Image Analysis and Understanding

Representation and Description

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Note: Artwork from
Digital Image Processing, 3rd ed.
Gonzalez & Woods

Outline

- Representation
- Description
  - Boundary
  - Regional
  - PCA
  - Relational

Preamble

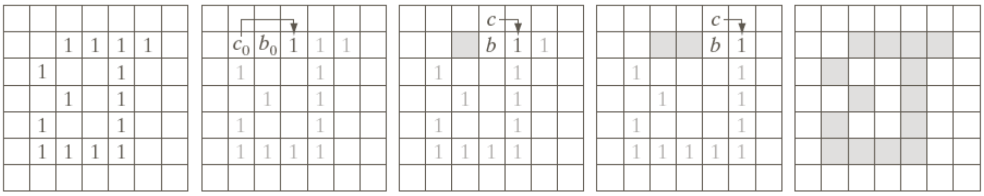
- Segmentation is a crucial but intermediate image processing phase
- What's the real meaning of the segmented regions?
- How to make them usefull for further computer processing?
- Shall we represent a region by
  - External characteristics
    - Boundary
  - Internal characteristics
    - Pixel aggregates

Representation versus Description

- Choosing a representation scheme is only a part of making segmentation data useful
- The next step consists on describing the region in manner consistent with the chosen representation
- A representation scheme should lead to a compact data description
- An external representation is chosen when the primary focus is on shape features
  - Length, orientation, etc ...
- An internal representation is chosen when the primary focus is on regional features
  - Intensity, color, texture, etc …
- Often hybrid representations and descriptions are needed

Boundary tracking

- Many description algorithms need to sort boundary points according to a given orientation
- Problem: Given a binary region or its boundary how can we follow the boundary in clockwise (or acw) order?
- The Moore boundary tracking algorithm



  - Boundary CW traversal: initial steps
  - Works also for regions
  - Outer boundary extraction

The Moore boundary tracking algorithm

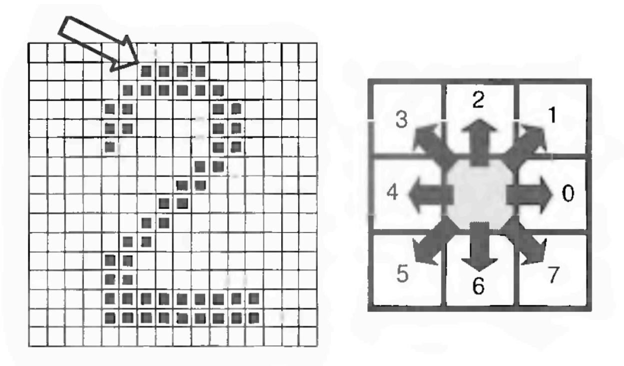
1. Locate b_0 the uppermost, leftmost foreground point
   Let c_0 be the west neighbor (background point)
   Inspect the 8 neighbors of b_0 starting at c_0 and proceed CW.
   Let b_1 be the first foreground neighbor and c_1 the background neighbor preceding b_1 in the sequence.
   Store b_0 and b_1
2. Let $b=b_1$ and $c=c_1$
3. Let the 8 neighbors of b starting at c and proceeding CW be denoted by $n_1,n_2,...,n_8$. Find the first n_k foreground pixel.
4. Let $b=n_k$ and $c=n_{k-1}$
5. Repeat steps 3 and 4 until $b=b_0$ and the next boundary point found is b_1.
   The obtained sequence of points constitutes the set of boundary CW ordered points.

Chain Codes

- Chain codes (and Freeman codes) are often used for the description of object borders, or other one-pixel-wide lines in images.
- The border is defined by the co-ordinates of its reference pixel and the sequence of symbols corresponding to the line of the unit length in several pre-defined orientations.
- A chain code is of a relative nature; data are expressed with respect to some reference point.
- Symbols may vary but they normally indicate some neighborhood coordinates

