

**Fig. 3** Cycle length design

a The design principle  
b Convergence time over

Applying the ECR over the standalone FIR-LMS filter (AF1) leads to the weighted variance relation [4], which provides a model for the AF1 transient evolution, assuming a real valued Gaussian white input  $u(i)$ ; i.e.

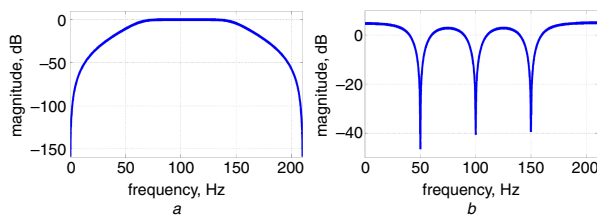
$$E\|\tilde{\mathbf{w}}_{1,i-1}\|^2 = \gamma E\|\mathbf{w}_1^0\|^2 + \mu_1 \sigma_v^2 \sigma_u^2 (M_1 + 1) \sum_{k=0}^{i-1} \gamma^k \quad (10)$$

where  $\gamma \triangleq 1 - 2\mu\sigma_u^2 + \mu^2\sigma_u^4(M_1 + 4)$ ,  $\mathbf{w}_1^0$  is the FIR truncated from the IIR  $H^0(z)$ ,  $\tilde{\mathbf{w}}_{1,i} = \mathbf{w}_1^0 - \mathbf{w}_{1,i}$ . Equation (10) can be approximated by  $E\|\tilde{\mathbf{w}}_{1,i-1}\|^2 \simeq \gamma^i E\|\mathbf{w}_1^0\|^2$ . As  $e_1(i) = \mathbf{u}_i^T \tilde{\mathbf{w}}_{1,i-1} + v(i)$ , the MSE  $Ee_1^2(i)$  equals  $\sigma_u^2 E\|\tilde{\mathbf{w}}_{1,i-1}\|^2 + \sigma_v^2$ . The quantity  $\|\mathbf{w}_1^0\|^2$  can be estimated from  $\|\mathbf{w}_1^0\|^2 = (\sigma_d^2 - \sigma_v^2)/\sigma_u^2$  [4], such that  $Ee_1^2(i) \simeq \gamma^i (\sigma_d^2 - \sigma_v^2)$  and the AF1 approximate transient model becomes (in dB)

$$\text{MSE}_{1,\text{dB}}(i) \simeq 10i \log \gamma + 10 \log(\sigma_d^2 - \sigma_v^2) \quad (11)$$

A good estimate for  $L$  is then  $L = \{i | \text{MSE}_1(i) = 10 \log \sigma_v^2\}$  (see Fig. 3a). This simple design is quite efficient, and robust as  $L$  varies over a wide range, as Fig. 3b depicts. Therein, the convergence time of an FIIR cell is depicted as a function of  $L$ , and it is defined as the time required for the combination to reach 85% of the AF2 steady-state error.

**Simulation results:** In the simulations below, the FIIR cell identifies the plants, the frequency responses of which are in Figs. 4a and b. Both are particularly hard due to their peculiar poles-zeros setup [6, 13]: the first is a Butterworth system with clustered poles and the second is a notch with underdamped poles. In both cases,  $M=6$ ,  $u(i)$  is a zero mean Gaussian white signal with power  $\sigma_u^2 = 1$  and  $v(i)$  has a power of  $\sigma_v^2 = 1 \times 10^{-3}$ .

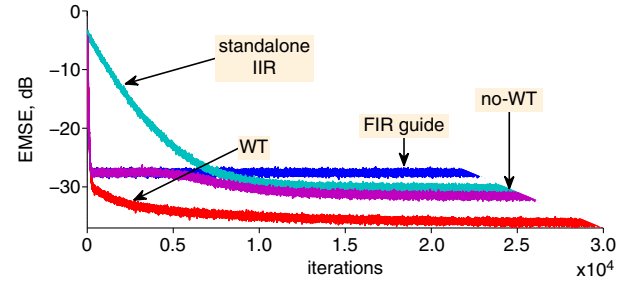


**Fig. 4** Frequency response of simulation scenarios

a Butterworth scenario  
b Notch scenario

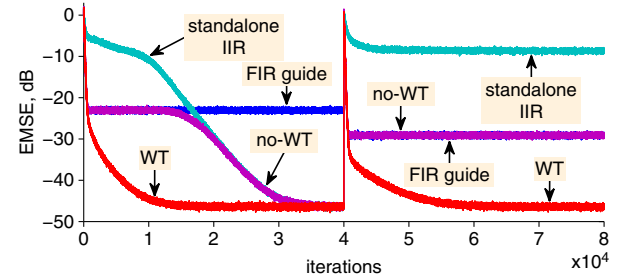
The plots that follow show the EMSE of AF1 ('guide'), the FIIR cell with weight transfers ('WT', see (7)) and without ('no-WT'), where the superiority of the former is clear. For reference, the EMSE of a standalone IIR AF exhibits near constant error regions around the global minimum on which gradient-based algorithms adapt slowly [6]. The combination considerably improves the performance, as AF1 guides AF2 towards the vicinity of the minimum rapidly. Only the first iterations are shown as the standalone IIR takes too long to reach the combination's steady-state.

In Fig. 5, the combination identifies the Butterworth system with  $M_1 = 10$ ,  $\mu_1 = 0.02$  and  $\mu_2 = 0.005$ . In this case, the error surface of a regular IIR AF exhibits near constant error regions around the global minimum on which gradient-based algorithms adapt slowly [6]. The combination considerably improves the performance, as AF1 guides AF2 towards the vicinity of the minimum rapidly. Only the first iterations are shown as the standalone IIR takes too long to reach the combination's steady-state.



**Fig. 5** Stationary scenario

In Fig. 6, the combination identifies the notch with  $M_1 = 15$ ,  $\mu_1 = 0.007$  and  $\mu_2 = 0.03$ . When  $i = 4 \times 10^4$ , the notch frequency changes suddenly (abrupt change in  $\mathbf{w}^0$ ). Although the standard IIR nearly stagnated, the FIIR cell responded fast, with a much superior performance.



**Fig. 6** Non-stationary scenario

**Conclusion:** The FIIR cells are robust, have a superior convergence rate and handle abrupt non-stationarities better than a standard IIR AF while avoiding the stability checks. In stationary scenarios, a properly designed  $L$  minimises the number of transfers needed to accelerate AF2 and decreases the overall complexity of the mapping  $\mathcal{P}$ . The current work involves fast Fourier transform (FFT)-based maps to efficiently replace the function  $\mathcal{P}$  employed here.

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One or more of the Figures in this Letter are available in colour online.

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