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An adaptive dynamic controller for autonomous mobile robot trajectory tracking

Felipe N. Martins [a,*,1], Wanderley C. Celeste [a], Ricardo Carelli [b], Mário Sarcinelli-Filho [a], Teodiano F. Bastos-Filho [a]

[a] Department of Electrical Engineering, Federal University of Espírito Santo (UFES), Av. Fernando Ferrari, 514 Campus Universitário, ES. 29075-910, Vitória, Brazil
[b] Institute of Automatics (INAUT), National University of San Juan (UNSJ), Av. San Martin Oeste 1112 J5400ARL, San Juan, Argentina

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ABSTRACT

This paper proposes an adaptive controller to guide an unicycle-like mobile robot during trajectory tracking. Initially, the desired values of the linear and angular velocities are generated, considering only the kinematic model of the robot. Next, such values are processed to compensate for the robot dynamics, thus generating the commands of linear and angular velocities delivered to the robot actuators. The parameters characterizing the robot dynamics are updated on-line, thus providing smaller errors and better performance in applications in which these parameters can vary, such as load transportation. The stability of the whole system is analyzed using Lyapunov theory, and the control errors are proved to be ultimately bounded. Simulation and experimental results are also presented, which demonstrate the good performance of the proposed controller for trajectory tracking under different load conditions.

1. Introduction

Among different mobile robot structures, unicycle-like platforms are frequently adopted to accomplish different tasks, due to their good mobility and simple configuration. Nonlinear control for this type of robot has been studied for several years (de Wit & Sordalen, 1992; Kanayama, Kimura, Miyazaki, & Noguchi, 1990), and such robot structure has been used in various applications, such as surveillance (Birk & Kenn, 2002; Patel, Sanyal, & Sobh, 2006) and floor cleaning (Prassler, Ritter, Schaeffer, & Fiorini, 2000). Other applications, like industrial load transportation using automated guided vehicles (AGVs) (Stouten & Graaf, 2004), automatic highway maintenance and construction (Feng & Velinsky, 1997), and autonomous wheelchairs (Frizera Neto, Celeste, Bastos-Filho, Martins, & Sarcinelli-Filho, 2006; Rao et al., 2002), also make use of the unicycle-like structure. Some authors (Antonini, Ippoliti, & Longhi, 2006; Corradini & Orlando, 2002; Feng & Velinsky, 1997; Frizera Neto et al., 2006; Kanayama et al., 1990; Rao et al., 2002; Stouten & Graaf, 2004) have addressed the problem of trajectory tracking, a quite important functionality that allows a mobile robot to describe a desired trajectory when accomplishing a task.

An important issue in the nonlinear control of AGVs is that most controllers designed so far are based only on the kinematics of the mobile robot (Carelli, Secchi, & Mut, 1999; de Wit & Sordalen, 1992; Freire & Carelli, 2003; Kanayama et al., 1990; Künhe, Gomes & Fetter, 2005; Wu, Chen, Wang, & Woo, 1999). However, when high-speed movements and/or heavy load transportation are required, it becomes essential to consider the robot dynamics, in addition to its kinematics. Thus, some controllers that compensate for the robot dynamics have been proposed. As an example, Fierro and Lewis (1995) proposed a combined kinematic/torque control law for nonholonomic mobile robots taking into account the modeled vehicle dynamics. The control commands they used were torques, which are hard to deal with when regarding most commercial robots. Moreover, only simulation results were reported. Fierro and Lewis (1997) also proposed a robust-adaptive controller based on neural networks to deal with disturbances and non-modeled dynamics, although not reporting experimental results. Das and Kar (2006) showed an adaptive fuzzy logic-based controller in which the uncertainty is estimated by a fuzzy logic system and its parameters were tuned on-line. The dynamic model included the actuator dynamics, and the commands generated by the controller were voltages for the robot motors.

An adaptive trajectory tracking controller based on a dynamic model that generates torques was proposed by Fukao, Nakagawa and Adachi (2000). Nevertheless, only simulation results were

* Corresponding author. Tel./fax: +55 27 3335 2644.
E-mail addresses: felipe.n.martins@gmail.com (F.N. Martins), wanderley@ele.ufes.br (W.C. Celeste), rcarelli@inaut.unsj.edu.ar (R. Carelli), mario.sarcinelli@ele.ufes.br (M. Sarcinelli-Filho), tfbastos@ele.ufes.br (T.F. Bastos-Filho).
[1] On leave from the Department of Automation and Control Engineering, Centro Leste College (UCL).

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shown. Other trajectory tracking controllers based on robot dynamics were developed by Dong and Huo (1999), Liu, Zhang, Yang, and Yu (2004) and Dong and Guo (2005), but just simulation results were presented. By its turn, Kim, Shin, and Lee (2000) proposed a two-part robust-adaptive controller for a mobile robot, whose first part is based on the robot kinematics and is responsible for generating the references for the second one, which compensates for the modeled dynamics. Once more, no experimental results were reported.

Thus, control signals generated by most dynamic controllers reported in the literature are torques (Corradini & Orlando, 2002; Dong & Guo, 2005; Dong & Huo, 1999; Fierro & Lewis, 1995, 1997; Fukao et al., 2000; Kim et al., 2000; Liu et al., 2004) or voltages (Das & Kar, 2006) for the robot motors, while commercial robots usually receive velocity commands. Antonini et al.(2006) proposed a switching controller with on-line learning and hierarchical architecture, investigating Neural Network-based methodologies to compensate the effects of non-modeled phenomena. The Neural Networks were used for identification and control, and the control signals were linear and angular velocities, but the real-time implementation of their solution required a high-performance computer architecture based on a multiprocessor system. On the other hand, De La Cruz and Carelli (2006) proposed a dynamic model using linear and angular velocities as inputs, and showed the design of a trajectory tracking controller based on their model. One advantage of their controller is that its parameters are directly related to the robot parameters. However, if the parameters are not correctly identified or if they change with time, for example, due to load variation, the performance of their controller will be severely affected.

To reduce performance degradation, on-line parameter adaptation becomes quite important in applications in which the robot dynamic parameters may vary, such as load transportation. It is also useful when the knowledge of the dynamic parameters is limited or does not exist at all.

In this paper, an adaptive trajectory-tracking controller based on the robot dynamics is proposed, and its stability property is proved using the Lyapunov theory. The design of the controller was divided in two parts (Kim et al., 2000), each part being a controller itself. The first one is a kinematic controller, which is based on the robot kinematics, and the second one is a dynamic controller, which is based on the robot dynamics. The dynamic controller is capable of updating the estimated parameters, which are directly related to physical parameters of the robot. Both controllers working together form a complete trajectory-tracking controller for the mobile robot. The controllers have been designed based on the model of a unicycle-like mobile robot proposed by De La Cruz and Carelli (2006). A σ-modification term (Sastry & Bodson, 1989; Aström & Wittenmark, 1995) is applied to the parameter-updating law to prevent possible parameter drift. The asymptotic stability of both the kinematic and the dynamic controllers is proven. Simulation results show that parameter drift does not arise even when the system works for a long period of time. Experimental results regarding such a controller are also presented and show that the proposed controller is capable of updating its parameters in order to reduce the tracking error. An experiment dealing with the case of load transportation is also presented, and the results show that the proposed controller is capable of guiding the robot to follow a desired trajectory with a quite small error even when its dynamic parameters change.

The main contributions of the paper are: (1) the use of a dynamic model whose input commands are velocities, which is usual in commercial mobile robots, while most of the works in the literature deals with torque commands; (2) the design of an adaptive controller with a σ-modification term, which makes it robust, with the corresponding stability study for the whole adaptive control system; and (3) the presentation of experimental results showing the good performance of the controller in a typical industrial application, namely load transportation.

The paper is hereinafter organized in the following sections: Section 2, which shows the mathematical representation of the complete unicycle-like robot model; Sections 3 and 4, where the kinematic and dynamic controllers are discussed, respectively; Section 5, which presents some simulations and experimental results; and, finally, Section 6, where some conclusions are highlighted.

2. Dynamic model

In this section, the dynamic model of the unicycle-like mobile robot proposed by De La Cruz and Carelli (2006) is reviewed. Fig. 1 depicts the mobile robot, its parameters and variables of interest. u and ω are the linear and angular velocities developed by the robot, respectively, G is the center of mass of the robot, C is the position of the castor wheel, E is the location of a tool onboard the robot, h is the point of interest with coordinates x and y in the XY plane, ψ is the robot orientation, and a is the distance between the point of interest and the central point of the virtual axis linking the traction wheels (point B). The complete mathematical model is written as (De La Cruz & Carelli, 2006)

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \\ \dot{u} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} u\cos\psi - a\omega\sin\psi \\ u\sin\psi + a\omega\cos\psi \\ \omega \\ \dfrac{\theta_3}{\theta_1}\omega^2 - \dfrac{\theta_4}{\theta_1}u \\ -\dfrac{\theta_5}{\theta_2}u\omega - \dfrac{\theta_6}{\theta_2}\omega \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \dfrac{1}{\theta_1} & 0 \\ 0 & \dfrac{1}{\theta_2} \end{bmatrix} \begin{bmatrix} u_{ref} \\ \omega_{ref} \end{bmatrix} + \begin{bmatrix} \delta_x \\ \delta_y \\ 0 \\ \delta_u \\ \delta_\omega \end{bmatrix}, \qquad (1)$$

where u_{ref} and ω_{ref} are the desired values of the linear and angular velocities, respectively, and represent the input signals of the system.

A vector of identified parameters and a vector of parametric uncertainties are associated with the above model of the mobile robot, which are, respectively,

$$\boldsymbol{\theta} = [\theta_1 \ \ \theta_2 \ \ \theta_3 \ \ \theta_4 \ \ \theta_5 \ \ \theta_6]^{\mathrm{T}}$$

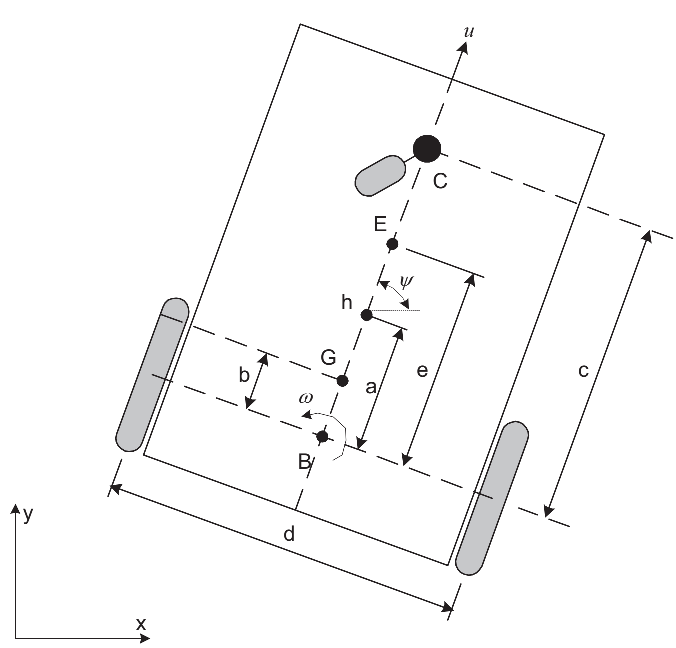

Fig. 1. The unicycle-like mobile robot.

and

$$\boldsymbol{\delta} = [\delta_x \;\; \delta_y \;\; 0 \;\; \delta_u \;\; \delta_\omega]^{\mathrm{T}},$$

where δ_x and δ_y are functions of the slip velocities and the robot orientation, δ_u and δ_ω are functions of physical parameters as mass, inertia, wheel and tire diameters, parameters of the motors and its servos, forces on the wheels, etc., and are considered as disturbances.

The parameters included in vector $\boldsymbol{\theta}$ are functions of some physical parameters of the robot, such as its mass m, its moment of inertia I_z at G, the electrical resistance R_a of its motors, the electromotive constant k_b of its motors, the constant of torque k_a of its motors, the coefficient of friction B_e, the moment of inertia I_e of each group rotor-reduction gear-wheel, the radius r of the wheels, the nominal radius R_t of the tires, and the distances b and d. It is assumed that the robot servos have PD controllers to control the velocities of each motor, with proportional gains $k_{PT}>0$ and $k_{PR}>0$, and derivative gains $k_{DT}\geqslant 0$ and $k_{DR}\geqslant 0$. It is also assumed that the motors associated to the driven wheels are DC motors with identical characteristics, with negligible inductance. The equations describing the parameters $\boldsymbol{\theta}$ were firstly presented in (De La Cruz & Carelli, 2006), and are reproduced here for convenience. They are

$$\theta_1 = \frac{\left[\dfrac{R_a}{k_a}(mR_t r + 2I_e) + 2rk_{DT}\right]}{(2rk_{PT})},$$

$$\theta_2 = \frac{\left[\dfrac{R_a}{k_a}(I_e d^2 + 2R_t r(I_z + mb^2)) + 2rdk_{DR}\right]}{(2rdk_{PR})},$$

$$\theta_3 = \frac{R_a}{k_a}\frac{mbR_t}{2k_{PT}},$$

$$\theta_4 = \frac{\frac{R_a}{k_a}\left(\frac{k_a k_b}{R_a} + B_e\right)}{(rk_{PT})} + 1,$$

$$\theta_5 = \frac{R_a}{k_a}\frac{mbR_t}{dk_{PR}},$$

$$\theta_6 = \frac{R_a}{k_a}\left(\frac{k_a k_b}{R_a} + B_e\right)\frac{d}{2rk_{PR}} + 1.$$

It should be stressed that $\theta_i>0$, $i = 1,2,4,6$. Parameters θ_3 and θ_5 will be null if, and only if, the center of mass G is exactly in the central point of the virtual axis linking the traction wheels (point B), i.e., $b = 0$. In this paper it is assumed that $b\neq 0$.

The robot's model presented in (1) is partitioned into a kinematic part and a dynamic part, as shown in Fig. 2. Therefore, two controllers are implemented, based on feedback linearization, for both the kinematic and dynamic models of the robot.

3. The kinematic controller

3.1. Design

The design of the kinematic controller is based on the kinematic model of the robot, assuming that the disturbance term in (1) is a zero vector. From (1), the robot's kinematic model is given by

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \cos\psi & -a\sin\psi \\ \sin\psi & a\cos\psi \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ \omega \end{bmatrix},$$

whose output are the coordinates of the point of interest, thus meaning $\mathbf{h} = [x \;\; y]^{\mathrm{T}}$. Hence

$$\dot{\mathbf{h}} = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} \cos\psi & -a\sin\psi \\ \sin\psi & a\cos\psi \end{bmatrix} \begin{bmatrix} u \\ \omega \end{bmatrix} = \mathrm{A}\begin{bmatrix} u \\ \omega \end{bmatrix} \tag{2}$$

with

$$\mathbf{A} = \begin{bmatrix} \cos\psi & -a\sin\psi \\ \sin\psi & a\cos\psi \end{bmatrix},$$

