08-Determinant-Computation-Example

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```
In [2]: from latools import *
        from sympy import *
        init_printing(use_latex=True)
```

1 Computing Determinants Using Row Operations

To compute the determinant of a matrix A using Row operations, proceed as follows:

- Use row operations to reduce the matrix to a triangular matrix T. Use only the following two kinds of operation:
  - Type 1: `Ri` $*$ (`c`) $+$ `Rj` $=>$ `Rj` - Multiply a row by a scalar and add to another row (`i` $\neq$ `j`)
  - Type 3: `Ri` $<=>$ `Rj`: - Swap rows `i` and `j` (`i` $\neq$ `j`)
- The determinant of A is then given by:

$$(-1)^{\text{(Number of row swaps)}} \times (\text{Product of diagonal entries of } T)$$

The following cells show how to compute the determinant of the 5×5 matrix

$$A = \begin{bmatrix} 3 & -2 & 1 & \frac{1}{4} & 0 \\ \frac{2}{3} & -\frac{4}{9} & -4 & 2 & 3 \\ 4 & -1 & \frac{1}{2} & 5 & 2 \\ -2 & 2 & 1 & 0 & 1 \\ 2 & \frac{5}{2} & -2 & 3 & -3 \end{bmatrix}$$

```
In [3]: A = matrix_to_rational([[   3,    -2,    1, 1/4,   0],
                                [2/3, -4/9,   -4,    2,   3],
                                [   4,    -1, 1/2,    5,   2],
                                [ -2,      2,    1,    0,   1],
                                [   2,   5/2,   -2,    3,  -3]])
        A
```

Out[3]:

$$\begin{bmatrix} 3 & -2 & 1 & \frac{1}{4} & 0 \\ \frac{2}{3} & -\frac{4}{9} & -4 & 2 & 3 \\ 4 & -1 & \frac{1}{2} & 5 & 2 \\ -2 & 2 & 1 & 0 & 1 \\ 2 & \frac{5}{2} & -2 & 3 & -3 \end{bmatrix}$$

```
In [4]: A1 = rop(A, 'R1*(-2/9)+R2=>R2', 'R1*(-4/3)+R3=>R3', 'R1*(2/3)+R4=>R4', 'R1*
        A1
```

Out[4]:

$$\begin{bmatrix} 3 & -2 & 1 & \frac{1}{4} & 0 \\ 0 & 0 & -\frac{38}{9} & \frac{35}{18} & 3 \\ 0 & \frac{5}{3} & -\frac{5}{6} & \frac{14}{3} & 2 \\ 0 & \frac{2}{3} & \frac{5}{3} & \frac{1}{6} & 1 \\ 0 & \frac{23}{6} & -\frac{8}{3} & \frac{17}{6} & -3 \end{bmatrix}$$

```
In [5]: A2 = rop(A1, 'R2<=>R3')    # 1 row swap
        A2
```

Out[5]:

$$\begin{bmatrix} 3 & -2 & 1 & \frac{1}{4} & 0 \\ 0 & \frac{5}{3} & -\frac{5}{6} & \frac{14}{3} & 2 \\ 0 & 0 & -\frac{38}{9} & \frac{35}{18} & 3 \\ 0 & \frac{2}{3} & \frac{5}{3} & \frac{1}{6} & 1 \\ 0 & \frac{23}{6} & -\frac{8}{3} & \frac{17}{6} & -3 \end{bmatrix}$$

```
In [6]: A2 = rop(A2, 'R2*(-2/5)+R4=>R4', 'R2*(-23/10)+R5=>R5')
        A2
```

Out[6]:

$$\begin{bmatrix} 3 & -2 & 1 & \frac{1}{4} & 0 \\ 0 & \frac{5}{3} & -\frac{5}{6} & \frac{14}{3} & 2 \\ 0 & 0 & -\frac{38}{9} & \frac{35}{18} & 3 \\ 0 & 0 & 2 & -\frac{17}{10} & \frac{1}{5} \\ 0 & 0 & -\frac{3}{4} & -\frac{79}{10} & -\frac{38}{5} \end{bmatrix}$$

```
In [7]: A3 = rop(A2, 'R3*(9/19)+R4=>R4', 'R3*(-27/152)+R5=>R5')
        A3
```

Out[7]:

$$\begin{bmatrix} 3 & -2 & 1 & \frac{1}{4} & 0 \\ 0 & \frac{5}{3} & -\frac{5}{6} & \frac{14}{3} & 2 \\ 0 & 0 & -\frac{38}{9} & \frac{35}{18} & 3 \\ 0 & 0 & 0 & -\frac{74}{95} & \frac{154}{95} \\ 0 & 0 & 0 & -\frac{12533}{1520} & -\frac{6181}{760} \end{bmatrix}$$

Up to now, we did the arthtmetic in our heads. But now let's use `sympy` to do the operations with fractions, in order to compute the multiplier for row 4:

```
In [8]: -Rational(-12533,1520) * (Rational(95,-74))
```

Out[8]:

$$-\frac{12533}{1184}$$

```
In [9]: A4 = rop(A3, 'R4*(-12533/1184)+R5=>R5')
        A4
```

Out[9]:

$$\begin{bmatrix} 3 & -2 & 1 & \frac{1}{4} & 0 \\ 0 & \frac{5}{3} & -\frac{5}{6} & \frac{14}{3} & 2 \\ 0 & 0 & -\frac{38}{9} & \frac{35}{18} & 3 \\ 0 & 0 & 0 & -\frac{74}{95} & \frac{154}{95} \\ 0 & 0 & 0 & 0 & -\frac{14973}{592} \end{bmatrix}$$

We finally got the matrix in triangular form, so now all we have to do is to multiply the diagonal entries. Also, remember that we did one row swap, so we have to change the sign:

```
In [10]: -prod([A4[i,i] for i in range(5)])
```

Out[10]:

$$\frac{4991}{12}$$

Conclusion: the determinant of A is

$$\frac{4991}{12}$$

```
In [ ]:
```