第 *3-10* 讲*:* 图的连通性

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评分： ______　　**评阅：** ______

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请独立完成作业，不得抄袭。 若得到他人帮助, 请致谢。 若参考了其它资料，请给出引用。 鼓励讨论，但需独立书写解题过程。

1 作业（必做部分）

题目 1 (CZ 5.4)

证明：若 v 是图 G 的割点，则 v 一定不是 G^{-} 的各点

解答:

For the node u that is different from v, w

If there are different connected branches of $G-v$, it means that the (u,w) edge does not exist in G, so there is the (u,v) edge in $\overline{G}$. So in $\overline{G}$, there is a pathway between u and w that does not go through v.

If in the same connected branch of $G-v$, for a point p of another connected branch, there are no two edges in G (u,p) and (w,p), so there are two edges in $\overline{G}$. So in $\overline{G}$, there is a pathway between u and w that does not go through v.

In summary, there is a path between any two points u and w in $\overline{G}$ that are different from v. Therefore, v is not a cut point.

题目 2 (CZ 5.8)

(a) 设 G 是非平凡的连通图. 证明: 若 v 是 G 的生成树的一个端点，则 v 不是 G 的割点.

(b) 利用 (a) 的结论给出下面事实的另一种证明: 任意一个非平凡的连通图至少包含两个不是割点的顶点.

(c) 设 v 是非平凡连通图 G 的一个顶点证明: 存在 G 的一个生成树, 它包含 G 中所有与 v 相关联的边.

(d) 证明: 若连通图 G 恰有两个非割点的顶点，则 G 是一条路.[提示: 若树 T 包含一个度大于 2 的顶点, 则 T 包含两个以上的端点]

解答:

(a) In the spanning tree T of G , the theorem shows that its endpoint has a degree of 1 , not a cut point.

In the process of reducing from T to G, it is a process of continuous edges, and the connectivity of the graph is enhanced. If a point is not a cut point in T, it must not be a cut point in G.

(b) The spanning tree of any nontrivial graph contains at least two endpoints , and it can be seen from (a) that these two endpoints must not be cut points in the original nontrivial graph , so the number of cut points is at least 2.

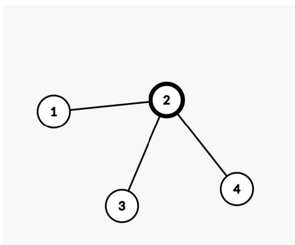
(c) For any spanning tree T of G, if there is an edge associated with v in G that is not in T, add T to that edge, and delete an edge in the resulting ring where neither endpoint is v.
After this operation, the built graph is still a spanning tree of G.
Therefore, by continuously constructing through this operation, a tree can be formed that meets the above conditions.
(d) From (c) it can be seen that at any point v, there is a spanning tree T of G, containing all the edges associated with v in G.
As can be seen from the hint, none of the vertices in T has a degree of more than 2.
Therefore, all vertices in the figure G do not exceed 2.
Combined with G connectivity and exactly two non-cut endpoints, it is concluded that G is a path.

题目 3 (CZ 5.10)

解答:
Assuming that $(u, v)(x, y)$ is not contiguous, then 4 points are different. Because G is indivisible, any two points are on a ring in the diagram.
If there is a pathway between u and x that does not go through v and y, then there must be a pathway between v and y that does not go through u and x. At this time, these two pathways and the ring formed by these two edges meet the requirements of the topic.
Otherwise, if there is a path between u and y that does not go through x and v, then there must be a path between x and v that does not go through u and y. At this time, these two pathways and the ring formed by these two edges meet the requirements of the topic.
Thus there exists a ring satisfying that all four vertices are on that ring.
In summary, these two sides are on a circle.

题目 4 (CZ 5.12)

解答:
1，2
when k = 1:
When k = 2:



if$k$>2,the number of parts is at least 4. So this assumption is clearly not valid

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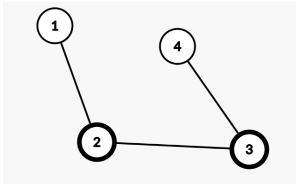
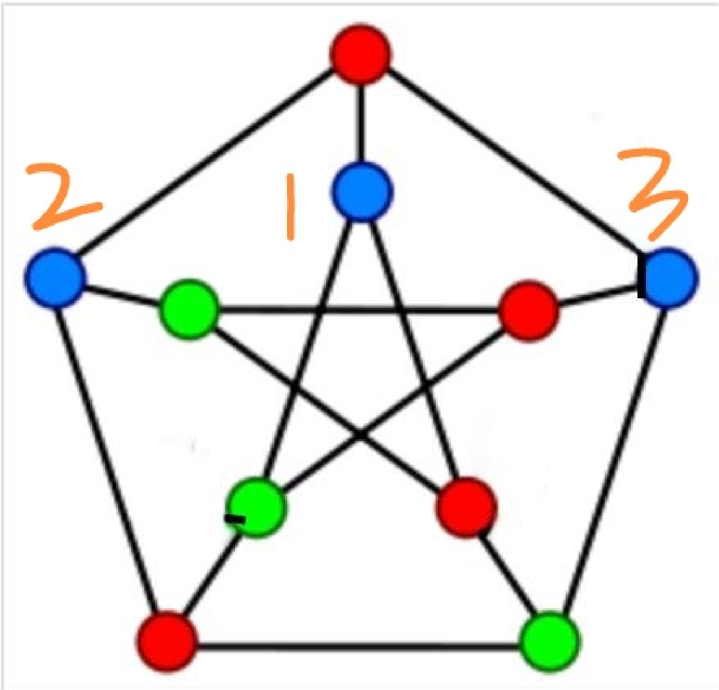


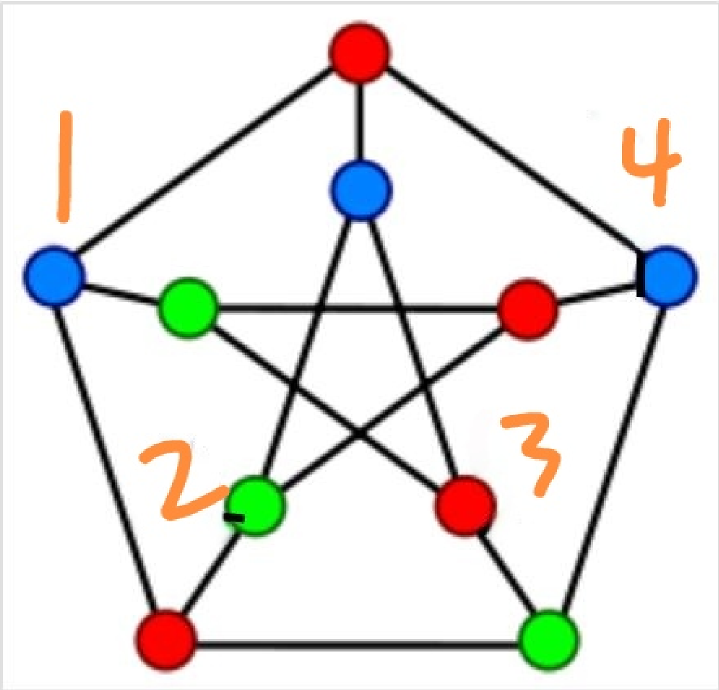
题目 5 (CZ 5.18)

解答:

(a)



(b)



题目 6 (CZ 5.22)

解答：

(a) Proof by contradiction.

Suppose the point connectivity of $G-e$ is p($p<k-1$). The connectivity of points with G in the title is at least k.

Let H be the plot obtained by removing the smallest set of cut points of graph $G-e$ in G. Then in H, the edge e is the cut edge. (After removing the side e, the figure is not connected).

Then in H, you only need to remove one of the two points connected by e to make H unconnected. Therefore, it can be inferred that the connectivity of G is at most $p+1$.

$p+1<k$ from $p<k-1$ is obtained, which contradicts the point connectivity of G at least k. Therefore, it is not true.

Therefore, if G is k connected, then $G-e$ is $k-1$ connected

(b) Proof by contradiction.

Suppose the edge connectivity of $G-e$ is p($p<k-1$). The connectivity of edges with G in the title is at least k.

Let the edge cut set of size p in $G-e$ be S, then $S+e$ must be a side cut set of G.

Therefore, the edge connectivity of G is at most $p+1$

$p+1<k$ from $p<k-1$ is obtained, which contradicts the edge connectivity of G being at least k. Therefore, it is not true.

Therefore, if G is k edge-connected, then $G-e$ is $k-1$ edge-connected

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题目 7 (CZ 5.26)

解答:
Let S be the edge cut set of G. Then $G-S$ is divided into $G1, G2$ two maximal connected subgraphs. Assume $|V(G_1)| < |V(G_2)|$, then there is $|V(G_1)| = k < n/2$
Then there are sides from G_1 to G_2 greater than or equal to $k(\delta(G)-(k-1))$, that is, $\lambda(G) \geq k(\delta(G)-(k-1))$.
From the nature of the quadratic function, the left equation is the smallest when $k=1$, so there is $\lambda(G) \geq \delta(G)$
From the textbook nature $\lambda(G) \leq \delta(G)$, so $\lambda(G) = \delta(G)$

题目 8 (CZ 5.34)

解答:
In figure H, for any point u, v that differs from w, there are at least k disjoint $u-v$ paths in figure G. Obviously, in the increase point and edge H chart, at least k bars do not intersect $u-v$ paths.
If one of the points is w, it is not generally assumed that u is w, and it is now necessary to prove that there are k disjoint $u-v$ paths from v to u.
Suppose k points in S are $n_1, n_2, ..., n_k$.
From v to n_1 there are k roads that do not intersect with each other.
Suppose one of them passes $n_i(ineq1)$, then its and $k-1$ must not pass n_i. In summary, there are at least 1 roads in it, and the point in the S passed through is only n_1.
Similarly, for any point n_i in S, there is at least one path such that from v to n_i does not pass through the total other points of S. The path from v to n_i, plus the side of (n_i, w), forms the k disjoint $u-v$ path from v to u.
In summary, Figure H is k connected.

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2 Open Topics

Open Topics 1 (Tarjan's Algorithm)
[参考资料: R. Tarjan, "Depth-first search and linear graph algorithms," 12th Annual Symposium on Switching and Automata Theory (swat 1971), East Lansing, MI, USA, 1971, pp. 114-121, doi: 10.1109/SWAT.1971.10.]

Open Topics 2 (证明 Harary 图是 r-连通的)

3 反馈