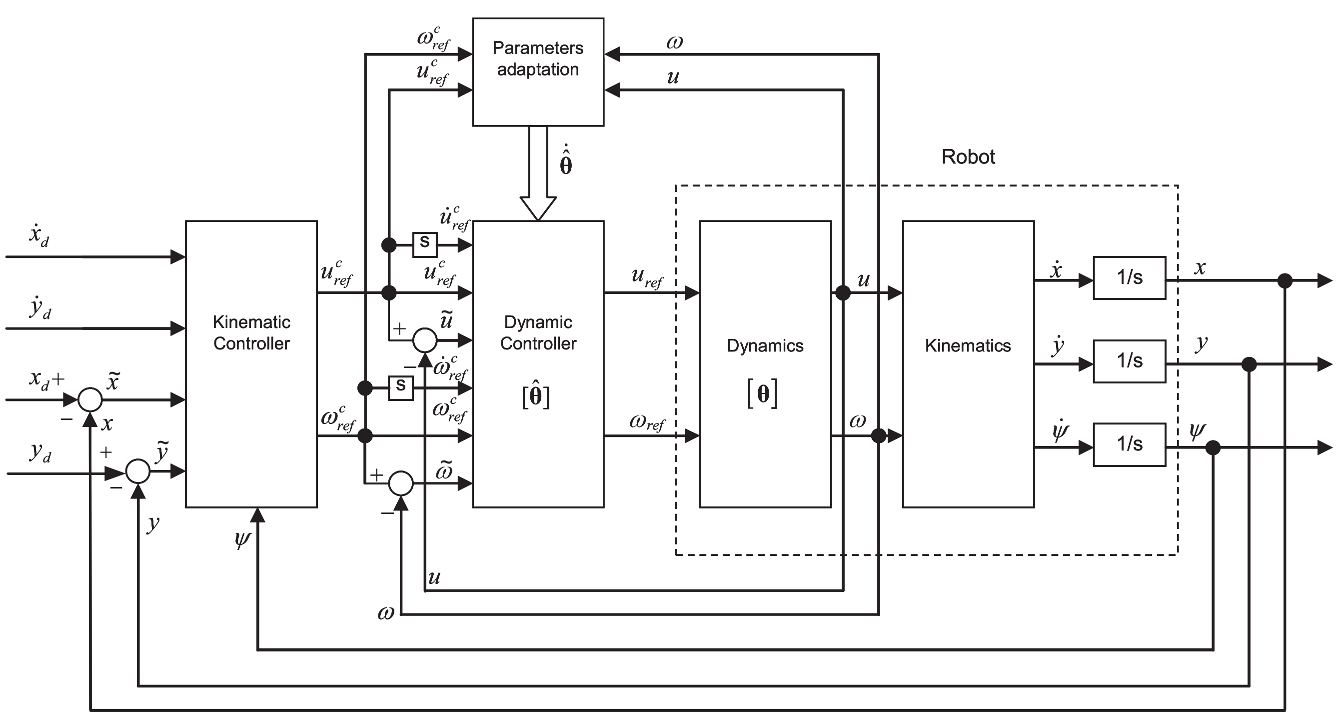

Fig. 2. Control structure.

whose inverse is

$$\mathbf{A}^{-1} = \begin{bmatrix} \cos\psi & \sin\psi \\ -\frac{1}{a}\sin\psi & \frac{1}{a}\cos\psi \end{bmatrix}.$$

Therefore, the inverse kinematics is given by

$$\begin{bmatrix} u \\ \omega \end{bmatrix} = \begin{bmatrix} \cos\psi & \sin\psi \\ -\frac{1}{a}\sin\psi & \frac{1}{a}\cos\psi \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} \tag{3}$$

and the kinematic control law proposed to be applied to the robot is given by

$$\begin{bmatrix} u^c_{ref} \\ \omega^c_{ref} \end{bmatrix} = \begin{bmatrix} \cos\psi & \sin\psi \\ -\frac{1}{a}\sin\psi & \frac{1}{a}\cos\psi \end{bmatrix} \begin{bmatrix} \dot{x}_d + l_x \tanh\left(\frac{k_x}{l_x}\tilde{x}\right) \\ \dot{y}_d + l_y \tanh\left(\frac{k_y}{l_y}\tilde{y}\right) \end{bmatrix}. \tag{4}$$

Here, $\tilde{x} = x_d - x$, and $\tilde{y} = y_d - y$ are the current position errors in the axes X and Y, respectively, $k_x > 0$ and $k_y > 0$ are the gains of the controller, $l_x \in \Re$, and $l_y \in \Re$ are saturation constants, and (x,y) and (x_d,y_d) are the current and the desired coordinates of the point of interest, respectively. The objective of such a controller is to generate the references of linear and angular velocities for the dynamic controller, as shown in Fig. 2.

3.2. Stability analysis

In this analysis it is supposed a perfect velocity tracking, allowing to equate (3) and (4) under the assumption of $u \equiv u^c_{ref}$ and $\omega \equiv \omega^c_{ref}$, which means that the dynamic effects of the robot are ignored. Then, the closed-loop equation, in terms of the velocity errors, is

$$\begin{bmatrix} \dot{\tilde{x}} \\ \dot{\tilde{y}} \end{bmatrix} + \begin{bmatrix} l_x & 0 \\ 0 & l_y \end{bmatrix} \begin{bmatrix} \tanh\left(\frac{k_x}{l_x}\tilde{x}\right) \\ \tanh\left(\frac{k_y}{l_y}\tilde{y}\right) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \tag{5}$$

Now, defining the output error vector $\tilde{\mathbf{h}} = [\tilde{x} \;\; \tilde{y}]^T$, Eq. (5) can be written as

$$\dot{\tilde{\mathbf{h}}} = -\left[l_x \tanh\left(\frac{k_x}{l_x}\tilde{x}\right) \quad l_y \tanh\left(\frac{k_y}{l_y}\tilde{y}\right) \right]^T,$$

which has a unique equilibrium point at the origin.

To conclude on the stability of such equilibrium point, the Lyapunov candidate function

$$V = \tfrac{1}{2}\tilde{\mathbf{h}}^T\tilde{\mathbf{h}} > 0$$

is considered, whose first time derivative

$$\dot{V} = \tilde{\mathbf{h}}^T\dot{\tilde{\mathbf{h}}} = -\tilde{x} l_x \tanh\left(\frac{k_x}{l_x}\tilde{x}\right) - \tilde{y} l_y \tanh\left(\frac{k_y}{l_y}\tilde{y}\right) < 0$$

is negative definite.

Hence, one can straightforwardly conclude that the system characterized so far, when controlled through the kinematic controller here proposed, has an asymptotically stable equilibrium at the origin, which means that $\tilde{x}(t) \to 0$ and $\tilde{y}(t) \to 0$ as $t \to \infty$.

Note 1: Considering the case in which the reference remains constant for a while, the robot tends to reach such a reference point and to stop there. Nevertheless, it should be also guaranteed that the robot orientation given by the variable ψ keeps bounded. It can be seen from (4) that $\omega^c_{ref} = 0$ when $\tilde{x} = 0$, $\dot{x}_d = 0$, $\tilde{y} = 0$ and $\dot{y}_d = 0$. Under the assumption that $\omega \equiv \omega^c_{ref}$, it can be concluded that $\dot{\psi} = \omega = 0$, and $\psi(t) \to \psi_{constant}$.

Note 2: The stability of the whole system will be revisited in the next section, in which an adaptive dynamic controller is added to the kinematic controller in order to implement the whole control scheme of Fig. 2.

4. The adaptive dynamic controller

4.1. Design

The dynamic controller receives from the kinematic controller the references for linear and angular velocities, and generates another pair of linear and angular velocities to be delivered to the robot servos, as shown in Fig. 2.

