

第 3-5 讲: 树

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评分: _____ 评阅: _____

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请独立完成作业, 不得抄袭。
若得到他人帮助, 请致谢。
若参考了其它资料, 请给出引用。
鼓励讨论, 但需独立书写解题过程。

1 作业 (必做部分)

题目 1 (CZ 4.4)

Let G be a connected graph and let e_1 and e_2 be two edges of G . Prove that $G - e_1 - e_2$ has three components if and only if both e_1 and e_2 are bridges in G .

解答:

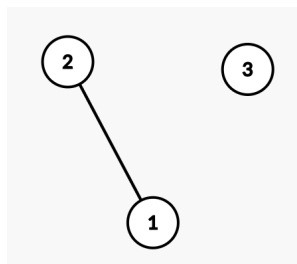
若 e_1 和 e_2 为割边, $G - e_1$ 为有两个连通分支 G_1 和 G_2 。假设边 e_2 在连通分支 G_1 中。在这个连通分支中将 e_2 去掉, 则该连通分支会新成两个连通分支 G_3, G_4 。此时即为 $G - e_1 - e_2$, 共有 G_2, G_3, G_4 三个连通分支。

若 $G - e_1 - e_2$ 有三个连通分量, 由于 G 连通, 破坏连通性的方法为去掉割边, 且去掉一条割边, 只会多生成一个连通分支。因此, e_1, e_2 必为割边。
综上, 得证。

题目 2 (CZ 4.9)

Show that a graph of order n and size $n - 1$ need not be a tree.

解答:



$n = 3$

题目 3 (CZ 4.14)

A certain tree T of order 35 is known to have 25 vertices of degree 1, two vertices of degree 2, three vertices of degree 4, one vertex of degree 5 and two vertices of degree 6. It also contains two vertices of the same (unknown) degree x . What is x ?

解答:

由图论第一定理, $25 \cdot 1 + 2 \cdot 2 + 3 \cdot 4 + 1 \cdot 5 + 2 \cdot 6 + 2 \cdot x = 2 \cdot 34$

解得 $x = 5$

题目 4 (CZ 4.22)

Let T be a tree of order n . Show that the size of the complement of \overline{T} is the same as the size of K_{n-1} .

解答:

K_{n-1} 的边数是 $\frac{(n-1)(n-2)}{2}$

\overline{T} 的边数是 $\frac{n(n-1)}{2} - (n-1) = \frac{(n-1)(n-2)}{2}$

题目 5 (CZ 4.26)

Prove that an edge e of a connected graph is a bridge if and only if e belongs to every spanning tree of G .

解答:

假设最后的生成树中不含这条割边, 则根据定义, 去掉这条割边后的图不连通。

这与生成树的连通性矛盾。

故生成树上一定有这条割边

题目 6 (CZ 4.28)

Apply both Kruskal's and Prim's Algorithms to find a minimum spanning tree in the weighted graph in Figure 4.12. In each case, show how this tree is constructed, as in Figures 4.8 and 4.9.

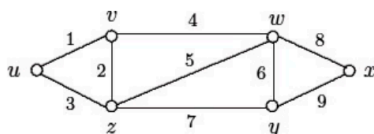


Figure 4.12: The weighted graph in Exercise 4.28

解答:

Kruskal 的加边顺序: $(u, v, 1), (v, z, 2), (v, w, 4), (w, y, 6), (w, x, 8)$

Kruskal 的加边顺序 (假设初使点为 u): $(u, v, 1), (v, z, 2), (v, w, 4), (w, y, 6), (w, x, 8)$

题目 7 (CZ 4.30)

Let G be a connected weighted graph and T a minimum spanning tree of G . Show that T is a unique minimum spanning tree of G if and only if the weight of each edge e of G that is not in T exceeds the weight of every other edge on the cycle in $T + e$.


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2-10-10000000000000
-12-10000000000000
0-14-100000000-10-10
-10-14-10-100000000
000-12-10000000000
0000-12-100000000
000-10-14-10-100000
000000-12-1000000
0000000-12-100000
000000-10-14-1-1000
000000000-120-100
00-1000000-104-10-1
0000000000-1-1200
00-100000000002-1
00000000000-10-12

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其矩阵 C 如上。

经过计算，其共有 3840 个生成树。

2 作业 (选做部分)

3 Open Topics

Open Topics 1 (Chu-Liu/Edmonds algorithm)

- In graph theory, an arborescence is a directed graph in which, for a vertex u called the root and any other vertex v , there is exactly one directed path from u to v .
- In graph theory, Edmonds' algorithm or Chu-Liu/Edmonds' algorithm is an algorithm for finding a spanning arborescence of minimum weight (sometimes called an optimum branching).

参考资料: https://en.wikipedia.org/wiki/Edmonds%27_algorithm

Open Topics 2 (Minimum bottleneck spanning tree)

In mathematics, a minimum bottleneck spanning tree (MBST) in an undirected graph is a spanning tree in which the most expensive edge is as cheap as possible.

参考资料: https://en.wikipedia.org/wiki/Minimum_bottleneck_spanning_tree

4 反馈