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**Problem 1.** (10 points) The values of a “mystery” linear transformation  $L : \mathbb{R}^3 \rightarrow \mathbb{R}^2$  are known for three vectors:

$$L\left(\begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} -3 \\ -1 \end{bmatrix}, \quad L\left(\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 3 \end{bmatrix}, \quad L\left(\begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

**Part (a).** Given an arbitrary vector, find scalars  $c_1$ ,  $c_2$  and  $c_3$  such that:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = c_1 \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$$

Your answer should give the values of  $c_1$ ,  $c_2$  and  $c_3$  as a function of  $x$ ,  $y$  and  $z$ . Notice that the transformation  $L$  is not used to solve this part of the problem.

(This problem continues on the next page.)

**Part (b).** Use the previous item to evaluate:

$$L\left(\begin{bmatrix} 3 \\ -2 \\ 3 \end{bmatrix}\right)$$

To receive full credit, your solution must show how to use the answer to the previous item and the linearity property of  $L$  to obtain the result.