第 *3-14* 讲*:* 平面图与图的染色

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评分：_____　　**评阅：**_____

2023 年 *2* 月 *5* 日

请独立完成作业，不得抄袭。
若得到他人帮助, 请致谢。
若参考了其它资料，请给出引用。
鼓励讨论，但需独立书写解题过程。

1　作业 (必做部分)

题目 1 (CZ 9.3)

解答：
We can know that:

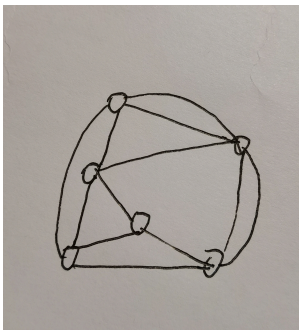
$$p = \begin{cases} 1, & f(x_1) \leq f(x) \\ e^{-\frac{f(x_1)-f(x)}{T}}, & f(x_1) > f(x) \end{cases} \tag{1}$$

(a) $m = \frac{(3+4+4+4+5+6+6)}{2} = 16, n = 6.$
For $m > 3n - 6$, so G is nonplanar.

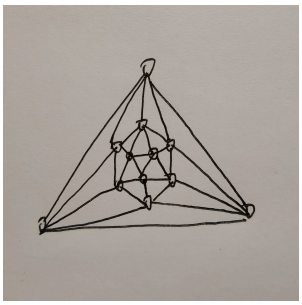
(b) $m = \frac{(4+4+4+5+5+5+6+6+6+7+7+7)}{2} = 33, n = 12.$
For $m > 3n - 6$, so G is nonplanar.

题目 2 (CZ 9.5)

解答：
(a)

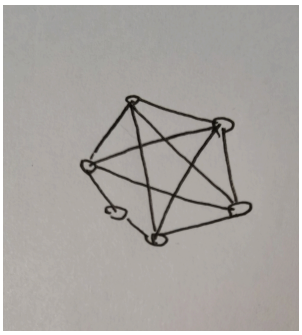


K_5
(b)
K_6
(c)Since $\frac{r \times n}{2} \leq 3n - 6$, we can know that there is no r-regular planar graph for $r \geq 6$.

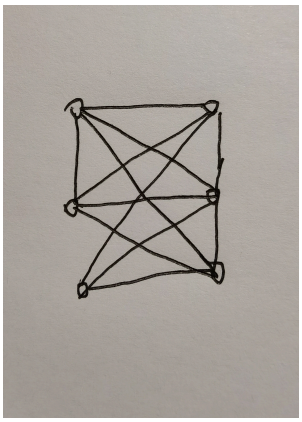


题目 3 (CZ 9.7)

解答:
(a)C_4
(b)There is no such graph.The graph of order 4 doesn't include $K_{3,3}, K_5$ and subgraphs for their segmentation.
(c)

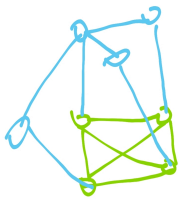


(d)Since $n = 5, m = 10$, we can know it is K_5. So it can not be a planar graph.
(e)C_3
(f)



题目 4 (CZ 9.8)

解答:
$G = K_4 \times K_2$ can not be a planar graph.
The subgraph of $K_4 \times K_2$ contains a breakdown of K_5, as shown in the figure.



题目 5 (CZ 10.2)

解答:
(a) If the maximum degree of the Petersen diagram is 3 and contains odd circles, its color number is 3.
(b) n-cube is a bipartite with a color number of 2
(c) When $n = 1$, it is the number of color 2.
$n > 1$ and n is even, the number of colors is 3.
$n > 1$ and n is odd, the color number is 4.

题目 6 (CZ 10.3)

解答:
If there is only 1 point, the number of colors is 1. Otherwise, the number of colors is 2.

题目 7 (CZ 10.4)

解答:
(a) No, K_4 is a counterexample.
(b) No, C_4 is a counterexample.
(c) No, K_2 is a counterexample.
(d) No, $K_{3,3}$ is a counterexample.

题目 8 (CZ 10.5)

解答:
Suppose it can be divided into three independent sets of V_1, V_2, V_3.
Generally assumes $|v_1| \geq |v_2| \geq |V_3|$.
If $|v_1| \geq 3$, the number of edges is $\leq |k_6| - |k_{v_1}| = 12$
Otherwise, $|v_1| = |v_2| = |v_3| = 2$, then number of edges $\geq |K_6| - 3 = 12$

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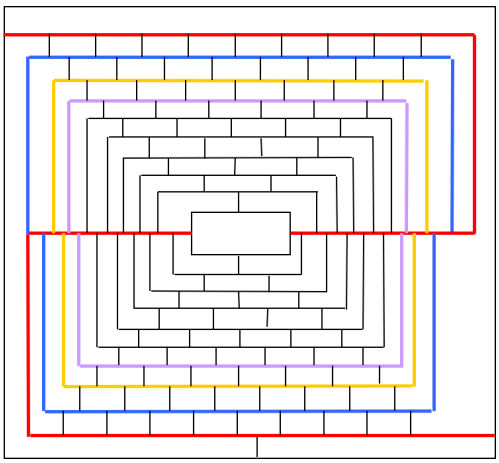
2 Open Topics

Open Topics 1 (请证明 Brooks 定理)
(Brooks' Theorem) For every connected graph G that is not an odd cycle or a complete graph, $\chi(G) \leq \Delta(G)$

Open Topics 2 (Martin Gardner 的愚人节礼物)
《科学美国人》即《Scientific American》，是美国出版的一种著名科学杂志，在国际上极富声誉。该刊 1975 年 4 月号上登载了著名数学专栏作家，马丁 · 加德纳（Martin Gardner）的一篇文章。文章附了一张有着 110 个区域的地图:



加德纳在该图下赫然写道:“四色定理被推翻了!”正文中他还语气肯定地说:该地图不能用少于 5 种颜色使相邻区域着不同颜色。

请问:四色定理真的被推翻了么?

3 反馈