

Name: _____

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Problem 1. (10 points) The values of a “mystery” linear transformation $L : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ are known for three vectors:

$$L\left(\begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ -1 \end{bmatrix}, \quad L\left(\begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \quad L\left(\begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

Part (a). Given an arbitrary vector, find scalars c_1 , c_2 and c_3 such that:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = c_1 \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$$

Your answer should give the values of c_1 , c_2 and c_3 as a function of x , y and z . Notice that the transformation L is not used to solve this part of the problem.

(This problem continues on the next page.)

Part (b). Use the previous item to evaluate:

$$L\left(\begin{bmatrix} -7 \\ -3 \\ 1 \end{bmatrix}\right)$$

To receive full credit, your solution must show how to use the answer to the previous item and the linearity property of L to obtain the result.