Exam Cover Sheet

Instructors: Please complete sections 1-3 and return with all test materials to Testing Services.
Location: Rhodes West #215 **Extension: 2272** **E-mail: testingservices@csuohio.edu**

Section 1: Student and Course Information

Eric Lief
Student's Name

MTH288
Course Name/ Section Number

Martins
Instructor's Name

Instructor's Contact Information

Exam deadline
(last date student is allowed to take test)

60 mins
Time allowed for class
(Please do not calculate extended time)

Section 2: Materials allowed- Please check all that apply

- ☐ Open Book
- ☐ Blue Scantron
- ☐ Internet Access
- ☐ Computer Access
- ☐ Open Note
- ☐ Green Scantron
- ☑ Calculator
- Other:

Additional instructions for proctor:
The student can access the course software from his computer

Section 3: Completed test return method

Please note that delivery is not provided

☐ I will pick up in testing services (ID required)

Sign here upon pick-up:

☐ A designated person will pick up the test from Testing Services (ID Required)

Name of Individual:

Sign here upon pick-up:

☑ Send test via e-mail to my CSU account l.martins@csuohio.edu

Hard copies sent via e-mail must be picked up from Testing Services by the end of the semester

☐ Score the test with the rest of the class (bubble sheet exams only)

Testing Services Use Only:

☐ Time and a Half ☑ Double Time Time Allowed: 2hrs Other: Seat# 9

Date Received: 5/4
Method Received: email
Initials: DA

Date Taken: 5/4/16
Start time: 10:29 End time: 12:02
Proctor Initials: ES ES

Date Returned:
Method Returned:
Initials:

Re: Exam

Luiz F Martins

Tue 5/3/2016 8:12 PM

To:testingservices <testingservices@csuohio.edu>;

1 attachment (118 KB)

test03-mth288-spring2016.pdf;

Enclosed please find the test. This is a 60 min test, and the student can access the course software from his computer.

Thanks, Felipe Martins

From: testingservices
Sent: Tuesday, May 3, 2016 1:03:50 PM
To: Eric Lief; Luiz F Martins
Cc: Eric A Lief
Subject: Re: Exam

Hello Instructor Martins,

We had an error in our scheduling. We would like you to know that Eric Lief is taking his exam in out office tomorrow 5/4/16 at 10:15 a.m. If you could please email us a copy of the exam and cover sheet or drop off a copy at our office, it would be greatly appreciated. Thank you.

Best Regards,
Cleveland State University
Testing Services
2124 Chester Avenue
Rhodes Tower West #215
Cleveland Ohio 44115
216-687-2272 - Phone
216-687-2212 - Fax

From: Eric Lief <ericlief@me.com>
Sent: Tuesday, May 3, 2016 11:17 AM
To: testingservices; Luiz F Martins
Cc: Eric A Lief
Subject: Exam

Hi I am scheduled to take an exam for nth 288 tomorrow at 10:15 and am confirming this.
Thank you.

Best,

Eric

On Apr 18, 2016, at 12:20 PM, testingservices <testingservices@csuohio.edu> wrote:

> Hello,
> We are now scheduling final exams in Testing Services. Please confirm that the following appointments are correct:
>
> | Martins (MTH 288/1 Final Exam): 5/9/16 10:15 |
> |---|
>
> Best Regards,
> Cleveland State University
> Testing Services
> 2124 Chester Avenue
> Rhodes Tower West #215
> Cleveland Ohio 44115
> 216-687-2272 - Phone
> 216-687-2212 - Fax

Name and Student ID: Eric Lief

Instructions. All solutions must be justified, unless otherwise stated. Show all work leading to your answer in each problem. Solutions without appropriate work that supports it will receive no credit. All work must be written in the test. Do not attach computer printouts to the test. If not enough space is provided for an answer, continue the solution on the back of the page.

Unless where specified otherwise, you can use Python to do the following computations:

- Compute the determinant of a matrix.
- Factor polynomials or solve equations.
- Do any matrix operations, including inversion and solving systems

If you use the software, please state where and how you are using it. Remember that you are still required to completely justify your answers, with a careful description of the solution process. Also remember that all solutions should be given as *exact* values, not decimal approximations.

Please identify your final answer to each problem by surrounding it with a rectangle.

Total score: 100 points

Problem 1.(30 points.) Let A be the matrix:

30 points

$$A = \begin{bmatrix} -19 & -10 & 20 \\ -5 & -14 & 10 \\ -15 & -15 & 21 \end{bmatrix}$$

(a) (15 points) Find all the eigenvalues of A, and determine the multiplicity of each eigenvalue.

$\det(A - \lambda I_3) = -\lambda^3 - 12\lambda^2 + 27\lambda + 486$

$= -(\lambda - 6)(\lambda + 9)^2 = [-9, 6]$

$\lambda_1 = 6$ (mult 1) $\lambda_2 = -9$ (multiplicity 2)

For $\lambda_1 = 6$:

$A - 6I = 0$

$\begin{bmatrix} -25 & -10 & 20 \\ -5 & -20 & 10 \\ -15 & -15 & 15 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -2/3 \\ 0 & 1 & -1/3 \\ 0 & 0 & 0 \end{bmatrix}$ 1 free var $\dim(A) = 1$

$x_1 = 2/3\, x_3 = (2/3)\, 3s = 2s$

$x_2 = 1/3\, x_3 = (1/3)\, 3s = 1s$ $X = s\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$

$x_3 = 3s$

Eigenspace $\left(\lambda_1, \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}\right)$ dim 1

For $\lambda_2 = -9$

$\begin{bmatrix} -10 & -10 & 20 \\ -5 & -5 & 10 \\ -15 & -15 & 30 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ 2 free vars $\Rightarrow \dim(A) = 2$

$x_1 + x_2 - 2x_3 = 0$

$x_1 = -x_2 + 2x_3 = -s_1 + 2s_2$

$x_2 = 1s_1 + 0s_2$

$x_3 = 0s_1 + 1s_2$

$X = s_1\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + s\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$

(This problem continues on the next page.)

