Alohomora

Luis F. Recalde†

***Abstract*— This project presents the implementation of attitude estimation using three different filtering methodologies: pure gyroscope estimation, accelerometer-based estimation, and the complementary filter. The data is sourced from the *ArduIMU+ V2*, a six-degree-of-freedom Inertial Measurement Unit (IMU). The results are compared using Euler angles. The findings demonstrate that the complementary filter offers the most accurate and reliable performance among the evaluated methods.**

I. Phase I: IMU Attitude Estimation

A. Problem Statement

In this phase of the project, we focus on implementing three different attitude estimation methods: gyroscope-based, accelerometer-based, and complementary filter. These methods were evaluated using three different test sets and the results were presented appropriately.

B. Data Processing

The IMU data were collected using the well-known sensor *ArduIMU+ V2*, which provides measurements of angular velocity $\boldsymbol{\omega} = \begin{bmatrix}\omega_x & \omega_y & \omega_z\end{bmatrix}^T$ and linear acceleration $\mathbf{a} = \begin{bmatrix}a_x & a_y & a_z\end{bmatrix}^T$, both expressed with respect to the IMU's local reference frame.

However, the raw values returned by the *ArduIMU+ V2* are not expressed in standard physical units. Therefore, an appropriate unit conversion must be applied as follows:

$$\tilde{\mathbf{a}} = \mathbf{S}_a\mathbf{a} + \mathbf{b}_a \tag{1}$$

$$\boldsymbol{\omega} = 0.3\frac{3300}{1023}\frac{\pi}{180}(\boldsymbol{\omega} - \mathbf{b}_\omega) \tag{2}$$

$$\tag{3}$$

where $\mathbf{S}_a$ is a diagonal matrix that contains the scale values $\begin{bmatrix}s_x & s_y & s_z\end{bmatrix}$, $\mathbf{b}_a$ is a vector with the bias of the acceleration $\begin{bmatrix}b_{ax} & b_{ay} & b_{az}\end{bmatrix}$ and $\mathbf{b}_\omega$ is the bias associated to the angular velocity. We presented the linear acceleration and angular velocity of experiment 1 in Fig. 1 and Fig. 2.

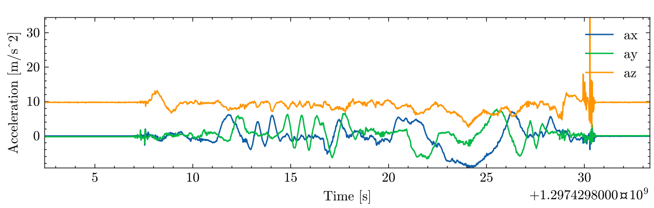

Fig. 1. Linear acceleration of experiment 4.

† Luis F. Recalde is PhD student in Robotics Engineering at Worcester Polytechnic Institute `lrecalde@wpi.edu`

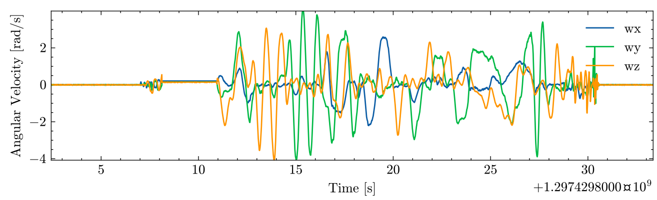

Fig. 2. Angular velocity of experiment 4.

Additionally, ground truth data are obtained from a Vicon motion capture system, which provides time-stamped rotation matrices $\mathbf{R} \in \mathrm{SO}(3)$, representing the estimated orientation with respect to the global reference frame.

C. Sensor Calibration

The Vicon and IMU data streams are not hardware-synchronized; therefore, software-based synchronization using time stamps is required. To address temporal misalignment, interpolation is employed to estimate IMU measurements at Vicon time instances. Additionally, it is important to account for the fact that the IMU and Vicon operate in different coordinate frames. As such, a frame alignment procedure is necessary to transform the data into a common reference frame.

D. Attitude Estimation Based on Gyroscope

This section describes the implementation of attitude estimation based solely on gyroscope measurements using integration methods. For numerical stability and computational efficiency, the integration is performed using quaternions, expressed as follows:

$$\dot{\mathbf{q}} = \frac{1}{2}\mathbf{q} \otimes \boldsymbol{\omega} \tag{4}$$

where $\otimes$ denotes multiplication of quaternions.

To integrate this differential equation, we adopt the fourth-order Runge–Kutta method, as described in [1]. This approach provides accurate, singularity-free numerical integration of rotational motion over extended time intervals, while preserving the geometric structure of $\mathrm{SO}(3)$. Finally, we can calculate the Euler angles from the quaternions as presented in [2].

E. Attitude Estimation Based on Accelerometers

We can provide an estimation of the attitude using the accelerometer measurements by considering the following

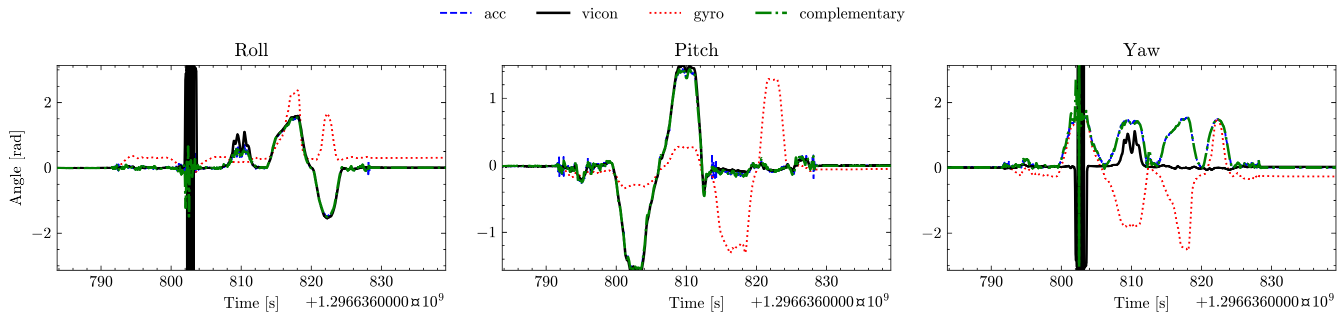

Fig. 3. Attitude estimation of the three methods considering the experiment 1.

expression:

$$\phi = \tan^{-1} \frac{a_y}{\sqrt{a_x^2 + a_z^2}} \tag{5}$$

$$\theta = \tan^{-1} \frac{-a_x}{\sqrt{a_y^2 + a_z^2}} \tag{6}$$

$$\psi = \tan^{-1} \frac{\sqrt{a_x^2 + a_y^2}}{a_z} \tag{7}$$

$$\tag{8}$$

However, due to the inherent symmetry of the accelerometer with respect to the Z axis, it is not possible to accurately estimate the yaw angle ψ using accelerometer data alone. Therefore, more advanced techniques are required to achieve the full attitude estimation.

F. Complementary Filter

This section provides insights into the complementary filter. We can combine both estimations presented in (4) and (5) in a unified manner as follows:

$$\begin{bmatrix}\psi\\ \theta\\ \psi\end{bmatrix}_{CF} = \begin{bmatrix}\alpha & 0 & 0\\ 0 & \beta & 0\\ 0 & 0 & \gamma\end{bmatrix}\begin{bmatrix}\psi\\ \theta\\ \psi\end{bmatrix}_{acc} + \tag{9}$$

$$\begin{bmatrix}1-\alpha & 0 & 0\\ 0 & 1-\beta & 0\\ 0 & 0 & 1-\gamma\end{bmatrix}\begin{bmatrix}\psi\\ \theta\\ \psi\end{bmatrix}_{gyro} \tag{10}$$

where the α, β, γ values are mixing parameters that have been carefully selected to increase the accuracy of the estimation. For most data sets, these values were chosen as $\alpha = 0.9$, $\beta = 0.85$, and $\psi = 0.9$.

G. Result

This section presents the results of the three estimation methods considering two data sets. The results of experiments 1 and 2 are shown in Fig. 3 and Fig. 4, respectively.

A short video demonstrating the different performance of each method is presented in Link.

References

[1] C. Rucker, "Integrating rotations using nonunit quaternions," *IEEE Robotics and Automation Letters*, vol. 3, no. 4, pp. 2979–2986, 2018.

[2] E. Coutsias and L. Romero, "The quaternions with applications to rigid body dynamics," 01 1999.

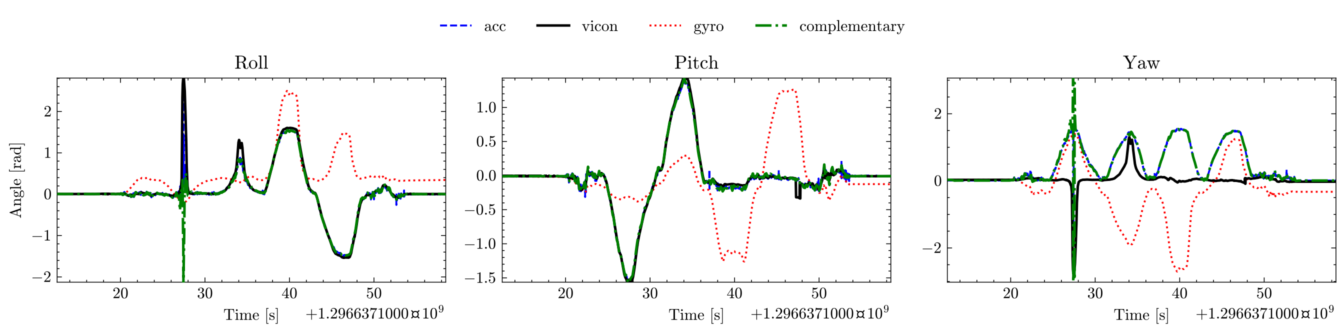

Fig. 4. Attitude estimation of the three methods considering the experiment 2.

