Universität Stuttgart

Luiz F. O. Chamon

robust learning

IMPRS BootCamp
Sep. 15th, 2023

ellis

SimTech Cluster of Excellence

Who am I?

Luiz F. O. Chamon

2022–: ELLIS-SimTech research group leader
IMPRS-IS faculty

2021–2022: Simons Institute, UC Berkeley (Postdoc)

2020: University of Pennsylvania (PhD)

< 2015: University of São Paulo, Brazil (BSc. & MSc.)

- I speak 4.5 languages (German = 0)

Disclaimer

https://luizchamon.com/pdf/imprs_bootcamp.pdf

What is about to follow...

- ...is **very** opinionated. Change my mind!
- ...contains ~~affiliated links~~ **shameless** plugs to my own work
- ...contains images and illustrations that I do not own: **do not share publicly**
- ...is **not** exhaustive

Agenda

What is "robust learning"?

Why are models brittle?

How to learn robustly?

What else are adversaries good for?

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What is learning?

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- **Learning:** "model-free statistics" (data-driven)
- **Inference:** "model-driven statistics" (~~data-free~~)

[Rostamizadeh, Talwalkar, Mohri. Foundations of machine learning, 2012]; [Ben-David, Shalev-Shwartz. Understanding machine learning..., 2014]

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Examples

- How good is an estimator (sample mean)? ⇒ confidence interval vs. cross-validation

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- Statistical optimization: $P^\star = \min_\theta \mathbb{E}_{(x,y)\sim\mathfrak{D}}\left[\ell\left(f_\theta(x), y\right)\right]$
  - ℓ is the bounded, Lipschitz continuous, possibly non-convex loss
  - f_θ is the (possibly nonlinear) parametrization [e.g., logistic classifier, (G)(C)NN]

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- Statistical optimization: $\hat{P}^\star = \min_\theta \frac{1}{N}\sum_{n=1}^{N} \ell\left(f_\theta(x_n), y_n\right), \quad (x_n, y_n) \sim \mathfrak{D}$ (i.i.d.)
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What is learning theory?

$$\min_\theta \frac{1}{N}\sum_{n=1}^{N} \text{Loss}\left(f_\theta(x_n), y_n\right) \xrightarrow{\text{"LLN"}} \min_\theta \mathbb{E}\left[\text{Loss}\left(f_\theta(x), y\right)\right]$$

- Is this possible with a finite number of samples (learnability)? When?
  e.g., linear functions, smooth functions (finite RKHS norm, bandlimited), NNs...
- How many samples do we need to get a good approximation (sample complexity)?
  $\approx 1/\epsilon^2$

Robustness in learning

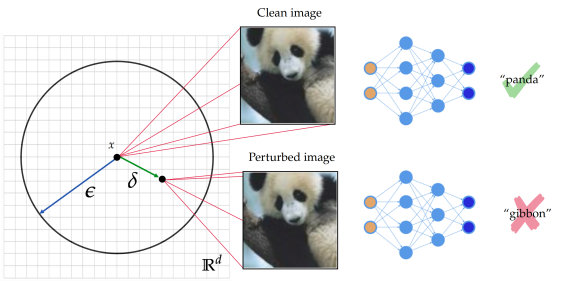
"Since such external effects are rarely expected to help achieve desired system behavior [...] they are commonly referred as *disturbances*.

One of the main objectives of feedback is to render a system insensitive to such disturbances."

[Dullerud, Paganini. A course in robust control theory, 2005.]

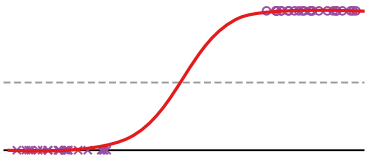
- Strong against input disturbances (≈ classical control: stability)
- Property of model/estimator/classifier

Robustness in learning



Smoothness vs. Robustness

Smoothness ⇒ Robustness



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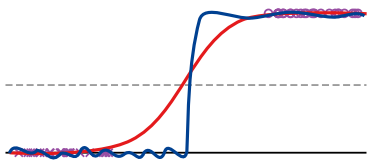

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Robustness in ~~statistics~~ inference

"*robustness signifies insensitivity to small deviations from the assumptions.*

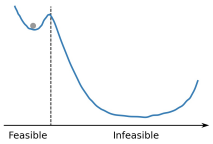
[...] the shape of the true underlying distribution deviates slightly from the assumed model (usually the Gaussian law)."

[Huber and Ronchetti. Robust Statistics, 2009.]

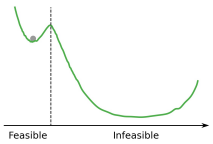
- Strong against model misspecification (≈ robust control)
  strong against "model misspecification" ≡ (*resistant*) [Hampel's theorem]
- Property of statistical problem

Resilience vs. Robustness

Robustness
(hard to break)



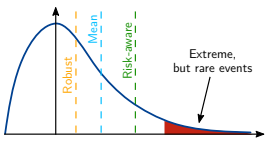
Resilience
(easy to fix)



Risk vs. Robustness

	Frequent	Rare
Relevant	Model	*Risk*
Irrelevant	Noise	Outlier

Robustness



- Value at risk (VaR, V@R), conditional VaR (CVaR, CV@R), expected shortfall…

[Shapiro et al. Lectures on Stochastic Programming, 2014]; [Kalogerias, **C.**, Pappas, Ribeiro. Better Safe Than Sorry…, IEEE ICASSP'20 (best paper)]

Summary

…in learning

- Strong against input disturbances
- Property of estimator
- "Test-time" distribution shift

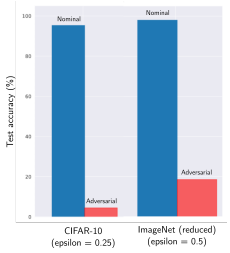
≠ …in inference (property of statistical problem)

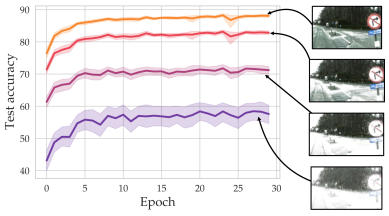
≠ smoothness

≠ resilience

≠ risk

Is it actually an issue?





[Ilyas et al. Adversarial examples are not bugs…, NeurIPS'19]; [Robey, Hassani, Pappas. Model-based robust deep learning…, arXiv'20]

Agenda

What is "robust learning"?

Why are models brittle?

How to learn robustly?

What else are adversaries good for?

Why are models brittle?

We don't really know
[long awkward pause]

- Overfitting
- Margin
- Features

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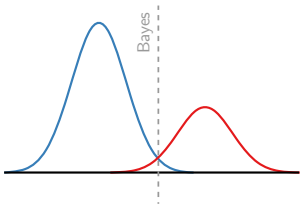
Binary classification



$$y = \begin{cases} +1, & \text{with prob. } \pi \\ -1, & \text{with prob. } 1-\pi \end{cases}$$

$$x \mid y \sim \text{Normal}(y\mu, \sigma^2 \boldsymbol{I})$$

$$\Delta = \{\boldsymbol{\delta} \mid \|\boldsymbol{\delta}\|_2 \leq \epsilon\}$$

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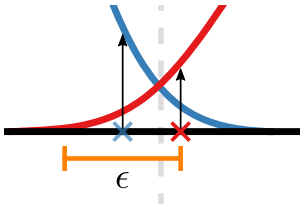

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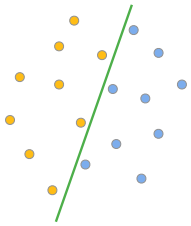
The margin argument

Data points close to the decision boundary are not robust



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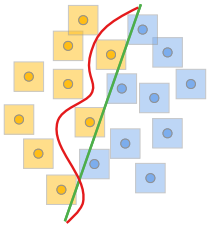

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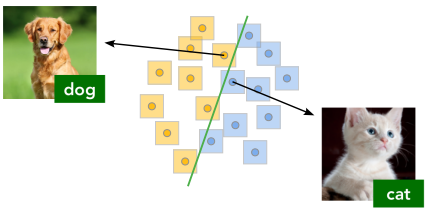

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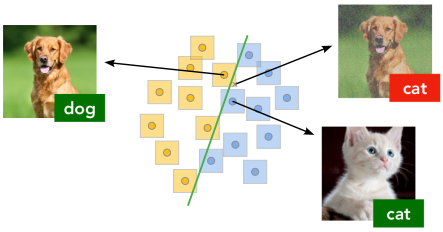

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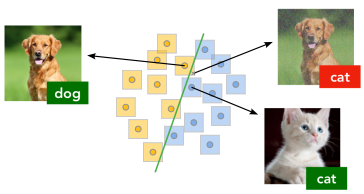

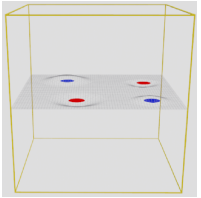

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We don't really know
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- Margin
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The features argument

Some features are more robust than others

- Adversarial examples arise from "spurious" features

[Ilyas, Santurkar, Tsipras, Engstrom, Tran, Mądry. Adversarial examples are not bugs, they are features, NeurIPS'19]

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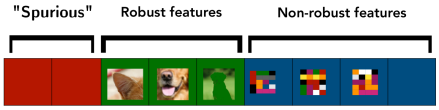

- Adversarial examples arise from "spurious" features
- Useful, non-robust features

[Ilyas, Santurkar, Tsipras, Engstrom, Tran, Mądry. Adversarial examples are not bugs, they are features, NeurIPS'19]

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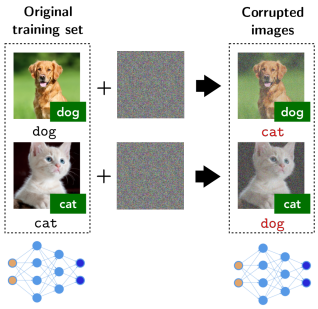

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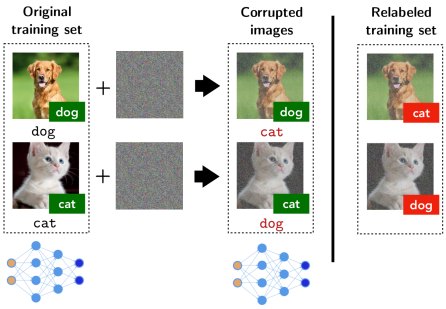

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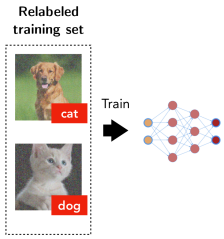

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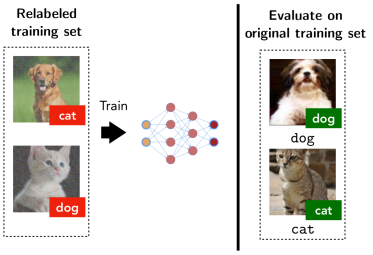

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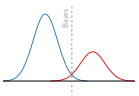
Summary



~~Overfitting~~

Margin

Features

Why are models brittle?

Learning is doing exactly what we asked for...