The design of the adaptive dynamic controller is based on the parameterized dynamic model of the robot. After neglecting the disturbance terms δ_u and δ_ω the dynamic part of Eq. (1) is

$$\begin{bmatrix} \dot{u} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} \frac{\theta_3}{\theta_1}\omega^2 - \frac{\theta_4}{\theta_1}u \\ -\frac{\theta_5}{\theta_2}u\omega - \frac{\theta_6}{\theta_2}\omega \end{bmatrix} + \begin{bmatrix} \frac{1}{\theta_1} & 0 \\ 0 & \frac{1}{\theta_2} \end{bmatrix} \begin{bmatrix} u_{ref} \\ \omega_{ref} \end{bmatrix},$$

or

$$\begin{bmatrix} \dot{u} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} \frac{\theta_3}{\theta_1}\omega^2 - \frac{\theta_4}{\theta_1}u + \frac{u_{ref}}{\theta_1} \\ -\frac{\theta_5}{\theta_2}u\omega - \frac{\theta_6}{\theta_2}\omega + \frac{\omega_{ref}}{\theta_2} \end{bmatrix}.$$

By rearranging the terms, the linear parameterization of the dynamic equation can be expressed as

$$\begin{bmatrix} u_{ref} \\ \omega_{ref} \end{bmatrix} = \begin{bmatrix} \dot{u} & 0 & -\omega^2 & u & 0 & 0 \\ 0 & \dot{\omega} & 0 & 0 & u\omega & \omega \end{bmatrix} \times [\theta_1 \;\; \theta_2 \;\; \theta_3 \;\; \theta_4 \;\; \theta_5 \;\; \theta_6]^T,$$

which can also be rewritten as

$$\begin{bmatrix} u_{ref} \\ \omega_{ref} \end{bmatrix} = \begin{bmatrix} \theta_1 & 0 \\ 0 & \theta_2 \end{bmatrix} \begin{bmatrix} \dot{u} \\ \dot{\omega} \end{bmatrix} + \begin{bmatrix} 0 & 0 & -\omega^2 & u & 0 & 0 \\ 0 & 0 & 0 & 0 & u\omega & \omega \end{bmatrix} \times [\theta_1 \;\; \theta_2 \;\; \theta_3 \;\; \theta_4 \;\; \theta_5 \;\; \theta_6]^T,$$

or, in a compact form,

$$\mathbf{v_{ref}} = \mathbf{D}\dot{\mathbf{v}} + \boldsymbol{\eta}, \tag{6}$$

where $\mathbf{v_{ref}} = [u_{ref} \;\; \omega_{ref}]^T$, $\mathbf{v} = [u \;\; \omega]^T$,

$$\boldsymbol{\eta} = \begin{bmatrix} 0 & 0 & -\omega^2 & u & 0 & 0 \\ 0 & 0 & 0 & 0 & u\omega & \omega \end{bmatrix} \times [\theta_1 \;\; \theta_2 \;\; \theta_3 \;\; \theta_4 \;\; \theta_5 \;\; \theta_6]^T,$$

and $\mathbf{D} = diag(\theta_1, \theta_2)$.

Based on the inverse dynamics, it is proposed the control law given by

$$\begin{bmatrix} u_{ref} \\ \omega_{ref} \end{bmatrix} = \begin{bmatrix} \theta_1 & 0 \\ 0 & \theta_2 \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 & -\omega^2 & u & 0 & 0 \\ 0 & 0 & 0 & 0 & u\omega & \omega \end{bmatrix} \times [\theta_1 \;\; \theta_2 \;\; \theta_3 \;\; \theta_4 \;\; \theta_5 \;\; \theta_6]^T,$$

which can be rewritten as

$$\mathbf{v_{ref}} = \mathbf{D}\boldsymbol{\sigma} + \boldsymbol{\eta}, \tag{7}$$

where

$$\boldsymbol{\sigma} = [\sigma_1 \;\; \sigma_2]^T,$$

with

$$\sigma_1 = \dot{u}^c_{ref} + k_u\tilde{u}, \;\; k_u > 0,$$

$$\sigma_2 = \dot{\omega}^c_{ref} + k_\omega\tilde{\omega}, \;\; k_\omega > 0,$$

$$\tilde{u} = u^c_{ref} - u \;\text{ and }\; \tilde{\omega} = \omega^c_{ref} - \omega.$$

Eq. (7) can also be written as

$$\mathbf{v_{ref}} = \mathbf{G}(\sigma_1, \sigma_2, u, \omega)\boldsymbol{\theta}, \tag{8}$$

where

$$\mathbf{G} = \begin{bmatrix} \sigma_1 & 0 & -\omega^2 & u & 0 & 0 \\ 0 & \sigma_2 & 0 & 0 & u\omega & \omega \end{bmatrix}$$

and

$$\boldsymbol{\theta} = [\theta_1 \;\; \theta_2 \;\; \theta_3 \;\; \theta_4 \;\; \theta_5 \;\; \theta_6]^{\mathrm{T}}.$$

However, if there is any uncertainty in the parameters of the robot, the control law

$$\mathbf{v_{ref}} = \mathbf{G}\hat{\boldsymbol{\theta}} = \mathbf{G}\boldsymbol{\theta} + \mathbf{G}\tilde{\boldsymbol{\theta}} = \mathbf{D}\boldsymbol{\sigma} + \boldsymbol{\eta} + \mathbf{G}\tilde{\boldsymbol{\theta}} \tag{9}$$

should be considered, instead of (8), where $\boldsymbol{\theta}$ and $\hat{\boldsymbol{\theta}}$ are the real and estimated parameters of the robot, respectively, whereas $\tilde{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}} - \boldsymbol{\theta}$ is the vector of parameter errors.

4.2. Stability analysis

From (6) and (9) it follows that

$$\mathbf{D}\dot{\mathbf{v}} + \boldsymbol{\eta} = \mathbf{D}\boldsymbol{\sigma} + \boldsymbol{\eta} + \mathbf{G}\tilde{\boldsymbol{\theta}},$$

which is equivalent to

$$\mathbf{D}(\boldsymbol{\sigma} - \dot{\mathbf{v}}) = -\mathbf{G}\tilde{\boldsymbol{\theta}}.$$

Considering $\boldsymbol{\sigma} - \dot{\mathbf{v}} = \dot{\tilde{\mathbf{v}}} + \mathbf{K}\tilde{\mathbf{v}}$, where $\tilde{\mathbf{v}} = \mathbf{v}^{\mathbf{c}}_{ref} - \mathbf{v}$ and $\mathbf{v}^{\mathbf{c}}_{ref} = [u^c_{ref} \;\; \omega^c_{ref}]^{\mathrm{T}}$, with $\mathbf{K} = diag(k_u, k_\omega) > 0$, it results that

$$\mathbf{D}(\dot{\tilde{\mathbf{v}}} + \mathbf{K}\tilde{\mathbf{v}}) = -\mathbf{G}\tilde{\boldsymbol{\theta}}$$

or

$$\dot{\tilde{\mathbf{v}}} = -\mathbf{D}^{-1}\mathbf{G}\tilde{\boldsymbol{\theta}} - \mathbf{K}\tilde{\mathbf{v}}, \tag{10}$$

which represents the error equation of the control system.

