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L4DC tutorial
July 15, 2024

# supervised and reinforcement learning under requirements

# Constrained reinforcement learning

## Agenda

Constrained reinforcement learning

CMDP duality

CRL algorithms



## Reinforcement learning

- Model-free framework for decision-making in Markovian settings



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- Model-free framework for decision-making in Markovian settings

$$\Pr\left(s_{t+1} \mid \{s_u, a_u\}_{u \leq t}\right) = \Pr\left(s_{t+1} \mid s_t, a_t\right) = p(s_{t+1} \mid s_t, a_t)$$


- MDP: $\mathcal{S}$ (state space), $\mathcal{A}$ (action space), $p$ (transition kernel)



## Reinforcement learning

- Model-free framework for decision-making in Markovian settings

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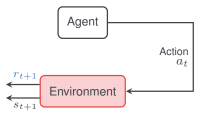


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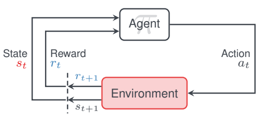


$$\underset{\pi \in \mathcal{P}(\mathcal{S})}{\text{maximize}}\ V(\pi) \triangleq \mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T} \gamma^t r(s_t, a_t)\right] \tag{P-RL}$$

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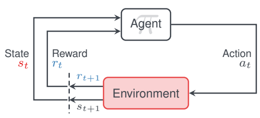


$$\underset{\pi \in \mathcal{P}(\mathcal{S})}{\text{maximize}}\ V(\pi) \triangleq \mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T} \gamma^t r(s_t, a_t)\right] \tag{P-RL}$$

- (P-RL) can be solved using policy gradient and/or Q-learning type algorithms
[W'92, WD'92, BT'96, KT'00, JFEPF'14, HKSC'15, NFPIY'15, AJFR'17, PP'18, SB'18, B'19, KCP'19. . . ]

## Constrained RL

$$\begin{aligned} \underset{\pi \in \mathcal{P}(\mathcal{S})}{\text{maximize}} \quad & V_0(\pi) \triangleq \mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T-1} \gamma^t r_0(s_t, a_t)\right] \\ \text{subject to} \quad & V_i(\pi) \triangleq \mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T-1} \gamma^t r_i(s_t, a_t)\right] \geq c_i, \quad i = 1, \dots, m \end{aligned} \tag{P-CRL}$$

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[Altman'99; Achiam et al., ICML'17; Paternain, Chamon, Calvo-Fullana, Ribeiro, NeurIPS'19; Paternain, Calvo-Fullana, Chamon, Ribeiro, IEEE TAC'23. . . ]



## Safe navigation

Problem
Find a control policy that navigates the environment effectively and safely

$$\underset{\pi \in \mathcal{P}(\mathcal{S})}{\text{maximize}} \quad V(\pi)$$

$$r(s, a) =$$



## Safe navigation

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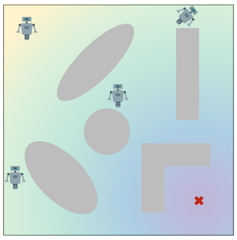



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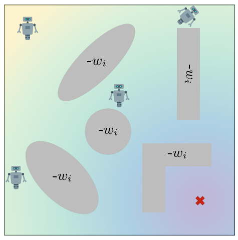




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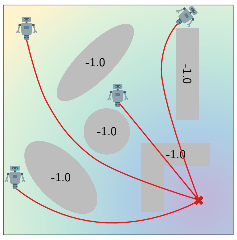




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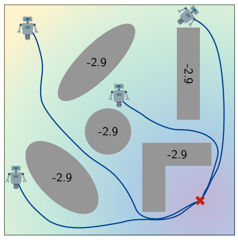




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$$\begin{aligned} \underset{\pi \in \mathcal{P}(\mathcal{S})}{\text{maximize}} \quad & \text{Task reward} \\ \text{subject to} \quad & \Pr\left(\text{Not colliding with } \mathcal{O}_i\right) \geq 1 - \delta, \quad i = 1, 2, \dots \end{aligned}$$



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Find a control policy that navigates the environment effectively and safely

$$\begin{aligned} \underset{\pi \in \mathcal{P}(\mathcal{S})}{\text{maximize}} \quad & V_0(\pi) \triangleq \mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T-1} r_0(s_t, a_t)\right] \\ \text{subject to} \quad & \Pr\left(\text{Not colliding with } \mathcal{O}_i\right) \geq 1 - \delta, \quad i = 1, 2, \dots \end{aligned}$$

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- Probabilistic version of control invariant sets



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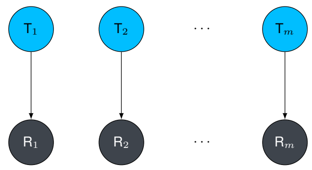
- Probabilistic version of control invariant sets
- Constraint tightening: $\Pr\left(\bigcap_{t=0}^{T-1}\mathcal{E}_t\right) \geq 1-\delta \Longleftarrow \sum_{t=0}^{T-1}\Pr(\mathcal{E}_t) \geq T-\delta$