Chain Codes

- Example

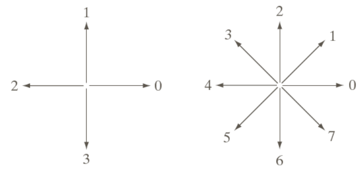


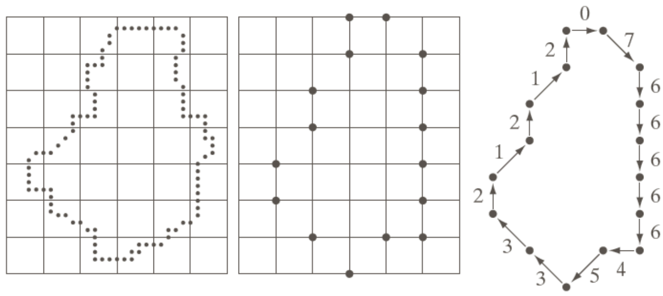
Adapted from Image Processing , Analysis, and Machine Vision 3Ed,Sonka et al

- Code from pointed pixel
  - 000776655555566000000064444444422211111122344456 5221

Chain Codes

- Effects of noise
  - Boundary down-sampling
  - Smoothing



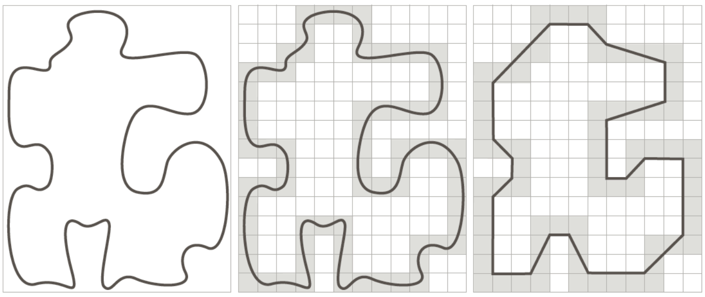


Polygonal Approximations

- A digital boundary can be approximated with arbitrary accuracy by a polygon.
- Ideal approximation for a close curve
  - Make the number of segments in the polygon equal to the number of points in the boundary so that each pair of adjacent points defines a segment in the polygon.
- Practical goal
  - Capture the "essence" of the boundary shape with the fewest possible polygonal segments.
- Several polygonal approximation techniques of modest complexity and processing requirements are well suited for image processing applications.

Minimum Perimeter Polygon (MPP)

- Idea:
  - Enclose the boundary by a set of concatenated cells
  - Make the boundary shrink constrained by the inner an outer walls of the bounding region defined by the cells
  - The “shrinking” process leads to a polygon of minimum perimeter with respect to a particular cell arrangement



Minimum Perimeter Polygon (MPP)

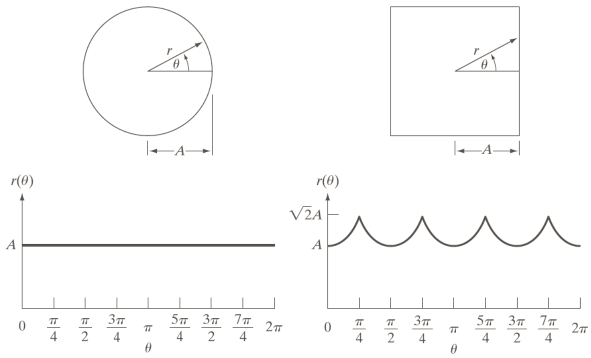
- Example

a b c
d e f
g h i

FIGURE 11.8 (a) 566 × 566 binary image. (b) 8-connected boundary. (c) through (i), MMPs obtained using square cells of sizes 2, 3, 4, 6, 8, 16, and 32, respectively (the vertices were joined by straight lines for display). The number of boundary points in (b) is 1900. The numbers of vertices in (c) through (i) are 206, 160, 127, 92, 66, 32, and 13, respectively.

Signatures

- A signature is a 1-D functional representation of a boundary
- Reduce the boundary representation to a 1-D function, which presumably is easier to describe than the original 2-D boundary.
- Several ways to create this functional:
  - Plot of the distance from the centroid to the boundary as a function of angle

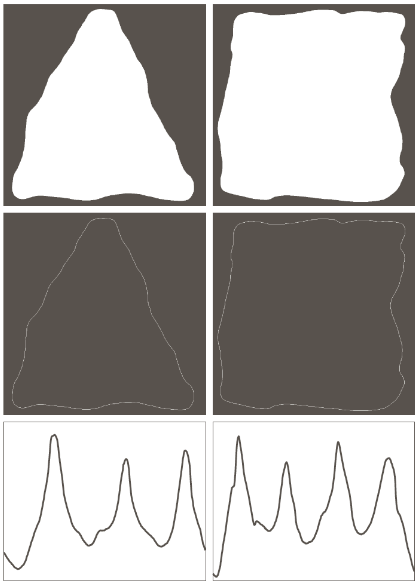


Signatures

- Translation invariant
- Rotation and scaling dependent
- Normalization w.r.t rotation
  - Select the starting point
    - Farthest from the centroid
    - Farthest from the centroid on the eigen axis
- Normalization w.r.t scaling
  - Scale the function by the variance of the signature
- Other signature function
  - The angle between a line tangent to the boundary at that point and a reference line.

