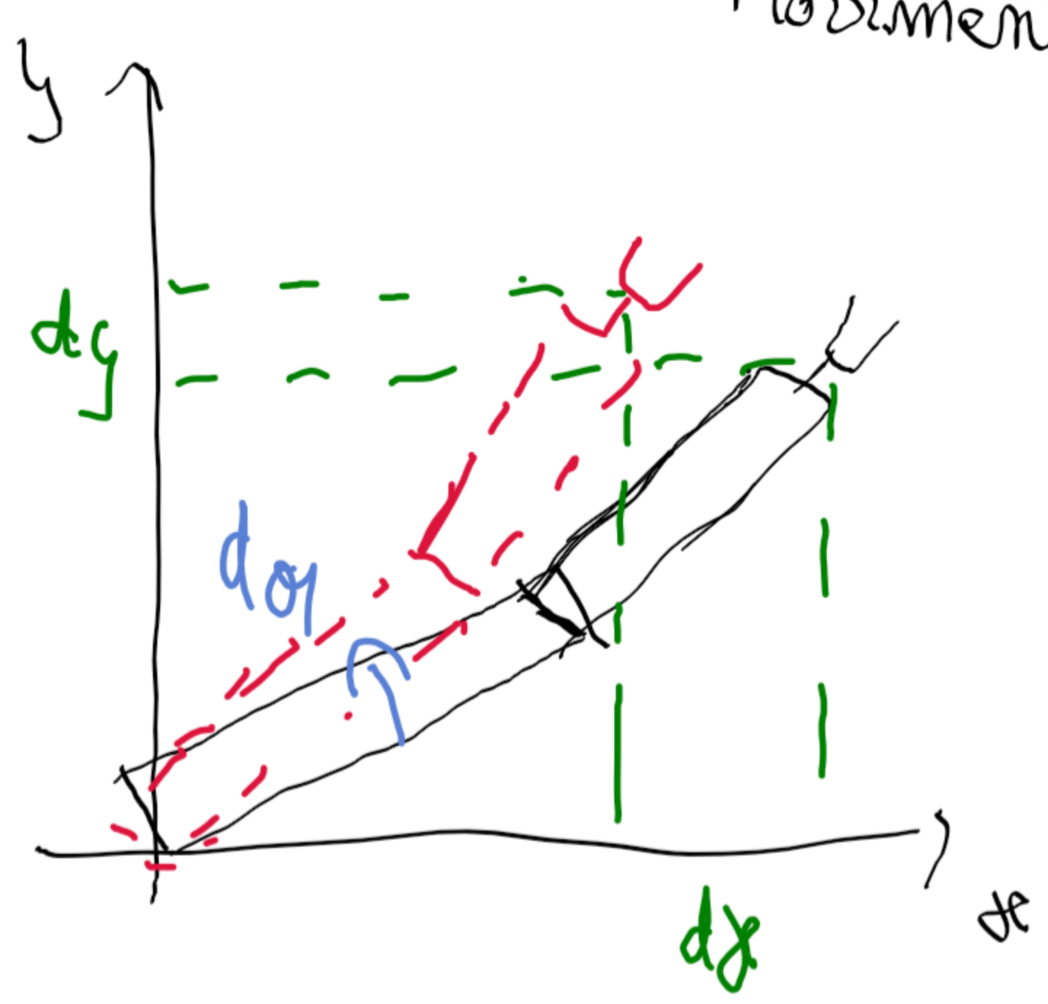
Movimentos têm de ser pequeninos



$$\begin{cases} x = L_1C_1 + L_2C_{12} \\ y = L_1S_1 + L_2S_{12} \end{cases}$$

$$\begin{bmatrix} dx \\ dy \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{bmatrix} d\theta_1 \\ d\theta_2 \end{bmatrix}$$

$$J_{11} = \frac{\partial x}{\partial \theta_1} = -L_1S_1 - L_2S_{12}$$

$$J_{12} = \frac{\partial x}{\partial \theta_2} = -L_2S_{12}$$

$$J_{21} = \frac{\partial y}{\partial \theta_1} = L_1C_1 + L_2C_{12}$$

$$J_{22} = \frac{dy}{d\theta_2} = L_2C_{12}$$

$$\begin{bmatrix} dx \\ dy \end{bmatrix} = \underbrace{\begin{bmatrix} -L_1S_1 - L_2S_{12} & -L_2S_{12} \\ L_1C_1 + L_2C_{12} & L_2C_2 \end{bmatrix}}_{J} \begin{bmatrix} d\theta_1 \\ d\theta_2 \end{bmatrix}$$

RR 3D

$$\begin{cases} x = L_2C_1C_2 \\ y = L_2S_1C_2 \\ z = L_2S_2 + L_1 \end{cases}$$

$$\begin{bmatrix} dx \\ dy \\ dz \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \\ J_{31} & J_{32} \end{bmatrix} \begin{bmatrix} d\theta_1 \\ d\theta_2 \end{bmatrix}$$

$$J = \begin{bmatrix} -L_2S_1C_2 & -L_2C_1S_2 \\ L_2C_1C_2 & -L_2S_1S_2 \\ 0 & L_2C_2 \end{bmatrix}$$

$$\vec{q} = F_I\,\vec{x} \Rightarrow d\vec{q} = J_I\, d\vec{x} \quad \Big|\; J_I = ?$$

1ª Inversão analítica

$$J_I = J^{-1} \quad \text{tem de ser invertível}$$

$$A^{-1} = \frac{Adj(A)}{det(A)} \;;$$

Para RR planar

$$J^{-1} = \frac{\begin{bmatrix} J_{22} & -J_{21} \\ -J_{12} & J_{11} \end{bmatrix}^T}{det(J)}$$

$$det(J) = |J| = J_{11}J_{22} - J_{21}J_{12} = -(L_1S_1 + L_2S_1S_2)L_2C_{12} + (L_1C_1 + L_2C_{12})L_2S_{12} =$$

$$= -L_1S_1L_2C_{12} - \cancel{L_2S_1S_2L_2C_{12}} + L_1C_1L_2S_{12} + \cancel{L_2C_{12}L_2S_{12}}$$

$$= L_1L_2(\underbrace{S_{12}C_1 - C_{12}S_1}_{\sin(\theta_1+\theta_2-\theta_1)}) = L_1L_2S_2$$

2º diferenciar cinemática inversa

RR a 3D

① $\theta_1 = \operatorname{arctg} \frac{y}{x}$

$\theta_2 = \operatorname{arctg} \frac{z - L_1}{xC_1 + yS_1}$

possível mas não é usual

forma alternativa

①

$$\frac{S_1}{C_1} = \frac{y}{x} \Leftrightarrow xS_1 - yC_1 = 0$$

$$S_1 dx + xC_1 d\theta_1 - C_1 dy + yS_1 d\theta_1 = 0 \Leftrightarrow d\theta_1 (xC_1 + yS_1) = -S_1 dx + C_1 dy$$

$$\Leftrightarrow d\theta_1 = \frac{-S_1 dx + C_1 dy}{xC_1 + yS_1}$$