A SIMPLE NEURAL MODEL FOR FUZZY REASONING

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***ABSTRACT*-A very simple neural architecture for fuzzy reasoning is presented. It is shown that fuzzy rules may be implemented with such nets. The net is layered and the concept of variables and predicates may be associated with areas in those layers. It is the density of activated neurones which defines the membership grades. Fuzzy logic operations are induced in a natural way by the random connections of the neurones from layer to layer. The layered structure of the model, its simplicity and the randomness of the connections makes this model adequate for representing natural systems.**

I. INTRODUCTION

Binary neural nets have been used for different purposes such as pattern recognition (1,2), image processing(3) and universal logic function construction (4).

Here a very simple binary neural model (that is, where the nodes are Boolean functions) for fuzzy reasoning is presented. It is shown that fuzzy production rules may be implemented by a layered structure of binary neurones, where the neurones are of two types only : those which implement logic AND's and those which implement logic OR's. These processing levels (where the rules are implemented) may interact with an input layer at one end (with sensor neurones) and with an output layer at the other end. It is shown that, if neurones are aggregated in areas associated with the different variable predicates and if the density of active neurones in those areas represent the membership grade of those predicates, the fuzzy logic operations are naturally induced by the atomic logic operations of the neurones. It is the topology of the net (which is organised at a macro level but random at the level of neurone connections) which defines the membership functions of the fuzzy sets associated with the different predicates.

II. MODEL DEFINITION

A *Definition of Fuzzy Logic operations*

As will follow in the text, membership grades of fuzzy variables will be associated with densities of activated neurones in given areas or with the probability of finding activated a randomly chosen neurone at that area.

The fuzzy operations defined are then:

The Fuzzy complement is:

$$c(p) = 1-p$$

The axioms for the complement are obeyed:

1. $c(o)=1$; $c(1)=0$ since $1-0=1$ and $1-1=0$

2. if $pa<pb$ then $c(pa)>c(pb)$ since $1-pa>1-pb$

3. $c(c(p))=p$ since $1-(1-p)=p$

The fuzzy AND (t norm) is defined by the probability product pa.pb . This also obeys the axioms for the fuzzy intersection:

1. $pa \; t \; pb = pb \; t \; pa$ since $pa.pb=pb.pa$

2. $(pa \; t \; pb) \; t \; pc = pa \; t \; (pb \; t \; pc)$ since $(pa.pb).pc=pa.(pb.pc)$

3. if $pa<=pb$; $pc<=pd$, $pa \; t \; pc <= pb \; t \; pd$ since $pa.pc<=pb.pd$

4. $p \; t \; 0 = 0$, $p \; t \; 1 = p$ since $p.0=0$ and $p.1=p$

This operation is also Archimedean and strict.

For the fuzzy union (s norm) the definition is :
$pa+pb-pa.pb$, known as the probability union which obeys also the axioms for fuzzy union:

1. $pa<=pb$; $pc<=pd$, $pa \; s \; pc <= pb \; s \; pd$ since $pa+pc-pa.pc<=pb+pd-pb.pd$

2. $pa \; s \; pb = pb \; s \; pa$ since $pa+pb-pa.pb=pb+pa-pb.pa$

3. $(pa \; s \; pb) \; s \; pc=pa \; s \; (pb \; s \; pc)$ since $(pa+pb-pa.pb)+pc-(pa+pb-pa.pb).pc=$
$=pa+(pb+pc-pb.pc)-pa.(pb+pc-pb.pc)$

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4. p s 0 = p since p+0-0=p

p s 1 = 1 since p+1-p=1

B *Variables*

In our model it is considered that different areas (or regions) are allocated to different variables. These areas are dense regions of binary neurones. Moreover sub-areas are considered, one for each possible fuzzy set associated with that variable (including fuzzy predicates).

It is the average density of activated neurones in each of these areas which defines the membership grade of the variable to the fuzzy set associated with that area.

To visualise this concept, in Fig. 1 three areas are shown with different densities to define the membership grades of the variable "age" to the fuzzy sets "child", "young" and "old" for a person with 20 years of age.

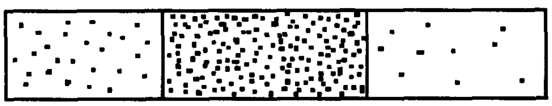

Fig. 1 Neurone activation in different areas

C *Membership-Functions*

Membership functions for each of the fuzzy sets are defined entirely by the topology of connections. In particular one possible model for defining piece wise linear membership functions is presented in the section E.

