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Nitrogen non-stoichiometric stabilization of UN₂

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ABSTRACT

Ab initio quantum chemical calculations are employed to investigate the electronic, mechanical, elastic and thermodynamic properties of UN_2 . The influence of nitrogen non-stoichiometry on the electronic, structural and dynamical properties of cubic uranium dinitride (UN_2) is evaluated. The equilibrium lattice parameters, electronic band structure, density of state (DOS), elastic properties and phonon frequencies for the UN_{2-x} (where x=0, 0.125, 0.25, 0.5) compounds are determined and analyzed in comparison with available experimental data. The introduction of nitrogen vacancies in UN_2 structure is accompanied by pronounced DOS changes due to the shifting of the various electronic orbitals as well as the appearance of novel vacancy states in the near-Fermi region. We find that $UN_{1.75}$, which is a 12.5% nitrogen deficient structure, is observed to be dynamically and elastically stable in good agreement with experimental observations.

1. Introduction

Actinide based nitride compounds have unique, complex and interesting properties, which are the subject of continuous theoretical and experimental investigations [1–8]. These properties are caused mainly by the presence of partially filled 5*f* orbitals, that give rise to strong electron–electron interactions, and the large number of electrons that results in heavy fermionic effects.

It has been realized that the electron–electron interactions in the partially filled 5f orbitals cannot be properly described using local density approximation (LDA) and generalized gradient approximation (GGA) exchange correlation functionals. The shortcomings of the above-mentioned functionals can be overcome using various approaches such as the self-interaction correction (SIC), DFT hybrid approach, DFT+U, GW, etc. The DFT+U (LDA+U or GGA+U) methodology effectively corrects a number of the deficiencies observed in this class of materials concerning their band gap [9–11], hence it is used to resolve this problem. This implies that accounting for the 5f electrons is essential for most of the early actinide compounds. Worth noting is that, irrespective of the underlying basis in estimating the U parameter, it still remains an $ad\ hoc$ parameter.

The uranium-based nitrides are a class of ceramic materials. They comprise uranium mononitride (UN), uranium sesquinitride (U_2N_3) and uranium dinitride (UN₂). Here nitrogen having a -3 oxidation state is bounded to uranium metal. From both a scientific and technological point of view, the understanding of uranium-based

nitride is of significant interest [12,13]. Scientifically, it provides the avenue for an in-depth understanding of the electron—electron interactions in this class of compounds. Technologically, it provides a means for clean and safe energy generation at a competitive cost. Based on the significance of these uranium nitrides from both a scientific and technological point of view, the understanding of the electronic and structural properties is essential to effectively harness, control, and optimally utilize these materials.

Evarestov et al. [14] investigated the electronic structure of crystalline UN, U₂N₃ and UN₂. However, they did not consider the dynamic stability of these compounds. Recently, we showed [3] that most of the early actinide dinitride compounds (ThN2, PaN2, NpN2 and PuN2) are stoichiometric compounds, as well as thermodynamically stable. A notable exception was UN2 that exhibits some degree of instability, which is clearly observed in the phonon dispersion plot. Our theoretical observation is in line with the experimental studies dating back to the 1970's by Rundle et al. [15], Tagawa et al. [16,17] and Silva et al. [18]. Recent experimental evidence by Poineau et al. [8] in 2012 showed that a stoichiometric compound of UN2 cannot be synthesized. Long et al. [19,20] also investigated the electronic structure of U₂N₃ using both experimental and theoretical methods. The various experimental observations show that the correct description for the non-stoichiometric UN₂ structure may be between UN_{1.75} and UN_{1.78}. The CaF structure was hypothetical assigned to the uranium dinitride compound on the basis of experimental data from the actinide dinitrides with N/U=1.75-1.78. The assertion is that the non-stoichiometry

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arises from the presence of nitrogen vacancies in UN₂. Therefore, we investigate the influence of nitrogen non-stoichiometry on the electronic and mechanical properties of uranium dinitride compound to properly elucidate the stability of the structure and understand the physics of this nitride. Various degree of nitrogen vacancies are introduced into UN₂ compound in the ferromagnetic fluorite structure to understand the reason behind stoichiometric UN₂ instability.

