Name: ____________________

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Problem 1. (10 points) The values of a "mystery" linear transformation $L : \mathbb{R}^3 \to \mathbb{R}^2$ are known for three vectors:

$$L\left(\begin{bmatrix}-3\\-5\\3\end{bmatrix}\right) = \begin{bmatrix}2\\-3\end{bmatrix}, \quad L\left(\begin{bmatrix}1\\2\\-1\end{bmatrix}\right) = \begin{bmatrix}3\\-3\end{bmatrix}, \quad L\left(\begin{bmatrix}1\\3\\-2\end{bmatrix}\right) = \begin{bmatrix}3\\3\end{bmatrix}$$

Part (a). Given an arbitrary vector, find scalars c_1, c_2 and c_3 such that:

$$\begin{bmatrix}x\\y\\z\end{bmatrix} = c_1\begin{bmatrix}-3\\-5\\3\end{bmatrix} + c_2\begin{bmatrix}1\\2\\-1\end{bmatrix} + c_3\begin{bmatrix}1\\3\\-2\end{bmatrix}$$

Your answer should give the values of c_1, c_2 and c_3 as a function of x, y and z. Notice that the transformation L is not used to solve this part of the problem.

(This problem continues on the next page.)

Part (b). Use the previous item to evaluate:

$$L\left(\begin{bmatrix} 7 \\ 12 \\ -7 \end{bmatrix}\right)$$

To receive full credit, your solution must show how to use the answer to the previous item and the linearity property of L to obtain the result.