respect to time
                l: laser object
                gts: time point to be plotted, [1, 100]
                title: figure title
                x1, x2: xlim
                y1, y2: ylim
            """

            lstyle = ['-', '--', ':', '-.', '-', '--']
            lwidth = [1, 2, 2, 2, 2, 2]
            fig, ax = plt.subplots(sharex=True, figsize=(12, 4))
            t_list = l.t_list
            pns_all = l.get_pns()

            for i in range(len(gts)):
                pns = pns_all[gts[i]]
                ax.plot(np.arange(N_max), pns, \
                        linestyle=lstyle[i], linewidth=lwidth[i], \
```

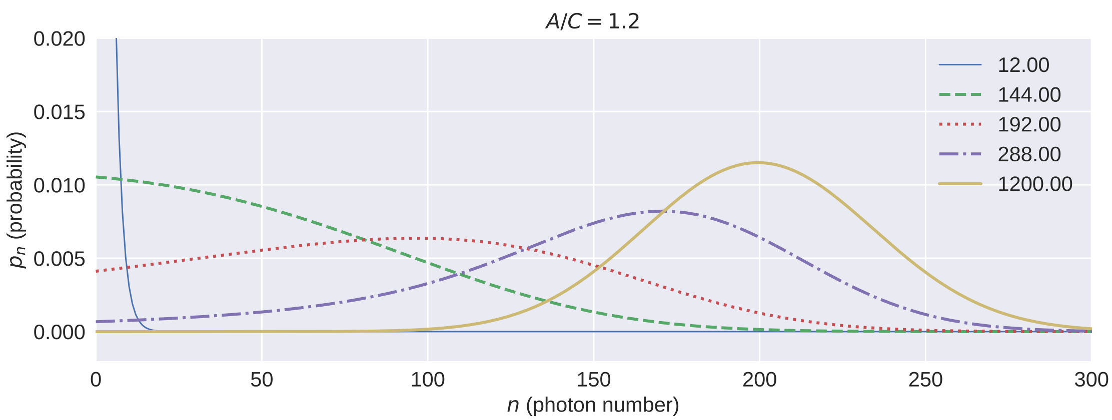
```
                         label='{:4.2f}'.format(t_list[gts[i]] * G))

        ax.set_xlim(x1, x2)
        ax.set_ylim(y1, y2)
        ax.set_xlabel(r'$n$ (photon number)', fontsize=14)
        ax.set_ylabel(r'$p_n$ (probability)', fontsize=14)
        ax.tick_params(labelsize=14)
        ax.legend(fontsize=14)
        plt.title(title, fontsize=14);
```

0.0.2 $A/C = 1.05, 1.2$

```
In [7]: laser_s = np.load('./data/vacu_200/200_vacu_l1.npz')['lasers'].flatten()[0]
        ls1 = laser_s['1.05']
        ls2 = laser_s['1.20']
```

```
In [52]: # plot_photon_statistics_evolution(ls1, (1, 15, 25, 50, 100), \
         #                                  r'$A/C=1.05$', 0, 300, -0.002, 0.02)
```

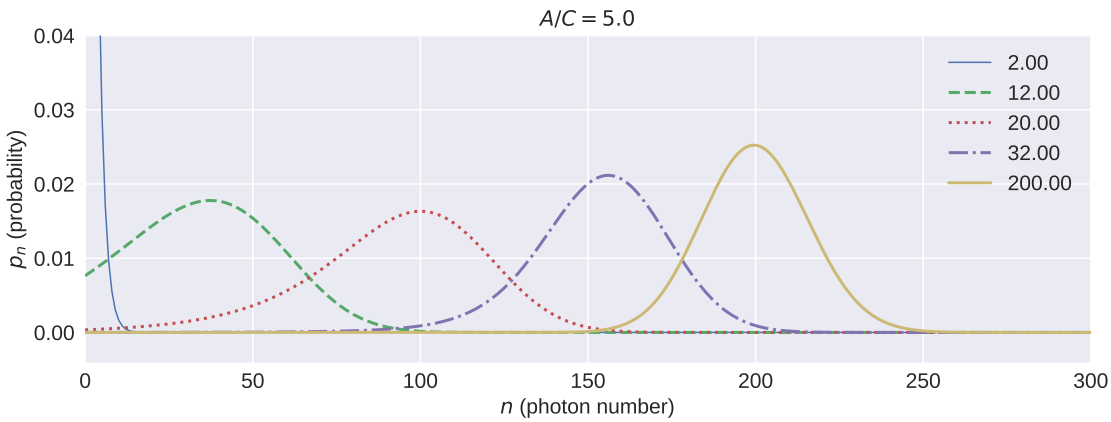
```
In [9]: plot_photon_statistics_evolution(ls2, (1, 12, 16, 24, 100), \
                                         r'$A/C=1.2$', 0, 300, -0.002, 0.02)
```



0.0.3 $A/C = 5$

