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# The out of plane a.c. fluctuation conductivity of superconductor under magnetic field

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ABSTRACT

The time-dependent Ginzburg-Landau model with thermal noise is used to calculate the out of plane a.c. fluctuation conductivity of type-II superconductor under magnetic field. We assume that thermal fluctuations, represented by the Langevin white noise, are strong enough to melt the Abrikosov vortex lattice created by the magnetic field into a moving vortex liquid. The nonlinear interaction term in the time-dependent Ginzburg-Landau equation is treated within the self-consistent Gaussian approximation. The real and imaginary parts of the out of plane a.c. fluctuation conductivity formula are summation over all Landau levels to treat arbitrary magnetic fields. Our results indicate that high frequencies can be effectively used to suppress the out-of-plane a.c. fluctuation conductivity in high-temperature superconductors. The results also have refined analytical form and are fitted well to experimental data on high-$T_c$ superconductor. The results in two-dimensional and three-dimensional limits are directly inferred from the corresponding result of the layered model.

## 1. Introduction

After the discovery of high-$T_c$ superconductors (HTSC), effects of fluctuations on transport properties have been interesting subject for many years. The fluctuations in these compounds were more pronounced than in the classical low-temperature superconductors due to short coherence lengths and high transition temperatures, but anisotropy and the degree of doping may also play a role [1,2]. It was found that high-$T_c$ superconductors could give experimental access to a study of critical fluctuations [3]. The penetration depth measurements in $YBa_2Cu_3O_{7-\delta}$ revealed critical behavior as wide as $\pm 10$ K from $T_c$ [4]. This wide range of critical fluctuations was confirmed by measurements of thermal expansivity [5,6] and two-coil inductive measurements [7], while d.c. fluctuation conductivity measurements still claimed very narrow critical regions [8,9]. A study of the microwave fluctuation conductivity yielded two experimental curves, one for the real part $\sigma_1(T)$, and the other for the imaginary part $\sigma_2(T)$, with different shapes but ensuing from the same physics. Advantage of the a.c. fluctuation conductivity is that the imaginary part $\sigma_2(T)$ has no contribution from the normal electrons so that its analysis is free from subtraction problems often encountered in d.c. conductivity studies.

The out of plane and in-plane d.c. fluctuation conductivity were calculated in the framework of the microscopic (BCS) theory by Aslamasov and Larkin [10,11], the approach fast becomes too cumbersome in more complicated situations involving external magnetic field, layered structure, etc. and a more phenomenological Ginzburg–Landau approach is more effective. The expressions for the out of plane and in-plane a.c. fluctuation conductivity in the normal phase in zero magnetic field in the Gaussian fluctuations regime have been very early obtained within the time-dependent Ginzburg–Landau (TDGL) equation theory [12,13]. If one only use the Gaussian fluctuations to calculate the properties of a type-II superconductor, one would predict similar nonanalytic behavior in the thermodynamic and transport properties, in conflict with the experimental results [14]. Fluctuations become more important in HTSC. Therefore, one expects that interactions between the fluctuations are important near $T_c(H)$, and that these interactions remove the nonanalyticities present in zero magnetic field. Calculations of the specific heat and the transport properties of a superconductor in magnetic field [15,16] which treat the interaction terms within the Hartree approximation, find that the specific heat is smooth through the mean-field transition temperature, in accordance with the above expectations.

In this paper we calculate the out of plane a.c. fluctuation conductivity of type-II superconductor under magnetic field by using TDGL approach with thermal fluctuations conveniently modeled by the Langevin white noise. The interaction term in TDGL equation is treated in self-consistent Gaussian approximation which is similar in structure to the Hartree approximation [11,14,17,18]. The self-consistent

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Gaussian approximation used in this paper is consistent to leading order with perturbation theory, see Ref. [19] in which it is shown that dependence on the ultraviolet (UV) cutoff is the same. One can use Hartree procedure only when UV issues are unimportant. The main results of our work are that the real and imaginary parts of the out of plane a.c. fluctuation conductivity formula are summation over all Landau levels and independent of UV cutoff as it should be as the standard $|\Psi|^4$ theory is renormalizable.

The paper is organized as follows. The model is defined in Section 2. The vortex liquid within the self-consistent Gaussian approximation is described in Section 3. The out of plane a.c. fluctuation conductivity is calculated in Section 4. The comparison with experiment are described in Section 5 while Section 6 contains conclusions.

