Agenda

I. Constrained supervised learning
  - Constrained learning theory
  - Constrained learning algorithms
  - Resilient constrained learning

Break (10 min)

II. Constrained reinforcement learning
  - Constrained RL duality
  - Constrained RL algorithms

Q&A and discussions

https://luizchamon.com/imprs2024

Universität Stuttgart

IMPRS tutorial
Sep. 19, 2024

supervised and reinforcement learning under requirements

Luiz F. O. Chamon

ellis

SimTech Cluster of Excellence

KI institute

Constrained reinforcement learning

Agenda

Constrained reinforcement learning

CMDP duality

CRL algorithms

Reinforcement learning

- Model-free framework for decision-making in Markovian settings

Reinforcement learning

- Model-free framework for decision-making in Markovian settings

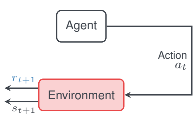
$$\mathbb{P}\left(s_{t+1} \mid \{s_u, a_u\}_{u \leq t}\right) = \mathbb{P}\left(s_{t+1} \mid s_t, a_t\right) = p(s_{t+1} \mid s_t, a_t)$$

- MDP: $\mathcal{S}$ (state space), $\mathcal{A}$ (action space), p (transition kernel)

Reinforcement learning

- Model-free framework for decision-making in Markovian settings

$$\mathbb{P}\left(s_{t+1} \mid \{s_u, a_u\}_{u \leq t}\right) = \mathbb{P}\left(s_{t+1} \mid s_t, a_t\right) = p(s_{t+1} \mid s_t, a_t)$$

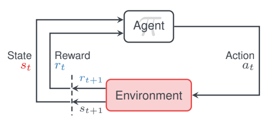


- MDP: $\mathcal{S}$ (state space), $\mathcal{A}$ (action space), p (transition kernel), $r : \mathcal{S} \times \mathcal{A} \to [0, B]$ (reward)

Reinforcement learning

- Model-free framework for decision-making in Markovian settings

$$\mathbb{P}\left(s_{t+1} \mid \{s_u, a_u\}_{u \leq t}\right) = \mathbb{P}\left(s_{t+1} \mid s_t, a_t\right) = p(s_{t+1} \mid s_t, a_t)$$



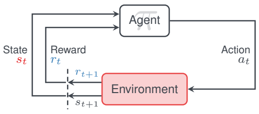
$$\underset{\pi \in \mathcal{P}(\mathcal{S})}{\text{maximize}}\ V(\pi) \triangleq \mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T} \gamma^t r(s_t, a_t)\right] \qquad \text{(P-RL)}$$

- MDP: $\mathcal{S}$ (state space), $\mathcal{A}$ (action space), p (transition kernel), $r : \mathcal{S} \times \mathcal{A} \to [0, B]$ (reward)
- $\mathcal{P}(\mathcal{S})$: space of probability measures parameterized by $\mathcal{S}$
- T (horizon) (possibly $T \to \infty$) and $\gamma < 1$ (discount factor) (possibly $\gamma = 1$)

Reinforcement learning

- Model-free framework for decision-making in Markovian settings

$$\mathbb{P}\left(s_{t+1} \mid \{s_u, a_u\}_{u \leq t}\right) = \mathbb{P}\left(s_{t+1} \mid s_t, a_t\right) = p(s_{t+1} \mid s_t, a_t)$$



$$\underset{\pi \in \mathcal{P}(\mathcal{S})}{\text{maximize}} \; V(\pi) \triangleq \mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T} \gamma^t r(s_t, a_t)\right] \qquad \text{(P-RL)}$$

- (P-RL) can be solved using policy gradient and/or Q-learning type algorithms
  [W'92, WD'92, BT'96, KT'00, JFEPF'14, HKSC'15, NFPIY'15, AJFR'17, PP'18, SB'18, B'19, KCP'19...]

Constrained RL

$$\begin{aligned} &\underset{\pi \in \mathcal{P}(\mathcal{S})}{\text{maximize}} && V_0(\pi) \triangleq \mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T-1} \gamma^t r_0(s_t, a_t)\right] \\ &\text{subject to} && V_i(\pi) \triangleq \mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T-1} \gamma^t r_i(s_t, a_t)\right] \geq c_i, \quad i = 1, \ldots, m \end{aligned} \qquad \text{(P-CRL)}$$

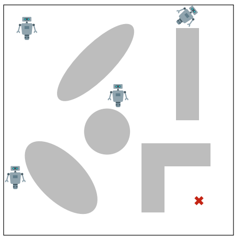
- MDP: $\mathcal{S}$ (state space), $\mathcal{A}$ (action space), p (transition kernel), $r_i : \mathcal{S} \times \mathcal{A} \to [0, B]$ (reward)
- $\mathcal{P}(\mathcal{S})$: space of probability measures parameterized by $\mathcal{S}$
- T (horizon) (possibly $T \to \infty$) and $\gamma < 1$ (discount factor) (possibly $\gamma = 1$)

[Altman'99; Achiam et al., ICML'17; Paternain, Chamon, Calvo-Fullana, Ribeiro, NeurIPS'19; Paternain, Calvo-Fullana, Chamon, Ribeiro, IEEE TAC'23...]

Safe navigation

Problem
Find a control policy that navigates the environment effectively and safely

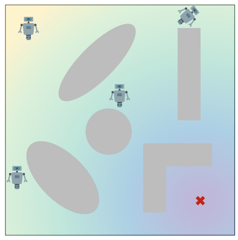
$$\underset{\pi \in \mathcal{P}(\mathcal{S})}{\text{maximize}} \; \mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T-1} \qquad \right]$$



Safe navigation

Problem
Find a control policy that navigates the environment effectively and safely

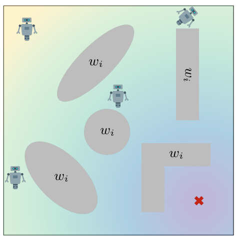
$$\underset{\pi \in \mathcal{P}(\mathcal{S})}{\text{maximize}} \; \mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T-1} \underbrace{-\|s - s_{\text{goal}}\|^2}_{r_0} \qquad \right]$$



Safe navigation

Problem
Find a control policy that navigates the environment effectively and safely

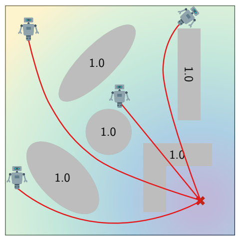
$$\underset{\pi \in \mathcal{P}(\mathcal{S})}{\text{maximize}} \; \mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T-1} \underbrace{-\|s - s_{\text{goal}}\|^2}_{r_0} - \sum_{i=1}^{5} w_i \underbrace{\mathbb{I}(s_t \in \mathcal{O}_i)}_{r_i}\right]$$



Safe navigation

Problem
Find a control policy that navigates the environment effectively and safely

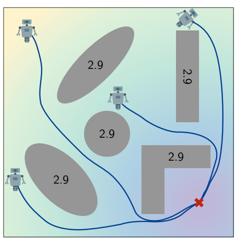
$$\underset{\pi \in \mathcal{P}(\mathcal{S})}{\text{maximize}} \; \mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T-1} \underbrace{-\|s - s_{\text{goal}}\|^2}_{r_0} - \sum_{i=1}^{5} w_i \underbrace{\mathbb{I}(s_t \in \mathcal{O}_i)}_{r_i}\right]$$



Safe navigation

Problem
Find a control policy that navigates the environment effectively and safely

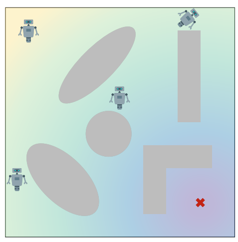
$$\underset{\pi \in \mathcal{P}(\mathcal{S})}{\text{maximize}} \; \mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T-1} \underbrace{-\|s - s_{\text{goal}}\|^2}_{r_0} - \sum_{i=1}^{5} w_i \underbrace{\mathbb{I}(s_t \in \mathcal{O}_i)}_{r_i}\right]$$



