Robust Control with Redundancy Resolution and Dynamic Compensation for Mobile Manipulators

Víctor Andaluz, Flavio Roberti and Ricardo Carelli
Instituto de Automática, Universidad Nacional de San Juan
Av. San Martin 1109 (OESTE), San Juan (5400), Argentina
{vandaluz,froberti,rcarelli}@inaut.unsj.edu.ar

Abstract- **This paper presents the tracking control problem of a mobile manipulator system to maintain maximum manipulability and including the obstacle avoidance. The design of the controller is based on two cascaded subsystems: a minimum norm kinematic controller with command saturation, and a controller that compensates for the dynamics of the mobile manipulator system. Robot commands are defined in terms of reference velocities. Stability and robustness to parametric uncertainties are proved by using Lyapunov's method. Experimental results show a good performance of the proposed controller as proved by the theoretical design.**

I. Introduction

A mobile manipulator system consists of a manipulator robot (or arm) mounted on a mobile platform (or vehicle). The mobile platform extends the arm workspace. Such systems offer multiple applications in different industrial and productive areas for instance mining, construction or people asistence [1, 2].

In recent years, plenty of research on the development of strategies for the tracking of trajectories of mobile manipulator systems has been done. The control schemes can be classified into two categories: the first one through a decentralized control law, in which there is a controller for the mobile platform and another for the manipulator arm; and a second one that uses a single control law for all the mobile manipulator system [3,4].

Reference [5] presents a controller based on a neural-network for the dynamic compensation and the coordinated control of the mobile manipulator. A cascaded controller for trajectory tracking by using a dynamic model with inputs being torques is developed in [6]. Simulations are shown for the proposed controller. Manipulability, which means the capacity to produce an arbitrary change of orientation and position at the end effector of the manipulator, is a subject addressed in many works on robot manipulators. Reference [7] decompose the motions of the mobile manipulator into decoupled base and manipulator subsystems. The base is then controlled so as to bring the manipulator to a preferred configuartion (using criteria such as the manipulability measure). In [8] an analysis of the concept of manipulability applied to mobile manipulator systems is presented. Reference [9] presents a path planning algorithm for nonholonomic mobile manipulators that computes the robot trajectory by maximizing a performance index that combines manipulability and pose stability measures, the proposed method is validated by simulation examples. Experimental evaluation of a mobile manipulator is performed in [10], by tracking the trajectories with obstacle avoidance using an algorithm with two reference signals, one for the mobile platform and another for end-effector of the manipulating arm. Both signals represent torques of the actuators.

In this paper, we consider a robot manipulator mounted on a non holonomic mobile platform. A dynamic model for the mobile manipulator is developed which, differently to previous works, has reference velocities as control signals to the system, as it is common in commercial robots. A tracking controller for the mobile manipulator system is presented, which maximizes the manipulability and is able to avoid obstacles. The design of the controller is based on two cascaded subsystems: a minimum norm kinematic controller with saturation of velocity commands, and one inverse dynamics controller, which receives as inputs the velocities calculated by the kinematic controller. It is worth noting that, differently to the work in [10], we use a single reference for the end-effector of the mobile manipulator system. Additionally, it is proved the stability and robustness properties to parametric uncertainties in the dynamic model of the mobile manipulator system, by using Lyapunov's method. To validate the proposed control algorithm, experimental results are included and discussed.

This paper is organized as follows. In Section II it is obtained the kinematic and dynamic models which have as inputs the velocities of the mobile manipulator. Section III presents the design of the kinematic controller and the system dynamic compensation. In section IV, an analysis of the system's stability and robustness is developed. The experimental results are presented and discussed in Section V. Finally, conclusions and future work are given in Section VI.

II. System Models

A. *Kinematic Model*

Fig. 1, shows the scheme of the mobile manipulator system which is taken into account in this article. The mobile platform has two driven wheels which are controlled independently by two DC motors. The position of the mobile platform, which is of the unicycle type, is given by point G, representing the center

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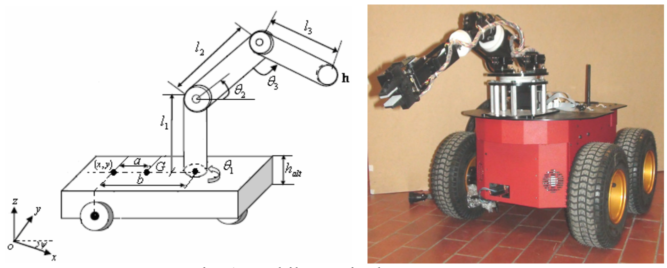

Fig. 1. Mobile manipulator system

of mass which is located at a distance a from the center between the driven wheels. The manipulator is placed at a distance b from this center between the wheeks. Each of the manipulator links is moved by independent DC motors.

Point $\mathbf{h} = [x_{ee} \quad y_{ee} \quad z_{ee}]^T$ defines the end-effector of the mobile manipulator, h_{alt} is the height of the mobile platform at which the base of the manipulator robot is positioned in relation to the absolute coordinate system. Hence, this kinematic model of the mobile manipulator system is obtained:

$$\begin{aligned} x_{ee} &= x + bC_{\psi} + C_{\psi\theta_1}(l_2C_{\theta_2} + l_3C_{\theta_2\theta_3}) \\ y_{ee} &= y + bS_{\psi} + S_{\psi\theta_1}(l_2C_{\theta_2} + l_3C_{\theta_2\theta_3}) \\ z_{ee} &= h_{alt} + l_1 + l_2S_{\theta_2} + l_3S_{\theta_2\theta_3} \end{aligned} \tag{1}$$

where, ψ is the orientation of the mobile platform, θ_i $(i = 1,2,3)$ are the joint positions of the manipulator, $C_{\alpha\beta\lambda} = \cos(\alpha+\beta+\lambda)$ and $S_{\alpha\beta\lambda} = \sin(\alpha+\beta+\lambda)$.

The non holonomic velocity constraint of the mobile platform determines that it can only move perpendicularly to the wheels axis,

$$\dot{x}\sin(\psi) - \dot{y}\cos(\psi) - a\dot{\psi} = 0 \tag{2}$$

By letting $\mathbf{q}(t) = [x \quad y \quad \psi \quad \theta_1 \quad \theta_2 \quad \theta_3]^T$, (2) can be written as,

$$\mathbf{A(q)\dot{q} = 0} \tag{3}$$

where, $\mathbf{A(q)} = [\sin(\psi) \quad -\cos(\psi) \quad -a \quad 0 \quad 0 \quad 0]$. The kinematic model of the platform is represented as,

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \cos\psi & -a\sin\psi \\ \sin\psi & a\cos\psi \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ \omega \end{bmatrix} \tag{4}$$

where, u is the rotational velocities and ω is the longitudinal velocities. Deriving (1) with respect to x, y, ψ, θ_1, θ_2, θ_3 and substituting (4), we obtain the kinematic model of the mobile manipulator system.

$$\begin{bmatrix} \dot{x}_{ee} \\ \dot{y}_{ee} \\ \dot{z}_{ee} \end{bmatrix} = \begin{bmatrix} C_{\psi} & -aS_{\psi} - bS_{\psi} - S_{\psi\theta_1}(l_2C_{\theta_2} + l_3C_{\theta_2\theta_3}) & -S_{\psi\theta_1}(l_2C_{\theta_2} + l_3C_{\theta_2\theta_3}) & -C_{\psi\theta_1}(l_2S_{\theta_2} + l_3S_{\theta_2\theta_3}) & -l_3C_{\psi\theta_1}S_{\theta_2\theta_3} \\ S_{\psi} & aC_{\psi} + bC_{\psi} + C_{\psi\theta_1}(l_2C_{\theta_2} + l_3C_{\theta_2\theta_3}) & C_{\psi\theta_1}(l_2C_{\theta_2} + l_3C_{\theta_2\theta_3}) & -S_{\psi\theta_1}(l_2S_{\theta_2} + l_3S_{\theta_2\theta_3}) & -l_3S_{\psi\theta_1}S_{\theta_2\theta_3} \\ 0 & 0 & 0 & l_2C_{\theta_2} + l_3C_{\theta_2\theta_3} & l_3C_{\theta_2\theta_3} \end{bmatrix} \begin{bmatrix} u \\ \omega \\ \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} \tag{5}$$

Equation (5) can be written in compact form as

$$\dot{\mathbf{h}}(t) = \mathbf{J(q)v}(t) \tag{6}$$

where, $\dot{\mathbf{h}}(t) = [\dot{x}_{ee} \quad \dot{y}_{ee} \quad \dot{z}_{ee}]^T$ is the vector of the end-effector velocity, $\mathbf{J(q)}$ represents the Jacobian matrix, $\mathbf{v}(t) = [u \quad \omega \quad \dot{\theta}_1 \quad \dot{\theta}_2 \quad \dot{\theta}_3]^T$ is the vector of velocities of the mobile manipulator. It is considered that the system works outside the singularity points.

B. Dynamic Model

The mathematic model that represents the dynamics of a mobile manipulator can be obtained from Lagrange's dynamic equations, which are based on the difference between the kinetic and potential energy of each of the joints of the robot (energy balance) [11]. The dynamic equation of the mobile manipulator can be represented as follows,

$$\mathbf{M(q)\,\dot{v} + C(q,v)v + G(q) + d = B(q)\,\boldsymbol{\tau}} \tag{7}$$

where, $\mathbf{q} = [q_1,...,q_n]^T \in \Re^n$ is the general coordinate system vector of the mobile manipulator, $\mathbf{v} = [v_1,....,v_m]^T \in \Re^m$ is the velocity vector of the mobile manipulator, $\mathbf{M(q)} \in \Re^{\mathrm{mxm}}$ is a symmetrical positive definite matrix that represents the system's inertia, $\mathbf{C(q,v)\,v} \in \Re^m$ represents the components of the centripetal and Coriolis forces, $\mathbf{G(q)} \in \Re^m$ represent the gravitational forces, $\mathbf{d}$ denotes bounded unknown disturbances including unmodeled dynamics, $\boldsymbol{\tau} \in \Re^m$ is the torque input vector, $\mathbf{B(q)} \in \Re^{mxm}$ is the transformation matrix of the control actions. For more details on the model see [12].

Most of the robots available in the market have low level PID controllers in order to follow the reference velocity inputs, thus not allowing to control the voltages of the motors directly. Therefore, it becomes useful to express the dynamic model of the mobile manipulator in a more appropriate way, taking the rotational and longitudinal reference velocities as the control signals. To do so, the velocity controllers are included in the model.

It is considered that the motors driving both wheels of the mobile platform are similar, thus the models of the motors without taking into account the inductance voltages are

$$\tau_r = \frac{K_a}{R_a}(v_r - K_b\omega_r) \qquad \tau_l = \frac{K_a}{R_a}(v_l - K_b\omega_l) \tag{8}$$

$$\tau_i = \frac{K_{a_i}}{R_{a_i}}(v_i - K_{b_i}\omega_i), \qquad \text{con } i = 1,2,3 \tag{9}$$

where, v_r and v_l are the input voltages applied to the right and left motor of the mobile platform, v_i is the input voltage applied to each of the manipulator´s joint motor, k_b and k_{b_i} are the electromotor constants multiplied by the reduction constant, R_a and R_{a_i} are the electric resistances constant, τ_r and τ_l are the torque of the left and right motors of the mobile platform multiplied by the reduction constant, τ_i are the torques at each of the joint manipulator, multiplied by their gear ratio respectively, k_a and k_{a_i} are the torque constants multiplied by the gear ratio.

