CLEVELAND STATE UNIVERSITY 1964 TESTING SERVICES

Exam Cover Sheet

Total: 83

Instructors: Please complete sections 1-3 and return with all test materials to Testing Services.

Location: Rhodes West #215 Extension: 2272 E-mail: testingservices@csuohio.edu

Section 1: Student and Course Information

ERIC LIEF	MTH 288
Student's Name	Course Name/ Section Number
Martins	x4683
Instructor's Name	Instructor's Contact Information
	2 hrs.
Exam deadline *(last date student is allowed to take test)*	Time allowed for class *(Please do not calculate extended time)*

Section 2: Materials allowed- Please check all that apply

☐ Open Book ☐ Blue Scantron ☐ Internet Access ☐ Computer Access

☐ Open Note ☐ Green Scantron ☒ Calculator Other: No books or notes

Additional instructions for proctor:

can use the software in his computer

Section 3: Completed test return method

Please note that delivery is not provided

☐ I will pick up in testing services (ID required)

Sign here upon pick-up: ______

☐ A designated person will pick up the test from Testing Services (ID Required)

Name of Individual: ______

Sign here upon pick-up: ______

☒ Send test via e-mail to my CSU account l.martins@csuohio.edu

Hard copies sent via e-mail must be picked up from Testing Services by the end of the semester

☐ Score the test with the rest of the class (bubble sheet exams only)

Testing Services Use Only:

☐ Time and a Half ☒ Double Time Time Allowed: 4hrs. Other: ______ 20

Date Received: 5/9	Date Taken: 5-9	Date Returned: 5-9
Method Received: email	Start time: 10:44 End time: 2:20	Method Returned: email
Initials: SL	Proctor Initials: MC ES	Initials: SL

Stop at 2:44

Name and Student ID: Eric Lief 2667664

Instructions. All solutions must be justified, unless otherwise stated. Show all work leading to your answer in each problem. Solutions without appropriate work that supports it will receive no credit. All work must be written in the test. Do not attach computer printouts to the test. If not enough space is provided for an answer, continue it in the back of the page.

Please identify your final answer to each problem by surrounding it with a rectangle.

Problem 1. (15 points.) Determine the set of solutions of the system:

7 points

No eqv system, no solution set, no number of solutions

$$\begin{bmatrix} 1 & 2 & 2 & 2 & -3 \\ 2 & 0 & -3 & 3 & 2 \\ 5 & 3 & 4 & 1 & -2 \\ 4 & 9 & 16 & 1 & -15 \\ 7 & 4 & 9 & -3 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 14 \\ -32 \\ 37 \\ 143 \\ 92 \end{bmatrix}$$

Solve this problem by using elementary row operations to find the reduced row echelon form the system's augmented matrix. You can use the compute to perform the row operations, but *you must report all the row operations performed and the resulting matrix of each step of the solution process.* Finally, determine if the system has zero, one or infinitely many solutions.

$R1 * (-2) + R2 \Rightarrow R2$
$R1 * (-5) + R3 \Rightarrow R3$
$R1 * (-4) + R4 \Rightarrow R4$
$R1 * (-7 + R5 \Rightarrow R5$

$$\sim \begin{bmatrix} 1 & 2 & 2 & 2 & -3 \\ 0 & -4 & -7 & -1 & 8 \\ 0 & -7 & -6 & -9 & 13 \\ 0 & 1 & 8 & -7 & -3 \\ 0 & -10 & -5 & -17 & 18 \end{bmatrix}$$

$R2 * (-7/4) + R3 \Rightarrow R3$
$R2 * (1/4) + R4 \Rightarrow R4$
$R2 * (-10/4) + R5 \Rightarrow R5$

$$\sim \begin{bmatrix} 1 & 2 & 2 & 2 & -3 \\ 0 & -4 & -7 & -1 & 8 \\ 0 & 0 & 25/4 & -29/4 & -1 \\ 0 & 0 & 25/4 & -29/4 & -1 \\ 0 & 0 & 25/2 & -29/2 & -2 \end{bmatrix}$$

(Extra space for Problem 1.)

$R3 * (-1) + R4 \Rightarrow R4$
$R3 * (-2) + R5 \Rightarrow R5$

$$\sim \begin{bmatrix} 1 & 2 & 2 & 2 & -3 \\ 0 & -4 & -7 & -1 & 8 \\ 0 & 0 & 25/4 & -29/4 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R3 * (28/25) + R2 \Rightarrow R2$
$R3 * (-8/25) + R1 \Rightarrow R1$

$$\begin{bmatrix} 1 & 2 & 0 & \frac{108}{25} & \frac{-67}{25} \\ 0 & -4 & 0 & -228/25 & \frac{172}{25} \\ 0 & 0 & 25/4 & -29/4 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R2 * (1/2) + R1 \Rightarrow R1$

$$\begin{bmatrix} 1 & 0 & 0 & -6/25 & \frac{19}{25} \\ 0 & -4 & 0 & -227/25 & 172/25 \\ 0 & 0 & 25/4 & -29/4 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R2 * (-\frac{1}{4}) \Rightarrow R2$
$R3 * (4/25) \Rightarrow R3$

$$\begin{bmatrix} 1 & 0 & 0 & -6/25 & 19/25 \\ 0 & 1 & 0 & 57/25 & -43/25 \\ 0 & 0 & 1 & -29/25 & -4/25 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Problem 2. (10 points.) Determine if the vectors below linearly independent. Show all computations, and explain your solution in terms of the definition of linear independence.

10 points

$$\begin{array}{ccc} u_1 & v_2 & v_3 \\ \begin{bmatrix} 2 \\ -3 \\ 2 \\ 4 \end{bmatrix}, & \begin{bmatrix} 1 \\ -1 \\ 2 \\ 2 \end{bmatrix}, & \begin{bmatrix} -2 \\ 5 \\ 2 \\ -4 \end{bmatrix} \end{array}$$

$$\text{let } A = \begin{bmatrix} 2 & 1 & -2 \\ -3 & -1 & 5 \\ 2 & 2 & 2 \\ 4 & 2 & -4 \end{bmatrix} \overset{rop^*}{\sim} \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\left.\begin{aligned} x_1 &= 3x_3 = 3s \\ x_2 &= -4x_3 = -4s \\ x_3 &= s \end{aligned}\right\}$$

$$X = s\begin{bmatrix} 3 \\ -4 \\ 1 \end{bmatrix}$$

Infinite solutions, let $s = 1 \Rightarrow X = \begin{bmatrix} 3 \\ -4 \\ 1 \end{bmatrix}$

$$\Rightarrow \quad 3u_1 - 4v_2 + v_3 = 0$$

$$u_3 = -3u_1 + 4v_2$$

$$\begin{bmatrix} -2 \\ 5 \\ 2 \\ 4 \end{bmatrix} = -3\begin{bmatrix} 2 \\ -3 \\ 2 \\ 4 \end{bmatrix} + 4\begin{bmatrix} 1 \\ -1 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ 5 \\ 2 \\ -4 \end{bmatrix}$$

Thus v_3 is <u>linearly dependent</u> on the other two vectors

and the set $\{v_1, v_2, v_3\}$ is LD.

Problem 3. (15 points) Let:

15 points

$$\mathbf{u} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} 4 \\ k \\ k \end{bmatrix}$$

Find a value of k such that $\mathbf{w}$ is in the span of $\mathbf{u}$, $\mathbf{v}$. Your solution must contain an explanation of how you found k.

