For each of the items below, do the following:

1. Write the augmented matrix for the given system.
2. Use elementary row operations to transform the matrix into reduced row echelon form.
3. Write the system that corresponds to the reduced row echelon form.
4. Write the solution set of the system (using set notation).

Exercises:

1.

$$\begin{aligned} -3x_3 - 6x_4 - 2x_5 + 16x_7 &= 4 \\ x_1 + 4x_2 - 3x_3 - 7x_4 - 2x_5 - 3x_6 + 19x_7 &= 8 \\ -2x_1 - 8x_2 - 2x_3 - 2x_4 - 3x_5 - x_6 + 22x_7 &= 19 \\ -3x_1 - 12x_2 + x_3 + 5x_4 + x_5 + 3x_7 &= 13 \end{aligned}$$

2.

$$\begin{aligned} -3x_1 + x_2 - 2x_3 - 14x_4 &= -8 \\ -2x_2 + x_3 + 7x_4 &= -5 \\ x_3 + 3x_4 &= 3 \end{aligned}$$

3.

$$\begin{aligned} -2x_1 + 2x_2 + 8x_3 - 2x_4 - 2x_5 + 2x_6 - 6x_7 - 2x_8 &= 10 \\ 2x_1 - 2x_2 - 8x_3 + 2x_4 - 3x_5 - 22x_6 + 11x_7 &= -12 \\ -2x_1 + 2x_2 + 8x_3 - 2x_4 + 2x_5 + 18x_6 - 10x_7 &= 10 \end{aligned}$$

4.

$$\begin{aligned} x_1 - 4x_3 - 2x_4 + x_5 - 7x_8 &= -1 \\ x_2 - 4x_3 + 2x_4 - 3x_5 + 2x_6 - 6x_7 + 10x_8 &= -14 \\ 2x_1 - x_2 - 4x_3 - 6x_4 + 2x_5 - 2x_6 - 3x_7 - 9x_8 &= 9 \\ -3x_1 + x_2 + 8x_3 + 8x_4 - 2x_5 - 3x_6 + x_7 + 16x_8 &= 18 \end{aligned}$$

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5.

$$\begin{aligned}
x_2 \qquad\qquad\qquad &= -10 \\
-2x_1 - 3x_2 - 2x_3 + 2x_4 &= 20 \\
x_1 \qquad + 2x_3 - x_4 &= -8 \\
-3x_1 + x_2 - 3x_3 + 2x_4 &= 0 \\
-x_1 - 3x_2 - x_3 + x_4 &= 11
\end{aligned}$$

6.

$$\begin{aligned}
2x_1 - 10x_2 + x_3 - 4x_4 + 2x_5 &= 8 \\
-3x_3 - 12x_4 - x_5 &= 5 \\
2x_3 + 8x_4 - x_5 &= -5 \\
-2x_1 + 10x_2 + x_3 + 12x_4 - 3x_5 &= -13 \\
2x_1 - 10x_2 - x_3 - 12x_4 + x_5 &= 11
\end{aligned}$$

7.

$$\begin{aligned}
-2x_1 - 3x_2 - 3x_3 \qquad &= -13 \\
-2x_2 - x_3 - 2x_4 &= -4 \\
x_1 - 3x_2 + x_3 + 2x_4 &= -2 \\
-2x_2 - x_3 + 2x_4 &= -8
\end{aligned}$$

8.

$$\begin{aligned}
-2x_1 + 8x_2 - 4x_3 + 4x_4 - 6x_5 - 3x_6 - 3x_7 - 12x_8 \qquad &= 10 \\
-2x_1 + 8x_2 - 4x_3 + 4x_4 - 6x_5 \qquad - 2x_7 + 2x_8 \qquad &= 2 \\
-3x_1 + 12x_2 - 6x_3 + 6x_4 - 9x_5 - 2x_6 + x_7 + 3x_8 + 14x_9 &= -1 \\
2x_1 - 8x_2 + 4x_3 - 4x_4 + 6x_5 + x_6 - 2x_7 - 6x_8 - 13x_9 &= 4
\end{aligned}$$

9.

$$\begin{aligned}
-2x_1 + 6x_2 - 8x_3 + 8x_4 - 3x_6 + 2x_7 + 25x_8 + 23x_9 &= 4 \\
-3x_6 - 2x_7 + 7x_8 + 11x_9 &= 6 \\
-x_1 + 3x_2 - 4x_3 + 4x_4 - x_6 + 2x_7 + 14x_8 + 11x_9 &= 5 \\
-x_1 + 3x_2 - 4x_3 + 4x_4 - x_6 + x_7 + 10x_8 + 9x_9 &= 8
\end{aligned}$$

10.

