Information Theory

Problem set 02 - Probability, entropy and inference

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1. (a) The frequentist interpretation of probability is, obviously, associated with the concept of frequency. It states that the probability $p(s)$ of a event s should reflect the frequency that the event s happens compared to the rest of the events in the sample space, as the number of experiments goes to infinity. On the other hand, the bayesian interpretation of probability is more subjective, as it assumes the probability $p(s)$ of a event s is the degree of belief we should have that the outcome of an experiment in the sample space will be s.

   (b) b

   (c) c

2. Obviously, the probability that the first ball is white, P_f, is equal to $\frac{w}{w+b}$, since there are $w + b$ balls and w of them are white. Let's calculate now the event of the second ball being white. We should consider that the first ball can be either white or black in this case, and each outcome for the first ball can affect the outcome for the second ball. So, we have that the probability P_s of the second ball being white is defined as the sum of the probability that the second ball is white given the fact the first ball was white and the probability that the second ball is white given that the first

ball was black. In mathmatical terms, it would be:

$$P_s = P_f \frac{w-1}{w-1+b} + (1-P_f)\frac{w}{w+b-1} \tag{1}$$

$$P_s = P_f \frac{w}{w+b-1} - \frac{P_f}{w+b-1} + \frac{w}{w+b-1} - P_f \frac{w}{w+b-1} \tag{2}$$

$$P_s = \frac{w-P_f}{w+b-1} \tag{3}$$

$$P_s = \frac{w-\frac{w}{w+b}}{w+b-1} \tag{4}$$

$$P_s = \frac{w(w+b)-w}{(w+b-1)(w+b)} \tag{5}$$

$$P_s = \frac{w(w+b-1)}{(w+b-1)(w+b)} \tag{6}$$

$$P_s = \frac{w}{w+b} \tag{7}$$

At the end, we have what we expected, $P_f = P_s = \frac{w}{w+b}$.

3.