

Levitation force measurement on a switchable track for superconducting levitation systems

Tilo Espenhahn , Dietmar Berger, Ludwig Schultz, Kornelius Nielsch  and Ruben Hühne 

IFW Dresden, PF 27 01 16, D-01171 Dresden, Germany

E-mail: t.espenhahn@ifw-dresden.de

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Abstract

Whereas current superconducting levitation systems typically use permanent magnetic tracks, additional switchable components are required to realize fast turnouts or crossings. Therefore, a prototype for a switchable track segment was realized by a combination of permanent magnets with a superconducting coil, which enables switching of the magnetic field above the track from active (magnetic field is present) to inactive (significantly reduced magnetic field) and back. Previous studies of the magnetic flux density distribution above both track types showed only a small mismatch between the standard permanent magnetic setup and the new switchable design. Here, we present measurements of the levitation as well as of the lateral forces for both track types. The results show that the active switchable track leads to comparable force values as for the standard track. The differences lead only to a slight reduction of the levitation height above the switchable track. In contrast, the measured forces are significantly reduced, if the track is switched in the inactive mode, i.e. the values are no longer sufficient to maintain a stable levitation. Furthermore, a typical hysteresis for the force values was observed for both systems. The lateral asymmetry of the measured force values was traced back to inhomogeneities of the used superconductors.

Keywords: large scale application, superconducting levitation, track design, force measurement

(Some figures may appear in colour only in the online journal)

1. Introduction

In recent years, a number of large test facilities based on superconducting levitation were realized worldwide [1–5]. In general, these systems utilize either plain permanent magnetic tracks (PMT) [2, 6, 7] or v-shaped magnet arrangements [3] to provide the required magnetic field for a stable levitation of the superconductor. Consequently, most effort was focused on the improvement of the magnetic field configuration in order to maximize the generated levitation force by optimizing the track geometry and in particular the arrangement of the permanent magnets [8–11].

However, none of these systems can switch the strong magnetic field above the track on or off. Yet, switchable track segments are essential to build complex transportation

networks such as fast turnouts and crossings. Therefore, our goal was to design and test such a switchable track. The basic requirements for such devices are to realize a fast switching as well as to replicate the magnetic field distribution originating from the standard PMT. Both features are important to enable high vehicle densities and smooth transitions between different track segments.

One option for building such switchable tracks is to replace the permanent magnets by coils. However, suitable copper-based coils arranged together with an iron yoke or another flux collector require too much space to implement them effectively in a full-scale levitation system [12–14]. Instead, a combination of permanent magnets, flux collectors and a superconducting coil was chosen in a so-called combined switchable track (CST) to control the magnetic flux

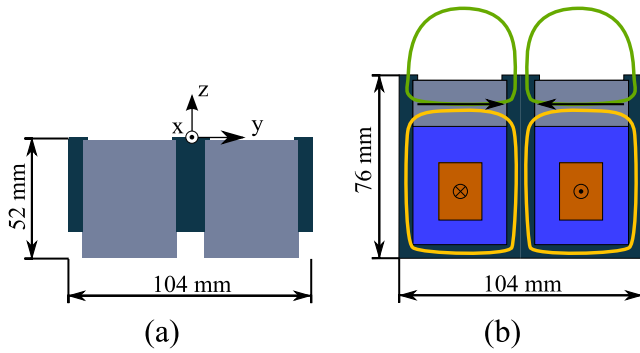


Figure 1. Comparison between the design of: (a) the permanent magnetic track and (b) the switchable track. The track coordinate system is shown on the PMT, the switchable track is shown in active mode, i.e. the magnetic flux above the track is similar to the PMT.

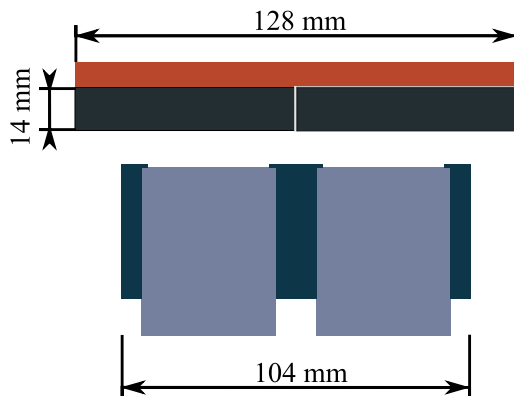


Figure 2. Comparison of the dimensions for the used YBCO bulk superconductors and the PMT without a lateral displacement.

density above the track [15]. The new design was derived from the current PMT of the SupraTrans test facility (figure 1(a)). To do so, the PMT was extended by longer flux collectors, an additional coil made from YBCO coated conductors beneath the permanent magnets and an iron yoke at the bottom of the track (figure 1(b)). The detailed design of the switchable track is described in more detail in [15].

