第 2 讲: 算法的效率

姓名: 林凡琪　　**学号:** 211240042

评分: ______　　**评阅:** ______

2022 年 3 月 3 日

请独立完成作业，不得抄袭。
若得到他人帮助, 请致谢。
若参考了其它资料，请给出引用。
鼓励讨论，但需独立书写解题过程。

1 作业 (必做部分)

题目 1 (DH Problem 6.18: $\log_m n$)

Algorithm 1 $\log_m n$

解答:

```
function LG1(m, n)
    k = 0
    res = 1
    while res * m <= n do
        res = res * m
        k = k + 1
    end while
    return res
end function
```

Time complexity: $O(\log_m n)$
Space complexity: $O(1)$

题目 2 (DH Problem 6.19: $\log_m n$)

Time complexity:$O((\log\log n)^2)$
Space complexity: $O(1)$

题目 3 (DH Problem 6.20 (a): $\log_m n$)

Algorithm 2 $\log_m n$

解答:

```
function LG2(m, n)
    k = 0
    a = 1
    while m * a <= n do
        k_1 = 1
        b = m * m
        c = m * a
        while a * b <= n do
            c = a
            a = b * c
            k_1 = 2k_1
            b = b * b
        end while
        a = c
        k = k + k_1
    end while
    return k
end function
```

Algorithm 3 LG2

解答:

```
procedure LG2(m,n)
    res[0] = m
    count = 0
    while res[count] * res[count] <= n do
        res[count + 1] = res[count] * res[count]
        count = count + 1
    end while
    sum = res[count]
    ans = pow(2, count)
    while count >= 1 do
        if sum * res[count - 1] <= n AND sum * res[count - 1] * m * m > n then
            ans = ans + pow(2, count - 1)
            sum = sum * res[count - 1]
        end if
        count = count - 1
    end while
    return ans
end procedure
```

Time complexity:$O(\log\log n)$
Space complexity: $O(\log n)$

题目 4 (TC Exercise 3.1-6)

解答:
充分性:
设该算法的运行时间为 $T(n) = \Theta(n) \to \exists c_1, c_2, n_0, \forall n > n_0, c_1 n <= T(n) <= c_2 n$
所以 $\exists c_1, n_0, \forall n > n_0, c_1 n <= T(n) \to T(n) = \Omega(n)$ 即最好情况运行时间是 $\Omega(n)$
同样，$\exists c_2, n_0, \forall n > n_0, T(n) <= c_2 n \ \to T(n) = O(n)$ 即最坏情况运行时间为 $O(n)$
必要性:
设该算法运行时间为 $T(n) \in O(n)$ 并且 $T(n) \in \Omega(n)$
即 $\exists c_1, n_0, \forall n > n_0, c_1 n <= T(n) \to T(n) = \Omega(n)$ 且 $\exists c_2, n_0, \forall n > n_0, T(n) <= c_2 n \ \to T(n) = O(n)$.
综合两点可得 $T(n) = \Theta(n) \to \exists c_1, c_2, n_0, \forall n > n_0, c_1 n <= T(n) <= c_2 n$
综上, 证毕.

题目 5 (TC Exercise 3.1-7)

解答:
$o(g(n)) = \{f(n) | \lim_{n\to\infty} \frac{f(n)}{g(n)} = 0\}$
$\omega(g(n)) = \{f(n) | \lim_{n\to\infty} \frac{f(n)}{g(n)} \to \infty\}$
$\Rightarrow o(g(n)) \cap \omega(g(n)) = \{f(n) | \lim_{n\to\infty} \frac{f(n)}{g(n)} \to \infty, \lim_{n\to\infty} \frac{f(n)}{g(n)} = 0\}$
显然,$0 \neq \infty$. 所以一定不存在符合条件的 f(n), 所以 $o(g(n)) \cap \omega(g(n))$ 为空集

题目 6 (TC Problem 3-3 (a))

解答:
答案被移到下面去了...

---

2 作业 (选做部分)

题目 1 (DH Problem 6.13)

解答:

等价类序号	函数
1	$g_1 = 2^{2^{n+1}}$
2	$g_2 = 2^{2^n}$
3	$g_3 = (n+1)!$
4	$g_4 = n!$
5	$g_5 = e^n$
6	$g_6 = n2^n$
7	$g_7 = 2^n$
8	$g_8 = (\frac{3}{2})^n$
9	$g_9 = n^{\lg\lg n}, g_10 = (\lg n)^{\lg n}$
10	$g_{11} = (\lg n)!$
11	$g_{12} = n^3$
12	$g_{13} = n^2, g_{14} = 4^{\lg n}$
13	$g_15 = n\lg n, g_{16} = lg(n!)$
14	$g_{17} = n, g_{18} = 2^{\lg n}$
15	$g_{19} = (\sqrt{2})^{\lg n}$
16	$g_{20} = 2^{\sqrt{2\lg n}}$
17	$g_{21} = \lg^2 n$
18	$g_{22} = \ln n$
19	$g_{23} = \sqrt{\lg n}$
20	$g_{24} = \ln\ln n$
21	$g_{25} = 2^{\lg^* n}$
22	$g_{26} = \lg^*(\lg n), g_{27} = \lg^* n$
23	$g_{28} = \lg(\lg^* n)$
24	$g_{29} = n^{1/\lg n}$

3 Open Topics

本次 OT 介绍两种证明问题下界的常用技术。

Open Topics 1 (Decision Trees)
介绍 Decison Trees (决策树) 的概念以及在证明问题下界时的应用 (包括但不限于本次选做题 DH 6.13)。

参考资料:

- Decision tree model @ wiki
- lecture-note by jeffe

Open Topics 2 (Adversary Argument)
介绍 Adversary Argument (对手论证) 的概念以及在证明问题下界时的应用。

- lecture-note by jeffe

4 反馈