Signatures

- Example



a b
c d
e f

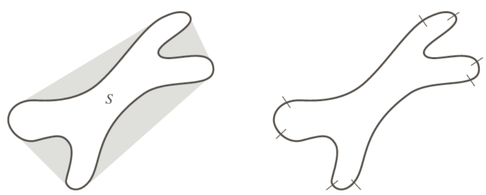
FIGURE 11.11 Two binary regions, their external boundaries, and their corresponding $r(\theta)$ signatures. The horizontal axes in (e) and (f) correspond to angles from 0° to 360°, in increments of 1°.

Signatures

- Exercises
  - Load the image “redmaple.jpg”
  - Study the IPT Matlab functions
    - `bwboundaries,bwlabel,regionprops`
    - Compute the boundary of only the leaf portion of the image
      - Use auxiliary morphological operators
    - Compute the $r(\theta)$ boundary signature relative to the centroid

Boundary Segments

- Decomposing a boundary into segments often is useful.
  - Reduces boundary complexity
  - Simplifies the description process
  - Attractive when significant concavities exist
    - Important data structure: Convex Hull
      - The convex hull *H* of an arbitrary set *S* is the smallest convex set containing *S*.
      - The set difference *H*-*S* is called the convex deficiency *D* of the set *S*.

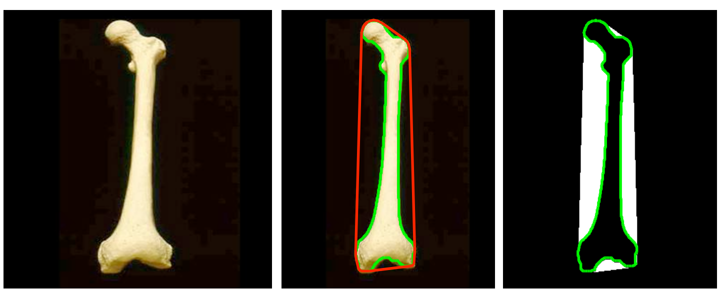


Follow the contour of *S* and mark the transitions into or out of components of the convex deficiency.

The marks are natural boundary segment limiters

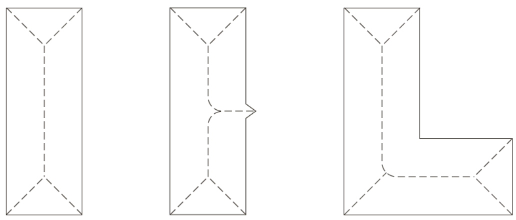
Boundary Segments

- Exercise
  - Load the image “femur.jpg”
  - Study the IPT Matlab functions
    - `Convhull, poly2mask`
    - Compute the boundary, the convex hull and convex deficiency of the femur region
    - Display the results as shown bellow



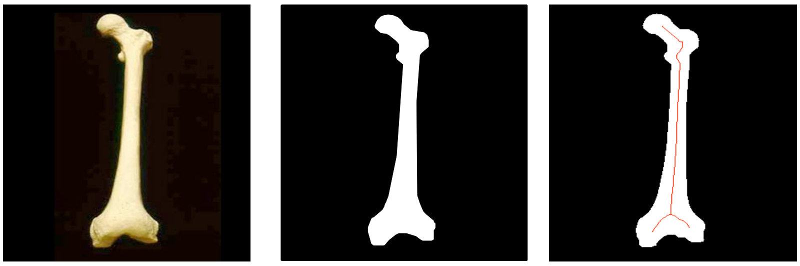
Skeletons

- The skeleton of a region may be defined via the medial axis transformation (MAT)
- MAT construction
  - For each point *p* in region *R*, find its closest neighbor in B. If *p* has more than one such neighbor, it is said to belong to the medial axis (skeleton) of *R*.
  - The basics of skeletonizing relies on morphology.
    - Euclidean distance and maximum disk algorithms



Skeletons

- Exercise
  - Load the image “femur.jpg”
  - Study the IPT Matlab functions
    - `bwmorph(BW,'skel',N),bwmorph(BW,'spur,'N)`
    - Compute the image skeleton
      - Try with several iterations
    - Display the results as shown bellow



Boundary descriptors

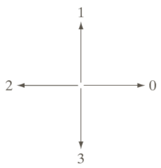
- Possible taxonomy
  - Simple descriptors
  - Shape numbers
  - Fourier descriptors
  - Statistical moments

Simple boundary descriptors

- Length
  - The number of pixels along a boundary gives a rough approximation of its length.
  - Chain coded curve with unit spacing in both directions
    - $L = Nh + Nv + \sqrt{2}\,Nd$ (*v:* vertical, *h*: horizontal, *d*: diagonal)
- Diameter
  - Once a distance function is defined for all *p* boundary points
    - $Diam(B) = \max_{i,j}[D(p_i, p_j)]$
  - Associated descriptors
    - Major axis, Minor axis
    - Basic rectangle
    - Eccentricity of the boundary

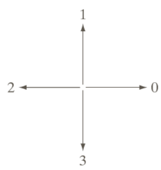
Shape Numbers

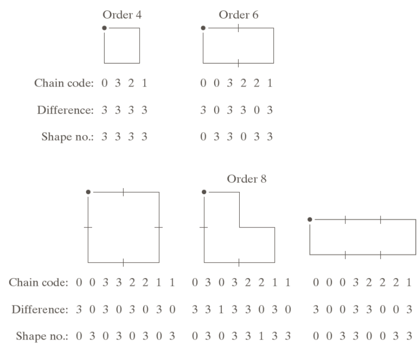
- The shape number of a boundary is a sequence digits that convey information about the first difference of a chain code representation
- The first difference in a chain code is obtained by counting the number of direction changes (ACW for example) between adjacent code elements.
- Example
  - Chain code: 10103322
  - 1rst difference: 3133030



Shape Numbers

- The shape number of boundary, is defined as the sequence of first differences starting by the smallest.
- The order n of a shape number is defined as the number of digits in its representation.
- n is even for a closed boundary, and its value limits the number of possible different shapes



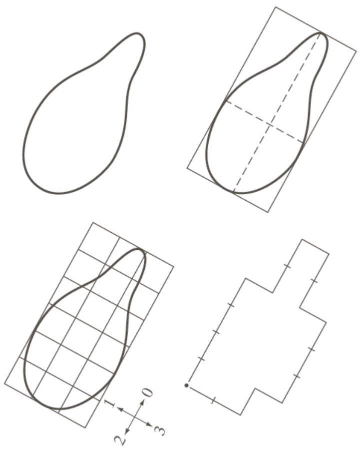


Shape numbers

- Although the first difference of a chain code is independent of rotation, in general the coded boundary depends on the orientation of the grid.
- One way to normalize the grid orientation is by aligning the chain-code grid with the sides of the basic rectangle defined in the previous section.
- In practice given a shape order the rectangle with whose eccentricity best approximates that of the basic rectangle is chosen.
- This new rectangle is resampled with an adequate grid size
  - eg. *n*=12: grids $2\times4,\ 3\times3,\ 1\times5$

Shape numbers

- Example, n=18



Chain code: 0 0 0 0 3 0 0 3 2 2 3 2 2 2 1 2 1 1

Difference: 3 0 0 0 3 1 0 3 3 0 1 3 0 0 3 1 3 0

Shape no.: 0 0 0 3 1 0 3 3 0 1 3 0 0 3 1 3 0 3

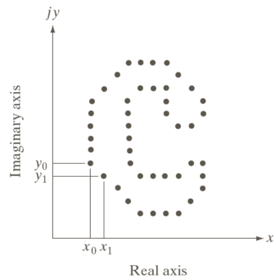
Fourier Descriptors

- A closed closed boundary may be given the alternative representation of the figure
- Traversing the boundary in CW a sequence of ordered pairs $s(k)$ may be formed as

$$s(k)=[x(k),y(k)],k=0,1,2,\ldots,K-1$$

- Mapping each pair into a complex number a formal $2\mathrm{D}\rightarrow 1\mathrm{D}$ reduction is achieved
- $$s(k)=x(k)+jy(k)$$
- A Fourier description is now possible with the DFT

$$a(u)=\sum_{0}^{K-1} s(k)\mathrm{e}^{-j2\pi uk/K}, \quad s(k)=\frac{1}{K}\sum_{0}^{K-1} a(u)\mathrm{e}^{j2\pi uk/K}$$