Safe navigation

Problem
Find a control policy that navigates the environment effectively and safely

$$\begin{aligned} &\underset{\pi \in \mathcal{P}(\mathcal{S})}{\text{maximize}} && \mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T-1} r_0(s_t, a_t)\right] \\ &\text{subject to} && \mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T-1} \underbrace{\mathbb{I}(s_t \notin \mathcal{O}_i)}_{r_i}\right] \geq 1 - \frac{\delta_i}{T} \end{aligned}$$



[Paternain, Calvo-Fullana, Chamon, Ribeiro, IEEE TAC'23]

Safe navigation

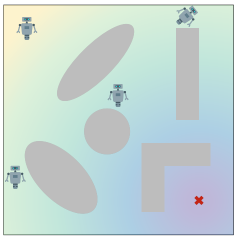
Problem

Find a control policy that navigates the environment effectively and safely

$$\begin{aligned}&\underset{\pi\in\mathcal{P}(\mathcal{S})}{\text{maximize}} && \mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T-1} r_0(s_t,a_t)\right]\\ &\text{subject to} && \mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T-1} \underbrace{\mathbb{I}(s_t\notin\mathcal{O}_i)}_{r_i}\right]\geq 1-\frac{\delta_i}{T}\end{aligned}$$

- Safety guarantee:

$$\sum_{t=0}^{T-1}\mathbb{P}(\mathcal{E}_t)\geq T-\delta \implies \mathbb{P}\left(\bigcap_{t=0}^{T-1}\mathcal{E}_t\right)\geq 1-\delta$$



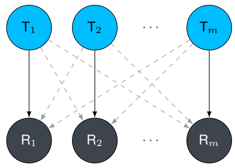
[Paternain, Calvo-Fullana, Chamon, Ribeiro, IEEE TAC'23]

Wireless resource allocation

Problem

Allocate the least transmit power to m device pairs to achieve a communication rate

$$\max_{\pi\in\mathcal{P}(\mathcal{S})}\ \mathbb{E}_{\boldsymbol{h},\boldsymbol{p}\sim\pi(\boldsymbol{h})}\left[\frac{1}{T}\sum_{t=0}^{T-1}\underbrace{-\sum_{i=1}^{m}p_{i,t}}_{r_0}-\sum_{i=1}^{m}w_i\underbrace{\text{Rate}_i(\boldsymbol{p}_t,\boldsymbol{h}_t)}_{r_i}\right]$$



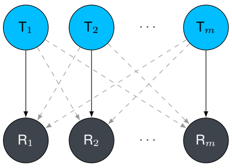
[Eisen, Zhang, Chamon, Lee, and Ribeiro, IEEE TSP'19]

Wireless resource allocation

Problem

Allocate the least transmit power to m device pairs to achieve a communication rate

$$\begin{aligned}&\underset{\pi\in\mathcal{P}(\mathcal{S})}{\text{maximize}} && \mathbb{E}_{\boldsymbol{h},\boldsymbol{p}\sim\pi(\boldsymbol{h})}\left[\frac{1}{T}\sum_{t=0}^{T-1}-\sum_{i=1}^{m}p_{i,t}\right]\\ &\text{s. to} && \mathbb{E}_{\boldsymbol{h},\boldsymbol{p}\sim\pi(\boldsymbol{h})}\left[\frac{1}{T}\sum_{t=0}^{T-1}\text{Rate}_i(\boldsymbol{p}_t,\boldsymbol{h}_t)\right]\geq c_i\end{aligned}$$



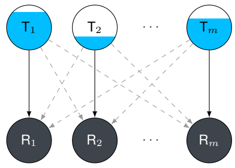
[Chowdhury, Paternain, Verma, Swami, Segarra, Asilomar'23]

Wireless resource allocation

Problem

Allocate the least transmit power to m device pairs to achieve a communication rate

$$\begin{aligned}&\underset{\pi\in\mathcal{P}(\mathcal{S})}{\text{maximize}} && \mathbb{E}_{(\boldsymbol{h},\boldsymbol{b}),\boldsymbol{p}\sim\pi(\boldsymbol{h},\boldsymbol{b})}\left[\frac{1}{T}\sum_{t=0}^{T-1}-\sum_{i=1}^{m}\mathbb{I}(b_{i,t}=0)\right]\\ &\text{s. to} && \mathbb{E}_{(\boldsymbol{h},\boldsymbol{b}),\boldsymbol{p}\sim\pi(\boldsymbol{h},\boldsymbol{b})}\left[\frac{1}{T}\sum_{t=0}^{T-1}\text{Rate}_i(\boldsymbol{p}_t,\boldsymbol{h}_t)\right]\geq c_i\end{aligned}$$



[Chowdhury, Paternain, Verma, Swami, Segarra, Asilomar'23]

Constrained RL

$$\begin{aligned}&\underset{\pi\in\mathcal{P}(\mathcal{S})}{\text{maximize}} && V_0(\pi)\triangleq\mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T-1}\gamma^t r_0(s_t,a_t)\right]\\ &\text{subject to} && V_i(\pi)\triangleq\mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T-1}\gamma^t r_i(s_t,a_t)\right]\geq c_i,\quad i=1,\dots,m\end{aligned}\tag{P-CRL}$$

- MDP: $\mathcal{S}$ (state space), $\mathcal{A}$ (action space), p (transition kernel), $r_i:\mathcal{S}\times\mathcal{A}\to[0,B]$ (reward)
- $\mathcal{P}(\mathcal{S})$: space of probability measures parameterized by $\mathcal{S}$
- T (horizon) (possibly $T\to\infty$) and $\gamma<1$ (discount factor) (possibly $\gamma=1$)

[Altman'99; Achiam et al., ICML'17; Paternain, Chamon, Calvo-Fullana, Ribeiro, NeurIPS'19; Paternain, Calvo-Fullana, Chamon, Ribeiro, IEEE TAC'23. . .]

RL $\subsetneq$ CRL

Proposition

There exist environments in which every task cannot be unambiguously described by a reward

- There are tasks that CRL can tackle and RL cannot

$$\underset{\pi\in\mathcal{P}(\mathcal{S})}{\text{maximize}}\ V(\pi)\quad\subsetneq\quad\begin{aligned}&\underset{\pi\in\mathcal{P}(\mathcal{S})}{\text{maximize}} && V_0(\pi)\\ &\text{subject to} && V_i(\pi)\geq c_i\end{aligned}$$

[Calvo-Fullana, Paternain, Chamon, and Ribeiro, IEEE TAC'24]

RL $\subsetneq$ CRL

Proposition

There exist ~~environments~~ in which every ~~task~~ cannot be ~~unambiguously described by a reward~~

(MDPs) (occupation measure) (induced by a unique $\pi^\star$ that maximizes a reward)

- There are tasks that CRL can tackle and RL cannot

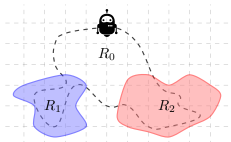
$$\underset{\pi\in\mathcal{P}(\mathcal{S})}{\text{maximize}}\ V(\pi)\quad\subsetneq\quad\begin{aligned}&\underset{\pi\in\mathcal{P}(\mathcal{S})}{\text{maximize}} && V_0(\pi)\\ &\text{subject to} && V_i(\pi)\geq c_i\end{aligned}$$

[Calvo-Fullana, Paternain, Chamon, and Ribeiro, IEEE TAC'24]

Monitoring task

Problem

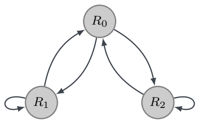
Find a policy that maximizes the time in R_0 while monitoring R_1 and R_2 at least $1/3$ of the time each



[Calvo-Fullana, Paternain, Chamon, Ribeiro, IEEE TAC'24]

Monitoring task

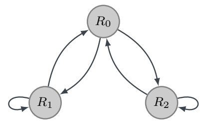
Problem
Find a policy that maximizes the time in R_0 while monitoring R_1 and R_2 at least $1/3$ of the time each



✓ $\pi^\star$ = draw actions uniformly at random

Monitoring task

Problem
Find a policy that maximizes the time in R_0 while monitoring R_1 and R_2 at least $1/3$ of the time each

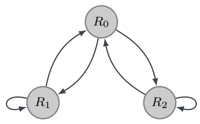


$$\underset{\pi\in\mathcal{P}(\mathcal{S})}{\text{maximize}} \ \lim_{T\to\infty} \mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T-1} r(s_t)\right]$$

