Exam Cover Sheet

CLEVELAND STATE UNIVERSITY 1964 TESTING SERVICES

Instructors: Please complete sections 1-3 and return with all test materials to Testing Services.
Location: Rhodes West #215 **Extension: 2272** **E-mail: testingservices@csuohio.edu**

Section 1: Student and Course Information

Eric Lief
Student's Name

MTH 288
Course Name/ Section Number

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Instructor's Name

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Instructor's Contact Information

Exam deadline
(last date student is allowed to take test)

60 min
Time allowed for class
(Please do not calculate extended time)

Section 2: Materials allowed- Please check all that apply

☐ Open Book ☐ Blue Scantron ☐ Internet Access ☑ Computer Access

☐ Open Note ☐ Green Scantron ☐ Calculator Other: can use software on his computer

Additional instructions for proctor:

Section 3: Completed test return method

Please note that delivery is not provided

☐ I will pick up in testing services (ID required)

Sign here upon pick-up:

☐ A designated person will pick up the test from Testing Services (ID Required)

Name of Individual:

Sign here upon pick-up:

☑ Send test via e-mail to my CSU account

Hard copies sent via e-mail must be picked up from Testing Services by the end of the semester

☐ Score the test with the rest of the class (bubble sheet exams only)

Testing Services Use Only:

☐ Time and a Half ☑ Double Time Time Allowed: 2hrs Other: 9

Date Received: 4/13
Method Received: email
Initials: DA

Date Taken: 4/13/16
Start time: 10:49 End time: 12:49
Proctor Initials: ES ES

Date Returned:
Method Returned:
Initials:

Name and Student ID: Eric Lief 2667664

Instructions. All solutions must be justified, unless otherwise stated. Show all work leading to your answer in each problem. Solutions without appropriate work that supports it will receive no credit. All work must be written in the test. Do not attach computer printouts to the test. If not enough space is provided for an answer, continue it in the back of the page.

Please identify your final answer to each problem by surrounding it with a rectangle.

Total: 87

Problem 1.(20 points.) Find a basis for the subspace of $\mathbb{R}^3$ spanned by the vectors:

10 points

$$u_1 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} \quad u_2 = \begin{bmatrix} -4 \\ 4 \\ -8 \end{bmatrix} \quad u_3 = \begin{bmatrix} 2 \\ 0 \\ -3 \end{bmatrix} \quad u_4 = \begin{bmatrix} -4 \\ -2 \\ 13 \end{bmatrix}$$

pivots | dependent vars

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1/2 \\ 0 & 0 & 1 & -3 \end{bmatrix} \xleftarrow{\text{row}^*} \begin{bmatrix} 1 & -4 & 2 & -4 \\ -2 & 4 & 0 & -12 \\ 2 & -8 & -3 & 13 \end{bmatrix}$$

free var $\underline{s}$

$x_4 = s$

$x_3 = 3x_4 = 3s$

$x_2 = 1/2\, x_4 = 1/2\, s$

$x_1 = 0$

Reduced row echelon form of matrix is incorrect: -5

Obtaining basis from reduced row echelon form incorrect: -5

$$x = s\begin{bmatrix} 0 \\ 1/2 \\ 3 \\ 1 \end{bmatrix}$$

So $B = \left\{ \begin{bmatrix} 0 \\ 1/2 \\ 3 \\ 1 \end{bmatrix} \right\}$

0
2
6
2

Problem 2.(20 points.) Answer the following items for the matrix:

$$A = \begin{bmatrix} 1 & 3 & 2 & 0 \\ 3 & 11 & 7 & 1 \\ 1 & 1 & 4 & 0 \end{bmatrix}$$

(a) (10 points.) Find a basis for the range of A, and determine the dimension of the range.

$$B \underset{\text{row}^*}{\sim} \begin{bmatrix} \underbrace{\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}}_{\text{pivots}} & \underbrace{\begin{matrix} -5/3 \\ 1/3 \\ 1/3 \end{matrix}}_{\text{free var}} \end{bmatrix}$$

Since there are 3 nonzero rows/pivots the dimension of row and col space is $\mathbb{R}^3$; rank = 3

The range = col (A) & corresponds to independent (pivot columns) of original matrix A = subspace spanned by columns :

$$\text{range}(A) = \text{span}\left\{ \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 11 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 7 \\ 4 \end{bmatrix} \right\}$$

(Problem continues on next page).

