



## Communication

## Massless Dirac fermions in semimetal HgCdTe

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## ABSTRACT

Magneto-transport results obtained for the strained 100 nm thick  $\text{Hg}_{1-x}\text{Cd}_x\text{Te}$  ( $x=0.135$ ) layer grown by MBE on the CdTe/GaAs substrate are interpreted by the  $8\times 8$   $\mathbf{k}\mathbf{p}$  model with the in-plane tensile strain. The dispersion relation for the investigated structure proves that the Dirac point is located in the gap caused by the strain. It is also shown that the fan of the Landau Levels (LL's) energy calculated for topological protected surface states for the studied HgCdTe alloy corresponds to the fan of the LL's calculated using the graphene-like Hamiltonian which gives excellent agreement with the experimental data for velocity on the Fermi level equal to  $v_f \approx 0.85 \times 10^6$  m/s. That characterized strained  $\text{Hg}_{1-x}\text{Cd}_x\text{Te}$  layers ( $0.13 < x < 0.14$ ) are a perfect Topological Insulator with good perspectives of further applications.

## 1. Introduction

In the last decade one of the most surprising discoveries at the intersection of the condensed matter physics and materials science is the realization that relativistic effects, strong for electrons in heavy elements (strong spin-orbital interaction) what lead to striking elementary excitations, such as Dirac, Majorana, and Weyl fermions in several classes of solids and nanostructures [1]. An interest phenomena have generated in new state of quantum matter - topological insulators (TI) - which behaves as an insulator in its interior and as a metal on its boundaries [2–11]. The surface states of a three-dimensional (3D) TI such as 3D strained HgTe layers [9]  $\text{Bi}_2\text{Se}_3$ ,  $\text{Bi}_2\text{Te}_3$ , and  $\text{Sb}_2\text{Te}_3$  [12–15], consist of a single massless Dirac cone due to the crossing of two branches of bands at the surface. That are known as topologically protected surface states (TPSS) [13] - resistant against the time reversal invariant perturbation. Frequently, there are two edge states as cones belonging to the top and the bottom surfaces for the whole sample on which the transport of Dirac fermions is realized over the conductance with the Berry phase of  $\pi$  [16–19].

For the case of the 3D HgTe the uniaxial strain along (001) axis lifts the degeneracy of the  $\Gamma_8$  bands by breaking the cubic symmetry at the  $\Gamma$ -point of the Brillouin zone (BZ) and opens an insulating gap which makes such systems as 3D TI: the tensile strain caused by the mismatch between the substrate - CdTe and HgTe lattice is enough to open a gap between  $\Gamma_8^{3/2}$  and  $\Gamma_8^{1/2}$  [9]. After that the TPSS are left inside the energy gap, what allows to receive a truly insulating bulk system with the metallic phase at the surface.

In our previous work it was shown that analogous situation as for

the pure HgTe, is realized in a strained layer of the semimetal  $\text{Hg}_{1-x}\text{Cd}_x\text{Te}$  alloys ( $x < 0.16$  at  $T=4.2$  K) [20]. The additional degree of freedom exists in the case of semimetal HgCdTe: possibility to change the energy dispersion of carriers using the composition as a parameter. The latter one enables to reach as close as possible to the so-called 3D Dirac point where the  $\Gamma_6$  and  $\Gamma_8$  are crossing in the BZ and a linear disperse in all directions around the critical point in the 3D momentum space takes place [21]. The key questions are: i) how it will affect to TPSS; ii) how it can influence the charges transport on the TPSS in case of high intrinsic electron density in the investigated samples [8,20]. These questions are really important for the realization of a Weyl semimetal phase based on the HgCdTe, e.g. in  $\text{Hg}_{1-x-y}\text{Cd}_x\text{Mn}_y\text{Te}$  [22].

In this paper the experimental results on magnetotransport – Integer Quantum Hall Effect (IQHE) and Shubnikov-de Haas oscillations (SdH) – obtained for the strained 100 nm wide  $\text{Hg}_{0.865}\text{Cd}_{0.135}\text{Te}$  layer (it still corresponds to a semi-metallic type of a band-structure but it is close to the 3D Dirac point) are interpreted on the basis of the  $8\times 8$   $\mathbf{k}\mathbf{p}$  model. It will be shown that the fan of the Landau Levels (LL's) calculated by  $\mathbf{k}\mathbf{p}$  model for TPSS of the studied HgCdTe alloy corresponds to the fan of the LL's calculated using the graphene-like Hamiltonian which gives excellent agreement with experimental data for velocity on the Fermi level  $v_f \approx 0.85 \times 10^6$  m/s what is important for future applications of this material as TI.