Simulations of the generated force were performed for such a CST with the COMSOL AC/DC package. The results indicated only a slightly lower levitation force generation for the CST in comparison to the PMT at a constant height above the track [16]. To investigate if the designed CST can provide similar levitation and lateral forces as the PMT, force measurements were conducted on a full-scale CST demonstration track segment. The measurements were carried out while the CST is in an active or inactive mode (i.e. using a coil current or not) and were compared afterwards with results of a standard PMT track segment.

2. Method

The force measurements were performed with a setup using YBCO bulk superconductors. Therefore, two three-seeded bulks (size: 64 mm × 32 mm × 14 mm) were fixed to a copper mount and form a double bulk (compare sketch in

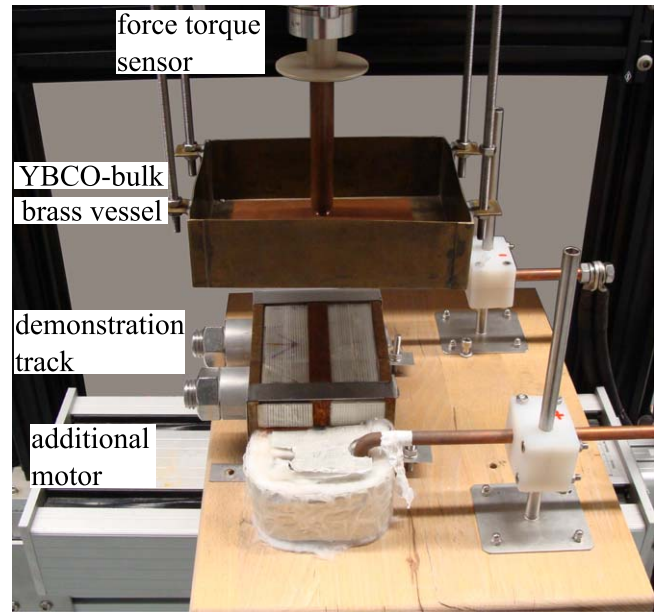


Figure 3. Setup for force measurements with the superconductor inside a brass vessel and the switchable track below.

figure 2) resulting in an overall size of the superconductor of 128 mm × 32 mm × 14 mm. This superconductor arrangement was mounted on a three-axis force and torque sensor. Additionally, the force sensor can be moved vertically (z) and rotated around z (ϕ). Lateral displacements along y (i.e. perpendicular to the track direction) were realized with an additional motor, which moves the complete track below the sensor (figure 3). The YBCO-double bulk was positioned inside a brass vessel and cooled with liquid nitrogen. The investigations were performed on a 150 mm long PMT segment of the SupraTrans system and on the switchable demonstration track segment described in detail previously [15]. Figure 3 shows this CST demonstration segment in the measurement setup.

The measurements were carried out in the zero field cooling (ZFC) as well as in the field cooling (FC) mode. The used superconductor movements for each measurement procedure are summarized in figure 4. For ZFC measurements, only the studied track (either PMT or CST) was used in the measurement setup. In that case, the superconductor was cooled in a distance of 100 mm above the track to avoid any flux trapping [17].

For FC measurements, the superconductor was always cooled above the PMT with a distance of 40 mm. If the measurements were performed above the CST, the superconductor was moved from the PMT to the CST after cooling (figure 4 bottom). To do this, the PMT was placed parallel to the CST, which enabled an easy transfer of the bulk from one track to the other with a simple movement. After the transfer, the PMT was removed from the setup. FC over the PMT was chosen, as it is the usual application mode for a superconducting levitation system, where the switchable track will only replace short segments of the PMT (i.e. at crossing or turnouts). The superconductor temperature was kept at liquid nitrogen temperature all the time.

