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Fluctuations electrical conductivity in a granular s-wave superconductor

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ABSTRACT

The present study tries to evaluate the fluctuation electrical conductivity in a granular s-wave superconductor at the temperature near to the critical temperature. The evaluation is conducted under the condition of limited tunneling conductance between the grains and small impurity concentration. All the first order fluctuation corrections, involving the nonlocal scattered electron in a granular s-wave superconductor, are calculated in three dimensions and in the limit of clean. Using Green's function theory initially, the Cooperon (impurity vertex), $\lambda(\boldsymbol{q}, \varepsilon_1, \varepsilon_2)$, and the fluctuation propagator, $L_k(\boldsymbol{q}, \Omega_k)$, are calculated in the presence of impurities. Then, the three distinct contributions of Aslamazov-Larkin, Maki-Thompson, and Density of states are calculated by means of the Kubo formula. Analysis shows that the terms of Aslamazov-Larkin and anomalous Maki-Thompson have positive contributions to the conductivity in the clean limit, whereas the terms of Density of state and the regular Maki-Thompson have negative signs, leading to the reduction of total fluctuation conductivity.

1. Introduction

Granular superconductors are usually regarded as a random network of superconducting grains coupled by Josephson weak links and grains coated with an insulator layer [1–3]. The granules are large enough to possess a distinct electronic structure, but sufficiently small to be mesoscopic in nature and exhibit effects of quantized electronic level of confined electrons [4–6]. A granular superconductor consists of many superconducting islands, called grains. The contacts where the grains touch each other act as weak links, interconnecting the grains to arrange a complex network. Such a system has the properties of a multiply connected superconductor [7–9]. The Josephson Effect is certainly one of the most intriguing phenomena in superconductivity. It is a result of the coherent tunneling between the two superconducting condensates, each of which is represented by a complex macroscopic wave function, which is the order parameter of superconductivity [1,10]. On a microscopic level, this effect can be defined as the tunneling of Cooper pairs from the pairing state on one side of the junction to that of the other side. In a tunneling process, electrons moving perpendicularly to the interface make the largest contribution. It follows that the power of Josephson tunneling will depend on a weighted average over the pairing wave function, weighted in favor of electronic momenta in this perpendicular direction. Therefore, the Josephson Effect is a direction-sensitive phenomenon connected with the orientation of the junction and with the crystal axis of the superconductor on each side. This fact is of trivial importance for conventional s-wave superconductors with an essentially isotropic pair wave function [2,11]. However, in the case of non-s-wave superconductivity, where pair wave functions contain an internal angular structure, this property can lead to intriguing new effects [12–14].

Each isolated grain is characterized by the mean level spacing δ. Provided the hopping amplitudes are not very large, the macroscopic transport in the system of the granules will be determined by the ratio of hopping amplitudes t to δ. The dimensional conductance for the system of grains is [15]

$$g_T \sim \left(\frac{t}{\delta}\right)^2. \tag{1}$$

It is quite clear that in the limit $t \gg \delta$, the discreteness of the spectrum in a single grain is not perceived, and the electron motion is diffusive through many grains. This limit corresponds to macroscopically weak disorder. In the opposite limit, $t \ll \delta$, electrons are almost completely localized in granules, and this is the strong disorder limit. The metal-insulator transition occurs at values of the macroscopic conductance g_T, the of the order of unity. At such values, the calculations are very difficult. The problem becomes even more complicated because of the coulomb interaction. At small values of g_T, system must be an insulator. However the present study consider the region of large conductance, $g_T \gg 1$, where the system would be a good metal [16]. In this region all localization effects and the Coulomb interaction are allowed to be ignored. The first analysis of superconductivity fluctuation corrections has been carried out on electrical

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conductivity where the pairing leads to three different contributions of the Aslamazov-Larkin (AL), Maki-Thompson (MT), and Density of state (DOS) [17,18]. In the first correction, the formation of cooper pair leads to the parallel superconducting channel in the normal phase, the second considers the coherent scattering of the electrons forming a cooper pair on impurities; and, the third correction is due to the rearrangement of the states close to the Fermi surface, since the electrons involved in the pair transport are no longer available for the single particle transport. Both the AL and MT terms lead to increasing of the conductivity aboveT_c. Moreover, the DOS correction is of opposite sign. These superconducting fluctuations strongly affect both thermodynamic and transport properties. In recent years, they have been greatly studied both theoretically and experimentally [19,20]. In normal metals, in the presence of BCS interaction, electrons can form cooper pairs even for temperatures higher than the critical temperature of T_c. As $T \geq T_c$, the pairs have a finite lifetime, the Ginsburg-Landau (GL) time, inversely proportional to the distance from the critical temperature$\tau_{GL} \sim (T - T_c)^{-1}$. As in conventional (nongranular) bulk superconductors, we can write the superconducting fluctuation corrections to the classical electrical conductivity as a sum of corrections to the DOS, AL, MT, and the weak-localization (WL). The WL correction is of a purely quantum origin. This correction is the regime where interference effects between different plane waves, being treated independently, start to play a role. The interference of plane waves leads to an increase of the probability to find an electron at a certain place, and this effect in turn results in a reduction of the conductivity. In this letter, there is a focus on the nature of the Josephson coupling in granular s-wave superconductors. In contrast to d-wave superconductors, which have a superconducting gap which is anisotropic and is identically zero at four line nodes located at the diagonals of the Brillouin zone, the s-wave superconductors have a superconducting gap which is isotropic in all directions. The s-wave gaps can have variations in magnitude around the Fermi surface or even accidental nodes where the gaps goes to zero, but not in a symmetry-protected way, such that small perturbations can turn the node into a finite gap [21–24]. In this letter, the fluctuation propagator, $L_k(\boldsymbol{q}, \Omega_k)$, and the cooperon,$\lambda(\boldsymbol{q}, \varepsilon_1, \varepsilon_2)$, have been evaluated in the present of impurities in granular s-wave superconductors, using Green's function theory. It is assumed that the momentum of an electron is completely randomized after the tunneling. Also, it is assumed that the sample is a good metal ($g_T \gg 1$). Another assumptions are not being in the Coulomb blockade regime and the long range Coulomb interactions can be safely neglected. In the previous works [9–11], the scattering electron by impurities through the averaged over impurity positions were considered to be in the standard form of one-electron Green's functions. Those work tended to use the local form of the fluctuation propagator and Cooperons. Also previously fluctuations electrical conductivity in granular supercon-ductors is studied in dirty limit in low temperature, and in the presence of magnetic field [14]. The present study focuses on the fluctuation hopping conductivity in granular s-wave superconductors in the clean limit near the critical temperature. Moreover the present study takes into account all fluctuation corrections of the first order in the case of the arbitrary impurity concentrations (note that that impurity concentrations are low so that the localization length is larger than the grain size) involving nonlocal electron scattering in the clean superconductor. The present study is to the region of temperatures near the critical temperature in the absence of magnetic field. The effect of nonlocal on the opposite (low temperatures and strong magnetic field) is under consideration and will be published elsewhere.

2. Formulation

The focus is on a simplified model where superconducting grains are coupled to each other, inserted by an isolator. The grains are not perfect, and there can be impurities both inside the grains and on the surface. Each grain is separated from its nearest neighbors by tunneling barriers. It is assumed that electrons can hop from one grain to another one, and electrons can interact with phonons. In the model proposed by present study the system can be explained by a dimensionless tunneling conductance g_T on a scale much bigger than the typical linear dimension of the grains, a, but smaller than the macroscopic dimension of the entire sample. The Hamiltonian of the system can be written as [19]

$$\hat{H} = \hat{H}_o + \hat{H}_T, \tag{2}$$

where $\hat{H}_o$ is the Hamiltonian of the free electron gas with an electron-phonon interaction:

$$\hat{H}_o = \sum_{i,k} E_{i,k} a^{\dagger}_{i,k} a_{i,k} + \hat{H}_{e-ph}, \tag{3}$$

where $\hat{H}_{e-ph}$ illustrates the electron-phonon interaction on each grain and is given by

$$\hat{H}_{e-ph} = -|\lambda| \sum_{i,k,k'} a^{\dagger}_{i,k} a^{\dagger}_{i,-k} a_{i,-k'} a^{\dagger}_{i,k'}, \tag{4}$$

where i stands for the grains; $k \equiv (\boldsymbol{k}, \uparrow)$, $-k \equiv (-\boldsymbol{k}, \downarrow)$; λ is the interaction constant; $a^{\dagger}_k$, a_k are the fermion creation and annihilation operators. The term $\hat{H}_T$ in Eq. (2) represents the tunneling from one grain to another grain and has the follow from:

$$\hat{H}_T = \Sigma_{(i,j,p,q)} t_{ij} a^{+}_{ip} a_{jq} + H.\,C.\,, \tag{5}$$

where t_{ij} is the tunneling matrix element corresponding to the points of contact of ith and jth grains, and p, q represent the states in the grains. The Kubo formula is used to obtain a term for the hopping conductivity in the granular superconductors. In the Matsubara imaginary time formalism, one can find [20,25]

$$\sigma_{\alpha\beta}(\omega) = \frac{i}{\omega}[\Pi_{\alpha\beta}(\omega)], \tag{6}$$

The electromagnetic response operator $\Pi_{\alpha\beta}(\omega)$, that is defined on the Matsubara frequencies, can exhibit the correlator of two one-electron Green's function averaged over impurity positions, accounting for interactions, here, the particle-particle interaction in the cooper channel. The Scattering of electrons inside the grains by impurities is occurs in the Born approximation, giving rise to a scattering mean free time τ. With taking into account of magnetic field $\boldsymbol{A}(\boldsymbol{r})$, the Green's functions change and include the effects of orbital quantization. However, in the presence of strong disorder $\omega_c \tau \ll 1$ (ω_c is the cyclotron frequency) or at relatively high temperatures, the discrete Landau levels are smeared out and the effects of the magnetic field can be treated semicla-ssically. This means that the Green's function in the coordinate representation has the following form

$$\Im(i\varepsilon_n, \boldsymbol{r}, \boldsymbol{r}') = \Im^{(0)}(i\varepsilon_n, \boldsymbol{r} - \boldsymbol{r}') exp\left(\frac{ie}{c}\int_{\boldsymbol{r}}^{\boldsymbol{r}'} \boldsymbol{A} d\boldsymbol{r}\right), \tag{7}$$

where $\varepsilon_n = \pi T(2n+1)$ is the fermionic Matsubara frequency, and $\Im^{(0)}(i\varepsilon_n, \boldsymbol{r} - \boldsymbol{r}')$ is the Green's function in the absence of magnetic field. In the momentum representation, it can be written as

$$\Im^{(0)}(i\varepsilon_n, \boldsymbol{p}) = \frac{1}{i\varepsilon_n - \xi(\boldsymbol{p}) - iIm\Sigma(i\varepsilon_n, \boldsymbol{p})}, \tag{8}$$

where the self-energy $\Sigma(i\varepsilon_n, \boldsymbol{p})$ is given by [26]:

$$\Sigma(i\varepsilon_n, \boldsymbol{p}) = -T \sum_{q,\Omega_k} \Im^0(\boldsymbol{p} - \boldsymbol{q}, i\Omega_k - i\varepsilon_n) L(\boldsymbol{q}, i\Omega_k), \tag{9}$$

where $\Omega_k = 2\pi Tk$ is the bosonic Matsubara's frequency reflecting the bosonic nature of the cooper pairs, and $\xi(\boldsymbol{p}) = p^2/2m - \mu$, μ is the Fermi level.

Here in the present study, Cooperon, $\lambda(\boldsymbol{q}, \varepsilon_1, \varepsilon_2)$, and the fluctuation propagator, $L_k(\boldsymbol{q}, \Omega_k)$, is evaluated in presence of dilute impurities.

The propagator is generally defined by means of diagrams in Fig. 1 [27]. All these diagrams can be summed up as shown in the bottom line, allowing to write a Dyson's equation for the fluctuation propagator

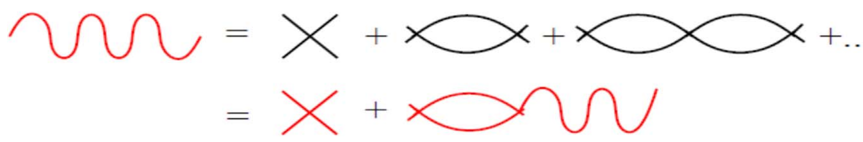

Fig. 1. The cooper pair fluctuation propagator without tunneling. The first diagram in the right-hand side on the top line represents the BCS electron-electron interaction, and it is expressed by a constant $g > 0$. The second and third diagrams take into account the corrections induced by the fluctuations. The diagrams on the bottom line graphically represent Dyson's equation for the fluctuation propagator, Eq. (10).

as

$$L^{-1}(\boldsymbol{q},\ \Omega_\mu)=g^{-1}-P(\boldsymbol{q},\Omega_k), \tag{10}$$

where the polarization $P(\boldsymbol{q},\Omega_k)$ is expressed as a loop of two single-electron Green's functions:

$$P(q,\ \Omega_k)=T\sum\nolimits_{\varepsilon_n}\int\frac{d^3p}{(2\pi)^3}\Im(\boldsymbol{p}+\boldsymbol{q},\ \varepsilon_{n+k})\Im(-\boldsymbol{p},\ \varepsilon_{-n}), \tag{11}$$

where $\boldsymbol{q}$ is the momentum of the pair, and $\boldsymbol{p}$ is the momentum of the electron. Propagator (9) can be evaluated by means of the one – electron Green's functions Eq. (8). The correlator of two one-electron Green's functions is illustrated as follow:

$$\Xi(\boldsymbol{q},\ \varepsilon_1,\ \varepsilon_2)=\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}\Im(\boldsymbol{p}+\boldsymbol{q},\ \varepsilon_1)\Im(-\boldsymbol{p},\ \varepsilon_2)$$

$$=2\pi N(0)\Theta(-\varepsilon_1\varepsilon_2)\left\langle\frac{1}{|\tilde{\varepsilon}_1-\tilde{\varepsilon}_2|+isgn(\varepsilon_1)\Delta\xi(\boldsymbol{p},\ \boldsymbol{q})\ |_{|\boldsymbol{p}|=P_F}}\right\rangle_{F.S.}, \tag{12}$$

where $\Theta(x)$ is the Heaviside step-function, $\tilde{\varepsilon}_1=\varepsilon_1+(1/2\tau)sgn(\varepsilon_1)$, $\tilde{\varepsilon}_2=\varepsilon_2+(1/2\tau)sgn(\varepsilon_2)$, and N(0) is the one- electron Density of states on the Fermi surface, $\langle\ldots\rangle_{F.S.}=\int\frac{d\Omega_p}{4\pi}$ indicate the averaging over the Fermi surface and

$$\Delta\xi(\boldsymbol{p},\ \boldsymbol{q})|_{|P|=P_F}=[\xi(\boldsymbol{q}+\boldsymbol{p})-\xi(-\boldsymbol{p})]|_{|P|=P_F}\approx(\boldsymbol{v}_p\boldsymbol{q})_{\xi(p)=0}. \tag{13}$$

The last approximation is valid as it is not too far from the Fermi Surface, i.e. when $(\boldsymbol{v}_p\boldsymbol{q})_{\xi(p)=0}\ll E_F$. It is impossible to carry out the angular averaging in Eq. (12) for a general anisotropic spectrum. Nevertheless, in the following calculation of fluctuation effects in the vicinity of the critical temperature, only small momenta $\boldsymbol{v}_p\boldsymbol{q}\ll T$ will be involved in the integrations. Here, therefore, only this region can be considered, and the integrand can be expanded in powers of $\boldsymbol{v}_p\boldsymbol{q}$. The first term in $\boldsymbol{v}_p\boldsymbol{q}$ will apparently be averaged out, so with a quadratic accuracy, one can find:

$$\Xi(\boldsymbol{q},\ \varepsilon_1,\ \varepsilon_2)=2\pi N(0)\frac{\theta(-\varepsilon_1,\ \varepsilon_2)}{|\varepsilon_1-\varepsilon_2|}\left(1-2\frac{<(\boldsymbol{v}_p\boldsymbol{q})^2>_{F.S.}}{|\varepsilon_1-\varepsilon_2|^2}\right). \tag{14}$$

Now, the polarization operator can be calculated:

$$P(\boldsymbol{q},\ \Omega_k)=T\sum_{\varepsilon_n}\Xi(\boldsymbol{q},\varepsilon_{n+k},\varepsilon_{-n})=N(0)\left[\sum\nolimits_{n\geq0}\frac{1}{n+1/2+\frac{|\Omega_k|}{4\pi T}}-\frac{<(\boldsymbol{v}_p\boldsymbol{q})^2>_{F.S.}}{(4\pi T)^2}\times\right.$$

$$\left.\sum\nolimits_{n=0}^{\infty}\frac{1}{(n+1/2+\frac{|\Omega_k|}{4\pi T})^3}+\eta(\boldsymbol{q})\right], \tag{15}$$

where, $\eta(\mathbf{q})=\frac{2}{\pi}g_T\delta\sum_{i=1}^{3}(1-cosq_id)$, $\mathbf{q}$ is the quasi- momen-tum, $d=2R$ is distance of the center of neighboring grain, R is the radius of the grain, and g_T is the dimensionless conductance that describes the tunneling of electrons from one grain to another grain. The calculation of the sums in (15) can be done in terms of the logarithmic derivations of the Γ-function $\psi^n(x)$. It is worth mentioning that the first sum is well-known in the BCS theory, and one can identify in the so-called " Cooper logarithm", its logarithmic divergence at the upper limit $(\psi(x\gg1)\cong lnx)$ is cut off by the Debye energy ($N_{max}=\frac{\omega_D}{2\pi T}$), and we have:

$$\frac{1}{N(0)}P(\boldsymbol{q},\ \Omega_k)=\psi\left(1/2+\frac{|\Omega_k|}{4\pi T}+\frac{\omega_D}{2\pi T}\right)-\psi\left(1/2+\frac{|\Omega_k|}{4\pi T}\right)$$

$$-\frac{<(\boldsymbol{v}_p\boldsymbol{q})^2>_{F.S.}}{2(4\pi T)^2}\psi''\left(1/2+\frac{|\Omega_k|}{4\pi T}\right)+\eta(\boldsymbol{q}). \tag{16}$$

The critical temperature in the BCS theory is determined as the temperature T_c at which the pole of $L(0,0,T_c)$ takes place:

$$L^{-1}(q=0,\Omega_k=0,T_c)=g^{-1}-P(0,\ 0,T_c)=0,$$

$$T_c=\frac{2\gamma_E}{\pi}E_F exp\left(-\frac{1}{N(0)g}\right), \tag{17}$$

where $\gamma_E=1.78$ is the Euler constant. From the condition $\frac{1}{\epsilon N(0)}P(0,0)=(gN(0))^{-1}$ for the system under consideration, and by showing the reduced temperature $\epsilon=\ln(T/T_c)$, from Eq. (10), one can write the propagator as

$$L^{-1}(\boldsymbol{q},\ \Omega_\mu)=-N(0)\left[\epsilon+\psi\left(\frac{1}{2}+\frac{|\Omega_k|}{4\pi T}\right)-\psi\left(\frac{1}{2}\right)\right.$$

$$\left.-\frac{\langle(\boldsymbol{v}_p\boldsymbol{q})^2\rangle_{F.S}}{2(4\pi T)^2}\psi''\left(\frac{1}{2}+\frac{|\Omega_k|}{4\pi T}\right)+\eta(q)\right]. \tag{18}$$

The propagator of superconducting fluctuations in Eq. (18) is given by the sum of all diagrams with two incoming and two outgoing lines showing in Fig. 2 [27].

Another effect of the coherent scattering on the identical impurity by both electrons forming a cooper pair is the renormalization of the vertex part $\lambda(\boldsymbol{q},\ \varepsilon_1,\ \varepsilon_2)$ in the particle- particle channel. The renormalized vertex $\lambda(\boldsymbol{q},\ \varepsilon_1,\ \varepsilon_2)$ is specified by a graphical equation of the ladder type (see Fig. 3) [28].

$$\lambda^{-1}(\boldsymbol{q},\ \varepsilon_1,\ \varepsilon_2)=1-\frac{1}{2\pi N(0)\tau}\Xi(\boldsymbol{q},\ \tilde{\varepsilon}_1,\ \tilde{\varepsilon}_2), \tag{19}$$

where $\Xi(\boldsymbol{q},\ \tilde{\varepsilon}_1,\ \tilde{\varepsilon}_2)$ was defined by Eq. (14).

The proper exact expression for Cooperon in the proposed granular system can be written as

$$\lambda(\boldsymbol{q},\ \varepsilon_n,\ \Omega_k-\varepsilon_n)=2\pi\tau N(0)\frac{|\frac{1}{\tau}-\Omega_k+2\varepsilon_n|}{|2\varepsilon_n-\Omega_k|+\frac{\left[(\eta(q))^2+\frac{(v_f q)^2}{3}\right]^{1/2}}{2\tau\left|\frac{1}{\tau}-\Omega_k+2\varepsilon_n\right|^2}}. \tag{20}$$

The total superconducting fluctuation conductivity is given by [28]

$$\sigma=\sigma^{(AL)}+\sigma^{(Dos)}+\sigma^{(MT)}. \tag{21}$$

All the diagrams indicating the conductivity of the granular metal are shown in Fig. 4. The first diagram in Fig. 4 represents the correction to the electrical conductivity as result of the AL contribution. The electromagnetic response operator tensor for the AL contribution can be written as

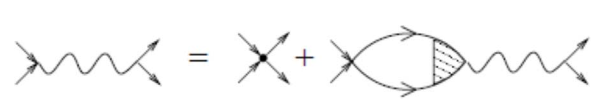

Fig. 2. The Dyson's equation for the propagator of the superconducting fluctuations in the ladder approximation. The black point shows the coupling constant, and the shaded three-point vertex represents the renormalized impurity vertex.

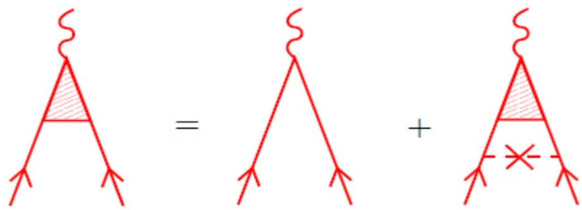

Fig. 3. The Cooperon correction. The possibility of coherent scattering by both the electrons forming the cooper pair of the identical impurities. Solid lines are one-electron Green's functions, while the dashed lines show the scattering by both electrons on the identical impurity represented by the cross.

$$\Pi^{AL}_{\alpha\beta}(\varepsilon_\nu)=-4e^2T\sum\nolimits_{\Omega_k}\int\frac{d^3\boldsymbol{q}}{(2\pi)^3}B_\alpha(\boldsymbol{q},\,\Omega_k,\,\varepsilon_\nu)L(\boldsymbol{q},\Omega_k)$$

$$\times B_\beta(\boldsymbol{q},\,\Omega_k,\,\varepsilon_\nu)L(\boldsymbol{q},\Omega_k+\varepsilon_\nu), \tag{22}$$

where $B_\alpha(\boldsymbol{q},\,\Omega_k,\,\varepsilon_n)$ describes the block of Green's functions and is given by

$$B_\alpha(\boldsymbol{q},\,\Omega_k,\,\varepsilon_n)=T\sum\nolimits_{\varepsilon_m}\lambda(\boldsymbol{q},\,\varepsilon_n+\varepsilon_m,\,\Omega_k-\varepsilon_n)\lambda(\boldsymbol{q},\,\varepsilon_n,\,\Omega_k-\varepsilon_n)$$

$$\times\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}v_\alpha(\boldsymbol{p})\Im(\boldsymbol{p},\varepsilon_n+\varepsilon_m)\Im(\boldsymbol{p},\varepsilon_n+\varepsilon_m))$$

$$\times\Im(\boldsymbol{p},\varepsilon_n)\Im(\boldsymbol{q}-\boldsymbol{p},\,\Omega_k-\varepsilon_n). \tag{23}$$

By expanding $\Im(\boldsymbol{q}-\boldsymbol{p},\,\Omega_k-\varepsilon_n)$ over $\boldsymbol{q}$, one can find that the angular integration over the Fermi surface cancels the first term and only allow to remain the second term of the expansion nonzero. Therefore, one can write Eq. (23) in the following form:

$$B_\alpha(\boldsymbol{q},\,\Omega_k,\,\varepsilon_n)=-N(0)\frac{\gamma}{v_f^2}\langle v_\alpha q_\beta v_\beta\rangle_{F.S}$$

$$\frac{8\pi}{\pi\varepsilon_n}\left[\psi\left(\frac{1}{2}+\frac{|\Omega_k|+\varepsilon_n}{4\pi T}\right)\psi\left(\frac{1}{2}+\frac{|\Omega_k|}{4\pi T}\right)\psi\left(\frac{1}{2}+\frac{|\Omega_{k+n}|+\varepsilon_n}{4\pi T}\right)\right.$$
$$\left.\psi\left(\frac{1}{2}+\frac{|\Omega_{k+n}|}{4\pi T}\right)\right], \tag{24}$$

where

$$\gamma=-\frac{v_f^2\tau^2}{3}\left[\psi\left(\frac{1}{2}+\frac{1}{4\pi T\tau}\right)-\psi\left(\frac{1}{2}\right)-\frac{1}{4\pi T\tau}\psi'\left(\frac{1}{2}\right)\right], \tag{25}$$

where ψ and ψ' are the digamma function and its derivative, respectively. Inserting Eq. (24) into Eq. (22), one can obtain

$$\sigma^{AL}=\frac{e^2\pi^2\nu^2}{d}\int\frac{d^3\vec{q}}{(2\pi)^3}\frac{\boldsymbol{q}^2}{[(\gamma\boldsymbol{q}^2+\epsilon)(\gamma\boldsymbol{q}^2+\epsilon+r]^{3/2}}$$

$$=\frac{e^2}{16d}\frac{1}{[\epsilon(\epsilon+r)]^{1/2}}, \tag{26}$$

where

$$r=4\gamma\frac{\delta^2g_T^2}{v_f^2}. \tag{27}$$

This result has been obtained with only the assumption $\epsilon\ll1$, so it is acceptable for any frequency, any impurity concentra-tion, and any dimension of the fluctuation behavior. One can write Eq. (26) in the following form:

$$\sigma^{AL}=\frac{3e^2}{16\pi\tau^2v_f^2\epsilon^{1/2}}\frac{1}{[\psi(\frac{1}{2}+\frac{1}{4\pi T\tau})-\psi(\frac{1}{2})-\frac{1}{4\pi T\tau}\psi(\frac{1}{2})]}. \tag{28}$$

For clean limit, $T\tau\gg1$, one can write

$$\sigma^{AL}=\frac{3\pi e^2T^2}{7\xi(3)v_f^2\epsilon^{1/2}}. \tag{29}$$

With respect to the classical conductivity of the granular metal, σ^0, one can write

$$\frac{\sigma^{AL}}{\sigma^0}=\frac{3g_T}{28}\frac{\delta^2dT^2}{\xi(3)(tv_f)^2\epsilon^{1/2}}. \tag{30}$$

It can be seen that σ^{AL} represents a positive contribution to the fluctuation conductivity and is proportional to T^2. This means that, at sufficiently low temperatures, this correction is small and has a considerable value only near the transition temperature. The correction of electrical conductivity due to suppression of the DOS can be considered according to diagrams 5–10 of Fig. 4. The DOS contribution appears from corrections to the DOS on account of fluctuations of the normal quasiparticles to the superconducting state. In the dirty limit, the calculation of the contributions to the longitudinal fluctuation conductivity from such diagrams had been considered in previous studies. [27,28]. Diagrams 9 and 10 in $\epsilon\ll1$ are not singular at all and can be ignored. In the clean limit, the chief contributions from the DOS fluctuations can be seen in diagrams 5 and 6.

The DOS contribution to the electromagnetic response operator tensor $\Pi^{Dos}_{\alpha\beta}(\varepsilon_n)$ due to diagram 5 can be written as

$$\Pi^{Dos}_{\alpha\beta}(\varepsilon_n)=2e^2T\sum_{\Omega_k}\int\frac{d^3\boldsymbol{q}}{(2\pi)^3}L(\boldsymbol{q},\,\Omega_k)$$

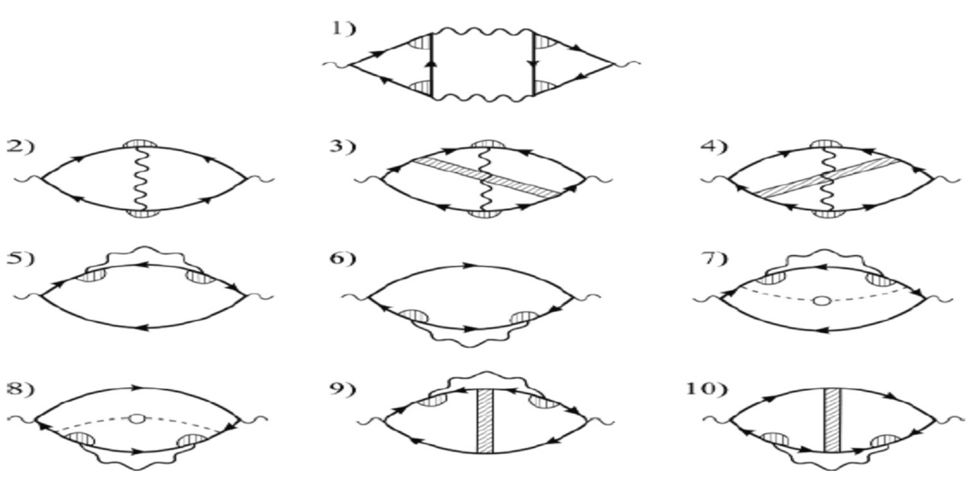

Fig. 4. Diagrams for the heading order contribution to the fluctuation conductivity of granular metals. Wavy lines represent the propagator of the superconducting fluctuations, thin solid lines with arrows show the normal state Green's function averaged over impurity positions, and shaded semicircles are vertex corrections originating from impurities. Dashed lines with central crosses are additional impurity renormalizations, and shaded blocks show impurity ladders. Diagram 1 is the Aslamazov-Larkin (AL) contribution, and diagram 2 is the Maki-Thompson (MT). 5, 6, 7 and 8 show the density of states (DOS) diagrams. Diagrams 3, 4 and 9, 10 originate when we average the DOS and MT diagrams over impurities.

$$\times T\sum_{\varepsilon_m}\lambda^2(\boldsymbol{q},\ \varepsilon_n,\Omega_k-\varepsilon_n)I^{Dos}_{\alpha\beta}(\Omega_k,\ \varepsilon_m+\varepsilon_n),\qquad(31)$$

where

$$I^{Dos}_{\alpha\beta}(\Omega_k,\ \varepsilon_m+\varepsilon_n)=\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}v_\alpha(\boldsymbol{p})v_\beta(\vec{p})\Im^2(\boldsymbol{p},\ \varepsilon_m)$$

$$\times\Im(\boldsymbol{q}-\boldsymbol{p},\ \Omega_k-\varepsilon_m)\Im(\boldsymbol{p},\ \varepsilon_n+\varepsilon_m).\qquad(32)$$

Diagram 6, in turn, gives an identical contribution. One can select the propagator frequency Ω_k=0, close to the critical temperature, even for the case of an arbitrary external frequency. The DOS contribution to the fluctuation conductivity with respect to Diagrams 5 and 6 is given by

$$\sigma^{5+6}_{\alpha\beta}=-\frac{\pi e^2}{2d}F_1\gamma\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}\frac{1}{[(\gamma\vec{q}^{\,2}+\epsilon)(\gamma q^2+\epsilon+r]^{\frac{1}{2}}}$$

$$\approx-\frac{F_1e^2}{8d}ln\left(\frac{2}{\sqrt{\epsilon}+\sqrt{\epsilon+r}}\right),\qquad(33)$$

where

$$F_1=\frac{(v_f\tau)^2}{\gamma\pi^2}\left[\psi'\left(\frac{1}{2}+\frac{1}{4\pi T\tau}\right)-\frac{3}{4\pi T\tau}\psi''\left(\frac{1}{2}\right)\right].\qquad(34)$$

In a similar manner, the equal contribution from Diagrams 7 and 8 can be given as

$$\sigma^{7+8}_{\alpha\beta}=-\frac{\pi e^2}{2d}F_2\gamma\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}\frac{1}{[(\gamma\boldsymbol{q}^2+\epsilon)(\gamma\boldsymbol{q}^2+\epsilon+r]^{\frac{1}{2}}}$$

$$\approx-\frac{F_2e^2}{8d}ln\left(\frac{2}{\sqrt{\epsilon}+\sqrt{\epsilon+r}}\right),\qquad(35)$$

where

$$F_2=\frac{(v_f\tau)^2}{2\pi^3T\tau}\psi''\left(\frac{1}{2}\right).\qquad(36)$$

The contribution of other diagrams (9 and 10 of Fig. 4) can be treated in a similar manner. From Eqs. (32) and (31), and given Eqs. (6), (33), and (35), the total DOS contribution to fluctuation conductivity is given by

$$\sigma^{Dos}_{\alpha\beta}=-\frac{e^2}{2d}F(T\tau)ln\left(\frac{2}{\sqrt{\epsilon}+\sqrt{\epsilon+r}}\right).\qquad(37)$$

where

$$F(T\tau)=F_1+F_2=\frac{-\psi'(\frac{1}{2}+\frac{1}{4\pi T\tau})+\frac{1}{2\pi T\tau}\psi''(\frac{1}{2})}{\pi^2[\psi(\frac{1}{2}+\frac{1}{4\pi T\tau})-\psi(\frac{1}{2})-\frac{1}{4\pi T\tau}\psi'(\frac{1}{2})]}.\qquad(38)$$

In the clean limit, Eq. (38) has the following form:

$$F(T\tau)=\frac{8\pi^2(T\tau)^2}{[7\xi(3)]}\simeq 9.384(T\tau)^2,\qquad(39)$$

where $F(T\tau)$ is a function of $T\tau$ only. As will be shown below at the upper limit $T\tau\sim 1/\sqrt{\epsilon}$, the DOS contribution reaches the value of the other fluctuation contributions, and in the limit of $T\tau\to\infty$ precisely remove the MT one. The DOS contribution to the fluctuation conductivity can be written as

$$\frac{\sigma^{(Dos)}}{\sigma_0}=-\frac{9.384(T\tau)^2}{32\pi^6}\left(\frac{d\delta^2g_T}{tv_f}\right)^2\psi''\left(\frac{1}{2}\right)ln\left[\frac{24T}{\delta^2\tau g_T^2}\right].\qquad(40)$$

It can be seen that the DOS gives a negative contribution to the fluctuation conductivity. Another term, which usually increases the fluctuation conductivity, is the MT contribution, shown in Diagram 2 of Fig. 4. The contributions from Diagrams 3 and 4 of Fig. 4 of the MT type are insignificant, because they are less singular inϵ. The electromagnetic response operator tensor for the MT contribution to fluctuation conductivity is given by

$$\Pi^{MT}_{\alpha\beta}(\varepsilon_n)=2e^2T\sum_{\Omega_k}\int\frac{d^3\boldsymbol{q}}{(2\pi)^3}L(\vec{q},\ \Omega_k)B_{\alpha\beta}(\boldsymbol{q},\ \Omega_k,\ \varepsilon_n),\qquad(41)$$

where

$$B_{\alpha\beta}(\boldsymbol{q},\ \Omega_k,\ \varepsilon_n)=T\sum_{\varepsilon_m}\lambda(\boldsymbol{q},\ \varepsilon_n+\varepsilon_m,\ \Omega_{k-n-m})\lambda(\boldsymbol{q},\ \varepsilon_n,\ \Omega_{k-m})$$

$$\times\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}v_\alpha(\boldsymbol{p})v_\beta(\boldsymbol{q}-\boldsymbol{p})\Im(\vec{p},\ \varepsilon_{n+m})\Im(\boldsymbol{p},\ \varepsilon_m)$$

$$\times\Im(\boldsymbol{q}-\boldsymbol{p},\ \Omega_{k-m-n})\Im(\boldsymbol{q}-\boldsymbol{p},\ \Omega_{k-m}).\qquad(42)$$

In calculating the sum over the Matsubara frequency ε_m in (42), it is useful to break up the sum into two parts. In the first part, ε_m belongs to the domains $[-\infty,\varepsilon_n]$ and $[0,\ \infty]$, which finally yield two equal contributions. This leads to the regular part of the MT diagram. The second anomalous part of the MT diagram originates from the summation over ε_n in the domain $[-\varepsilon_n,0]$. By using this, one can carry out the sum over the ε_m and after integration over the momenta, one can write the function B as a sum of an anomalous B^{an} and a regular B^{reg} contribution to the MT diagram. It is assumed that the scattering lifetime τ and the pair-breaking lifetime τ_ϕ are arbitrary, but satisfying $\tau_\phi>\tau$. Close to the critical temperature, it is possible to restrict the assumption to the static limit of the MT diagram, by setting Ω_k=0 in Eq. (42), one can write

$$B_{\alpha\beta}(q,0,\varepsilon_n)=B^{an}_{\alpha\beta}(q,\ \varepsilon_n)+B^{reg}_{\alpha\beta}(q,\ \varepsilon_n)$$

