Information Theory   **DCC/ICEx/UFMG**
Prof. Mário S. Alvim   **2024/1**

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PROBLEM SET
DEPENDENT RANDOM VARIABLES
(MACKAY - CHAPTER 8)

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Necessary reading for this assignment:

- *Information Theory, Inference, and Learning Algorithms* (MacKay): *Information Theory, Inference, and Learning Algorithms* (MacKay):

    - Chapter 8.1: *More about entropy*

Note: The exercises are labeled according to their level of difficulty: [Easy], [Medium] or [Hard]. This labeling, however, is subjective: different people may disagree on the perceived level of difficulty of any given exercise. Don't be discouraged when facing a hard exercise, you may find a solution that is simpler than the one the instructor had in mind!

Review questions.

1. The entropy $H(X) = -\sum_x p(x) \log p(x)$ can be interpreted as the uncertainty one has about the random variable X. With that in mind, for each of the items below, give its name, its mathematical formula and explain its meaning in terms of uncertainty.

    (a) $H(X, Y)$.
    (b) $H(X \mid Y)$.
    (c) $I(X; Y)$.
    (d) $I(X; Y \mid Z)$.

2. State the following "laws" of information theory.

    (a) The chain rule for entropy $H(X_1, X_2, \ldots, X_n)$.
    (b) The chain rule for mutual information $I(X_1, X_2, \ldots, X_n; Y)$.
    (c) The data-processing inequality (DPI), and explain what it intuitively means.

Exercises.

3. (MacKay 8.1) [Medium]

4. (MacKay 8.2) [Medium]

5. (MacKay 8.5) [Hard]

6. (MacKay 8.6) [Easy]

7. (MacKay 8.7) [Medium]

8. (MacKay 8.10) [Medium]