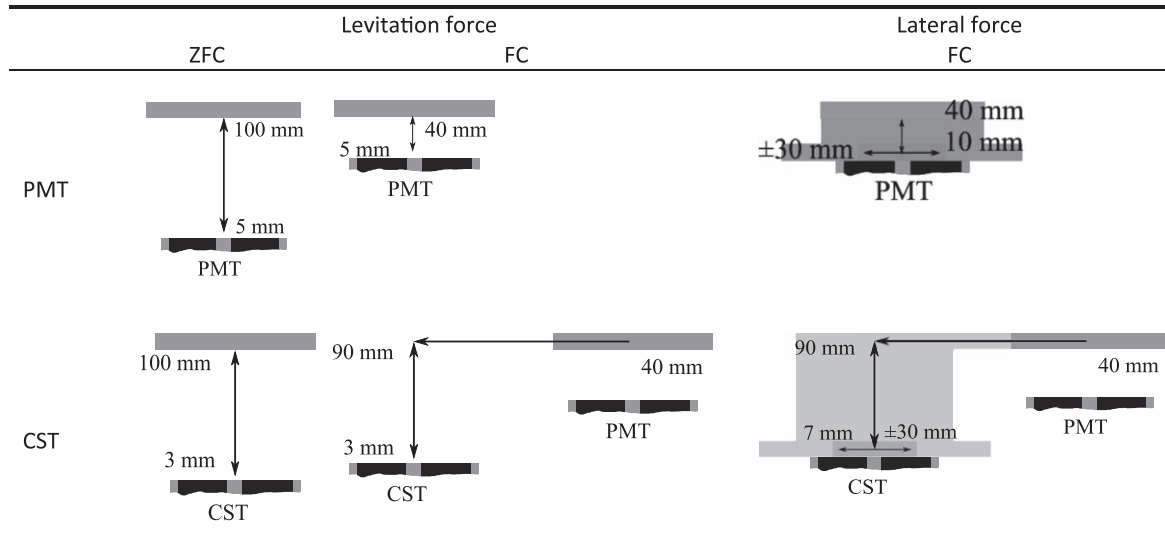


Figure 4. YBCO movements for force measurements above both track types. The light gray area in the lateral force measurements indicate areas passed by the superconductor during measurement.

Different measurement procedures were used to determine the levitation and lateral forces. Such independent procedures were necessary due to the fact, that both levitation and lateral force depend on the history of the applied field [18]. The superconductor was heated up after each measurement until no restoring force was recorded anymore and cooled down again above the PMT. This heating step is required to eliminate any influence of the internal magnetization resulting from previous movements and corresponding field changes [19].

During the measurements, a computer with a data acquisition card collected the data and controlled the motor movements. A constant holding time of 0.5 s between movement and measurement were used to minimize the influence of the motor's inertia on the force measurement.

2.1. Levitation force

ZFC measurements started from the cooling height $z = 100$ mm above the studied track type moving the superconductor down to minimal height and back to the initial position. The minimal height was 5 mm above the PMT and 3 mm above the CST.

Levitation force measurements in FC mode used vertical movement from $z = 40$ mm down to the minimal height (3 mm or 5 mm, respectively) and back to 40 mm. The different minimum heights originates from differences in the generated magnetic field profile measured above the CST in comparison to the PMT [15, 16]. Based on these studies it was assumed that the CST will require a smaller levitation height to generate the same levitation force as the PMT.

2.2. Lateral force

Lateral force measurements were only conducted in the FC mode, as significant lateral forces are only induced above a plain track, if the pinned flux density is altered through an external field change [20]. The cooling of the superconductor took place above the PMT, as mentioned before. The working height for the

lateral measurements was not identical to the minimum height used for the levitation force measurements. For the PMT a working height of 10 mm was chosen, as this is the typical value used for the SupraTrans system. For the CST, the working height was chosen differently (i.e. 7 mm) based on the results of the levitation force measurements, where a lower levitation height above the CST was observed for the same force value. Each lateral force measurement is divided into different steps:

- FC of the YBCO 40 mm above the PMT.
- Transfer of the superconductor to the CST, if necessary.
- Movement of the superconductor down to the desired working height of 10 mm over the PMT and 7 mm over the CST. The used levitation heights result from the levitation force measurements giving smaller values for the CST compared to the PMT (compare section 3.1 and figure 5).
- Measurement of forces during the lateral movements in loops from $y = 0$ mm to $y = 30$ mm, afterwards to $y = -30$ mm and back to $y = 0$ mm while keeping a constant levitation height.
- Lift up of the superconductor to the initial height.