Eigenspace $\left(\lambda_2, \left\{\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}\right\}\right)$ Basis dim 2

$\dim\left(\left\{\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}\right\}\right) = 3 \in \mathbb{R}^3$

(b) (15 points) Find a basis for the eigenspace associated to each of the eigenvalues of A.

check:

$\lambda_1 = 6$

$u_1 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$

$$A\lambda_1 = A\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 12 \\ 6 \\ 18 \end{bmatrix} = 6\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} = 6u_1 = \lambda_1 u_1$$

$$A\lambda_2 = A\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 9 \\ -9 \\ 0 \end{bmatrix} = -9\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} = -9u_2 = \lambda_2 v_2$$

$$A\lambda_3 = A\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -18 \\ 0 \\ -9 \end{bmatrix} = -9\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} = -9v_2 = \lambda_2 v_3$$

Problem 2. (30 points.) Answer the following items for the following basis of $\mathbb{R}^3$:

30 points

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \right\}$$

(a) (15 points) Convert the following vector to a coordinate vector with respect to the standard basis:

$$[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} = A[X]_B = 2u_1 - 1u_2 + 3u_3$$

$$= \begin{bmatrix} 2+1+3 \\ 0-2-3 \\ 0-1 \quad 0 \end{bmatrix} = \begin{bmatrix} 6 \\ -5 \\ -1 \end{bmatrix}$$

we can check .

$$[X]_B = A^{-1} X = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} \checkmark$$

(This problem continues on the next page.)

(b) (15 points) Convert the following vector to a coordinate vector with respect to the basis $\mathcal{B}$:

$$\mathbf{x} = \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$$

$$[X]_B = A^{-1}X = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -1 \\ 0 & 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$$

$$[X]_B = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 0 & 1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \\ 3 \end{bmatrix}$$

We can check:

$$X = A[x]_B$$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} -3 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix} \checkmark$$

Problem 3. (30 points.) Let A be a 4×4 matrix. The following information is given about the eigenvalues and corresponding eigenspaces of the matrix A:

30 points

Eigenvalue: $\lambda_1 = 1$; Basis for eigenspace: $\left\{ \begin{bmatrix} 2 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \\ 1 \end{bmatrix} \right\}$

Eigenvalue: $\lambda_2 = -2$; Basis for eigenspace: $\left\{ \begin{bmatrix} 0 \\ 1 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 4 \\ 1 \end{bmatrix} \right\}$

Is it possible to determine the matrix A using only this information? If possible, find the matrix A. If not, explain why.

Let $P = \begin{bmatrix} 2 & 0 & 0 & 3 \\ 0 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

Since $A \in R^{4\times4}$ & we have 4 linearly independent eigenvectors $\Rightarrow \mathcal{B} = \{v_1, v_2, v_3, v_4\} \in \mathbb{R}^4$, i.e. the basis spans all of $\mathbb{R}^4$, it is diagonalizable:

Let $D = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}$

Furthermore $\det(P) = -3 \Rightarrow$ invertible

$P^{-1} = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 1/3 & 8/3 & -1 & 1/3 \\ -2/3 & -7/3 & 1 & 1/3 \\ 1/3 & 2/3 & 0 & -2/3 \end{bmatrix}$

$A = PDP^{-1} = \begin{bmatrix} -2 & -6 & 0 & 6 \\ 1 & 6 & -3 & 1 \\ 2 & 13 & -8 & 5 \\ 1 & 5 & -3 & 2 \end{bmatrix}$

Check

$P^{-1}AP = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}$ ✓

10 points

Problem 4. (10 points.) Let a be a scalar and consider the matrix:

$$A = \begin{bmatrix} a & 1 \\ 0 & a \end{bmatrix}$$

Answer the following items for the matrix A. Notice that your answers will depend on the unspecified scalar a

(a) Determine the eigenvalues of the matrix A, and specify the multiplicity of each eigenvalue.

$$\det(A-\lambda I) = \det\left\{\begin{bmatrix} a-\lambda & 1 \\ 0 & a-\lambda \end{bmatrix}\right\}$$

$$= (a-\lambda)^2 - 1 = (a-\lambda)(a-\lambda) = a^2 - 2\lambda a + \lambda^2$$

$$= (a-\lambda)(a-\lambda) = (a-\lambda)^2$$

$\lambda = a$ with multiplicity $= 2$

(b) Find a basis for the eigenspace corresponding to each of the eigenvalues you found in the previous item.

$$A - \lambda I = 0$$

$$A - aI = \begin{bmatrix} a & 1 \\ 0 & a \end{bmatrix} - \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = 0$$

$$x_2 = 0 \Rightarrow x_2 = 0s$$

$$x_1 = s$$

$$x = s\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\mathcal{B} = \left\{\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right\}$$

Check:

$$Au = \lambda u$$

$$\begin{bmatrix} a & 1 \\ 0 & a \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} a \\ 0 \end{bmatrix} = a\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} a \\ 0 \end{bmatrix}$$

(This problem continues on the next page.)

(c) Determine if the matrix A is diagonalizable or not, and justify your answer.

The dimension of the basis in (b)
is one. Thus, it does not
span $\mathbb{R}^2$ ($A^{2\times2}$). It
is therefore not diagonalizable.
In order for it to be diagonalizable,
we would need the dimension of the
eigenspace to equal the multiplicity
(here equal to 2).