[Paternain, Calvo-Fullana, Chamon, Ribeiro, IEEE TAC'23]



# Wireless resource allocation

**Problem**
Allocate the least transmit power to $m$ device pairs to achieve a communication rate



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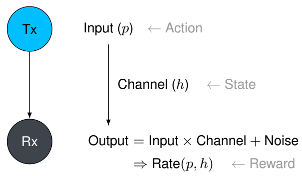




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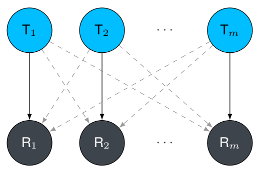


$$\begin{aligned} \min_{\boldsymbol{p}} \quad & \sum_{i=1}^{m}\mathbb{E}\left[\frac{1}{T}\sum_{t=0}^{T-1} p_i(h_t)\right] \\ \text{s. to} \quad & \mathbb{E}\left[\frac{1}{T}\sum_{t=0}^{T-1}\text{Rate}_i\big(\boldsymbol{p}(h_t),h_t\big)\right] \geq c_i \end{aligned}$$

[Eisen, Zhang, Chamon, Lee, and Ribeiro, IEEE TSP'19]



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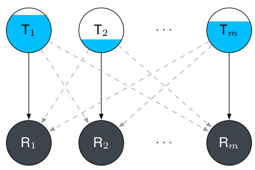


$$\begin{aligned} \min_{\boldsymbol{p}} \quad & \sum_{i=1}^{m}\Pr\left[\bigcap_{t=0}^{T-1}\{b_{i,t}=0\}\right] \\ \text{s. to} \quad & \mathbb{E}\left[\frac{1}{T}\sum_{t=0}^{T-1}\text{Rate}_i\big(\boldsymbol{p}(h_t),h_t\big)\right] \geq c_i \end{aligned}$$

[Chowdhury, Paternain, Verma, Swami, Segarra, Asilomar'23]



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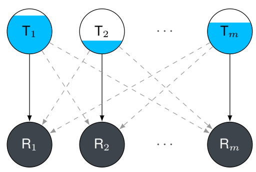


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[Altman'99; Achiam et al., ICML'17; Paternain, Chamon, Calvo-Fullana, Ribeiro, NeurIPS'19; Paternain, Calvo-Fullana, Chamon, Ribeiro, IEEE TAC'23 . . . ]

# CRL methods

$$\underset{\pi \in \mathcal{P}(\mathcal{S})}{\text{maximize}} \quad \mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T-1}\gamma^t r_0(s_t,a_t)\right]$$
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- Reward shaping ≈ penalty methods
  - ✗ Manual, time-consuming, domain-dependent
  - ✗ Trade-offs, training plateaux
- Prior knowledge ≈ projection methods
  e.g., safe exploration [Berkenkamp et al., NeurIPS'17, Dalal et al., arXiv'18]
  - ✗ Requires set of safe actions or safe policies
  - ✗ Intractable projections
- Linearization and convex surrogates
  e.g., CPO [Achiam et al., ICML'17]
  - ✗ No approximation guarantee
  - ✗ Approximate problem may be infeasible



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- Linearization and convex surrogates
  e.g., CPO [Achiam et al., ICML'17]
- Duality
  [Bhatnagar et al., JOTA'12; Tesler et al., ICRL'19; PCCR, NeurIPS'19; Ding et al., NeurIPS'20; PCCR, IEEE TAC'23 ...]
  - ✓ Domain independent
  - ✓ Tractable
  - ✗ Approximation guarantee [non-convexity]



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# Strong Duality of CRL

- Define the dual problem as

$$D = \min_{\lambda \mathbb{R}_+^m} \max_{\pi \in \mathcal{P}(\mathcal{S})} \mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T}\gamma^t r_0(s_t,a_t)\right] + \lambda^\top\left(\mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T}\gamma^t r(s_t,a_t)\right] - c\right)$$

**Theorem (Paternain, Chamon, Calvo-Fullana, Ribeiro'19)**

Assume that there exist a strictly feasible policy $\pi^\dagger$ such that $V(\pi^\dagger) < c$. Then, the constrained reinforcement learning problem has zero duality gap $P = D$

- There is some sort of hidden convexity in CRL problems ⇒ Occupancy measure reformulation

[Paternain, Chamon, Calvo-Fullana, Ribeiro, NeurIPS'19; Paternain, Calvo-Fullana, Chamon, Ribeiro, IEEE TAC'23]



# Occupancy Measure Reformulation

- The occupancy measure of policy $\pi$ is the accumulated probability of visiting each state action pair

$$\rho_\pi(s,a) = (1-\gamma)\sum_{t=0}^{T-1}\gamma^t \mathbb{P}_\pi\big(s_t = s, a_t = a\big) \;\Rightarrow\; \pi(a|s) = \rho_\pi(s,a) \times \left[\int_{\mathcal{A}} \rho_\pi(s,a)\,da\right]^{-1}$$