A detailed first principles study on the nitrogen non-stoichiometric stabilization of UN_{2-x} structures (where x=0, 0.125, 0.25, 0.5) using the density functional theory approach within the generalized gradient approximation for the exchange-correlation potential plus the Hubbard U (GGA + U) with spin polarization (SP) and spin-orbit coupling (SOC) as implemented in the Vienna Ab-initio simulation package (VASP) is lacking. We calculate the electronic, elastic, structural and dynamic properties of these compounds. Overall, we show that a non-stoichiometric structure of UN_2 is stabilized from a purely quantum-mechanical based calculation. The presence of meta-stable and different magnetic configuration with respect to the Hubbard U are not considered in this current communication.

A brief description of the computational methodology used to analyze the structural and electronic properties of UN_{2-x} structures is presented in Section 2. The details of the UN_{2-x} structures are presented in Section 3. In Section 4, we discuss the effect of the various nitrogen vacancies on the electronic band structure, elastic properties, charge density distribution and phonon frequencies on these structures. Finally, we summarize our conclusions in Section 7.

2. Methodology

All the electronic, structural, elastic and dynamics computations were performed using density functional theory [21] as implemented in the VASP code [22]. The electron wave functions of the uranium and nitrogen atoms are described using the projector augmented wave (PAW) method of Blöchl in the implementation of Kresse and Joubert [23]. The PBE [24] form of the GGA exchange-correlation potentials is used together with its GGA + U variants. The Hubbard-U parameter of 2 eV for the onsite correlation energy was taken from our previous studies on the structure of actinide dinitrides [3]. It was determined by optimizing the lattice parameter with respect to the U. An adequately converged kinetic energy cutoff of 550 eV was chosen to ensure fully converged total energies for the uranium nitride structures. A spacing of k-points of 0.2 Å⁻¹ for the Monkhorst-Pack [25] grid is used to sample the Brillouin zone, and Methfessel-Paxton smearing [26] with a width of 0.2 eV was used to integrate over the bands at the Fermi level. The unit cell parameters and atomic positions are optimized with an energy and force tolerance of 10^{-7} eV and 10^{-4} Å/eV respectively. The total energy, electronic band structure and density of states (DOS) were calculated using the tetrahedron integration method with Blöchl corrections. The ferromagnetic (FM) spin polarized configuration was

A method employing a least-squares fit [27], as implemented in the MedeA-MT module is used to determine the elastic properties (bulk, shear and Young's moduli). The MedeA-MT module also uses the tetrahedron method for the Brillouin zone integrations. The elastic properties are calculated from the Hill values using the MedeA-MT, which are a geometric mean of the Voigt and Reuss values. The eigenvalues of the stiffness matrix give an indication of the mechanical stability of the structures under consideration, which is used to obtain the elastic constants. The Hill values [28] are used in the estimation of longitudinal, shear and mean sound velocities, and the Debye temperatures [29].

The MedeA PHONON package [30] is used to determine the phonon dispersion and phonon densities of states. It was also applied during the execution of the individual normal modes of vibration for the computation of the atom trajectories. The symmetry analysis of the

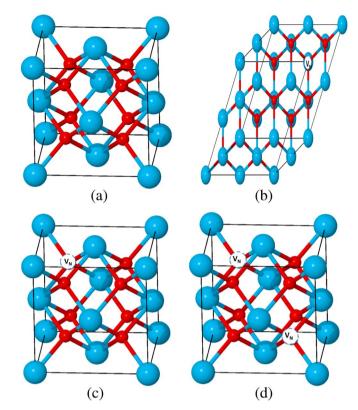


Fig. 1. (Color online) The crystallographic unit cells of the various uranium nitride structures UN_{2-x} considered (a) UN_2 , (b) $UN_{1.875}$, (c) $UN_{1.75}$ and (d) $UN_{1.5}$. Blue spheres depict uranium atoms, the red spheres depict the nitrogen atoms and the white spheres with V_N indicates a missing nitrogen atom.

modes of vibration for the contents of the spectroscopic unit cell is provided by the software and follows the usual prescriptions of symmetry theory. We have shown previously that the influence of the Hubbard U parameter is negligible on the phonon frequencies for the early actinide based nitrides [3]. Therefore, we have only presented results for the phonon frequencies within the DFT (GGA) approximation.

3. Structures

A precise prescription for the structural determination would require a uniformly large super cell size of approximately 64–128 atoms. Subsequently, different nitrogen vacancies would be introduced. Followed by the estimation of the lowest energy structure, which would be used to determine the specific UN_{2-x} structure of interest. However, this procedure is not tractable as the permutations for each structure would be highly computationally intensive. A tractable and concise approach was to consider the smallest super cell size with the lowest energy for each non-stoichiometric structure.

Fig. 1 shows the various nitrogen non-stoichiometric UN_{2-x} crystal structures considered. In Fig. 1a, the stoichiometric fluorite UN_2 structure is presented. It is known to possess an underlying cubic motif with the Fm-3m space group. The experimental and calculated lattice parameter within GGA + U for UN_2 are 5.308 Å [15] and 5.302 Å respectively. Fig. 1b shows the $UN_{1.875}$ supercell structure, which consists of a $(2\times2\times2)$ primitive unit cell of UN_2 with a total of 23 atoms (8 uranium and 15 nitrogen atoms). $UN_{1.875}$ is 6.25% nitrogen deficient compared to UN_2 . Fig. 1c shows the $UN_{1.75}$ supercell structure, which consists of a UN_2 in the fluorite structure with a total of 11 atoms (4 uranium and 7 nitrogen atoms). $UN_{1.75}$ is 12.5% nitrogen deficient compared to UN_2 . Fig. 1d shows the $UN_{1.5}$ supercell structure, which consists of a UN_2 in the fluorite structure with a total of 10 atoms (4 uranium and 6 nitrogen atoms). $UN_{1.5}$ is 25% nitrogen deficient

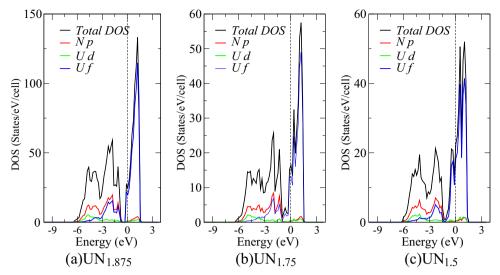


Fig. 2. (Color online) The computed total and projected densities of states (PDOS) for (a) UN_{1.875} (b) UN_{1.75} (c) UN_{1.5} within the description of the GGA + U (U=2 eV). The Fermi energy level is set at zero and is represented using dashed line.

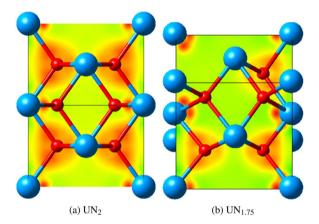


Fig. 3. (Color online) Contour map of plane-averaged charge density distribution for (a) UN $_2$ and (b) UN $_{1.75}$.

compared to UN₂. The structure is determined by taking symmetry constraints into account and subsequently determining the minimum energy configuration. Fig. 1d, which is relatively metastable compared to α or $\beta-U_2N_3$ structure is considered to ensure uniformity. All the structures have an underlying cubic symmetry. Further explanation is provided in Section 6.

4. Electronic properties

To understand the influence of non-stoichiometric nitrogen on the various UN_{2-x} based structure, we investigate the electronic properties of these structures.

An insulating character with a band gap ranging from about 0.83 eV [3] using GGA+U (with a Hubbard U of 2 eV) to 0.94 eV using the all electron [31] approach has been observed from various studies carried out on UN₂ to understand its quantum mechanical properties. These studies demonstrate that the electronic properties of UN₂ are governed mainly by the 5f states, while the top of the valence band (VB) and bottom of the conduction band (CB) are largely characterized by the hybridization of the U 5f and N 2p states.

Fig. 2 shows the computed electronic total density of states (DOS) and projected densities of states (PDOS) for the various non-stoichiometric UN_2 structures investigated in this current study. The DOS provides an indication of how closely packed the electronic energy levels are distributed within the energy interval. Furthermore, the atomic projections of the uranium p, d, and f states along with the

nitrogen 2p states are given to elucidate their individual electronic signature. In Fig. 2a, the metallicity of UN_{1.875} results largely from the shift in the 5f electrons in the bottom of the CB in stoichiometric UN₂ into the Fermi level. Fig. 2b shows metallic UN_{1.75}, which is dominated by the 5f electrons. Furthermore, Fig. 2c shows metallic character with the Fermi level dominated by the 5f electrons. This observation is in line with our previous investigation on stoichiometric UN₂ [3] that the uranium 5f electrons primarily characterize the system. The general features of the energy bands are metallic for the non-stoichiometric structures investigated. The metallicity of these structures from a purely computational point of view is attributed to the presence of various degree of nitrogen vacancy, which results in extra bonding

Worth noting is the influence of spin–orbit coupling in the accurate qualitative description of heavy elements as demonstrated in our previous studies [32,3]. For the various non-stoichiometric alloys considered in this investigation, half-metallicity is obtained taking into account only the presence of magnetization. However, the inclusion of SOC results in metallic structures with zero total magnetization. Recent experimental studies confirm our theoretical prediction that the UN_{2-x} systems are metallic structures [20].