...and robustness is NOT what we asked for!

$$\min_{\theta} \mathbb{E}_{(x,y)\sim\mathcal{D}}\left[\text{Loss}(f_\theta(x), y)\right]$$

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Robust learning(ish)

Post-processing

- Filtering

$$f_{\theta\uparrow} \longrightarrow f_{\theta\uparrow} \circ \varphi$$

[Das, Shanbhogue, Chen, Hohman, Li, Chen, Kounavis, Chau. SHIELD: Fast, Practical..., KDD'18]
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$$f_{\theta\uparrow}(\cdot) \longrightarrow \mathbb{E}_{z\sim\mathrm{m}}\left[f_{\theta\uparrow}(\cdot + z)\right]$$

[Lecuyer, Atlidakis, Geambasu, Hsu, and Jana. Certified robustness to adversarial examples..., IEEE SSP'19]
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Robust learning

Data augmentation

- Random augmentation

$$\min_{\theta} \frac{1}{N}\sum_{n=1}^{N} \text{Loss}\left(f_\theta(x_n), y_n\right) \longrightarrow \min_{\theta} \frac{1}{NK}\sum_{n=1}^{N}\sum_{k=1}^{K} \text{Loss}\left(f_\theta(x_n + \delta_k), y_n\right), \quad \delta_k \sim \mathrm{m}$$

[Holmstrom, Koistinen. Using additive noise in back-propagation training, IEEE TNN'92]
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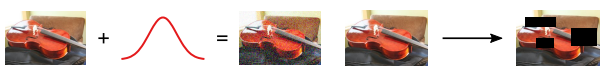

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- "Pseudolabeling"

Pretrained network: $h^\dagger$
Unlabeled dataset: $\{z\}$

$$\longrightarrow \min_{\theta} \frac{1}{N}\sum_{n=1}^{N} \text{Loss}\left(f_\theta(x_n), y_n\right) + \frac{1}{K}\sum_{k=1}^{K} \text{Loss}\left(f_\theta(z_n), h^\dagger(z_n)\right)$$

[Carmon, Raghunathan, Schmidt, Duchi, Liang. Unlabeled data improves adversarial robustness. NeurIPS'19]
[Gowal, Rebuffi, Wiles, Stimberg, Calian, Mann. Improving robustness using generated data. NeurIPS'21]

Robust learning

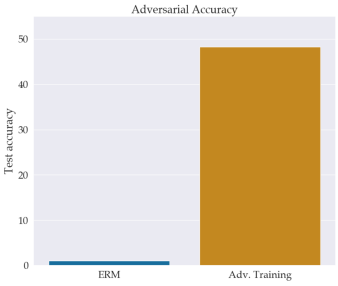
Robust optimization

- Adversarial training

$$\min_{\theta} \mathbb{E}_{(x,y)\sim\mathcal{D}}\Big[\text{Loss}\big(f_\theta(\boldsymbol{x}),y\big)\Big] \longrightarrow \min_{\theta} \mathbb{E}_{(x,y)\sim\mathcal{D}}\Big[\max_{\delta\in\Delta} \text{Loss}\big(f_\theta(\boldsymbol{x}+\delta),y\big)\Big]$$

[Goodfellow, Shlens, Szegedy. Explaining and harnessing adversarial examples, ICLR'15]
[Mądry, Makelov, Schmidt, Tsipras, Vladu. Towards deep learning models resistant to adversarial attacks, ICLR'18]
[HKSC'16, AJFR'17, PPAK'18, SBFZ'19, SCB'19, KCP'20, BTT'21, KTM'22, JFEPF'22, WDJTM'23, NFPIY'23…] (RobustBench)

Adversarial training



[Robey*, C.*, Pappas, Ribeiro, Hassani. Adversarial robustness with semi-infinite constrained learning, NeurIPS'21]

Robust learning

Robust optimization

- Adversarial training

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Robust optimization

- Adversarial training

$$\min_{\theta} \mathbb{E}_{(x,y)\sim\mathcal{D}}\Big[\text{Loss}\big(f_\theta(\boldsymbol{x}),y\big)\Big] \longrightarrow \min_{\theta} \mathbb{E}_{(x,y)\sim\mathcal{D}}\Big[\max_{\delta\in\Delta} \text{Loss}\big(f_\theta(G_\delta(\boldsymbol{x})),y\big)\Big]$$

[Goodfellow, Shlens, Szegedy. Explaining and harnessing adversarial examples, ICLR'15]
[Mądry, Makelov, Schmidt, Tsipras, Vladu. Towards deep learning models resistant to adversarial attacks, ICLR'18]
e.g., [Robey, Hassani, Pappas. Model-based robust deep learning: Generalizing to natural..., arXiv'20]

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- Distributional robustness

$$\min_{\theta} \mathbb{E}_{(x,y)\sim\mathcal{D}}\Big[\text{Loss}\big(f_\theta(\boldsymbol{x}),y\big)\Big] \longrightarrow \min_{\theta} \sup_{p\in\mathcal{P}} \mathbb{E}_{(x,y)\sim p}\Big[\text{Loss}\big(f_\theta(\boldsymbol{x}),y\big)\Big]$$

[Ben-Tal, Nemirovski, El Ghaoui. Robust Optimization, 2009]
[Sinha, Namkoong, Volpi, Duchi. Certifying some distributional robustness with principled adversarial..., ICLR'18]

Summary

- Post-processing: filtering ($f_{\theta\uparrow} \circ \varphi$), smoothed classifiers ($\mathbb{E}_{z\sim\mathrm{m}}[f_{\theta\uparrow}(\cdot + z)]$)
  - ✅ No need to retrain
  - ❌ Poor empirical performance, weak guarantees
- (Randomized) data augmentation: $\min_{\theta} \mathbb{E}_{(x,y)\sim\mathcal{D}}\Big[\mathbb{E}_{\delta\sim\mathrm{m}}\Big[\text{Loss}\big(f_\theta(\boldsymbol{x}+\delta),y\big)\Big]\Big]$
  - ✅ Low complexity
  - ❌ Poor empirical performance, no guarantees
- Adversarial training: $\min_{\theta} \mathbb{E}_{(x,y)\sim\mathcal{D}}\Big[\max_{\delta\in\Delta} \text{Loss}\big(f_\theta(\boldsymbol{x}+\delta),y\big)\Big]$
  - ✅ Great(!) empirically performance
  - ❌ …

Break
(10 min)