Monitoring task

Problem
Find a policy that maximizes the time in R_0 while monitoring R_1 and R_2 at least $1/3$ of the time each

✗ $r(R_0) > r(R_1), r(R_2)$
$\pi^\dagger$ s.t. $\mathbb{P}[s \in R_0] = 1/2$



$$\underset{\pi\in\mathcal{P}(\mathcal{S})}{\text{maximize}} \ \lim_{T\to\infty} \mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T-1} r(s_t)\right]$$

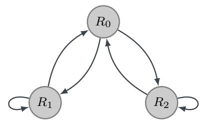
Monitoring task

Problem
Find a policy that maximizes the time in R_0 while monitoring R_1 and R_2 at least $1/3$ of the time each

✗ ~~$r(R_0) > r(R_1), r(R_2)$~~

✗ $r(R_1) > r(R_0), r(R_2)$
$\pi^\dagger$ s.t. $\mathbb{P}[s \in R_1] = 1$

✗ $r(R_2) > r(R_0), r(R_1)$
$\pi^\dagger$ s.t. $\mathbb{P}[s \in R_2] = 1$



$$\underset{\pi\in\mathcal{P}(\mathcal{S})}{\text{maximize}} \ \lim_{T\to\infty} \mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T-1} r(s_t)\right]$$

Monitoring task

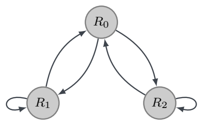
Problem
Find a policy that maximizes the time in R_0 while monitoring R_1 and R_2 at least $1/3$ of the time each

✗ ~~$r(R_0) > r(R_1), r(R_2)$~~

✗ ~~$r(R_1) > r(R_0), r(R_2)$~~

✗ ~~$r(R_2) > r(R_0), r(R_1)$~~

✓ $r(R_0) = r(R_1) = r(R_2)$
all $\pi \in \mathcal{P}(\mathcal{S})$ are optimal

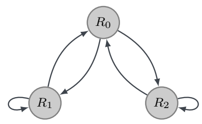


$$\underset{\pi\in\mathcal{P}(\mathcal{S})}{\text{maximize}} \ \lim_{T\to\infty} \mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T-1} r(s_t)\right]$$

Monitoring task

Problem
Find a policy that maximizes the time in R_0 while monitoring R_1 and R_2 at least $1/3$ of the time each

$$\begin{aligned}
\underset{\pi\in\mathcal{P}(\mathcal{S})}{\text{maximize}} \quad & \lim_{T\to\infty} \mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T-1} \mathbb{I}\left(s_t \in R_0\right)\right] \\
\text{s. to} \quad & \lim_{T\to\infty} \mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T-1} \mathbb{I}\left(s_t \in R_1\right)\right] \geq \frac{1}{3} \\
& \lim_{T\to\infty} \mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T-1} \mathbb{I}\left(s_t \in R_2\right)\right] \geq \frac{1}{3}
\end{aligned}$$



RL $\subsetneq$ CRL

Proposition
There exist ~~environments~~ in which every ~~task~~ cannot be ~~unambiguously described by a reward~~
(MDPs) (occupation measure) (induced by a unique $\pi^\star$ that maximizes a reward)

- There are tasks that CRL can tackle and RL cannot

$$\underset{\pi\in\mathcal{P}(\mathcal{S})}{\text{maximize}} \ V(\pi) \quad \subsetneq \quad \begin{aligned} \underset{\pi\in\mathcal{P}(\mathcal{S})}{\text{maximize}} \quad & V_0(\pi) \\ \text{subject to} \quad & V_i(\pi) \geq c_i \end{aligned}$$

$\Rightarrow$ Regularized RL cannot solve *all* CRL problems

RL $\subsetneq$ CRL

Proposition
There exist ~~environments~~ in which every ~~task~~ cannot be ~~unambiguously described by a reward~~
(MDPs) (occupation measure) (induced by a unique $\pi^\star$ that maximizes a reward)

- There are tasks that CRL can tackle and RL cannot

$$\underset{\pi\in\mathcal{P}(\mathcal{S})}{\text{maximize}} \ V(\pi) \quad \subsetneq \quad \begin{aligned} \underset{\pi\in\mathcal{P}(\mathcal{S})}{\text{maximize}} \quad & V_0(\pi) \\ \text{subject to} \quad & V_i(\pi) \geq c_i \end{aligned}$$

$\Rightarrow$ Regularized RL cannot solve *all* CRL problems

- How can we tackle CRL problems?

CRL methods

$$\begin{aligned} &\underset{\pi \in \mathcal{P}(\mathcal{S})}{\text{maximize}} && \mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T-1}\gamma^t r_0(s_t,a_t)\right] \\ &\text{subject to} && \mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T-1}\gamma^t r_i(s_t,a_t)\right] \geq c_i \end{aligned}$$

- Reward shaping ≈ penalty methods
  - ✖ Manual, time-consuming, domain-dependent
  - ✖ Trade-offs, training plateaux
- Prior knowledge ≈ projection methods
  e.g., safe exploration [Berkenkamp et al., NeurIPS'17, Dalal et al., arXiv'18]
  - ✖ Requires set of safe actions or safe policies
  - ✖ Intractable projections
- Linearization and convex surrogates
  e.g., CPO [Achiam et al., ICML'17]
  - ✖ No approximation guarantee
  - ✖ Approximate problem may be infeasible

CRL methods

$$\begin{aligned} &\underset{\pi \in \mathcal{P}(\mathcal{S})}{\text{maximize}} && \mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T-1}\gamma^t r_0(s_t,a_t)\right] \\ &\text{subject to} && \mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T-1}\gamma^t r_i(s_t,a_t)\right] \geq c_i \end{aligned}$$

- Reward shaping ≈ penalty methods
- Prior knowledge ≈ projection methods
  e.g., safe exploration [Berkenkamp et al., NeurIPS'17, Dalal et al., arXiv'18]
- Linearization and convex surrogates
  e.g., CPO [Achiam et al., ICML'17]
- Duality
  [Bhatnagar et al., JOTA'12; Tesler et al., ICRL'19; PCCR, NeurIPS'19; Ding et al., NeurIPS'20; PCCR, IEEE TAC'23 . . .]
  - ✔ Domain independent
  - ✔ Tractable
  - ✖ Approximation guarantee [non-convexity]

Agenda

Constrained reinforcement learning

CMDP duality

CRL algorithms

CMDP duality

$$D^\star = \min_{\lambda \succeq 0} \max_{\pi\in\mathcal{P}(\mathcal{S})} \overbrace{\mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T-1}\gamma^t r_0(s_t,a_t)\right] + \lambda\,\mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T-1}\gamma^t r_1(s_t,a_t)\right]}^{L(\pi,\lambda)}$$

DUAL

$$\not\Updownarrow$$

$$P^\star = \max_{\pi\in\mathcal{P}(\mathcal{S})} \mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T-1}\gamma^t r_0(s_t,a_t)\right] \text{ subject to } \mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T-1}\gamma^t r_1(s_t,a_t)\right] \geq 0$$

PRIMAL

- ✔ Domain independent ⇐ No hyperparameters tuning
- ✔ Tractable ⇐ Equivalent to solving a sequence of unconstrained RL problems
- ✖ Approximation guarantee ⇐ ~~$D^\star = P^\star$ (strong duality)~~ ~~[e.g., convex optimization]~~

CMDP duality

$$D^\star = \min_{\lambda \succeq 0} \max_{\pi\in\mathcal{P}(\mathcal{S})} \overbrace{\mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T-1}\gamma^t r_0(s_t,a_t)\right] + \lambda\,\mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T-1}\gamma^t r_1(s_t,a_t)\right]}^{L(\pi,\lambda)}$$

DUAL

$$\Updownarrow$$

$$P^\star = \max_{\pi\in\mathcal{P}(\mathcal{S})} \mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T-1}\gamma^t r_0(s_t,a_t)\right] \text{ subject to } \mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T-1}\gamma^t r_1(s_t,a_t)\right] \geq 0$$

PRIMAL

Theorem
If there exists $\pi^\dagger \in \mathcal{P}(\mathcal{S})$ such that $V_i(\pi^\dagger) > c_i$ for all $i = 1, \ldots, m$, then $D^\star = P^\star$ (strong duality).