```
In [10]: laser_m = np.load('./data/vacu_200/200_vacu_l2.npz')
         lm = laser_m['lasers'].flatten()[0]['5.00']
```

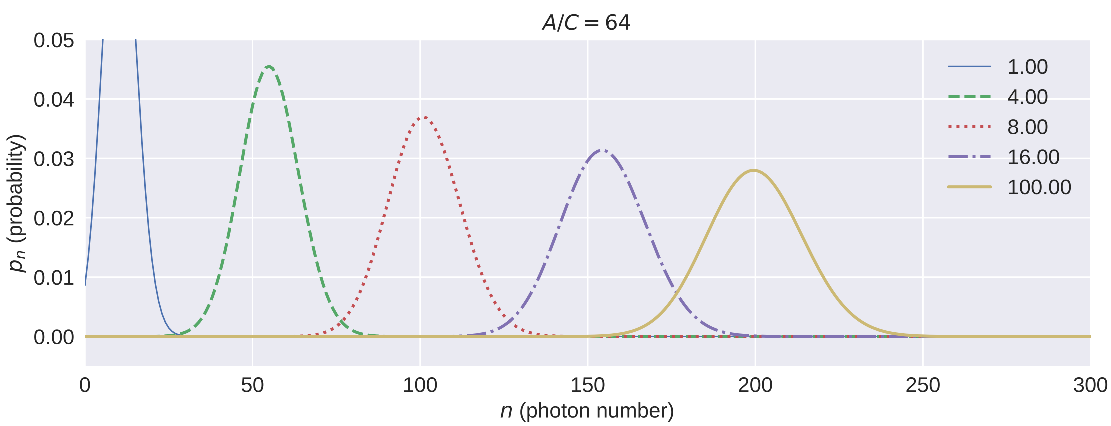
```
In [11]: plot_photon_statistics_evolution(lm, (1, 6, 10, 16, 100), \
                                          r'$A/C=5.0$', 0, 300, -0.004, 0.04)
```



0.0.4 $A/C = 64$

```
In [12]: laser_l = np.load('./data/vacu_200/200_vacu_l3.npz')
         ll = laser_l['lasers'].flatten()[0]['64.00']
```

```
In [13]: plot_photon_statistics_evolution(ll, (1, 4, 8, 16, 100), \
                                          r'$A/C=64$', 0, 300, -0.005, 0.05)
```



0.0.5 Variance and Entropy

```
In [14]: np.arange(10) / 0.5
```

```
Out[14]: array([  0.,   2.,   4.,   6.,   8.,  10.,  12.,  14.,  16.,  18.])
```

```
In [15]: def calc_var_n(l, start, end):
             pns_all = l.get_pns()[start:end]
             var_n_all = []
```

```
    for pns in pns_all:
        aver_n = sum([pns[i] * i for i in range(N_max)])
        aver_n2 = sum([pns[i] * i**2 for i in range(N_max)])
        var_n_all.append(aver_n2 - aver_n**2)
    return np.array(var_n_all)


from scipy.special import erfc
def calc_entr_vec(mean, varn):
    """ Calculate the vector of entropy
        given on the vector of the mean and the variance
    """
    result = np.log(np.sqrt(2.0 * np.pi * varn)) + 0.5
    result +=  np.log(0.5 * erfc(- mean / np.sqrt(2.0 * varn)))
    nn = - mean * np.exp(- mean**2 / 2.0 / varn)
    dd = np.sqrt(2.0 * np.pi * varn) * erfc(- mean / np.sqrt(2.0 * varn))
    result += nn / dd
    return result
```

In [45]:
```
def plot_varn_entr(l, title, ax, start, end, approx=False):
    varn = calc_var_n(l, start, end)
    mean = l.get_ns()[start:end]
    if approx:
        entr = np.log(np.sqrt(2.0 * np.pi) * np.sqrt(varn)) + 0.5
    else:
        entr = calc_entr_vec(mean, varn)
    t_list = l.get_tlist()[start:end] * l.g

