1 Gaussian Elimination Example

$$\begin{aligned} x + \frac{1}{2}y + 2z &= 6 \\ 2x - 3y + 4z &= 20 \\ x + 2y + z &= 1 \end{aligned}$$

Gaussian Elimination:

Augmented Matrix	Row Operations
$\begin{bmatrix} 1 & \frac{1}{2} & 2 & 6 \\ 2 & -3 & 4 & 20 \\ 1 & 2 & 1 & 1 \end{bmatrix}$	R1 * (−2) + R2 => R2, R1 * (−1) + R3 => R3
$\begin{bmatrix} 1 & \frac{1}{2} & 2 & 6 \\ 0 & -4 & 0 & 8 \\ 0 & \frac{3}{2} & -1 & -5 \end{bmatrix}$	R2 * (−1/4) => R2
$\begin{bmatrix} 1 & \frac{1}{2} & 2 & 6 \\ 0 & 1 & 0 & -2 \\ 0 & \frac{3}{2} & -1 & -5 \end{bmatrix}$	R2 * (−1/2) + R1 => R1, R2 * (−3/2) + R3 => R3
$\begin{bmatrix} 1 & 0 & 2 & 7 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & -1 & -2 \end{bmatrix}$	R3 * (−1) => R3
$\begin{bmatrix} 1 & 0 & 2 & 7 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 2 \end{bmatrix}$	R3 * (−2) + R1 => R1
$\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 2 \end{bmatrix}$	

The system corresponding to the last augmented matrix is:

$$\begin{aligned} 1x + 0y + 0z &= 3 \\ 0x + 1y + 0z &= -2 \\ 0x + 0y + 1z &= 2 \end{aligned}$$

This yields the solution:

$$\begin{aligned} x &= 3 \\ y &= -2 \\ z &= 2 \end{aligned}$$

In vector form:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 2 \end{bmatrix}$$

We could still say that the solution set of the system is:

$$\left\{ \begin{bmatrix} 3 \\ -2 \\ 2 \end{bmatrix} \right\}$$

(This denotes a set that has only one vector.)