Fig. 3a and b shows the contour map of plane-averaged charge density distributions along the (011) plane of the stoichiometric UN2 and non-stoichiometric UN_{1.75} crystal structures. The charge density distributions are presented as a superposition on the ball-and-stick representation of the corresponding structural model. The charge density distribution for the two structures are presented to show the influence of atomic vacancy on the UN2 lattice. Fig. 3a and b shows that the charge density localization along the U-N bond length is nonuniform. It has maxima at both U and N atomic centers but progressively decreases towards the bond center, with the global minimum localized at hollow sites. The concentration of the charge density shifts towards the nitrogen atom along the U-N bond. We ascribe this uranium-to-nitrogen charge transfer in the fluorite UN_{2-x} compound to nitrogen being more electronegative than uranium. Therefore, the U-N bonds show strong ionic character with weak covalent character. Furthermore, the only observed difference in the charge density distribution of Fig. 3a and b is charge depletion around the nitrogen vacancy region which is attributed to the missing atom. For more in-depth analysis of these structures, we now evaluate their elastic properties.

Table 1 The bulk, shear and Young's moduli (calculated from Hill values in GPa), acoustic velocities (m s⁻¹) and Debye temperature (K) using GGA+ U within the SOC scheme with a Hubbard U of 2 eV.

Compound	B_H	G_H	Y_H	v_s	v_{I}	v_m	θ_D
$\begin{array}{c} UN_{1.5} \\ UN_{1.75} \\ UN_{1.875} \\ UN_{2} \end{array}$	138.19	49.71	131.87	2116	4292	2376	283.7
	199.67	89.96	234.44	2791	5261	3119	387.8
	228.97	95.12	250.53	2853	5517	3194	403.7
	251.71	89.71	238.96	3740	5585	3078	396.0

5. Elastic properties

Table 1 shows the calculated elastic properties of the various UN_{2-x} compounds considered with a Hubbard U parameter of 2 eV. This allows for consistency and comparison of all the UN_{2-x} structures. The inclusion of the U parameter results in better agreement of the calculated elastic properties with other available theoretical and experimental data. The calculated bulk, shear and Young's moduli for the structures are positive definite, which are determined from their elastic constants. This indicates that the structures are all elastically stable. Note that the elastic stability is related to sum rules. Therefore, negative elastic constants do not necessarily signify elastic instability. The elastic properties determined for UN2, UN1.875 and UN1.75 are all comparable. From the calculated values of shear (transverse) velocity and longitudinal velocity, we determine the mean velocity. The mean velocity is used to determine the Debye temperature (θ_D), which gives an indication of the material stiffness. UN1.75, UN1.875 and UN2 have numerically similar stiffness compared to UN1.5, which can form as a stoichiometric compound with chemical formula U₂N₃. However, the calculated Debye temperature for the various structures does not follow a linear trend. Hence, we evaluate the phonon modes to determine their dynamic stability.

6. Phonon dispersion

Our previous investigation of actinide-based nitrides shows the presence of imaginary phonon frequency in stoichiometric UN $_2$ [3]. The theoretical findings are in agreement with experimental results corroborating the non-stoichiometric nature of UN $_2$ compound. As mentioned earlier, the presence of nitrogen vacancies in UN $_2$ could lead to dynamic stabilization of the compound. However, we applied various degree of isotropic pressure to stoichiometric UN $_2$ structure with the aim of structural stabilization. The imaginary phonon modes remained with no stabilization or phase transition with pressure. A number of factors could be responsible such as unit cell size, actual number of atoms, huge energy barriers, etc. However, these factors were not explored further due to computational constraints. Next, the effect of nitrogen non-stoichiometry on the phonon modes of UN $_{2-x}$ structures are investigated to determine the possibility of structural stabilization.