Fourier Descriptors

- The complex numbers $a(u)$ are called the boundary Fourier descriptors.
- Band-limiting the description is the same as making

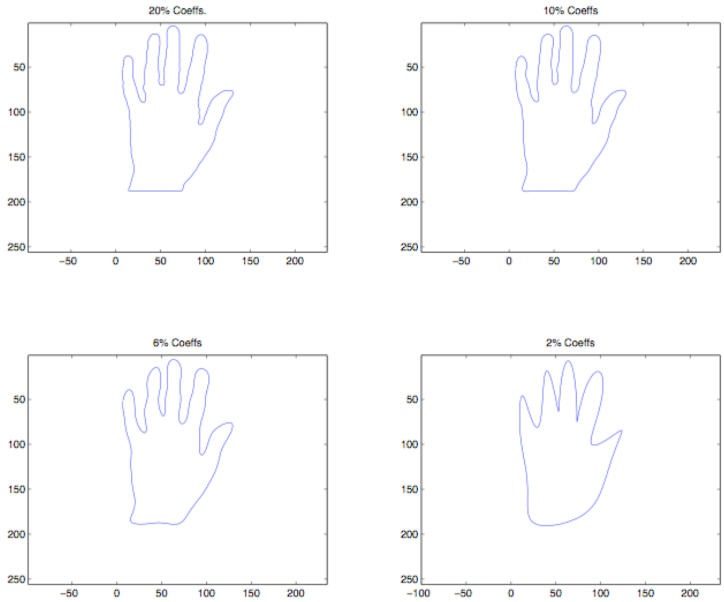
$$a(u)=0, \quad u=P,P+1,\ldots,K-1$$

$$\hat{s}(k)=\frac{1}{K}\sum_{0}^{P-1} a(u)\mathrm{e}^{j2\pi uk/K}, \quad k=0,1,\ldots,K-1$$

- Note that the reconstruction is still with *K* points but now only *P* terms are involved
- High-frequency components account for fine detail, and low-frequency components determine global shape. Thus the smaller *P* becomes, the more detail that is lost on the boundary.

Fourier Descriptors

- Example



Fourier Descriptors

- Few Fourier descriptors can be used to capture the gross essence of a boundary.
- Fourier descriptors are not directly insensitive to geometrical changes, but the changes in these parameters can be related to simple transformations on the descriptors.

Transformation	Boundary	Fourier Descriptor
Identity	$s(k)$	$a(u)$
Rotation	$s_r(k) = s(k)e^{j\theta}$	$a_r(u) = a(u)e^{j\theta}$
Translation	$s_t(k) = s(k) + \Delta_{xy}$	$a_t(u) = a(u) + \Delta_{xy}\delta(u)$
Scaling	$s_s(k) = \alpha s(k)$	$a_s(u) = \alpha a(u)$
Starting point	$s_p(k) = s(k - k_0)$	$a_p(u) = a(u)e^{-j2\pi k_0 u/K}$

$$s_t(k) = s(k) + \Delta_{xy} = [x(k) + \Delta X] + j[y(k) + \Delta y]$$

Statistical Moments

- The shape of boundary segments (and of signature waveforms) can be described quantitatively by using simple statistical moments, such as the mean, variance, and higher-order moments.
  - Assume there is a 1-D descriptor that can be modeled by a function g(r).
  - Assume that this function has unit area and g(r) can be treated as a probability density.
  - Descriptors based on the statistical moments of g(r) can be obtained by computing

$$\mu_n = \sum_{i=0}^{K-1} (r_i - m)^n g(r_i) \quad \text{where} \quad m = \sum_{i=0}^{K-1} (r_i) g(r_i)$$

Statistical Moments

- Problems
  - Recall the maple leaf example.