## 2. The time dependent GL Lawrence-Doniach model

The layered structure of HTSC is described by using the Lawrence-Doniach model which consists of superconducting planes separated by a distance $d'$, with a Josephson coupling between the planes. The free energy is

$$F_{GL} = s'\sum_l \int d^2r\left\{\frac{\hbar^2}{2m_{ab}}|\mathbf{D}\Psi_l|^2 + \frac{\hbar^2}{2m_c d'^2}|\Psi_l-\Psi_{l+1}|^2 + a|\Psi_l|^2 + \frac{b'}{2}|\Psi_l|^4\right\}. \tag{1}$$

Here $s'$ is the (effective) layer thickness, $m_{ab}$ and $m_c$ are effective Cooper pair masses in the ab plane and along the c axis, respectively. It is simple to assume that $a = a_0 T_c^{mf}(t-1)$, $t \equiv T/T_c^{mf}$. Note that the "mean field" critical temperature $T_c{}^{mf}$ is higher than real transition temperature $T_c$ due to strong thermal fluctuations on the mesoscopic scale and depends on UV cutoff, $\tau_c$, specified later. The covariant derivative is defined by $\mathbf{D} \equiv \nabla + i(2\pi/\Phi_0)\mathbf{A}$ and $\mathbf{A}$ is the vector potential describes constant and homogeneous magnetic field which is choosen in the Landau gauge $\mathbf{A} = (-By, 0)$ and $\Phi_0 = hc/e^*$ is the flux quantum with $e^* = 2|e|$.

It is necessary to introduce the equation of motion for the order parameter; the simplest is to use the Langevin approach to the gauge-invariant relaxational TDGL equation [11,14,20,21] governing the critical dynamics of the superconducting order parameter in the $l$th superconducting plane:

$$\Gamma_0^{-1}\left(\frac{\partial}{\partial\tau} - i\frac{e^*}{\hbar}\phi_l\right)\Psi_l = -\frac{1}{s'}\frac{\delta F_{GL}}{\delta\Psi_l^*} + \zeta_l. \tag{2}$$

For our purpose, we consider the electric field E as being applied along the c-axis, and generated by the scalar potential $\phi_l = -Ed'l\exp(-i\omega\tau)$. The Langevin forces $\zeta_l(\mathbf{r},\tau)$ are chosen to have the Gaussian white-noise law

$$\langle\zeta_l^*(\mathbf{r},\tau)\zeta_{l'}(\mathbf{r}',\tau')\rangle = 2\Gamma_0^{-1}k_BT\delta(\mathbf{r}-\mathbf{r}')\delta(\tau-\tau')\delta_{ll'}/s'. \tag{3}$$

In the chosen gauge, the Josephson current density between the $l$th and ($l$+1)th layers take a form

$$J_c = \frac{i\hbar e^*}{2m_c d'}[\langle\Psi_l^*\Psi_{l+1}\rangle - \langle\Psi_l\Psi_{l+1}^*\rangle] \tag{4}$$

Through the paper we use the upper critical field $H_{c2} = \Phi_0/2\pi\xi^2$ as the magnetic field unit, coherence length $\xi = \hbar/\sqrt{2m_{ab}\alpha T_c}$ as the unit of length and $\varphi = \sqrt{\beta/2\alpha_0 T_c^{mf}}\,\Psi$ as the dimensionless order parameter, so the GL Boltzmann factor in scaled units has a form,

$$\frac{F_{GL}}{T} = \frac{s}{2\eta^{mf}t^{mf}}\sum_l\int d^2r\{|\mathbf{D}\varphi_l|^2 + d^{-2}|\varphi_l-\varphi_{l+1}|^2 - (1-t^{mf})|\varphi_l|^2 + |\varphi_l|^4\}. \tag{5}$$

Here the dimensionless fluctuation's strength coefficient is $\eta^{mf} = \sqrt{2Gi^{mf}}\,\pi$ with the Ginzburg number being defined by $Gi^{mf} = \frac{1}{2}(8e^2\kappa^2\xi^2 k_B T_c^{mf}\gamma/c^2\hbar^2)^2$, $t^{mf} = T/T_c^{mf}$, $b = B/H_{c2}$ are the dimensionless temperature and induction, and $\gamma = \sqrt{m_c/m_{ab}}$ is the anisotropy, $d = d'\gamma/\xi$, $s = s'\gamma/\xi$. The covariant derivative in the dimensionless units is $\mathbf{D} = \nabla + i\mathbf{A}$ with $\mathbf{A} = (-by, 0)$.

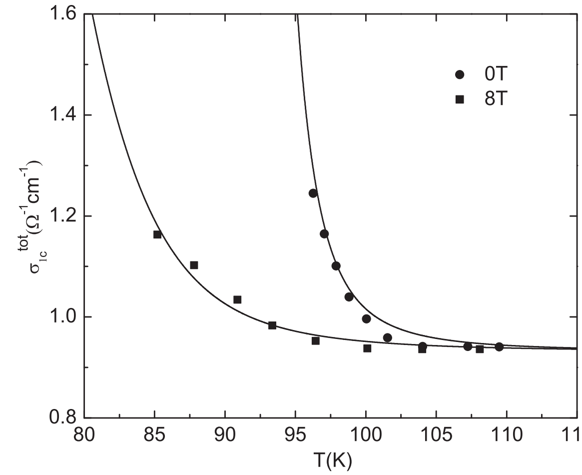


**Fig. 1.** The real part of the out of plane a.c. fluctuation conductivity in HBCO as a function of temperature, for zero field and 8 T, at frequencies of 15.15 GHz.

In the dimensionless units, TDGL equation (2) takes a form

$$\left\{\frac{\partial}{\partial\tau} - \frac{1}{2}\mathbf{D}^2 + i\mathcal{E}\exp(-i\omega\tau)dl\right\}\varphi_l + \frac{1}{2d^2}(2\varphi_l - \varphi_{l+1} - \varphi_{l-1}) - \frac{1-t^{mf}}{2}\varphi_l + |\varphi_l|^2\varphi_l = \overline{\zeta_l}, \tag{6}$$

while the Gaussian white-noise correlation (3) becomes

$$\langle\overline{\zeta_l}^*(\mathbf{r},\tau)\overline{\zeta_{l'}}(\mathbf{r}',\tau')\rangle = \frac{2\eta^{mf}t^{mf}}{s}\delta(\mathbf{r}-\mathbf{r}')\delta(\tau-\tau')\delta_{ll'}. \tag{7}$$

where $\overline{\zeta}$ is the thermal noise in dimensionless units and $\mathcal{E} = E/E_{GL}$ with $E_{GL} = H_{c2}\xi/c\tau_{GL}$ as a unit of electric field and $\tau_{GL} = m_{ab}\Gamma_0^{-1}\xi^2/\hbar^2$ as a unit of time.

The dimensionless electric current density along $c$-direction is $J_c = J_{GL}j_c$ where

$$j_c = \frac{i}{2d}[\langle\varphi_l^*\varphi_{l+1}\rangle - \langle\varphi_l\varphi_{l+1}^*\rangle], \tag{8}$$

with $J_{GL} = cH_{c2}/(2\pi\xi\gamma\kappa^2)$ is the unit of the current density. The unit of electrical conductivities will be defined by $\sigma_{GL} = J_{GL}/E_{GL} = c^2\Gamma_0^{-1}/(4\pi\xi\gamma\kappa^2)$. This unit is close to the normal state conductivity $\sigma_n$ in dirty limit superconductors [22]. In general there is a factor $k$ of order one relating $\sigma_n$ and $\sigma_{GL}$: $\sigma_n = k\sigma_{GL}$.

## 3. The self-consistent Gaussian approximation for solution of TDGL

In order to calculate the out of plane a.c. fluctuation conductivity, it is necessary to employ some approximation to deal with the cubic term in the TDGL equation (6). A simple approximation which captures the interesting fluctuation effects is the self-consistent Gaussian approximation, in which $|\varphi_l|^2\varphi_l$ is replaced by $2\langle|\varphi_l|^2\rangle\varphi_l$. This results in a linear TDGL equation:

$$\left(\frac{\partial}{\partial\tau} - \frac{1}{2}\mathbf{D}^2 + i\mathcal{E}\exp(-i\omega\tau)dl - \frac{b}{2}\right)\varphi_l + \frac{1}{2d^2}(2\varphi_l - \varphi_{l+1} - \varphi_{l-1}) + \varepsilon\varphi_l = \overline{\zeta_l}, \tag{9}$$

where

$$\varepsilon = -a_h + 2\langle|\varphi_l|^2\rangle, \tag{10}$$

with the constant being defined as $a_h = (1 - t^{mf} - b)/2$. The average $\langle|\varphi_l|^2\rangle$ is expressed via the parameter $\varepsilon$ below and will be determined self-consistently together with $\varepsilon$.

We proceed further by introducing the Fourier transform with

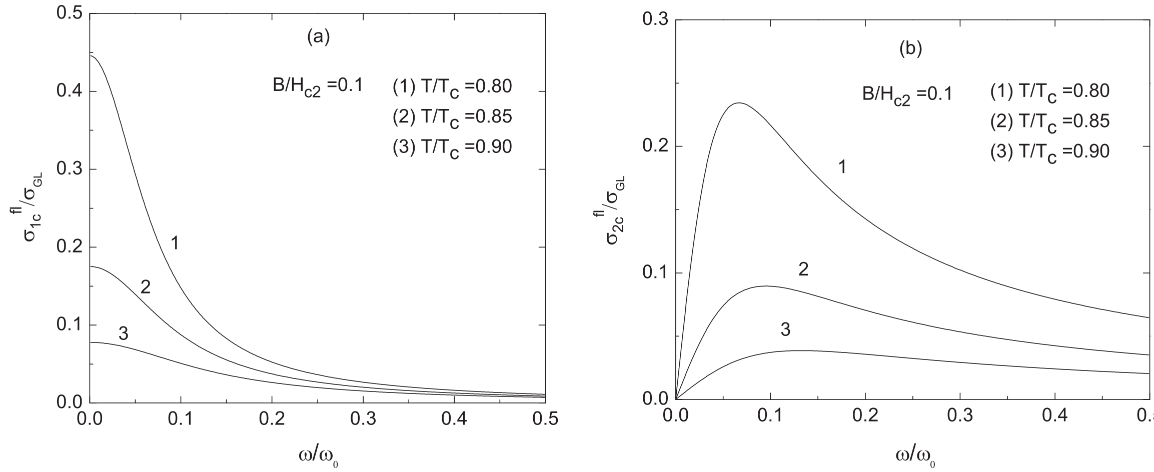


**Fig. 2.** The real and imaginary part of the out of plane a.c. fluctuation below $T_c$ as a function of frequency, for different temperatures, at fixed magnetic field.

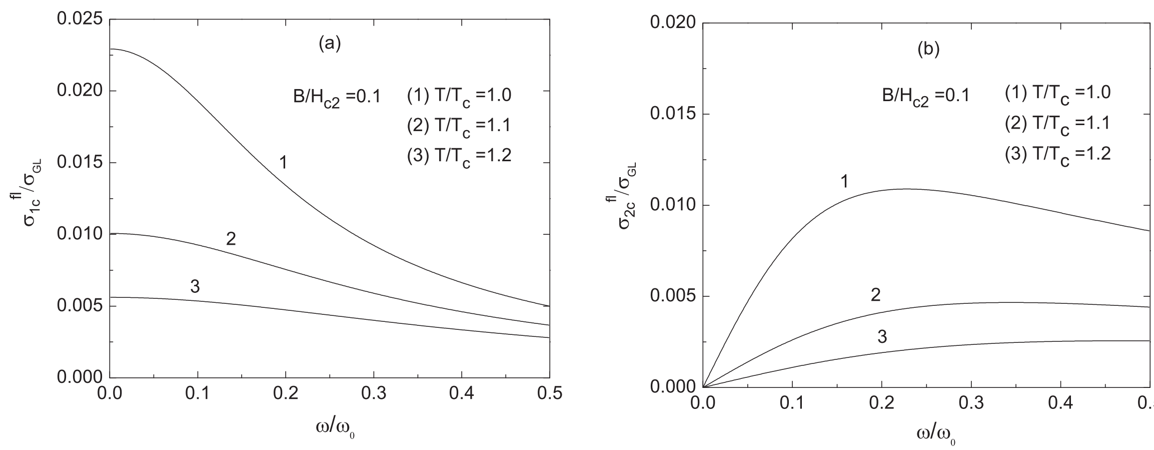


**Fig. 3.** The real and imaginary part of the out of plane a.c. fluctuation conductivity above $T_c$ as a function of frequency, for different temperatures, at fixed magnetic field.

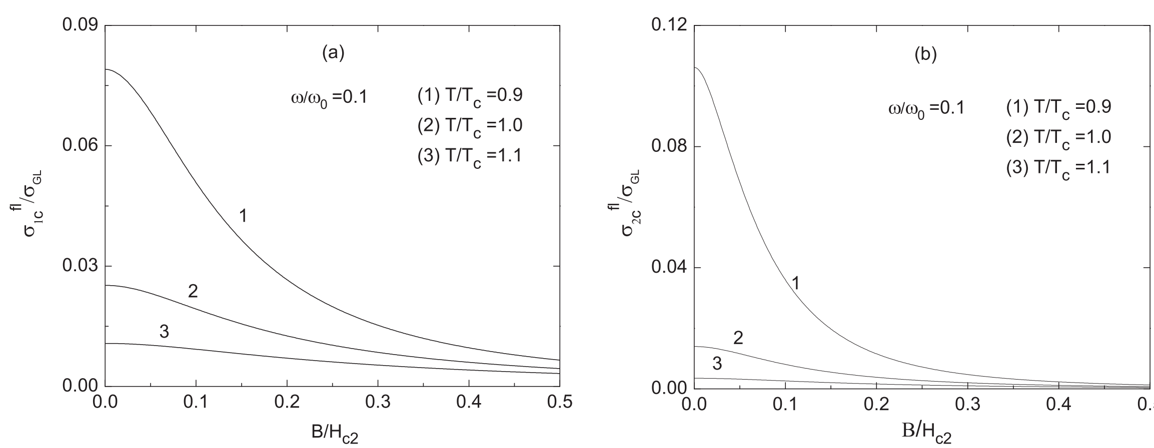


**Fig. 4.** The real and imaginary part of the out of plane a.c. fluctuation conductivity below and above $T_c$ as a function of magnetic field, for different temperatures, at fixed frequency.

respect to the two in-plane coordinates and the layer index, respectively, through the relations

$$\varphi_l(\mathbf{r},\tau) = \int_0^{2\pi/d} \frac{dk_z}{2\pi} e^{-ilk_z d}\varphi_{k_z}(\mathbf{r},\tau),\ \varphi_{k_z}(\mathbf{r},\tau) = d\sum_l e^{ilk_z d}\varphi_l(\mathbf{r},\tau), \tag{11}$$

and similar transformation for $\overline{\zeta}$. Therefore, the TDGL equation (9) will write in the new variables:

$$\left\{\frac{\partial}{\partial\tau} - \frac{1}{2}\mathbf{D}^2 + \mathcal{E}\exp(-i\omega\tau)\frac{\partial}{\partial k_z} - \frac{b}{2} + \frac{1}{d^2}[1-\cos(k_z d)] + \varepsilon\right\}\times\varphi_{k_z}(\mathbf{r},\tau) = \overline{\zeta_{k_z}}(\mathbf{r},\tau), \tag{12}$$

where the new noise terms $\overline{\zeta_{k_z}}$ are delta-correlated as

$$\langle \overline{\zeta_{k_z}}^*(\mathbf{r},\tau)\overline{\zeta_{k'_z}}(\mathbf{r}',\tau')\rangle = 4\pi\eta t\frac{d}{s}\delta(\mathbf{r}-\mathbf{r}')\delta(\tau-\tau')\delta(k_z-k'_z). \tag{13}$$

The solution of Eq. (12) is

$$\varphi_l(\mathbf{r},\tau)=\int_0^{2\pi/d}\frac{dk_z}{2\pi}e^{-ilk_zd}\int d\mathbf{r}'\int d\tau' G_{k_z}(\mathbf{r},\tau;\mathbf{r}',\tau')\overline{\zeta_{k_z}}(\mathbf{r}',\tau'),\tag{14}$$

where $G_{k_z}(\mathbf{r},\tau;\mathbf{r}',\tau')$ is the Green function for Eq. (12).

To find the Green function to linear order in the electric field we write $G_{k_z}=G^0_{k_z}+G^1_{k_z}$, where $G^0_{k_z}$ is the equilibrium Green function and $G^1_{k_z}$ is first order in the electric field. Substituting this expansion into the TDGL equation, Eq. (12), we find for $G^1_{k_z}$

$$G^1_{k_z}(\mathbf{r},\tau;\mathbf{r}',\tau')=-\mathcal{E}\int d\mathbf{r}_1\int d\tau_1 G^0_{k_z}(\mathbf{r},\tau;\mathbf{r}_1,\tau_1)\exp(-i\omega\tau_1)\times\frac{\partial}{\partial k_z}G^0_{k_z}(\mathbf{r}_1,\tau_1;\mathbf{r}',\tau'),\tag{15}$$

where the equilibrium Green function $G^0_{k_z}$ satisfies

$$\left\{\frac{\partial}{\partial\tau}-\frac{1}{2}\mathbf{D}^2-\frac{b}{2}+\frac{1}{d^2}[1-\cos(k_zd)]+\varepsilon\right\}G^0_{k_z}(\mathbf{r},\mathbf{r}',\tau-\tau')=\delta(\mathbf{r}-\mathbf{r}')\delta(\tau-\tau').\tag{16}$$

The equilibrium Green function has the Gaussian form as follows

$$G^0_{k_z}(\mathbf{r},\mathbf{r}',\tau'')=C_{k_z}(\tau'')\theta(\tau'')\exp\left[\frac{ib}{2}X(y+y')-\frac{X^2+Y^2}{2\beta}\right],\tag{17}$$

with $X=x-x'$, $Y=y-y'$, $\tau''=\tau-\tau'$. $\theta(\tau'')$ is the Heaviside step function, $C$ and $\beta$ are coefficients which are determined as follows:

$$C=\frac{b}{4\pi}\exp\left\{-\left(\varepsilon-\frac{b}{2}+\frac{1}{d^2}[1-\cos(k_zd)]\right)\tau''\right\}\sinh^{-1}\left(\frac{b\tau''}{2}\right).\tag{18}$$

$$\beta=\frac{2}{b}\tanh(b\tau''/2),\tag{19}$$

In equilibrium, the correlation function between the order parameter in two layers $l$ and $l'$ is given by

$$\langle\varphi_l^*\varphi_{l'}\rangle=\frac{\eta bt}{2\pi s}\int_{\tau_c}^{\infty}\sinh^{-1}(b\tau'')\exp\left\{-\left(2\varepsilon-b+\frac{2}{d^2}\right)\tau''\right\}I_0\left(\frac{2\tau''}{d^2}\right)\times e^{-i(l-l')k_zd}.\tag{20}$$

It is necessary to introduce an ultraviolet (UV) cutoff $\tau_c$ for regularization because the expression (20) is divergent at small $\tau$. Now the self-consistent Gaussian equation can be obtained by substituting expression (20) into Eq. (10). After renormalization, physically the renormalization corresponds to reduction in the critical temperature by the thermal fluctuations from $T_c^{mf}$ to $T_c$, the self-consistent Gaussian equation takes a form as

$$\varepsilon=-a_h^r-\frac{\eta t}{\pi s}\int_0^{\infty}\ln[\sinh(b\tau)]\frac{d}{d\tau}\left[\frac{g(\varepsilon,\tau)}{\cosh(b\tau)}\right]+\frac{\eta t}{\pi s}\{\gamma_E-\ln(bd^2)\}.\tag{21}$$

Here $a_h^r=\frac{1-b-T/T_c}{2}$, $t=T/T_c$, $\gamma_E=0.577$ is Euler constant, $\eta=\sqrt{2Gi}\,\pi$ with $Gi=\frac{1}{2}(8e^2\kappa^2\xi T_c\gamma/c^2\hbar^2)^2$ ($T_c{}^{mf}$ is now replaced by $T_c$), and

$$g(\varepsilon,\tau)=e^{-(2\varepsilon-b+2/d^2)\tau}I_0\left(\frac{2\tau''}{d^2}\right),\tag{22}$$

with $I_0(x)=(1/2\pi)\int_0^{2\pi}e^{x\cos\theta}d\theta$ being the modified Bessel function.

## 4. The out of plane a.c. fluctuation conductivity

Starting from the correlation function (20) for $l'=l+1$, one can obtain the current density definition (8) in physical unit

$$J_c=\frac{\eta bt}{2\pi s}\sigma_{GL}\mathcal{E}\int_0^{\infty}\frac{\cos(\omega\tau)-\cos[\omega(\tau-\tau'')]+\tau''\omega\sin(\omega\tau)}{\omega^2\tau''\sinh(b\tau'')}\times\exp\left\{-\left(2\varepsilon-b+\frac{2}{d^2}\right)\tau''\right\}I_1\left(\frac{2\tau''}{d^2}\right),\tag{23}$$

where $I_1$ the modified Bessel function of first order.

