Neural Activation Ratio Based Fuzzy Reasoning

José A. B. Tomé
IST(Technical University of Lisbon)/INESC
email: jose.tome@inesc.pt

Abstract

This work presents a new class of binary neural nets which seem to be similar to the natural neural nets in which concerns topologic and functional issues. It is the medium activity among the neurons in a given neural area which represents the "amplitude" of the associated variable, making the net insensible to individual errors. It is proved that fuzzy reasoning is an emergent property of such nets, if predefined membership functions are considered. Macroscopic (net level) fuzzy reasoning emerges from microscopic (neuron level) boolean operations . Strategies for teaching with real experiments are proposed resulting in a global learning of the net from local neural operations.

1. Introduction

Functional systems based simultaneously on fuzzy logic and neural networks are becoming more and more common, taking advantage of the benefits of the two paradigms, and creating thus a synergistic co-operation. The two most common ways of doing this co-operation are Neural Fuzzy systems, where the neural networks learning capabilities are used to constitute the components of the fuzzy inference system, [15], [7], [11], [6], and Fuzzy Neural nets where a fuzzification of the neural net (e.g. fuzzy parameters instead of crisp weights) takes place with the objective of achieving the traditional properties of flexibility and robustness in the neural network, [3], [4], [8], [10]. Other architectures where both neural nets and fuzzy systems cooperate, more or less independently, have also been proposed in a hybrid way, [2], [1]. An excellent presentation of these and other architectures may be found in [9].

Boolean neural networks with random connections between neurons (when observed at a micro level) and exhibiting a macroscopic fuzzy behaviour have been also investigated by the author, [12], [13], [14].

In this paper a class of boolean nets where the concept of fuzzy variable is associated with an area of neurons is presented. Moreover there is no need to create neuron sub-areas (one for each variable predicate or linguistic term) as in former models, [12], [13]. In fact, the uncertainty or fuzziness about a given variable predicate exists only at the macroscopic level due to the different "opinions" of sets of neurons, which altogether form the variable area. This corresponds to a philosophical view where fuzziness is a natural and emergent property of complexity.

Random connections between the neuron outputs of the different antecedent areas and neuron inputs of the consequent area ensure a reasoning capability for these nets.

Moreover, teaching can be achieved from real experiments, by establishing *intra*-neural connections (in fact establishing their functions) which depend only on local neural activation.

It is noticeable the similarities of the model with natural neural systems (unlike the classic back propagation neural nets) : partition of neurons in areas with their own significance (variables), dense meshes of connections between those areas , individual random connections, "amplitude" of variables given by the density of neural activation in the correspondent areas. Moreover it can be stated that these nets have an inherent fuzzy reasoning capability , which has not been there explicitly placed . In other words: Fuzziness is an emergent property of the model here presented and probably of natural systems.

2.The Network Architecture

Hypothetically, the network implements qualitative reasoning through production rules of the type (it will be shown that this hypothesis is verified):

Rule: IF A is A1 AND B is B1 AND....THEN C is C1, where A and B are Antecedent variables , C is the Consequent variable, and A1, B1 and C1 are linguistic terms. If fuzzy reasoning is considered, these linguistic terms correspond to fuzzy sets.

0-7803-4863-X/98 $10.00©1998 IEEE

In order to implement this goal, neurons are aggregated in areas (one area per variable) and connections are established between the outputs of antecedent neurons (those on antecedent areas) to the inputs of the neurons on the consequent area. The model here considered postulates that each consequent neuron is an N.m-input neuron, where N is the number of antecedents and m the number of inputs coming from the same antecedent area. Each input I_{kn} (k=1,N; n=1,m) is connected to a **randomly** chosen neuron output from antecedent area k. Here the number m is considered the same for every antecedent for explanation simplicity only.

2.1. The Neurons

Neurons are binary (inputs and outputs are restricted to 1 or 0) and a neuron may be viewed as an Universal Logic Function (ULF) of its inputs. It may be reduced, however, to a simpler counting function as shown in what follows.

