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Energy density and energy flow of magnetoplasmonic waves on graphene

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ABSTRACT

By means the linearized magnetohydrodynamic theory, expressions for energy density and energy flow are derived for the p-polarized surface magnetoplasmon polaritons on graphene in the Voigt configuration, where a static magnetic field is normal to the graphene surface. Numerical results show that the external magnetic field has significant impact on the energy density and energy transport velocity of magnetoplasmon waves in the long-wavelength region, while total power flow vary only weakly with magnetostatic field. The velocity of energy propagation is proved to be identical with group velocity of the surface waves.

1. Introduction

Since the discovery of graphene by Novoselov et al [1], optical properties of this truly two-dimensional (2D) electronic system have received a great deal of attention from both theoretical [2–4] and experimental [5] points of view. As mentioned in [6,7], graphene can support p-polarized surface plasmon polariton (SPP) waves, or transverse magnetic (TM) surface waves, with subwavelength localization in the direction perpendicular to the surface. Furthermore, Mikhailov and Ziegler [8] predicted that graphene also supports unusual transverse electric (TE) waves in a well defined and narrow frequency window. These excitation are possible only if the imaginary part of the conductivity of a thin sheet of material is negative. Then, Bludov et al [9] found that nonlinear TE-polarized SPPs may propagate on graphene.

On the other hand, by employing the random-phase approximation, Berman et al [10] calculated the dispersion equations for magnetoplasmons in a graphene in a perpendicular magnetic field. Also, Roldan et al [11] studied the electrodynamics of doped graphene in a magnetic field by means a collisionless hydrodynamic approach. In the presence of an external magnetic field perpendicular to the graphene layer, they obtained new differences between the conventional 2D electron gas of massive electrons and graphene layer. Nevertheless, no explicit calculation can be found for Poynting vector (which is the electromagnetic energy flow vector) associated with the magnetoplasmonic waves, density of the electromagnetic energy, and the energy transport velocity on graphene.

To get deeper insight into the physics of magnetoplasmonic waves of graphene, in the present paper for the first time we investigate the Poynting vector and density of energy of p-polarized magnetoplasmonic waves on graphene in the Voigt configuration, where a static magnetic field is normal to the graphene surface. To do this, we use the linearized magnetohydrodynamic (MHD) theory for electronic excitations on graphene surface and extend the classical theory for the continuity equation of the electromagnetic. Then, we obtain energy flow and density of the electromagnetic energy for magnetoplasmonic waves of graphene.

2. Basic equations and dispersion relation

As a simple model of graphene, we assume a 2D electron gas in equilibrium that is uniformly distributed in the xy plane in a Cartesian coordinate system with coordinates (x, y, z) and has the surface density (density per unit area) of n_0 corresponding to the free electrons per carbon atom in graphene. Also, we assume a static magnetic field $\mathbf{B}_0 = B_0\mathbf{e}_z$ that is normal to the graphene plane (Voigt configuration). Furthermore, a substrate with dielectric constant ϵ_1 is supposed to occupy the region $z < 0$ underneath the graphene, whereas the region $z > 0$ is assumed to be a semi-infinite insulator with dielectric constant ϵ_2.

Now, assuming that $n(x, t)$ is the first-order perturbed density (per unit area) of the homogeneous electron fluid on the graphene surface, due to the propagating p-polarized magnetoplasmonic wave parallel to the interface z=0 along the x-direction. We note the p-polarized or TM magnetoplasmonic wave have frequency ω and E_x, H_y and E_z components. Based on the linearized MHD theory [11,12], the electronic excitations on graphene surface can be described by the continuity equation

$$\partial_t n(x, t) + n_0\partial_x v_x(x, t) = 0, \quad (1)$$

and the momentum-balance equation,

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$$(\partial_t + i\omega_c^2/\omega)v_x(x, t) = -\frac{e}{m}E_x\Big|_{z=0} - \frac{\alpha}{n_0}\partial_x n(x, t), \tag{2}$$

where e is the element charge and m is the effective electron mass, $\omega_c = eB_0/m$ is the cyclotron frequency of the electron and $v_x(x, t)$ is the first-order perturbed values of velocity field. In the right-hand side of Eq. (2), the first term is the force on electrons due to the tangential component of the electric field, evaluated at the graphene surface z=0, the second term is the force due to the internal interaction in the electron fluid, with $\alpha = v_F^2/2$ that is the square of the speed of propagation of density disturbances in a uniform 2D homogeneous electron fluid, where $v_F = \hbar k_F/m$ and k_F is the well-known Fermi wavenumber.

