00-Useful-Commands

January 6, 2017

```
In [1]: from latools import *
        from sympy import *
        init_printing(use_latex=True)
```

1 Gram-Schmidt

Input: $\{\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_k\}$ a linearly independent set of vectors in $\mathbb{R}^n$
Output: $\{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_k\}$, and orthogonal basis of span$\{\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_k\}$.
Algorithm:

- Let $\mathbf{v}_1 = \mathbf{u}_1$
- Let $\mathbf{v}_j = \mathbf{u}_j - \dfrac{\mathbf{u}_j \cdot \mathbf{v}_1}{\mathbf{v}_1 \cdot \mathbf{v}_1}\mathbf{v}_1 - \dfrac{\mathbf{u}_j \cdot \mathbf{v}_2}{\mathbf{v}_2 \cdot \mathbf{v}_2}\mathbf{v}_2 - \cdots - \dfrac{\mathbf{u}_j \cdot \mathbf{v}_{j-1}}{\mathbf{v}_{j-1} \cdot \mathbf{v}_{j-1}}\mathbf{v}_{j-1}$ for $j = 2, \dots, k$

The first three steps are, more explicitly:

$$\mathbf{v}_1 = \mathbf{u}_1$$

$$\mathbf{v}_2 = \mathbf{u}_2 - \frac{\mathbf{u}_2 \cdot \mathbf{v}_1}{\mathbf{v}_1 \cdot \mathbf{v}_1}\mathbf{v}_1$$

$$\mathbf{v}_3 = \mathbf{u}_3 - \frac{\mathbf{u}_3 \cdot \mathbf{v}_1}{\mathbf{v}_1 \cdot \mathbf{v}_1}\mathbf{v}_1 - \frac{\mathbf{u}_3 \cdot \mathbf{v}_2}{\mathbf{v}_2 \cdot \mathbf{v}_2}\mathbf{v}_2$$

2 Matrices

2.1 Matrix Input

```
In [2]: A = matrix_to_rational([[1,2,3],
                                [4,5,6],
                                [7,8,9]])

        A
```

Out[2]:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

2.2 Row Operations

```
In [3]: A = matrix_to_rational([[1,2,3],[4,5,6],[7,8,9]])
        A
```

Out[3]:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

2.2.1 Add a multiple of a row to another row:

Notice that you can put more than one row opeartion in a single command.

```
In [4]: A1 = rop(A, 'R1*(-4)+R2=>R2', 'R1*(-7)+R3=>R3')
        A1
```

Out[4]:

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & -6 & -12 \end{bmatrix}$$

2.2.2 Multiply a row by a scalar

```
In [5]: A2 = rop(A1, 'R2*(-1/3)=>R2')
        A2
```

Out[5]:

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -6 & -12 \end{bmatrix}$$

2.2.3 Swap rows

```
In [6]: A = matrix_to_rational([[0,2,1],[-1,0,3],[2,4,-2]])
        A
```

Out[6]:

$$\begin{bmatrix} 0 & 2 & 1 \\ -1 & 0 & 3 \\ 2 & 4 & -2 \end{bmatrix}$$

Since the entry a_{11} is zero, to get a pivot on row 1 we need to swap rows:

```
In [7]: A1 = rop(A, 'R1<=>R2')
        A1
```

Out[7]:

$$\begin{bmatrix}-1 & 0 & 3\\0 & 2 & 1\\2 & 4 & -2\end{bmatrix}$$

2.3 Identity Matrix

```
In [8]: eye(5) # Identity matrix, 5x5
```

Out[8]:

$$\begin{bmatrix}1 & 0 & 0 & 0 & 0\\0 & 1 & 0 & 0 & 0\\0 & 0 & 1 & 0 & 0\\0 & 0 & 0 & 1 & 0\\0 & 0 & 0 & 0 & 1\end{bmatrix}$$