    ln, = ax.plot(t_list, entr, linestyle='--', label='Gaussian entropy')
    ax.plot(t_list, l.get_entrs()[start:end], linestyle='-', label ='numeric entropy')
    ax.plot(t_list, entr - l.get_entrs()[start:end], label ='difference')
    ax.legend(fontsize=14, loc=4)
    ax.set_title("Entropy for A/C = " + title)

    return ln


def plot_varn_entr_all(lasers, start, end, approx=False):
    fig, ax_array = plt.subplots(2, 2, figsize=(12, 8), sharey=True)
    ax = np.ravel(ax_array)
    i = 0
    for ratio, l in sorted(lasers.items()):
        plot_varn_entr(l, ratio, ax[i], start, end, approx)
        i += 1
```

0.1 Approximation of entropy based on the mean and the variance of photon numbers

A Poisson distribution can be appproximates by a Gaussian distribution. For a Gaussain distribution with variance $\sigma^2 = A$, mean 0, and truancated at R, the entropy is given by

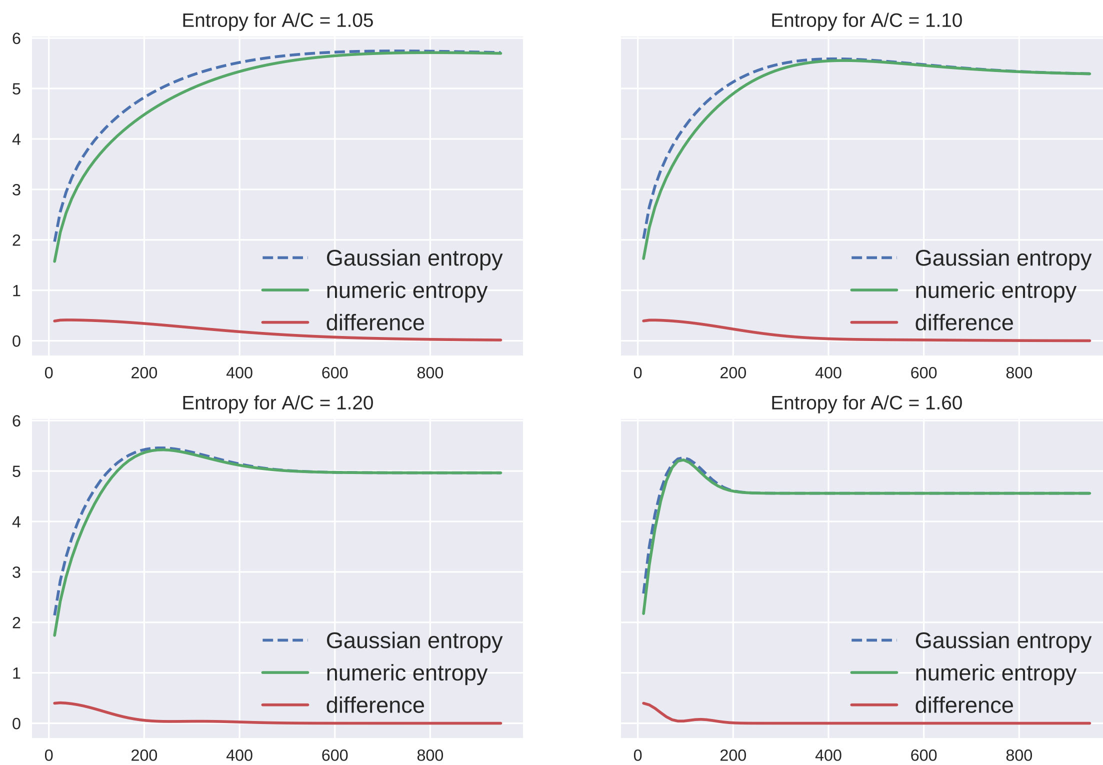
$$S = \log\sqrt{2\pi\sigma^2} + \frac{1}{2} + \log\left(\frac{1}{2}\mathrm{erfc}\frac{R}{\sqrt{2\sigma^2}}\right) + \frac{R\exp(-\frac{R^2}{2\sigma^2})}{\sqrt{2\pi\sigma^2}\ \mathrm{erfc}\frac{R}{\sigma^2}}$$

When the laser is at the steady state, $\sigma^2 \approx \frac{A^2}{BC}$, and $R = \frac{A}{B} - \frac{A^2}{BC}$. Before the laser reach the steady state, the truncated position R can be approximated by $-\bar{n}$. In the following figures, the mean $\bar{n}$ and variance σ^2 are calculated given on the photon statistics $p(n)$ numerically.

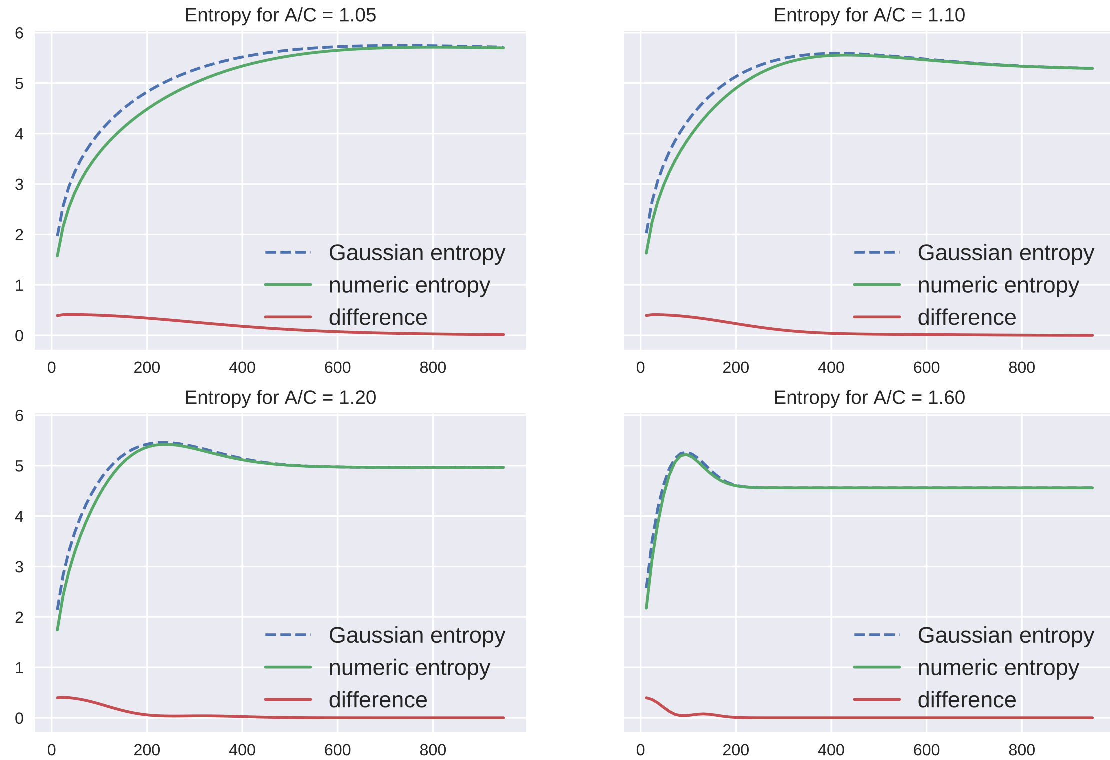
0.1.1 Approximation using variance (keep first two terms)