After doing the Fourier transform expression (23) with respect to frequency one then obtains the out of plane a.c. fluctuation conductivity as:

$$\sigma_c^{fl}(\omega)=\frac{J_c(\omega)}{\mathcal{E}(\omega)}=\frac{\eta bt}{4\pi s}\sigma_{GL}\int_0^{\infty}\frac{1-e^{-i\tau''\omega}-i\tau''\omega}{\omega^2\tau''\sinh(b\tau'')}\exp\left\{-\left(2\varepsilon-b+\frac{2}{d^2}\right)\tau''\right\}\times I_1\left(\frac{2\tau''}{d^2}\right).\tag{24}$$

Consequently the real and imaginary parts of the out of plane a.c. fluctuation conductivity are given by

$$\sigma_{1c}^{fl}(\omega)=\frac{\eta bt}{4\pi s}\sigma_{GL}\int_0^{\infty}\frac{1-\cos(\tau''\omega)}{\omega^2\tau''\sinh(b\tau'')}\exp\left\{-\left(2\varepsilon-b+\frac{2}{d^2}\right)\tau''\right\}I_1\left(\frac{2\tau''}{d^2}\right),\tag{25}$$

$$\sigma_{2c}^{fl}(\omega)=\frac{\eta bt}{4\pi s}\sigma_{GL}\int_0^{\infty}\frac{\sin(\tau''\omega)-\tau''\omega}{\omega^2\tau''\sinh(b\tau'')}\exp\left\{-\left(2\varepsilon-b+\frac{2}{d^2}\right)\tau''\right\}I_1\left(\frac{2\tau''}{d^2}\right).\tag{26}$$

The integrals in Eqs. (25) and (26) are convergent provided $2\varepsilon-b+2/d^2>0$. However, this condition is assured while solving Eq. (21) for the parameter $\varepsilon$ at any temperature $T$.

In the 2D regime ($\varepsilon\gg 1/d^2$) the general expression (24) is reduced to

$$\sigma_{c(2D)}^{fl}(\omega)=\frac{\eta t}{4\pi sd^2\omega^2}\sigma_{GL}\left[\psi\left(\frac{2\varepsilon+i\omega}{2b}\right)-\psi\left(\frac{\varepsilon}{b}\right)-\frac{i\omega}{2b}\psi'\left(\frac{\varepsilon}{b}\right)\right]\tag{27}$$

In limit case $B\to 0$ and the 2D regime, the out of plane a.c. fluctuation conductivity can be easily obtained by taking the limit $b\to 0$ in Eq. (27), in which the real and imaginary parts of the out of plane a.c. fluctuation conductivity takes thus the form

$$\sigma_{1c(2D)}^{fl}=\frac{\eta t}{4\pi sd^2\omega^2}\sigma_{GL}\log\left[1+\left(\frac{\omega}{\varepsilon}\right)^2\right],\tag{28}$$

$$\sigma_{2c(2D)}^{fl}=\frac{\eta t}{4\pi sd^2\omega^2}\sigma_{GL}\left[arctan\left(\frac{\omega}{\varepsilon}\right)-\frac{\omega}{\varepsilon}\right].\tag{29}$$

The expressions (28) and (29) thus matches the result previously obtained within the diagrammatic microscopic approach, for Gaussian fluctuations (i.e., $\varepsilon=\ln(T/T_c)$) Ref. [11].

In the 3D limit, the out of plane a.c. fluctuation conductivity can be directly inferred from the corresponding result of the layered model, if one takes the 3D limit $d\to 0$ and the asymptotic expression of the modified Bessel functions of first order in Eq. (24) $I_1(z)\approx e^z/\sqrt{2\pi z}$ for large arguments $z\to\infty$

$$\sigma_{c(3D)}^{fl}(\omega)=\frac{\eta bt}{8\pi\sqrt{\pi}}\sigma_{GL}\int_0^{\infty}\frac{1-e^{-i\tau''\omega}-i\tau''\omega}{\omega^2\tau''\sqrt{\tau''}\sinh(b\tau'')}\exp\{-(2\varepsilon-b)\tau''\}.\tag{30}$$

In limit case $B\to 0$, namely for vanishing magnetic field, the general expressions (30) takes a form

$$\sigma_{1c(3D)}^{fl}=\frac{\eta t}{6\sqrt{2}\pi s\omega^2}\sigma_{GL}\left(\sqrt{2}\varepsilon^{3/2}-\varepsilon\sqrt{\sqrt{\varepsilon^2+\omega^2}+\varepsilon}+\omega\sqrt{\sqrt{\varepsilon^2+\omega^2}-\varepsilon}\right),\tag{31}$$

$$\sigma_{2c(3D)}^{fl}=\frac{\eta t}{12\pi s\omega^2}\left[3\sqrt{\varepsilon}\omega-2(\varepsilon^2+\omega^2)^{3/4}\sin\left(\frac{3}{2}arctan\left(\frac{\omega}{\varepsilon}\right)\right)\right].\tag{32}$$

