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A new approach to the current distribution in field cooled superconductors disks

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Abstract

The Bean model considers that in field cooled superconducting cylinders with diameter R, the currents flow over all the thickness of the superconductor along circular paths, the minimum radius of which depends on the magnetizing field and the critical current density. A combination of trapped field and levitation force measurements reported recently has shown, however, that in YBCO and MgB_2 disks the current flows in fact in a restricted region with thickness t of the superconductor. In this contribution, from measurements carried out on two YBCO and two MgB_2 disks, we report the dependence on temperature of t and J_p, the current density in this region, as well as that of the field trapped by the samples. The results confirm that t decreases as the temperature decreases. This behaviour is ascribed to the conservation of the magnetic energy stored in the superconductor, which depends on the magnetizing source and not on the measurement temperature. As a consequence, t behaves as ${J_p}^{-2/3}$, while the field trapped along the axis of the cylinder behaves as ${J_p}^{1/3}$. These claims are substantiated by the experimental results. The possibility that J_p is equal to the depairing current is investigated.

Keywords: current distribution, depairing current, field cooled bulks

(Some figures may appear in colour only in the online journal)

1. Introduction

According to the Bean model, the field B_T trapped by a field cooled superconducting disk with thickness h and radius R is proportional to both R and the critical current density of the material, J_c [1]. The current lines generating the field are circular, and penetrate the disk along its radius on a distance $r \leqslant R$ depending both on the magnetizing field and the critical current density of the sample and flow over all the thickness of the disk [2]. Otherwise, an expression for the calculation of the vertical component of B_T along the axis of cylinders was proposed by Chen *et al* [3], also assuming that the superconducting current flows along circular paths over all the thickness of the disks. In recent works [4, 5] however, levitation force measurements carried out on $YBa_2Cu_3O_7$ (YBCO) and MgB_2 disks in a wide range of temperatures were analysed with a mean field approach. This approach is based on the assumption that if the applied field and the field gradient are averaged along the axis of the disk, the levitation force can be estimated using relations proposed by Sanchez *et al* [6] for the levitation force in an uniform field gradient and by Brandt [7] for the magnetic moment of a disk in an uniform field. Consistently with some numerical simulations [8–10], the results have suggested that the superconducting current flows in fact with density $J_{p,}$ in a layer of thickness $t < h$ perpendicular to the magnetizing field. It was shown in addition that t is an increasing function of the temperature. The validity of the mean field approach and of the determination of J_p and t was supported by the reproduction of the field trapped by the investigated disks along their axis. We emphasize that in [4, 5] J_p was designated as the critical current density of the samples. For the reasons detailed in section 4, this was a mistake.

In this contribution, we firstly highlight that B_T is in fact proportional to the surface density $J_p^s = J_p t$ of the current flowing in the sample. Then, we show that the magnetic energy stored in the superconductor does not depend on the temperature at which the B_T measurements are carried out and we show that $t \propto J_p^{-\frac{2}{3}}$ and $B_T \propto J_p^{\frac{1}{3}}$ as a result.

The paper is organized as follows. In the second section, we briefly describe the levitation force and trapped field measurements we have carried out to determine J_p^s, J_p and t.