D *Production Rules*

The implementation of fuzzy rules in this model is a very simple operation. Neurone outputs of the areas associated with the antecedents are linked to the area associated with the consequent. Every neurone output is randomly mapped to an input of a neurone at the reception area. The operations performed in the neurones of the consequent area are simply AND/OR Boolean operations. More specifically, for each neurone in the consequent area each one of its inputs is taken as the output of a randomly chosen neurone from each one of the areas associated with the antecedents.

See Fig. 2 to visualise the following rules (only two neurones per area are shown):

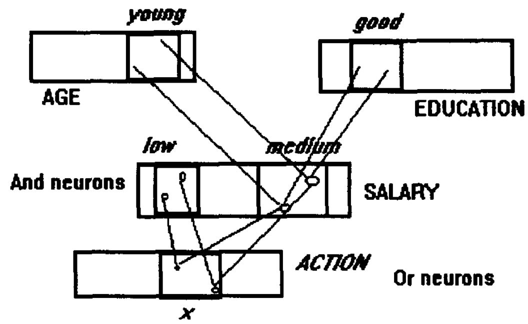

Fig 2 Connections for rule implementation

IF Age is Young AND Education Good THEN Salary Medium

IF Salary Medium OR Salary Low THEN Action X

These type of connections and logic operations induce fuzzy logic AND and fuzzy logic OR operations, (as defined in a previous section) as follows:

The average density of active neurones in a region **i** is defined by Ni/N, where Ni is the number of active (output 1) neurones in that area and N is the total number of neurones in it. This is the same as the probability **pi** that a randomly chosen neurone in the area is activated.

The probability that a given neurone in the consequent area is activated (also the activated neurone density in that area) is:

$px = \Pi pi$ for all $i \varepsilon I$, and I is the enumeration set of the antecedent areas.

This is so because the atomic operation in each consequent neurone is a logic AND, and only when every input is a "1" will the AND give a "1".

Then, the above defined (section A) fuzzy logic AND is obtained by the model implementation of antecedent ANDING.

Also, if the atomic operation realised in the consequent area is taken as the logic OR, the probability that one given neurone in that area is activated is (for antecedent areas i and j):

$$px = pi+pj-pi.pj$$

This is obvious since a neurone in the consequent area is 1 if one one or both of the two inputs are 1.

This operation may be extensible for any number of input neurones (pz = px+pk-px.pk, for antecedent areas i,j and k).

Again, this implements the fuzzy logical OR defined in the section A.

E *Input-Output Layers*

One of the basic assumptions of the model is that neurones are aggregated in areas (or regions) according to predicates of variables (e.g. low, medium, high for the variable temperature), which seems a good representation for fuzzy reasoning but does not seem to agree with the peripheral levels of neural processing. At these levels (input/output) it appears more natural that the intensity of a given variable may be represented by the number of activated neurones in the area associated with the variable (for example, the number of activated neurones proportional to the value or amplitude of the variable).

For this purpose possible input/output layers and interaction with the processing levels (rule implementation levels) are presented.

1) *Input-Level*

At this level it is supposed that there are sets of sensor neurones, with each set responding to different amplitude input values. It is also supposed that the output of these neurones saturate at "1" with those input amplitudes (see Fig. 3).

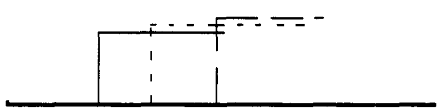

Fig. Neurone outputs with different inputlevels

If the number of sensor neurones which fire in every Δx ($\Delta x \rightarrow 0$) is the same, the total number of activated sensor neurones is proportional to the input value of x.

Linear piece wise membership functions may be obtained simply by connecting the outputs of these sensor neurones to the first level of processing neurones - to the area associated with the membership function . This is made trough AND neurones at this processing level. Each neurone at this level has two randomly chosen inputs, one from a neurone in the area associated with the amplitude interval corresponding to the increasing part of the membership function and the other from the decreasing part.

Fig. 4 illustrates this concept. Here sensor neurones which fire at different xi are shown, for the sake of simplicity, in different strip slices.