(b) (10 points.) Find a basis for the kernel of A, and determine the dimension of the kernel.

null space

$Ax = 0$

Solve system

$$A = \begin{bmatrix} 1 & 3 & 2 & 0 \\ 3 & 11 & 7 & 1 \\ 1 & 1 & 4 & 0 \end{bmatrix}$$

$$\overset{\text{row}^*}{\sim} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -5/3 & 0 \\ 0 & 1 & 0 & 1/3 & 0 \\ 0 & 0 & 1 & 1/3 & 0 \end{array}\right]$$

$x_4 = s$

$x_3 = -1/3 x_4 = -1/3\, s$

$x_2 = -1/3 x_4 = -1/3\, s$

$x_1 = 5/3 x_4 = 5/3\, s$

$\Rightarrow$

3 s

−1 s

−1 s

5 s

removing fraction) assume $x_4 = 3s$

$$X = s \begin{bmatrix} 3 \\ -1 \\ -1 \\ 5 \end{bmatrix}$$

20 points

$$\ker(A) = \operatorname{null}(A) = \operatorname{span}\left\{ \begin{bmatrix} 3 \\ -1 \\ -1 \\ 5 \end{bmatrix} \right\}$$

This means that any linear combination of this vector will result in a b (Combining) = 0 ?

$$\begin{bmatrix} 1 & 3 & 2 & 0 \\ 3 & 11 & 7 & 1 \\ 1 & 1 & 4 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \\ -1 \\ 5 \end{bmatrix} = \begin{bmatrix} 3 - 3 - 2 + 0 \\ \\ \end{bmatrix}$$

Problem 3. (16 points.) Find scalars a and b that make the following matrix identity true.

16 points

$$\begin{bmatrix} a & 1 & 2 \\ 2 & 2 & b \\ -1 & 4 & 1 \end{bmatrix}\begin{bmatrix} 2 & 2 \\ 1 & -1 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} 8 & -4 \\ 11 & -\frac{4}{3} \\ 5 & -8 \end{bmatrix}$$

$$\begin{bmatrix} 2a + 1 + 6 & 2a - 1 - 4 \\ 4 + 2 + 3b & 4 - 2 - 2b \\ -2 + 4 + 3 & -2 - 4 - 2 \end{bmatrix} = \begin{bmatrix} 8 & -4 \\ 11 & -4/3 \\ 5 & -8 \end{bmatrix}$$

$$\begin{bmatrix} 2a + 7 & 2a - 5 \\ 6 + 3b & 2 - 2b \\ 5 & -8 \end{bmatrix}$$

$2a + 7 = 8$

$6 + 3b = 11$

$2a - 5 = -4$

$2 - 2b = -4/3$

$2a = 1$

$\boxed{a = 1/2}$

$3b = 5$

$\boxed{b = 5/3}$

$-2b = -4/3 - 2 = -4/3 + -\frac{6}{3} = \frac{-10}{3} \cdot \frac{1}{2} = \frac{10}{6} = \frac{5}{3}$

$b =$

Problem 4. (16 points.) Find the determinants for the following matrices. Explain the method you used to find the determinant, showing all computations. Do not use Python for this problem.

(a) (8 points.) $\det \begin{bmatrix} 1 & 0 & 3 \\ 2 & -2 & 1 \\ 3 & 4 & 1 \end{bmatrix}$

16 points

cofactor 1 2 3

$(-1)^{1+1}(1)\begin{vmatrix} -2 & 1 \\ 4 & 1 \end{vmatrix} + (-1)^{1+2}(0)\begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} + (-1)^{1+3}(3)\begin{vmatrix} 2 & -2 \\ 3 & 4 \end{vmatrix}$

1) $(1)\begin{vmatrix} -2 & 1 \\ 4 & 1 \end{vmatrix} = -2 - 4 = -6$

2) $(-1)(0)\begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} = 0$

3) $(1)(3)\begin{vmatrix} 2 & -2 \\ 3 & 4 \end{vmatrix} = (3)[8-(-6)] = 3(14) = 42$

$\det(A) = -6 + 0 + 42 = 36$

(b) (8 points.) $\det \begin{bmatrix} 2 & 3 & -5 & 9 & 0 \\ 0 & -1 & 3 & -4 & 2 \\ 0 & 0 & 3 & 7 & -11 \\ 0 & 0 & 0 & \frac{1}{2} & -1 \\ 0 & 0 & 0 & 0 & 10 \end{bmatrix}$

Upper triangular matrix

det = diagonal

$= 2(-1)(3)(\frac{1}{2})(10) = -30$

15 points

Problem 5. (18 points.) Determine if each of the following statements is true or false, and provide a brief justification for your answer.

