Jet Processing

Question 1.

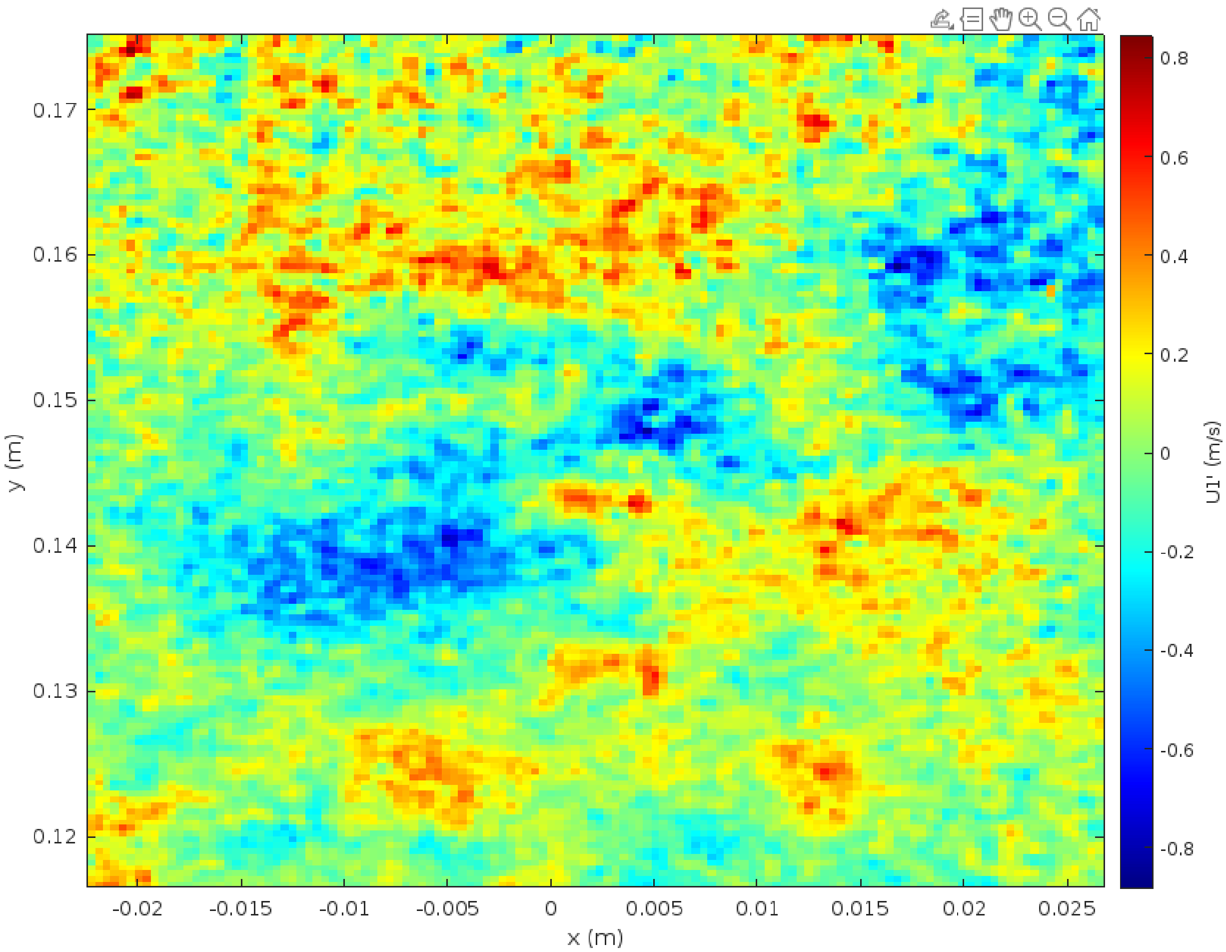

Figure 1: Fluctuating component of u1

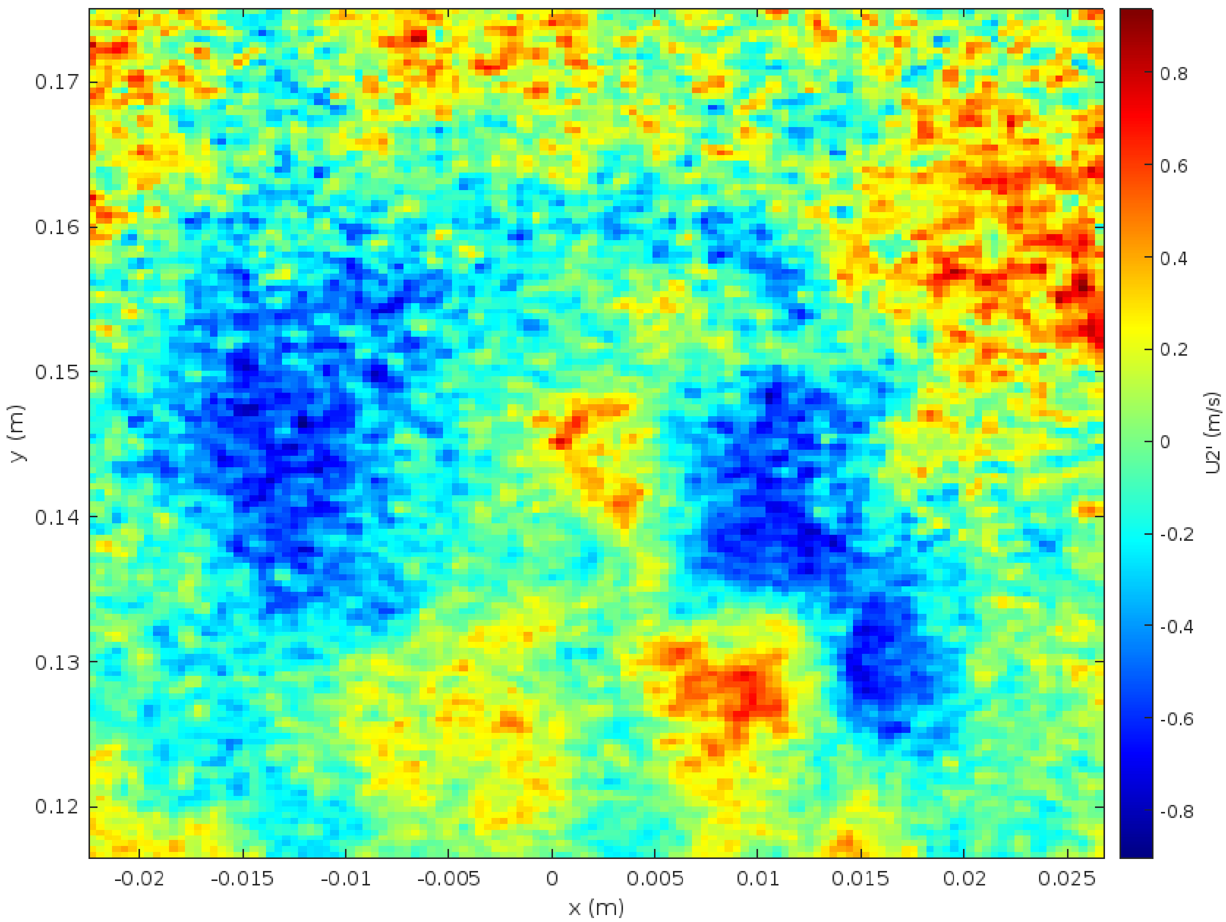