- The value functions $V_i(\pi)$ can be rewritten as expectations with respect to the occupancy measure

$$V_i(\rho) = \mathbb{E}_{(s,a)\sim\rho}\Big[r_i(s,a)\Big] = \int_{\mathcal{S}\times\mathcal{A}} r(s,a)\,\rho_\pi(s,a)\,da\,ds$$

- Thus, value functions $V_i(\rho)$ are linear with respect to the occupancy measure variable



# A Non-Proof of Strong Duality

- CRL is a nonconvex program in policy variables but a linear program on occupancy measure variables

$$P = \max_{\pi} \quad V_0(\pi) := \mathbb{E}_{s,a\sim\pi}\left[\sum_{t=0}^{\infty}\gamma^t r_0(s_t,a_t)\right] \qquad = P_\rho = \max_{\rho} \quad V_0(\rho) := \mathbb{E}_{(s,a)\sim\rho}\Big[r_0(s,a)\Big]$$
$$\text{subject to}\, V(\pi) := \mathbb{E}_{s,a\sim\pi}\left[\sum_{t=0}^{\infty}\gamma^t r(s_t,a_t)\right] \geq c \qquad \text{subject to}\, V(\rho) := \mathbb{E}_{(s,a)\sim\rho}\Big[r(s,a)\Big] \geq c$$

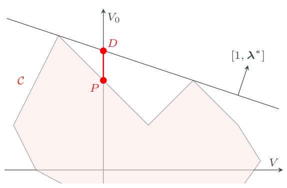
- CRL formulated in terms of occupancy measure variables has no duality gap because it is an LP

$$P_\rho = D_\rho = \min_{\boldsymbol{\lambda}} \max_{\rho} V_0(\rho) + \boldsymbol{\lambda}^T\Big(V(\rho) - c\Big)$$

- Primal equivalence $\neq$ dual equivalency ⇒ CRL with policy variables may still have a duality gap



# A Proof Sketch of Strong Duality




- Epigraph of policy CRL need not be convex

$$\mathcal{C} = \left\{\Big[V_0(\pi);\, V(\pi)\Big] \text{ for some } \pi\right\}$$



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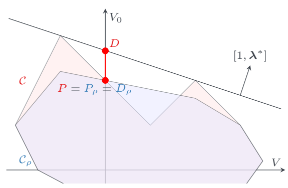


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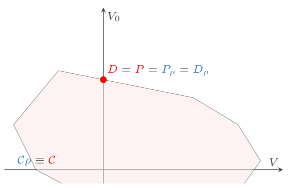


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- Epigraph of occupancy measure CRL is convex

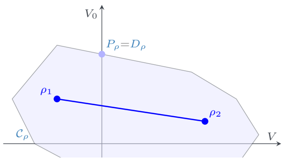
$$\mathcal{C}_\rho = \left\{ \left[ V_0(\rho);\ V(\rho) \right] \text{ for some } \rho \right\}$$

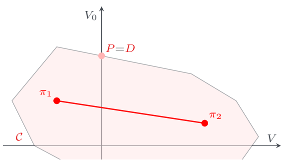
- These two sets are the same $\Rightarrow \mathcal{C}_\rho \equiv \mathcal{C}$



# Epigraphs are Convex in Different Ways

- The epigraphs $\mathcal{C}_\rho$ and $\mathcal{C}$ of occupancy measure and policy CRL are convex in different ways




$$V\left[\alpha\rho + (1-\alpha)\rho'\right] = \alpha V(\rho) + (1-\alpha)V(\rho')$$

There exist $\pi_\alpha$ such that $V\left[\pi_\alpha\right] = \alpha V(\pi) + (1-\alpha)V(\pi')$

- The policy $\pi_\alpha$ is not a convex combination of $\pi$ and $\pi'$ challenges convergence of dual methods



# Learning Parameterization

- Strong duality, $D = P$, despite having value functions $V_0(\pi)$ and $V(\pi)$ that are not concave on $\pi$

$$P = D = \min_{\boldsymbol{\lambda} \geq 0} \max_{\pi} \mathbb{E}_{s,a\sim\pi}\left[\sum_{t=0}^{\infty} \gamma^t \left( r_0(s_t, a_t) + \boldsymbol{\lambda}^T r(s_t, a_t) \right)\right] + \lambda^T c$$

- In practice, policies are functions of learning parameterizations $\Rightarrow$ Choose actions as $a \sim \pi_\theta$

$$D_\theta = \min_{\boldsymbol{\lambda} \geq 0} \max_{\pi_\theta} \mathbb{E}_{s,a\sim\pi_\theta}\left[\sum_{t=0}^{\infty} \gamma^t \left( r_0(s_t, a_t) + \lambda^T r(s_t, a_t) \right)\right] + \lambda^T c$$