## 2. Numerical calculations

The  $\mathbf{k}\mathbf{p}$  model used an eight band description of the band structure including all second-order terms representing the remote-band con-

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**Table 1**  
Band structure parameters of HgTe and CdTe (according to Refs. [23,24]).

	HgTe	CdTe		HgTe	CdTe
$E_g$	-0.303 eV	1.606 eV	$C$	-3.83 eV	-4.06 eV
$E_v$	0	-570 meV	$a$	0	-0.7 eV
$\Delta$	1.08 eV	0.91 eV	$b$	-1.5 eV	-1.17 eV
$E_p$	18.8 eV	18.8 eV	$d$	-2.08 eV	-3.2 eV
$F$	0	-0.09	$C_{11}$	53.6 GPa	53.6 GPa
$\gamma_1$	4.1	1.47	$C_{12}$	36.6 GPa	37.0 GPa
$\gamma_2$	0.5	-0.28	$C_{44}$	21.2 GPa	19.9 GPa
$\gamma_3$	1.3	0.03			
$\kappa$	-0.4	-1.31			

tributions with the first-order terms attributed to the  $\Gamma_8^{3/2}$  - heavy hole,  $\Gamma_8^{1/2}$  - light hole bands, and the  $\Gamma_6^{1/2}$  - conduction band as well as  $\Gamma_7^{1/2}$  - cleaved by the spin-orbital interaction [23,24]. The available band-structure parameters used in the eight band description, are presented in Table 1. The detailed descriptions of the calculation and the results for the mixed  $\text{Hg}_{1-x}\text{Cd}_x\text{Te}$  3D TI for the different width of the layers and different tensile strain are presented in Ref. [25]. The calculations of the energy spectrum are provided in the framework of the envelope function approach (EFA) where  $z$  - axis coincides with the growth direction of the 3D system. The finite difference method (FDM) with the common central difference form is particularly employed in the discretization procedure.

The results of the calculations are presented in Figs. 1a)–d). For the strained 75 nm thick HgTe layer (Fig. 1a) the dispersion relations calculated by the above mentioned method look the same as those ones in the work [9]. The exposed TPSS appear here in the gap induced by the strain as a small upper part of the Dirac cones which are formed inside of the  $\Gamma_8^{3/2}$  band.

Similar like in the Ref. [9] (supp. material) and in the [26] the dispersion relation as well as the charge distribution for investigated material were detailed analysis in Ref. [25]. It was shown that in the

case of the mixed 3D HgCdTe TI the shape of the  $|\Psi(z)|^2$  in the mini gap region (where the linear dispersion relation is presented) fulfills the definition of edge states [26] for this materials.

The band-structure calculations within the eight-band  $\mathbf{k}\mathbf{p}$  model for different values of  $x$  ( $0.135 < x < 0.15$ ) and at  $T=4.2$  K with the 100 nm thick strained layers shows also that the Dirac point can be visible in the strained gap (Figs. 1b, c).