```
In [43]: laser_s = np.load('./data/vacu_200/200_vacu_l1.npz')['lasers'].flatten()[0]
         laser_m = np.load('./data/vacu_200/200_vacu_l1.npz')['lasers'].flatten()[0]
         laser_l = np.load('./data/vacu_200/200_vacu_l3.npz')['lasers'].flatten()[0]
```

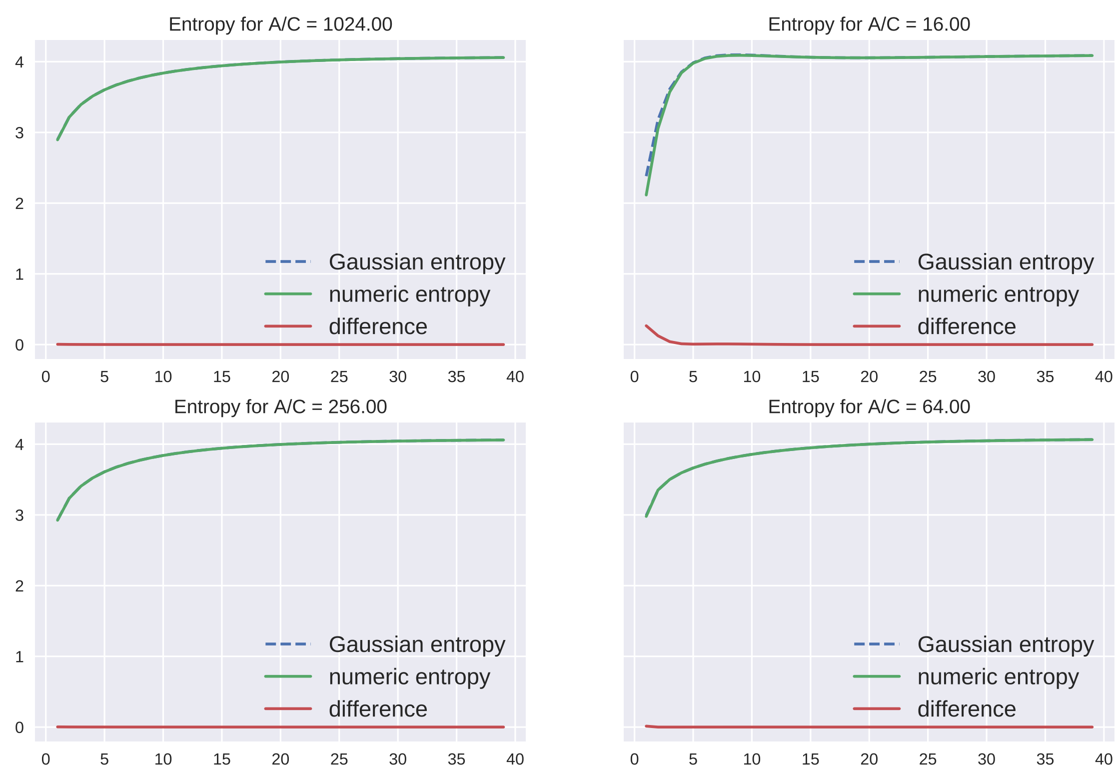
```
In [46]: plot_varn_entr_all(laser_s, 1, 80, True)
```




