For each of the items below, do the following:

1. Write the augmented matrix for the given system.
2. Use elementary row operations to transform the matrix into reduced row echelon form.
3. Write the system that corresponds to the reduced row echelon form.
4. Write the solution set of the system (using set notation).

Exercises:

1.

$$\begin{aligned} x_3 - x_4 + x_5 + 5x_6 &= 0 \\ x_1 - 2x_2 + x_3 - x_4 + x_6 &= -8 \\ -3x_1 + 6x_2 + 2x_3 - 2x_4 - x_5 + 10x_6 &= 6 \\ -2x_1 + 4x_2 - 3x_5 - 2x_6 &= 1 \end{aligned}$$

2.

$$\begin{aligned} x_1 - x_2 - 2x_3 + 2x_4 + x_5 - 3x_6 &= -1 \\ x_1 + 2x_2 + 2x_3 + 2x_4 - 28x_5 - x_6 &= 7 \\ -3x_1 - x_2 + 2x_3 - 3x_4 + 14x_5 - x_6 &= 17 \\ -2x_1 - x_2 - 2x_3 - x_4 + 22x_5 + x_6 &= -1 \\ -x_2 - 3x_4 + 18x_5 + x_6 &= -5 \end{aligned}$$

3.

$$\begin{aligned} -2x_1 - 2x_2 - 2x_3 + x_4 &= 5 \\ -x_1 - x_2 - 3x_3 - 3x_4 &= 21 \\ 2x_1 - 3x_2 - 2x_3 - 3x_4 &= 2 \\ 2x_1 + 2x_2 - 2x_3 - 2x_4 &= 14 \\ -2x_1 - 2x_2 - x_3 - x_4 &= 7 \end{aligned}$$

Solutions:

1.
$$\left\{ \begin{bmatrix} -5 \\ 0 \\ -3 \\ 0 \\ 3 \\ 0 \end{bmatrix} + \lambda_1 \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \lambda_2 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + \lambda_3 \begin{bmatrix} 2 \\ 0 \\ -3 \\ 0 \\ -2 \\ 1 \end{bmatrix} \ : \ \lambda_1, \lambda_2, \lambda_3 \in \mathbb{R} \right\}$$

2.
$$\left\{ \begin{bmatrix} -4 \\ 2 \\ 2 \\ 0 \\ 0 \\ -3 \end{bmatrix} + \lambda_1 \begin{bmatrix} 2 \\ 3 \\ 5 \\ 5 \\ 1 \\ 0 \end{bmatrix} \ : \ \lambda_1 \in \mathbb{R} \right\}$$

3.
$$\left\{ \begin{bmatrix} -3 \\ 3 \\ -4 \\ -3 \end{bmatrix} : \in \mathbb{R} \right\}$$