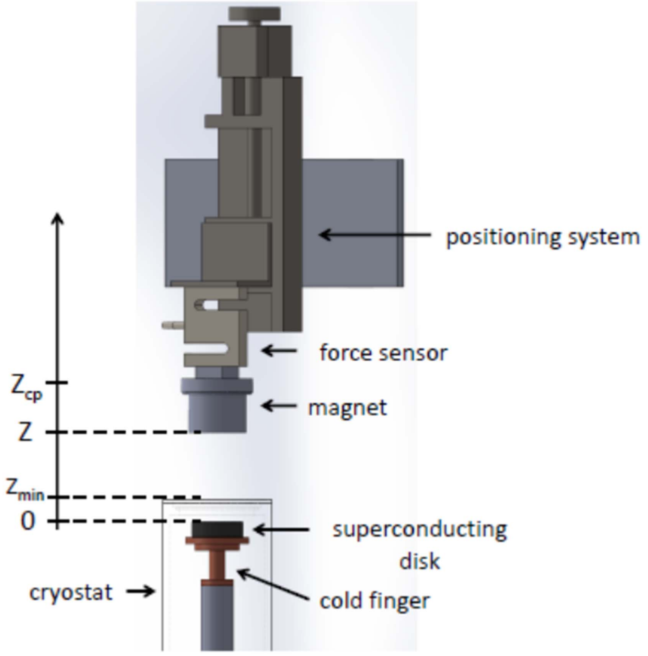

Figure 1. Levitation force measurement set up. From the bottom up: the cryostat with the superconductor inside the magnet, the force sensor and the magnet positioning system; the upper surface of the superconducting disk is located at $Z = 0$; Z_{cp} is the cooling height and Z_{min} the distance between the sample and the top of the cryostat. Reproduced from [5]. © IOP Publishing Ltd. All rights reserved.

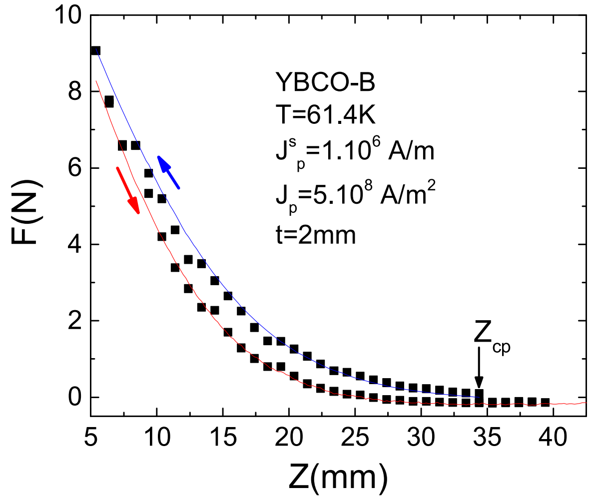

Figure 2. Levitation force cycle measured at 61.4 K after field cooling disk YBCO-B at distance $Z_{cp} = 35$ mm from the magnet. The black squares are the forces measured while the continuous lines are obtained with the mean field approach described in [4, 5] and the data reported in the figure.

In section 3 we describe the investigated samples and present the obtained results. Section 4 is dedicated to the demonstration of the proposed power laws and section 5 to a discussion of the possibility that J_p is the depairing current density of the sample.

2. Experimental techniques

For this work we have carried out levitation force and trapped field measurements on field cooled superconducting disks. The levitation force measurements and the procedure used for analyzing the results are detailed in [4, 5]. Briefly, the investigated sample was fastened in the normal state in a cryostat. A 45 mm diameter NdFeB magnet was located above the superconductor at distance Z_{cp} from the top of the sample. The superconductor was cooled down to the measurement temperature T, and, after stabilization, the magnet was moved down vertically in the direction of the disk.

Once the magnet was put into contact with the top of the cryostat at Z_{min}, its direction of motion was reversed and it was moved up to a distance $Z \geqslant Z_{cp}$ (see figure 1). The force of interaction between the magnet and the superconductor was recorded as a function of Z during the whole process and resulted in a force hysteresis loop (see an example in figure 2). As reported in [5] for both YBCO and MgB_2 disks, after increasing strongly as the temperature decreases, the levitation force at a given distance from the disk tends towards a maximum value at low temperature. Analyzing the hysteresis loops with the mean field approach has led to the determination of J_p and t. A striking result was that t is an increasing function of the temperature. We emphasize that the validity of the mean field approach we have used is restricted to a domain of temperature with an upper and a lower limits determined respectively by the conditions $t < \frac{h}{2}$ and that the levitation force is not at saturation.