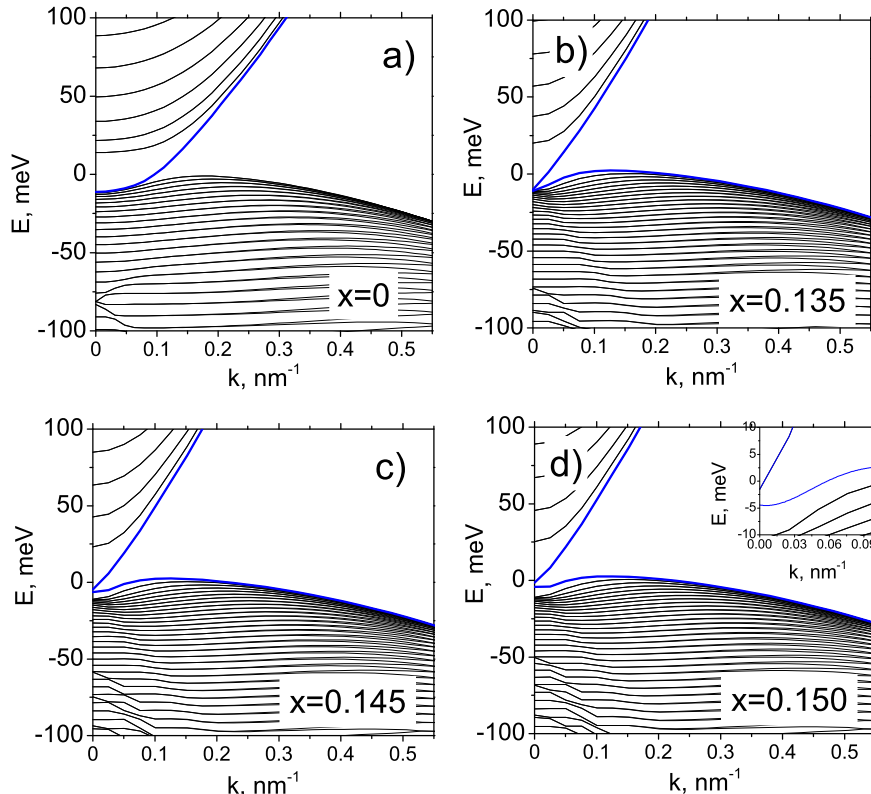
The numerical calculations performed for  $\text{Hg}_{0.865}\text{Cd}_{0.135}\text{Te}$  (Fig. 1b) show that the smaller inverted energy gap together with the uniaxial tensile strain along (001) direction (for 100 nm thick layers) make the Dirac point for TPSS located a little bit above the top of the  $\Gamma_8^{3/2}$  band. Beside that the gap between  $\Gamma_8^{3/2}$  and  $\Gamma_8^{1/2}$  which is induced by the strain is greater than for the pure 3D HgTe TI and is equal to about 30 meV for  $x=0.135$  at the  $\Gamma$  point of BZ (though the mismatch is reduced by  $\approx 13\%$  but the effect of the tensile is much greater due to the energy dispersion closed to the linear one in a bulk part of a sample).

The position of the Dirac point goes up together with the increase of the  $x$ -value. For  $x=0.135$  it is located around -10 meV (Fig. 1b)), and it practically touches the top of the band (with a small gap about 0.5 meV). For the higher values of  $x$  the Dirac point is being created inside the strained gap, for  $x=0.145$  it is located about 7 meV above  $\Gamma_8^{3/2}$  - Fig. 1c). It is necessary to underline that the position of the Dirac point in the energy scale is responsible for the shape of the energy dispersion for the TPSS: it can be expected that the linear dispersion law is applicable to a wider range of  $E(\mathbf{k})$  for the case when  $x$ -value is close to 0.15.

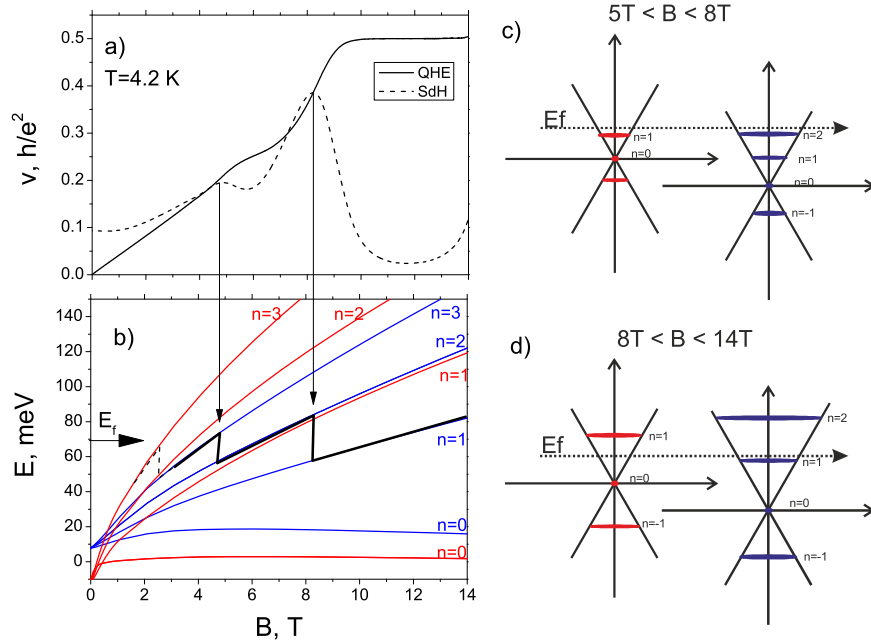
A different situation takes place in the same temperature for the  $\text{Hg}_{1-x}\text{Cd}_x\text{Te}$  layers with the  $x \geq 0.150$ . For  $x = 0.155$  the Dirac point at surfaces is violated: a gap of about 4 meV between the upper and the lower part of the Dirac cone appears (see inset Fig. 1d) and then these states do not represent the TPSS anymore.