Fig. 4 shows the phonon dispersion plot for non-stoichiometric UN_{1.5} structure. We observe imaginary phonon modes (negative phonon frequency) around the Γ -point. This indicates that the non-stoichiometric UN_{1.5} structure is dynamically unstable. The slight imaginary phonon modes might be suggestive of a metastable structure, which have a likelihood of structural transformation to the stoichiometric α -U₂N₃ or β -U₂N₃ structure rather than the defective considered UN_{1.5} structure under certain conditions, which are beyond the scope of the current investigation [20]. Uranium atoms govern the dynamics of the low lying frequency, hence, they contribute solely to the imaginary modes. In addition, the nitrogen atoms govern the dynamics of the higher lying frequencies, which are much lighter than the uranium atoms. The proportion of the nitrogen vacancies in the non-stoichiometric UN_{1.5} structure is relatively large, hence, dynamic stabilization is not achieved.

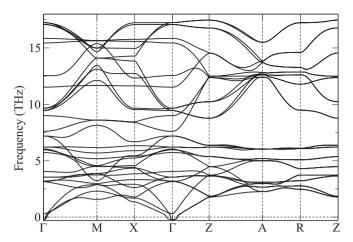


Fig. 4. The phonon dispersion plot of $UN_{1.5}$ structure showing the lattice dynamics of U and N atoms. Note the presence of imaginary (negative) phonon frequencies at the Γ -point demonstrating structural instability.

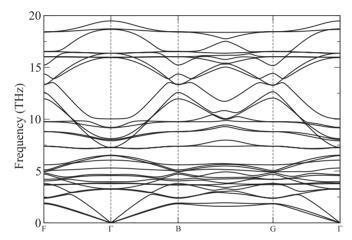


Fig. 5. The phonon dispersion plot of $UN_{1.75}$, showing the lattice dynamics of U and N atoms.

In Fig. 5, we observed no imaginary phonon modes in the phonon dispersion plot of the non-stoichiometric $\mathrm{UN}_{1.75}$ structure. This implies that the structure is dynamically stable. As mentioned above the uranium atoms govern the dynamics of the low-lying frequencies, whereas the nitrogen atoms govern those of the higher lying frequencies. Hence, the introduction of 12.5% nitrogen vacancies in the UN_2 structure results in its dynamic stabilization.

Fig. 6 shows the phonon dispersion plot for non-stoichiometric UN_{1.875} structure. Several imaginary phonon frequencies are observed at different high symmetry points. This implies that the structure is unstable. The presence of imaginary frequencies in Fig. 6 is attributed to the constrained dynamics of the uranium atoms, which govern the low-lying frequencies. The minute proportion of nitrogen vacancies in the non-stoichiometric UN_{1.875} structure (which is 6.25% compared to the UN₂ structure) constrains the phonon dynamics of uranium atoms in the UN_{1.875} structure resulting in the instability.

It is worth noting that the presence of nitrogen vacancies in the UN_2 structure allows for increased mobility of the uranium atom leading to reduction in the imaginary phonon frequency. In essence, this structure needs a certain amount of nitrogen vacancy to attain dynamic and elastic stability.

7. Conclusions

The structural, mechanical and electronic properties of nitrogen non-stoichiometric UN_{2-x} have been studied within the DFT approach.

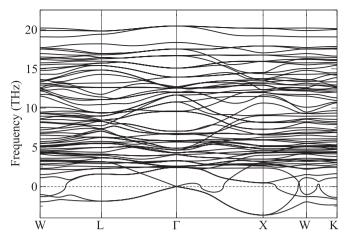


Fig. 6. The phonon dispersion plot of $UN_{1.875}$, showing the lattice dynamics of U and N atoms. Note the presence of imaginary (negative) phonon frequencies demonstrating structural instability.

Previous studies [3,32] demonstrated that for these heavy elements without taking into account the spin–orbit interactions, we would obtain a qualitative incorrect description. Within this scheme, we derive that only the $\mathrm{UN}_{1.75}$ structure is dynamically stable as shown by the phonon dispersion data. The UN_{2-x} based compounds are elastically stable. For all the non-stoichiometric alloys investigated, half-metallicity is obtained taking into account only the presence of magnetization (SP). However the inclusion of SOC gives metallic structures. Generally, the calculated DOS show significant uranium d and f states hybridization with the nitrogen 2p in the UN based compound. Our current theoretical prediction is in line with previous experimental data, which proposes that certain percentage of nitrogen vacancy in the vicinity of (12.5–15%) is needed to stabilize UN_2 structure resulting in a nitrogen non-stoichiometric structure.

Acknowledgments

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