The superconductors have to pass a number of lateral movement loops to reach a steady state during the lateral force measurement [21]. Additionally, the levitation force was recorded while moving the superconductor laterally, as a change in levitation force was expected as described previously in [22]. Similar levitation and lateral forces were expected for both track types, as the same concept for flux concentration is used to maximize the magnetic flux density above the track [2].

3. Results and discussion

3.1. Levitation force

Figure 5(a) shows the measured levitation force above the PMT in comparison to the active as well as the inactive CST

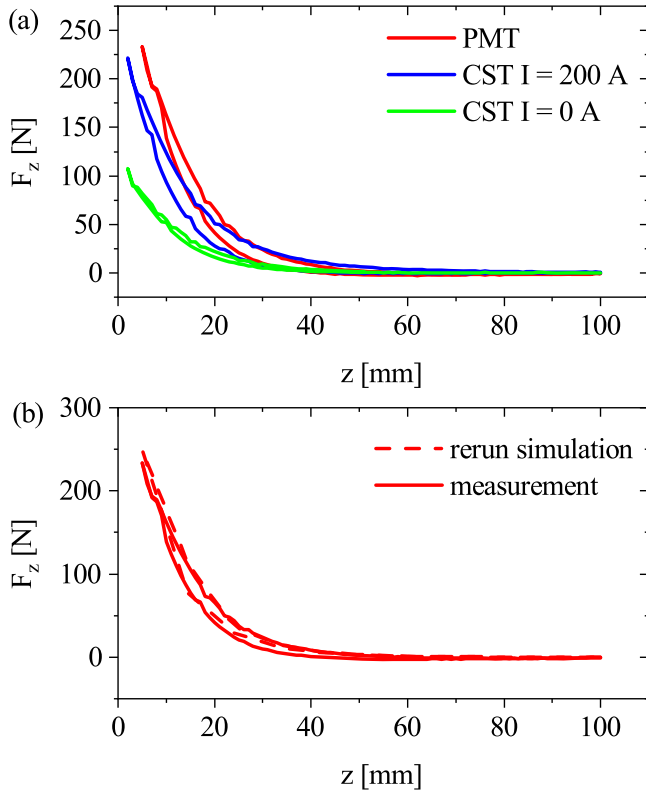


Figure 5. (a) Zero field cooled levitation force above the permanent magnetic track (PMT) as well as the inactive (CST, $I = 0$ A) and active (CST, $I = 200$ A) switchable track. (b) Comparison between a levitation force simulation of the PMT and the measurement.

for the ZFC case. The maximum varies between 230 N at 5 mm above the PMT and 75 N at 3 mm above the inactive CST. The levitation force of the active CST (225 N at 3 mm) is only slightly below the value for the PMT. It is also visible that the cooling height $z = 100$ mm is sufficient to prevent any flux from being pinned inside the superconductor (figure 5(a)), as the onset of the levitation force is just detected for a height of around 80 mm. In figure 5(b), the measurement above the PMT is compared with the simulation. It should be noted that the result of the simulation differs in the maximum force value from the results in [16] due to differences in material parameters used in the original simulation. The new simulation takes into account the actual used material parameter of the real system.

After the ZFC measurements, FC measurements were carried out to compare both track types using typical operation conditions. Figure 6 shows the corresponding results. As expected, the overall levitation force is reduced compared to the ZFC mode. As it is well known, the levitation force directly depends on the change of magnetic flux density applied to the superconductor [23]. In the case of FC this flux density difference is less than the difference in ZFC. In FC the difference is between field cooled flux density at cooling height (40 mm) and the flux density at minimal levitation height (10–3 mm). In the ZFC case the difference is between the minimal magnetic flux density at ZFC height (100 mm)

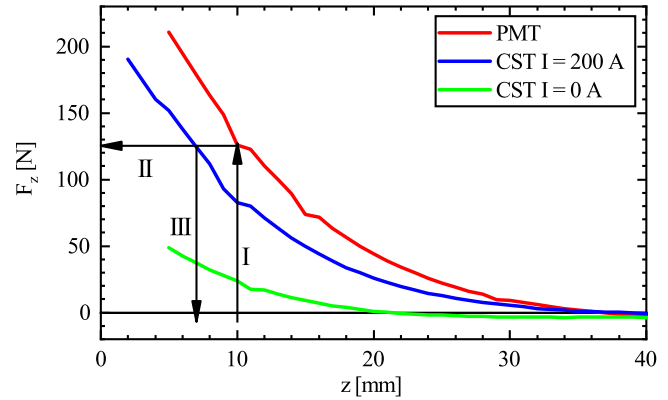


Figure 6. Field cooled levitation force above the permanent magnetic as well as the inactive and active switchable track while approaching the track (i.e. movement from 40 mm to 5 mm or 2 mm, respectively). Lines I to III illustrate the estimation of the levitation height above the CST.