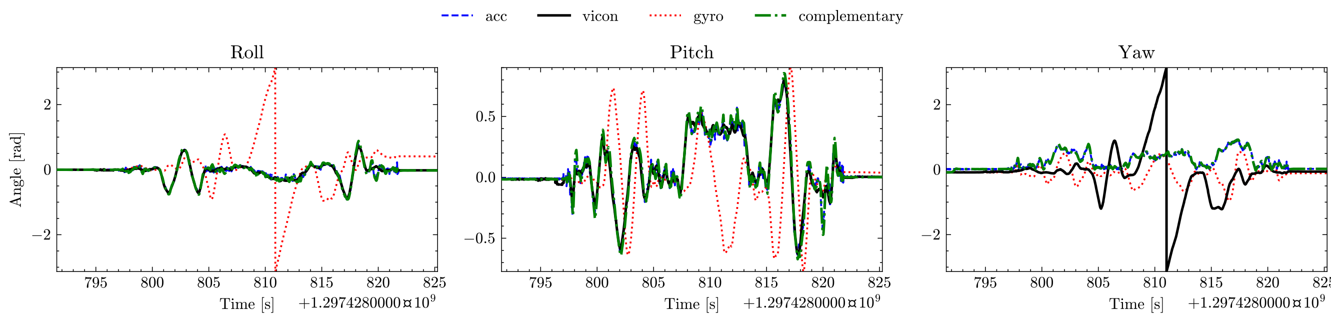

Fig. 5. Attitude estimation of the three methods considering the experiment 3.

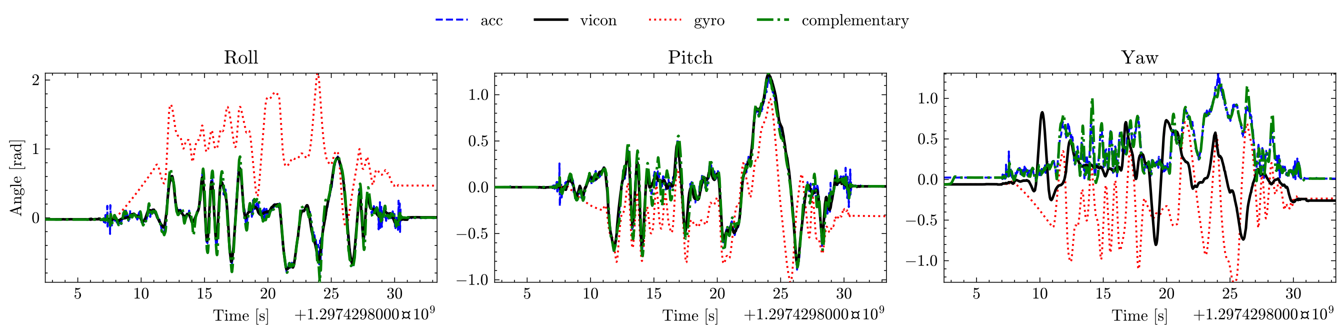

Fig. 6. Attitude estimation of the three methods considering the experiment 4.

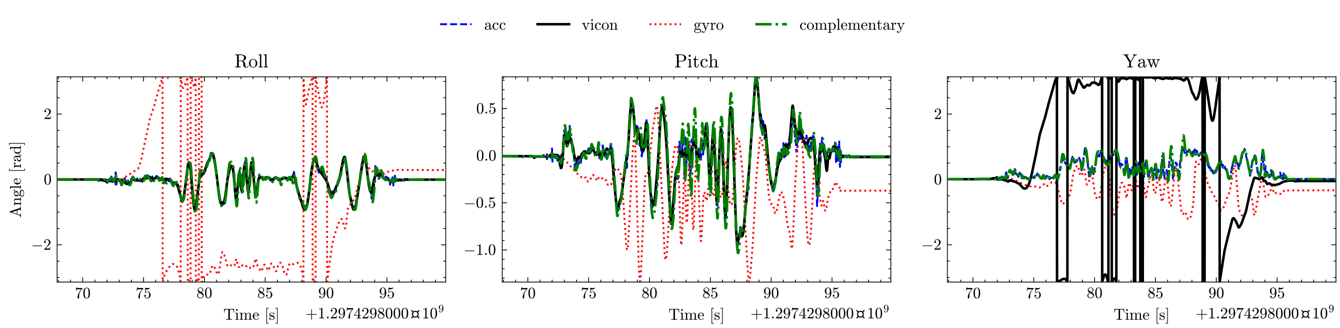

Fig. 7. Attitude estimation of the three methods considering the experiment 5.