Robust learning

Robust optimization

- Adversarial training

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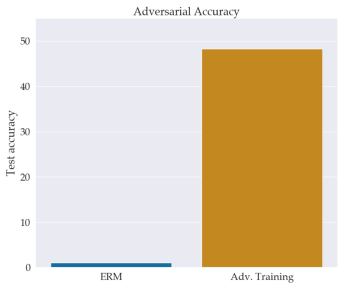
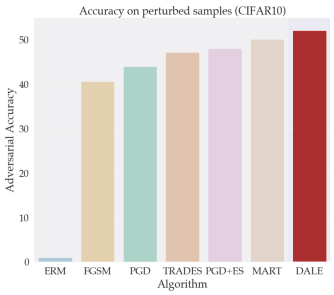
[HKSC'16, AJFR'17, PPAK'18, SBFZ'19, SCB'19, KCP'20, BTT'21, KTM'22, JFEPF'22, WDJTM'23, NFPIY'23…] (RobustBench)

Adversarial training



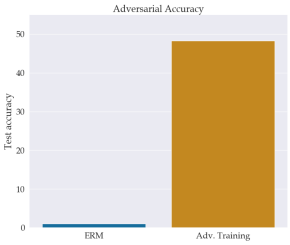
[Robey*, C.*, Pappas, Ribeiro, Hassani. Adversarial robustness with semi-infinite constrained learning, NeurIPS'21]

Adversarial training



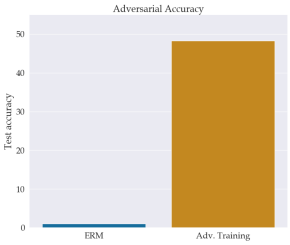
[Robey*, C.*, Pappas, Ribeiro, Hassani. Adversarial robustness with semi-infinite constrained learning, NeurIPS'21]

Adversarial training



- Great(!) empirical performance
- Grounded on a *robust* literature
- Statistical complexity
- Performance trade-offs
- Computational complexity

Adversarial training



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Statistical complexity

$$\min_{\boldsymbol{\theta}} \frac{1}{N}\sum_{n=1}^{N}\left[\max_{\boldsymbol{\delta}\in\Delta} \text{Loss}\Big(f_{\boldsymbol{\theta}}(\boldsymbol{x}_n+\boldsymbol{\delta}), y_n\Big)\right] \xrightarrow{\;?\;} \min_{\boldsymbol{\theta}} \mathbb{E}_{(\boldsymbol{x},y)}\left[\max_{\boldsymbol{\delta}\in\Delta} \text{Loss}\Big(f_{\boldsymbol{\theta}}(\boldsymbol{x}+\boldsymbol{\delta}), y\Big)\right]$$

- Is robust learning harder than non-robust learning? Do we need more samples?

Statistical complexity

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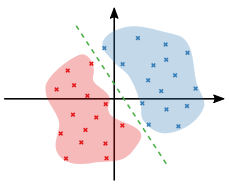
- Is robust learning harder than non-robust learning? Do we need more samples?
  A: YES *and* NO

[Cullina, Bhagoji, Mittal. PAC-learning in the presence of evasion adversaries, NeurIPS'18]
[Yin, Ramchandran, Bartlett. Rademacher Complexity for Adversarially Robust Generalization, ICML'19]
[Montasser, Hanneke, Srebro. VC Classes are Adversarially Robustly Learnable, but Only Improperly, COLT'19]
[Awasthi, Frank, Mohri. Adversarial Learning Guarantees for Linear Hypotheses and Neural Networks, ICML'20]
[Montasser, Hanneke, Srebro. Adversarially robust learning: A generic minimax optimal learner & characterization, NeurIPS'22]

Halfspace classifiers with 0-1 loss

$$P^\star = \min_{\boldsymbol{\theta}} \Pr_{(\boldsymbol{x},y)}\Big[f_{\boldsymbol{\theta}}(\boldsymbol{x}) \neq y\Big]$$

$$\hat{P}^\star = \min_{\boldsymbol{\theta}} \frac{1}{N}\sum_{n=1}^{N}\mathbb{I}\Big[f_{\boldsymbol{\theta}}(\boldsymbol{x}_n) \neq y_n\Big]$$



- 0-1 loss, i.e., probability of error
- Halfspace classifiers, i.e., $f_{\boldsymbol{\theta}}(\boldsymbol{x}) = \mathbb{I}\left[\boldsymbol{\theta}^\top \boldsymbol{x} + \bar{\theta} \geq 0\right]$
- $\left|P^\star - \hat{P}^\star\right| \leq \epsilon$ w.h.p. $\Leftrightarrow N \approx \frac{d+1}{\epsilon^2}$

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Halfspace classifiers with 0-1 loss

$$P_r^\star = \min_{\boldsymbol{\theta}} \Pr_{(\boldsymbol{x},y)}\Big[\exists \boldsymbol{\delta} \in \Delta \text{ s.t. } f_{\boldsymbol{\theta}}(\boldsymbol{x}+\boldsymbol{\delta}) \neq y\Big]$$

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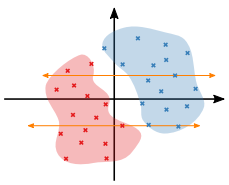

- Robust 0-1 loss
- $\Delta = \{\boldsymbol{\delta} \mid \boldsymbol{\delta} = c\boldsymbol{z}, c \in \mathbb{R}\}$ for fixed $\boldsymbol{z}$

[Cullina, Bhagoji, Mittal. PAC-learning in the presence of evasion adversaries, NeurIPS'18]

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$$P_r^\star = \min_\theta \Pr_{(x,y)}\left[\exists \delta \in \Delta \text{ s.t. } f_\theta(x+\delta) \neq y\right]$$

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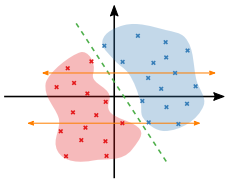

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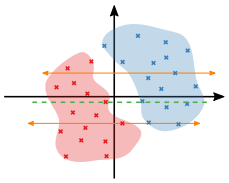

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- $\left|P_r^\star - \hat{P}_r^\star\right| \leq \epsilon$ w.h.p. $\Leftrightarrow N \approx \frac{d+1}{\epsilon^2}$ $(P_r^\star \gg P^\star)$

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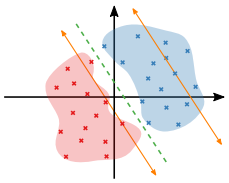

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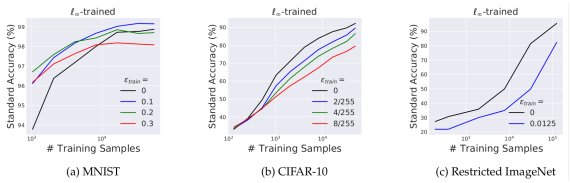
Nominal performance of robust models



(a) MNIST (b) CIFAR-10 (c) Restricted ImageNet

[Tsipras, Santurkar, Engstrom, Turner, Mądry. Robustness May Be at Odds with Accuracy, ICLR'19]

"Softer" robustness

- Softmax or *log-sum-exp*

$$\min_\theta \mathbb{E}_{(x,y)}\left[\frac{1}{\tau}\log\left(\mathbb{E}_{\delta\sim\mathrm{m}}\left[e^{\tau\cdot\mathrm{Loss}\left(f_\theta(x+\delta),y\right)}\right]\right)\right]$$

  - $\tau \to 0$: classical learning (with randomized data augmentation)
  - $\tau \to \infty$: adversarial robustness (ess sup)

[Li, Beirami, Sanjabi, Smith. Tilted empirical risk minimization, ICLR'21]

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[Li, Beirami, Sanjabi, Smith. Tilted empirical risk minimization, ICLR'21]

- L_p norms

$$\min_\theta \mathbb{E}_{(x,y)}\left[\mathbb{E}_{\delta\sim\mathrm{m}}\left[\left|\mathrm{Loss}\left(f_\theta(x+\delta),y\right)\right|^\tau\right]^{1/\tau}\right]$$

  - $\tau = 1$: classical learning (with randomized data augmentation)
  - $\tau \to \infty$: adversarial robustness (ess sup)

[Rice, Bair, Zhang, Kolter. Robustness between the worst and average case, NeurIPS'21]

"Softer" robustness

- Probabilistic robustness

$$\min_\theta \mathbb{E}_{(x,y)}\left[\underset{\delta\in\Delta}{\tau\text{-esssup}}\ \mathrm{Loss}\left(f_\theta(x+\delta),y\right)\right]$$

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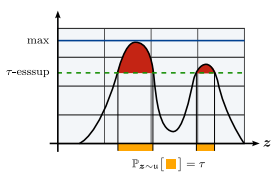
  - $\tau = 0$: adversarial robustness (ess sup)



[Robey, C., Pappas, Hassani. Probabilistically robust learning: Balancing average- and worst-case..., ICML'22 (spotlight)]

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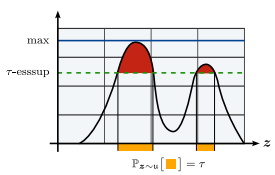
- Potentially better sample complexity
  [Robey, C., Pappas, Hassani, ICML'22 (spotlight)]
  [Raman, Tewari, Subedi, NeurIPS ML Safety Workshop'22]



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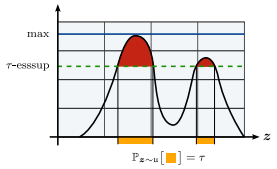
  [Raman, Tewari, Subedi, NeurIPS ML Safety Workshop'22]
- Better performance trade-off
  [Robey, C., Pappas, Hassani, ICML'22 (spotlight)]



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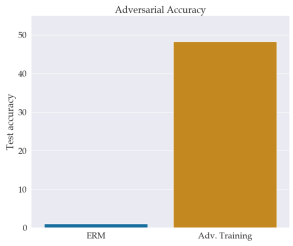
Adversarial training



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Nominal vs. adversarial performance

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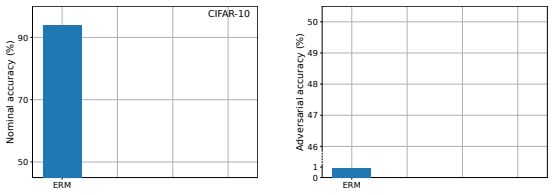

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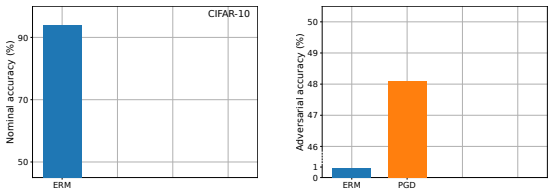

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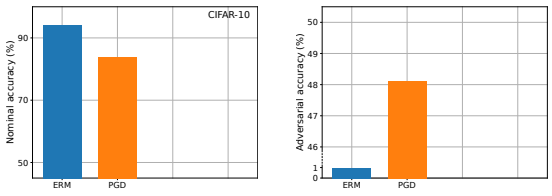

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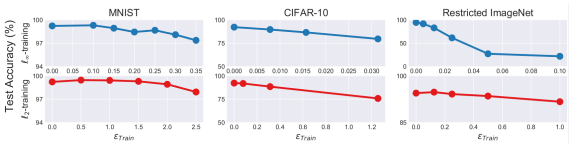
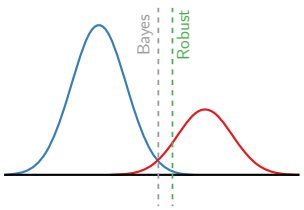


[Tsipras, Santurkar, Engstrom, Turner, Mądry. Robustness may be at odds with accuracy, ICLR'19]

Binary classification



$$y = \begin{cases} +1, & \text{with prob. } \pi \\ -1, & \text{with prob. } 1-\pi \end{cases}$$

$$x \mid y \sim \text{Normal}(y\mu, \sigma^2 \boldsymbol{I})$$

$$\Delta = \{\delta \mid \|\delta\|_2 \le \epsilon\}$$

[Dobriban, Hassani, Hong, Robey. Provable tradeoffs in adversarially robust classification, IEEE TIT'22]

Nominal vs. adversarial performance

Learning is doing exactly what we asked for!

Nominal vs. adversarial performance

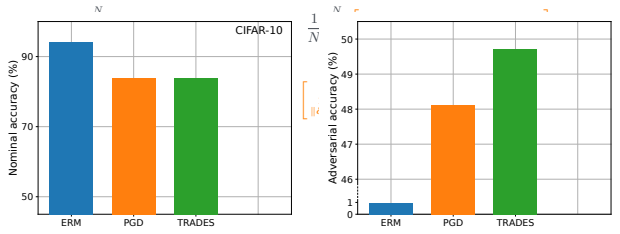
(Penalized) adversarial training

$$\min_\theta \frac{1}{N}\sum_{n=1}^{N} \text{Loss}\big(f_\theta(\boldsymbol{x}_n), y_n\big) \longrightarrow \min_\theta \frac{1}{N}\sum_{n=1}^{N} \left[\max_{\|\boldsymbol{\delta}\|_\infty \leq \epsilon} \text{Loss}\big(f_\theta(\boldsymbol{x}_n + \boldsymbol{\delta}), y_n\big)\right]$$

$$\min_\theta \frac{1}{N}\sum_{n=1}^{N} \text{Loss}\big(f_\theta(\boldsymbol{x}_n), y_n\big) + \lambda \left[\max_{\|\boldsymbol{\delta}\|_\infty \leq \epsilon} \text{Loss}\big(f_\theta(\boldsymbol{x}_n + \boldsymbol{\delta}), y_n\big)\right]$$

Nominal vs. adversarial performance

(Penalized) adversarial training



[Zhang, Yu, Jiao, Xing, El Ghaoui, Jordan. Theoretically principled trade-off between robustness and accuracy, ICML'19]

Nominal vs. adversarial performance

(Penalized) adversarial training

$$\min_\theta \frac{1}{N}\sum_{n=1}^{N} \text{Loss}\big(f_\theta(\boldsymbol{x}_n), y_n\big) + \lambda \left[\max_{\|\boldsymbol{\delta}\|_\infty \leq \epsilon} \text{Loss}\big(f_\theta(\boldsymbol{x}_n + \boldsymbol{\delta}), y_n\big)\right]$$

- No straightforward relation between λ and adversarial loss
- λ depends on the values of the losses (dataset, model, performance measure)
- Generalization

Nominal vs. adversarial performance

Constrained learning

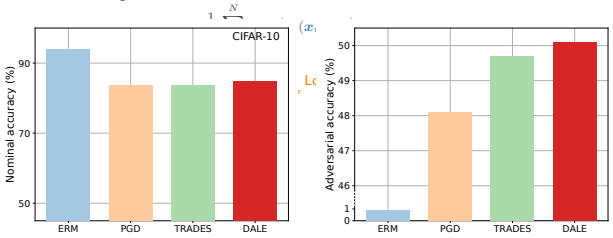
$$\min_\theta \frac{1}{N}\sum_{n=1}^{N} \text{Loss}\big(f_\theta(\boldsymbol{x}_n), y_n\big)$$

$$\text{subject to } \frac{1}{N}\sum_{n=1}^{N} \left[\max_{\|\boldsymbol{\delta}\|_\infty \leq \epsilon} \text{Loss}\big(f_\theta(\boldsymbol{x}_n + \boldsymbol{\delta}), y_n\big)\right] \leq c$$

[C. and Ribeiro, NeurIPS'20]; [Robey*, C.*, Pappas, Hassani, and Ribeiro, NeurIPS'21]; [C., Paternain, Calvo-Fullana, and Ribeiro, IEEE TIT'23]

Nominal vs. adversarial performance

Constrained learning



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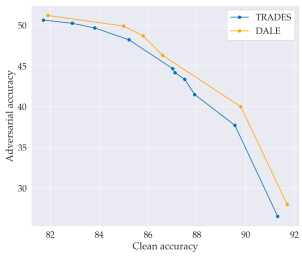

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- Does constrained learning generalize?
- How many samples do we need?
- How do we solve constrained learning problems?

Nominal vs. adversarial performance

Constrained learning

$$\min_\theta \frac{1}{N}\sum_{n=1}^{N} \text{Loss}\big(f_\theta(\boldsymbol{x}_n), y_n\big)$$

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- Does constrained learning generalize? ⇒ yes, same as unconstrained learning
- How many samples do we need? ⇒ essentially the same as unconstrained learning
- How do we solve constrained learning problems? ⇒ using duality (despite non-convexity)

[C., Paternain, Calvo-Fullana, and Ribeiro. The empirical duality gap of constrained..., IEEE ICASSP'20 (best student paper)]
[C. and Ribeiro. Probably approximately correct constrained learning, NeurIPS'20]
[C., Paternain, Calvo-Fullana, and Ribeiro. Constrained learning with non-convex losses, IEEE TIT'23]

Nominal vs. adversarial performance

Dual empirical constrained learning

$$\max_{\lambda\ge 0}\ \min_{\boldsymbol{\theta}}\ \frac{1}{N}\sum_{n=1}^{N}\text{Loss}\big(f_{\boldsymbol{\theta}}(\boldsymbol{x}_n),y_n\big)+\lambda\left[\max_{\|\boldsymbol{\delta}\|_\infty\le\epsilon}\text{Loss}\big(f_{\boldsymbol{\theta}}(\boldsymbol{x}_n+\boldsymbol{\delta}),y_n\big)-c\right]$$

Nominal vs. adversarial performance

Dual empirical constrained learning

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Nominal vs. adversarial performance

Dual empirical constrained learning

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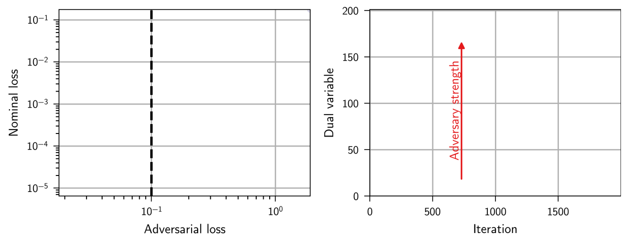
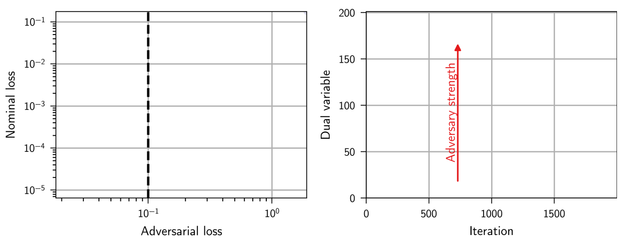
- c (static) is part of the problem, λ (dynamic) is part of the algorithm
- $\neq$ penalty/regularization (static λ)

[Byrd, Lipton. What is the effect of importance weighting in deep learning?, ICML'19]
[Sagawa*, Koh*, Hashimoto, Liang. Distributionally robust neural networks, ICLR'20]
[Robey*, C.*, Pappas, Ribeiro, Hassani. Adversarial robustness with semi-infinite constrained learning, NeurIPS'21]
[C., Paternain, Calvo-Fullana, and Ribeiro. Constrained learning with non-convex losses, IEEE TIT'23]

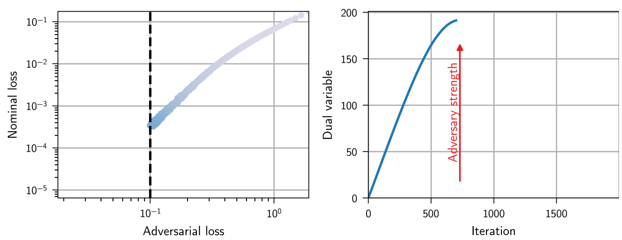
Robust image recognition



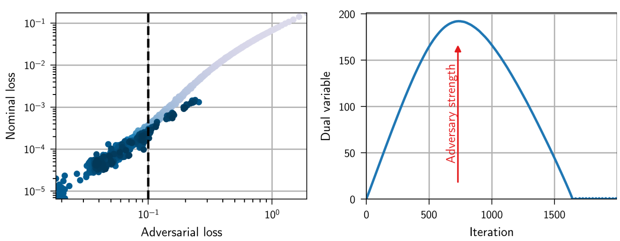
Robust image recognition



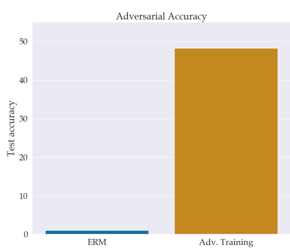
Robust image recognition



Robust image recognition



Adversarial training



- ✅ Great(!) empirical performance
- ✅ Grounded on a *robust* literature
- ❌ Statistical complexity
- ❌ Performance trade-offs
- ❌ Computational complexity

Computational complexity

$$\min_{\theta} \mathbb{E}_{(x,y)} \left[\max_{\delta \in \Delta} \text{Loss}\Big(f_\theta(x+\delta), y \Big) \right]$$

- Hard to evaluate/optimize

Computational complexity

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- Hard to evaluate/optimize

$$\text{“}\nabla_\theta\text{”} \left[\max_{\delta \in \Delta} \text{Loss}\Big(f_\theta(x+\delta), y \Big) \right] \text{“}=\text{”} \nabla_\theta \text{Loss}\Big(f_\theta(x+\delta^\star), y \Big)$$

$$\delta^\star \in \operatorname*{argmax}_{\delta \in \Delta} \text{Loss}\Big(f_\theta(x+\delta), y \Big)$$

  - Typically non-convex, e.g., $f_\theta(\cdot) = \sum_{m=1}^{M} \theta_m \kappa(\cdot, c_m)$
  - Often underparametrized ($\theta \in \mathbb{R}^p$, $x \in \mathbb{R}^n$, and $p \gg n$)

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- Hard to evaluate/optimize

$$\partial_\theta \left[\max_{\delta \in \Delta} \text{Loss}\Big(f_\theta(x+\delta), y \Big) \right] \ni \nabla_\theta \text{Loss}\Big(f_\theta(x+\delta^\star), y \Big)$$

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“PGD”

$$\min_{\theta} \mathbb{E}_{(x,y)} \left[\max_{\delta \in \Delta} \text{Loss}\Big(f_\theta(x+\delta), y \Big) \right]$$

- “PGD” [Mądry et al., ICLR'18] (alternating optimization, gradient ascent-descent)

1: $\delta^1 \leftarrow \delta_{t-1}$
2: **for** $k = 1, \dots, K$
3: $\quad \delta^{k+1} \leftarrow \operatorname*{proj}_{\Delta} \left[\delta^k + \eta \operatorname{sign}\Big(\nabla_\delta \text{Loss}\big(f_{\theta^k}(x+\delta^k), y \big) \Big) \right]$
4: **end**
5: $\delta_t \leftarrow \delta^{K+1}$
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- Random initialization
- Restarts
- Pruning
- Adaptive step size

[Dhillon et al., ICLR'18]; [Carmon et al., NeurIPS'19]; [Wu et al., NeurIPS'20]; [Cheng et al., IJCAI'22]

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5: $\delta_t \leftarrow \delta^{K+1}$
6: $\theta_{t+1} = \theta_t - \eta \nabla_\theta \text{Loss}\Big(f_\theta(x+\delta_t), y \Big)$ $\quad \approx$ data augmentation