If w is in the span of u, v, then it is linearly dependent and there is a solution to

$Ax = W$

where $A = [u \ v]$. Solving the system:

$$\begin{bmatrix} 1 & -1 & 4 \\ 2 & 3 & K \\ 1 & 2 & K \end{bmatrix} \overset{R1*(-2)+R2 \Rightarrow R2}{\sim} \begin{bmatrix} 1 & -1 & 4 \\ 0 & 5 & K-8 \\ 1 & 2 & K \end{bmatrix}$$

$$\overset{R1*(-1)+R3 \Rightarrow R3}{\sim} \begin{bmatrix} 1 & -1 & 4 \\ 0 & 5 & K-8 \\ 0 & 3 & K-4 \end{bmatrix} \overset{(-3/5)R2+R3 \Rightarrow R3}{\sim} \begin{bmatrix} 1 & -4 & 4 \\ 0 & 5 & K-8 \\ 0 & 0 & 2/5K+4/5 \end{bmatrix}$$

$(-3/5)(K-8)$
$-3/5K+3/5$
$-3/5K$

$$x_1 = 4x_2 - 4x_3$$

$$x_2 = \frac{-K+8}{5} = \frac{8-K}{5}$$

$$x_3 = \frac{3/5K}{2/5K+4/5} =$$

$$x_3 = \frac{3K}{2K+4} = \frac{3K}{2K} + \frac{3K}{4} = \frac{3}{2} + \frac{1}{4}K$$

back

Problem 4. (15 points.) Find all values of the scalar a for which the matrix below is singular:

15 points

$$A = \begin{bmatrix} a & 2 & 1 \\ 3 & 1 & -2 \\ 2 & 1 & a \end{bmatrix}$$

a matrix A is singular iff det (A) = 0

$$\begin{array}{ccccc} a & 2 & 1 & a & 2 \\ 3 & 1 & -2 & 3 & 1 \\ 2 & 1 & a & 2 & 1 \end{array}$$

$$\det(A) = a \cdot 1 \cdot a + 2(-2)(2) + 1(3) \cdot 1 - 2(1)(1) - 1(-2)a - 3(2)a = 0$$

$$a^2 - 8 + 3 - 2 + 2a - 6a = 0$$

$$a^2 - 4a - 7 = 0$$

$$(a + 4)(a + 3) = 0 \quad \text{can't factor}$$

$$\frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-7)}}{2(1)} = \frac{\sqrt{16 + 28}}{2} \quad \frac{4 \pm \sqrt{44}}{2}$$

$$= \frac{4 \pm \sqrt{11 \cdot 4}}{2} = \frac{4 \pm 2\sqrt{11}}{2} = 2 \pm \sqrt{11}$$

Thus for $a = (2 + \sqrt{11},\ 2 - \sqrt{11})$

8 points

Problem 5. (12 points.) Determine if each of the following statements is true or false, and provide a brief justification for your answer. *Solutions without justivication will receive no credit.*

(a) (2 points.) A homogeneous linear system with 3 equations and 5 unknowns always has infinitely many solutions.

Circle one: True False

Justification:

$$\begin{array}{cc} 3 & 2 \\ \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} & \text{free vars} \end{array}$$

There are 3 ind. vars & 2 "free" vars chosen freely yielding ∞ solutions

(b) (3 points.) The columns space of the matrix below has dimension 3:

$$\begin{bmatrix} 1 & 2 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

independent vars $\Rightarrow$ dim = 2
pivot colums

Circle one: True False

Justification:

The col & row space of a matrix depend on pivot rows/cols only. Thus the dimension of both col & row space here is 2 (not 3).

(c) (2 points.) The matrix below has rank 2:

$$\begin{bmatrix} 1 & 2 & 0 & -1 & 0 & 4 \\ 0 & 0 & 1 & 1 & 0 & -2 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Circle one: True False

Justification:

pivot cols (independent) $\Rightarrow$
range has dimension 2
$\Rightarrow$ rank 2

(d) (2 points.) If a set of vectors $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ are linearly dependent and $\mathbf{u}_5$ is an arbitrary vector, then the vectors $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4, \mathbf{u}_5\}$ also are linearly dependent.

Circle one: True False

Justification:

Because the original set is not linearly independent, expanding it w/ any vector (i.e v_5) will not change the fact (even if v_5) is not linearly dependent on $u_1 - v_4$.