In order to measure the field trapped by the disks, the magnet was located at $Z_{cp} = 5$ mm and the samples were cooled down to the measurements temperature. Then, the magnet was slowly moved up far away from the superconductors and the vertical component of the field generated along their axis by the superconducting current, $B_T(Z)$, was measured. Assuming, for the sake of symmetry, that the layers carrying the current were located in the middle of the disks, $B_T(Z)$ was reproduced using the relation proposed by Chen *et al* [3] for a disk completely penetrated by the magnetizing field:

$$B_T(Z) = \frac{\mu_0 J_p}{2}\Bigg[(Z^* + t)Ln\left(\frac{R}{Z^*+t} + \sqrt{1 + \frac{R^2}{(Z^*+t)^2}}\right) - Z^* Ln\left(\frac{R}{Z^*} + \sqrt{1 + \frac{R^2}{Z^{*2}}}\right)\Bigg]. \quad (1)$$

In equation (1), $Z^* = Z + \frac{h-t}{2}$ is the separation between the magnetic probe and the upper current loops. Difficulties arose because the determination of J_p and t was not univocal. Different couples of J_p and t values could reproduce the measurements. A good fitting curve was obtained only on the condition that the surface current density J_p^s be equal to that resulting from the levitation force measurements. As an example, figure 3 shows the field trapped by a YBCO disk at 61.4 K. The results can be reproduced with equation (1) and the J_p and t resulting from the levitation force measurements carried out at the same temperature [$J_p = 5.10^8$ A cm^{-2} and $t = 2$ mm] (see figure 2) as well as with $J_p = 2.5.10^8$ A cm^{-2}

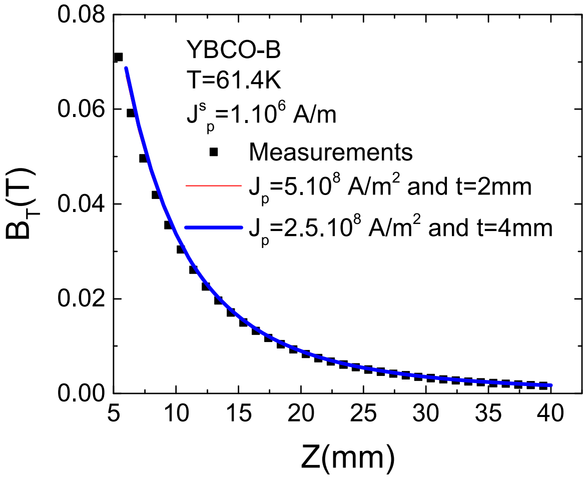

Figure 3. Field trapped along the axis of disk YBCO-B at 61.4 K after field cooling at distance $Z_{cp} = 5$ mm from the magnet. The black squares are the fields measured while the continuous lines are obtained with equation (1) and the data reported in the figure.

Table 1. Radius, thickness and critical temperature of the investigated samples.

Sample	R (mm)	h (mm)	T_c (K)
YBCO-A	8.7	11.6	88.5
YBCO-B	9.3	11.4	83
MgB_2-A	14.8	14.1	34.5
MgB_2-B	14.9	11.9	33

and $t = 4$ mm. As a conclusion, J_p^s is proportional to the trapped field and is the only quantity that can be determined unambiguously from the trapped field measurements.

3. Results

3.1. Samples

We report results from trapped field and levitation force measurements carried out on (i) two YBCO disks labelled as YBCO-A and YBCO-B and (ii) two MgB2 disks labelled as MgB_2-A and MgB_2-B. The YBCO disks were fabricated by a top-seeded melt growth process [11–13] in slightly different conditions. Trapped field mappings have shown that they were single-grain. The MgB_2 disks were fabricated by rapid spark plasma sintering according to the process reported in [14]. The dimensions and the critical temperatures of the samples are reported in table 1.

3.2. Results of the measurements

The current densities J_p resulting from the levitation force measurements are reported in figure 4 as functions of the temperature, while the thicknesses of the layers carrying the currents are reported in figure 5.

As expected, thickness t is an increasing function of the temperature for all the samples. The trapped fields measured above the top of the cryostat at 6 mm from the superconductors are reported in figure 6. They increase as the temperature decreases, except YBCO-A for which the trapped field levels off below 65 K, showing that the magnetizing field was not large enough to fully penetrate the superconductor in this domain of temperatures.

4. Modelling

In this section, we write a vortex depinning condition consistent with the restriction to a region of thickness t of the volume penetrated by the current and we establish power laws for $t(J_p)$ and $B_T(J_p)$ from considerations on the magnetic energy stored in the samples.

In the single vortex model, depinning occurs if the Lorentz force due to the superconducting current is equal to the vortex pinning force [15]. We have:

$$J_c \phi_0 = f_p \tag{2}$$

where J_c is the critical current density of the sample and f_p the pinning force per unit length. At a given temperature, f_p depends on the composition of the sample and the impurities it contains as well as on its micro-structural properties [16–19]. While the defects responsible for pinning are distributed along the whole length of the vortices, the results in section 3 suggest that the current flows with density J_p in a layer of thickness t only. As a result we can rewrite equation (2) as:

$$J_p \phi_0 t = h f_p. \tag{3}$$

Contrary to the claim in [4, 5], J_p is different from J_c that, according to equations (2) and (3), is the depinning current density averaged over the thickness of the sample. From another point of view, we emphasize that since the product $J_p^s = J_p t$ depends on f_p and h only, it depends on the sample, not on the magnetizing process.

We show now that $t(T)$ and $B_T(T)$ behave as $J_p{}^{-2/3}$ and $J_p{}^{1/3}$, respectively. Let us consider a cylindrical bulk at $T < T_c$ fully penetrated by the field of the magnet located at the cooling point. As long as the magnet is motionless, the applied magnetic field is canalized along vortex lines and no current flows in the superconductor. For trapped field measurements, while the magnet is moved up, current is generated in order to keep the mean field in the superconductor unchanged. The energy E_m required to generate this current is provided by the motion of the magnet. If the magnet motion is low enough to make negligible energy losses due to magnetic relaxation we have:

$$W = \int_{Z_{cp}}^{\infty} F dZ = -E_m \tag{4}$$

where W is the work of the force acting on the magnet. The force acting on a 45 mm diameter magnet located above a 30 mm diameter YBCO disk when it is moved up from $Z_{cp} = 15$ mm to $Z = 40$ mm is reported from 35 K to 80 K in figure 7. The force at a given distance from the superconductor is almost a constant below 70 K, showing that in

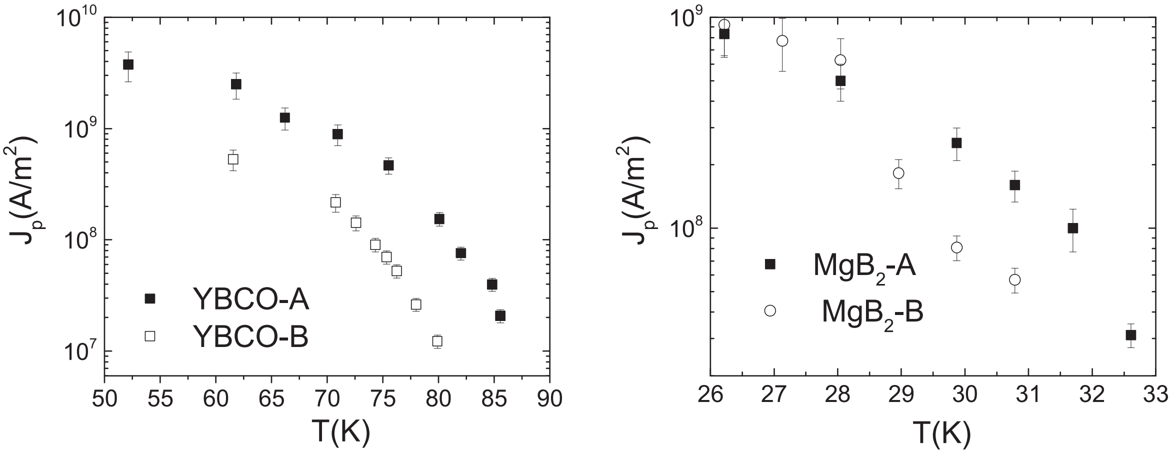

Figure 4. Current densities of the YBCO and the MgB_2 disks obtained with the levitation force measurements as functions of temperature.

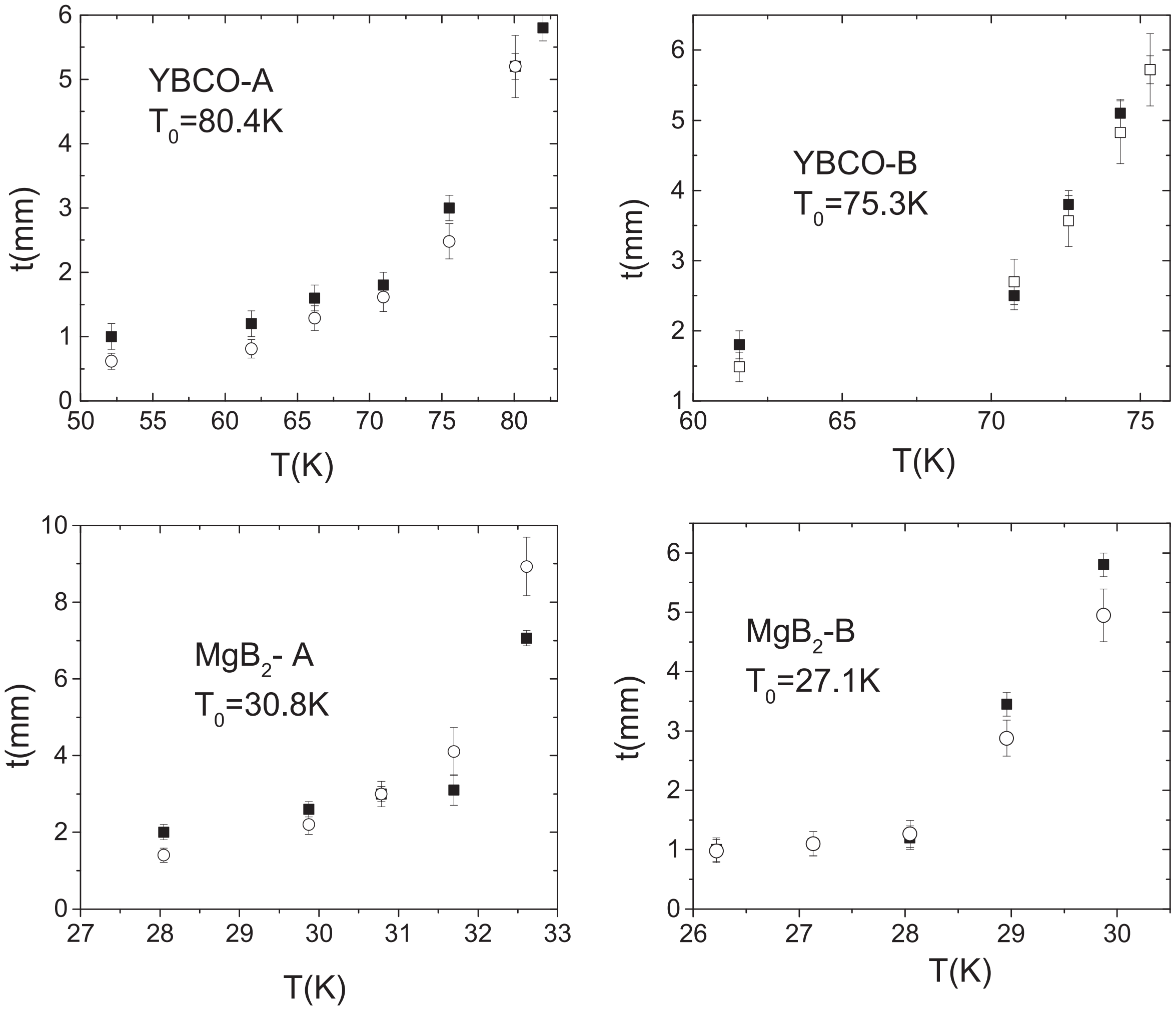

Figure 5. Thickness of the layer carrying the currents in the investigated disks; full symbols: values resulting from the levitation force measurements; open symbols: values calculated with equation (6) taking for J_{p0} and t_0 the current densities and thicknesses measured at the temperature T_0 shown in the figures. The errors in the values calculated with equation (6) result from the errors in the determination of J_p.

this domain of temperatures E_m is also a constant. The reason is that, as long as the current density in the sample is large enough to generate a field matching the field that was generated by the magnet, the magnetic energy depends only on the field applied when the sample was cooled down. Magnetic energy E_m can be written as:

$$E_m = \frac{1}{2}LI^2 = \frac{1}{2}LJ_p^2 t^2 R^2. \quad (5)$$

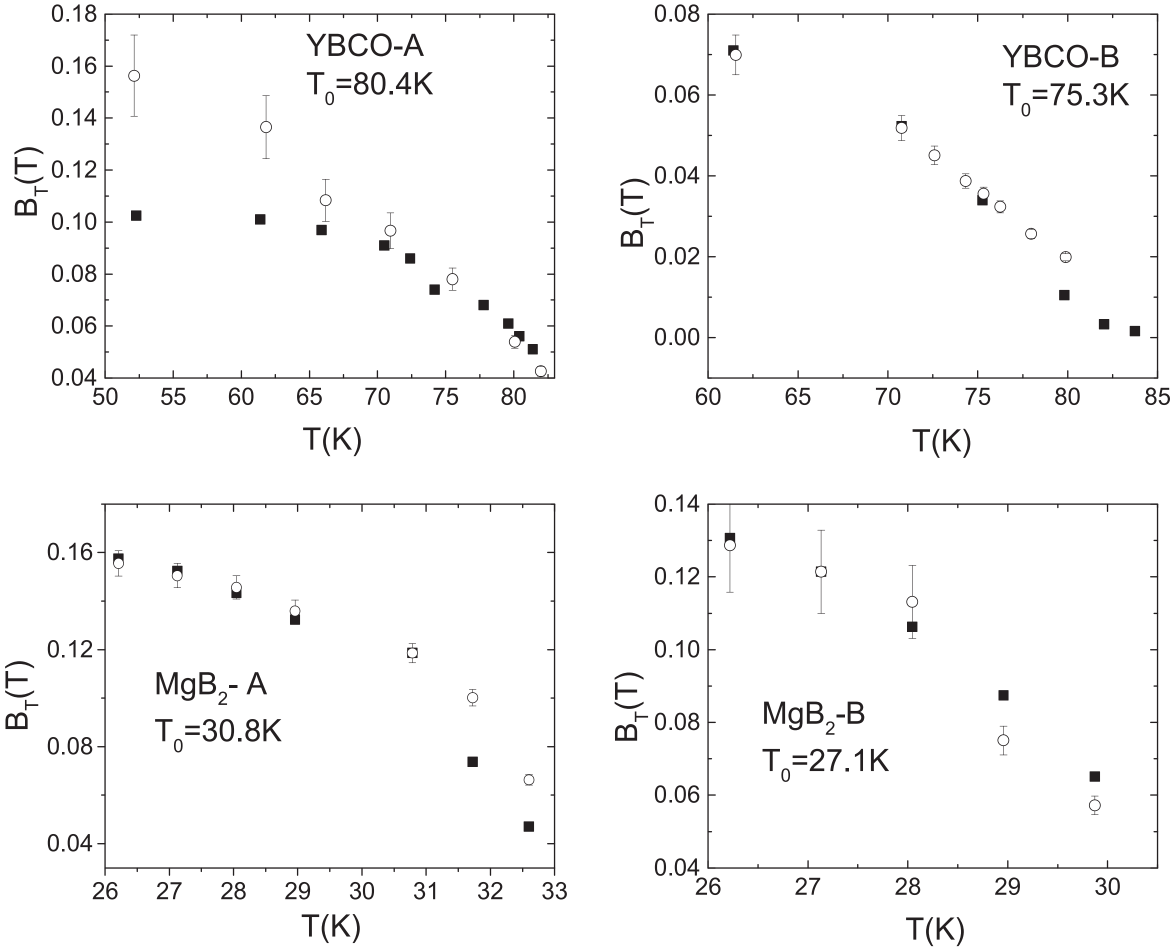

Figure 6. Vertical component of the field trapped at 6 mm from the top of the disks along their axis; full symbols: measurements; open symbols: values calculated with equation (7) taking for J_{p0} and B_{T0} the current densities and trapped fields measured at the temperature T_0 shown in the figures. The errors in the values calculated with equation (7) result from the errors in the determination of J_p.

In equation (5), I is the current flowing in the superconductor and L the superconductor inductance. We assume $L \propto t$, because the larger t, the larger the energy required to keep the vortex lines parallel to each other on a thickness in the range of t. Let's consider a couple of values (J_{p0}, t_0) satisfying equation (3) at temperature T_0. From the conservation of the magnetic energy we obtain at temperature $T \neq T_0$:

$$t = t_0\left(\frac{J_p}{J_{p0}}\right)^{-\frac{2}{3}}. \tag{6}$$

As the temperature decreases, J_p increases and the thickness of the layer carrying the superconducting currents decreases as $J_p^{-\frac{2}{3}}$. Otherwise, although the stored magnetic energy is independent of T and J_p in the domain of temperatures we are interested in, the trapped field increases as $J_p^{\frac{1}{3}}$, since it is proportional to$J_p^s = J_p t$. We can write:

$$B_T = B_{T0}\left(\frac{J_p}{J_{p0}}\right)^{\frac{1}{3}} \tag{7}$$

where B_{T_0} is the field trapped by the bulk at temperature T_0. In figure 5 the thicknesses determined from the measurements carried out on the YBCO and MgB_2 disks are compared to the t values given by equation (6). In figure 6 the field trapped at 6 mm from the top of the disks is compared to the values given by equation (7). Both for t and B_T, there is a very good agreement between the values determined by the mean field approach and those calculated with equation (6) and equation (7), except for the field trapped at the lowest temperatures by disk YBCO-A. As discussed above, this can be ascribed to the incomplete penetration of the sample by the magnetizing field in in this range of temperatures.

5. Discussion and conclusions

One of the main conclusions of this work is that the classical picture of the current flowing over all the thickness of a field cooled superconducting disk with density J_c should be replaced by one in which the current flows with density J_p in a restricted part of the volume. In this section, we firstly

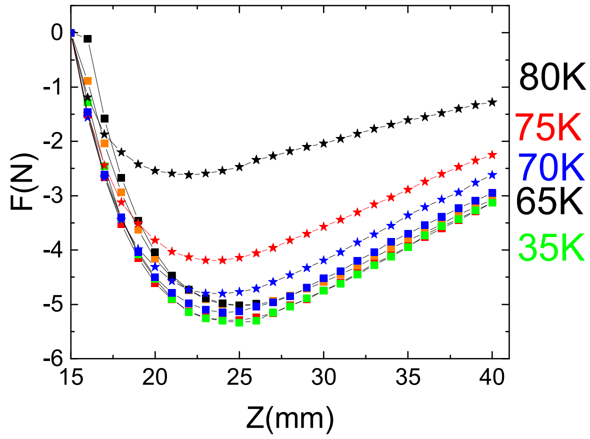

Figure 7. Force acting on a 45 mm diameter magnet located above a 30 mm diameter YBCO disk between 35 K and 80 K when it is moved up from $Z_{cp} = 15$ mm to $Z = 40$ mm.

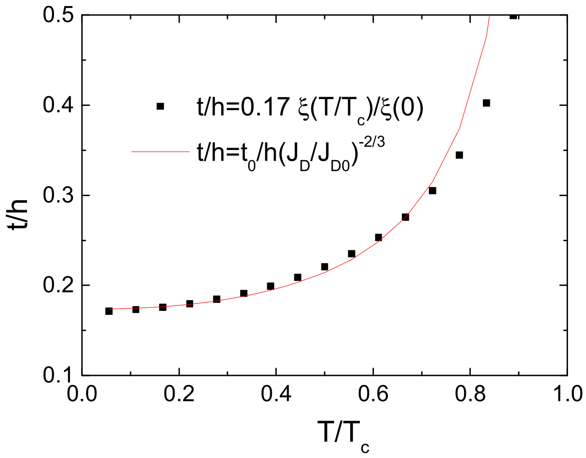

Figure 8. $\frac{t}{h}$ ratios calculated with equations (12) and (13), taking $\frac{t}{h} = 0.5$ at $\frac{T}{T_c} = 0.9$. The red line is obtained with equation (6).

investigate the possibility that J_p is the depairing current density, J_D, then we sum up the results of this work.

