Exam Cover Sheet

CLEVELAND STATE UNIVERSITY 1964 TESTING SERVICES

Instructors: Please complete sections 1-3 and return with all test materials to Testing Services.
Location: Rhodes West #215 **Extension: 2272** **E-mail: testingservices@csuohio.edu**

Section 1: Student and Course Information

Eric Lief
Student's Name

MTH 288
Course Name/ Section Number

Luiz Martins
Instructor's Name

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Instructor's Contact Information

Exam deadline
(last date student is allowed to take test)

55 min
Time allowed for class
(Please do not calculate extended time)

Section 2: Materials allowed- Please check all that apply

☐ Open Book ☐ Blue Scantron ☐ Internet Access ☐ Computer Access

☐ Open Note ☐ Green Scantron ☑ Calculator Other: ____

Additional instructions for proctor:
can use course software installed on his computer

Section 3: Completed test return method

Please note that delivery is not provided

☐ I will pick up in testing services (ID required)

Sign here upon pick-up: ____

☐ A designated person will pick up the test from Testing Services (ID Required)

Name of Individual: ____

Sign here upon pick-up: ____

☒ Send test via e-mail to my CSU account ____
Hard copies sent via e-mail must be picked up from Testing Services by the end of the semester

☐ Score the test with the rest of the class (bubble sheet exams only)

Testing Services Use Only:

☐ Time and a Half ☑ Double Time Time Allowed: 1h 50 min Other: ____ seat 9

Date Received: 2/17	Date Taken: 2-17	Date Returned: 2/19
Method Received: email	Start time: 10:22 End time: 12:13	Method Returned: email
Initials: DA	Proctor Initials: DA ES	Initials: DA

Name and Student ID: Eric Lief 2667664

Instructions. All solutions must be justified, unless otherwise stated. Show all work leading to your answer in each problem. Solutions without appropriate work that supports it will receive no credit. All work must be written in the test. Do not attach computer printouts to the test. If not enough space is provided for an answer, continue it in the back of the page.

Please identify your final answer to each problem by surrounding it with a rectangle.

Problem 1. (42 points.) Answer the following items based on the linear system below:

$$\begin{aligned} 2x_1 + \ 6x_2 - 9x_3 - 4x_4 &= 0 \\ -3x_1 - 11x_2 + 9x_3 - \ x_4 &= 0 \\ x_1 + \ 4x_2 - 2x_3 + \ x_4 &= 0 \end{aligned}$$

(a) (8 points.) Write the augmented matrix corresponding to this system.

$$\left[\begin{array}{cccc|c} 2 & 6 & -9 & -4 & 0 \\ -3 & -11 & 9 & -1 & 0 \\ 1 & 4 & -2 & 1 & 0 \end{array}\right]$$

(b) (16 points) Use a sequence of elementary row operations to find a matrix in reduced row echelon form that is equivalent to the matrix in Part (a). Write, in the space provided below, all row operations and intermediate matrices in your computations.

$R1*(3/2)+R2 \Rightarrow R2$

$$\begin{bmatrix} 2 & 6 & -9 & -4 & 0 \\ 0 & -2 & -9/2 & -7 & 0 \\ 1 & 4 & -2 & 1 & 0 \end{bmatrix}$$

$R1*(-1/2)+R3 \Rightarrow R3$

$$\sim \begin{bmatrix} 2 & 6 & -9 & -4 & 0 \\ 0 & -2 & -9/2 & -7 & 0 \\ 0 & 1 & 5/2 & 3 & 0 \end{bmatrix}$$

$R2*(1/2)+R3 \Rightarrow R3$

$$\sim \begin{bmatrix} 2 & 6 & -9 & -4 & 0 \\ 0 & -2 & -9/2 & -7 & 0 \\ 0 & 0 & 1/4 & -1/2 & 0 \end{bmatrix}$$

$R3*(18)+R2 \Rightarrow R2$

$$\sim \begin{bmatrix} 2 & 6 & -9 & -4 & 0 \\ 0 & -2 & 0 & -16 & 0 \\ 0 & 0 & 1/4 & -1/2 & 0 \end{bmatrix}$$

$R2*(3)+R1 \Rightarrow R1$

$$\sim \begin{bmatrix} 2 & 0 & -9 & -52 & 0 \\ 0 & -2 & 0 & -16 & 0 \\ 0 & 0 & 1/4 & -1/2 & 0 \end{bmatrix}$$

$R3*(36)+R1 \Rightarrow R1$

$$\sim \begin{bmatrix} 2 & 0 & 0 & -70 & 0 \\ 0 & -2 & 0 & -16 & 0 \\ 0 & 0 & 1/4 & -1/2 & 0 \end{bmatrix}$$

$R1*(1/2) \Rightarrow R1$

$$\sim \begin{bmatrix} 1 & 0 & 0 & -35 & 0 \\ 0 & -2 & 0 & -16 & 0 \\ 0 & 0 & 1/4 & -1/2 & 0 \end{bmatrix}$$

$R2*(1/2) \Rightarrow R2$

$$\sim \begin{bmatrix} 1 & 0 & 0 & -35 & 0 \\ 0 & 1 & 0 & 8 & 0 \\ 0 & 0 & 1/4 & -1/2 & 0 \end{bmatrix}$$

$R3 * (4) \Rightarrow R3$

$$\begin{bmatrix} 1 & 0 & 0 & -35 & 0 \\ 0 & 1 & 0 & 8 & 0 \\ 0 & 0 & 1 & -2 & 0 \end{bmatrix}$$

(Extra space for Problem 1, Part b.)

(c) (8 points.) Write the linear system that corresponds to the reduced row echelon form matrix from the previous item.

x_4 free var

$$x_1 + 0x_2 + 0x_3 - 35x_4 = 0$$
$$0x_1 + x_2 + 0x_3 + 8x_4 = 0$$
$$0x_1 + 0x_2 + x_3 - 2x_4 = 0$$

$$x_1 \quad - 35x_4 = 0$$
$$x_2 \quad + 8x_4 = 0$$
$$x_3 \quad - 2x_4 = 0$$

$s_1 = x_4$

(d) (10 points) Determine the set of solution of the system. If there are infinitely many solutions, write the solutions in parametric form, that is, using a set of independent variables (s_1, ..., etc.) to express the values of x_1, x_2, x_3 and x_4. Finally, determine if the system has zero, one or infinitely many solutions.

$s_1 = x_4$

$$x_3 = 2x_4 = 2s_1$$
$$x_2 = -8x_4 = -8s_1$$
$$x_1 = 35x_4 = 35s_1$$

$$X = \begin{bmatrix} 35s_1 \\ -8s_1 \\ 2s_1 \end{bmatrix}$$

Any $\mathbb{R}$ value for s_1 : $s_1 \in \mathbb{R}$

Thus there are infinite solutions to the system.

E.g. for $s_1 = 1, 2, \ldots \infty$

Problem 2. (10 points.) When propane burns in oxygen, it produces carbon dioxide and water:

x_1 x_2 x_3 x_4

$$C_3H_8 + O_2 \longrightarrow CO_2 + H_2O$$

We want to balance this chemical equation. Write a system of linear equations that can be used to solve this problem, using the following variables:

x_1 = (amount of propane C_3H_8 in the reagents)
x_2 = (amount of oxygen O_2 in the reagents)
x_3 = (amount of carbon dioxide CO_2 in the products)
x_4 = (amount of water H_2O in the products)

4 unknowns / vars

(molecules)

It is not necessary to solve the linear system.