According to [13] the rotational and longitudinal velocities u and ω respectively, without including the slip speed are,

$$u = \frac{r}{2}(\omega_r + \omega_l) \quad ; \quad \omega = \frac{r}{d}(\omega_r - \omega_l) \tag{10}$$

where, r is the right and left wheel radius; ω_r and ω_l are the angular velocities of the right and left wheels and d is the distance between the wheels.

Besides, it is known that

$$\tau_u = \frac{1}{2}(\tau_r + \tau_l) \quad ; \quad \tau_w = \frac{1}{2}(\tau_r - \tau_l) \tag{11}$$

To simplify the model, PD controllers are considered and these are described by the following expression

$$v_u = k_p\left(u_{ref} - u\right) - k_d\dot{u} \quad ; \quad v_\omega = k_p\left(\omega_{ref} - \omega\right) - k_d\dot{\omega} \tag{12}$$

$$\omega_i = k_{p_i}\left(\omega_{ref_i} - \omega_i\right) - k_{d_i}\dot{\omega}_i \tag{13}$$

where, u and ω are the longitudinal and rotational velocities of the mobile platform, while ω_i is the velocity of each of the joints of the manipulator robot. To simplify the model it is considered that: $\dot{u}_{ref} = 0$, $\dot{\omega}_{ref} = 0$ and $\dot{\omega}_{ref_i} = 0$.

With equations (8) to (13) we get the dynamic model of the mobile manipulator, having as control signals the reference velocities of the system. It can be represented as follows,

$$\bar{\mathbf{M}}(\mathbf{q})\dot{\mathbf{v}} + \bar{\mathbf{C}}(\mathbf{q},\mathbf{v})\mathbf{v} + \bar{\mathbf{G}}(\mathbf{q}) + \bar{\mathbf{d}} = \mathbf{v_{ref}} \tag{14}$$

where, $\bar{\mathbf{M}}(\mathbf{q}) = \mathbf{H}^{-1}(\mathbf{M}+\mathbf{D})$, $\bar{\mathbf{C}}(\mathbf{q},\mathbf{v}) = \mathbf{H}^{-1}(\mathbf{C}+\mathbf{P})$, $\bar{\mathbf{G}}(\mathbf{q}) = \mathbf{H}^{-1}\mathbf{G}(\mathbf{q})$, $\bar{\mathbf{d}} = \mathbf{H}^{-1}\mathbf{d}$ and $\mathbf{v_{ref}} = [u_{ref} \;\; \omega_{ref} \;\; \dot{\theta}_{1_{ref}} \;\; \dot{\theta}_{2_{ref}} \;\; \dot{\theta}_{3_{ref}}]^T$. Thus, $\bar{\mathbf{M}}(\mathbf{q}) \in \Re^{mxm}$ is a positive definite matrix, $\bar{\mathbf{C}}(\mathbf{q},\mathbf{v})\mathbf{v} \in \Re^m$ represents the components of the centripetal and Coriolis forces, $\bar{\mathbf{G}}(\mathbf{q}) \in \Re^m$ is the gravitational vector, $\bar{\mathbf{d}}$ denotes bounded unknown disturbances including unmodeled dynamics and $\mathbf{v_{ref}} \in \Re^m$ is the vector of control velocity signals, $\mathbf{H} \in \Re^{mxm}$, $\mathbf{D} \in \Re^{mxm}$ and $\mathbf{P} \in \Re^{mxm}$ are constant symmetrical diagonal matrices, positive definite, that contain the physical parameters of the mobile manipulator, motors, velocity controllers of both the mobile platform and the manipulator robot.

Property 1. Matrix $\bar{\mathbf{M}}$ is positive definite, additionally it is know that

$$\left\|\bar{\mathbf{M}}(\mathbf{q})\right\| < k_M \tag{15}$$

Property 2. Furthermore, the following inequalities are also satisfied

$$\left\|\bar{\mathbf{C}}(\mathbf{q},\mathbf{v})\right\| < k_c\left\|\mathbf{v}\right\| \tag{16}$$

Property 3. Vector $\bar{\mathbf{G}}(\mathbf{q})$ and $\bar{\mathbf{d}}$ are bounded

$$\left\|\bar{\mathbf{G}}(\mathbf{q})\right\| < k_G \quad ; \quad \left\|\bar{\mathbf{d}}\right\| < k_d \tag{17}$$

where, k_c, k_M, k_G and k_d denote some positive constants.

The full mathematical model of the mobile manipulator system is represented by: (6) the kinematic model and (14) the dynamic model, taking the reference velocities of the system as control signals.

III. Controllers Design

A. Minimal Norm Kinematic Controller

Rewriting (6) with $\mathbf{v} \equiv \mathbf{v_c}$, $\mathbf{v_c} = [u_c \;\; \omega_c \;\; \dot{\theta}_{1_c} \;\; \dot{\theta}_{2_c} \;\; \dot{\theta}_{3_c}]^T$ by assuming, by now, perfect velocity tracking,

$$\dot{\mathbf{h}} = \mathbf{J}\mathbf{v_c} \tag{18}$$

To obtain the control actions $\mathbf{v_c}$ that correspond to the end-effector, the right pseudo-inverse Jacobian matrix $\mathbf{J}$ is used [11].

$$\mathbf{v_c} = \mathbf{J}^{+}\dot{\mathbf{h}} \tag{19}$$

where, $\mathbf{J}^{+} = \mathbf{W}^{-1}\mathbf{J}^T(\mathbf{J}\mathbf{W}^{-1}\mathbf{J}^T)^{-1}$, being $\mathbf{W}$ a definite positive matrix that weighs the control actions of the system,

$$\mathbf{v_c} = \mathbf{W}^{-1}\mathbf{J}^T(\mathbf{J}\mathbf{W}^{-1}\mathbf{J}^T)^{-1}\dot{\mathbf{h}} \tag{20}$$

The following control law is proposed for the mobile manipulator system. It is based on a minimal norm solution, which means that, at any time, the mobile manipulator will attain its navigation target with the smallest number of possible movements,

$$\mathbf{v_c} = \mathbf{J}^{+}\left(\dot{\mathbf{h}}_\mathbf{d} + \mathbf{L_K}\tanh\left(\mathbf{L_K^{-1}}\mathbf{K}\,\tilde{\mathbf{h}}\right)\right) + \left(\mathbf{I} - \mathbf{J}^{+}\mathbf{J}\right)\mathbf{L_D}\tanh\left(\mathbf{L_D^{-1}}\mathbf{D}\,\mathbf{v}_0\right) \tag{21}$$

where, $\dot{\mathbf{h}}_\mathbf{d} = [\dot{x}_{ee_d} \;\; \dot{y}_{ee_d} \;\; \dot{z}_{ee_d}]^T$ is the desired velocities vector of the end-effector $\mathbf{h}$, $\tilde{\mathbf{h}} = [\tilde{x}_{ee} \;\; \tilde{y}_{ee} \;\; \tilde{z}_{ee}]^T$ is the vector of control errors with: $\tilde{x}_{ee} = x_{ee_d} - x_{ee}$, $\tilde{y}_{ee} = y_{ee_d} - y_{ee}$, $\tilde{z}_{ee} = z_{ee_d} - z_{ee}$, $\mathbf{K}$, $\mathbf{D}$, $\mathbf{L_K}$ and $\mathbf{L_D}$ are definite positive diagonal matrices that weigh the error vector $\tilde{\mathbf{h}}$ and vector $\mathbf{v}_0$.

The second term represents the projection on the null space of the mobile manipulator, where $\mathbf{v}_0$ is an arbitrary vector which contains the velocities associated to the mobile manipulator. Therefore, any value given to $\mathbf{v}_0$ will have effects on the internal structure of the manipulator only, and will not affect the final control of the end-effector at all. By using this term different control objectives can be achieved: maximum manipulability and obstacle avoidance for instance.

In order to include an analytical saturation of velocities in the mobile manipulator system, the tanh(.) function, which limits the error in $\mathbf{h}$ and the magnitude of vector $\mathbf{v}_0$, is proposed. The expressions $\tanh(\mathbf{L_K^{-1}}\mathbf{K}\,\tilde{\mathbf{h}})$ and $\tanh(\mathbf{L_D^{-1}}\mathbf{D}\,\mathbf{v_0})$ denote a component by component operation.

Now, substituting (21) in (18), we have

$$\dot{\tilde{\mathbf{h}}} + \mathbf{L_K}\tanh(\mathbf{L_K^{-1}}\mathbf{K}\,\tilde{\mathbf{h}}) = \mathbf{0} \tag{22}$$

For the stability analysis the following Lyapunov candidate function is considered

$$V(\tilde{\mathbf{h}}) = \tfrac{1}{2}\tilde{\mathbf{h}}^{\mathrm{T}}\tilde{\mathbf{h}} \tag{23}$$

Its time derivative on the trajectories of the system is,

$$\dot{V}(\tilde{\mathbf{h}}) = -\tilde{\mathbf{h}}^{\mathbf{T}}\mathbf{L_K}\tanh(\mathbf{L_K^{-1}K}\,\tilde{\mathbf{h}}) < 0 \qquad (24)$$

which implies that the equilibrium on the closed-loop (22) is asymptotically stable, thus the position error of the end-effector verifies $\tilde{\mathbf{h}}(t) \to \mathbf{0}$ asymptotically.

B. Manipulability

Much research has been done in order to quantify the ability of a manipulator, seen from the dynamics and cinematic point of view, [14,15,16]. A quantity w is defined as the measure of *manipulability* for a system of the form $\dot{\mathbf{h}}(t) = \mathbf{J}(\mathbf{q})\mathbf{v}(t)$, where w is defined as follows:

i) For redundant manipulators ($n > m$)

$$w = \sqrt{\det(\mathbf{J}(\mathbf{q})\mathbf{J}^T(\mathbf{q}))} \qquad (25)$$

ii) For non-redundant manipulators ($m = n$)

$$w = \left|\det \mathbf{J}(\mathbf{q})\right| \qquad (26)$$

iii) In i) and ii) $w \geq 0$. On the other hand $w = 0$ only if

$$\operatorname{rank}\mathbf{J}(\mathbf{q}) < m \qquad (27)$$

being m the length of the workspace and n the number of degrees of freedom.

The mobile manipulator considered in this article is redundant, thus manipulability will be calculated with (25).

C. Obstacle Avoidance

Fig. 2 shows the obstacle evasion scheme. It is proposed a method in which the angular velocity of the mobile platform will be affected by a fictitious repulsion force. Such a force will depend on the angle α_i, which represents the incidence angle on the obstacle, and distance d to the obstacle. This way, the following control law is proposed:

$$\omega_{obs} = k_{obs}(r-d)sign(\alpha_i)\left[\pi/2 - abs(\alpha_i)\right] \qquad (28)$$

where, r is the radius which determines the distance at which the obstacle starts to be avoided, k_{obs} is a positive adjustment gain and the sign function allows defining to which side the obstacle is to be avoided. The closer the platform is to the obstacle, the bigger the gain of angular velocity.