Figure 2: Fluctuating component of u2

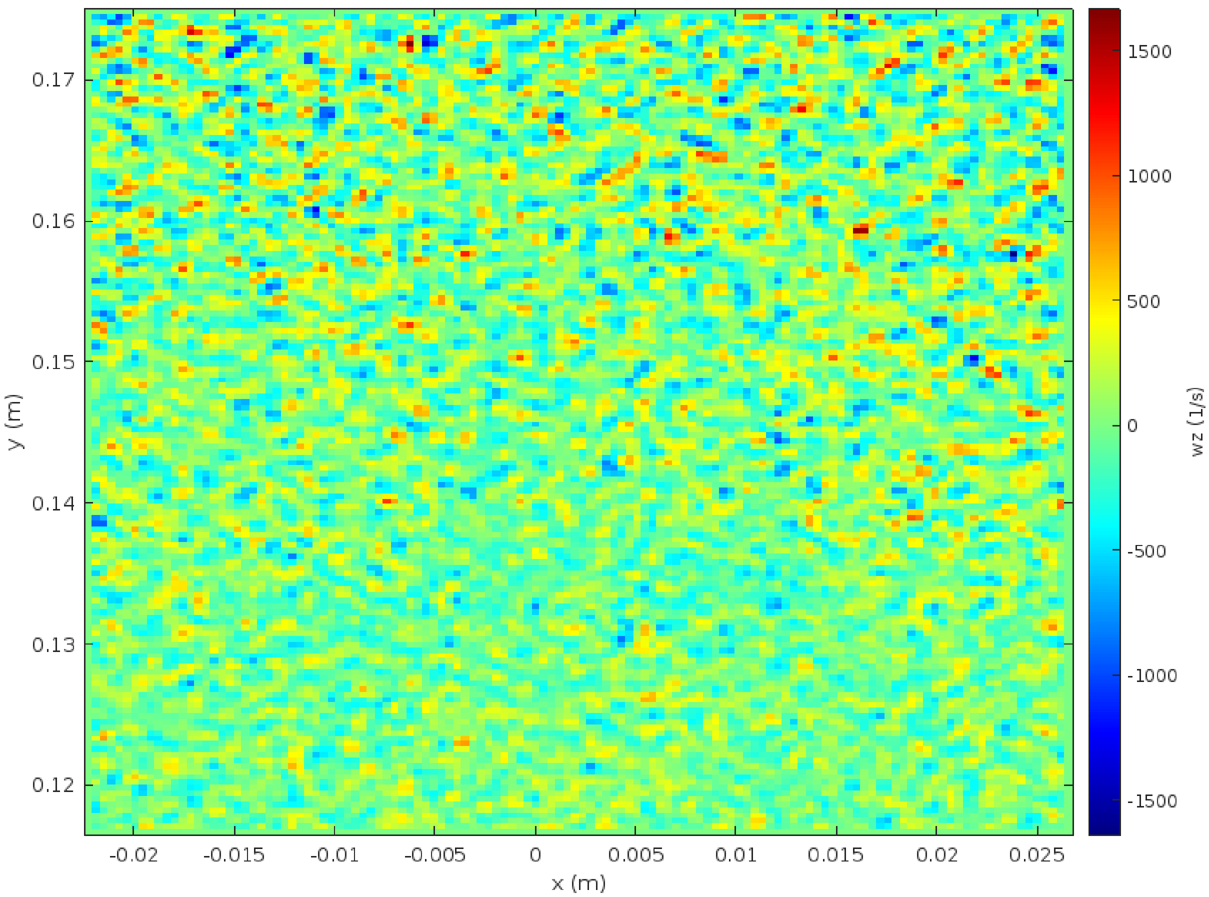

Figure 3: Vorticity in z-direction

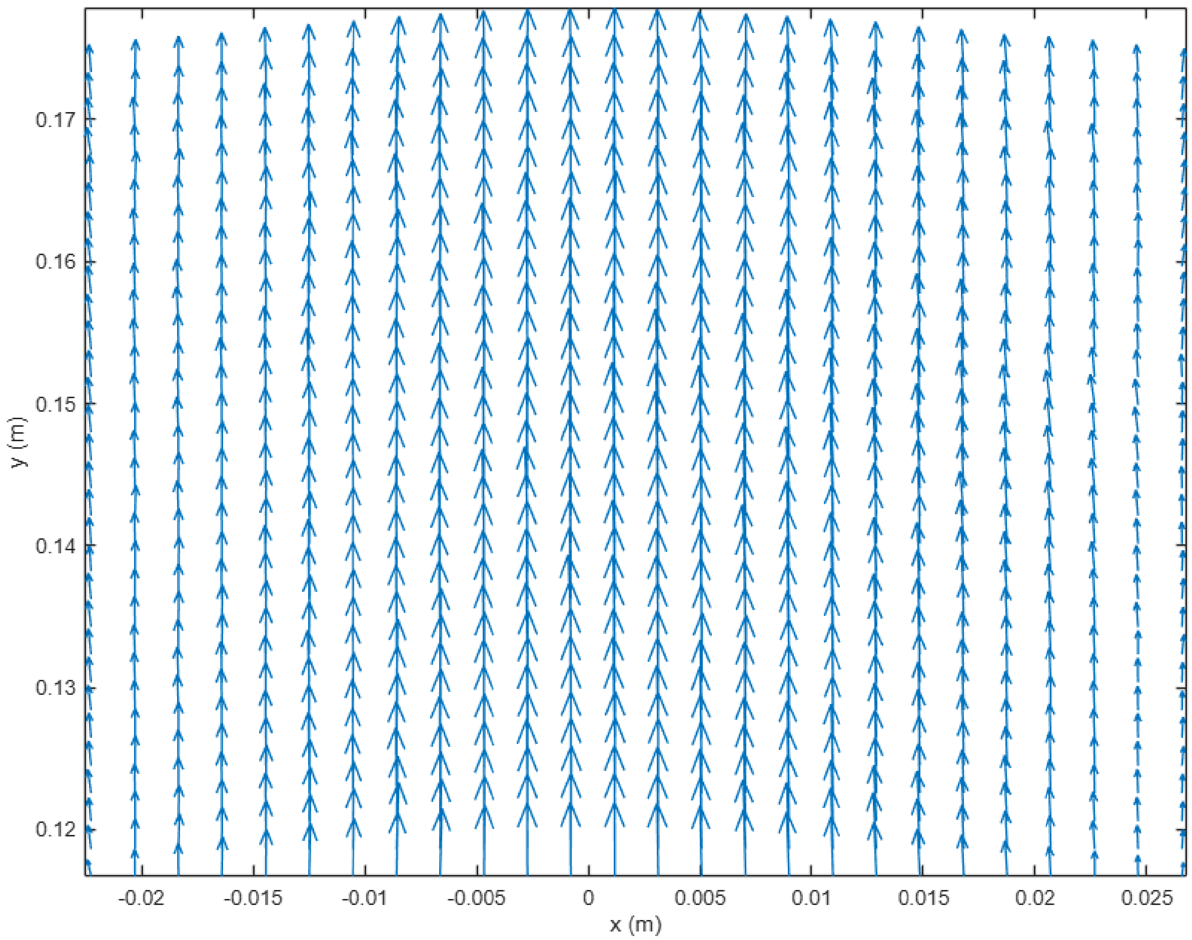

Figure 4: Streamline plot of jet