It is now considered the Lyapunov candidate function

$$V = \tfrac{1}{2}\tilde{\mathbf{v}}^{\mathbf{T}}\mathbf{D}\tilde{\mathbf{v}} + \tfrac{1}{2}\tilde{\boldsymbol{\theta}}^{\mathbf{T}}\gamma\tilde{\boldsymbol{\theta}} \tag{11}$$

with $\gamma \in \Re^{6x6}$ being a positive definite diagonal matrix and $\mathbf{D} > \mathbf{0}$. Using $\tilde{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}} - \boldsymbol{\theta}$ or $\dot{\tilde{\boldsymbol{\theta}}} = \dot{\hat{\boldsymbol{\theta}}}$ (the vector of real parameters $\boldsymbol{\theta}$ is considered constant), the time derivative of the Lyapunov candidate function can be written as

$$\dot{V} = -\tilde{\mathbf{v}}^{\mathbf{T}}\mathbf{D}\mathbf{K}\tilde{\mathbf{v}} - \tilde{\mathbf{v}}^{\mathbf{T}}\mathbf{G}\tilde{\boldsymbol{\theta}} + \tilde{\boldsymbol{\theta}}^{\mathbf{T}}\gamma\dot{\hat{\boldsymbol{\theta}}}. \tag{12}$$

Two parameter-updating laws are considered for the proposed adaptive controller. The first one is given by

$$\dot{\hat{\boldsymbol{\theta}}} = \gamma^{-\mathbf{1}}\mathbf{G}^{\mathbf{T}}\tilde{\mathbf{v}}. \tag{13}$$

Therefore, by substituting (13) in (12),

$$\dot{V} = -\tilde{\mathbf{v}}^{\mathbf{T}}\mathbf{D}\mathbf{K}\tilde{\mathbf{v}} \leqslant 0, \tag{14}$$

which allows verifying the stability of the equilibrium at the origin of the error system (10). This implies that $\tilde{\mathbf{v}}$ and $\tilde{\boldsymbol{\theta}}$ are bounded signals.

Now, by integrating (14) one gets

$$V(T) - V(0) = -\int_0^T \tilde{\mathbf{v}}^{\mathbf{T}}\mathbf{D}\mathbf{K}\tilde{\mathbf{v}}\,\mathrm{d}t$$

and by neglecting $V(T)$, the condition

$$V(0) \geqslant \int_0^{\mathrm{T}} \tilde{v}^{\mathrm{T}}\mathbf{D}\mathbf{K}\tilde{\mathbf{v}}\,\mathrm{d}t \tag{15}$$

is verified.

As $\mathbf{DK}$ is symmetric and positive definite,

$$\lambda_{\min}(\mathbf{DK})\|\tilde{\mathbf{v}}\|^2 \leqslant \tilde{\mathbf{v}}^{\mathbf{T}}\mathbf{D}\mathbf{K}\tilde{\mathbf{v}} \leqslant \lambda_{\max}(\mathbf{DK})\|\tilde{\mathbf{v}}\|^2, \tag{16}$$

with $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ representing the minimum and the maximum eigenvalues of the matrix.

From (15) and (16) one can immediately obtain that

$$\int_0^{\mathrm{T}} \|\tilde{\mathbf{v}}\|^2\,\mathrm{d}t \leqslant \frac{V(0)}{\lambda_{\min}(\mathbf{DK})}, \quad \forall T,$$

which means that $\tilde{\mathbf{v}}$ is a square-integrable signal. Besides, $\dot{\tilde{\mathbf{v}}}$ is bounded, as it can be seen from $\dot{\tilde{\mathbf{v}}} = -\mathbf{D}^{-1}(\mathbf{G}\tilde{\boldsymbol{\theta}}) - \mathbf{K}\tilde{\mathbf{v}}$ and noting that $\tilde{\mathbf{v}}, \tilde{\boldsymbol{\theta}}$and $\mathbf{G}$ are all bounded. Now, according to the Barbalat lemma (Aström & Wittenmark, 1995), one concludes that $\tilde{\mathbf{v}} \to \mathbf{0}$ as $t \to \infty$, which guarantees the asymptotic convergence of the control errors to zero.

However, the parameter-updating law in (13) works as an integrator and, therefore, can cause robustness problems in the presence of measurement errors, noise or disturbances. One possible way to prevent parameter drift is then to turn off the parameter-updating when the error is smaller than a boundary value, as shown in (Martins, Celeste, Carelli, Sarcinelli Filho, & Bastos Filho, 2007). Another well known way to avoid parameter drift is to change the parameter-updating law by introducing a σ-modification (Kaufman, Barkana, & Sobel, 1998; Sastry & Bodson, 1989). Nasisi and Carelli (2003) have presented an adaptive visual servo controller with σ-modification applied to a robot manipulator.

Thus, the parameter-updating law

$$\dot{\hat{\boldsymbol{\theta}}} = \gamma^{-1}\mathbf{G}^{\mathrm{T}}\tilde{\mathbf{v}} - \gamma^{-1}\boldsymbol{\Gamma}\hat{\boldsymbol{\theta}} \tag{17}$$

is considered here, instead of (13), to include the σ-modification, with $\boldsymbol{\Gamma} \in \Re^{6x6}$ a positive diagonal gain matrix. Such a parameter-updating law can also be written as

$$\dot{\tilde{\boldsymbol{\theta}}} = \gamma^{-1}\mathbf{G}^{\mathrm{T}}\tilde{\mathbf{v}} - \gamma^{-1}\boldsymbol{\Gamma}\tilde{\boldsymbol{\theta}} - \gamma^{-\mathbf{1}}\boldsymbol{\Gamma}\boldsymbol{\theta}. \tag{18}$$

Now, after substituting (18) in (12) it results that

$$\dot{V} = -\tilde{\mathbf{v}}^{\mathrm{T}}\mathbf{D}\mathbf{K}\tilde{\mathbf{v}} - \tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\Gamma}\tilde{\boldsymbol{\theta}} - \tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\Gamma}\boldsymbol{\theta}. \tag{19}$$

Considering the constants

$$\mu_{DK} = \chi(\mathbf{DK}) \quad \text{and} \quad \mu_\Gamma = \chi(\boldsymbol{\Gamma}),$$

where $\chi(\mathbf{A}) = \sqrt{\lambda_{\min}(\mathbf{A}^{\mathrm{T}}\mathbf{A})}$ is the minimum singular value of a matrix $\mathbf{A}$, with $\lambda_{\min}(\cdot)$ being the smallest eigenvalue of a matrix, it results that

$$\dot{V} \leqslant -\mu_{DK}\|\tilde{\mathbf{v}}\|^2 - \mu_\Gamma\|\tilde{\boldsymbol{\theta}}\|^2 + \mu_\Gamma\|\tilde{\boldsymbol{\theta}}\|\|\boldsymbol{\theta}\|. \tag{20}$$

Considering the difference square

$$\left(\frac{1}{\xi}\|\tilde{\boldsymbol{\theta}}\| - \xi\|\boldsymbol{\theta}\|\right)^2 = \frac{1}{\xi^2}\|\tilde{\boldsymbol{\theta}}\|^2 - 2\|\tilde{\boldsymbol{\theta}}\|\|\boldsymbol{\theta}\| + \xi^2\|\boldsymbol{\theta}\|^2,$$

it can be written that

$$\|\tilde{\boldsymbol{\theta}}\|\|\boldsymbol{\theta}\| = \frac{1}{2\xi^2}\|\tilde{\boldsymbol{\theta}}\|^2 + \frac{\xi^2}{2}\|\boldsymbol{\theta}\|^2 - \frac{1}{2}\left(\frac{1}{\xi}\|\tilde{\boldsymbol{\theta}}\| - \xi\|\boldsymbol{\theta}\|\right)^2$$

with $\xi \in \Re^+$.