(e) (2 points.) If $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is a basis of $\mathbb{R}^3$ and A is a 3×3 matrix, then $\{A\mathbf{u}_1, A\mathbf{u}_2, A\mathbf{u}_3\}$ is also a basis of $\mathbb{R}^3$

Circle one: True False

Justification:

Since $\{v_1, v_2, v_3\}$ span $\mathbb{R}^3$

The transformation $T(v) = Av = b$

Since it is $A^{3\times3}$ will guarantee that b ~~$\in$~~ $\mathbb{R}^3$ spans

i.e. $\mathbb{R}^3 \to \mathbb{R}^3$

(f) (2 points.) The two matrices below have the same determinant.

$$\begin{bmatrix} a & b & c \\ d & f & g \\ h & i & j \end{bmatrix} \quad \begin{bmatrix} b+2a & a & c \\ f+2d & d & g \\ i+2h & h & j \end{bmatrix}$$

Circle one: True False

Justification:

A multiple of a col is added to col 2, which (col 1) wouldn't change the determinant, but then cols 1 & 2 are swapped, changing the sign of the determinant.

Problem 6. (10 points) Find a basis for the range of the matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 & 0 \\ -1 & 1 & 2 & -4 \end{bmatrix} \underset{\sim}{\text{row*}} \overset{\text{RRE}}{\begin{bmatrix} 1 & -1 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{bmatrix}}$$

pivot | indep. cols

5 points

Thus rank = 2

$$x_1 = x_2 - 2x_4 = s_1 - 2s_2$$
$$x_3 = x_4 = 0s_1 + s_2$$
$$x_2 = s_1 = 1s_1 + 0s_2$$
$$x_4 = s_2 = 0s_1 + 1s_2$$

$$\Rightarrow s \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} -2 \\ 0 \\ 2 \\ 1 \end{bmatrix}$$

Thus range is span $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 2 \\ 1 \end{bmatrix} \right\}$

Basis of the range is a subspace of R^2, not R^4

Problem 7. (15 points) Determine if the matrix below is diagonalizable:

15 points

$$\begin{bmatrix} 2 & 0 & -4 & 0 \\ -4 & -2 & 4 & 0 \\ 0 & 0 & -2 & 0 \\ 2 & 2 & -2 & 2 \end{bmatrix}$$

If the matrix is diagonalizable, find a diagonal matrix D and a matrix P such that $P^{-1}AP = D$.

$$\det(A - \lambda I) = \lambda^4 - 8\lambda^2 + 16 = (\lambda-2)^2(\lambda+2)^2$$

$\lambda_1 = 2$ multiplicity 2

$\lambda_2 = -2$ 〃

For $\lambda_1 = 2$

$A - 2I = 0$

$$\begin{bmatrix} 0 & 0 & -4 & 0 \\ -4 & -4 & 4 & 0 \\ 0 & 0 & -4 & 0 \\ 2 & 2 & -2 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} x_1 &= -x_2 &&= -1s_1 + 0s_2 \\ x_3 &= 0 &&= 0s_1 + 0s_2 \\ x_2 &= s_1 &&= 1s_1 + 0s_2 \\ x_4 &= s_2 &&= 0s_1 + 1s_2 \end{aligned} \Rightarrow s_1\begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + s_2\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\left(\lambda_1, \left\{\begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}\right)\right)$$

dim of eigenspace = 2

for $\lambda_2 = -2$

$A + 2I = 0$

$$\sim \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} x_1 &= x_3 &&= 1s_1 &&= 1s_1 + 0s_2 \\ x_2 &= -2x_4 &&= -2s_2 &&= 0s_1 - 2s_2 \\ x_3 &= s_1 & & &&= 1s_1 + 0s_2 \\ x_4 &= s_2 & & &&= 0s_1 + 1s_2 \end{aligned}$$

$$\left(\lambda_2, \left\{\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ 0 \\ 1 \end{bmatrix}\right\}\right)$$

dim = 2

Problem 8. (8 points) Find the general solution of the system of differential equations:

8 points

$$\mathbf{y}'(t) = \begin{bmatrix} -2 & -2 \\ 0 & 3 \end{bmatrix} \mathbf{y}(t)$$

