What is a System

This activity describes what is a system of ordinary differential equations.

After completing this section, students should be able to do the following:

- Recognize a system of ODEs.
- Interpret a system of ODEs as a vector field.
- Understand the representation of a vector field in the phase plane.

To introduce the concept of a system of ODEs, let's consider the *Lotka-Volterra predator-prey model*, which describes two animal species in the wild, traditionally referred to as "rabbits" (the prey) and "foxes" (the predator). Denoting by $R(t)$ and $F(t)$ the rabbit and fox populations, respectively, we use the following differential equations to model the dynamics of the two species in the wild:

$$R' = aR - bRF \tag{1}$$
$$F' = cF + dRF \tag{2}$$

How do ecologists come up with this model? This is not a simple question, since there usually are many possible ways of modeling a real-world system. As a rule of thumb, scientists try to use the simplest possible model as a first approximation. For the predator prey model, the following are reasonable hypothesis:

- In the absence of predators, the prey population grows without limits. We can model this by an exponential growth model:

  $$R' = aR \text{ where } a > 0.$$

- In the absence of prey, the predator population decays to zero, since they don't have any available food. This can be modeled by an exponential decay model:

  $$F' = -cF \text{ where } b > 0.$$

- To model the interactions between the two populations, we assume that the number of possible "predation situations" is proportional to the number of possible "encounters" between individuals of the two populations, which can be approximated by the product RF.

Learning outcomes:
Author(s):

- Since predation is disadvantageous to the prey population, we subtract a term proportional to RF to the prey equation:

$$R' = aR - bRF \text{ where } a, c > 0.$$

- Since predation is beneficial to the predator population, we add a term proportional to RF to the predator population:

$$F' = -cF + dRF \text{ where } b, d > 0.$$

Putting all together, we get the pair of equations 1, 2.

Problem 1 *Let's consider now consider two populations x and y that are in competition with each other. Let's get a system of ODEs for x, y based on some simple assumptions.*

Suppose that, in isolation, each of the populations grows exponentially. Write below expressions for the derivatives, using the variables $a > 0$ and $c > 0$, respectively, for the growth rates for x and y, respectively:

Question 1.1 $x' = \boxed{ax}$

Question 1.2 $y' = \boxed{bx}$