- There is some sort of hidden convexity in CRL ⇒ Occupation measure

[Paternain, Chamon, Calvo-Fullana, Ribeiro, NeurIPS'19; Paternain, Calvo-Fullana, Chamon, Ribeiro, IEEE TAC'23]

Occupation measure

- The occupation measure of a policy π is the (averaged) probability of visiting each state-action pair

$$\rho_\pi(s,a) = \frac{1-\gamma}{1-\gamma^T}\sum_{t=0}^{T-1}\gamma^t\,\mathbb{P}_{s,a\sim\pi}\big(s_t = s, a_t = a\big) \longleftrightarrow \pi(a|s) = \frac{\rho_\pi(s,a)}{\displaystyle\int_{\mathcal{A}}\rho_\pi(s,a)da}$$

- The value functions $V_i(\pi)$ can be written as an expectation with respect to the ρ_π

$$\begin{aligned} \mathbb{E}_{s,a\sim\pi}\left[\sum_{t=0}^{T-1}\gamma^t r_i(s_t,a_t)\right] = V_i(\pi) \propto V(\rho_\pi) &= \mathbb{E}_{(s,a)\sim\rho_\pi}\big[r_i(s,a)\big] \\ &= \int_{\mathcal{S}\times\mathcal{A}} r_i(s,a)\rho_\pi(s,a)\,dsda \end{aligned}$$

⇒ The value functions $V_i(\rho_\pi)$ are linear with respect to the occupation measure ρ_π

A non-proof of strong duality

$$\begin{aligned} P^\star = \max_{\pi\in\mathcal{P}(\mathcal{S})} \quad & V_0(\pi) = \mathbb{E}_{s,a\sim\pi}\left[\sum_{t=0}^{T-1}\gamma^t r_0(s_t,a_t)\right] \\ \text{s. to} \quad & V_1(\pi) = \mathbb{E}_{s,a\sim\pi}\left[\sum_{t=0}^{T-1}\gamma^t r_1(s_t,a_t)\right] \geq c \end{aligned}$$

$$\equiv$$

$$\uparrow$$

$$(P^\star = P^\star_\rho)$$

$$\begin{aligned} P^\star_\rho = \max_{\rho\in\mathcal{R}} \quad & V_0(\rho) = \int r_i(s,a)\rho(s,a)dsda \\ \text{s. to} \quad & V_1(\rho) = \int r_1(s,a)\rho(s,a)dsda \geq c \end{aligned}$$

- CRL is *non-convex* in policy space, but *linear* in occupation measure space

[Paternain, Chamon, Calvo-Fullana, Ribeiro, NeurIPS'19; Paternain, Calvo-Fullana, Chamon, Ribeiro, IEEE TAC'23]

A non-proof of strong duality

$$\begin{aligned} P^\star = \max_{\pi\in\mathcal{P}(\mathcal{S})} \quad & V_0(\pi) = \mathbb{E}_{s,a\sim\pi}\left[\sum_{t=0}^{T-1}\gamma^t r_0(s_t,a_t)\right] \\ \text{s. to} \quad & V_1(\pi) = \mathbb{E}_{s,a\sim\pi}\left[\sum_{t=0}^{T-1}\gamma^t r_1(s_t,a_t)\right] \geq c \end{aligned}$$

$$\equiv$$

$$\uparrow$$

$$(P^\star = P^\star_\rho)$$

$$\begin{aligned} P^\star_\rho = \max_{\rho\in\mathcal{R}} \quad & V_0(\rho) = \int r_i(s,a)\rho(s,a)dsda \\ \text{s. to} \quad & V_1(\rho) = \int r_1(s,a)\rho(s,a)dsda \geq c \end{aligned}$$

(strongly dual)

- CRL is *non-convex* in policy space, but *linear* in occupation measure space
- CRL in occupation measure space has *no duality gap* (LP)

$$P^\star_\rho = D^\star_\rho = \min_{\lambda\geq 0}\max_{\rho\in\mathcal{R}} V_0(\rho) + \lambda\big(V_1(\rho) - c\big)$$

[Paternain, Chamon, Calvo-Fullana, Ribeiro, NeurIPS'19; Paternain, Calvo-Fullana, Chamon, Ribeiro, IEEE TAC'23]

A non-proof of strong duality

$$P^\star = \max_{\pi\in\mathcal{P}(\mathcal{S})} \; V_0(\pi) = \mathbb{E}_{s,a\sim\pi}\left[\sum_{t=0}^{T-1}\gamma^t r_0(s_t,a_t)\right] \qquad P^\star_\rho = \max_{\rho\in\mathcal{R}} \; V_0(\rho) = \int r_i(s,a)\rho(s,a)dsda$$

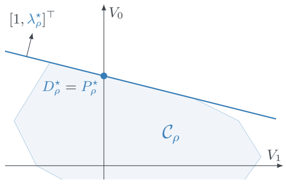
$$\text{s. to} \quad V_1(\pi) = \mathbb{E}_{s,a\sim\pi}\left[\sum_{t=0}^{T-1}\gamma^t r_1(s_t,a_t)\right] \geq c \qquad \equiv \qquad \text{s. to} \quad V_1(\rho) = \int r_1(s,a)\rho(s,a)dsda \geq c$$

~~(strongly dual)~~ $\not\Leftarrow$ $(P^\star = P^\star_\rho)$ + (strongly dual)

- CRL is *non-convex* in policy space, but *linear* in occupation measure space
- CRL in occupation measure space has *no duality gap* (LP)

$$P^\star_\rho = D^\star_\rho = \min_{\lambda\geq 0} \max_{\rho\in\mathcal{R}} \; V_0(\rho) + \lambda\big(V_1(\rho) - c\big)$$

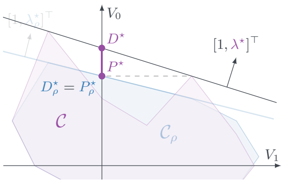
A ~~non~~-proof of strong duality



- Epigraph of CRL in occupation measure is convex

$$\mathcal{C}_\rho = \Big\{ \big[V_0(\rho); V_1(\rho)\big] \text{ for some } \rho\in\mathcal{R} \Big\}$$

A ~~non~~-proof of strong duality



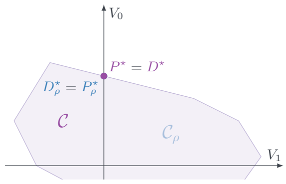
- Epigraph of CRL in occupation measure is convex

$$\mathcal{C}_\rho = \Big\{ \big[V_0(\rho); V_1(\rho)\big] \text{ for some } \rho\in\mathcal{R} \Big\}$$

- Epigraph of CRL in policy need not be convex

$$\mathcal{C} = \Big\{ \big[V_0(\pi); V_1(\pi)\big] \text{ for some } \pi\in\mathcal{P}(\mathcal{S}) \Big\}$$

A ~~non~~-proof of strong duality



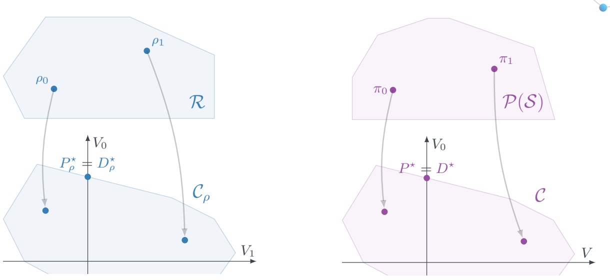
- Epigraph of CRL in occupation measure is convex

$$\mathcal{C}_\rho = \Big\{ \big[V_0(\rho); V_1(\rho)\big] \text{ for some } \rho\in\mathcal{R} \Big\}$$