Taking into account the maximum manipulability and obstacle avoidance, vector $\mathbf{v}_0$ is defined in this way,

$$\mathbf{v}_0 = \begin{bmatrix}0 & \omega_{obs} & \theta_{1d}-\theta_1 & \theta_{2d}-\theta_2 & \theta_{3d}-\theta_3\end{bmatrix}^T \qquad (29)$$

where, $\theta_{id}-\theta_i \quad (i = 1,2,3)$ is the configuration error of the mobile manipulator system, so when minimizing this index, the manipulator joints will be pulled to the desired θ_{id} values that maximize manipulability.

D. Dynamic Compensation

If there is no perfect velocity tracking, the velocity error is defined as,

$$\tilde{\mathbf{v}} = \mathbf{v_c} - \mathbf{v} \qquad (30)$$

This velocity error motivates the dynamic compensation process, which will be performed based on the inverse dynamics of the mobile manipulator. To this aim, we consider the exact model of the system and without including disturbances, thus obtaining the following control,

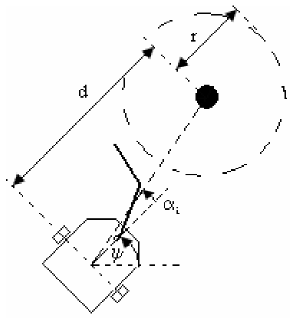

Fig. 2. Obstacle evasion scheme

$$\mathbf{v_{ref}} = \bar{\mathbf{M}}(\mathbf{q})\boldsymbol{\sigma} + \bar{\mathbf{C}}(\mathbf{q},\mathbf{v})\,\mathbf{v} + \bar{\mathbf{G}}(\mathbf{q}) \qquad (31)$$

where, $\mathbf{v_{ref}} = [u_d \quad \omega_d \quad \dot{\theta}_{1d} \quad \dot{\theta}_{2d} \quad \dot{\theta}_{3d}]^T$ and

$$\boldsymbol{\sigma} = \dot{\mathbf{v}}_\mathbf{c} + \mathbf{L_v}\tanh(\mathbf{L_v}^{-1}\mathbf{K_v}\tilde{\mathbf{v}}) \qquad (32)$$

Replacing (31) in (14) we have

$$\mathbf{0} = \dot{\tilde{\mathbf{v}}} + \mathbf{L_v}\tanh(\mathbf{L_v}^{-1}\mathbf{K_v}\tilde{\mathbf{v}}) \qquad (33)$$

The same way as it was made for (22), it can now be concluded that the error vector $\tilde{\mathbf{v}} \to 0$ asymptotically, provided that $\mathbf{K_v}$ and $\mathbf{L_v}$ are symmetrical positive definite matrices.

For the sake of simplicity, from now on we will write $\bar{\mathbf{M}} = \bar{\mathbf{M}}(\mathbf{q})$, $\bar{\mathbf{C}} = \bar{\mathbf{C}}(\mathbf{q},\mathbf{v})$ and $\bar{\mathbf{G}} = \bar{\mathbf{G}}(\mathbf{q})$.

IV. Stability and Robustness

A. Robustness analysis

The proposal developed above considers that the dynamic parameters of the mobile manipulator are exactly known. Nevertheless, this is not always possible in real contexts, which motivates to study the effect of possible parametric errors on the control performance. When such parametric errors are considered, control law (31) can be re-written as,

$$\mathbf{v_{ref}} = \hat{\bar{\mathbf{M}}}\boldsymbol{\sigma} + \hat{\bar{\mathbf{C}}}\mathbf{v} + \hat{\bar{\mathbf{G}}} \qquad (34)$$

where, $\hat{\bar{\mathbf{M}}}$, $\hat{\bar{\mathbf{C}}}$ and $\hat{\bar{\mathbf{G}}}$ are the dynamic matrices with the estimated parameters of the mobile manipulator.

Equating (34) and (14), the closed-loop equation for the inverse dynamics with uncertain model is obtained,

$$\bar{\mathbf{M}}\dot{\mathbf{v}} + \bar{\mathbf{C}}\mathbf{v} + \bar{\mathbf{G}} + \bar{\mathbf{d}} = \hat{\bar{\mathbf{M}}}\boldsymbol{\sigma} + \hat{\bar{\mathbf{C}}}\mathbf{v} + \hat{\bar{\mathbf{G}}} \qquad (35)$$

Now defining $\tilde{\bar{\mathbf{C}}} = \bar{\mathbf{C}} - \hat{\bar{\mathbf{C}}}$, $\tilde{\bar{\mathbf{M}}} = \bar{\mathbf{M}} - \hat{\bar{\mathbf{M}}}$ and $\tilde{\bar{\mathbf{G}}} = \bar{\mathbf{G}} - \hat{\bar{\mathbf{G}}}$, we can write

$$\bar{\mathbf{M}}(\boldsymbol{\sigma} - \dot{\mathbf{v}}) - \tilde{\bar{\mathbf{M}}}\boldsymbol{\sigma} = \tilde{\bar{\mathbf{C}}}\mathbf{v} + \tilde{\bar{\mathbf{G}}} + \bar{\mathbf{d}} \qquad (36)$$

Substituting (32) in (36)

$$\dot{\tilde{\mathbf{v}}} = \bar{\mathbf{M}}^{-1}\tilde{\bar{\mathbf{C}}}\mathbf{v} + \bar{\mathbf{M}}^{-1}(\tilde{\bar{\mathbf{G}}} + \bar{\mathbf{d}}) + \bar{\mathbf{M}}^{-1}\tilde{\bar{\mathbf{M}}}\boldsymbol{\sigma} - \mathbf{L_v}\tanh\left(\mathbf{L_v}^{-1}\mathbf{K_v}\tilde{\mathbf{v}}\right) \qquad (37)$$