By neglecting the negative term, the inequality

$$\|\tilde{\boldsymbol{\theta}}\|\|\boldsymbol{\theta}\| \leqslant \frac{1}{2\xi^2}\|\tilde{\boldsymbol{\theta}}\|^2 + \frac{\xi^2}{2}\|\boldsymbol{\theta}\|^2 \tag{21}$$

is obtained. Now, by replacing (21) in (20) one gets

$$\dot{V} \leqslant -\mu_{DK}\|\tilde{\mathbf{v}}\|^2 - \mu_\Gamma\|\tilde{\boldsymbol{\theta}}\|^2 + \mu_\Gamma\left(\frac{1}{2\xi^2}\|\tilde{\boldsymbol{\theta}}\|^2 + \frac{\xi^2}{2}\|\tilde{\boldsymbol{\theta}}\|^2\right),$$

or, equivalently,

$$\dot{V} \leqslant -\mu_{DK}\|\tilde{\mathbf{v}}\|^2 - \mu_\Gamma\left(1 - \frac{1}{2\xi^2}\right)\|\tilde{\boldsymbol{\theta}}\|^2 + \mu_\Gamma\frac{\xi^2}{2}\|\boldsymbol{\theta}\|^2. \tag{22}$$

In the sequence, the parameters

$$\alpha_1 = \mu_{DK} > 0 \quad \text{and} \quad \alpha_2 = \mu_\Gamma\left(1 - \frac{1}{2\xi^2}\right) > 0$$

are defined, with ξ conveniently selected. Then, (22) can be rewritten as

$$\dot{V} \leq -\alpha_1\|\tilde{\mathbf{v}}\|^2 - \alpha_2\|\tilde{\boldsymbol{\theta}}\|^2 + \rho, \tag{23}$$

where

$$\rho = \mu_\Gamma\frac{\xi^2}{2}\|\boldsymbol{\theta}\|^2.$$

Now, Eq. (11) can be stated as

$$V \leqslant \beta_1\|\tilde{\mathbf{v}}\|^2 + \beta_2\|\tilde{\boldsymbol{\theta}}\|^2, \tag{24}$$

where $\beta_1 = (1/2)\vartheta_D$, $\beta_2 = \vartheta_\gamma$, $\vartheta_\gamma = \kappa_{\max}(\boldsymbol{\gamma})$, $\vartheta_D = \kappa_{\max}(\mathbf{D})$, with $\kappa_{\max}(\mathbf{A}) = \sqrt{\lambda_{\max}(\mathbf{A}^{\mathrm{T}}\mathbf{A})}$ denoting the maximum singular value of $\mathbf{A}$. Then,

$$\dot{V} \leqslant -\Lambda V + \rho \tag{25}$$

with

$$\Lambda = \min\left\{\frac{\alpha_1}{\beta_1}, \frac{\alpha_2}{\beta_2}\right\}.$$

Since ρ is bounded, (25) implies that $\tilde{\mathbf{v}}(t)$ and $\tilde{\boldsymbol{\theta}}(t)$ are ultimately bounded. Therefore, the σ-modification makes the adaptation law more robust at the expense of increasing the error bound. As ρ is a function of the minimum singular value of the gain matrix $\boldsymbol{\Gamma}$ of the σ-modification term, and its values are arbitrary, then the error bound can be made small. In the limit, if $\boldsymbol{\Gamma} \to \mathbf{0}$, then $\tilde{\mathbf{v}} \to \mathbf{0}$ as $t \to \infty$, as shown previously.

Now, the behavior of the trajectory control errors $\tilde{\mathbf{h}}$ is revisited. After relaxing the perfect velocity tracking assumption of the kinematic controller in Section 3, Eq. (5) is written as

$$\begin{bmatrix}\dot{\tilde{x}} \\ \dot{\tilde{y}}\end{bmatrix} + \begin{bmatrix} l_x & 0 \\ 0 & l_y\end{bmatrix}\begin{bmatrix}\tanh\left(\frac{k_x}{l_x}\tilde{x}\right) \\ \tanh\left(\frac{k_y}{l_y}\tilde{y}\right)\end{bmatrix} = \begin{bmatrix}\varepsilon_1 \\ \varepsilon_2\end{bmatrix}. \tag{26}$$

By recalling Eqs. (3) and (4), one can see that the error vector $\boldsymbol{\varepsilon}$ can also be written as $\mathbf{A}\tilde{\mathbf{v}}$, where $\tilde{\mathbf{v}}$ is the velocity tracking error and matrix $\mathbf{A}$ is defined in Section 3.1. Rewriting (26) one gets

$$\dot{\tilde{\mathbf{h}}} + \mathbf{L}(\tilde{\mathbf{h}}) = \mathbf{A}\tilde{\mathbf{v}},$$

where

$$\mathbf{L}(\tilde{\mathbf{h}}) = \begin{bmatrix} l_x & 0 \\ 0 & l_y\end{bmatrix}\begin{bmatrix}\tanh\left(\frac{k_x}{l_x}\tilde{x}\right) \\ \tanh\left(\frac{k_y}{l_y}\tilde{y}\right)\end{bmatrix}.$$

It is now considered the same Lyapunov candidate function adopted in Section 3.2, namely $V = \frac{1}{2}\tilde{\mathbf{h}}^{\mathrm{T}}\tilde{\mathbf{h}} > 0$. Its first time derivative is now

$$\dot{V} = \tilde{\mathbf{h}}^{\mathrm{T}}\dot{\tilde{\mathbf{h}}} = \tilde{\mathbf{h}}^{\mathrm{T}}(\mathbf{A}\tilde{\mathbf{v}} - \mathbf{L}(\tilde{\mathbf{h}}))$$

and a sufficient condition for $\dot{V} < 0$ can be expressed as

$$\tilde{\mathbf{h}}^{\mathrm{T}}\mathbf{L}(\tilde{\mathbf{h}}) > |\tilde{\mathbf{h}}^{\mathrm{T}}\mathbf{A}\tilde{\mathbf{v}}|.$$

For small values of the control error $\tilde{\mathbf{h}}$, one can write

$$\mathbf{L}(\tilde{\mathbf{h}}) \approx \mathbf{K_{xy}}\tilde{\mathbf{h}},$$

where

$$\mathbf{K_{xy}} = \begin{bmatrix} k_x & 0 \\ 0 & k_y\end{bmatrix}.$$

Then, the sufficient condition for $\dot{V} < 0$ is expressed as

$$\tilde{\mathbf{h}}^{\mathrm{T}}\mathbf{K_{xy}}\tilde{\mathbf{h}} > |\tilde{\mathbf{h}}^{\mathrm{T}}\mathbf{A}\tilde{\mathbf{v}}|,$$

$$\min(k_x, k_y)\|\tilde{\mathbf{h}}\|^2 > \|\tilde{\mathbf{h}}\|\,\|\mathbf{A}\tilde{\mathbf{v}}\|$$

or

$$\|\tilde{\mathbf{h}}\| > \frac{\|\mathbf{A}\tilde{\mathbf{v}}\|}{\min(k_x, k_y)}. \tag{27}$$

For the case the updating law is given by Eq. (13), it was proved that $\tilde{\mathbf{v}}(t) \to \mathbf{0}$, which implies that condition (27) is asymptotically verified for any value of $\tilde{\mathbf{h}}$. Consequently, the tracking control error $\tilde{\mathbf{h}}(t) \to \mathbf{0}$, thus accomplishing the control objective.

On the other hand, by considering the more robust updating law of Eq. (17), which includes the σ-modification term, it was proved that $\tilde{\mathbf{v}}(t)$ is ultimately bounded, which means there is a bound R on a norm of the signal. Then, the conclusion is that the tracking control error will also be ultimately bounded by the bound $R\|\mathbf{A}\tilde{\mathbf{v}}\|/\min(k_x, k_y)$ on a norm of the control error.

Note 3: The reader should notice that the proposed controller does *not* guarantee that $\tilde{\boldsymbol{\theta}} \to \mathbf{0}$ as $t \to \infty$. In other words, estimated parameters might converge to values that are different from their true values. Actually, it is not required that $\tilde{\boldsymbol{\theta}} \to \mathbf{0}$ in order to make $\tilde{\mathbf{v}}$ converge to a bounded value.

Note 4: It is important to point out that a nonholonomic mobile robot must be oriented according to the tangent of the trajectory path to track a trajectory with small error. Otherwise, the control errors would increase. This is true because the nonholonomic platform restricts the direction of the linear velocity developed by the robot. So, if the robot orientation is not tangent to the trajectory, the distance to the desired position at each instant will increase. The fact that the control errors converge to a bounded value shows that robot orientation does not need to be explicitly controlled, and will be tangent to the trajectory path while the control errors remain small.