```
In [47]: plot_varn_entr_all(laser_m, 1, 80, True)
```

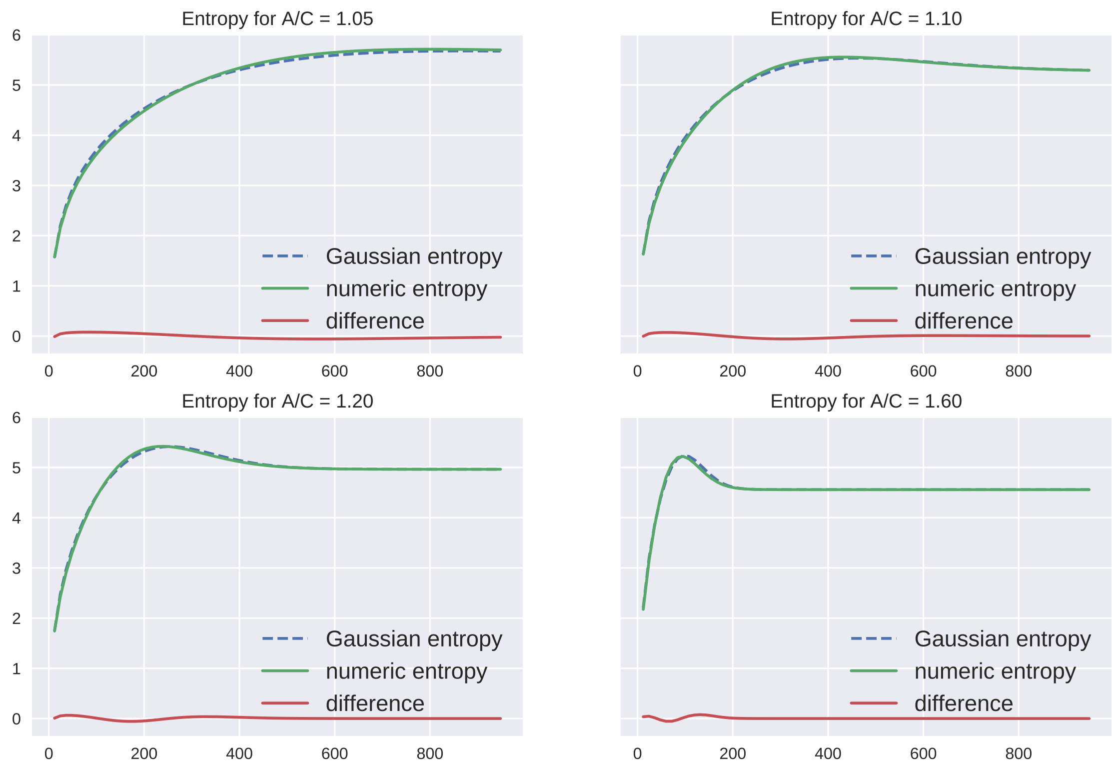