and again the flux density at minimal levitation height (10–3 mm). For the SupraTrans test facility, the superconductors are usually cooled down at $z = 40$ mm resulting in a levitation height z of around 10 mm due to the payload [2]. In our case, this working height of 10 mm corresponds to a levitation force of 126 N above to the PMT (see figure 6, line II). The same levitation force above the active CST requires a shorter distance to the track, resulting in a levitation height of 6.9 mm above the CST (figure 5, line III), i.e. a lateral transition of the superconductor from the PMT to the CST will reduce the distance to the track from 10 mm to 6.9 mm, in order to maintain the same levitation force.

The main reason for the varying levitation forces between the PMT and the CST is the differences in the magnetic flux density distribution above the two track types. The specific track design not only influences the flux density but also the gradient of the magnetic flux density. In particular, the CST exhibits a lower gradient and a lower absolute magnetic flux density at the position of the superconductor if compared with the PMT [15] due to its design. The increased track height of the CST leads to an increased flux ratio leaving the track at the side of the flux collector instead at the top of the pole and therefore reduces the absolute flux density inside the superconductor working area. The changed field distribution over the track height also leads to a larger stray field with a lower flux density gradient above the CST. These field differences result in the lower levitation force values above the switchable track and therefore reduce the levitation height. Despite the reduced levitation height, stable levitation is possible above the active CST with the used track design, i.e. a load representing a force of 126 N would not touch the track when passing from the PMT to the CST.

Apart from the differences in levitation force, both track show a hysteretic behavior when moving the superconductor in z direction. This common feature of superconducting levitation systems was already reported previously [18, 24–26] and indicates that the same mechanisms apply for the CST as for the PMT.

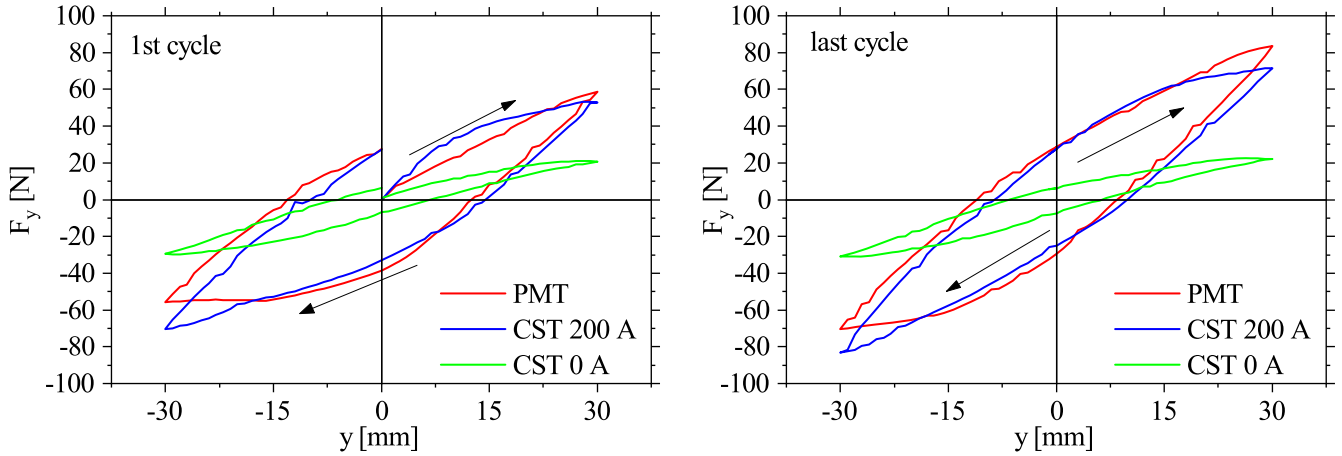


Figure 7. Measured lateral forces for the different track types, if the superconductor is laterally displaced. The first and the last measurement loop are shown.