As first step, we show that if $J_p = J_D$, $\frac{t}{h}$ shows the same dependence on temperature as the coherence length of the material and obeys equation (6) in a large range of temperatures. According to the Ginzburg–Landau (GL) model, we have [15]:

$$J_D \propto \frac{1}{\lambda^2 \xi} \tag{8}$$

where λ is the penetration length of the material. Otherwise, if vortex pinning is due to randomly dispersed pinning sites, the pinning force depends on a, the inter-vortex distance and on u, the energy difference between the situation where the pinning site is at the center of a vortex and that in which it is at the periphery of a vortex [20]. We can write:

$$f_p \propto \frac{u}{a}. \tag{9}$$

The inter-vortex distance depends on the applied field, not on the temperature, while the dependence on temperature of u is that of the vortex line energy. As a result, we have:

$$f_p \propto \frac{1}{\lambda^2} \tag{10}$$

and

$$\frac{t}{h} = \frac{f_p}{J_D \phi_0} \propto \xi. \tag{11}$$

As claimed, thickness t shows the same dependence on temperature as the coherence length. From equation (11) we can write:

$$\frac{t}{h} = \mathrm{a}\frac{\xi}{\xi(0)} \tag{12}$$

where a is a constant. Figure 8 shows $\frac{t}{h}$ as a function of the reduced temperature, $\frac{T}{T_c}$, taking for $\xi(T)$ the GL expression:

$$\xi(T) = \xi(0)\sqrt{\frac{1 + \left(\frac{T}{T_c}\right)^2}{1 - \left(\frac{T}{T_c}\right)^2}} \tag{13}$$

and selecting a in order that $\frac{t}{h} = 0.5$ at $\frac{T}{T_c} = 0.9$. The resulting curve is similar to the $t(T)$ curves in figure 5.

The $\frac{t}{h}$ in figure 8 can be reproduced with equation (6) in the lower temperature range, taking $\lambda^2(T) = \lambda^2(0)\left[1 - \left(\frac{T}{T_c}\right)^4\right]^{-1}$ for the calculation of J_D. These results support the suggestion that the density of the current flowing in the region of thickness t is the depairing current density. An argument against this proposition is that the J_p assigned to the YBCO samples are much lower than the depairing current densities expected for YBCO as well as the J_c measured both on thin films and coated conductors [21]. However, since $\frac{h}{t} = \frac{J_p}{J_c}$, figures 5 and 8 show that if $J_p = J_D$, we have $J_D \gg J_c$ at low temperature, as expected. From another point of view the depairing current density is proportional to the concentration of superconducting pairs. In single-grain bulks of the YBCO family, this concentration is strongly dependent on the oxygen content of the material, which can be far from optimal in large parts of the samples, as shown by studies carried out on thin walls bulks [16, 22]. As a result, the density of pairs and the depairing current of bulks samples are much lower than those of well-oxygenated thin films and coated conductors. However, more work is needed to decide the validity of the discussed proposition.

In this work, we have confirmed that the thickness t of the layer carrying the superconducting current in bulk superconductors decreases as the temperature decreases. We have shown that the field B_T trapped in a field cooled cylinder is proportional to the surface current density $J_p^s = J_p t$ and established the power laws $t \propto J_p^{-\frac{2}{3}}$ and $B_Z \propto J_p^{\frac{1}{3}}$. Although the proposition that J_p is equal to the depairing current density is doubtless not established, we are convinced that these results can have important consequences for the design of bulk superconductors dedicated to applications in the domains of field generation, magnetic levitation and superconducting motors and generators.

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