```
In [48]: plot_varn_entr_all(laser_l, 1, 40, True)
```

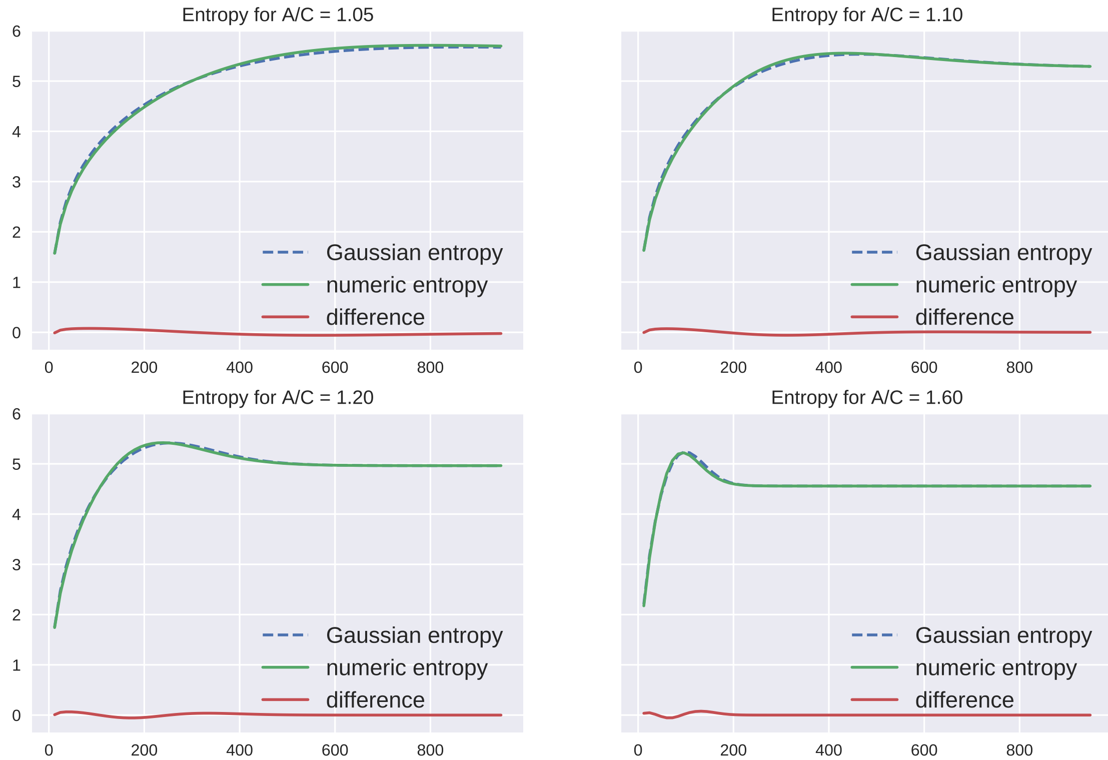


0.1.2 Approximation using variance and average

```
In [49]: plot_varn_entr_all(laser_s, 1, 80)
```




```
In [50]: plot_varn_entr_all(laser_m, 1, 80)
```




```
In [51]: plot_varn_entr_all(laser_l, 1, 40, True)
```