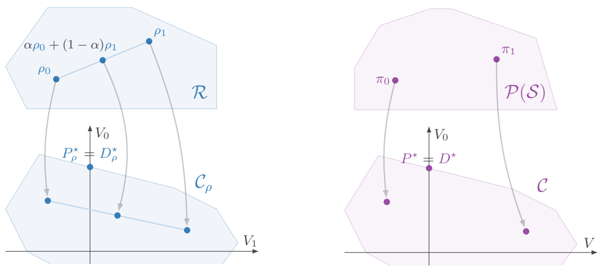
- Epigraph of CRL in policy need not be convex

$$\mathcal{C} = \Big\{ \big[V_0(\pi); V_1(\pi)\big] \text{ for some } \pi\in\mathcal{P}(\mathcal{S}) \Big\}$$

Epigraphs are "convex" in different ways

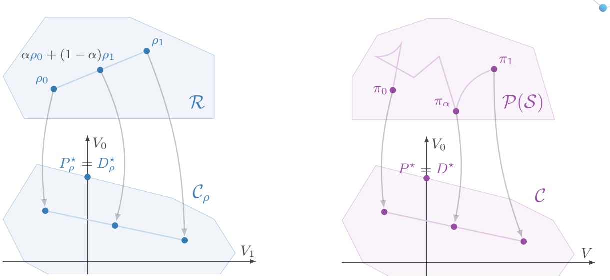


Epigraphs are "convex" in different ways



$$V\big(\alpha\rho_1 + (1-\alpha)\rho_2\big) = \alpha V(\rho_1) + (1-\alpha)V(\rho_2)$$

Epigraphs are "convex" in different ways



$V\big(\alpha\rho_1 + (1-\alpha)\rho_2\big) = \alpha V(\rho_1) + (1-\alpha)V(\rho_2)$ $\quad$ $\exists\, \pi_\alpha$ such that $V(\pi_\alpha) = \alpha V(\pi_1) + (1-\alpha)V(\pi_2)$

Strong duality in practice

$$P^\star = D^\star = \min_{\lambda\geq 0} \max_{\pi\in\mathcal{P}(\mathcal{S})} \; \mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T-1}\gamma^t r_0(s_t,a_t)\right] + \lambda\,\mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T-1}\gamma^t r_1(s_t,a_t)\right]$$

- Strong duality in policy space $\mathcal{P}(\mathcal{S})$ despite $V_0(\pi)$ and $V(\pi)$ being non-convex

Strong duality in practice

$$P^\star = D^\star = \min_{\lambda\succeq 0}\ \max_{\pi\in\mathcal{P}(\mathcal{S})}\ \mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T-1}\gamma^t r_0(s_t,a_t)\right] + \lambda\,\mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T-1}\gamma^t r_1(s_t,a_t)\right]$$

$$\updownarrow \Delta$$

$$D^\star_\theta = \min_{\lambda\succeq 0}\ \max_{\theta\in\Theta}\ \mathbb{E}_{s,a\sim\pi_\theta}\left[\frac{1}{T}\sum_{t=0}^{T-1}\gamma^t r_0(s_t,a_t)\right] + \lambda\,\mathbb{E}_{s,a\sim\pi_\theta}\left[\frac{1}{T}\sum_{t=0}^{T-1}\gamma^t r_1(s_t,a_t)\right]$$

- Strong duality in policy space $\mathcal{P}(\mathcal{S})$ despite $V_0(\pi)$ and $V(\pi)$ being non-convex
- But in practice, policies are parameterized (π_θ)
  - $\Rightarrow$ Introduces a duality gap Δ because standard parametrizations are not convex

[Paternain, Chamon, Calvo-Fullana, Ribeiro, NeurIPS'19; Paternain, Calvo-Fullana, Chamon, Ribeiro, IEEE TAC'23]

Duality gap of parametrized CRL

Theorem

Let π_θ be ν-universal, i.e.,

$$\min_{\theta\in\Theta}\ \max_{s\in\mathcal{S}} \int_{\mathcal{A}} \left|\pi(a|s)-\pi_\theta(a|s)\right| da \leq \nu, \text{ for all } \pi\in\mathcal{P}(\mathcal{S}).$$

Then,

$$\left|P^\star - D^\star_\theta\right| = \Delta \leq \frac{1+\|\boldsymbol{\lambda}^\star_\nu\|_1}{1-\gamma} B\nu$$

[Paternain, Chamon, Calvo-Fullana, Ribeiro, NeurIPS'19; Paternain, Calvo-Fullana, Chamon, Ribeiro, IEEE TAC'23]

Duality gap of parametrized CRL

Theorem

Let π_θ be ν-universal, i.e.,

$$\min_{\theta\in\Theta}\ \max_{s\in\mathcal{S}} \int_{\mathcal{A}} \left|\pi(a|s)-\pi_\theta(a|s)\right| da \leq \nu, \text{ for all } \pi\in\mathcal{P}(\mathcal{S}).$$

Then,

$$\left|P^\star - D^\star_\theta\right| = \Delta \leq \frac{1+\|\boldsymbol{\lambda}^\star_\nu\|_1}{1-\gamma} B\nu$$

Sources of error

parametrization richness (ν) requirements difficulty ($\lambda^\star_\nu$) horizon (γ)

[Paternain, Chamon, Calvo-Fullana, Ribeiro, NeurIPS'19; Paternain, Calvo-Fullana, Chamon, Ribeiro, IEEE TAC'23]

Duality gap of parametrized CRL

Theorem

Let π_θ be ν-universal, i.e.,

$$\min_{\theta\in\Theta}\ \max_{s\in\mathcal{S}} \int_{\mathcal{A}} \left|\pi(a|s)-\pi_\theta(a|s)\right| da \leq \nu, \text{ for all } \pi\in\mathcal{P}(\mathcal{S}).$$

Then,

$$\left|P^\star - D^\star_\theta\right| = \Delta \leq \frac{1+\|\boldsymbol{\lambda}^\star_\nu\|_1}{1-\gamma} B\nu$$

Sources of error

parametrization richness (ν) requirements difficulty ($\lambda^\star_\nu$) horizon (γ)

[Paternain, Chamon, Calvo-Fullana, Ribeiro, NeurIPS'19; Paternain, Calvo-Fullana, Chamon, Ribeiro, IEEE TAC'23]

Duality gap of parametrized CRL

Theorem

Let π_θ be ν-universal, i.e.,

$$\min_{\theta\in\Theta}\ \max_{s\in\mathcal{S}} \int_{\mathcal{A}} \left|\pi(a|s)-\pi_\theta(a|s)\right| da \leq \nu, \text{ for all } \pi\in\mathcal{P}(\mathcal{S}).$$

Then,

$$\left|P^\star - D^\star_\theta\right| = \Delta \leq \frac{1+\|\boldsymbol{\lambda}^\star_\nu\|_1}{1-\gamma} B\nu$$

Sources of error

parametrization richness (ν) requirements difficulty ($\lambda^\star_\nu$) horizon (γ)

[Paternain, Chamon, Calvo-Fullana, Ribeiro, NeurIPS'19; Paternain, Calvo-Fullana, Chamon, Ribeiro, IEEE TAC'23]

Agenda

Constrained reinforcement learning

CMDP duality

CRL algorithms

Primal-dual algorithm

$$D^\star_\theta = \min_{\lambda\succeq 0}\ \max_{\theta\in\Theta}\ \mathbb{E}_{s,a\sim\pi_\theta}\left[\frac{1}{T}\sum_{t=0}^{T-1}\gamma^t r_0(s_t,a_t)\right] + \lambda\left(\mathbb{E}_{s,a\sim\pi_\theta}\left[\frac{1}{T}\sum_{t=0}^{T-1}\gamma^t r_1(s_t,a_t)\right] - c_1\right)$$

Primal-dual algorithm

$$D^\star_\theta = \min_{\lambda\succeq 0}\ \max_{\theta\in\Theta}\ \mathbb{E}_{s,a\sim\pi_\theta}\left[\frac{1}{T}\sum_{t=0}^{T-1}\gamma^t r_0(s_t,a_t)\right] + \lambda\left(\mathbb{E}_{s,a\sim\pi_\theta}\left[\frac{1}{T}\sum_{t=0}^{T-1}\gamma^t r_1(s_t,a_t)\right] - c_1\right)$$