At this stage, we eliminate the velocity field $v_x(x, t)$ from Eqs. (1) and (2) and assume that all physical quantities (electron density perturbation and electromagnetic field) vary as $e^{i(kx-\omega t)}$ where k is the component of the wave vector along the x-axis. We obtain

$$N = -\frac{ien_0}{m}\frac{k}{\omega^2 - \omega_c^2 - \alpha k^2}E_x(z)\Big|_{z=0}, \tag{3}$$

and $n(x, t) = Ne^{i(kx-\omega t)}$. The solutions of Maxwell equations that are wavelike in the x-direction and whose amplitudes decay exponentially with increasing distance into each medium from the interface z=0 can be written as

$$E_{1z} = Ae^{\kappa_1 z}e^{i(kx-\omega t)}, \tag{4}$$

$$E_{1x} = -\frac{i\kappa_1}{k}Ae^{\kappa_1 z}e^{i(kx-\omega t)}, \tag{5}$$

$$H_{1y} = \frac{\omega\epsilon_0\epsilon_1}{k}Ae^{\kappa_1 z}e^{i(kx-\omega t)}, \tag{6}$$

in the region $z < 0$, and as

$$E_{2z} = Be^{-\kappa_2 z}e^{i(kx-\omega t)}, \tag{7}$$

$$E_{2x} = \frac{i\kappa_2}{k}Be^{-\kappa_2 z}e^{i(kx-\omega t)}, \tag{8}$$

$$H_{2y} = \frac{\omega\epsilon_0\epsilon_2}{k}Be^{-\kappa_2 z}e^{i(kx-\omega t)}, \tag{9}$$

in the region $z > 0$, where $\kappa_\lambda^2 = k^2 - k_\lambda^2$, $k_\lambda = \sqrt{\epsilon_\lambda}\omega/c$ (with $\lambda = 1, 2$) and c is the light speed.

We now apply the boundary conditions at the surface of the system. With the induced density, these boundary conditions can be written as

$$E_{1x}(z = 0) = E_{2x}(z = 0), \tag{10}$$

$$\epsilon_2 E_{2z}(z = 0) - \epsilon_1 E_{1z}(z = 0) = -\frac{eN}{\epsilon_0}, \tag{11}$$

where ϵ_0 is the permittivity of free space. Using the above equations, the retarded dispersion relation for the system is given as

$$\omega^2 = \omega_c^2 + \alpha k^2 + \frac{n_0 e^2}{m\epsilon_0}\left(\frac{\epsilon_1}{\kappa_1} + \frac{\epsilon_2}{\kappa_2}\right)^{-1}. \tag{12}$$

This is the frequency of magnetoplasmonic waves on graphene that is known in gas plasma physics as the upper hybrid mode. With the above equation, we will obtain the Poynting vector and energy density associated with magnetoplasmonic waves of the system in the following section.

3. Energy density and energy flow

To discuss power flow associated with the magnetoplasmonic waves, density of the electromagnetic energy, and the energy velocity on graphene, we begin with the Poynting theorem for energy in standard electrodynamics. We have:

$$\nabla\cdot\mathbf{S} + \partial_t u = -\mathbf{E}\cdot\mathbf{J}, \tag{13}$$

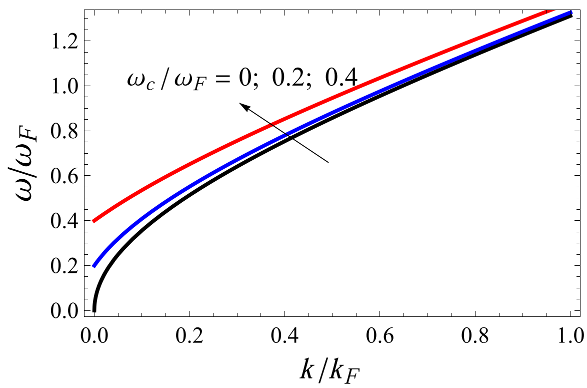

Fig. 1. Dispersion curves of the magnetoplasmonic waves (limited to the non-retardation regime) in the Voigt configuration. Three curves correspond to the three values of ω_c/ω_F, when $\epsilon_1 = \epsilon_2 = 1$.

where $\mathbf{S} = \mathbf{E}\times\mathbf{H}$ is known as the Poynting vector, which is a power density vector associated with an electromagnetic field and $u = \frac{1}{2}(\mathbf{E}\cdot\mathbf{D} + \mathbf{H}\cdot\mathbf{B})$ is the energy density of electromagnetic wave.

For the present system, we have $\mathbf{D} = \epsilon_0\epsilon_\lambda\mathbf{E}$ and $\mathbf{B} = \mu_0\mathbf{H}$ (where μ_0 is the permeability of free space and we put $\mu_\lambda = 1$). Using Eq. (13) for $z \neq 0$, the energy density u and energy flow density S_x associated with the surface waves in the insulator with dielectric constant ϵ_λ are

$$u_\lambda = \frac{1}{2}(\epsilon_0\epsilon_\lambda|\mathbf{E}_\lambda|^2 + \mu_0|\mathbf{H}_\lambda|^2), \tag{14}$$

$$S_{\lambda x} = (\mathbf{E}_\lambda \times \mathbf{H}_\lambda)_x. \tag{15}$$

On the other hand, the electric current density flowing on the graphene surface is given by Ohm law

$$J_x(x, t) = -en_0 v_x(x, t). \tag{16}$$

Now, using Eqs. (1)–(3) and after doing some algebra, we rewrite the right-hand side of Eq. (13) on graphene surface as:

$$\mathbf{E}\cdot\mathbf{J} = \frac{mn_0}{2}\left(1 + \frac{\omega_c^2}{\omega^2}\right)\partial_t v_x^2 + m\alpha\partial_x\mathbf{e}_x\cdot nv_x\mathbf{e}_x + \frac{m}{2n_0}\alpha\partial_t n^2. \tag{17}$$

Then, Eq. (13) together with Eq. (17) yields the energy density u_0 and the energy flow density S_{0x} on the graphene surface (z=0) in the forms, as:

$$u_0 = \frac{mn_0}{2}\left(1 + \frac{\omega_c^2}{\omega^2}\right)v_x^2 + \frac{m}{2n_0}\alpha n^2, \tag{18}$$

$$S_{0x} = m\alpha n v_x. \tag{19}$$

Eliminating v_x, and n in Eqs. (18) and (19) and using Eqs. (1)–(3), the energy and energy flow densities on graphene surface may be expressed, in complex notation, by:

$$u_0(z = 0) = \frac{e^2 n_0}{4\,m}\frac{\omega^2 + \omega_c^2 + \alpha k^2}{[\omega^2 - \omega_c^2 - \alpha k^2]^2}E_{\lambda x}E_{\lambda x}^*, \tag{20}$$

$$S_{0x}(z = 0) = \frac{e^2 n_0}{2\,m}\frac{\alpha k\omega}{[\omega^2 - \omega_c^2 - \alpha k^2]^2}E_{\lambda x}E_{\lambda x}^*. \tag{21}$$

In the insulators for $z \neq 0$, we have:

$$u_\lambda(z) = \frac{1}{4}(\epsilon_0\epsilon_\lambda|\mathbf{E}_\lambda|^2 + \mu_0 H_{\lambda y}H_{\lambda y}^*), \tag{22}$$

$$S_{\lambda x}(z) = -\frac{1}{2}Re[E_{\lambda z}H_{\lambda y}^*]. \tag{23}$$

The total energy density and flow of energy associated with the surface waves are determined by an integration over z. We have:

$$U = \int_{-\infty}^{0} u_1(z)dz + \int_0^{+\infty} u_2(z)dz + u_0(z = 0), \tag{24}$$

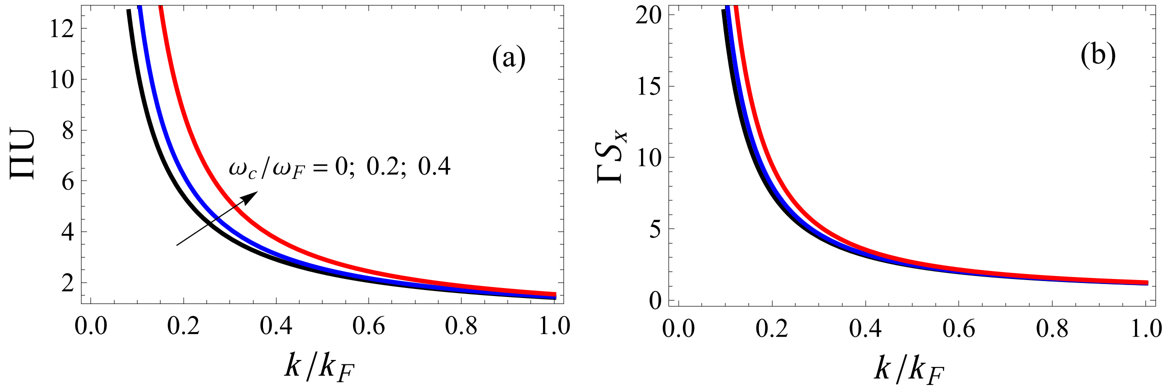

Fig. 2. (a) Integrated energy per unit length U [Eq. (26)] and (b) integrated power S_x [Eq. (27)] for three different values of ω_c/ω_F, when $\epsilon_1 = \epsilon_2 = 1$. Also $\Pi = k_F(\epsilon_0 A^2)^{-1}$ and $\Gamma = (k_F\omega_F\epsilon_0 A^2)^{-1}$.

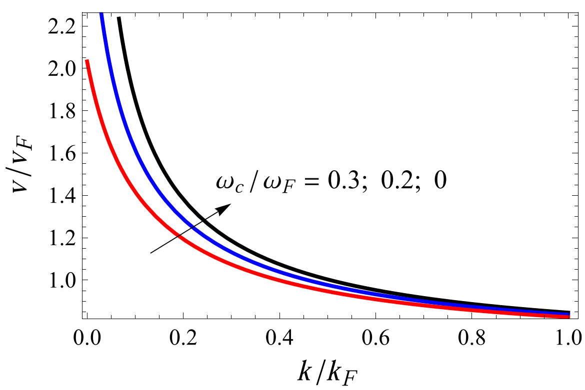

Fig. 3. Energy velocity $v_E(k,\omega) = S_z/U$ for three different values of ω_c/ω_F, when $\epsilon_1 = \epsilon_2 = 1$. This velocity equals the gradient of the corresponding dispersion curve in Fig. 1.

$$S_x = \int_{-\infty}^{0} S_{1x}(z)dz + \int_{0}^{+\infty} S_{2x}(z)dz + S_{0x}(z=0). \tag{25}$$