- Induces a duality gap because standard learning parameterizations are not convex



# Duality Gap in Parameterized CRL

- The learning parameterization is $\nu$-universal $\Rightarrow \min_\theta \ \max_s \int_{\mathcal{A}} \left| \pi(a|s) - \pi_\theta(a|s) \right| da \leq \nu$ for all $\pi$

**Theorem (Paternain, Chamon, Calvo-Fullana, Ribeiro'19)**

The difference between the CRL parameterized dual $D_\theta$ and the CRL primal $P$ is bounded by

$$\left| P - D_\theta \right| \leq \left( 1 + \|\lambda^\star\|_1 \right) \frac{B\nu}{1-\gamma}$$

- Duality gap depends on parameterization richness relative to discount factor and constraint difficulty

[Paternain, Chamon, Calvo-Fullana, Ribeiro, NeurIPS'19; Paternain, Calvo-Fullana, Chamon, Ribeiro, IEEE TAC'23]



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# Primal-dual algorithm

$$D_\theta^\star = \min_{\lambda \succeq 0} \max_{\boldsymbol{\theta}\in\Theta} \mathbb{E}_{s,a\sim\pi_{\boldsymbol{\theta}}}\left[\frac{1}{T}\sum_{t=0}^{T-1} \gamma^t r_0(s_t, a_t)\right] + \lambda \left( \mathbb{E}_{s,a\sim\pi_{\boldsymbol{\theta}}}\left[\frac{1}{T}\sum_{t=0}^{T-1} \gamma^t r_1(s_t, a_t)\right] - c_1 \right)$$



# Primal-dual algorithm

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- Maximize the primal ($\equiv$ vanilla RL)

$$\theta^\dagger \in \underset{\boldsymbol{\theta}\in\Theta}{\operatorname{argmax}} \ \mathbb{E}_{s,a\sim\pi_{\boldsymbol{\theta}}}\left[\frac{1}{T}\sum_{t=0}^{T-1} \gamma^t r_\lambda(s_t, a_t)\right]$$



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- Update the dual ($\equiv$ policy evaluation)

$$\lambda^+ = \left[ \lambda - \eta \left( \mathbb{E}_{s,a\sim\pi_{\boldsymbol{\theta}^\dagger}}\left[\frac{1}{T}\sum_{t=0}^{T-1} \gamma^t r_1(s_t, a_t)\right] - c_1 \right) \right]_+$$

## Primal-dual algorithm

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## In practice...

$$D_\theta^\star = \min_{\lambda \succeq 0} \max_{\boldsymbol{\theta}\in\Theta} \mathbb{E}_{s,a\sim\pi_{\boldsymbol{\theta}}}\left[\frac{1}{T}\sum_{t=0}^{T-1}\gamma^t r_0(s_t,a_t)\right] + \lambda\left(\mathbb{E}_{s,a\sim\pi_{\boldsymbol{\theta}}}\left[\frac{1}{T}\sum_{t=0}^{T-1}\gamma^t r_1(s_t,a_t)\right] - c_1\right)$$

- Maximize the primal ($\equiv$ vanilla RL): $\{s_t, a_t\} \sim \pi_{\boldsymbol{\theta}_k}$

$$\boldsymbol{\theta}_{k+1} = \boldsymbol{\theta}_k + \eta\left[\frac{1}{T}\sum_{t=0}^{T-1}\gamma^t r_\lambda(s_t,a_t)\right]\nabla_{\boldsymbol{\theta}}\log\left(\pi_{\boldsymbol{\theta}}(a_0|s_0)\right)$$

- Update the dual ($\equiv$ policy evaluation): $\{s_t, a_t\} \sim \pi_{\boldsymbol{\theta}_{k+1}}$

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## Dual CRL

Theorem

Suppose $\boldsymbol{\theta}^\dagger$ is a $\rho$-approximate solution of the regularized RL problem:

$$\boldsymbol{\theta}^\dagger \approx \operatorname*{argmax}_{\boldsymbol{\theta}\in\Theta} \mathbb{E}_{s,a\sim\pi_{\boldsymbol{\theta}}}\left[\frac{1}{T}\sum_{t=0}^{T-1}\gamma^t r_\lambda(s_t,a_t)\right].$$

Then, after $K = \left\lceil \frac{\|\boldsymbol{\lambda}^\star\|^2}{2\eta\nu}\right\rceil + 1$ dual iterations with step size $\eta \leq \frac{1-\gamma}{mB}$,

the iterates $\left(\boldsymbol{\theta}^{(T)}, \boldsymbol{\lambda}^{(T)}\right)$ are such that

$$\left|P^\star - L\left(\boldsymbol{\theta}^{(T)}, \boldsymbol{\lambda}^{(T)}\right)\right| \leq \frac{1 + \|\boldsymbol{\lambda}_\nu^\star\|_1}{1-\gamma} B\nu + \rho$$