- Maximize the primal ($\equiv$ vanilla RL)

$$\theta^\dagger \in \operatorname*{argmax}_{\theta\in\Theta}\ \mathbb{E}_{s,a\sim\pi_\theta}\left[\frac{1}{T}\sum_{t=0}^{T-1}\gamma^t r_{\lambda_k}(s_t,a_t)\right]$$

$$r_{\lambda_k}(s,a) = r_0(s,a) + \lambda_k r_1(s,a)$$

Primal-dual algorithm

$$D_\theta^\star = \min_{\lambda \succeq 0} \max_{\theta \in \Theta} \mathbb{E}_{s,a\sim\pi_\theta}\left[\frac{1}{T}\sum_{t=0}^{T-1}\gamma^t r_0(s_t,a_t)\right] + \lambda\left(\mathbb{E}_{s,a\sim\pi_\theta}\left[\frac{1}{T}\sum_{t=0}^{T-1}\gamma^t r_1(s_t,a_t)\right] - c_1\right)$$

- Maximize the primal ($\equiv$ vanilla RL)

$$\theta^\dagger \in \underset{\theta\in\Theta}{\operatorname{argmax}}\ \mathbb{E}_{s,a\sim\pi_\theta}\left[\frac{1}{T}\sum_{t=0}^{T-1}\gamma^t r_{\lambda_k}(s_t,a_t)\right]$$

$$r_{\lambda_k}(s,a) = r_0(s,a) + \lambda_k r_1(s,a)$$

- Update the dual ($\equiv$ policy evaluation)

$$\lambda_{k+1} = \left[\lambda_k - \eta\left(\mathbb{E}_{s,a\sim\pi_{\theta^\dagger}}\left[\frac{1}{T}\sum_{t=0}^{T-1}\gamma^t r_1(s_t,a_t)\right] - c_1\right)\right]_+$$

In practice...

$$D_\theta^\star = \min_{\lambda \succeq 0} \max_{\boldsymbol{\theta} \in \Theta} \mathbb{E}_{s,a\sim\pi_{\boldsymbol{\theta}}}\left[\frac{1}{T}\sum_{t=0}^{T-1}\gamma^t r_0(s_t,a_t)\right] + \lambda\left(\mathbb{E}_{s,a\sim\pi_{\boldsymbol{\theta}}}\left[\frac{1}{T}\sum_{t=0}^{T-1}\gamma^t r_1(s_t,a_t)\right] - c_1\right)$$

- Maximize the primal ($\equiv$ vanilla RL): $\{s_t, a_t\} \sim \pi_{\boldsymbol{\theta}_k}$

$$\boldsymbol{\theta}_{k+1} = \boldsymbol{\theta}_k + \eta\left[\frac{1}{T}\sum_{t=0}^{T-1}\gamma^t r_{\lambda_k}(s_t,a_t)\right]\nabla_{\boldsymbol{\theta}}\log\left(\pi_{\boldsymbol{\theta}}(a_0|s_0)\right)$$

- Update the dual ($\equiv$ policy evaluation): $\{s_t, a_t\} \sim \pi_{\boldsymbol{\theta}_k}$

$$\lambda_{k+1} = \left[\lambda_k - \eta\left(\frac{1}{T}\sum_{t=0}^{T-1}\gamma^t r_1(s_t,a_t) - c_1\right)\right]_+$$

Dual CRL

Theorem
Suppose $\boldsymbol{\theta}^\dagger$ is a ρ-approximate solution of the regularized RL problem:

$$\boldsymbol{\theta}^\dagger \approx \underset{\boldsymbol{\theta}\in\Theta}{\operatorname{argmax}}\ \mathbb{E}_{s,a\sim\pi_{\boldsymbol{\theta}}}\left[\frac{1}{T}\sum_{t=0}^{T-1}\gamma^t r_\lambda(s_t,a_t)\right].$$

Then, after $K = \left\lceil \frac{\|\boldsymbol{\lambda}^\star\|^2}{2\eta\nu}\right\rceil + 1$ dual iterations with step size $\eta \leq \frac{1-\gamma}{mB}$,

the iterates $\left(\boldsymbol{\theta}_K, \boldsymbol{\lambda}_K\right)$ are such that

$$\left|P^\star - L\left(\boldsymbol{\theta}_K, \boldsymbol{\lambda}_K\right)\right| \leq \frac{1+\|\boldsymbol{\lambda}_\nu^\star\|_1}{1-\gamma} B\nu + \rho$$

[Paternain, Chamon, Calvo-Fullana, and Ribeiro, NeurIPS'19; Calvo-Fullana, Paternain, Chamon, and Ribeiro, IEEE TAC'24]

Dual CRL

Theorem

$$\left|P^\star - L\left(\boldsymbol{\theta}_K, \boldsymbol{\lambda}_K\right)\right| \leq \frac{1+\|\boldsymbol{\lambda}_\nu^\star\|_1}{1-\gamma} B\nu + \rho$$

Theorem
The state-action sequence $\left\{s_t, a_t \sim \pi^\dagger(\boldsymbol{\lambda}_k)\right\}$ generated by dual CRL is $(\rho = \nu = 0)$

(i) almost surely feasible: $\lim_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} r_i(s_t,a_t) \geq c_i$ a.s., for all i

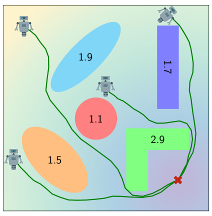
(ii) near-optimal: $\lim_{T\to\infty} \mathbb{E}\left[\frac{1}{T}\sum_{t=0}^{T-1} r_0(s_t,a_t)\right] \geq P^\star - \frac{\eta B^2}{2}$

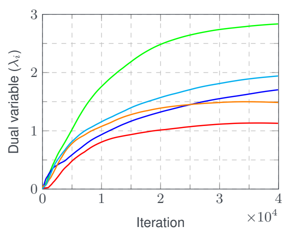
i.e., is a *solution* of the CRL problem (in fact, it is *stronger*: constraints are satisfied a.s.)

[Paternain, Chamon, Calvo-Fullana, and Ribeiro, NeurIPS'19; Calvo-Fullana, Paternain, Chamon, and Ribeiro, IEEE TAC'24]

Safe navigation

Problem
Find a control policy that navigates the environment effectively and safely

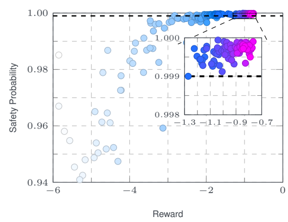


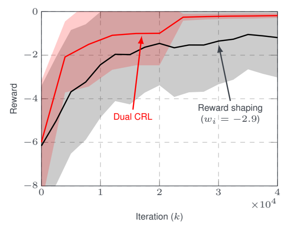


[Paternain, Calvo-Fullana, Chamon, Ribeiro, IEEE TAC'23]

Safe navigation

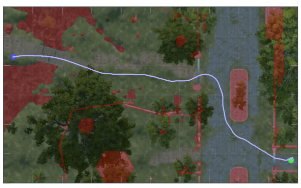
Problem
Find a control policy that navigates the environment effectively and safely





[Paternain, Calvo-Fullana, Chamon, Ribeiro, IEEE TAC'23]

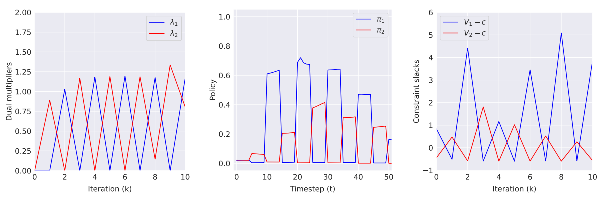
Safe navigation



[Paternain, Calvo-Fullana, Chamon, Ribeiro, IEEE TAC'23]

Wireless resource allocation

Problem
Allocate the least transmit power to m device pairs to achieve a communication rate

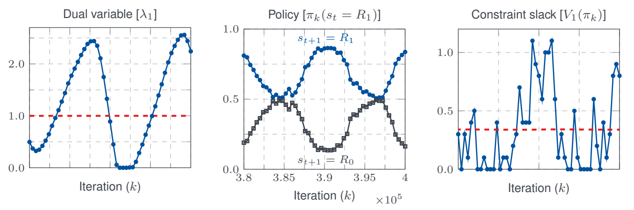


- The dual variables oscillate $\Rightarrow$ the policy switch $\Rightarrow$ constraint slacks to oscillate (feasible *on average*)

