This handout contains hints for some homework problems that explore the geometry of systems of linear equations. For most of these exercises, the strategy is to reformulate the problem as a linear system and then consider what can be concluded from the solution of the system.

1 Problem 7

This problem uses the notation:

$$\mathbf{i} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \mathbf{j} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad \mathbf{k} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

This notation is common in engineering and physics but not much in mathematics. We use $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$ instead.

2 Problem 8

Note: This problems represents most vectors as points, instead of a column vector. This makes no difference. They probably write it this way to save space on the page.

(a) Find the vector from the points $P = (-4, -1, 1)$ to the point $Q = (4, 3, 2)$.

Solution: The vector that connects two points is just the difference between the points, considered as vectors:

$$Q - P = (4 - (-4), 3 - (-1), 2 - 1) = (8, 4, 1)$$

(b) Consider the vector equation of the line through the two points listed above. For each equation listed below, answer **T** if the equation represents the line, and **F** if it does not.

Solution Recall that the vector form of the equation of a line in space is:

$$S + tv$$

where S is a point on the line, v is a vector parallel to the line, and t is a free parameter. All items can be answered using this characterization. Here are a few examples:

1. $(x, y, z) = (-4, -1, 1) + t(8, 4, 1)$: The "starting" point in this representation is $(-4, -1, 1)$, which is one of the two given points, so it is on the line connecting P and Q. The vector $(8, 4, 1)$ is the difference between the two given points, so it is parallel to the line.

2. $(4, 3, 2) + t(4, 2, 1/2)$: The "starting" point is $(4, 3, 2)$, which is on the line connecting P and Q. The vector $(4, 2, 1/2)$ is collinear to $(8, 4, 1)$ because:

$$(8, 4, 1) = 2(4, 2, 1/2)$$

So, this equation also represents the same line.

3 Problems 9, 12

Find the line of intersection of the planes $x + 4y + 3z = -4$ and $x + 4z = 0$.

Solution: “Intersection” means the set of points that are in both planes simultaneously, so we need to solve the system of equations:

$$\begin{aligned} x + 4y + 3z &= -4 \\ x \qquad + 4z &= 0 \end{aligned}$$

So, we can just go ahead and use Gaussian Elimination, find the RREF, and write the solution in vector form. That will be the equation of the intersecting line.

4 Problem 13

Find an equation of a plane containing the three points $(0, 4, -4)$, $(3, 3, -7)$, $(3, 4, -5)$ in which the coefficient of x is 1.

Solution: The equation of a plane is:

$$ax + by + cz = d$$

where a, b, c and d are scalars. All we have to do is to find these coefficients.

They tell us that the coefficient of x is 1, that is, $a = 1$. So the equation reduces to:

$$x + by + cz = d$$

We are given three points on the plane. So, each of the points must satisfy the equation. For example, plugging in $(0, 4, -4)$ on the equation we get:

$$0 + b \cdot 4 + c \cdot (-4) = d$$

This gives the equation:

$$4b - 4c - d = 0$$

By doing the same with the other two points, we get a linear system for the unknowns b, c and d, and from the solution of the system we can get the equation of the plane.