03-Linear-System-Practice-Worked-Example

January 6, 2017

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In [12]: from latools import *
         from sympy import *
         init_printing(use_latex=True)
```

1 Augmented Matrix

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In [13]: A = matrix_to_rational([[-2, -3, -9,  2, 0, -16,  0],
                                  [-1,  2,  6, -2, 1,  15,  6],
                                  [-1, -3, -9, -3, 1,  -1, 21],
                                  [2,   0,  0, -1, 0,   0,  1]])
         A
```

Out[13]:

$$\begin{bmatrix} -2 & -3 & -9 & 2 & 0 & -16 & 0 \\ -1 & 2 & 6 & -2 & 1 & 15 & 6 \\ -1 & -3 & -9 & -3 & 1 & -1 & 21 \\ 2 & 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix}$$

2 Row Operations

```
In [14]: A1 = rop(A, 'R1*(-1/2)=>R1')
         A1
```

Out[14]:

$$\begin{bmatrix} 1 & \frac{3}{2} & \frac{9}{2} & -1 & 0 & 8 & 0 \\ -1 & 2 & 6 & -2 & 1 & 15 & 6 \\ -1 & -3 & -9 & -3 & 1 & -1 & 21 \\ 2 & 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix}$$

```
In [15]: A2 = rop(A1, 'R1*(1)+R2=>R2', 'R1*(1)+R3=>R3', 'R1*(-2)+R4=>R4')
         A2
```

Out[15]:

$$\begin{bmatrix} 1 & \frac{3}{2} & \frac{9}{2} & -1 & 0 & 8 & 0 \\ 0 & \frac{7}{2} & \frac{21}{2} & -3 & 1 & 23 & 6 \\ 0 & -\frac{3}{2} & -\frac{9}{2} & -4 & 1 & 7 & 21 \\ 0 & -3 & -9 & 1 & 0 & -16 & 1 \end{bmatrix}$$

In [16]: A3 = rop(A2, 'R2*(2/7)=>R2')
         A3

Out[16]:

$$\begin{bmatrix} 1 & \frac{3}{2} & \frac{9}{2} & -1 & 0 & 8 & 0 \\ 0 & 1 & 3 & -\frac{6}{7} & \frac{2}{7} & \frac{46}{7} & \frac{12}{7} \\ 0 & -\frac{3}{2} & -\frac{9}{2} & -4 & 1 & 7 & 21 \\ 0 & -3 & -9 & 1 & 0 & -16 & 1 \end{bmatrix}$$

In [17]: A4 = rop(A3, 'R2*(-3/2)+R1=>R1', 'R2*(3/2)+R3=>R3', 'R2*(3)+R4=>R4')
         A4

Out[17]:

$$\begin{bmatrix} 1 & 0 & 0 & \frac{2}{7} & -\frac{3}{7} & -\frac{13}{7} & -\frac{18}{7} \\ 0 & 1 & 3 & -\frac{6}{7} & \frac{2}{7} & \frac{46}{7} & \frac{12}{7} \\ 0 & 0 & 0 & -\frac{37}{7} & \frac{10}{7} & \frac{118}{7} & \frac{165}{7} \\ 0 & 0 & 0 & -\frac{11}{7} & \frac{6}{7} & \frac{26}{7} & \frac{43}{7} \end{bmatrix}$$

In [18]: A5 = rop(A4, 'R3*(-7/37)=>R3')
         A5

Out[18]:

$$\begin{bmatrix} 1 & 0 & 0 & \frac{2}{7} & -\frac{3}{7} & -\frac{13}{7} & -\frac{18}{7} \\ 0 & 1 & 3 & -\frac{6}{7} & \frac{2}{7} & \frac{46}{7} & \frac{12}{7} \\ 0 & 0 & 0 & 1 & -\frac{10}{37} & -\frac{118}{37} & -\frac{165}{37} \\ 0 & 0 & 0 & -\frac{11}{7} & \frac{6}{7} & \frac{26}{7} & \frac{43}{7} \end{bmatrix}$$

In [19]: A6 = rop(A5, 'R3*(-2/7)+R1=>R1', 'R3*(6/7)+R2=>R2', 'R3*(11/7)+R4=>R4')
         A6

Out[19]:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -\frac{13}{37} & -\frac{35}{37} & -\frac{48}{37} \\ 0 & 1 & 3 & 0 & \frac{2}{37} & \frac{142}{37} & -\frac{78}{37} \\ 0 & 0 & 0 & 1 & -\frac{10}{37} & -\frac{118}{37} & -\frac{165}{37} \\ 0 & 0 & 0 & 0 & \frac{16}{37} & -\frac{48}{37} & -\frac{32}{37} \end{bmatrix}$$

In [20]: A7 = rop(A6, 'R4*(37/16)=>R4')
         A7

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -\frac{13}{37} & -\frac{35}{37} & -\frac{48}{37} \\ 0 & 1 & 3 & 0 & \frac{2}{37} & \frac{142}{37} & -\frac{78}{37} \\ 0 & 0 & 0 & 1 & -\frac{10}{37} & -\frac{118}{37} & -\frac{165}{37} \\ 0 & 0 & 0 & 0 & 1 & -3 & -2 \end{bmatrix}$$

In [21]: A8 = rop(A7, 'R4*(13/37)+R1=>R1', 'R4*(-2/37)+R2=>R2', 'R4*(10/37)+R3=>R3'
         A8

Out[21]:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -2 & -2 \\ 0 & 1 & 3 & 0 & 0 & 4 & -2 \\ 0 & 0 & 0 & 1 & 0 & -4 & -5 \\ 0 & 0 & 0 & 0 & 1 & -3 & -2 \end{bmatrix}$$