PROBLEM SET

Discrete Probability

(Supplemental material based on slide-set)

Necessary reading for this assignment:

- *Slide-set of the review of Discrete Probability*:

    - *An Introduction to Discrete Probability*
    - *Probability Theory*
    - *Bayes' Theorem*
    - *Expected Value and Variance*

Note: The exercises are labeled according to their level of difficulty: `[Easy]`, `[Medium]` or `[Hard]`. This labeling, however, is subjective: different people may disagree on the perceived level of difficulty of any given exercise. Don't be discouraged when facing a hard exercise, you may find a solution that is simpler than the one the instructor had in mind!

Review questions.

1. (Rosen Review Question 7-2) `[Easy]`

    (a) What conditions should be met by the probabilities assigned to the outcomes from a finite sample space?

    (b) What probabilities should be assigned to the outcome of heads and the outcome of tails if heads comes up three times as often as tails?

2. (Rosen Review Question 7-3) `[Easy]`

    (a) Define the conditional probability of an event E given an event F.

    (b) Suppose E is the event that when a die is rolled it comes up an even number, and F is the event that when a die is rolled it comes up 1, 2, or 3. What is the probability of F given E?

3. (Rosen Review Question 7-4) `[Easy]`

    (a) When are two events E and F independent?

    (b) Suppose E is the event that an even number appears when a fair die is rolled, and F is the event that a 5 or 6 comes up. Are E and F independent?

4. (Rosen Review Question 7-5) `[Easy]`

    (a) What is a random variable?

    (b) What are the possible values assigned by the random variable X that assigns to a roll of two dice the larger number that appears on the two dice? .

5. (Rosen Review Question 7-6) `[Easy]`

    (a) Define the expected value of a random variable X.

(b) What is the expected value of the random variable X that assigns to a roll of two dice the larger number that appears on the two dice?

6. (Rosen Review Question 7-8) [Easy]

   (a) What is meant by a *"Bernoulli trial"*?
   (b) What is the probability of k successes in n independent Bernoulli trials?
   (c) What is the expected value of the number of successes in n independent Bernoulli trials?

7. (Rosen Review Question 7-9) [Easy]

   (a) What does the linearity of expectations of random variables mean?

8. (Rosen Review Question 7-11) [Easy] State Bayes' theorem and use it to find $p(F \mid E)$ if $p(E \mid F) = 1/3$, $p(E \mid \overline{F}) = 1/4$, and $p(F) = 2/3$, where E and F are events from a sample space S.

9. (Rosen Review Question 7-13) [Easy]

   (a) What is the variance of a random variable?

Exercises.

10. (Rosen 7.2-11) [Hard] Suppose that E and F are events such that $p(E) = 0.7$ and $p(F) = 0.5$. Show that $p(E \cup F) \geq 0.7$ and $p(E \cap F) \geq 0.2$.

11. (Rosen 7.2-17) [Hard] If E and F are independent events, prove or disprove that $\overline{E}$ and F are necessarily independent events.

12. (Rosen 7.2-25) [Medium] What is the conditional probability that a randomly generated bit string of length four contains at least two consecutive 0s, given that the first bit is a 1? (Assume the probabilities of a 0 and a 1 are the same.)

13. (Rosen 7.2-27(b)) [Medium] Let E and F be the events that a family of 4 children has children of both sexes and has at most one boy, respectively. Are E and F independent?

14. (Rosen 7.3-9) [Medium] Suppose that 8% of the patients tested in a clinic are infected with HIV. Furthermore, suppose that when a blood test for HIV is given, 98% of the patients infected with HIV test positive and that 3% of the patients not infected with HIV test positive. What is the probability that

   (a) a patient testing positive for HIV with this test is infected with it?
   (b) a patient testing positive for HIV with this test is not infected with it?
   (c) a patient testing negative for HIV with this test is infected with it?
   (d) a patient testing negative for HIV with this test is not infected with it?