# 10-Homogeneous-Systems

January 6, 2017

```
In [1]: from latools import *
        from sympy import *
        init_printing(use_latex=True)
```

## 1 Homogeneous Systems.

This notebook has examples showing how to find the solution set of a homogeneous linear system.

### 1.1 Example 1.

$$\begin{bmatrix} 1 & 2 & -2 & 7 & 0 & 2 & 0 \\ -2 & 1 & 4 & 1 & -1 & 2 & 2 \\ 1 & -1 & -2 & -2 & 0 & 0 & 7 \\ 2 & 0 & -4 & 2 & 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

#### 1.1.1 Solution

**Step 1** Enter matrix $A$ and find its RREF equivalent matrix $R$:

```
In [2]: A = matrix_to_rational([[ 1,  2, -2,  7,  0, 2, 0],
                                [-2,  1,  4,  1, -1, 2, 2],
                                [ 1, -1, -2, -2,  0, 0, 7],
                                [ 2,  0, -4,  2,  0, 0, 4]])
        A
```

Out[2]:

$$\begin{bmatrix} 1 & 2 & -2 & 7 & 0 & 2 & 0 \\ -2 & 1 & 4 & 1 & -1 & 2 & 2 \\ 1 & -1 & -2 & -2 & 0 & 0 & 7 \\ 2 & 0 & -4 & 2 & 0 & 0 & 4 \end{bmatrix}$$

```
In [3]: R = reduced_row_echelon_form(A)
        R
```

Out[3]:

$$\left[\begin{matrix}1 & 0 & -2 & 1 & 0 & 0 & 2\\0 & 1 & 0 & 3 & 0 & 0 & -5\\0 & 0 & 0 & 0 & 1 & 0 & -3\\0 & 0 & 0 & 0 & 0 & 1 & 4\end{matrix}\right]$$

In [4]:
```
print(latex(R))
```

```
\left[\begin{matrix}1 & 0 & -2 & 1 & 0 & 0 & 2\\0 & 1 & 0 & 3 & 0 & 0 & -5\\0 & 0 &
```

**Step 2** Identify the free variables and write the pivot variables in terms of the free variables:

- Pivot columns: 1, 2, 5, 6.
- Non-pivot columns: 3, 4, 7. There are 3 free variables: $x_3$, $x_4$ and $x_7$

From the RREF matrix $R$ we get:

$$\begin{aligned}x_1 &= 2x_3 - x_4 - 2x_7\\ x_2 &= 0x_3 - 3x_4 + 5x_7\\ x_5 &= 0x_3 + 0x_4 + 3x_7\\ x_6 &= 0x_3 + 0x_4 + 4x_7\end{aligned}$$

**Step 3** Find the *basis of the solution set*:

1. Let

$$\begin{bmatrix}x_3\\x_4\\x_7\end{bmatrix} = \begin{bmatrix}1\\0\\0\end{bmatrix}$$

Then:

$$\begin{aligned}x_1 &= 2\\ x_2 &= 0\\ x_5 &= 0\\ x_6 &= 0\end{aligned}$$

We get the solution:

$$\begin{bmatrix}2\\0\\1\\0\\0\\0\\0\end{bmatrix}$$

2. Let
$$\begin{bmatrix} x_3 \\ x_4 \\ x_7 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$
Then:
$$\begin{aligned} x_1 &= -1 \\ x_2 &= -3 \\ x_5 &= 0 \\ x_6 &= 0 \end{aligned}$$
We get the solution:
$$\begin{bmatrix} -1 \\ -3 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

3. Let
$$\begin{bmatrix} x_3 \\ x_4 \\ x_7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$
Then:
$$\begin{aligned} x_1 &= -2 \\ x_2 &= 5 \\ x_5 &= 3 \\ x_6 &= 4 \end{aligned}$$
We get the solution:
$$\begin{bmatrix} -2 \\ 5 \\ 0 \\ 0 \\ 3 \\ 4 \\ 1 \end{bmatrix}$$

### 1.1.2 Step 3

Write the solution set, which is the span of the solutions we have found:

$$S = \left\{ \lambda_1 \begin{bmatrix} 2 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \lambda_2 \begin{bmatrix} -1 \\ -3 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \lambda_3 \begin{bmatrix} -2 \\ 5 \\ 0 \\ 0 \\ 3 \\ 4 \\ 1 \end{bmatrix} \; : \; \lambda_1, \lambda_2, \lambda_3 \in \mathbb{R} \right\}$$

```
In [ ]:
```