C: $3x_1 + 0x_2 = x_3 + 0x_4$
H: $8x_1 + 0x_2 = 0x_3 + 2x_4$
O: $0x_1 + 2x_2 = 2x_3 + x_4$

So
$$\begin{aligned} 3x_1 \quad - x_3 \quad &= 0 \\ 8x_1 \quad\quad -2x_4 &= 0 \\ 2x_2 - 2x_3 - x_4 &= 0 \end{aligned}$$

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Problem 3. (18 points.) Determine if each of the following statements is true or false, and provide a brief justification for your answer.

(a) (3 points.) A linear system with 3 equations and 3 unknowns always has exactly one solution.

Circle one: True False

$n = m$

Justification:

m

n $\left[\begin{array}{ccc|c} 1 & & & b_1 \\ & 1 & & b_2 \\ & & 0 & b_3 \end{array}\right]$

It is possible to solve for each variable, but if $0x_1 = b_3$ is one equation, then there is no solution if $b_3 \neq 0$.

$x_3 = b_1$

$x_2 = b_2$

$0x_1 = b_3$

(b) (3 points.) If a homogeneous system has 5 equations and 8 unknowns, then it must have infinitely many solutions.

Circle one: True False

Justification:

A homogeneous system with more unknowns than equations ($m > n$), has infinitely many solutions

(c) (3 points.) The matrix below is in reduced row echelon form:

$$\begin{bmatrix} 1 & 2 & 0 & -1 & 0 & 4 \\ 0 & 0 & 1 & 1 & 0 & -2 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Circle one: True False

Justification:

RRE

1) All nonzero rows are above any zero rows
2) Leading coefficient is one (1)
3) Zeros above and below leading coefficient

(d) (3 points.) If a set of vectors $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ are linearly dependent and $\mathbf{u}_5$ is an arbitrary vector, then the vectors $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4, \mathbf{u}_5\}$ also are linearly dependent.

Circle one: True False

Justification:

The vector u_5 can be written as $0 u_5$:

2) $x_1 u_1 + x_2 u_2 + x_3 u_3 + x_4 u_4 + 0 x_5 = 0$

1) $x_1 u_1 + x_2 u_2 + x_3 u_3 + x_4 u_4 = 0$

The above linear combination has a solution since 1 does for non zero x's

(e) (3 points.) A set of 8 vectors can span $\mathbb{R}^5$.

Circle one: True False

Justification:

$A = [v_1, v_2 \cdots v_8]$ in $\mathbb{R}^5$

$0 = x_1 v_1 + x_2 v_2 + x_3 v_3 + x_4 v_4 + x_5 v_5 + x_6 v_6 + x_7 v_7 + x_8 v_8$

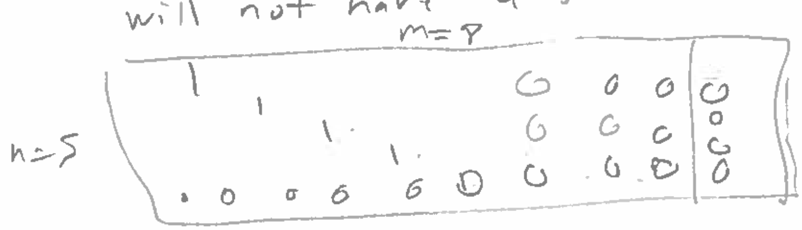
can be solved if any three variables (e.g. (x_2, x_4, x_6)) are chosen arbitrarily, given the remaining leading variables in the system solution

(f) (3 points.) Any set of 8 vectors spans $\mathbb{R}^5$.

Circle one: True False

Justification:

If x_2, x_4, x_6 are zeros the system will not have a solution or if a row is all zeros



Problem 4. (20 points.) To answer the following items, consider the following 3 vectors in $\mathbb{R}^3$:

$$\mathbf{u}_1 = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} \quad \mathbf{u}_2 = \begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix} \quad \mathbf{u}_3 = \begin{bmatrix} 7 \\ 8 \\ 12 \end{bmatrix}$$

(a) (10 points.) Determine if these vectors are linearly independent in $\mathbb{R}^3$. Show all computations, and explain your solution in terms of the definition of linear independence.