The simplest neuron, receiving m inputs from a single antecedent area (from m random neurons on that area) is an m input ULF or a counting modified ULF (MULF) as shown in Figure 1.

For the above purpose it is supposed that the set of integers (0-m) is partitioned on C disjoint subsets.

For example, if m=8, one could consider C as 3, with *cr(1)* the Oring of minterms with 0,1 or 2 "ones", *cr(2)* the Oring of minterms with 3,4 or 5 "ones" and *cr(3)* with 6,7 or 8 "ones". In this example, *cr(1)* is the set of counts {0, 1, 1, 1, 1, 1, 1, 1, 1, 2,...,2}, where 2 is repeated 28 times.

During teaching phase it is possible to modify the boolean value stored in flip-flop *FF* according its input *mr* (or *cr*). During test/use phase the output of the flip-flop addressed by its corresponding *mr* or *cr* is activated and thus the output of the ULF (or MULF) is set to the same value. This is so because one and only one of the *mr*'s (or *cr*'s) is activated.

For consequent areas with N antecedents, the neurons have a similar diagram as seen in Figure 2.

In Figure 2, mr(i,..,j) means a minterm where the configuration of input variables coming from the first antecedent area (I11..I1m) is the i^{th} ,..., and the configuration of the input variables coming from the N^{th} antecedent area is the j^{th}. cr(i,..,j) means the Oring of the minterms which have a number of ones in the inputs coming from the first antecedent area belonging to the i^{th} subset of the integers 0-m, ..., and a number of ones in the inputs coming from the N^{th} antecedent area belonging to the j^{th} subset. For simplicity, the number of inputs coming from every antecedent area is supposed to be the same (m). Also, when necessary , cr(i,...j) is used to denote the subset of joint counts in the various inputs, where each joint count is repeated a number of times equal to the number of minterms which enter in the correspondent above defined Oring.

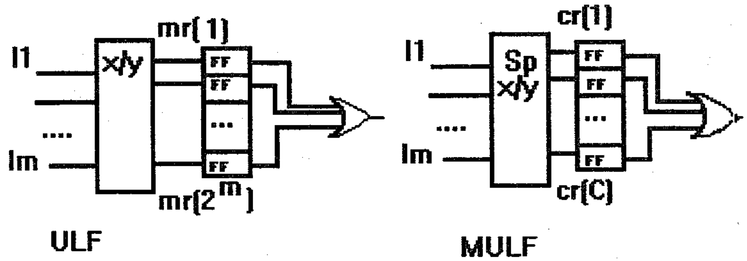

Figure 1. Single antecedent neurons

In Figure 1 each intermediate output, *mr(i)*, represents a minterm and each *cr(i)* represents the Boolean Oring of the minterms with a given number k of "true" (or 1) values on the input variables, k belonging to a given subset of possible counts on m inputs. When necessary *cr(i)* also designates this subset of counts, where each count is repeated a number of times equal to the number of minterms with k "ones"; this should be clear from the context. The ***Sp x/y***, in Figure 1, is a modified decoder to implement this counting logic

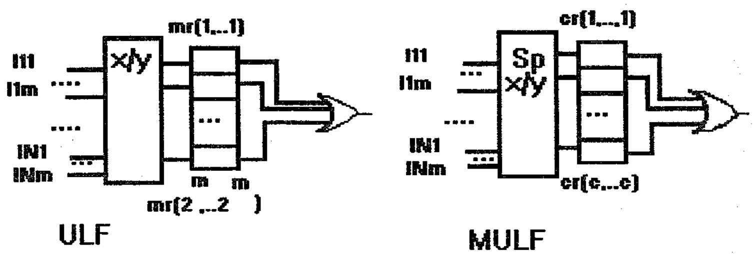

Figure 2. Multiple antecedent neurons

2.2. Neural Area Activation

For each area it is considered that the **significant** parameter, for the description of the variable value associated with that area, is the ratio between activated neurons and total number of neurons on that area: the **activation ratio**. This is the same as the probability of a randomly chosen neuron on that area to be at "1".