(a) (3 points.) A linear transformation $T : \mathbb{R}^8 \to \mathbb{R}^4$ must be onto.

All correct except (a)

Circle one: True False

Justification:

$$4\begin{bmatrix} 1 & a & a & \cdots \\ a & & & \cdots \\ a & a & & \cdots \\ a & a & a & \cdots \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_8 \end{bmatrix} 8 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

range $\in \mathbb{R}^4$

So if $x = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{bmatrix}$ $T(x) = b = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$

$\forall x \in \mathbb{R}^8$ and $b \in \mathbb{R}^4$; b will always map to at least one vector $x \Rightarrow$ onto

Also the cols of A span $\mathbb{R}^4$ = range

(b) (3 points.) A linear transformation $T : \mathbb{R}^8 \to \mathbb{R}^4$ can be one-to-one.

Circle one: True False

Justification:

For a b such as $\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$ there can be multiple x's, e.g.

$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ \vdots \\ 10 \\ 11 \\ 12 \\ 13 \end{bmatrix}$, etc. $\Rightarrow$ $T : \mathbb{R}^8 \to \mathbb{R}^4$ cannot be 1-to-1

(c) (3 points.) There is exactly one value of a for which the matrix below is singular:

= not invertible

$\Rightarrow$ det = 0

$$A = \begin{bmatrix} a & 2 \\ 2 & a \end{bmatrix}$$

Circle one: True False

Justification:

$$\det(A) = a^2 - 4 = 0$$

$$a^2 = 4$$

$$\left\{ \begin{array}{l} a_1 = 2 \\ a_2 = -2 \end{array} \right\}$$

(d) (3 points.) The two matrices below have the same determinant.

$$\begin{bmatrix} a & b & c \\ d & f & g \\ h & i & j \end{bmatrix} \quad \begin{bmatrix} a & b+2a & c \\ d & f+2d & g \\ h & i+2h & j \end{bmatrix}$$

add multiple of column 1 to column 2 $\Rightarrow$ doesn't change det(A)

Circle one: True False

Justification:

(e) (3 points.) If $\mathbf{u}_1$, $\mathbf{u}_2$ and $\mathbf{u}_3$ are distinct vectors in $\mathbb{R}^4$, then the subspace spanned by $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ has dimension 3.

Circle one: True False

Justification:

The subspace may be smaller if the vecs aren't linearly independent

(e) (3 points.) If all entries of a matrix are positive, then the matrix is invertible.

Circle one: True False

Justification:

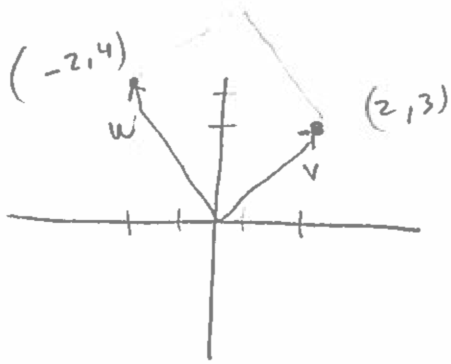
They can still result in 0:

$\det\left\{\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}\right\} = 2 - 2 = 0 \Rightarrow$ non invertible / singular

Problem 6. (10 points.) Find the area of the parallelogram in $\mathbb{R}^2$ determined by the vectors

10 points

$$\begin{bmatrix} 2 \\ 3 \end{bmatrix} \text{ and } \begin{bmatrix} -2 \\ 4 \end{bmatrix}$$



$$A = \begin{pmatrix} u_1 & u_2 \\ v_1 & v_2 \end{pmatrix} = \begin{bmatrix} -2 & 4 \\ 2 & 3 \end{bmatrix}$$

$$\text{Area} = |\det(A)| = |-6 - 8| = |-14| = 14$$