$R1 \times (-2) + R2 \Rightarrow R2$ $\qquad$ $R1 * (3) + R3 \Rightarrow R3$

$$\left[\begin{array}{ccc|c} 1 & 3 & 7 & 0 \\ 2 & 4 & 8 & 0 \\ -3 & 2 & 12 & 0 \end{array}\right] \sim \begin{bmatrix} 1 & 3 & 7 & 0 \\ 0 & -2 & -6 & 0 \\ -3 & 2 & 12 & 0 \end{bmatrix} \sim$$

$R2 * (11/2) + R3 \Rightarrow R3$ $\qquad$ $R2 * 3/2 + R1 \Rightarrow R1$

$$\begin{bmatrix} 1 & 3 & 7 & 0 \\ 0 & -2 & -6 & 6 \\ 0 & 11 & 33 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 7 & 0 \\ 0 & -2 & -6 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim$$

$R2 * \left(-\frac{1}{2}\right) \Rightarrow R2$

$$\begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & -2 & -6 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$x_1 + 0x_2 - 2x_3 = 0$$
$$0x_1 + x_2 + 3x_3 = 0$$
$$0 = 0$$

If the free variable is chosen arbitrarily, there are infinite solutions to the system, implying linear dependency. The lack of a trivial solution rules out linear independence.

(b) (10 points.) Determine if the vector

$$\mathbf{v} = \begin{bmatrix} 1 \\ 4 \\ -14 \end{bmatrix}$$

is in the span of the vectors $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$. Show all computations, and explain your solution in terms of the definition of span.

$$x_1 v_1 + x_2 u_2 + x_3 u_3 = v$$

$$x_1 \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} + x_2 \begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix} + x_3 \begin{bmatrix} 7 \\ 8 \\ 12 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ -14 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 7 & 1 \\ 2 & 4 & 8 & 4 \\ -3 & 2 & 12 & -14 \end{bmatrix} \sim$$ Same operations giving equations in (a):

$$x_1 + \quad -2x_3 = 1$$

$$x_2 + 3x_3 = 4$$

$$\boxed{0 = -14}$$

Since $0 = -14$ implies no solution, v cannot be in span(u_1, u_2, u_3)

Problem 5. (10 points.) Suppose that $\mathbf{v}$ is in the span of the vectors $\{\mathbf{u}_1, \mathbf{u}_2\}$. In other words, we are assuming that $\mathbf{v}$ can be written as a linear combination of $\mathbf{u}_1$ and $\mathbf{u}_2$:

$$\mathbf{v} = c_1\mathbf{u}_1 + c_2\mathbf{u}_2.$$

Show that v is in the span of $\{\mathbf{u}_1 + \mathbf{u}_2, \mathbf{u}_1 - \mathbf{u}_2\}$.

$V = X_1(u_1 + u_2) + X_2(u_1 - u_2)$

$V = X_1u_1 + X_1u_2 + X_2u_1 - X_2u_2$

$\underbrace{u_1(X_1 + X_2)}_{c_1} + \underbrace{u_2(X_1 - X_2)}_{c_2}$

$X_1, X_2 \in \mathbb{R} \Rightarrow$ solutions for all chosen X.

Thus, V is in span of $\{u_1 + u_2, u_1 - u_2\}$

e.g. $X_1 = 1$, $X_2 = 2$

$V = 3u_1 - u_2$, with $c_1 = 3$, $c_2 = -1$

(Eric Lief Scratch paper)

$\frac{-9/2}{1/4} =$ $4 \cdot \frac{9}{2} = 18$

$1/4$

$9 \cdot 4 = 36$

$2(35) + 6(-8) - 9(2) = 0$ $-4 \cancel{5}$

$70 \cancel{+} 48 - 18$

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