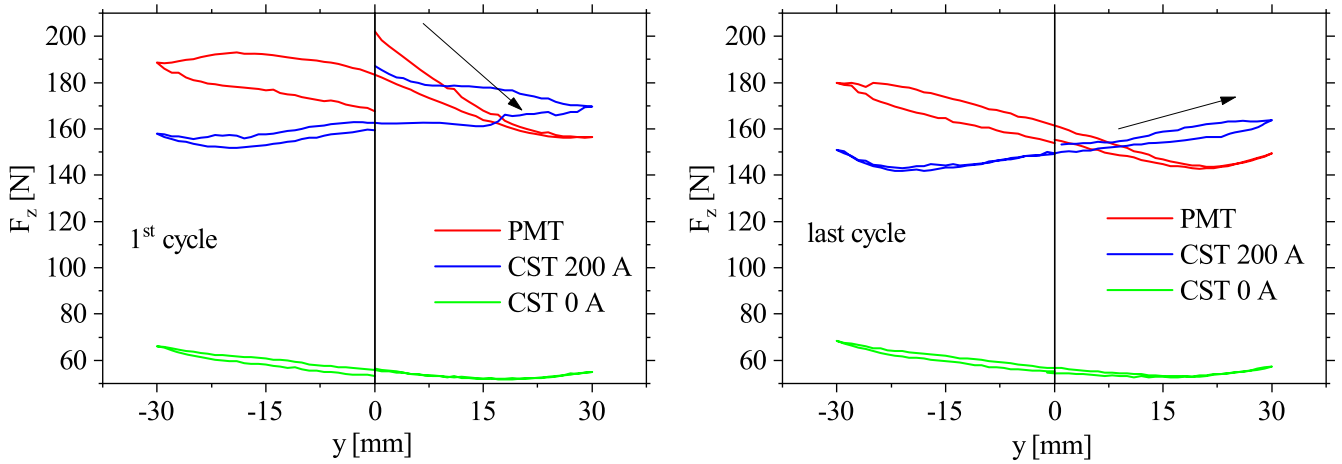


Figure 8. Levitation force above the different track types, if the field cooled superconductor is laterally displaced. The first and the last measurement loop are shown.

A change of the coil and the permanent magnet parameter (i.e. critical current or dimensions) can increase the levitation force in active state up to the values of the PMT. However, this will also influence the residual magnetic field in the inactive state.

In the inactive state (i.e. with $I = 0$ A), the generated force is not sufficient to maintain stable levitation as the required force is not achieved even at lower distances of $z < 5$ mm (compare line II in figure 5). In this case, the load of 126 N would press the superconductor onto the track. However, a residual levitation force remains even in the inactive mode, i.e. the magnetic field above the track is not completely switched off. A negative winding current can further decrease this residual force as shown previously [15]. Nevertheless, a general capability to switch the levitation force is proven with this setup by switching the CST from an active to an inactive state.

3.2. Lateral force

Figures 7 and 8 show the measured force while moving the superconductor laterally above both track types. For each

track type multiple subsequent lateral loops were carried out. In case of the PMT and the inactive CST, five loops were measured, in case of the active CST, only three loops were determined. Figure 7 depicts the lateral force and figure 8 the levitation force between the superconductor and both track types for the first displacement loop (solid lines) and the last loop (dotted lines), respectively.

The inactive CST induces the smallest lateral restoring force $F_y = 30$ N, while moving the superconductor (figure 7). In contrast, the active CST shows a lateral force of $F_y = 85$ N, which is almost like the value measured on the PMT ($F_y = 87$ N). Furthermore, all lateral force measurements show a distinct hysteresis, which is in good agreement with results published in [23].

Comparing the different hysteresis loops, an increased peak value is noticed for the lateral force F_y above the active CST and the PMT (squares and circles in figure 9). In contrast, no significant change in the peak value was found for the inactive CST (green symbols in figure 9). The change in the maximal lateral force can be explained by an optimized flux line arrangement, due to the repeated lateral loops and the associated magnetization changes.