For a test experiment where a single antecedent is used , the probability of an intermediate output mr(i) of an arbitrary consequent neuron (as in Figure 1) to become at "1", with the addressed flip-flop activated is:

$(p^{ci}.(1\text{-}p)^{m\text{-}ci}).pmr(i)$ or simply

$P^{a}mr(i).pmr(i)$

where p is the activation ratio of the antecedent, ci is the number of ones in the minterm mr(i) ,m the n° of inputs, and pmr(i) the probability of flip-flop FFi in the ULF to be at "1".

For the same arbitrary consequent neuron the probability of giving "1" at its output is then (since only one of the *mr*'s is activated) :

$$\sum_{i} (p^{ci}.(1\text{-}p)^{m\text{-}ci}).pmr(i) \qquad \text{or}$$

$$\sum_{i} P^{a}mr(i).pmr(i) \qquad (1)$$

This is also, by definition, the **activation ratio** of the consequent area.

In the same way the activation ratio of the consequent area , when MULF neuron type is used is:

$$\sum_{i=1}^{c} \sum_{k \in cr(i)} \binom{m}{k} p^{k}.(1\text{-}p)^{m\text{-}k}.\ pcr(i) \qquad \text{or}$$

$$\sum_{i=1}^{c} P^{a}cr(i).pcr(i) \qquad (2)$$

where C is the number of subsets of counts and pcr(i) is the probability of FFi in the MULF to be at "1".

The concept of activation ratio can also be extended to the $P^{a}mr$ and $P^{a}cr$. It is defined, for a given $P^{a}mr(i)$ ($P^{a}cr(i)$) as the ratio between the number of neurons at the consequent layer with the associated decoder (modified decoder) output activated and the total number of neurons in that layer.

It should be noticed that, if pcr(i) takes the same value as the various pmr(j) for the minterms mr(j) taken into account in the counting subset i, then expression (2) is not more than a rearrangement of terms of expression (1).

Similarly for N antecedent rules the **consequent activation ratio** are as follows.

For ULF neurons:

$$\sum_{i1=1}^{2^m} \cdots \sum_{iN=1}^{2^m} \prod_{j=1}^{N} [\ p_j^{Cij}.(1-p_j)^{m-Cij}\ ]pmr(i1,..,iN) \qquad \text{or} \quad (3)$$

$$\sum_{i1=1}^{2^m} .. \sum_{iN=1}^{2^m} P^{a}mr(i1,..,iN).pmr(i1,..,iN)$$

where Cij represents the number of ones for minterm numbered i in the subset of inputs of a consequent neuron coming from antecedent j ,and pmr(i1,..,iN) the probability of flip-flop (i1,..,iN) to be at "1".

For MULF neurons:

$$\sum_{i1=1}^{C} .. \sum_{iN=1}^{C} \sum_{ki1,..,kiN \in cr(i1,..iN)} \prod_{j=1}^{N} \binom{m}{k_{ij}} p_j^{\,kij} . \qquad \text{or} \qquad (4)$$

$$.(1-p_j)^{m-kij}.pcr(i1,..,iN)$$

$$\sum_{i1=1}^{C} .. \sum_{iN=1}^{C} P^{a}cr(i1,..iN).pcr(i1,..,iN)$$

where kij represents the number of ones in the subset of inputs coming from antecedent j and belonging to a set of joint count of ones cr(i1,...,iN).

Likewise, expression (4) is a rearrangement of (3) if each pcr(i1,..,iN) is equal to every pmr(j1,..,jN) with mr(j1,..,jN) being the set of minterms with joint counts belonging to cr(i1,..,iN).

Notice, also, that:

$P^{a}mr(i1,..,iN) = P^{a}mr(i1).\ ..\ .P^{a}mr(iN)$ and

$P^{a}cr(i1,..,iN) = P^{a}cr(i1).\ ...\ .P^{a}cr(iN)$

3. Emergent Reasoning

As already indicated in section 2, it is supposed that this net implements some kind of reasoning, more specifically fuzzy reasoning. This is shown in what follows.

Consider again rules of the type:

IF A is A1 AND B is B1 AND.... THEN C is C1.