Dual Adversarial LEarning

$$\min_{\theta} \mathbb{E}_{(x,y)} \left[\max_{\delta \in \Delta} \text{Loss}\Big(f_\theta(x+\delta), y\Big) \right]$$

$$\updownarrow \approx$$

$$\min_{\theta} \mathbb{E}_{(x,y)\sim\mathfrak{D}} \left[\mathbb{E}_{\delta\sim\mu(\delta|x,y)} \left[\text{Loss}\Big(f_\theta(x+\delta), y\Big) \right] \right]$$

- For any approximation error, $\exists\, \gamma(x,y)$ such that

$$\mu(\delta|x,y) \propto \Big[\text{Loss}\big(f_\theta(x+\delta), y\big) - \gamma(x,y)\Big]_+$$

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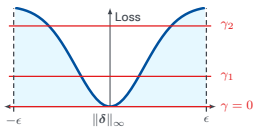

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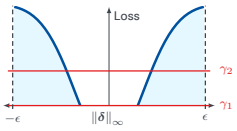

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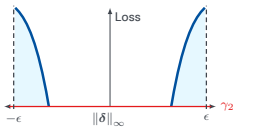

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- Use MCMC techniques (e.g., Langevin Monte Carlo)

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- Use MCMC techniques (e.g., Langevin Monte Carlo) [DALE → "PGD" as $T \to 0$, ...]
- Sampling works under milder conditions than optimization

[Robey*, C.*, Pappas, Ribeiro, Hassani. Adversarial robustness with semi-infinite constrained learning, NeurIPS'21] 58

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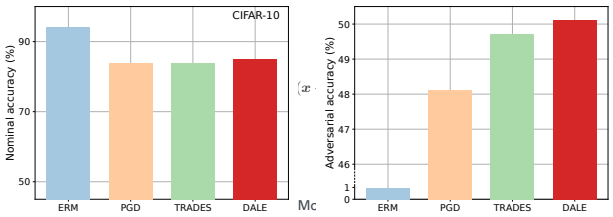

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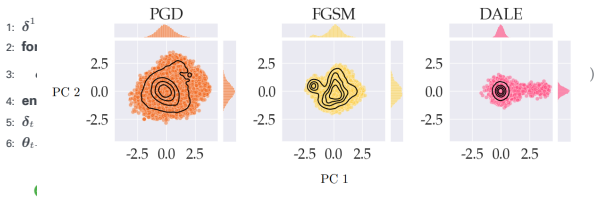

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Summary

- Adversarial training

$$\min_{\theta} \mathbb{E}_{(x,y)\sim\mathcal{D}} \Big[\text{Loss}\big(f_\theta(x), y \big) \Big] \longrightarrow \min_{\theta} \mathbb{E}_{(x,y)\sim\mathcal{D}} \left[\max_{\delta \in \Delta} \text{Loss}\Big(f_\theta\big(x+\delta\big), y \Big) \right]$$

- Great(!) empirically performance
- Statistical complexity
  ⇒ change what robustness means (e.g., probabilistic robustness)
- Performance trade-offs
  ⇒ constrained learning
- Computational complexity
  ⇒ sampling and DALE

Agenda

What is "robust learning"?

Why are models brittle?

How to learn robustly?

What else are adversaries good for?

Friendly adversaries

- Non-robustness robust properties
  - Invariance [Hounie, C., Ribeiro. Automatic data augmentation via invariance-constrained learning, ICML'23]
  - Smoothness [Cervino, C., Haeffele, Vidal, Ribeiro. Learning Globally Smooth Functions on Manifolds, ICML'23]
  - ...

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$$\text{Loss}\big(f_\theta(gx), y \big) = \text{Loss}\big(f_\theta(x), y \big), \forall g \in \mathcal{G} \Rightarrow \max_{g \in \mathcal{G}} \text{Loss}\big(f_\theta(gx), y \big) - \text{Loss}\big(f_\theta(x), y \big) \leq \epsilon$$

$$\vdots$$

  - Smoothness [Cervino, C., Haeffele, Vidal, Ribeiro. Learning Globally Smooth Functions on Manifolds, ICML'23]
  - ...

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Friendly adversaries

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  - Smoothness [Cervino, C., Haeffele, Vidal, Ribeiro. Learning Globally Smooth Functions on Manifolds, ICML'23]

$$\min_{\theta} \quad \max_{z \in \mathcal{M}} \left\| \nabla_{\mathcal{M}} f_\theta(z) \right\|^2 \quad \text{(Lipschitz constant)}$$
$$\text{subject to} \quad \mathbb{E}_{(x,y)\sim\mathfrak{D}} \Big[\text{Loss}\big(f_\theta(x), y\big) \Big] \leq c$$

  - ...

Friendly adversaries

- Density estimation (GANs) [Goodfellow et al. Generative adversarial nets, NeurIPS'14]

$$\min_{\theta} \max_{d} \mathbb{E}_{x\sim\mathfrak{D}} \Big[\log\big(d(x)\big) \Big] + E_{z\sim\mathfrak{m}} \Big[\log\big(1 - d(f_\theta(z))\big) \Big]$$
$$\big[d : x \leftarrow 1 \text{ and } f_\theta(z) \leftarrow 0 \big]$$

Friendly adversaries

- Density estimation (GANs) [Goodfellow et al. Generative adversarial nets, NeurIPS'14]

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$$\big[d : x \leftarrow 1 \text{ and } f_\theta(z) \leftarrow 0 \big]$$

- Semi-supervised and unsupervised learning
  [Cervino, C., Haeffele, Vidal, Ribeiro. Learning Globally Smooth Functions on Manifolds, ICML'23]
  [Chen, Kornblith, Norouzi, Hinton. A simple framework for contrastive learning of visual representations, ICML'20] (SimCLR)
  [Bochkovskiy, Wang, Liao. YOLOv4: Optimal Speed and Accuracy of Object Detection, arXiv'20]
- ...
- ...

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- So are we done with *adversarial training*?
  Absolutely not!

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robust learning