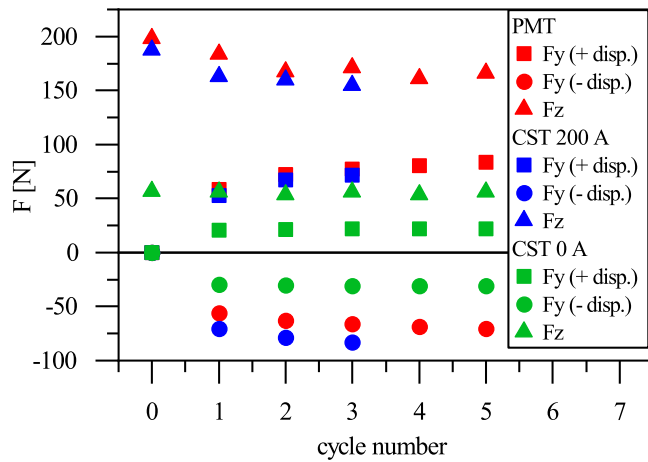


Figure 9. Peak lateral and levitation forces over the displacement cycles for both track types.

For each measurement, a shift of the position for $F_y = 0$ N is found either to more positive or negative y values in dependence of the displacement direction (figure 7). When displacing the superconductor laterally, the trapped magnetic flux is moved into the superconductor. The changed trapped flux modifies the superconductor track interaction by shifting the point of zero lateral force [18]. This behavior is typical for superconducting levitation systems and was reported earlier [18, 22, 27–29]. In figure 7, the direction of the shift depends on the movement history, i.e. directional changes cancel out previous displacements. The amplitude of the shift also depends on the movement history, but here directional changes lead to a saturation effect. We expect an accumulation of these shifts, if only displacements in one direction occur. The effect also depends on the magnetic field strength of the track. The inactive CST generates the lowest magnetic flux density. Since the inactive CST generates a highly reduced magnetic flux density compared to the PMT, it exhibits a lower shift compared to the active CST or PMT. As this effect is no special feature of the CST, we do not expect any negative influence on the usability of the CST.

In general, the active CST as well as the PMT generates significant steeper force changes for a certain displacement if compared with the inactive CST. Corresponding to [18], the gradient of a lateral force measurement indicates the stability and stiffness of the levitation system. Applying this concept on the curves in figure 7 leads to the conclusion that the active CST and the PMT show a similar stiffness against lateral movement of the superconductor.

Considering the force gradient at $y = \pm 30$ mm, the force change of the inactive CST is almost zero or even negative, which indicates that further displacements lead to instable lateral levitation conditions [28]. Therefore, the maximum displacement of 30 mm is a threshold value for the stability of the inactive CST. In contrast, there is no indication that the lateral force gradient goes to zero or even to negative values at $y = \pm 30$ mm for the active CST or PMT. This indicates that still no stability limit is reached, while the superconductor operates above the PMT or active CST. Therefore, it can be

assumed that a superconductor levitates stable in an interval of $y = \pm 30$ mm above the active CST as well as the PMT. The stiffness and stability values of the active CST are comparable to the values of the PMT. Therefore, these points do not influence the usability negatively.

Figure 8 shows the measured levitation force while moving the superconductor laterally. The observed behavior corresponds to FC measurements published in [28]. The increase of the levitation force during the lateral displacement of the double bulk might be explained with the misalignment between the track poles and the pinned magnetic field inside the FC superconductor. The levitation force decay over multiple lateral loops was also reported in [21, 30]. We assume that it originates from the flux movements inside the superconductor due to the lateral displacement loops. The value of the levitation force reduction for both operation modes of the CST as well as for the PMT differs from each other (PMT: 44, 7 N, active CST: 37, 8 N, inactive CST: 1, 5 N). The highest decrease was recorded for the PMT followed by the active CST and the inactive CST. After multiple loops, the active CST and the PMT generate an almost similar levitation force, which will reduce the differences in the levitation height when passing from a PMT segment to a CST track segment.

The measurements above all tracks show a nonlinear levitation force dependence with the lowest value for $y \neq 0$ mm. This minimum indicates the position to which a displaced superconductor ($y \sim 30$ mm) will return after removing the displacing force. As in the case of the levitation force measurements, the inactive CST induces the smallest levitation force, whereas the active CST shows force values, which are comparable to the values of the PMT. The results for the lateral as well as for the levitation forces of a lateral displaced superconductor differ from the results in [28] in their symmetry. Therefore, an additional measurement was carried out, to investigate the origin of this asymmetric behavior.