Let $A = \begin{bmatrix} -2 & -2 \\ 0 & 3 \end{bmatrix}$

Find Eigenvals:

$\det(A - \lambda I) = \lambda^2 - \lambda - 6 = (\lambda - 3)(\lambda + 2)$

$\lambda_1 = 3$

$\lambda_2 = -2$

Eigenvecs:

$A - 3I = \begin{bmatrix} -5 & -2 \\ 0 & 0 \end{bmatrix}$

$-5x_1 = 2x_2$

$x_1 = -\frac{2}{5}x_2 = -2s$

$x_2 = 5s$

$\left(3, \begin{bmatrix} -2 \\ 5 \end{bmatrix}\right) \Rightarrow s\begin{bmatrix} -2 \\ 5 \end{bmatrix} = v_1$

$A + 2I = \begin{bmatrix} 0 & -2 \\ 0 & 5 \end{bmatrix}$

$x_2 = 0$

$x_1 = s$

$\left(-2, \begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) \Rightarrow s\begin{bmatrix} 1 \\ 0 \end{bmatrix} = v_2$

$y = c_1 e^{\lambda_1 t} v_1 + c_2 e^{\lambda_2 t} v_2$

$= c_1 e^{3t}\begin{bmatrix} -2 \\ 5 \end{bmatrix} + c_2 e^{-2t}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$$y = c e^{t} + u$$
$$y(t) = P w(t)$$
$$w(t) = P^{-1} A P w(t)$$
$$w(t) = D w(t)$$

$$y' = \begin{bmatrix} 2 \\ 0 \end{bmatrix} y_1 + \begin{pmatrix} -2 \\ 3 \end{pmatrix} y_2$$

$$P = \begin{bmatrix} -2 & \cancel{5} \\ 5 & \cancel{0} \end{bmatrix}$$

$$D = \begin{bmatrix} 3 & \\ & -2 \end{bmatrix}$$

$$\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ k \\ k \end{bmatrix}$$

$$x_1 = 4 + x_2$$
$$2x_1 + 3x_2 = 1x_1 + \cancel{2x_2}$$
$$\cancel{x_2 = x_1 + 2x_2 - 2x_1}$$
$$x_2 = -x_1$$
$$x_1 = 4 + (-x_1)$$
$$x_1 = 2$$
$$x_2 = -2$$

$$2\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} - 2\begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix} = \begin{pmatrix} 4 \\ -2 \\ -2 \end{pmatrix}$$
$$\begin{pmatrix} 2 \\ 4 \end{pmatrix} + \begin{pmatrix} 2 \\ -6 \\ -4 \end{pmatrix} =$$

$0 \quad 0 \quad 0 =$

$\mathbb{R}^3 \to \mathbb{R}^3$

$\det(A - \lambda I)$

$$P = \begin{bmatrix} -1 & 0 & 1 & 0 \\ 1 & 0 & 0 & -2 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 2 & & & \\ & 2 & & \\ & & -2 & \\ & & & -2 \end{bmatrix}$$

Singular $\Rightarrow$ no inverse

det = 0

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a=1$ $b=-4$ $c=-7$

$a^2 - 4a - 7$

$2 \cdot 11$

$11 \cdot 4$

$11 \cdot 22$

$2 \cdot 22$

$\frac{4}{7}$ 7

$\frac{7}{4}$ -7

$(-4)\frac{7}{4}$ -7

$25/4$

$\frac{2}{25/4} = 2 \cdot \frac{4}{25} = \frac{8}{25}$

$\frac{25/2}{25/4}$

$\frac{25}{2} \cdot \frac{4}{25} = 2$

$\frac{7}{25/4} = 7 \cdot \frac{4}{25} = \frac{28}{25}$

-7 $\frac{28}{25}$

$-3/5\,k + \frac{3}{5} \cdot 8 + k - 4$

$K - 3/5K + \frac{24}{5} - 4$

$K - 3/5K + \frac{4}{5}$

$2/5\,K + 4/5$