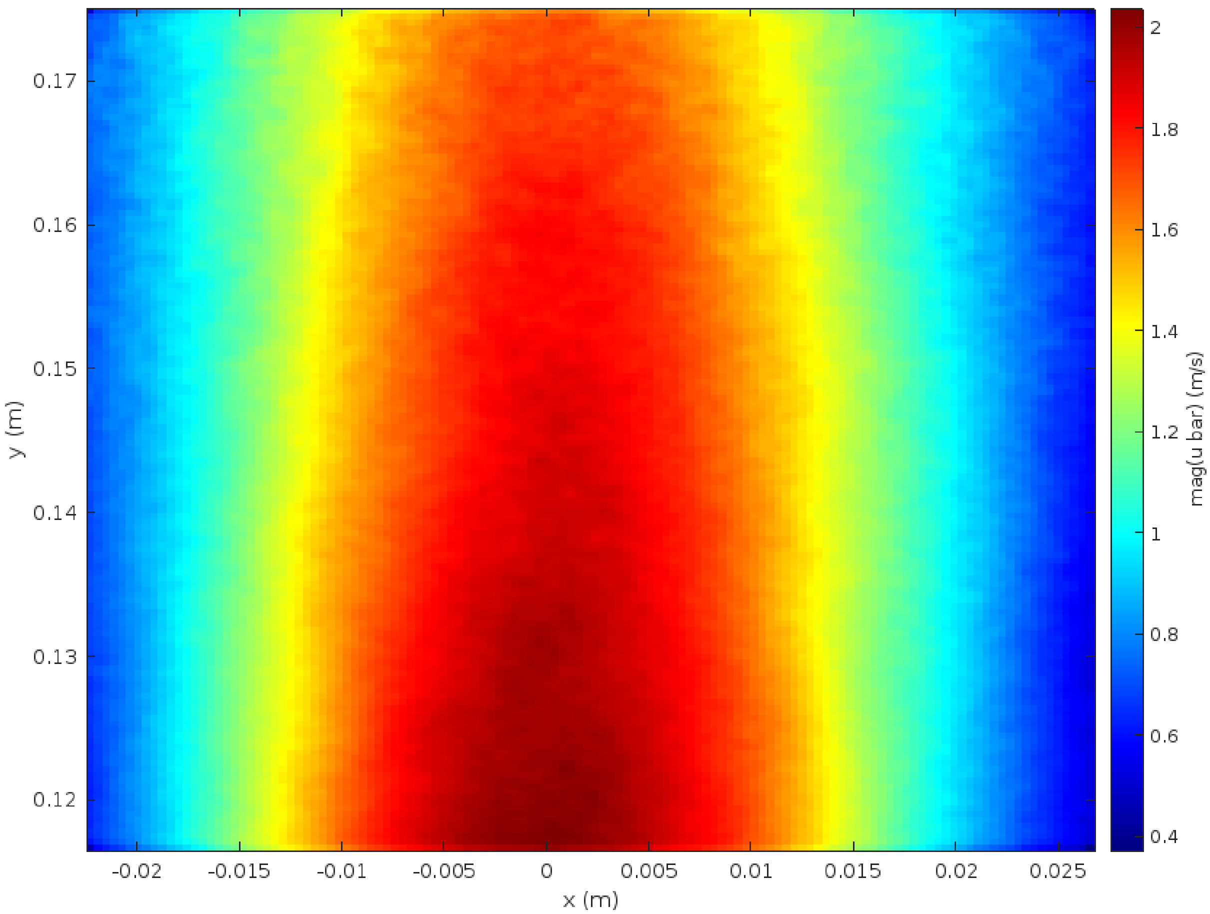

Figure 5: Plot of magnitude of average of u

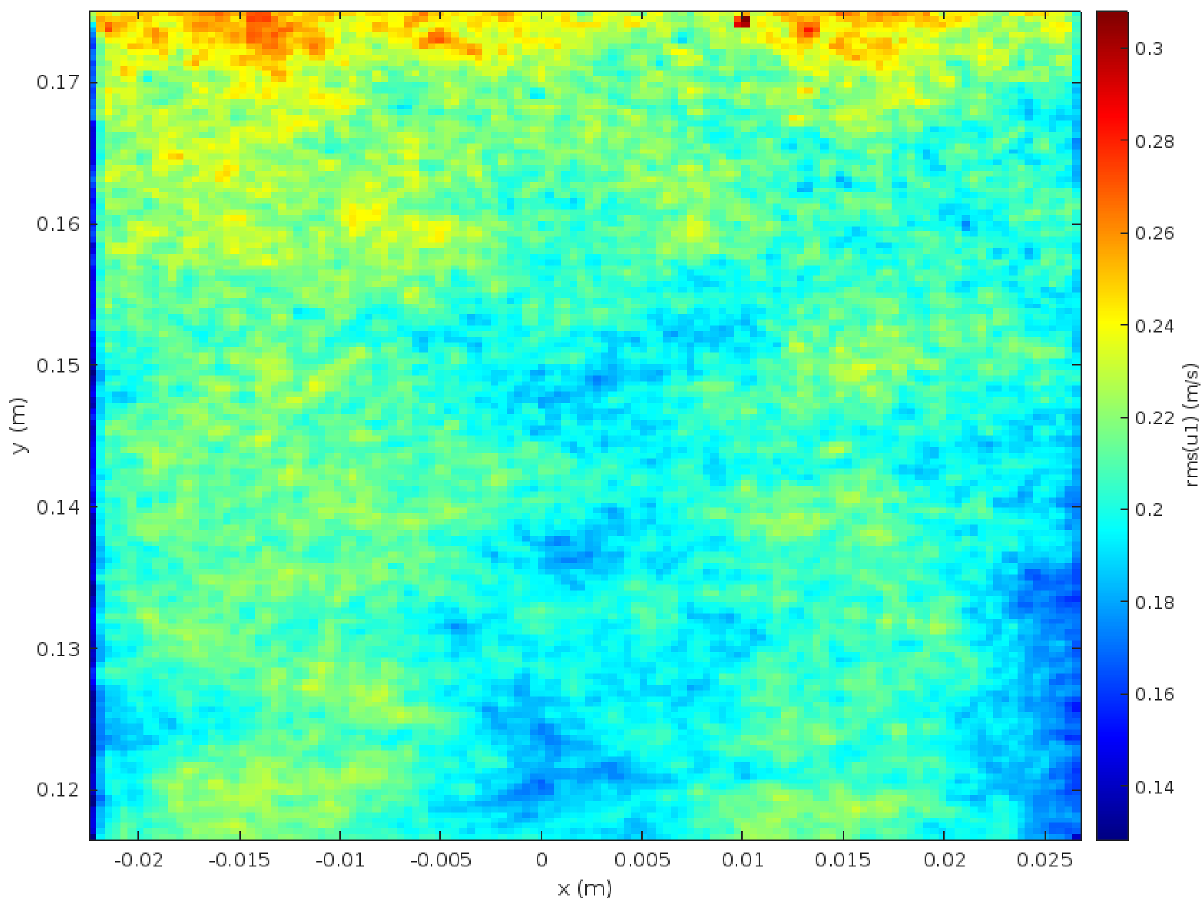

Figure 6: Plot of rms of u1