  - Compute the first 3 moments of the distance to centroid signature function $r(\theta)$

Regional Descriptors

- Simple Descriptors
- Topological
- Texture

Simple Regional Descriptors

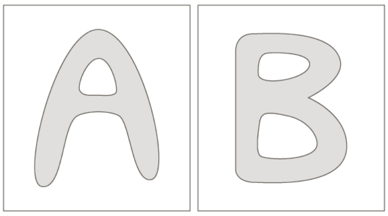
- Area (A)
- Perimeter (P)
- Compactness $\frac{P^2}{A}$
- Circularity
  - Ratio: area of region and area of circle with same perimeter
  - Show that for a region of perimeter *P*

$$\text{Circularity} = 4\pi \frac{A}{P^2}$$

- Histogram related
  - Mean, Median, Mode, Standard Deviation, Min, Max, etc ...

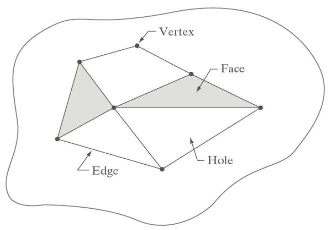
Topological descriptors

- Topological properties are useful for global descriptions of regions in the image plane.
- Simply defined, topology is the study of properties of a figure that arc unaffected by any deformation, as long as there is no rubber-sheet distortion.
  - Number of connected components (C)
  - Number of holes (H)
- Euler Number $E = C - H$
  - Compute *E* for the figures



Topological descriptors

- Polygonal networks
  - Regions delimited by straight-line segments

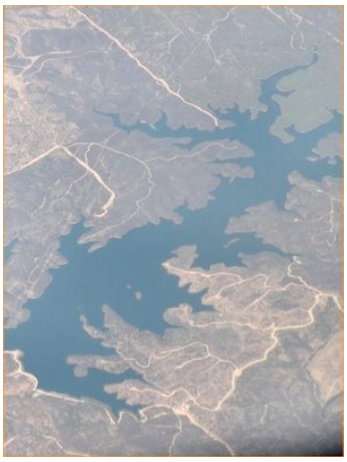


  - It can be shown that (Euler Formula)
    - V: nº of Vertices
    - Q: nº of Edges
    - F: nº of Faces

$$V - Q + F = C - H = E$$

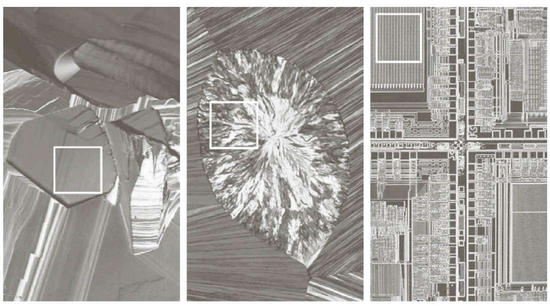
Topological descriptors

- Problem
  - Using the concept of largest connected component try to segment the river area in the following image
  - Important: after thresholding study the function `bwlabel`



Texture

- Although no formal definition of texture exists, intuitively this descriptor provides measures of properties such as smoothness, coarseness, and regularity.
- Approaches
  - Statistical
  - Structural
  - Spectral



a b c

FIGURE 11.28 The white squares mark, from left to right, smooth, coarse, and regular textures. These are optical microscope images of a superconductor, human cholesterol, and a microprocessor. (Courtesy of Dr. Michael W. Davidson, Florida State University.)

Statistical approach

- Basic data structure
  - histogram of a region or image with $p(z_i), i=0,\dots,L-1$ as the probability of occurrence of gray level z_i
- Central moments of *z*

$$\mu_n=\sum_{i=0}^{L-1}(z_i-m)^n p(z_i) \quad \text{where} \quad m=\sum_{i=0}^{L-1}(z_i)\,p(z_i)$$

- Note that $\mu_0(z)=1$ and $\mu_1(z)=0$
- The second moment (variance) is of particular importance in texture description. It is a measure of gray-level contrast that can be used to establish descriptors of relative smoothness such as *R*. What be inferred when *R=0* or when *R=1*?