A Lyapunov candidate function is proposed as $V(\tilde{\mathbf{v}}) = \frac{1}{2}\tilde{\mathbf{v}}^{\mathbf{T}}\tilde{\mathbf{v}}$, which temporal derivative is,

$$\dot{V}(\tilde{\mathbf{v}}) = \tilde{\mathbf{v}}^{\mathrm{T}}\left(\bar{\mathbf{M}}^{-1}\tilde{\bar{\mathbf{C}}}\mathbf{v} + \bar{\mathbf{M}}^{-1}(\tilde{\bar{\mathbf{G}}} + \bar{\mathbf{d}}) + \bar{\mathbf{M}}^{-1}\tilde{\bar{\mathbf{M}}}\boldsymbol{\sigma} - \mathbf{L_v}\tanh(\mathbf{L_v}^{-1}\mathbf{K_v}\tilde{\mathbf{v}})\right) \qquad (38)$$

A sufficient condition for $\dot{V}(\tilde{\mathbf{v}})$ to be negative defined is,

$$\left|\tilde{\mathbf{v}}^{\mathrm{T}}\mathbf{L_v}\tanh(\mathbf{L_v}^{-1}\mathbf{K_v}\tilde{\mathbf{v}})\right| > \left|\tilde{\mathbf{v}}^{\mathrm{T}}\bar{\mathbf{M}}^{-1}\tilde{\bar{\mathbf{C}}}\mathbf{v} + \tilde{\mathbf{v}}^{\mathrm{T}}\bar{\mathbf{M}}^{-1}(\tilde{\bar{\mathbf{G}}} + \bar{\mathbf{d}}) + \tilde{\mathbf{v}}^{\mathrm{T}}\bar{\mathbf{M}}^{-1}\tilde{\bar{\mathbf{M}}}\boldsymbol{\sigma}\right| \qquad (39)$$

For large velocity errors it can be expressed that: $\mathbf{L}_\mathrm{v}\tanh(\mathbf{L}_\mathrm{v}^{-1}\mathbf{K}_\mathrm{v}\tilde{\mathbf{v}}) \approx \mathbf{L}_\mathrm{v}$

$$\lambda_{\min}(\mathbf{L}_\mathrm{v}) > \|\mathbf{v}\|\left\|\bar{\mathbf{M}}^{-1}\tilde{\mathbf{C}}\right\| + \left\|\bar{\mathbf{M}}^{-1}(\tilde{\bar{\mathbf{G}}} + \bar{\mathbf{d}})\right\| + \|\boldsymbol{\sigma}\|\left\|\bar{\mathbf{M}}^{-1}\tilde{\bar{\mathbf{M}}}\right\| \qquad (40)$$

It is known that $\mathbf{v}_\mathbf{c}$ defined by (21) is bounded, and it can be proven that $\|\dot{\mathbf{v}}_\mathbf{c}\| \le k_{vc}\|\mathbf{v}\|$, thus we obtain

$$\|\boldsymbol{\sigma}\| \le k_\sigma \|\mathbf{v}\| \qquad (41)$$

where, k_{vc} y k_σ denote some positive constants.

Substituting (16), (30) and (41) in (40) we have,

$$\lambda_{\min}(\mathbf{L}_\mathrm{v}) > k_c\|\mathbf{v}\|^2\left\|\bar{\mathbf{M}}^{-1}\right\| + \left\|\bar{\mathbf{M}}^{-1}(\tilde{\bar{\mathbf{G}}} + \bar{\mathbf{d}})\right\| + k_\sigma\|\mathbf{v}\|\left\|\bar{\mathbf{M}}^{-1}\tilde{\bar{\mathbf{M}}}\right\| \qquad (42)$$

which represents a design condition that guarantees decreasing velocity error $\tilde{\mathbf{v}}$.

On the other hand, for small velocity errors it can be expressed that: $\mathbf{L}_\mathrm{v}\tanh\left(\mathbf{L}_\mathrm{v}^{-1}\mathbf{K}_\mathrm{v}\tilde{\mathbf{v}}\right) \approx \mathbf{K}_\mathrm{v}\tilde{\mathbf{v}}$, and (39) can be re-written as,

$$\lambda_{\min}(\mathbf{K}_\mathrm{v})\|\tilde{\mathbf{v}}\|^2 > \|\tilde{\mathbf{v}}\|\left\|\bar{\mathbf{M}}^{-1}\tilde{\mathbf{C}}\mathbf{v}\right\| + \|\tilde{\mathbf{v}}\|\left\|\bar{\mathbf{M}}^{-1}(\tilde{\bar{\mathbf{G}}} + \bar{\mathbf{d}})\right\| + \|\tilde{\mathbf{v}}\|\left\|\bar{\mathbf{M}}^{-1}\tilde{\bar{\mathbf{M}}}\boldsymbol{\sigma}\right\| \qquad (43)$$

In the same way as in the case above, substituting (16), (30) and (41) in (43) we have

$$-k_c\left\|\bar{\mathbf{M}}^{-1}\right\|\|\tilde{\mathbf{v}}\|^2 + \left(\lambda_{\min}(\mathbf{K}_\mathrm{v}) - 2k_c\|\mathbf{v}_\mathbf{c}\|\left\|\bar{\mathbf{M}}^{-1}\right\| - k_\sigma\left\|\bar{\mathbf{M}}^{-1}\tilde{\bar{\mathbf{M}}}\right\|\right)\|\tilde{\mathbf{v}}\|$$
$$-\left(k_c\|\mathbf{v}_\mathbf{c}\|^2\left\|\bar{\mathbf{M}}^{-1}\right\| + \left\|\bar{\mathbf{M}}^{-1}(\tilde{\bar{\mathbf{G}}} + \bar{\mathbf{d}})\right\| + k_\sigma\|\mathbf{v}_\mathbf{c}\|\left\|\bar{\mathbf{M}}^{-1}\tilde{\bar{\mathbf{M}}}\right\|\right) > \mathbf{0} \qquad (44)$$