$$
\begin{aligned}
x_2 - 2x_3 - x_4 + 2x_5 - 6x_6 - 3x_7 - x_8 + x_9 &= -25 \\
2x_1 - 3x_2 + 14x_3 + 7x_4 + 2x_5 + 4x_6 - 3x_7 - x_8 - x_9 &= 3 \\
x_2 - 2x_3 - x_4 - x_5 - 3x_6 + 2x_7 + x_8 &= 5 \\
2x_1 - x_2 + 10x_3 + 5x_4 - x_5 - x_6 - x_7 + 2x_9 &= -3 \\
2x_1 - x_2 + 10x_3 + 5x_4 + x_5 - 3x_6 - 2x_7 + x_8 &= -3 \\
x_1 - 2x_2 + 8x_3 + 4x_4 + 5x_6 - 2x_7 - 2x_8 &= 2
\end{aligned}
$$

Solutions:

1.
$$\left\{\begin{bmatrix}-5\\0\\0\\0\\-2\\-3\\0\end{bmatrix}+\lambda_1\begin{bmatrix}-4\\1\\0\\0\\0\\0\\0\end{bmatrix}+\lambda_2\begin{bmatrix}1\\0\\-2\\1\\0\\0\\0\end{bmatrix}+\lambda_3\begin{bmatrix}3\\0\\4\\0\\2\\2\\1\end{bmatrix}\ :\ \lambda_1,\lambda_2,\lambda_3\in\mathbb{R}\right\}$$

2.
$$\left\{\begin{bmatrix}2\\4\\3\\0\end{bmatrix}+\lambda_1\begin{bmatrix}-2\\2\\-3\\1\end{bmatrix}\ :\ \lambda_1\in\mathbb{R}\right\}$$

3.
$$\left\{\begin{bmatrix}-3\\0\\0\\0\\2\\0\\0\\-4\end{bmatrix}+\lambda_1\begin{bmatrix}1\\1\\0\\0\\0\\0\\0\\0\end{bmatrix}+\lambda_2\begin{bmatrix}4\\0\\1\\0\\0\\0\\0\\0\end{bmatrix}+\lambda_3\begin{bmatrix}-1\\0\\0\\1\\0\\0\\0\\0\end{bmatrix}+\lambda_4\begin{bmatrix}5\\0\\0\\0\\-4\\1\\0\\0\end{bmatrix}+\lambda_5\begin{bmatrix}-4\\0\\0\\0\\1\\0\\1\\0\end{bmatrix}\ :\ \lambda_1,\lambda_2,\lambda_3,\lambda_4,\lambda_5\in\mathbb{R}\right\}$$

4.
$$\left\{\begin{bmatrix}-2\\-1\\0\\0\\1\\-5\\0\\0\end{bmatrix}+\lambda_1\begin{bmatrix}4\\4\\1\\0\\0\\0\\0\\0\end{bmatrix}+\lambda_2\begin{bmatrix}2\\-2\\0\\1\\0\\0\\0\\0\end{bmatrix}+\lambda_3\begin{bmatrix}3\\-1\\0\\0\\-3\\-1\\1\\0\end{bmatrix}+\lambda_4\begin{bmatrix}2\\3\\0\\0\\5\\1\\0\\1\end{bmatrix}\ :\ \lambda_1,\lambda_2,\lambda_3,\lambda_4\in\mathbb{R}\right\}$$

5.
$$\varnothing$$

6.
$$\left\{\begin{bmatrix}4\\0\\-2\\0\\1\end{bmatrix}+\lambda_1\begin{bmatrix}5\\1\\0\\0\\0\end{bmatrix}+\lambda_2\begin{bmatrix}4\\0\\-4\\1\\0\end{bmatrix}\ :\ \lambda_1,\lambda_2\in\mathbb{R}\right\}$$

7.

$$\left\{ \begin{bmatrix} -1 \\ 1 \\ 4 \\ -1 \end{bmatrix} : \in \mathbb{R} \right\}$$

8.

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ -2 \\ -2 \\ 0 \\ 0 \end{bmatrix} + \lambda_1 \begin{bmatrix} 4 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \lambda_2 \begin{bmatrix} -2 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \lambda_3 \begin{bmatrix} 2 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \lambda_4 \begin{bmatrix} -3 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \lambda_5 \begin{bmatrix} 3 \\ 0 \\ 0 \\ 0 \\ 0 \\ -4 \\ -2 \\ 1 \\ 0 \end{bmatrix} + \lambda_6 \begin{bmatrix} 3 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ -3 \\ 0 \\ 1 \end{bmatrix} \;:\; \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6 \in \mathbb{R} \right\}$$

9.

$$\varnothing$$

10.

$$\left\{ \begin{bmatrix} 0 \\ -5 \\ 0 \\ 0 \\ -3 \\ 0 \\ 3 \\ 1 \\ -4 \end{bmatrix} + \lambda_1 \begin{bmatrix} -4 \\ 2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \lambda_2 \begin{bmatrix} -2 \\ 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \lambda_3 \begin{bmatrix} 3 \\ 4 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \;:\; \lambda_1, \lambda_2, \lambda_3 \in \mathbb{R} \right\}$$