A,B and C, the variables, are in this model represented by distinct areas of neurons. A1, B1 and C1 , linguistic terms, are fuzzy sets defined in the Universe of Discourse of the activation ratios of those areas. It is expected that the activation ratio of area C will signify the result of the above qualitative rule. To show that this is really so it is necessary to prove that Fuzzy operations (as the AND used in the rules) are induced macroscopically by the neural logic operations implemented in the ULF's or MULF's.

The standard operations for fuzzy sets here considered are well known, [9], and defined as follows:

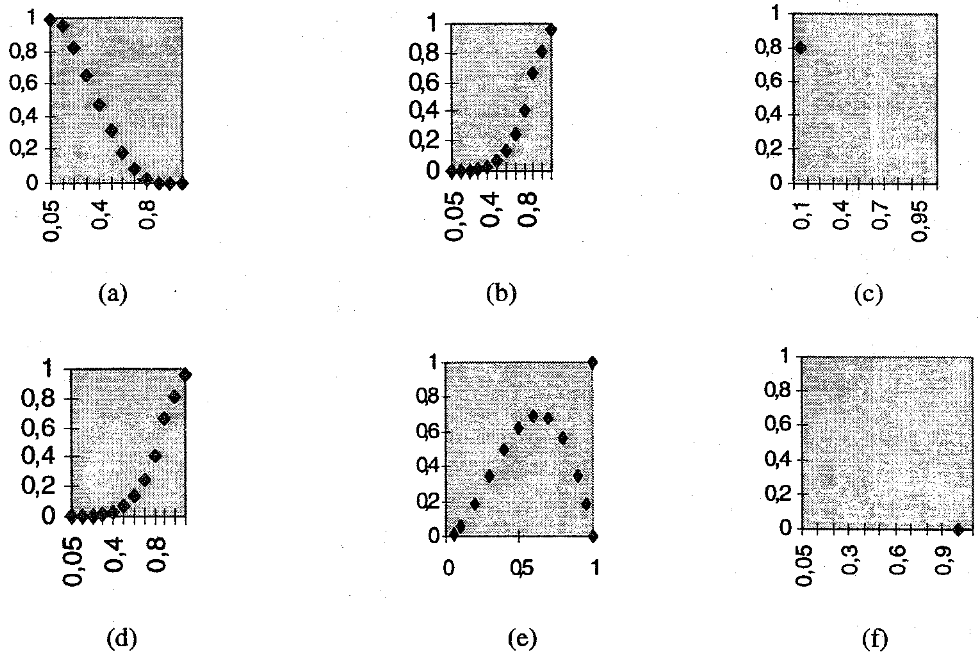

Figure 3. Example

$\mu_{\bar{A}}(x) = 1 - \mu_A(x) \qquad \forall x \in U$, for complement,

$\mu_{A \cap B}(x) = t\lfloor \mu_A(x), \mu_B(x) \rfloor = \mu_A(x).\mu_B(x)$ for t-norm

$\mu_{A \cup B}(x) = s\lfloor \mu_A(x), \mu_B(x) \rfloor = \mu_A(x) \oplus \mu_B(x)$

where $a \oplus b = \min(1, a+b)$, the bounded sum, for t conorm.

3.1. Induced Fuzzy Reasoning

Considering the activation ratios of antecedent areas j (the p_j) as the values of the input variables associated with those areas it follows that expressions (3) and (4) have a simple interpretation: they represent the defuzzification of the output (consequent) variable by the Centre of Area method, giving thus the value of that output variable - the activation ratio of the consequent.

Each vector (i1,..,iN) corresponds to a different production rule, as may be seen in what follows.

In expression (3) for each linguistic term (ij) of an antecedent j corresponds a membership function, given by $p_j^{Cij}.(1-p_j)^{m-Cij}$. The evaluation of the expression for a given p_j represents the membership degree of p_j in that fuzzy set. By definition of the t-norm above, P^amr(i1,..,iN) (the algebraic product of those terms) represents the fuzzy intersection of the antecedent part of the rule (i1,..,iN). The fuzzy set of the consequent variable (or area) for that rule is a singleton of "1", for the abscissa value of pmr(i1,..,iN). Since $\sum_{i1=1}^{2^m} .. \sum_{iN=1}^{2^m}$ P^amr(i1,..,iN) =1, and this expression is the denominator of the Centre of Area formula, expression (3) is indeed the defuzzification of the consequent.

