Information Theory **DCC/ICEx/UFMG**
Prof. Mário S. Alvim **2024/1**

PROBLEM SET

Decision Theory

(MacKay - Chapter 36)

Necessary reading for this assignment:

- *Information Theory, Inference, and Learning Algorithms* (MacKay):
  - Chapter 36.1: *Rational prospecting*
  - Chapter 36.2: *Further reading*
- *Additional material:*
  - *Slide-set of the lecture on Decision Theory*

Note: The exercises are labeled according to their level of difficulty: `[Easy]`, `[Medium]` or `[Hard]`. This labeling, however, is subjective: different people may disagree on the perceived level of difficulty of any given exercise. Don't be discouraged when facing a hard exercise, you may find a solution that is simpler than the one the instructor had in mind!

Review questions.

1. Answer formally the following questions. In a decision theory problem:
   (a) What do the set of *states of the world* model?
   (b) What do the set of *actions* model?
   (c) What does the *utility function* model?
   (d) What is the quantity we are trying to maximize?

Exercises.

2. `[Medium]` (The bicycle shop.) Zed and Adrian and run a small bicycle shop called "Z to A Bicycles". They must order bicycles for the coming season. Orders for the bicycles must be placed in quantities multiple of twenty (20). The cost per bicycle is \$70 if they order 20, \$67 if they order 40, \$65 if they order 60, and \$64 if they order 80. The bicycles will be sold for \$100 each. Any bicycles left over at the end of the season can be sold (for certain) at \$45 each. If Zed and Adrian run out of bicycles during the season, then they will suffer a loss of "goodwill" among their customers. They estimate this goodwill loss to be \$5 per customer who was unable to buy a bicycle. Zed and Adrian estimate that the demand for bicycles this season will be 10, 30, 50, or 70 bicycles with probabilities of 0.2, 0.4, 0.3, and 0.1 respectively.

   Decide Zed and Adrian's best possible action in this scenario, that is, determine the amount of bicycles they must order for the coming season in order to maximize their expected profit.

3. (MacKay 36.5) `[Hard]`

   (Hint: Model the problem as a decision theory problem (i.e., identify the states of the world, the possible actions, the probabilities involved) in which the utility function $U(x)$ is free. Then impose the restrictions that action A is preferred over B and that action D is preferred over C to derive a contradiction on the existence of $U(x)$.)