After doing some algebra, the energy density per unit surface area and the energy flow per unit width are given by

$$U = A^2\frac{\epsilon_0\kappa_1^2}{4k^2}\left\{k^2\left(\frac{\epsilon_1}{\kappa_1^3} + \frac{\epsilon_2}{\kappa_2^3}\right) + \frac{e^2 n_0}{\epsilon_0 m}\frac{\omega^2 + \omega_c^2 + \alpha k^2}{[\omega^2 - \omega_c^2 - \alpha k^2]^2}\right\}, \tag{26}$$

$$S_x = A^2\frac{\epsilon_0\omega\kappa_1^2}{2k}\left\{\frac{1}{2}\left(\frac{\epsilon_1}{\kappa_1^3} + \frac{\epsilon_2}{\kappa_2^3}\right) + \frac{e^2 n_0}{\epsilon_0 m}\frac{\alpha}{[\omega^2 - \omega_c^2 - \alpha k^2]^2}\right\}. \tag{27}$$

We note that the energy velocity of the magnetoplasmonic waves v_E is given as the ratio of the sums of the Poynting vectors and the energy densities in the three regions, as

$$v_E(k,\omega) = \frac{S_z}{U}. \tag{28}$$

The expression on the right-hand side of this equation can be written by using Eqs. (26) and (27). In an absence of damping, i.e., for real k, the expression for v_E is in essence the group velocity of the surface waves, defined by

$$v_g(k,\omega) = \frac{d\omega}{dk}. \tag{29}$$

4. Numerical results and discussion

We now present graphic illustrations of the results derived in Sections 2 and 3. The dispersion curves of the magnetoplasmonic waves in terms of the dimensionless variables are presented in Fig. 1, for three different values of ω_c/ω_F with $\omega_F = k_F v_F$, when $\epsilon_1 = \epsilon_2 = 1$. This figure clearly shows a significant contribution of the applied perpendicular magnetic field strength on the plasmon dispersion spectrum. Fig. 2 shows properties of (a) total energy U given by Eqs. (26) and (b) integrated power S_x [Eq. (27)] for three different values of ω_c/ω_F, when $\epsilon_1 = \epsilon_2 = 1$. It can be seen with increasing values of k, the curves decreases sharply that is an expected outcome, because this result corresponds to the result obtained by Nkoma et al [13] for a meta-insulator interface. Furthermore, we observe that total energy increase by increasing the magnetostatic field B_0 in the long-wavelength region, while total power flow vary only weakly with the external magnetic field.

On the other hand, according to Eq. (28), the energy velocity v_E is the ratio of the curves in Fig. 2. This ratio is shown in Fig. 3 in the dimensionless variables for different values of ω_c/ω_F, when $\epsilon_1 = \epsilon_2 = 1$; it is in quantitative agreement with the group velocity found from the corresponding dispersion curve in Fig. 1 by means of the usual formula [Eq. (29)]. This agreement is quite a stringent test of the accuracy of the underlying analytical and numerical work. To show this point analytically, we restrict our attention to the non-retardation limit of equations. Also we put $\epsilon_1 = \epsilon_2 = 1$ and $\alpha = 0$. In this case, from Eqs. (12) and (29) we find $v_g = (\omega^2 - \omega_c^2)/2\omega k$, where $\omega^2 = \omega_c^2 + (e^2 n_0/2\epsilon_0 m)k$. On the other hand, by using $v_E = S_x/U$ in conjunction with Eqs. (26) and (27) we obtain $v_E = (\omega^2 - \omega_c^2)/2\omega k$ which is identical to the group velocity. Also from Fig. 3, we observe a gradual decay of the energy velocity over the entire range of k. In general the magnitude of the energy velocity increases with the lowering of the magnetostatic field B_0 for long-wavelength region.

5. Conclusion

In summary, we have extended the classical Poynting theory for the magnetoplasmonic waves on graphene. We have found the generalized Poynting vector and energy density associated with surface waves of the system, using the linearized MHD theory for electronic excitations in graphene and applying Maxwell equations in conjunction with appropriate boundary conditions. We have found that the presence of a magnetic field causes significant alterations in the energy density and energy transport velocity of magnetoplasmon waves of the system in the long-wavelength region. Also, we have shown that the energy velocity of magnetoplasmonic waves on graphene surface is equal to the group velocity, where this agreement is quite a stringent test of the accuracy of the analytical results. The results presented here may be useful in the current studies of plasmonic properties of graphene and other related structures.

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