Linear-Algebra-Examples

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1 Loading the libraries.

Note: File `latools.py` must be in same directory as working notebook.

```
In [1]: from latools import *
        from sympy import *
        init_printing(use_latex=False)
```

2 Summary of Commands

2.1 Matrix input

To input matrices, use the function `matrix_to_rational()`. The argument is a two-dimensional array, as shown below:

```
A = matrix_to_rational([11,12,13],
                       [21,22,23],
                       [31,32,33]]);
```

2.2 Row operations

To do row operations on a matrix, use the function `rop()`:

```
output_matrix = rop(input_matrix, sequence_of_operations)
```

The syntax for row operations is:

- `'Ri*(c)+Rj=>Rj'`: Multiply row `i` by the scalar `c` and add to row `j`. The result is stored in row `j`.
- `'Ri*(c)=>Ri'`: Multiply row `i` by the scalar `c`. The result is stored in row `i`.
- `'Ri<=>Rj'`: Swap rows `i` and `j`

Notes:

- The parenthesis around the scalar `c` are always required.
- The row operations must be specified as strings, that is, they have to be surrounded by quotes.

2.3 Symbolic variables

To introduce symbolic variables, use the function `symbols()`. For example:

```
x, y, z = symbols('x,y,z')
```

This introduces three symbolic variables named `x`, `y`, `z`.

2.4 Examples

2.4.1 Example 1

```
In [2]: A = matrix_to_rational([[ 0, 0, -1, 2],
                                [ 2, 1,  1, 1],
                                [-3, 2,  0, 0]])
        A
```

Out[2]:

$$\begin{bmatrix} 0 & 0 & -1 & 2 \\ 2 & 1 & 1 & 1 \\ -3 & 2 & 0 & 0 \end{bmatrix}$$

```
In [3]: A1 = rop(A, 'R1<=>R2')
        A1
```

Out[3]:

$$\begin{bmatrix} 2 & 1 & 1 & 1 \\ 0 & 0 & -1 & 2 \\ -3 & 2 & 0 & 0 \end{bmatrix}$$

```
In [4]: A2 = rop(A1, 'R1*(1/2)=>R1')
        A2
```

Out[4]:

$$\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & -1 & 2 \\ -3 & 2 & 0 & 0 \end{bmatrix}$$

```
In [5]: A3 = rop(A2, 'R1*(3)+R3=>R3')
        A3
```

Out[5]:

$$\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & -1 & 2 \\ 0 & \frac{7}{2} & \frac{3}{2} & \frac{3}{2} \end{bmatrix}$$

```
In [6]: A4 = rop(A3, 'R2<=>R3')
        A4
```

Out[6]:

$$\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{7}{2} & \frac{3}{2} & \frac{3}{2} \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

```
In [7]: A5 = rop(A4, 'R2*(2/7)=>R2')
        A5
```

Out[7]:

$$\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & \frac{3}{7} & \frac{3}{7} \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

```
In [8]: A6 = rop(A5, 'R2*(-1/2)+R1=>R1')
        A6
```

Out[8]:

$$\begin{bmatrix} 1 & 0 & \frac{2}{7} & \frac{2}{7} \\ 0 & 1 & \frac{3}{7} & \frac{3}{7} \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

```
In [9]: A7 = rop(A6, 'R3*(-1)=>R3')
        A7
```

Out[9]:

$$\begin{bmatrix} 1 & 0 & \frac{2}{7} & \frac{2}{7} \\ 0 & 1 & \frac{3}{7} & \frac{3}{7} \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

```
In [10]: A8 = rop(A7, 'R3*(-2/7)+R1=>R1', 'R3*(-3/7)+R2=>R2')
         A8
```

Out[10]:

$$\begin{bmatrix} 1 & 0 & 0 & \frac{6}{7} \\ 0 & 1 & 0 & \frac{9}{7} \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

2.5 Example 2

An example with symbolic variables on the right-hand side of the system.

```
In [11]: x, y = symbols('x,y')
         A = matrix_to_rational([[ 1,2,x],
                                 [-2,3,y]])
         A
```

Out[11]:

$$\begin{bmatrix} 1 & 2 & x \\ -2 & 3 & y \end{bmatrix}$$

```
In [12]: A1 = rop(A, 'R1*(2)+R2=>R2')
         A1
```

Out[12]:

$$\begin{bmatrix} 1 & 2 & x \\ 0 & 7 & 2x + y \end{bmatrix}$$

```
In [13]: A2 = rop(A1, 'R2*(1/7)=>R2')
         A2
```

Out[13]:

$$\begin{bmatrix} 1 & 2 & x \\ 0 & 1 & \frac{2x}{7} + \frac{y}{7} \end{bmatrix}$$

```
In [15]: A3 = rop(A2, 'R2*(-2)+R1=>R1')
         A3
```

Out[15]:

$$\left[\begin{matrix}1 & 0 & \frac{3x}{7} - \frac{2y}{7}\\0 & 1 & \frac{2x}{7} + \frac{y}{7}\end{matrix}\right]$$

In [ ]: