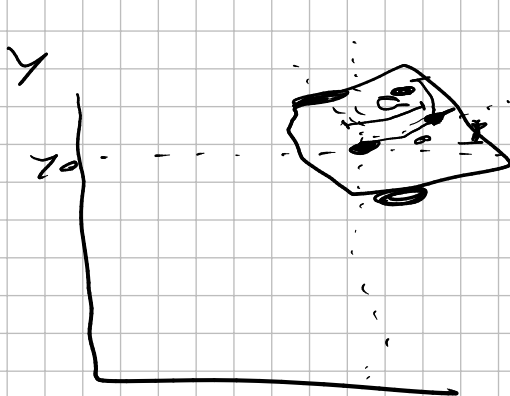
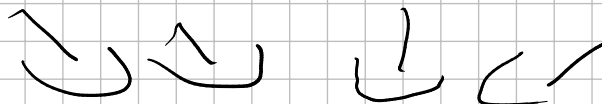


$$\begin{bmatrix} x \\ y \end{bmatrix} = R_z \begin{bmatrix} x_m \\ y_m \end{bmatrix}$$



$$\begin{aligned} x &= x_0 + a \cdot \cos(\theta) \\ y &= y_0 + a \cdot \sin(\theta) \end{aligned}$$

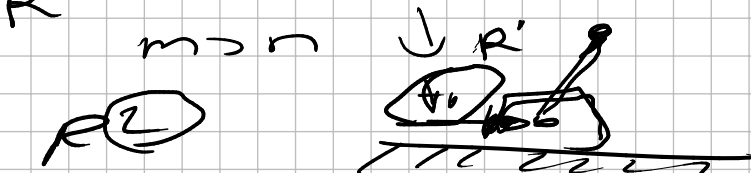


$$\begin{aligned} x &= x_0 + a \cdot \cos(\theta) - b \cdot \sin(\theta) \\ y &= y_0 + a \cdot \sin(\theta) + b \cdot \cos(\theta) \end{aligned}$$

$$\dot{x} = -a \cdot \sin(\theta) - b \cdot \cos(\theta) \cdot \omega$$

$$\dot{y} = a \cdot \cos(\theta) - b \cdot \sin(\theta) \cdot \omega$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -\sin(\theta) & -\cos(\theta) \\ \cos(\theta) & -\sin(\theta) \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \omega$$



$$\begin{bmatrix} x(t), \dot{x}(t) \\ y(t), \dot{y}(t) \end{bmatrix} \in \mathbb{R}^4$$

$$\begin{aligned} \mathbb{R}^2 &\subset \mathbb{R}^3 \\ \mathbb{R}^3 &\subset \mathbb{R}^4 \end{aligned}$$

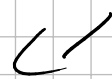
$$\dot{\mathbf{r}} = \dot{\mathbf{r}}_0 + \dot{\mathbf{r}}_1 \quad \dot{\mathbf{r}}_1 = \dot{\mathbf{r}}_0 + \dot{\mathbf{r}}_1 \quad \dot{\mathbf{r}}_1 = \dot{\mathbf{r}}_0 + \dot{\mathbf{r}}_1 \quad \dot{\mathbf{r}}_1 = \dot{\mathbf{r}}_0 + \dot{\mathbf{r}}_1$$

$$\dot{\mathbf{r}}_0 = \begin{bmatrix} \dot{x}_0(t) \\ \dot{y}_0(t) \end{bmatrix}$$

$$\mathbf{r} = \mathbf{r}_0 - \mathbf{r}_1 \rightarrow \tilde{\mathbf{r}} = \mathbf{r}_0 - \mathbf{r}_1$$

$$\mathbf{r} = \begin{bmatrix} x_0(t) \\ y_0(t) \end{bmatrix} - \begin{bmatrix} x_1(t) \\ y_1(t) \end{bmatrix} \quad \boxed{\tilde{\mathbf{r}} = \mathbf{r}_0 - \mathbf{r}_1}$$

$$\tilde{\mathbf{r}} = \begin{bmatrix} x_0(t) - x_1(t) \\ y_0(t) - y_1(t) \end{bmatrix}$$



$$x > 0$$

$$x < 0$$

$$V(x) \geq 0$$

$$x \neq 0$$

$$\frac{1}{2} x^2$$

$$\frac{1}{2} (-1)^2 = \boxed{\frac{1}{2}}$$

$$\frac{1}{2} (-1)^2 = \frac{1}{2} (1) = \boxed{\frac{1}{2}}$$

$$V(x) = \frac{1}{2} x^2$$

$$V(\tilde{\mathbf{r}}) = \frac{1}{2} \tilde{\mathbf{r}}^T \tilde{\mathbf{r}}$$

$$V(\tilde{\mathbf{r}}) = \frac{1}{2} \begin{bmatrix} \tilde{x} & \tilde{y} \end{bmatrix} \begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix}$$

$$V(\tilde{\mathbf{r}}) = \frac{1}{2} (\tilde{x}^2 + \tilde{y}^2)$$

$$V(\tilde{r}) = \frac{1}{2} (\tilde{x}^2 + \tilde{y}^2)$$

$$V > 0 \quad x \neq 0$$

$$V > 0 \quad \tilde{x} \neq 0$$

$$V(\tilde{r}) = \frac{1}{2} (\tilde{r}^T \tilde{r})$$

$$\tilde{y} = 0$$

$$\dot{V}(\tilde{r}) = \frac{1}{2} (2\tilde{x}\dot{\tilde{x}} + 2\tilde{y}\dot{\tilde{y}})$$

$$\dot{V}(\tilde{r}) = \tilde{x}\dot{\tilde{x}} + \tilde{y}\dot{\tilde{y}}$$

$$\dot{V}(\tilde{r}) = \tilde{r}^T \dot{\tilde{r}} \rightarrow \begin{bmatrix} \tilde{x} & \tilde{y} \end{bmatrix} \begin{bmatrix} \dot{\tilde{x}} \\ \dot{\tilde{y}} \end{bmatrix}$$

$\dot{\tilde{r}} = J \cdot v$

$$\dot{V}(\tilde{r}) = \tilde{r}^T \cdot \dot{\tilde{r}}$$

$$\tilde{r} = \tilde{r}_d - \tilde{r}$$

$$\dot{\tilde{r}} = \dot{\tilde{r}}_d - \dot{\tilde{r}}$$

$$\dot{\tilde{r}} = \dot{\tilde{r}}_d - Jv$$

$$\dot{V}(\tilde{r}) = \tilde{r}^T (\dot{\tilde{r}}_d - Jv)$$

$$v = \begin{bmatrix} u \\ w \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$2 \times 2 \quad 2 \times 2$

$$v = J^* (\dot{\tilde{r}}_d + \tilde{r})$$

$$\dot{V}(\tilde{r}) = \tilde{r}^T (\dot{\tilde{r}}_d - J(J^* (\dot{\tilde{r}}_d + \tilde{r})))$$

$$\dot{V}(\tilde{r}) = \tilde{r}^T (\dot{\tilde{r}}_d - \dot{\tilde{r}}_d - \tilde{r}) = \tilde{r}^T (-\tilde{r})$$

$$\dot{V}(\tilde{r}) = -\tilde{r}^T \tilde{r}$$

$$\dot{V}(\tilde{r}) = -\begin{bmatrix} \tilde{x} & \tilde{y} \end{bmatrix} \begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix}$$

$$\dot{V}(\tilde{r}) = -(\tilde{x}^2 + \tilde{y}^2)$$