If two different rules have the same consequent fuzzy set (pmr(i1,..,iN)), the definition of fuzzy **OR** above (bounded sum) is according expression (3), since it is sure that the sum is always less (equal on the limit where every rule has the same consequent) than 1.

If expression (4) is used, a similar demonstration could be used to show that it represents the deffuzification of the consequent. Now P^acr(i1,..iN) represents the antecedent part of the rule (i11,..,iN) - more specifically the fuzzy intersection of the different antecedent membership values - and pcr(i1,..,in) the abscissa for the consequent singleton.

Consider as an example the activation ratio of a consequent where the neurons have m=4, two antecedent areas and where C is equal to 3 (counts of 0 and 1 "ones",2 and 3 "ones" and 4"ones"):

$$\sum_{i1=1}^{3} \sum_{i2=1}^{3} P^a\text{cr}(i1,i2).\text{pcr}(i1,i2)$$

Take, for simplicity, only two rules. P^acr(1,3) and P^acr(3,2) represent the antecedent parts of those rules (define input variables as IN1 and IN2) and pcr(1,3) and pcr(3,2) the respective consequents. Take pcr(1,3) and pcr(3,2) as 0.1 and 0.9 and defined respectively as "low" and "high"(variable OUT).

If the sets of counts in m for the antecedents are classified as "low" for i1 or i2 =1 (counts "0" and "1"), "medium" for i1 or i2 =2 (counts "2" and "3") and "high" for i1 or i2 =3 (count "4") the expression

$\sum_{(i1,i2)=(1,3),(3,2)} P^a cr(i1,i2).pcr(i1,i2)$ represents the two rules:

R1 : IF IN1 is LOW AND IN2 is HIGH
THEN OUT is LOW

R2 IF IN1 is HIGH AND IN2 is MEDIUM
THEN OUT is HIGH

In Figure 3 (a), (d), (b) and (e) the membership functions for linguistic terms "low", and "high" for variable IN1 and "high" and "medium" for variable IN2 are shown respectively. In figure 3 (c) and (f) the results for R1 and R2 are shown, obtained from the fuzzy intersection of the antecedent part and the output singletons, when the activation ratios of IN1 and IN2 are respectively 0,05 and 0,95. In figure 3(c), for example, OUT-LOW=IN1-LOW(0,05).IN2-HIGH(0,95).1 = 0,803.

4. Learning

The learning phase is implemented through the establishment of logic values in the flip-flops of the ULFs or MULF's. This turns to be the setting of the ***pmr***'s or of the ***pcr***'s probabilities. To achieve this objective the following steps are implemented:

1. The network is activated (both in antecedent and consequent areas) by a set E of experiments.
2. For each experiment, each consequent neuron is obviously submitted to a particular input configuration, and thus a single flip-flop on the ULF (MULF) is also addressed. Updating of each MULF's (ULF's) flip-flop value depends on its selection (or not) and on the logic value of the consequent neuron.
3. Different strategies may be used to modify the state of the flip-flop, according the type learning. Here some possible strategies are proposed, but others may be implemented with minor modifications in the network. Those here treated are: a simple Hebbian Learning , Competitive Learning based and Grossberg based Learning. From these three, this paper will pay a little more attention to the last one.

4.1.Hebbian and other related types of Learning

Hebb, [5], hypothesised that synaptic changes in the brain depend on the correlation between activity on neurons previous to the synapses (antecedents) and those posterior to the synapses (the consequents).More specifically the synaptic strengths increase with that correlation . Here that hypothesis is taken for the Learning phase. Some minor adaptations to it are, however, allowed in order to achieve different types of learning.