$$=\frac{-nv_f^2}{2}\left[2\pi T\sum_{m=0}^{\infty}\frac{1}{(2\varepsilon_{m+n}+\chi)}\frac{1}{(2\varepsilon_m+\chi)}\frac{1}{(2\varepsilon_{m+n}+\varepsilon_n+\tau^{-1})}\right]$$

$$\times\left[\frac{\pi T}{\varepsilon_n+\tau^{-1}}\sum\nolimits_{m=-n}^{-1}\frac{1}{2\varepsilon_{m+n}+\chi}\frac{1}{-2\varepsilon_m+\chi}\right],\qquad(43)$$

where

$$\chi=\frac{v_f^2q^2}{3T}\left(\frac{1+2T\tau}{T\tau}\right)^{-1}.\qquad(44)$$

The anomalous part of the MT diagram can be written as

$$\sigma^{MT(an)}=8e^2\gamma T\int\frac{d^3q}{(2\pi)^3}\frac{Cosqd}{\left[\frac{1}{\tau_\phi}+\chi\right][\epsilon+\gamma q^2+\frac{r}{2}(1-Cosqd)]}\qquad.(45)$$

Eq. (45) is acceptable for the arbitrary value of $T\tau$. After doing the integration over in Eq. (45), one can obtain

$$\sigma^{MT(an)}=\frac{e^2}{4d(\epsilon+Y_\phi)}ln\left(\frac{\sqrt{\epsilon}+\sqrt{\epsilon+r}}{\sqrt{Y_\phi}+\sqrt{Y_\phi+r}}\right),\qquad(46)$$

where

$$Y_\phi=\frac{2\gamma}{v_f^2\tau\tau_\phi}\qquad(47)$$

For the clean limit $T\tau>1$, we have

$$\frac{\sigma^{MT(an)}}{\sigma_0}=\frac{\sqrt{3}d^2\delta^3g_TT}{(2\pi tv_f)^2\left(\frac{\tau}{\tau_\phi}\right)^{1/2}}.\qquad(48)$$

For the regular part of the MT, the fluctuation conductivity is given by

$$\sigma^{MT(reg)}=-\frac{e^2d^2\pi rK(T\tau)}{4}\int\frac{d^3q}{(2\pi)^3}\frac{Cosqd}{\epsilon+\gamma q^2+\frac{r}{2}(1-Cosqd)}.\qquad(49)$$

After doing the integration over Eq. (49), we have

$$\sigma^{MT\,(reg)} = -\frac{e^2}{2d} K(T\tau) ln \left(\frac{2}{\epsilon^{1/2} + (\epsilon + r)^{1/2}} \right), \tag{50}$$

where

$$K(T\tau) = \frac{-\psi\left(\frac{1}{2} + \frac{1}{4\pi\tau T}\right) + \psi'\left(\frac{1}{2}\right) + \frac{1}{4\pi\tau T}\psi''\left(\frac{1}{2}\right)}{\pi^2\left[\psi\left(\frac{1}{2} + \frac{1}{4\pi T\tau}\right) - \psi\left(\frac{1}{2}\right) - \frac{1}{4\pi T\tau}\psi'\left(\frac{1}{2}\right)\right]}. \tag{51}$$

For the clean limit

$$K(T\tau) = \frac{8}{\pi} T\tau = 2.\ 547(T\tau), \tag{52}$$

where Eq. (52) is only a function of $T\tau$. As it can be seen, the regular MT term is negative, as is the overal DOS contribution. Therefore, the regular part of the MT can be written as

$$\sigma^{MT\,(reg)} = -\frac{\delta^2 T\tau}{\pi^2} ln \left(\frac{2}{\varepsilon^{\frac{1}{2}} + (\varepsilon + r)^{\frac{1}{2}}} \right). \tag{53}$$

With respect to the classical conductivity of the granular metal, σ^0, one can write

$$\frac{\sigma^{MT\,(reg)}}{\sigma_0} = -\frac{2.\ 547T\tau}{32\pi^6} \left(\frac{d\delta^2 g_T}{tv_f} \right)^2 \psi''\left(\frac{1}{2}\right) ln \left[\frac{24T}{\delta^2 \tau g_T^2} \right]^{1/2}. \tag{54}$$

This term is smaller in magnitude than the DOS term, and therefore, it makes a relatively small contribution to the total function conductivity. It is usually a good approximation to ignore the regular term. From Eqs. (21), (30), (40), (48) and (54), one can drive the total electrical conductivity in granular supercon-ductors.

3. Conclusions

The fluctuation corrections of superconductivity were analytically studied by Feynman diagrams. This study calculates all of the fluctuation corrections of the first order in the case of small impurity concentration involving nonlocal electron scattering in the clean s-wave superconductor near the critical temperature. This study is restricted to the limit of large tunneling conductance in a way that the weak localization and charging effects have been neglected in the limit. By using Green's function method, Cooperon, and the fluctuation propagator have been calculated in the presence of dilute impurities. It has been demonstrated that the term σ^{AL} has a positive contribution to the fluctuation conductivity and is proportional to T^2. This means that, at sufficiently low temperatures, this correction is small and has a considerable value only near the transition temperature. It has been shown that the terms σ^{Dos} and σ_{reg}^{MT} give negative contributions to the fluctuation conductivity, whereas the terms σ^{AL} and σ_{an}^{MT} give a positive contribution to the conductivity. This leads to a competition between the positive and negative contributions.

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