3.3. Asymmetric force generation

Figure 10 shows the dependence of the levitation force for a lateral displacement above the inactive CST in the FC mode. The first measurement was done similar to the procedure described above. After heating up, the superconductor was rotated by 180° around the z -axis and the measurement was repeated. The symmetry of both curves with regard to the track center ($y = 0$ mm) leads to the assumption that the material characteristics of the used superconductor bulks are responsible for the asymmetric graphs.

Assuming two YBCO bulks with different critical current densities, the levitation force strongly depends on the moving direction. It drops, if the superconducting bulk with the lower critical current density is moved towards the track center, whereas the bulk with the higher value is laterally moved out of the magnetic field. As a result, the higher critical current density of that bulk does not contribute to the levitation force anymore. On the other hand, the levitation force is increased

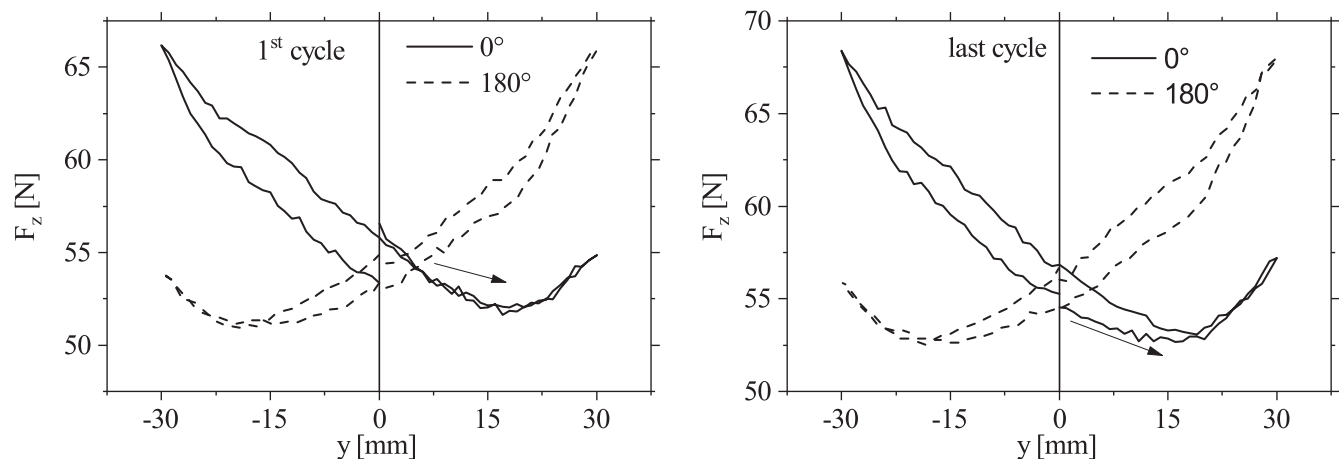


Figure 10. Levitation force for the first and sixth cycle for two different orientations of the used superconductor arrangement above the inactive CST (i.e. rotation around the z -axis by 180°).

if the superconductors are moved in the opposite direction. Thus, the generated levitation force is proportional to the spatial distribution of the critical current density for the studied system [31]. Due to this dependence, the minimum levitation force is shifted to the side of the stronger bulk as its contribution to the overall levitation force is higher. This result also indicates that it is crucial to use superconductors with similar properties to get a stable levitation above the center of the track.

4. Conclusion

The levitation and lateral forces for a double YBCO bulk were measured above a conventional PMT as well as above a newly designed switchable track having an active or an inactive state. The results for both track types were compared afterwards. It was shown that the switchable track design allows a switching of the levitation and lateral forces. The maximum of the measured levitation and lateral force for the active switchable track were in the same range as for the PMT of the SupraTrans test facility, which allows the simple replacement of a PMT segment by the CST to form crossings for other traffic systems, i.e. bicycles or cars. The obtained results are also a first step for the use of the CST in a turnout, where the coil edges need to be redesigned to form a switch frog point. Asymmetries in force measurements are the result of the different properties of the two superconductors in the used double bulk setup. Above the inactive track, the levitation force reaches only residual values, which are insufficient for the stable levitation in the used configuration.

ORCID iDs

Tilo Espenhahn <https://orcid.org/0000-0002-7621-8810>
 Kornelius Nielsch <https://orcid.org/0000-0003-2271-7726>
 Ruben Hühne <https://orcid.org/0000-0002-0030-6048>

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