### 3. Results

The results of the IQHE for the strained 100 nm wide  $\text{Hg}_{0.865}\text{Cd}_{0.135}\text{Te}$



**Fig. 1.** Dispersion relation for 100 nm wide strained 3D  $\text{Hg}_{1-x}\text{Cd}_x\text{Te}$  layers with: a)  $x=0$ , b)  $x=0.135$ , c)  $x=0.145$  and d)  $x=0.150$ . A blue line present the TPSS.



**Fig. 2.** a) The SdH oscillations and the IQHE for 4.2 K for 100 nm wide  $\text{Hg}_{0.865}\text{Cd}_{0.135}\text{Te}$  and b) the LL's fan calculated for two Dirac cones for two interfaces; c) and d) represent the LL's under the magnetic field region between 5 and 8 T and 8 and 14 T respectively.

Te layer obtained at 4.2 K are presented in Fig. 2a). The even filling factor  $\nu$  of 2, 4 and 6 means that two independent surfaces should be taken into account in the fermion transport - on the top and the bottom of a sample - its corresponds two Dirac cones model. As it is shown in Fig. 2c) and d) in order to get the filling factor  $\nu = 2$  it is necessary to have the Landau index  $N_b=1$  (the bottom surface) and  $N_t = 2$  for the top surface.

The LL's presented in Fig. 2 were calculated from the above mentioned eight band **kp** model but the experimental data can also be fitted by the LL's fan energy obtained from graphene-like Hamiltonian. In order to obtain a good agreement with the graphene-like LL's energies and experimental curves of  $R_{xy}$  and  $R_{xx}$  the difference between the positions of the Dirac points for two surfaces should be equal to about 20 meV. As it is shown in Figs. 2c,d it is worth to mention that the LL's fan calculated by the  $8 \times 8$  **kp** model with band-structure parameters presented in Table 1, are in excellent agreement with the one obtained using the graphene-like Hamiltonian if the Fermi level velocity is equal to about  $0.85 \times 10^6$  m/s.

The same calculations performed for pure HgTe ( $x=0$ ) lead to the Fermi level velocity  $v_f=0.43 \times 10^6$  m/s what confirms the result obtained by the authors of Ref. [9] ( $\hbar v_f=280$  meV nm).

The Fermi velocity can be directly received from  $E(k)$  relations  $\Delta E = \hbar v_f \Delta k$  (in the linear region of the dispersion relation) that are shown in Fig. 1. For the  $\text{Hg}_{0.865}\text{Cd}_{0.135}\text{Te}$  strained layers the velocity of the fermions for the TPSS obtained by this way is equal to  $0.85 \times 10^6$  m/s also.

Such velocity allows to observe the quantized Hall conductance on TPSS even without external gate voltage and on the background of the bulk states. It is interesting to remark that the TPSS are visible in magneto-transport for the wide range of the gate voltage applied into the pure HgTe 3D 70-nm-thick sample due to their remarkable screening properties according to Ref. [26]. This hypothesis could be applied to our  $\text{Hg}_{0.865}\text{Cd}_{0.135}\text{Te}$  sample also due to high electron density. Besides, the high temperature stability of the observed TPSS was shown in Ref. [20].

#### 4. Summary

To sum up briefly, it is shown that the LL's fan calculated for TPSS

for the studied HgCdTe alloy using the eight band **kp** model corresponds to the one calculated by graphene Hamiltonian and characterized by  $v_f \approx 0.85 \times 10^6$  m/s what is in excellent agreement with the experimental IQHE- and SdH-curves. On the other hand, this value of the Fermi velocity is two times greater than for the strained HgTe TI Ref. [9] and for other kind of TI - approximately  $5 \times 10^5$  m/s, for  $\text{Bi}_2\text{Se}_3$  [12], or  $3.4 \times 10^5$  m/s for the same compound according to Ref. [27].

Thus, the strained 3D  $\text{Hg}_{0.865}\text{Cd}_{0.135}\text{Te}$  is a Topological Insulator with high value of the Fermi velocity (close to the Fermi velocity value for graphene) which is a significant advantage. The last one concerns although the HgCdTe based quantum wells near the phase transition between the semi-metal and semiconductor [28]. Besides, the bigger strained energy gap inside the sample than in the case of strained HgTe, and a higher position of the Dirac points on the energy scale lead to an increase in the attractiveness of the Topological Insulators based on semimetal HgCdTe alloy for future applications. All these advantages follow from significantly lower value of the electron momentum on the Fermi level in comparison with pure HgTe and the energy dispersion is closer to linearity in the wider energy range.

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