[Paternain, C., Calvo-Fullana, and Ribeiro, NeurIPS'19; C. and Ribeiro, NeurIPS'20; C., Paternain, Calvo-Fullana, and Ribeiro, IEEE TIT'23]



## Dual gradient descent claims

Theorem (Calvo-Fullana et al'23)

The generated state-action sequences $\left(s_t, a_t \sim \pi^\dagger(\lambda_k)\right)$ are:

(i) Almost surely feasible: $\lim_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} r_i(s_t,a_t) \geq c_i$ a.s., for all $i$

(ii) Near-optimal: $\lim_{T\to\infty} \mathbb{E}\left[\frac{1}{T}\sum_{t=0}^{T-1} r_0(s_t,a_t)\right] \geq P^\star - \frac{\eta B^2}{2}$

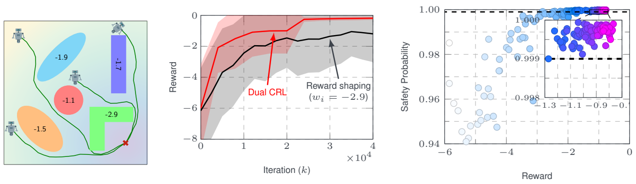
- The time average of the rewards of the sequence generated by rollout dual descent converges

  This sequence is a "solution" of the CRL problem. Stronger, in fact. Constraints satisfied a.s.



## Safe navigation

- Reach a target destination while avoiding collisions with a number of obstacles (w.h.p)



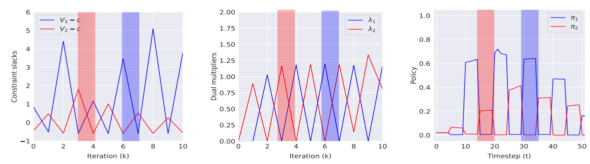

- Policy learned in dual domain outperforms optimal reward shaping policy (obstacle heterogeneity)

[Paternain, Calvo-Fullana, Chamon, Ribeiro, IEEE TAC'23]



## Wireless network

- Constraint slacks oscillate around zero $\Rightarrow$ They spend enough time below zero (feasibility claim)




- The slack oscillation is driven by multiplier oscillation which in turn drives policy switching

  The multipliers drive the policies to switch at the right rate

[Uslu, Doostnejad, Ribeiro, NaderiAlizadeh, arxiv:2102.11941]



## Dual gradient descent does not claim

Theorem (Calvo-Fullana et al'23)

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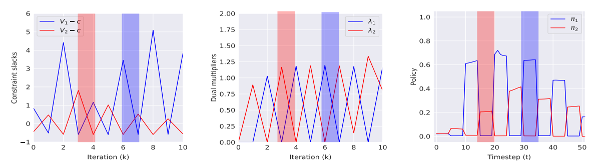
- No claim on optimal policy $\pi^\star$ $\Rightarrow$ Generate policies $\pi^\dagger(\lambda_k)$ that are samples of near optimal policies



## Optimal policy recovery

- DGD learns to allocate different users at different points in time with the right amount of power




- At any given epoch the policies $\pi^\dagger(\lambda_k)$ are not optimal $\Rightarrow$ Their combined action is "optimal"

  Would want to take the time average of policies $\Rightarrow$ Can't because $V_i(\pi)$ is not convex

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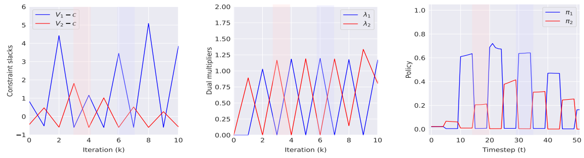


- At any given epoch the policies $\pi^\dagger(\lambda_k)$ are not optimal $\Rightarrow$ Their combined action is "optimal"
  Would want to take the time average of policies $\Rightarrow$ Can't because $V_i(\pi)$ is not convex

Cannot recover a near optimal policy $\pi^\star$ from sequence of Lagrangian maximizing policies $\pi^\dagger(\lambda_k)$



# Monitoring task

**Problem**
Find a control policy that maximizes the time in $R_0$
while monitoring $R_1$ and $R_2$ at least $1/3$ of the time each

$$\max_{\pi \in \mathcal{P}(\mathcal{S})} \lim_{T\to\infty} \mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T-1} \mathbb{I}\left(s_t \in R_0\right)\right]$$
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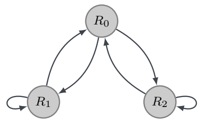


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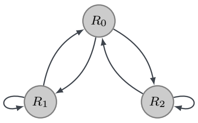


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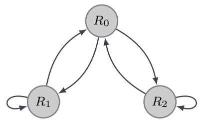
✔ $\pi^\star =$ draw actions uniformly at random

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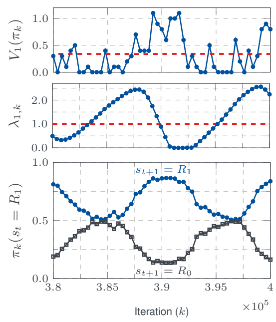
$$\max_{\pi \in \mathcal{P}(\mathcal{S})} \lim_{T\to\infty} \mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T-1} r_{\boldsymbol{\lambda}}(s_t)\right]$$
$$r_{\boldsymbol{\lambda}}(s) = \mathbb{I}\left(s \in R_0\right) + \lambda_1 \mathbb{I}\left(s \in R_1\right) + \lambda_2 \mathbb{I}\left(s \in R_2\right)$$

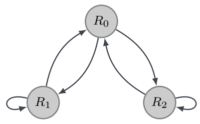
✔ $\lambda_1 = \lambda_2 = 1$: all $\pi \in \mathcal{P}(\mathcal{S})$ are optimal
✘ $\lambda_1, \lambda_2 < 1$: $\pi^\star$ s.t. $\Pr\left[s \in R_0\right] = 1/2$
✘ $\lambda_i > 1$ and $\lambda_i > \lambda_j$: $\pi^\star$ s.t. $\Pr\left[s \in R_i\right] = 1$

[Calvo-Fullana, Paternain, Chamon, Ribeiro, IEEE TAC'23]



# Monitoring task






# Primal recovery

- General issue with duality
  - (Primal-)dual methods: $f(\boldsymbol{\theta}_k) \not\to f(\boldsymbol{\theta}^\star)$ but $\frac{1}{K}\sum_{k=0}^{K-1} f(\boldsymbol{\theta}_k) \to f(\boldsymbol{\theta}^\star)$



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✔ Convex optimization $\Rightarrow$ dual averaging
  - Convexity: $f\left(\frac{1}{K}\sum_{k=0}^{K-1}\theta_k\right) \leq \frac{1}{K}\sum_{k=0}^{K-1} f(\theta_k)$ for all $K$ $\Rightarrow \boldsymbol{\theta}^\star = \lim_{K\to\infty} \frac{1}{K}\sum_{k=0}^{K-1}\boldsymbol{\theta}_k$

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- Non-convex optimization ⇒ randomization
  - $\boldsymbol{\theta}^\dagger \sim$ Uniform$(\boldsymbol{\theta}_k)$ ⇒ $\mathbb{E}\left[f(\boldsymbol{\theta}^\dagger)\right] = \frac{1}{K}\sum_{k=0}^{K-1} f(\boldsymbol{\theta}_k) \to f(\boldsymbol{\theta}^\star)$



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  - Must memorize the complete training sequence of policies



# So CRL is hard?

- There are tasks that CRL can tackle and RL cannot

$$\begin{array}{ll}\max_{\pi\in\mathcal{P}(\mathcal{S})} & V_0(\pi) \\ \text{subject to} & V_i(\pi) \ge c_i\end{array} \quad \supsetneq \quad \max_{\pi\in\mathcal{P}(\mathcal{S})} V(\pi)$$

- Regularized RL is unable to represent all CRL problems (cannot really "solve" them)
- How can we solve CRL?



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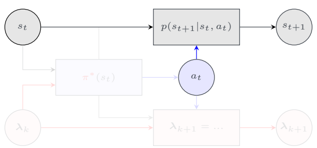
- How can we solve CRL?

  To solve CRL we augment the state with Lagrange multipliers and learn to maximize Lagrangians

$$\pi^\dagger(\lambda_k) \in \operatorname*{argmax}_{\pi} \lim_{T\to\infty} \mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T} r_{\lambda_k}(s_t,a_t)\right]$$



# State-augmented CRL

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- For a Markov decision process (MDP) we want to choose actions that solve a CRL problem



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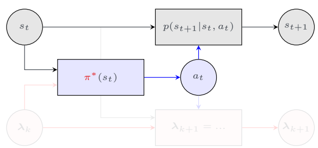


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- Requires finding optimal policy $\pi^\star$ ⇒ I do not know how to find it operating in policy space



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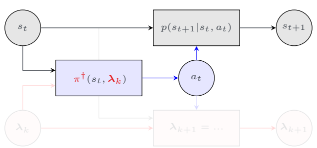


$$\pi^\dagger(s_t,\boldsymbol{\lambda}_k) \in \operatorname*{argmax}_{\pi} \lim_{T\to\infty} \mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T} r_{\boldsymbol{\lambda}_k}(s_t,a_t)\right]$$

- Find Lagrangian maximizing policies $\pi^\dagger(\boldsymbol{\lambda}_k)$ ⇒ Solve unconstrained RL with rewards $r_{\boldsymbol{\lambda}_k}(s_t,a_t)$



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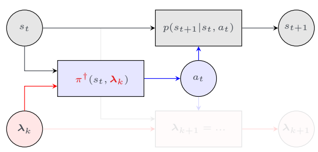