2.4 Reduced row echelon form

```
In [9]: R = reduced_row_echelon_form(A)
        R
```

Out[9]:

$$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

```
In [10]: x,y,z = symbols('x,y,z')
         B = matrix_to_rational([[1,2,3,x],
                                 [4,5,6,y],
                                 [7,8,9,z]])
         # Ignore last column of augmented matrix
         R = reduced_row_echelon_form(B, extra_cols=1)
         R
```

Out[10]:

$$\begin{bmatrix}1 & 0 & -1 & -\frac{5x}{3} + \frac{2y}{3}\\0 & 1 & 2 & \frac{4x}{3} - \frac{y}{3}\\0 & 0 & 0 & x - 2y + z\end{bmatrix}$$

2.5 Vector input

```
In [11]: v1 = Matrix([1,-1,1])
         v1
```

Out[11]:

$$\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

2.6 Vector with fractional entries

```
In [12]: v2 = Matrix([sympify('1/2'),sympify('-2/3'),4])
         v2
```

Out[12]:

$$\begin{bmatrix} \frac{1}{2} \\ -\frac{2}{3} \\ 4 \end{bmatrix}$$

2.7 Matrices with Symbolic variables

```
In [13]: x, y, z, t = symbols('x,y,z,t')
         A = matrix_to_rational([[1,-1,2,5,x],
                                 [0,1,-2,4,y],
                                 [1,-1,1,-3,z],
                                 [1,1,1,2,t]])
         A
```

Out[13]:

$$\begin{bmatrix} 1 & -1 & 2 & 5 & x \\ 0 & 1 & -2 & 4 & y \\ 1 & -1 & 1 & -3 & z \\ 1 & 1 & 1 & 2 & t \end{bmatrix}$$

```
In [14]: k = symbols('k') # Use "symbols" even if there is only one symbol
         A = matrix_to_rational([[1,k,2],
                                 [2,0,k],
                                 [1,1,5]])
         A
```

Out[14]:

$$\begin{bmatrix} 1 & k & 2 \\ 2 & 0 & k \\ 1 & 1 & 5 \end{bmatrix}$$

2.8 Characteristic polynomial

```
In [15]: A = matrix_to_rational([[1,1,1],[-1,-1,1],[2,2,3]])
         A
```

Out[15]:

$$\begin{bmatrix} 1 & 1 & 1 \\ -1 & -1 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$

In [16]:
```
lbd = symbols('lambda')
p = det(A - lbd*eye(3)) # eye(3) is the 3x3 identity
p
```

Out[16]:

$$-\lambda^3 + 3\lambda^2 + 4\lambda$$

In [17]:
```
factor(p)
```

Out[17]:

$$-\lambda\left(\lambda - 4\right)\left(\lambda + 1\right)$$

3 Matrix and vector operations

In [18]:
```
# Multiplication
A * v1
```

Out[18]:

$$\begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$$

In [19]:
```
A * v2
```

Out[19]:

$$\begin{bmatrix} \frac{23}{6} \\ \frac{25}{6} \\ \frac{35}{3} \end{bmatrix}$$

In [20]:
```
# Determinant
det(A)
```

Out[20]:

$$0$$

In [21]:
```
# Inversion
P = matrix_to_rational([[1,0,-1],
                        [2,1,3],
                        [2,1,1]])
P**(-1)
```

Out[21]:

$$\begin{bmatrix} 1 & \frac{1}{2} & -\frac{1}{2} \\ -2 & -\frac{3}{2} & \frac{5}{2} \\ 0 & \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

In [22]:
```
# Transposition
P.T
```

Out[22]:

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 1 \\ -1 & 3 & 1 \end{bmatrix}$$

In [23]:
```
# Dot product
v1.dot(v2)
```

Out[23]:

$$\frac{31}{6}$$

In [24]:
```
# Length (norm)
v1.norm()
```

Out[24]:

$$\sqrt{3}$$

In [ ]: