Research Log

Lucas Martin Fraile Vazquez

December 8, 2015

0.1 12/4/2015

First attempt to model the dynamics of the human body during an automotive accident is started. A simplified, two dimensional, head–on collision scenario is chosen. The passenger body is modeled as an inverted pendulum with masses m_o and m_u in the top and bottom ends. The three point seat belt is modeled as a Sash seat belt across the chess, which acts over m_o and a Lap seat belt which acts over m_u. Passenger slipping on the seat is accounted for as a displacement of the mass at the lower end, with friction force, between the seat and the passenger, acting against it.
L is the length between the passengers hip and center of mass of the torso. θ is the clockwise angle defined between the up-straight position of the seat and the passengers back. x is defined as the horizontal distance between the back of the seat and the passengers hip. The body mass of the passenger over the hip is represented by m_o and the mass under the hip by m_u.

Their respective coordinates are:

$$m_o = (Lsin(\theta) + x, Lcos(\theta)) \tag{1}$$

$$m_u = (x, 0) \tag{2}$$

Thus their velocities:

$$m_o = (Lcos(\theta)\frac{d\theta}{dt} + \frac{dx}{dt}, -Lsin(\theta)\frac{d\theta}{dt}) \tag{3}$$

$$m_u = (\frac{dx}{dt}, 0) \tag{4}$$

The Lagrangian is defined as:

$$L = T - V \tag{5}$$

$$T = \frac{m_o}{2}((Lcos(\theta)\dot{\theta} + \dot{x})^2 + (-Lsin(\theta)\dot{\theta})^2 + \frac{m_u}{2}\dot{x}^2 \tag{6}$$

$$V = m_o gLcos(\theta) + F_r x + V_{ssb} + V_{lsb} \tag{7}$$

$$\begin{aligned}\Rightarrow L &= \frac{m_o}{2}((Lcos(\theta)\dot{\theta} + \dot{x})^2 + (-Lsin(\theta)\dot{\theta})^2 + \frac{m_u}{2}\dot{x}^2 - (m_o gLcos(\theta) + F_r x + V_{ssb} + V_{lsb}) \\ &= \frac{m_o}{2}((L^2\dot{\theta}^2 + \dot{x}^2 + 2Lcos(\theta)\dot{\theta}\dot{x}) + \frac{m_u}{2}\dot{x}^2 - (m_o gLcos(\theta) + F_r x + V_{ssb} + V_{lsb})\end{aligned} \tag{8}$$

Where F_r models the "Friction Potential", V_{ssb} the "Sash seat belt Potential" and V_{lsb} the "Lap seat belt Potential".

Following the Euler–Lagrange Equation:

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} = \frac{\partial L}{\partial q} \tag{9}$$

We obtain:

$$m_o((L^2\ddot{\theta} + Lcos(\theta)\ddot{x} - Lsin(\theta)\dot{\theta}\dot{x}) - \frac{d}{dt}\frac{\partial V_{ssb}}{\partial \dot{\theta}} = m_o g Lsin(\theta) - \frac{dV_{ssb}}{d\theta} \tag{10}$$

$$m_o(\ddot{x} + Lcos(\theta)\ddot{\theta} - Lsin(\theta)\dot{\theta}^2) + m_u\ddot{x} = -F_r - \frac{dV_{lsb}}{dx} \tag{11}$$

Modeling the Sash seat belt as a mechanical damper of constant b, the Lap seat belt as a spring of constant k and the Friction force by a dynamic friction coefficient u_d times the normal force,, we obtain:

$$m_o((L^2\ddot{\theta} + Lcos(\theta)\ddot{x} - Lsin(\theta)\dot{\theta}\dot{x}) = m_o g Lsin(\theta) - b\dot{\theta} \tag{12}$$

$$m_o(\ddot{x} + Lcos(\theta)\ddot{\theta} - Lsin(\theta)\dot{\theta}^2) + m_u\ddot{x} = -u_d m_o g - kx \tag{13}$$