If Δx is a discrete value, the membership functions are approximated by stairs instead of a continuous slope, which presents no problem if Δx is sufficient low.

2) *Output-Layer*

Similarly, it is considered that the last processing level (always corresponding to a consequent area) is connected to an output level, where the number of activated neurones is proportional to the desired output (this corresponds to the defuzzification process).

A simple mapping is enough, as follows :

Taking K as the number of neurones of each of the areas of the processing level (and associated with a given predicate of one variable), R the number of such areas associated with a given variable (there is one area per label of the variable), and fi the number of activated neurones in area i, one randomly maps the neurones of these R areas to different output layer neurones of a total of K.R .

The neurones of area i are mapped in a fraction Xi (Xi<1). Thus the probability of a given output layer neurone to be activated (thus the density of active neurones at that level) is:

$$\Sigma \, Xi.fi/K.R$$

This is true if the mapping is one to one and the output rones without input (since Xi<1) will remain at 0.

If Xi represents a weight associated with strength of that area i this is a known defuzzification process. Fig. 5 tries to represent this mapping.

At this output level there is , thus, an area per output variable where the number of activated neurones represents the amplitude of that variable, resulting from the processing of the fuzzy rules.

III Conclusions

The main conclusion of this model is that fuzzy reasoning seems to be according simple neural topologies which have no need of complex computations to achieve their objectives. In fact, all it is needed to achieve the rules implementation is a structured layered net of binary neurones where some organisation is detected (neurone areas associated to the rule antecedents are linked together at an area associated to the consequent of the rule), but

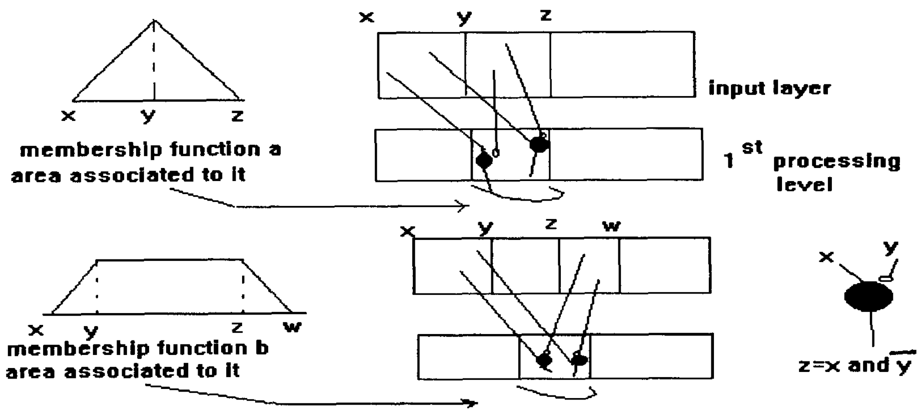

Fig 4 Linear membership functions implementation

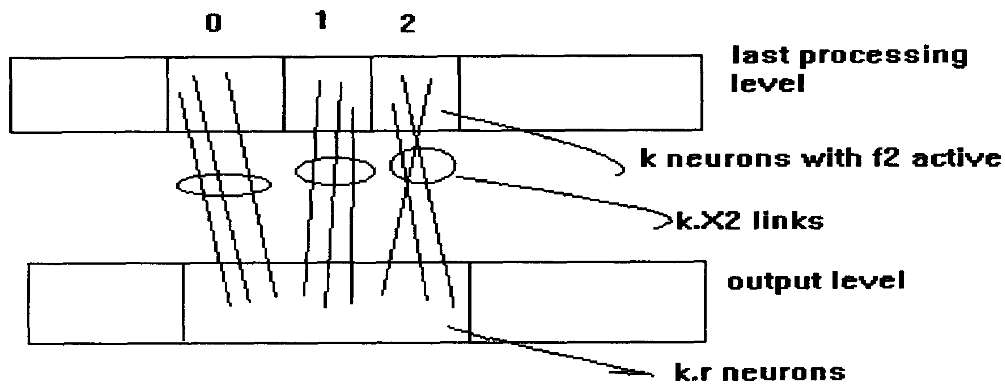

Fig. 5 Connection to output level

where randomness plays the most important place - in fact all neurone connections from area to area are random.

Therefore the model presents very attractive properties in order to represent real natural systems reasoning, among which the simplicity, layered structure and randomness of connections are probably the more important.

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