Equation (44) is written as

$$-a\|\tilde{\mathbf{v}}\|^2 + b\|\tilde{\mathbf{v}}\| - c > 0 \qquad a, c > 0 \qquad (45)$$

which implies that,

$$\frac{-b + \sqrt{b^2 - 4ac}}{-2a} < \|\tilde{\mathbf{v}}\| < \frac{-b - \sqrt{b^2 - 4ac}}{-2a} \qquad (46)$$

A sufficient condition for real positive solution of (45) is,

$$\lambda_{\min}(\mathbf{K}_\mathrm{v}) > 2\sqrt{a\,c} + 2k_c\|\mathbf{v}_\mathbf{c}\|\left\|\bar{\mathbf{M}}^{-1}\right\| + k_\sigma\left\|\bar{\mathbf{M}}^{-1}\tilde{\bar{\mathbf{M}}}\right\| \qquad (47)$$

It can be proved that $\mathbf{L}_\mathrm{v}\tanh(\mathbf{L}_\mathrm{v}^{-1}\mathbf{K}_\mathrm{v}\tilde{\mathbf{v}}) \approx \mathbf{K}_\mathrm{v}\tilde{\mathbf{v}}$, can be considered for $\|\tilde{\mathbf{v}}\| \ge \dfrac{-b - \sqrt{b^2 - 4ac}}{-2a}$ if

$$\lambda_{\min}(\mathbf{K}_\mathrm{v}) > \frac{b_k + \sqrt{b_k^2 + 4a_k c_k}}{2a_k} \qquad (48)$$

where, $a_k = 1 - \dfrac{c}{\lambda_{\max}(\mathbf{L}_\mathrm{v})}$, $b_k = 2k_c\|\mathbf{v}_\mathbf{c}\|\left\|\mathbf{M}^{-1}\right\| + k_\sigma\left\|\mathbf{M}^{-1}\tilde{\mathbf{M}}\right\|$ and $c_k = a\lambda_{\max}(\mathbf{L}_\mathrm{v})$

Therefore, if conditions (43), (47) and (48) are fulfilled, velocity error $\tilde{\mathbf{v}}$ will be ultimately bounded, with bound,

$$\|\tilde{\mathbf{v}}\| \le \frac{-b + \sqrt{b^2 - 4ac}}{-2a} \qquad (49)$$

B. Stability analysis

The behaviour of the tracking error of the end-effector $\mathbf{h}$ is now analyzed relaxing the assumption of non perfect velocity tracking. Equation (22) can now be written as,

$$\dot{\tilde{\mathbf{h}}} + \mathbf{L}_\mathbf{K}\tanh(\mathbf{L}_\mathbf{K}^{-1}\mathbf{K}\,\tilde{\mathbf{h}}) = \boldsymbol{\varepsilon} \qquad (50)$$

where, $\boldsymbol{\varepsilon} = [\varepsilon_x \quad \varepsilon_y \quad \varepsilon_z]^T$ is the velocity error vector of the mobile manipulator system. It can also be written as $\mathbf{J}\tilde{\mathbf{v}}$, being $\mathbf{J}$ the Jacobian matrix and $\tilde{\mathbf{v}}$ the velocity error vector, so that

$$\dot{\tilde{\mathbf{h}}} + \mathbf{L}_\mathbf{K}\tanh\left(\mathbf{L}_\mathbf{K}^{-1}\mathbf{K}\,\tilde{\mathbf{h}}\right) = \mathbf{J}\tilde{\mathbf{v}} \qquad (51)$$

A Lyapunov candidate function that equals (23) is proposed, and its time derivative is

$$\dot{V}(\tilde{\mathbf{h}}) = \tilde{\mathbf{h}}^\mathbf{T}\mathbf{J}\tilde{\mathbf{v}} - \tilde{\mathbf{h}}^\mathbf{T}\mathbf{L}_\mathbf{K}\tanh\left(\mathbf{L}_\mathbf{K}^{-1}\mathbf{K}\,\tilde{\mathbf{h}}\right) \qquad (52)$$

A sufficient condition for $\dot{V}(\tilde{\mathbf{h}})$ to be negative definite is

$$\left|\tilde{\mathbf{h}}^\mathbf{T}\mathbf{L}_\mathbf{K}\tanh(\mathbf{L}_\mathbf{K}^{-1}\mathbf{K}\,\tilde{\mathbf{h}})\right| > \left|\tilde{\mathbf{h}}^\mathbf{T}\mathbf{J}\tilde{\mathbf{v}}\right| \qquad (53)$$

For large values of $\tilde{\mathbf{h}}$, it can be considered that: $\mathbf{L}_\mathbf{K}\tanh(\mathbf{L}_\mathbf{K}^{-1}\mathbf{K}\,\tilde{\mathbf{h}}) \approx \mathbf{L}_\mathbf{K}$. $\dot{V}$ will be negative definite only if

$$\lambda_{\min}(\mathbf{L}_\mathbf{K}) > \|\mathbf{J}\|\|\tilde{\mathbf{v}}\| \qquad (54)$$

thus making the velocity errors $\tilde{\mathbf{h}}$ to decrease.