[Uslu, Doostnejad, Ribeiro, NaderiAlizadeh, arxiv:2405.05748]

Monitoring task

Problem
Find a policy that maximizes the time in R_0 while monitoring R_1 and R_2 at least $1/3$ of the time each



- The dual variables oscillate ⇒ the policy switch ⇒ constraint slacks to oscillate (feasible *on average*)

[Calvo-Fullana, Paternain, Chamon, and Ribeiro, IEEE TAC'24]

What dual CRL cannot do

Theorem

$$\left|P^\star - L\left(\theta_K, \lambda_T\right)\right| \leq \frac{1 + \|\lambda_\epsilon^\star\|_1}{1-\gamma} B\nu + \rho$$

Theorem
The state-action sequence $\{s_t, a_t \sim \pi^\dagger(\lambda_k)\}$ generated by dual CRL is $(\rho = \nu = 0)$

(i) almost surely feasible: $\lim_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} r_i(s_t, a_t) \geq c_i$ a.s., for all i

(ii) near-optimal: $\lim_{T\to\infty} \mathbb{E}\left[\frac{1}{T}\sum_{t=0}^{T-1} r_0(s_t, a_t)\right] \geq P^\star - \frac{\eta B^2}{2}$

i.e., is a *solution* of the CRL problem.

⇒ Cannot *effectively* obtain an optimal policy $\pi^\star$ from the sequence of Lagrangian maximizers $\pi^\dagger(\lambda_k)$

[Paternain, Chamon, Calvo-Fullana, and Ribeiro, NeurIPS'19; Calvo-Fullana, Paternain, Chamon, and Ribeiro, IEEE TAC'24]

Primal recovery

- General issue with duality
  - (Primal-)dual methods: $\frac{1}{K}\sum_{k=0}^{K-1} f(\theta_k) \to f(\theta^\star)$, but $f(\theta_k) \not\to f(\theta^\star)$

✔ Convex optimization ⇒ dual averaging

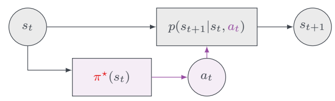
- $f\left(\frac{1}{K}\sum_{k=0}^{K-1}\theta_k\right) \leq \frac{1}{K}\sum_{k=0}^{K-1} f(\theta_k)$ for all K (convexity) ⇒ $\frac{1}{K}\sum_{k=1}^{K}\theta_k \to \theta^\star$

✘ Non-convex optimization ⇒ randomization

- $\theta^\dagger \sim$ Uniform(θ_k) ⇒ $\mathbb{E}\left[f(\theta^\dagger)\right] = \frac{1}{K}\sum_{k=1}^{K} f(\theta_k) \to f(\theta^\star)$

(requires memorizing the whole training sequence)

What we CANNOT do

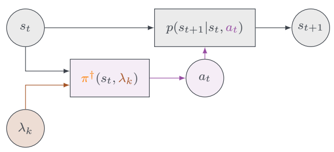


✘ We do not know how to find an optimal policy $\pi^\star$ in the policy space

$$\pi^\star \in \underset{\pi\in\mathcal{P}(\mathcal{S})}{\operatorname{argmax}} \quad \lim_{T\to\infty} \mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T-1} r_0(s_t, a_t)\right]$$
$$\text{subject to} \quad \lim_{T\to\infty} \mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T-1} r_1(s_t, a_t)\right] \geq c_1$$

[Calvo-Fullana, Paternain, Chamon, Ribeiro, IEEE TAC'23]

What we CAN do

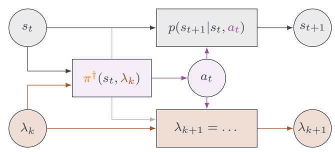


✔ Find Lagrangian maximizing policies $\pi^\dagger(\lambda_k)$ ⇒ unconstrained RL problem with reward $r_{\lambda_k}(s, a)$

$$\pi^\dagger(\lambda_k) \in \underset{\pi\in\mathcal{P}(\mathcal{S})}{\operatorname{argmax}} \quad \lim_{T\to\infty} \mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T-1} r_{\lambda_k}(s_t, a_t)\right]$$

[Calvo-Fullana, Paternain, Chamon, Ribeiro, IEEE TAC'23]

What we CAN do



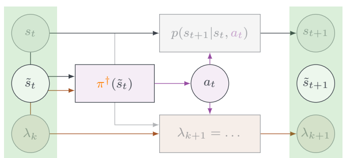
✔ Find Lagrangian maximizing policies $\pi^\dagger(\lambda_k)$ ⇒ unconstrained RL problem with reward $r_{\lambda_k}(s, a)$

✔ Update λ_k to generate a sequence of $\pi^\dagger(\lambda_k)$ that are "samples" from $\pi^\star$

$$\lambda_{k+1} = \left[\lambda_k - \eta\left(\mathbb{E}_{s,a\sim\pi^\dagger(\lambda_k)}\left[\frac{1}{T}\sum_{t=0}^{T-1} r_1(s_t, a_t)\right] - c_1\right)\right]_+$$

[Calvo-Fullana, Paternain, Chamon, Ribeiro, IEEE TAC'23]

State-augmented CRL



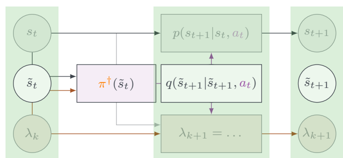
✔ Find Lagrangian maximizing policies $\pi^\dagger(\lambda_k)$ ⇒ unconstrained RL problem with reward $r_{\lambda_k}(s, a)$

✔ Update λ_k to generate a sequence of $\pi^\dagger(\lambda_k)$ that are "samples" from $\pi^\star$

⇒ equivalent to an MDP with (augmented) states $\tilde{s} = (s, \lambda)$

[Calvo-Fullana, Paternain, Chamon, Ribeiro, IEEE TAC'23]

State-augmented CRL



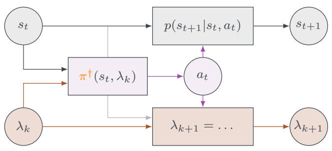
✔ Find Lagrangian maximizing policies $\pi^\dagger(\lambda_k)$ ⇒ unconstrained RL problem with reward $r_{\lambda_k}(s, a)$

✔ Update λ_k to generate a sequence of $\pi^\dagger(\lambda_k)$ that are "samples" from $\pi^\star$

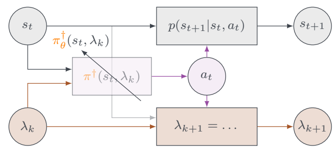
⇒ equivalent to an MDP with (augmented) states $\tilde{s} = (s, \lambda)$
and (augmented) transition kernel that includes the dual variables updates

[Calvo-Fullana, Paternain, Chamon, Ribeiro, IEEE TAC'23]

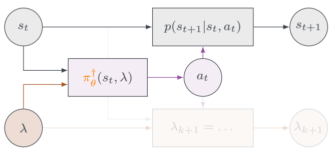
State-augmented CRL in practice



State-augmented CRL in practice

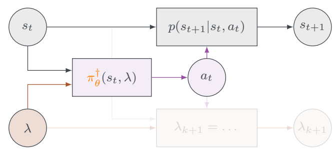


State-augmented CRL in practice



- **During training:** Learn a family of policies $\pi_\theta^\dagger(s, \lambda)$ that maximizes the Lagrangian for all (fixed) λ

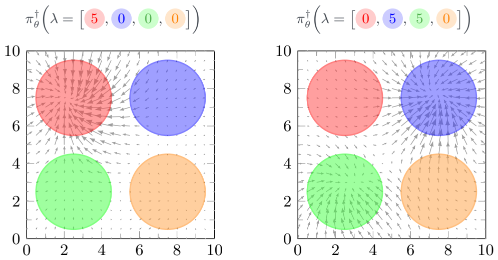
State-augmented CRL in practice



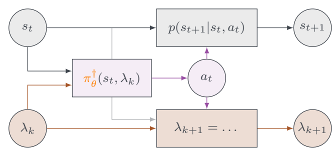
- **During training:** Learn a family of policies $\pi_\theta^\dagger(s, \lambda)$ that maximizes the Lagrangian for all (fixed) λ

$$\pi_\theta^\dagger(\lambda) \in \underset{\theta \in \Theta}{\operatorname{argmax}}\ \mathbb{E}_{\lambda \sim \mathfrak{m}} \left[\lim_{T \to \infty} \mathbb{E}_{s,a \sim \pi} \left[\frac{1}{T} \sum_{t=0}^{T-1} r_\lambda(s_t, a_t) \right] \right]$$

