Math 355 Problem Set #2 Soltuions
Spring 2012

p.15 #6. *Suppose lines ℓ and m intersect at the point Q, and let $\ell' = \sigma_m(\ell)$. Let P and R be points on ℓ and ℓ' that are distinct from Q and on the same side of m. Let S and T be the feet of the perpendiculars from P and R to m. Prove that $\angle PQS \cong \angle RQT$.*
Proof: Let $P' = \sigma_\ell(P)$ and $R' = \sigma_\ell(R)$; then by definition of σ_ℓ, $PS = SP'$ and $\angle QSP \cong \angle QSP'$ since both are right angles. Hence $\triangle PQS \cong \triangle P'QS$ by SAS, and $\angle SQP \cong \angle SQP'$ by CPCTC. But $\angle SQP' \cong \angle RQT$ since these are vertical angles.

p.17 #17. *Let l and m be lines. Prove that if $\sigma_m(l) = l$, then either $l \perp m$ or $l = m$.*
Proof: Assume $l \neq m$ and choose a point P on l and off m. Let $P' = \sigma_m(P)$; then $P \neq P'$ and m is the perpendicular bisector of $\overline{PP'}$, by definition of σ_m. But by assumption, $\sigma_m(l) = l$, hence P' is also on l and it follows that $l = \overleftrightarrow{PP'} \perp m$.

p.24 #5. Complete the proof of Theorem 49: *If P and Q are distinct points and $\ell = \overleftrightarrow{PQ}$, then $\tau_{\mathbf{PQ}}(\ell) = \ell$.*
Proof: Let A be a point on ℓ and let B be a point off ℓ. Since $\tau_{\mathbf{PQ}} = \tau_{P,Q}$, consider $A' = \tau_{P,Q}(A)$ and $B' = \tau_{P,Q}(B)$. Then by definition of $\tau_{P,Q}$, $\square PQB'B$ and $\square BB'A'A$ are parallelograms. Thus $\ell = \overleftrightarrow{PQ} \parallel \overleftrightarrow{BB'} \parallel \overleftrightarrow{AA'}$ so that $\ell \parallel \overleftrightarrow{AA'}$. Since A is on ℓ, it follows that $\overleftrightarrow{AA'} = \ell$. Hence A' is also on ℓ and $\tau_{\mathbf{PQ}}(\ell) = \tau_{P,Q}(\ell) = \ell$.