Now, for small values of $\tilde{\mathbf{h}}$, it can be expressed: $\mathbf{L}_\mathbf{K}\tanh(\mathbf{L}_\mathbf{K}^{-1}\mathbf{K}\,\tilde{\mathbf{h}}) \approx \mathbf{K}\tilde{\mathbf{h}}$, and (50) can be written as,

$$\|\tilde{\mathbf{h}}\| > \frac{\|\mathbf{J}\|\|\tilde{\mathbf{v}}\|}{\lambda_{\min}(\mathbf{K})} \qquad (55)$$

thus implying that the error $\tilde{\mathbf{h}}$ is bounded by,

$$\|\tilde{\mathbf{h}}\| \le \frac{\|\mathbf{J}\|\|\tilde{\mathbf{v}}\|}{\lambda_{\min}(\mathbf{K})} \qquad (56)$$

V. Experimental Results

The experimental system is composed of a mobile robot PIONEER 3AT and an arm robot one CYTON Alpha 7 DOF. It was planned to follow a spatial trajectory on which an obstacle is placed so that the mobile platform can avoid it. It is considered that the obstacle is placed at a maximum height that does not interfere with the manipulator arm, so that the arm can follow the desired trajectory even when the platform is avoiding the obstacle. The positions of the arm joints that maximize the arm's manipulability are obtained through numeric simulation. This way, the joints angles should be, $\theta_{1d} = 0.1745[\mathrm{rad}]$, $\theta_{2d} = 0.6065[\mathrm{rad}]$, and $\theta_{3d} = -1.2346[\mathrm{rad}]$.

Dynamic compensation is performed for the mobile platform only, because it presents the most significant dynamics of the whole mobile manipulator system. The dynamic model of the mobile platform, whose center of mass varies, can be represented as

$$\mathbf{M}_\mathbf{P}\dot{\mathbf{v}}_\mathbf{P} + \mathbf{C}_\mathbf{P} = \mathbf{v}_{\mathbf{erf}_\mathbf{P}} \qquad (57)$$

where,

$$\mathbf{M}_\mathbf{P} = \begin{bmatrix} \phi_1 & -\phi_7 \\ -\phi_8 & \phi_2 \end{bmatrix} \qquad \mathbf{C}_\mathbf{P} = \begin{bmatrix} u\phi_4 - \omega^2\phi_3 \\ \omega\phi_6 + u\omega\phi_5 \end{bmatrix}$$

$$\mathbf{v}_{\mathbf{P}} = [\dot{u} \quad \dot{\omega}]^T \qquad \mathbf{v}_{\mathbf{erf_P}} = [u_{ref} \quad \omega_{ref}]^T$$

Equation (54) has similar properties as (14).

In order to identify and verify this model, the robot arm CYTON Alpha 7DOF was mounted on a robot PIONEER 3AT, resulting in

$$\phi_1 = 0.3552 \quad \phi_2 = 0.3760 \quad \phi_3 = -0.0002 \quad \phi_4 = 1.0002$$
$$\phi_5 = -0.1050 \quad \phi_6 = 0.9406 \quad \phi_7 = -0.0010 \quad \phi_8 = -0.1355$$

Figs. 3 to 6, show the experimental results. Fig. 3, shows the stroboscopic movement on space *x-y-z*. It can be seen that the proposed controller works correctly, both when tracking trajectories and when evading obstacles. Figs. 4 and 5, show the control actions that do not surpass the maximum preset values, while Fig. 6, shows the tracking errors that.

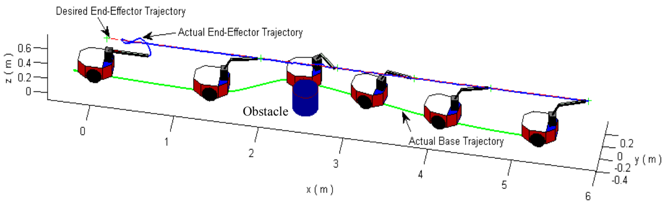

Fig. 3. Mobile manipulator tracking the prescribed trajectory

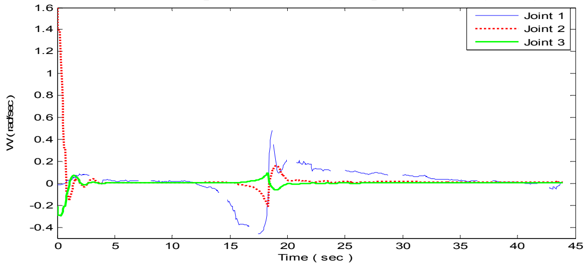

Fig. 4. Joint velocity of the manipulator robot

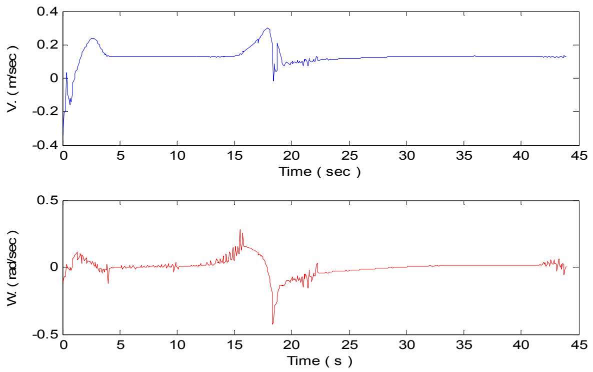

Fig. 5. Velocity of the mobile plataform

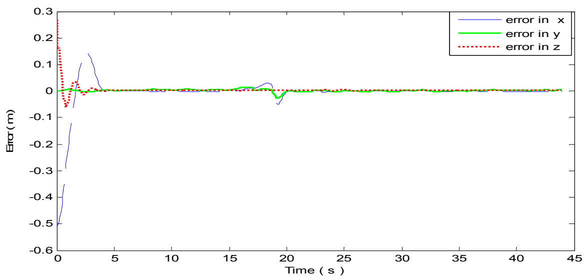

Fig. 6. Traking errors of the mobile manipulator

VI. CONCLUSIONS AND FUTURE WORK

A cascade control algorithm, which is composed of two subsystems: a kinematic controller and a dynamics compensator was implemented in order to track the trajectory of a mobile manipulator system with obstacle avoidance, maintaining the maximum manipulability of the arm. Stability and robustness of the system have been analytically proved. The results, which were obtained by experimentation, show a good performance of the proposed control system.

For future work, we will investigate the integration of the proposed method with an adaptive control strategy for mobile manipulator systems, considering parameter uncertainties in the mathematical model.

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