How is that correlation here considered? In this model, pre and post synaptic activities are, for each experiment in E,given by the ***activation ratios*** : p_j for antecedent area j and p_{out} for the consequent area. However, p_j is "modulated" through m random samples before it drives an output neuron. The m+1 activation ratios of the different countings are the meaningful parameters to take into account for pre synaptic activity. Thus, in a given experiment, the correlation between posterior synapse activity (p_{out}) and pre synaptic activity -the probability of a given cr(i,..,j) to be activated- can be represented by pcr(i,..,j), that is the probability of the MULF flip-flops to be activated. In practical terms, for each teaching experiment and for each consequent neuron, the state of flip-flop (i,..,j) is determined by, and only by, the boolean values of decoder output cr(i,..,j) and of the output neuron considered.

Considering then the pcr's as the synaptic strength, one may have the following types of learning (for simplicity the indexes i,...,j are omitted in what follows):

For the Simple Hebbian type:

$pcr(t+1)-pcr(t) = - pcr(t) + p_{out} . P^a cr$

where $P^a cr$ is the probability of activating, in the experiment, the decoder output associated with pcr. Notice that this type of learning is very simple: For each experiment, activated MULF flip-flops with activated output neurons are set , the others are reset.

For the Competitive Learning based type:

$pcr(t+1)-pcr(t) = p_{out} . P^a cr - p_{out} . pcr(t)$
$= p_{out} (P^a cr - pcr(t))$

This corresponds to setting to "1" selected MULF flip-flops with consequent neuron at "1", and resetting non-selected MULF flip-flops with consequent neuron at "1".

Finally for the Grossberg-based type:

$$pcr(t+1)-pcr(t) = P^a cr. (p_{out} - pcr(t)) \quad (5)$$

The interpretation is simple: non-selected MULF flip-flops maintain their state; selected MULF flip-flops take the value of consequent neuron.

Taking this last type of learning, it is easily seen that it converges, that is: if all experiments teach the same rule the net will learn it with zero limit error.

First consider that $pcr(t) = \mathbf{P_{out}}$ and another experiment takes place with the same rule, that is with consequent activation ratio of $\mathbf{P_{out}}$ and same $P^a cr$. pcr(t+1) should maintain the value. Indeed it comes:

$pcr(t+1)=pcr(t). (1- P^a cr) +P^a cr .\mathbf{P_{out}}$
$pcr(t+1)=\mathbf{P_{out}} (1- P^a cr) +P^a cr .\mathbf{P_{out}}$
$pcr(t+1) = \mathbf{P_{out}}$

Moreover considering an initial pcr different from $\mathbf{P_{out}}$., it will converge to $\mathbf{P_{out}}$ with coherent experiments - with $\mathbf{P_{out}}$ as consequent activation ratios and the same $P^a cr$.

To prove this take pcr(t+2)-pcr(t+1) , using again expression (5), and consider a consequent activation ratio of $\mathbf{P_{out}}$ for all experiments:

$pcr(t+2)-pcr(t+1) = P^a cr.$
$.(\mathbf{P_{out}} - pcr(t)-P^a cr. (\mathbf{P_{out}} - pcr(t)))$
$= (pcr(t+1)-pcr(t)) - (P^a cr)^2.(\mathbf{P_{out}} - pcr(t))$

It is a simple matter to verify that :

$|pcr(t+2)-pcr(t+1)| < |pcr(t+1)-pcr(t)|$ for any t

Thus it may be concluded that with a set of coherent experiments -teaching the same rule- the net converges. It will establish the pcr with the same value as $\mathbf{P_{out}}$, and in each experiment it will approach that value $\mathbf{P_{out}}$ proportionally to the distance between the present value of pcr and $\mathbf{P_{out}}$ itself, that is with approaching zero decreasing steps.

5. Conclusions

In this work a class of neural nets has been presented which functionality seems to be more similar to natural nets than the classic neural nets: variables or concepts are associated with different neural areas, meshes of links are established from areas to other areas depending on antecedent-consequent dependence, individual links are randomly established between neurons of those areas, individual neurons have only a two state space dimension (fire/do not fire) and the notion of "amplitude" of each variable is given by a natural activation ratio (that is the fraction of activated neurons on a given area). The net is totally digital (binary), there is no weights and individual neurons have no meaning - in fact any number of neurons can be discarded (provided the remaining neurons are enough to define the concept of activation ratio with accuracy) : this is a real robust network insensible to individual errors.