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- Needs dual variable $\boldsymbol{\lambda}_k$ as input. Also need to update $\boldsymbol{\lambda}_k$ to accumulate constraint violations

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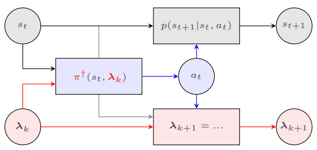


$$\boldsymbol{\lambda}_{k+1} = \left[\boldsymbol{\lambda}_k - \frac{\eta}{T_0}\sum_{t=kT_0}^{(k+1)T_0-1}\left[\mathbf{r}(s_t,a_t)-\mathbf{c}\right]\right]_+$$

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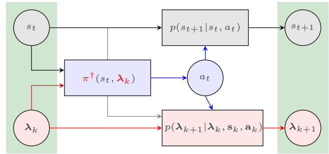


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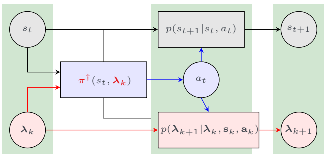
- This is equivalent to defining an augmented MDP with (augmented) state $\tilde{S}_t = (s_t, \boldsymbol{\lambda}_t)$

  And an augmented transition probability kernel that included the dual variable updates



## State-augmented CRL

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## State-augmented CRL

Training execution split goes here



## Learning Parameterization

- In practice, policies are functions of learning parameterizations ⇒ Choose actions as $a \sim \pi_\phi(s, \boldsymbol{\lambda})$

$$\pi_\phi^* \in \underset{\pi_\phi}{\operatorname{argmax}} \lim_{T\to\infty} \mathbb{E}_{\boldsymbol{\lambda}}\mathbb{E}_{s,a\sim\pi_\phi}\left[\frac{1}{T}\sum_{t=0}^{T} r_{\boldsymbol{\lambda}_t}(s_t,a_t)\right] \equiv \underset{\pi_\phi}{\operatorname{argmax}} \lim_{T\to\infty} \mathbb{E}_{\boldsymbol{\lambda}}\mathbb{E}_{s,a\sim\pi_\phi}\left[\frac{1}{T}\sum_{t=0}^{T} r(s_t,\boldsymbol{\lambda}_t,a_t)\right]$$

- Since this is an state augmented MDP we also need to take expectation over a $\boldsymbol{\lambda}$ distribution

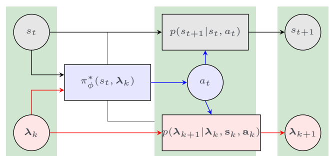
  Choosing this distribution presents the usual challenges of off-policy RL



## Parameterized State-augmented CRL

- Learn parameterized policy $\pi_\phi^*$ that maximizes the Lagrangian averaged over the dual distribution

  Execute policy $\pi_\phi^*$ while keeping track of dual variable updates ⇒ Generate optimal trajectory




$$\pi_\phi^*(s_t,\boldsymbol{\lambda}_k) \in \underset{\pi_\phi}{\operatorname{argmax}} \lim_{T\to\infty} \mathbb{E}_{\boldsymbol{\lambda}}\mathbb{E}_{s,a\sim\pi_\phi}\left[\frac{1}{T}\sum_{t=0}^{T} r(s_t,\boldsymbol{\lambda}_t,a_t)\right]$$

$$\boldsymbol{\lambda}_{k+1} = \left[\boldsymbol{\lambda}_k - \frac{\eta}{T_0}\sum_{t=kT_0}^{(k+1)T_0-1}\left[\mathbf{r}(s_t,a_t)-\mathbf{c}\right]\right]_+$$

$$\mathbf{s}_k = \left[s_{kT-0:(k+1)T_0-1}\right]$$

$$\mathbf{a}_k = \left[a_{kT-0:(k+1)T_0-1}\right]$$



## Dual Gradient Descent "Solves" CRL

(S1) At epoch $k$, choose policy ⇒ $\pi^\dagger(\boldsymbol{\lambda}_k) \in \underset{\pi}{\operatorname{argmax}} \lim_{T\to\infty} \mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T} r_{\boldsymbol{\lambda}}(s_t,a_t)\right]$

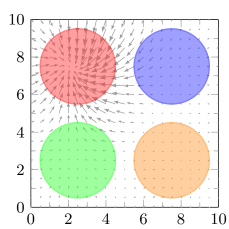
(S2) Choose actions $a_t \sim \pi^\dagger(\boldsymbol{\lambda}_k)$ between times $kT_0$ and $(k+1)T_0 - 1$

(S3) Update multiplier ⇒ $\boldsymbol{\lambda}_{k+1} = \left[\boldsymbol{\lambda}_k - \frac{\eta}{T_0}\sum_{t=kT_0}^{(k+1)T_0-1}\left[\mathbf{r}(s_t,a_t)-\mathbf{c}\right]\right]_+$