## 5. Comparison with experiment

Microwave measurements was used to determine both, the in-plane and out-of-plane conductivity of the high-Tc superconductor $HgBa_2CuO_{4+\delta}$ (HBCO) with $T_c=94.3$ K at frequencies of 15.15 GHz [23]. In order to compare the out of plane a.c. fluctuation conductivity with experimental data in HTSC, it is impossible to use the expression of relaxation time $\Gamma_0^{-1}$ in BCS theory which may be suitable for low-Tc superconductor. Instead of this, the factor $k = \sigma_n/\sigma_{GL}$ should be used as fitting parameter. The comparison of the real part of the out of plane a.c. fluctuation conductivity for zero magnetic fields and for 8 T is presented in Fig. 1. The curves were fitted with the formula $\sigma_{1c}^{tot} = \sigma_{1c}^{fl} + \sigma_n$. Here $\sigma_{1c}^{fl}$ is the out of plane a.c. fluctuation conductivity given by Eq. (25) and $\sigma_n = 0.93(\Omega^{-1}\,\mathrm{cm}^{-1})$ is the normal-state resistivity in Ref. [23]. The distance between the bilayers used the calculation is $d' = 9.5$ Å in Ref. [24]. From the fitting, we obtain the parameters: the upper critical field $H_{c2} = 176$ T, the Ginzburg-Landau parameter $\kappa$=65, the anisotropy parameter $\gamma = 32$, the order parameter effective thickness $s' = 5.6$ Å and the factor $k$=0.87. Our values are in good quantitative agreement with experimental data and the fitting value of $\gamma$ is agreement in magnitude with the fitting value of $\gamma$=30 in Ref. [25].

Using the parameters specified above, we obtain the Ginzburg number $Gi = 5.8 \times 10^{-3}$ (corresponding to dimensionless fluctuation's strength coefficient $\omega$=0.34) and plot the real and imaginary part of the out of plane a.c. fluctuation conductivity normalized to the unit of conductivity, $\sigma_{1c}^{fl}/\sigma_{GL}$, and $\sigma_{2c}^{fl}/\sigma_{GL}$, below $T_c$ in Fig. 2 and above $T_c$ in Fig. 3. In Fig. 2(a) shows that the real part increases dramatically with the decrease of frequency and temperature at fixed magnetic field. On the other hand, Fig. 2(b) demonstrates that the imaginary part increases with the decrease of frequency and temperature at fixed magnetic field, reaches the main peak at small frequency, and is linear in frequency at very small frequency. The main peak of the imaginary part shifts to lower frequencies with lower temperatures and its amplitude becomes larger. Fig. 3(a) and (b) show that contribution of the real and imaginary part to the normal state is apparently nonzero as an evidence of the pseudogap state. Fig. 4(a) and (b) show the real and imaginary part increases with the decrease of magnetic field and temperature at fixed frequency.

## 6. Conclusion

In conclusion, we have quantitatively studied the out of plane a.c. fluctuation conductivity of type-II superconductor under magnetic field by using the time-dependent Ginzburg-Landau equations in the presence of strong thermal fluctuations. The self-consistent Gaussian approximation has been used in order to take into account the fluctuation interaction. We have obtained the analytically explicit expressions for the real and imaginary part of the out of plane a.c. fluctuation conductivity in layered superconductors (as well as 2D and 3D models) summing over all Landau levels, so that the approach is valid for arbitrary values of the magnetic field not too close to $H_{c1}$. Our expressions have refined analytical form and could be probably easy to perform numerically. The results in 2D and 3D limits have been also derived and the 2D result has been found to reduce to the expressions already known in the limit case of a vanishing magnetic field. We have also compared the results of the real part of the out of plane a.c. fluctuation conductivity with the experimental data on HTSC. Our results are in good qualitative and even quantitative agreement with experimental data on HBCO material. Our results also have shown that there is important fluctuation suppression in the real and imaginary part of the out of plane a.c. fluctuation conductivity below and above $T_c$ for high frequencies and the out of plane a.c fluctuation conductivity has still been nonzero above $T_c$ in a wide range of frequencies.

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