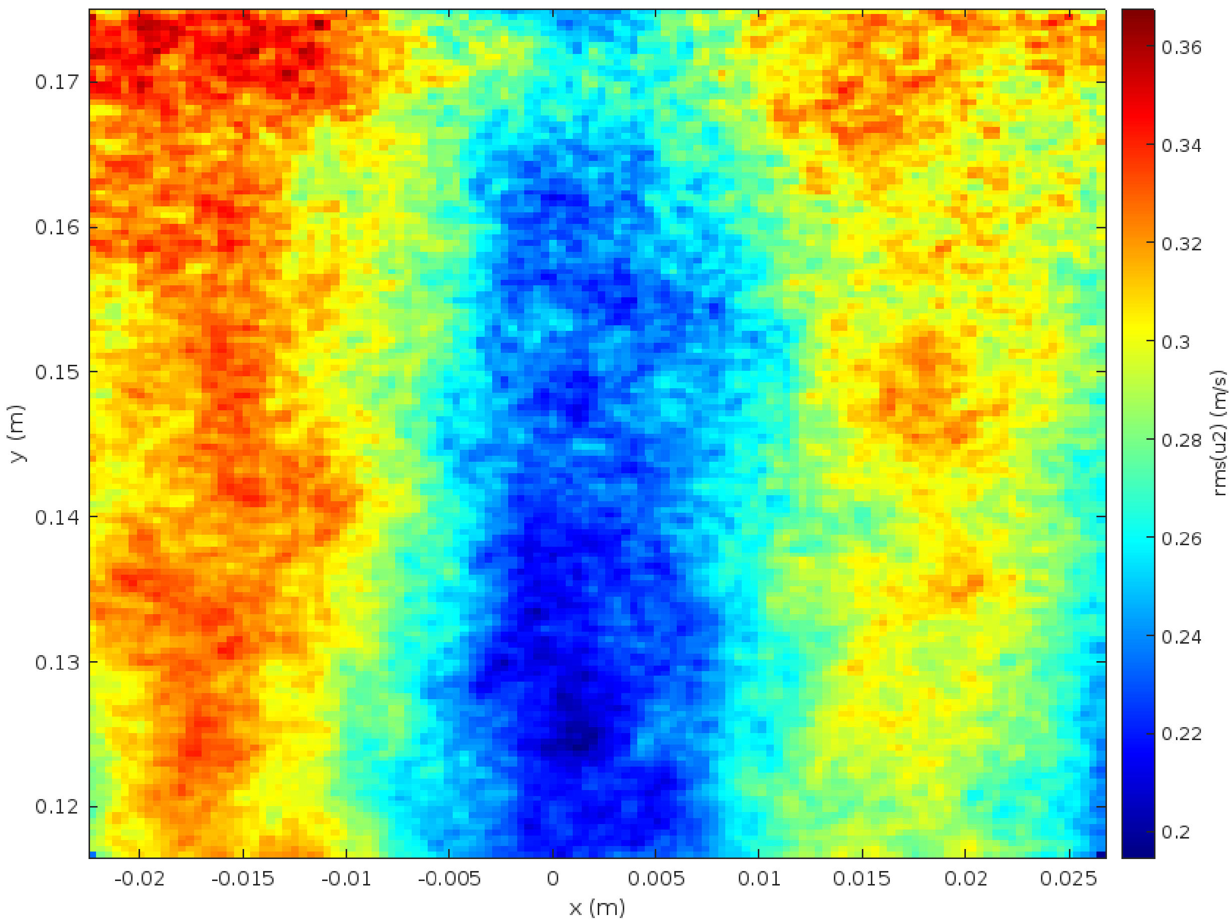

Figure 7: Plot of rms of u2

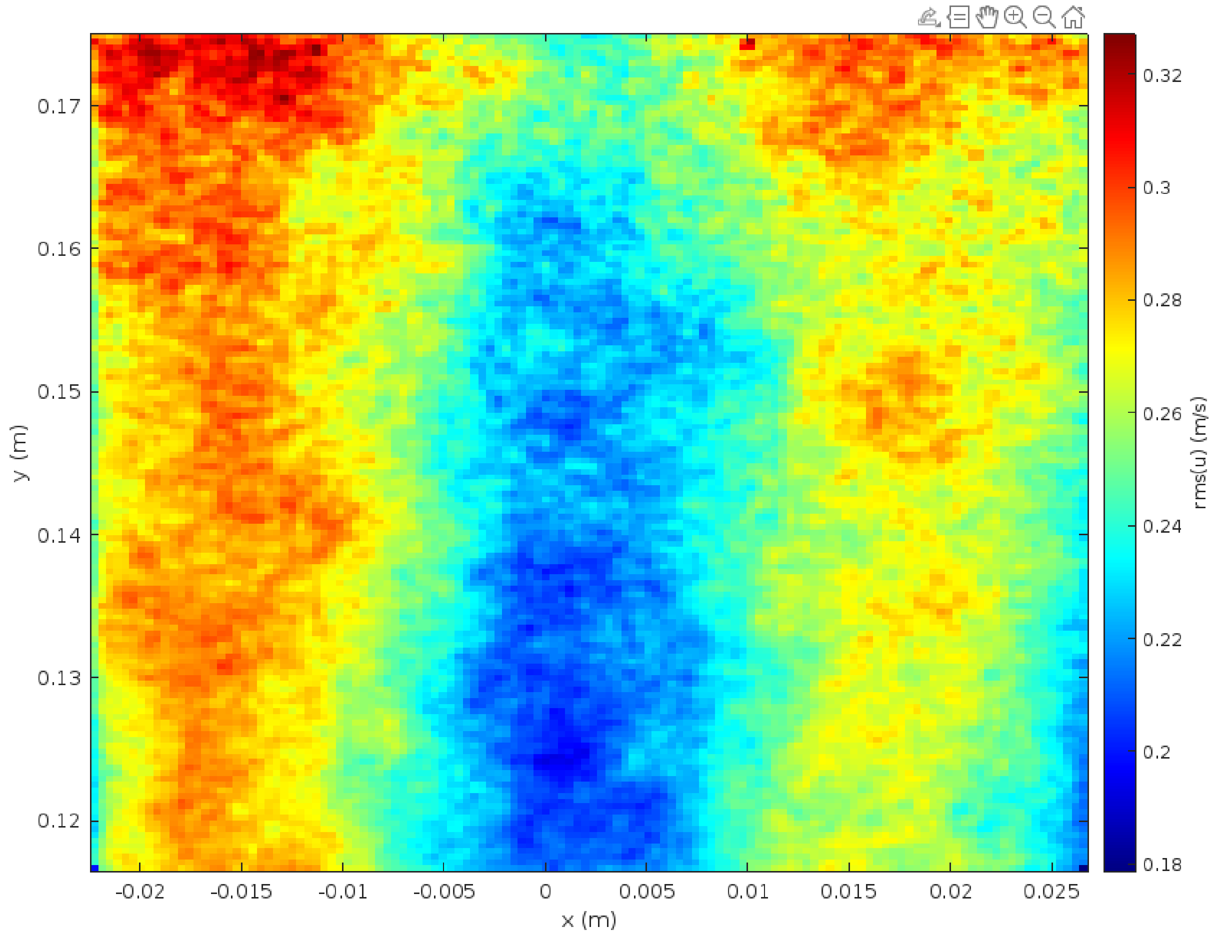

Figure 8: Plot of RMS of u

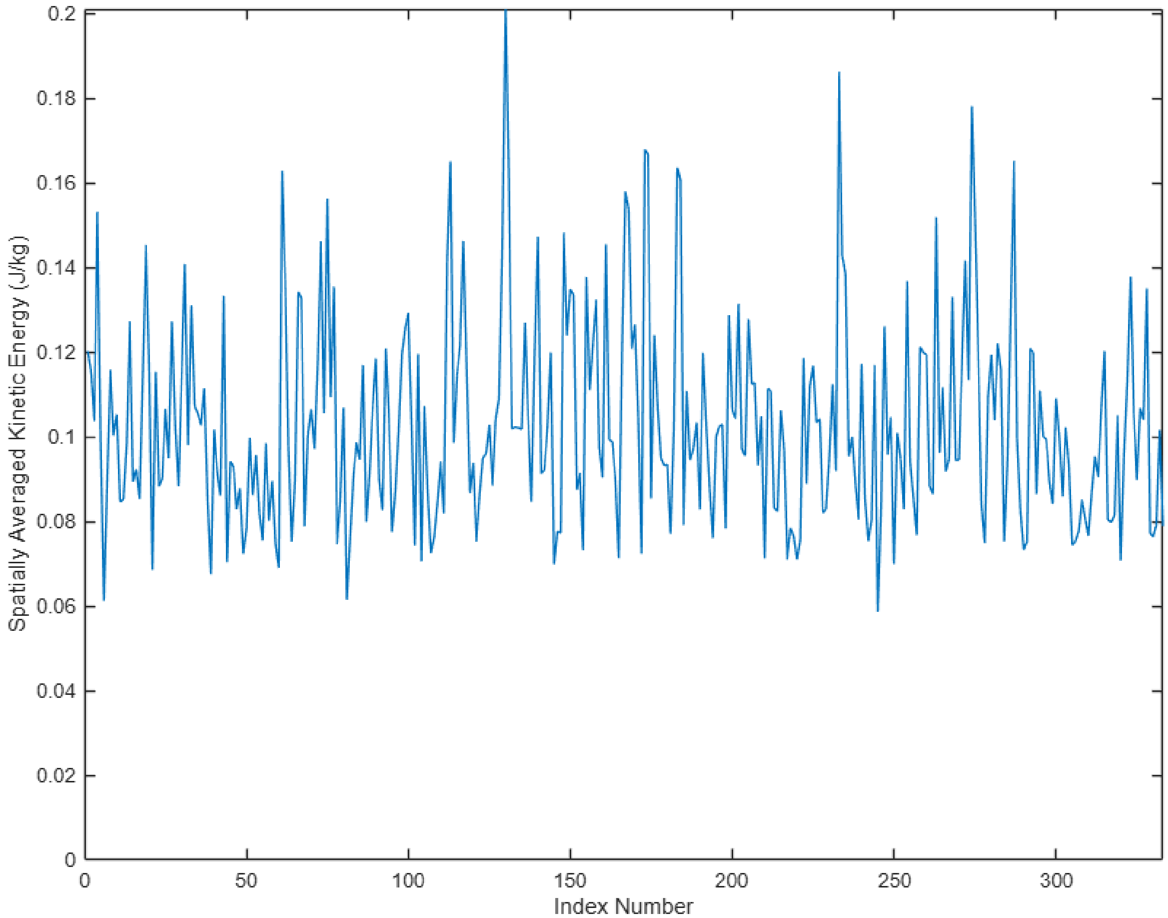

Figure 9: Plot of spatially averaged kinetic energy

Based on this plot of the spatial average of the kinetic energy, the flow can be considered steady. An unsteady turbulent flow would have some general trend in the kinetic energy. This flow has kinetic energy fluctuations, but no general increase or decrease.