Monitoring task



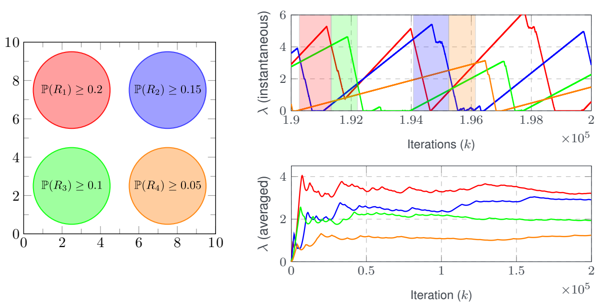
State-augmented CRL in practice



- **During training:** $\pi_\theta^\dagger(\lambda) \in \underset{\theta \in \Theta}{\operatorname{argmax}} \lim_{T \to \infty} \mathbb{E}_{s,a \sim \pi} \left[\frac{1}{T} \sum_{t=0}^{T-1} r_\lambda(s_t, a_t) \right]$, for all λ
- **During deployment:** Execute $a_t \sim \pi_\theta^\dagger(\lambda_k)$ for T_0 iterations and update λ_k

$$\lambda_{k+1} = \left[\lambda_k - \frac{\eta}{T_0} \sum_{t=kT_0}^{(k+1)T_0 - 1} \Big(r_1(s_t, a_t) - c_1 \Big) \right]_+$$

Monitoring task



Solving CRL

A-CRL:

$$\begin{cases} \textbf{Training:} & \pi_\theta^\dagger(\lambda) \in \operatorname{argmax}_{\theta \in \Theta} \lim_{T \to \infty} \mathbb{E}_{s,a \sim \pi} \left[\frac{1}{T} \sum_{t=0}^{T-1} r_\lambda(s_t, a_t) \right], \quad \text{for all } \lambda \\ \textbf{Deployment:} & \lambda_{k+1} = \left[\lambda_k - \frac{\eta}{T_0} \sum_{t=kT_0}^{(k+1)T_0 - 1} \Big(r_1(s_t, a_t) - c_1 \Big) \right]_+, \quad a_t \sim \pi_\theta^\dagger(\lambda_k) \end{cases}$$

- A-CRL solves (P-CRL) by generating state-action sequences $\{(s_t, a_t)\}$ that are (i) almost surely feasible and (ii) $O(\eta)$-optimal [Calvo-Fullana, Paternain, Chamon, Ribeiro, IEEE TAC'23]

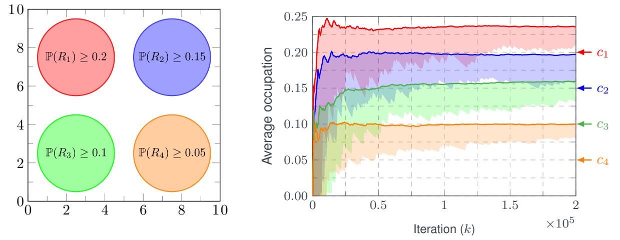
Solving CRL

$$\text{A-CRL:}\quad \begin{cases} \textbf{Training:} & \pi_\theta^\dagger(\lambda) \in \operatorname{argmax}_{\theta\in\Theta} \lim_{T\to\infty} \mathbb{E}_{s,a\sim\pi}\left[\frac{1}{T}\sum_{t=0}^{T-1} r_\lambda(s_t,a_t)\right], \quad \text{for all } \lambda \\ \textbf{Deployment:} & \lambda_{k+1} = \left[\lambda_k - \frac{\eta}{T_0}\sum_{t=kT_0}^{(k+1)T_0-1}\Big(r_1(s_t,a_t) - c_1\Big)\right]_+, \quad a_t \sim \pi_\theta^\dagger(\lambda_k) \end{cases}$$

- A-CRL solves (P-CRL) by generating state-action sequences $\{(s_t, a_t)\}$ that are (i) almost surely feasible and (ii) $O(\eta)$-optimal [Calvo-Fullana, Paternain, Chamon, Ribeiro, IEEE TAC'23]
- But A-CRL does not find a feasible and $\mathcal{O}(\eta)$-optimal policy $\pi^\star$
  $\Rightarrow$ It finds a policy $\pi_\theta^\dagger$ on an augmented MDP (s, λ) that generates the same trajectories as dual CRL on the original MDP (s)

[Calvo-Fullana, Paternain, Chamon, Ribeiro, IEEE TAC'23]

Monitoring task

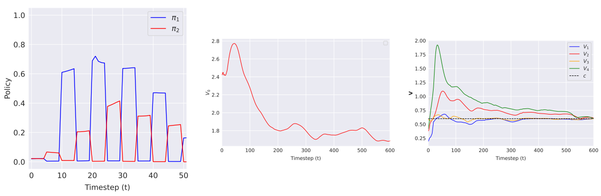


[Calvo-Fullana, Paternain, Chamon, Ribeiro, IEEE TAC'23]

Wireless resource allocation

Problem

Allocate the least transmit power to m device pairs to achieve a communication rate



[Uslu, Doostnejad, Ribeiro, NaderiAlizadeh, arxiv:2405.05748]

Summary

- **Constrained RL is the a tool for decision making under requirements**
- **Constrained RL is hard...**
- **...but possible. How?**

Summary

- **Constrained RL is the a tool for decision making under requirements**
  CRL is a natural way of specifying complex behaviors that cannot be handled by unconstrained RL
  $\Rightarrow$ (P-RL) $\subsetneq$ (P-CRL)
  e.g., safety [Paternain et al., IEEE TAC'23], wireless resource allocation [Eisen et al., IEEE TSP'19; Chowdhury et al., Asilomar'23], monitoring [Calvo-Fullana et al., IEEE TAC'24]
- **Constrained RL is hard...**
- **...but possible. How?**

Summary

- **Constrained RL is the a tool for decision making under requirements**
  CRL is a natural way of specifying complex behaviors that cannot be handled by unconstrained RL
  $\Rightarrow$ (P-RL) $\subsetneq$ (P-CRL)
  e.g., safety [Paternain et al., IEEE TAC'23], wireless resource allocation [Eisen et al., IEEE TSP'19; Chowdhury et al., Asilomar'23], monitoring [Calvo-Fullana et al., IEEE TAC'24]
- **Constrained RL is hard...**
  CRL is strongly dual (despite non-convexity), but that is not always enough to obtain feasible solutions
  $\Rightarrow$ ~~primal-dual methods~~
- **...but possible. How?**

Summary

- **Constrained RL is the a tool for decision making under requirements**
  CRL is a natural way of specifying complex behaviors that cannot be handled by unconstrained RL
  $\Rightarrow$ (P-RL) $\subsetneq$ (P-CRL)
  e.g., safety [Paternain et al., IEEE TAC'23], wireless resource allocation [Eisen et al., IEEE TSP'19; Chowdhury et al., Asilomar'23], monitoring [Calvo-Fullana et al., IEEE TAC'24]
- **Constrained RL is hard...**
  CRL is strongly dual (despite non-convexity), but that is not always enough to obtain feasible solutions
  $\Rightarrow$ ~~primal-dual methods~~
- **...but possible. How?**
  When combined with a *systematic state augmentation* technique, we can use policies that solve (P-RL) to solve (P-CRL)

Agenda

`https://luizchamon.com/imprs2024`

Universität Stuttgart

Survey:

forms.gle/Chtpw9FG1eDfnfeG7

www.luizchamon.com/imprs2024

ellis

SimTech Cluster of Excellence

KI institute

IMPRS tutorial
Sep. 19, 2024

supervised and reinforcement learning under requirements