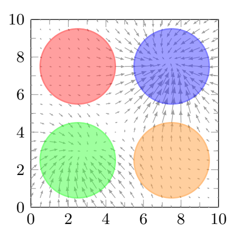
- The algorithm (S1)-(S3) "solves" CRL in the sense that it generates a state-action sequence $(s_t, a_t)$ that is almost surely feasible and $\mathcal{O}(\eta)$-optimal in expectation.
- This is not the statement we would like to prove ⇒ The algorithm (S1)-(S3) solves CRL in the sense that if finds a policy that is $\mathcal{O}(\eta)$-feasible and $\mathcal{O}(\eta)$-optimal in expectation.
- The price of the non-concavity of value functions ⇒ If the value function were convex we could prove that the ergodic average of policies $\lim_{K\to\infty}\frac{1}{K}\sum_{k=1}^{K}\pi^\dagger(\boldsymbol{\lambda}_k)$ is feasible and $\mathcal{O}(\eta)$-optimal.



## Monitoring task

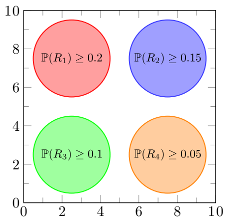


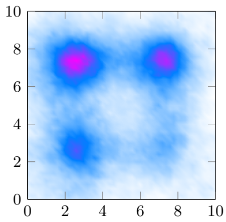

Policy learned for $\lambda = [5, 0, 0, 0]$




Policy learned for $\lambda = [0, 5, 5, 0]$

## Monitoring task

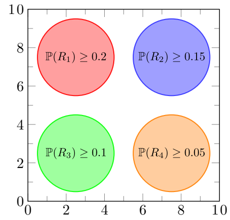


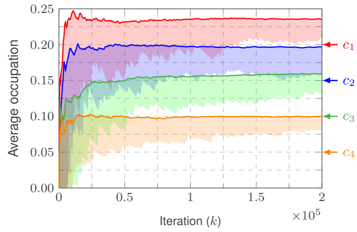



[Calvo-Fullana, Paternain, Chamon, Ribeiro, IEEE TAC'23]



## Monitoring task




[Calvo-Fullana, Paternain, Chamon, Ribeiro, IEEE TAC'23]



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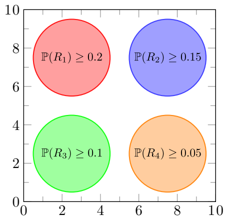
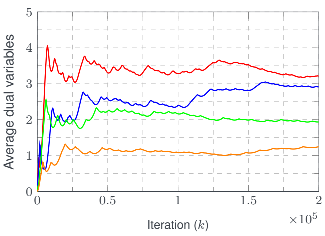
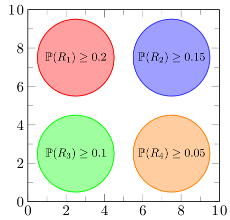


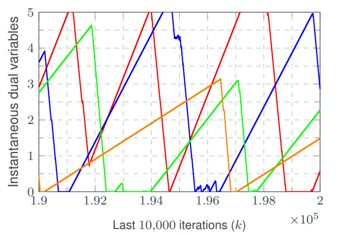




[Calvo-Fullana, Paternain, Chamon, Ribeiro, IEEE TAC'23]



## Monitoring task



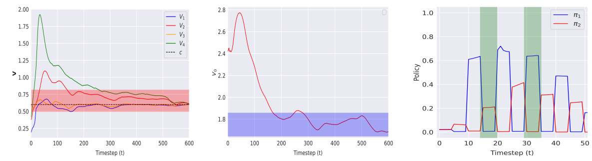




[Calvo-Fullana, Paternain, Chamon, Ribeiro, IEEE TAC'23]



## Wireless networks

- State augmented CRL learns policies that satisfy constraints and minimize objective on average




- Even though we still have policy switching ⇒ Multipliers drive policies to switch at the right rate



## Summary

- **Constrained RL is ~~the~~ a tool for decision making under requirements**

- **Constrained RL is hard. . .**

- **. . . but possible. How?**



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  CRL is a natural way of specifying complex behaviors that precludes fine tuning of rewards, e.g., safety [Paternain et al., IEEE TAC'23]

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  Although strong duality holds for CRL (despite non-convexity), that is not always enough to obtain feasible solutions ⇒ (P-RL) $\subsetneq$ (P-CRL)

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  Although strong duality holds for CRL (despite non-convexity), that is not always enough to obtain feasible solutions $\Rightarrow$ (P-RL) $\subsetneq$ (P-CRL)

- **...but possible. How?**
  When combined with a *systematic state augmentation* technique, we can use policies that solve (P-RL) to solve (P-CRL)



## Agenda

I. Constrained supervised learning
   - Constrained learning theory
   - Resilient constrained learning
   - Robust learning

Break (30 min)

II. Constrained reinforcement learning
   - Constrained RL duality
   - Constrained RL algorithms

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July 15, 2024

supervised and reinforcement learning under requirements