What Is A System

Another Ximera activity

To introduce the concept of a system of ODEs, let's consider the *Lotka-Volterra predator-prey model*, which describes two animal species in the wild, traditionally referred to as "rabbits" (the prey) and "foxes" (the predator). Denoting by $R(t)$ and $F(t)$ the rabbit and fox populations, respectively, we use the following differential equations to model the dynamics of the two species in the wild:

$$R' = aR - bRF \tag{1}$$

$$F' = cF + dRF \tag{2}$$

How do ecologists come up with this model? This is not a simple question, since there usually are many possible ways of modeling a real-world system. As a rule of thumb, scientists try to use the simplest possible model as a first approximation. For the predator prey model, the following are reasonable hypothesis:

In the absence of predators, the prey population grows without limits. We can model this by an exponential growth model:

$$R' = aR \text{ where } a > 0.$$

In the absence of prey, the predator population decays to zero, since they don't have any available food. This can be modeled by an exponential decay model:

$$F' = -cF \text{ where } b > 0.$$

To model the interactions between the two populations, we assume that the number of possible "predation situations" is proportional to the number of possible "encounters" between individuals of the two populations, which can be approximated by the product RF. Since predation is disadvantageous to the prey population, we subtract a term proportional to RF to the prey equation:

$$R' = aR - bRF \text{ where } a, c > 0.$$

Since predation is beneficial to the predator population, we add a term proportional to RF to the predator population:

$$F' = -cF + dRF \text{ where } b, d > 0.$$

Putting all together, we get the pair of equations 1, 2.

Problem 1 *Let's consider now consider two populations x and y that are in competition with each other. We make the following assumptions:*

Learning outcomes:
Author(s):

- *In isolation, each of the populations grows exponentially. We denote the growth rates for x and y by $a > 0$ and $c > 0$, respectively.*
- *The effect of competition to each population is proportional to the product of the populations sizes, xy. We denote the corresponding proportionality constants for populations x and y by $b > 0$ and $d > 0$, respectively.*

Question 1.1 $x' = \boxed{ax - bxy}$

Question 1.2 $y' = \boxed{cy - dxy}$