1 Questions

For each of the following statements determine if it is true or false, and provide a brief justification for your answer.

It is *guaranteed* that one or more of these questions will be in the test, in *exactly the same form* as they appear below.

1. The set of solutions of the linear system:

$$\begin{bmatrix} 1 & 2 & -1 \\ 2 & 3 & 5 \\ -4 & -2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

   is a linear subspace of the euclidean vector space $\mathbb{R}^3$.

2. If $T : \mathbb{R}^3 \to \mathbb{R}^4$ is a linear transformation, then T is one-to-one.
3. If $T : \mathbb{R}^4 \to \mathbb{R}^3$ is a linear transformation, then T is onto.
4. If $T : \mathbb{R}^8 \to \mathbb{R}^5$ is a linear transformation, then T can be onto.
5. If $T : \mathbb{R}^8 \to \mathbb{R}^5$ is a linear transformation, then T can be one-to-one.
6. If $T : \mathbb{R}^4 \to \mathbb{R}^4$ is a linear transformation, then T can be invertible.
7. If a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ is one-to-one, then T must also be onto.
8. If A and B are $n \times n$ matrices, then $(A + B)(A - B) = A^2 - B^2$.
9. If A is an $n \times n$ matrix and $A^3 = 0$, then $A = 0$.
10. If A and B are $n \times n$ matrices, then $(AB)^T = B^T A^T$
11. If A is an invertible $n \times n$ matrix, then the number of solutions to $A\mathbf{x} = \mathbf{b}$ is always greater than zero.
12. If the columns of an $n \times n$ matrix A span $\mathbb{R}^n$, then A is nonsingular.
13. If A and B are invertible, then $(AB)^{-1} = A^{-1}B^{-1}$.
14. The matrix

$$\begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix}$$

   is invertible.

15. If A is a 5×3 matrix, then $\mathrm{null}(A)$ forms a subspace of $\mathbb{R}^5$.
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17. If $T : \mathbb{R}^5 \to \mathbb{R}^8$ is a linear transformation, then $\operatorname{range}(T)$ forms a subspace of $\mathbb{R}^8$.
18. The dimension of a subspace of $\mathbb{R}^5$ is one of the integers 1, 2, 3 or 4.
19. Let $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ be any set of 3 vectors in $\mathbb{R}^5$. Then, it is always possible to add 2 vectors to the set to form a basis of $\mathbb{R}^5$.
20. The only subspace of $\mathbb{R}^8$ that has dimension 8 is $\mathbb{R}^8$ itself.
21. $\mathbb{R}^3$ has infinitely many subspaces of dimension 2.
22. The rank of a matrix cannot exceed the number of rows of A.
23. If A is not a square matrix, than the dimension of the row space of A is different of the dimension of the column space of A.
24. If the system $A\mathbf{x} = \mathbf{b}$ has solutions, than $\mathbf{b}$ is in the row space of A.
25. If A is an $n \times n$ matrix with all positive entries, then $\det(A)$ is positive.
26. If A is an upper triangular square matrix, then $\det(A) \neq 0$.
27. If A and B are $n \times n$ matrices, then $\det(A + B) = \det(A) + \det(B)$.
28. If A is an $n \times n$ matrix and c is a scalar, then $\det(cA) = c^n \det(A)$
29. If a and b are real numbers that are not both zero, then the matrix

$$\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

is nonsingular.

30. Let A be an $n \times n$ matrix such that $\det(A) \neq 0$. Then the columns of A are a basis for $\mathbb{R}^n$.