Moreover these nets are capable of reasoning, implementing qualitative rules. It appears that fuzzy reasoning is a natural emergent property of these networks. Mechanisms for learning those rules from real experiments have also been presented. The rules need not to be explicitly known (although it is a simple matter to teach the network, if they are) and different rules , with different experiments (where the rules are only implicit) can be taught .

It can be stated that these nets may be a possible , greatly simplified version, of real natural neural nets; at least they are similar in various ways with the emergent capability of reasoning.

The capability of these nets to implement any n-1 function (that is, of being Universal Approximators) is the subject of present investigation and will be published subsequently. Future work includes the possibility of introducing asymmetries in the network (that is introducing neurons with different meanings in the same area) in order to provide fuzzy reasoning with detailed capabilities and exploring the concept of Activation Ratio cellular automata.

6. Bibliography

[1] FUKUDA,T. and T.SHIBATA. "Hierarchical intelligent control for robotic motion by using fuzzy, artificial intelligence and neural networks" *Pro. Int. Joint Conf. Neural Networks*,volI,269-274,Baltimore, 1992.

[2] FUKUDA,T.,K.SHIMOJIMA, F.ARAI and H. MATSUURA."Multi-sensor integration system with fuzzy inference and neural network."*Proc. Int.Joint Conf. Neural Networks*,volII,757-762,Baltimore, 1992.

[3] GUPTA, M. and QI. "On fuzzy neuron models" *Proc. Int. Joint Conf. Neural Networks*, volII, 431-436, Seattle, 1991.

[4] HAYASHI, Y., E. CZOGALA and J.BUCKLEY. "Fuzzy neural controller" *Proc. IEEE Int. Conf. Fuzzy Systems*, 197-202,San Diego, 1992.

[5] HEBB,D. *The Organization of Behaviour:A Neuropsychological Theory.* New York:John Wiley, 1949

[6] HORIKAWA, S., T. FURUHASHI and Y. UCHIKAWA. "On fuzzy modeling using fuzzy neural networks with the back propagation algorithm." *IEEE Transactions Neural Networks* 3(5): 801-806, 1992.

[7] KELLER,J. , R. KRISHNAPURAM and F. RHEE. "Evidence aggregation networks for fuzzy logic interface." *IEEE Transactions Neural Networks* 3(5): 761-769, 1992.

[8] KELLER,J.M. and D.J.HUNT. "Incorporating fuzzy membership functions into the perceptron algorithm." *IEEE Transactions Pattern Anal.Mach.Intell.*. PAMI-7(6):693-699, 1985.

[9] LIN, CHIN-TEN and LEE,C.S. *A Neuro-Fuzzy Synergism to Intelligent Systems.*New Jersey : Prentice Hall., 1996.

[10] PEDRYCZ,W.. "Fuzzy neural networks with reference neurons as pattern classifiers" *IEEE Trans.Neural Networks* 3(5):770-775, 1992.

[11] TAKAGI,H. and HAYASHI,. "NN-driven fuzzy reasoning." *Int. J. Approximate Reasoning* 5(3):191-212, 1991.

[12] TOMÉ,J.A."A simple Neural Model for Fuzzy Reasoning."*Proc. IEEE Int. Conf. Fuzzy Systems*, San Francisco., 1993.

[13] TOMÉ,J.A."Learning in Binary Neural Nets which have Fuzzy Reasoning." *Proc. Int. Joint Conf. Neural networks*,Nagoya, 1993.

[14] TOMÉ,J.A."A connectionist model for recurrent fuzzy reasoning."*Proc. Int. Conf. Neural Information Processing*, Seul, 1994.

[15] YAGER,R. "OWA neurons: A new class of fuzzy neurons". *Proc. Int. Joint Conference Neural Networks*, vol I, 226-231, Baltimore,. 1992.