5. Experimental results

To show the performance of the proposed controller several experiments and simulations were executed. Some of the results are presented in this section. The proposed controller was implemented on a Pioneer 3-DX mobile robot, which admits linear and angular velocities as input reference signals, and for which the distance b in Fig. 2 is nonzero.

In the first experiment, the controller was initialized with the dynamic parameters of a Pioneer 2-DX mobile robot, weighing about 10 kg (which were obtained via identification). Both robots are shown in Fig. 3, where the Pioneer 3-DX has a laser sensor weighing about 6 kg mounted on its platform, which makes its dynamics significantly different from that of the Pioneer 2-DX.

In the experiment, the robot starts at $x = 0.2$ m and $y = 0.0$ m, and should follow a circular trajectory of reference. The center of the reference circle is at $x = 0.0$ m and $y = 0.8$ m. The reference trajectory starts at $x = 0.8$ m and $y = 0.8$ m and follows a circle

Fig. 3. The robots used in the experiments.

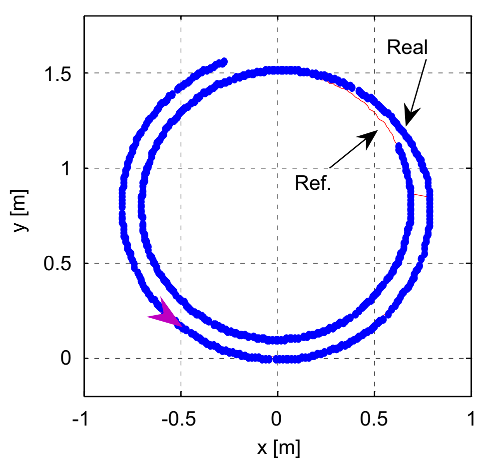

Fig. 4. Part of the reference and real circular trajectories.

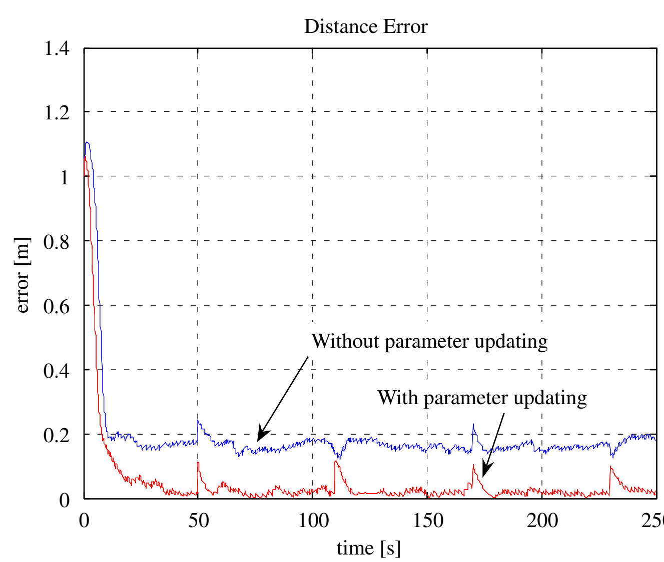

Fig. 5. Distance errors for experiments with and without parameter updating.

having a radius of 0.8 m. After 50 s, the reference trajectory suddenly changes to a circle of radius 0.7 m. After that, the radius of the reference trajectory alternates between 0.7 and 0.8 m each 60 s.

Fig. 4 presents the reference and the actual robot trajectories for a part of the experiment that includes a change in the trajectory radius. In this case, the parameter updating was active. Fig. 5 shows the distance errors for experiments using the proposed controller, with and without parameter updating, to follow the described reference trajectory. The distance error is defined as the instantaneous distance between the reference and the robot position. Notice the high initial error, which is due to the fact that the reference trajectory starts at a point that is far from the initial robot position. First, the proposed controller was tested with no parameter updating. It can be seen in Fig. 5 that, in this case, the trajectory tracking error exhibits a steady-state value of about 0.17 m, which does not vary even after the change in the radius of the reference trajectory. This figure also presents the distance error for the case in which the dynamic parameters are updated. By activating the parameter-updating, and repeating the same experiment, the trajectory tracking error achieves a much smaller value, in comparison with the case in which there is no parameter updating. The error values depicted in Fig. 5 were obtained when the parameter-updating law included the σ-modification. The evolution of the parameter estimates along time (Fig. 6) shows that all parameters converge, i.e., parameter drift was not present.

The whole system was also simulated to check its behavior for long-term operation. Simulation was performed using MATLAB/Simulink® software, employing the dynamic model of the unicycle-like mobile robot presented in Section 2 and the complete control structure using the σ-modification term, which were simulated for a period equivalent to almost 20 h of real running. The reference trajectory for the simulation was the same used in the experiment described above, namely a circle with a meaningful change in its radius each 60 s. Fig. 7 presents the evolution of the parameter estimates obtained for this simulation, and it can be seen that all parameters converge with no drifting.

In the previous experiment, the controller implemented on the Pioneer 3-DX with laser sensor used the identified parameters of

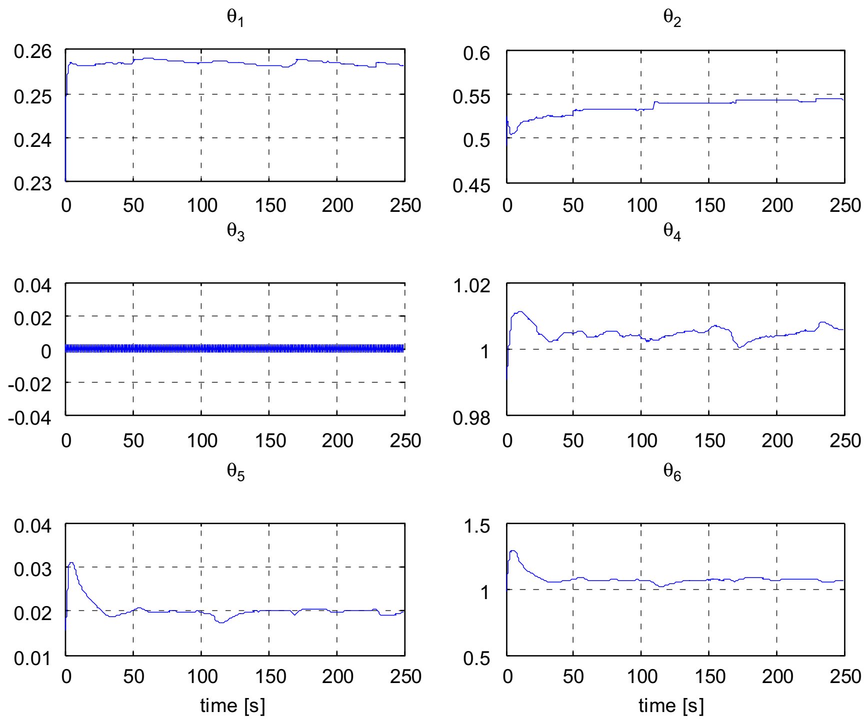

Fig. 6. Evolution of parameters estimates using σ-modification.