$$\mu_2(z)=\sigma^2(z) \quad R=1-\frac{1}{1+\sigma^2}$$

Statistical approach

- Other central moments
  - $\mu_3(z)$ is a measure of skewness of the histogram
  - $\mu_4(z)$ is a measure of flatness of the histogram
- Uniformity

$$U(z)=\sum_{i=0}^{L-1}p^2(z_i)$$

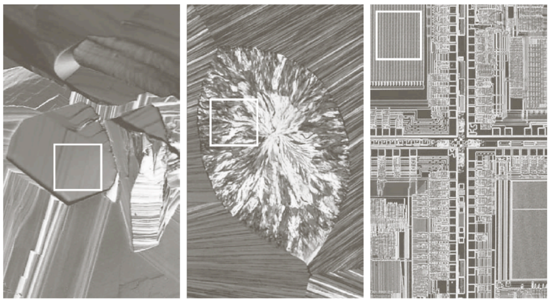
  - Study the behavior of *U*(*z*) with uniform probability values
- Entropy: average information quantity per gray level
  - Entropy is also a measure of variability

$$e(z)=-\sum_{i=0}^{L-1}p(z_i)\log_2(p(z_i))$$

  - What's the entropy of a constant image

Statistical descriptors

- Example of statistical texture metrics



a b c

FIGURE 11.28 The white squares mark, from left to right, smooth, coarse, and regular textures. These are optical microscope images of a superconductor, human cholesterol, and a microprocessor. (Courtesy of Dr. Michael W. Davidson, Florida State University.)

Texture	Mean	Standard deviation	R (normalized)	Third moment	Uniformity	Entropy
Smooth	82.64	11.79	0.002	−0.105	0.026	5.434
Coarse	143.56	74.63	0.079	−0.151	0.005	7.783
Regular	99.72	33.73	0.017	0.750	0.013	6.674

TABLE 11.2 Texture measures for the subimages shown in Fig. 11.28.

- Compute the above metrics

GLCM

- Histogram related texture measures carry no information about relative pixel positions
- Integrating intensity distribution with relative positions of pixels with similar intensity leads to the concept of the gray-level co-occurrence matrix.
- Let P be a position relation, and let **A** be matrix with entries a_{ij} such that a_{ij} is the number of occurrences or gray level pairs (i,j) for pixels positioned according to P.
- If P is "1 pixel to right and 1 pixel below":

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 2 \\ 1 & 1 & 0 & 1 & 1 \\ 2 & 2 & 1 & 0 & 0 \\ 1 & 1 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix} \qquad A = \begin{bmatrix} 4 & 2 & 1 \\ 2 & 3 & 2 \\ 0 & 2 & 0 \end{bmatrix}$$

Exercise

- Write a MATLAB function that computes the GLCM for a region with a given range of gray values and direction defined within the 4-neighborhood
- Reproduce the example below

1	1	7	5	3	2
5	1	6	1	2	5
8	8	6	8	1	2
4	3	4	5	5	1
8	7	8	7	6	2
7	8	6	2	6	2

Image f

	1	2	3	4	5	6	7	8
1	1	2	0	0	0	1	1	0
2	0	0	0	0	1	1	0	0
3	0	1	0	1	0	0	0	0
4	0	0	1	0	1	0	0	0
5	2	0	1	0	1	0	0	0
6	1	3	0	0	0	0	0	1
7	0	0	0	0	1	1	0	2
8	1	0	0	0	0	2	2	1

Co-occurrence matrix **G**

Adapted from Digital Image Processing 3rd Ed. Gonzalez, Woods

Probabilistic estimates with GLCM

- Let N be the number of point pairs satisfying P. Then

$$N = \sum_i \sum_j a_{ij}$$

- Making **C**=A/N, then a_{ij} is an estimate of the joint probability that a pair of pixels satisfying P will have gray-levels (i,j). The matrix **C** is called the gray-level co-occurrence matrix.
- There is a dependence on P so an appropriate position relation may be defined in order to “match” a particular patterned texture.
- How can **C** provide quantitative information about the texture of region?
- What is the meaning of diagonal elements with P being the identity relation?
- What can we expect from the elements far from the diagonal?

GLCM related measures

- Maximum probability

$$\max_{i,j}\left(c_{ij}\right)$$

- Element-wise difference of order *k*

$$\sum_i \sum_j (i-j)^k c_{ij}$$

- Inverse element-wise difference of order *k*

$$\sum_i \sum_j c_{ij} / (i-j)^k$$

- Uniformity

$$\sum_i \sum_j c_{ij}^2$$

- Entropy

$$-\sum_i \sum_j c_{ij} \log_2\left(c_{ij}\right)$$