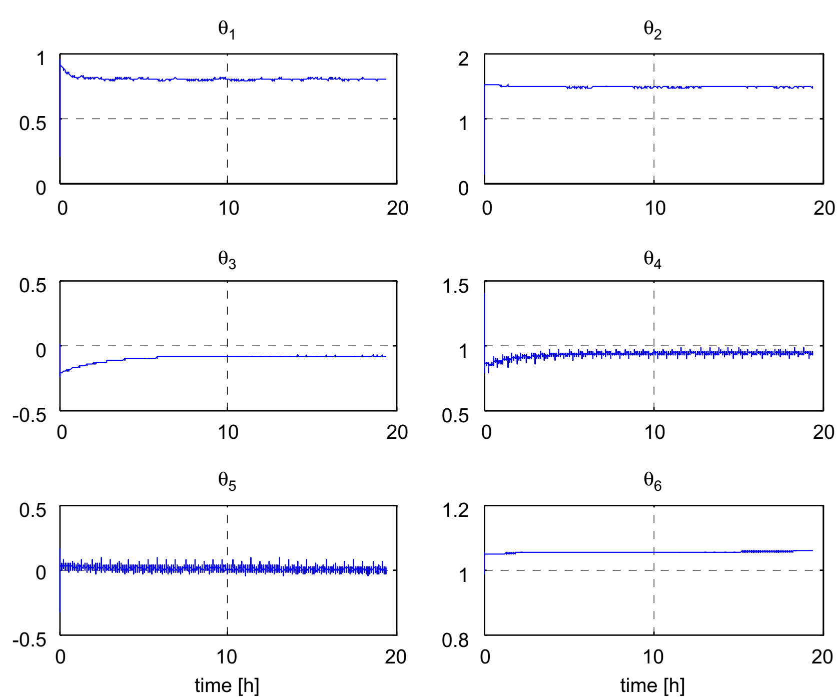

Fig. 7. Evolution of parameter estimates for a long-term simulation.

the Pioneer 2-DX, which is a much lighter robot. The presented results show that the error gets smaller when the parameter-updating law is active, which illustrates the importance of the adaptive controller for the case in which the robot parameters are unknown, or the identified parameters are not correct.

The adaptive controller is also useful even when the nominal robot parameters are correct. For the case of load transportation by an AGV, for example, vehicle mass and moment of inertia vary according to the load being transported, thus meaning that the model parameters are not constant, which is emphasized in the next experiments. Once more, the Pioneer 3-DX robot should track an eight-shape trajectory starting at the position (0.0, 0.0), but the parameters loaded in the dynamic controller are those identified for the robot to be used (the parameters uncertainty is minimum). In the very beginning, the parameter estimates are kept constant, and updating starts at $t = 50$ s. Fig. 8 shows the performance of the robot while tracking the eight-shape trajectory without any load. The small error presented before adaptation are due to small errors in the nominal parameters. It can be seen that the robot follows the reference trajectory with small error, and after adaptation is activated, the tracking error gets even smaller. Fig. 9 presents the reference and real trajectories for the eight-shape but, in this case, the robot was loaded with its full capacity of 23 kg. It can be noticed that the initial error is much higher than before, which is due to the change in the robot dynamics, caused by the increase in the total mass. After parameter updating, the tracking performance is very similar to the case in which the robot is not loaded. Fig. 10 depicts the distance error for these last two previous experiments, which emphasize the importance of on-line parameter updating when the robot dynamics is admitted to vary.

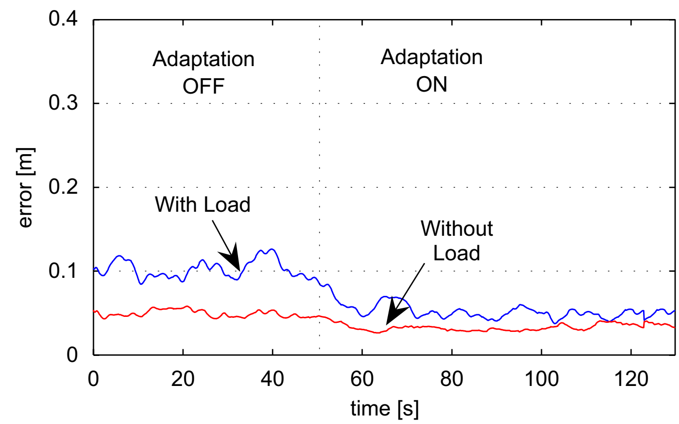

Fig. 10. Distance errors for the eight-shape trajectory tracking: with and without load.

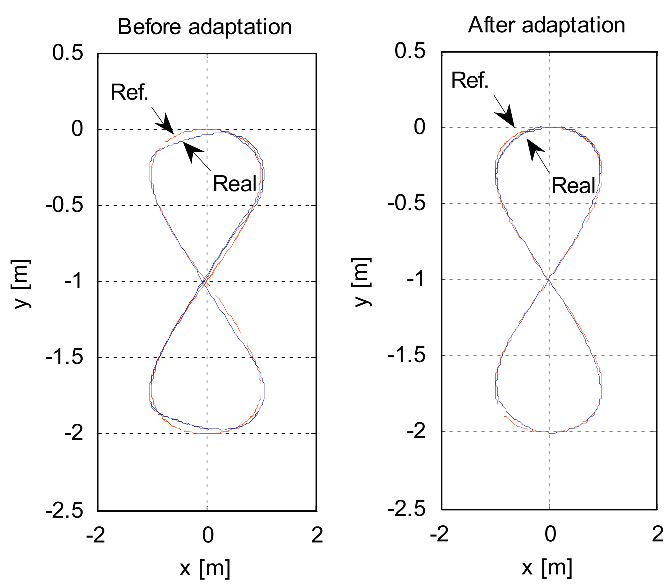

Fig. 8. Reference and real trajectories—without load.

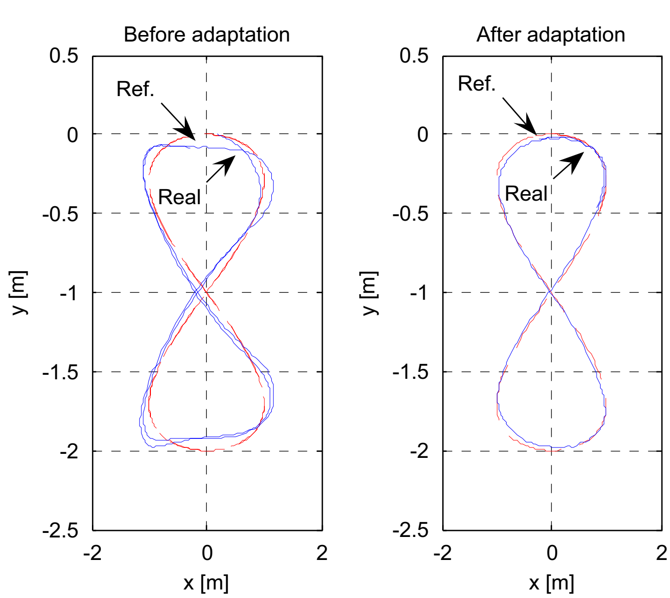

Fig. 9. Reference and real trajectories—with load.

6. Conclusion

An adaptive trajectory-tracking controller for a unicycle-like mobile robot was designed and fully tested in this work. Such a controller is divided in two parts, which are based on the kinematic and dynamic models of the robot. The model considered takes the linear and angular velocities as input reference signals, which is usual when regarding commercial mobile robots. It was considered a parameter-updating law for the dynamic part of the controller, improving the system performance. A σ-modification term was included in the parameter-updating law to prevent possible parameter drift. Stability analysis based on Lyapunov theory was performed for both kinematic and dynamic controllers. For the last one, stability was proved considering a parameter-updating law with and without the σ-modification term. Experimental results were presented, and showed the good performance of the proposed controller for trajectory tracking when applied to an experimental mobile robot. A long-term simulation result was also presented to demonstrate that the updated parameters converge even if the system works for a long period of time. The results proved that the proposed controller is capable of tracking a desired trajectory with a small distance error when the dynamic parameters are adapted. The importance of on-line parameter updating was illustrated for the cases where the robot parameters are not exactly known or might change from task to task. A possible application for the proposed controller is to industrial AGVs used for load transportation, because on-line parameter adaptation would maintain small tracking error even in the case of important changes in the robot load.

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