Question 2.

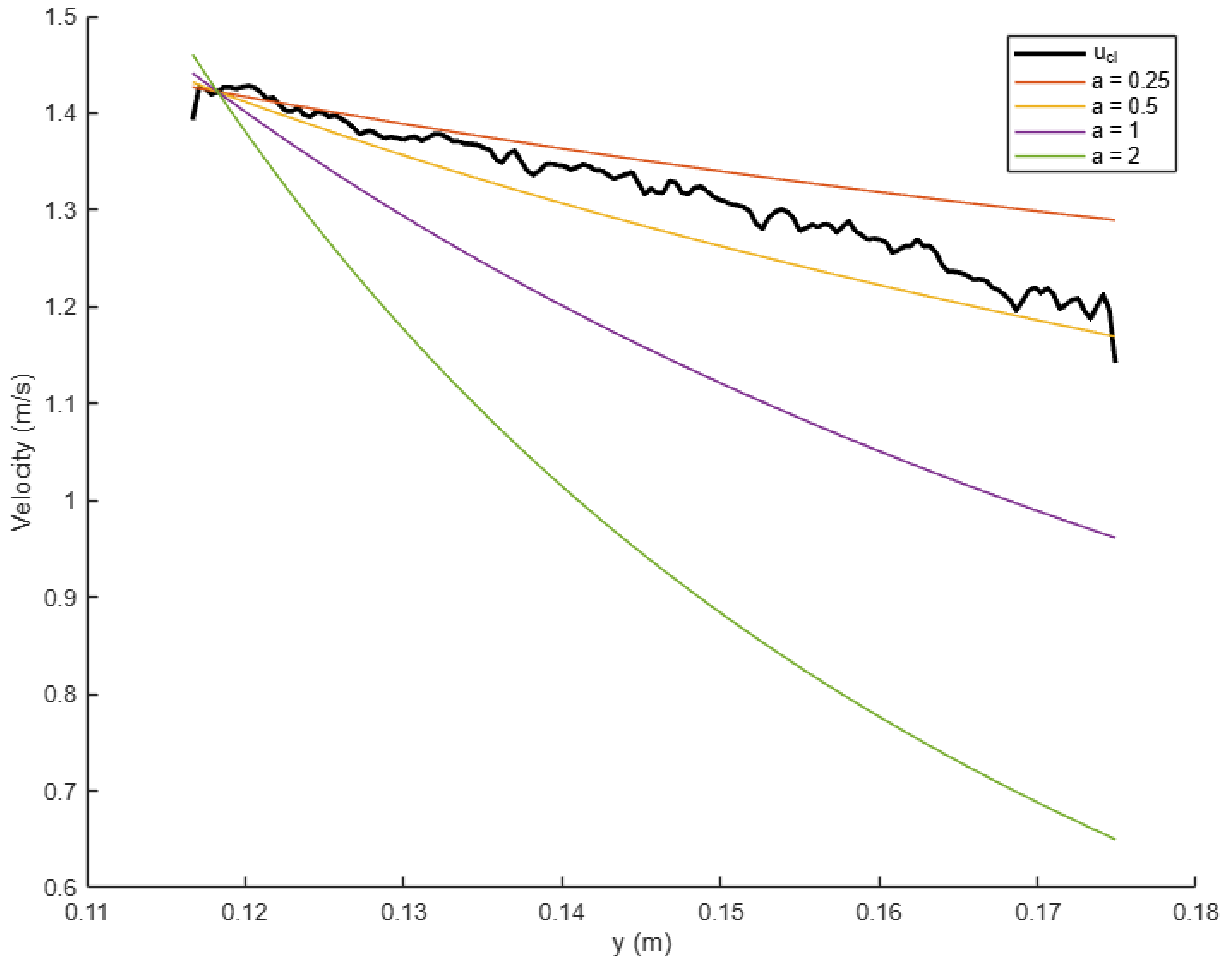

Figure 10: Centerline velocity and scaling

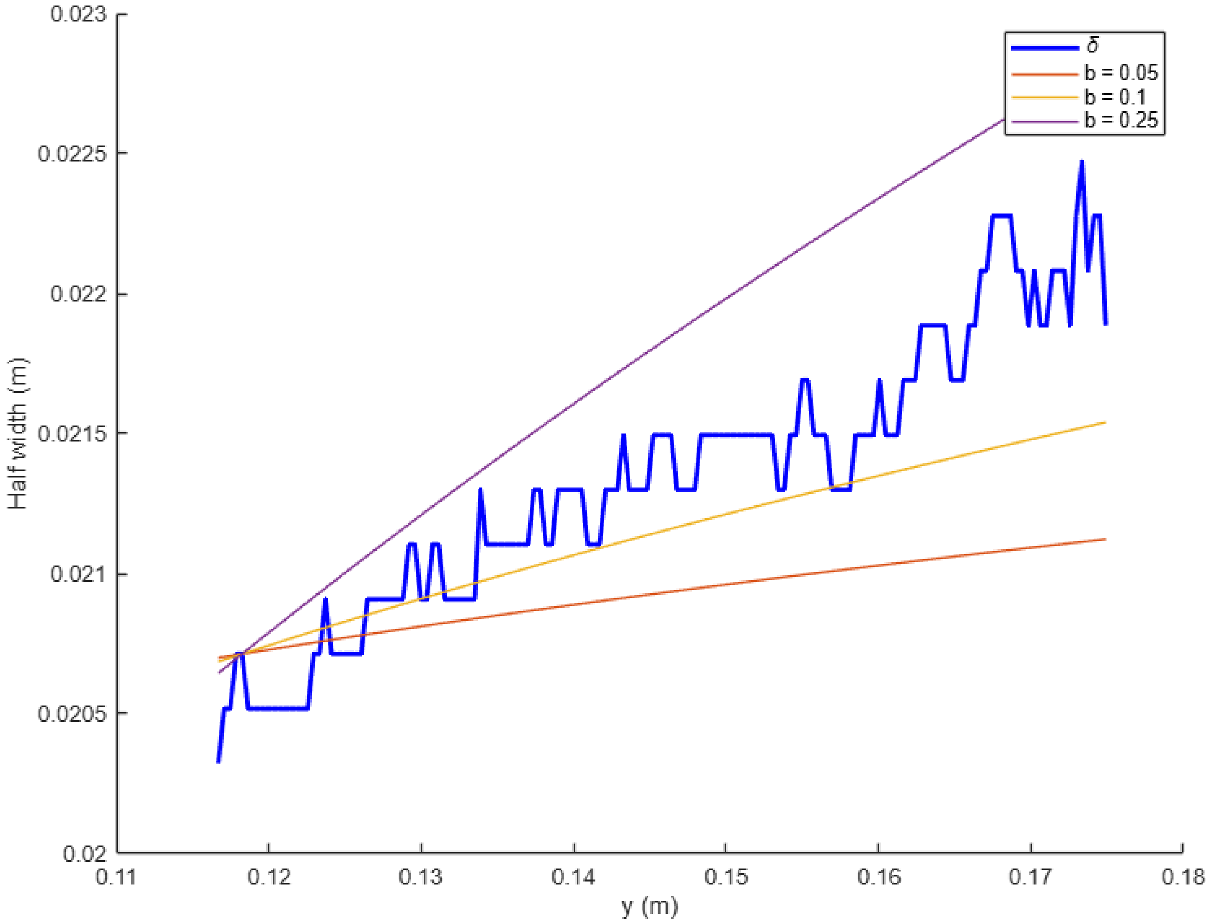

Figure 11: Jet half-width and scaling

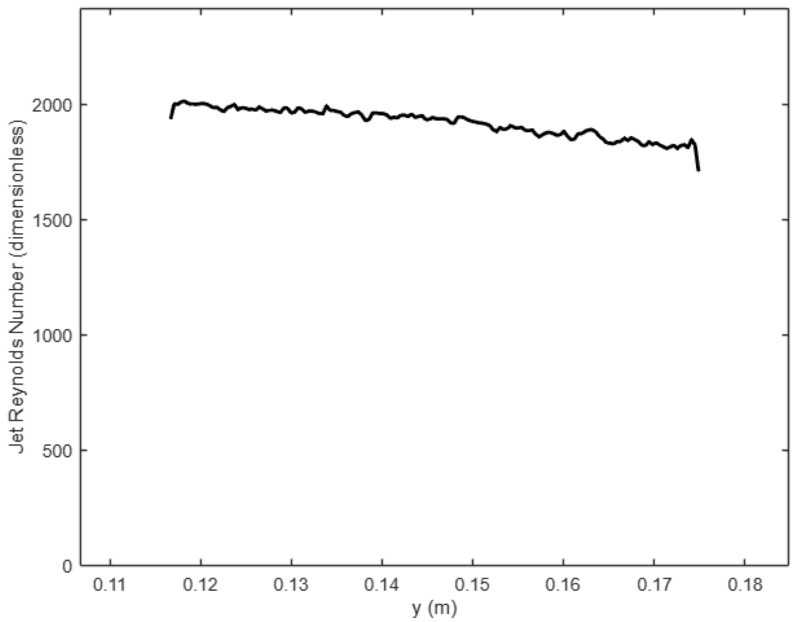

Figure 12: Plot of Jet Reynolds number as a function of downstream distance

Based on the scalings in Figure 10, the scaling for U_cl(y) is approximately a~0.5. From Table 6.1 in the book, a=0.55 for an axisymmetric jet. Based on Figure 11, the best scaling coefficient for the half-width, delta, is a b-value slightly higher than 0.1. This lines up with the b-value for axisymmetric jets in Table 6.1 of b=0.11. The jet Reynolds number for an axisymmetric jet is constant, according to Table 6.1.

The Reynolds number in Figure 12 is approximately constant but exhibits a decrease. As the jet slows down, the assumption of large Reynolds number becomes less appropriate as viscous effects begin to slow the jet. This decreases the jet Reynolds number. This effect might explain the decrease visible in Figure 12.

Overall, a,b, and the jet Reynolds number demonstrate scalings roughly in line with the scalings from Table 6.1 for an axisymmetric jet.

Question 3.

a & d.

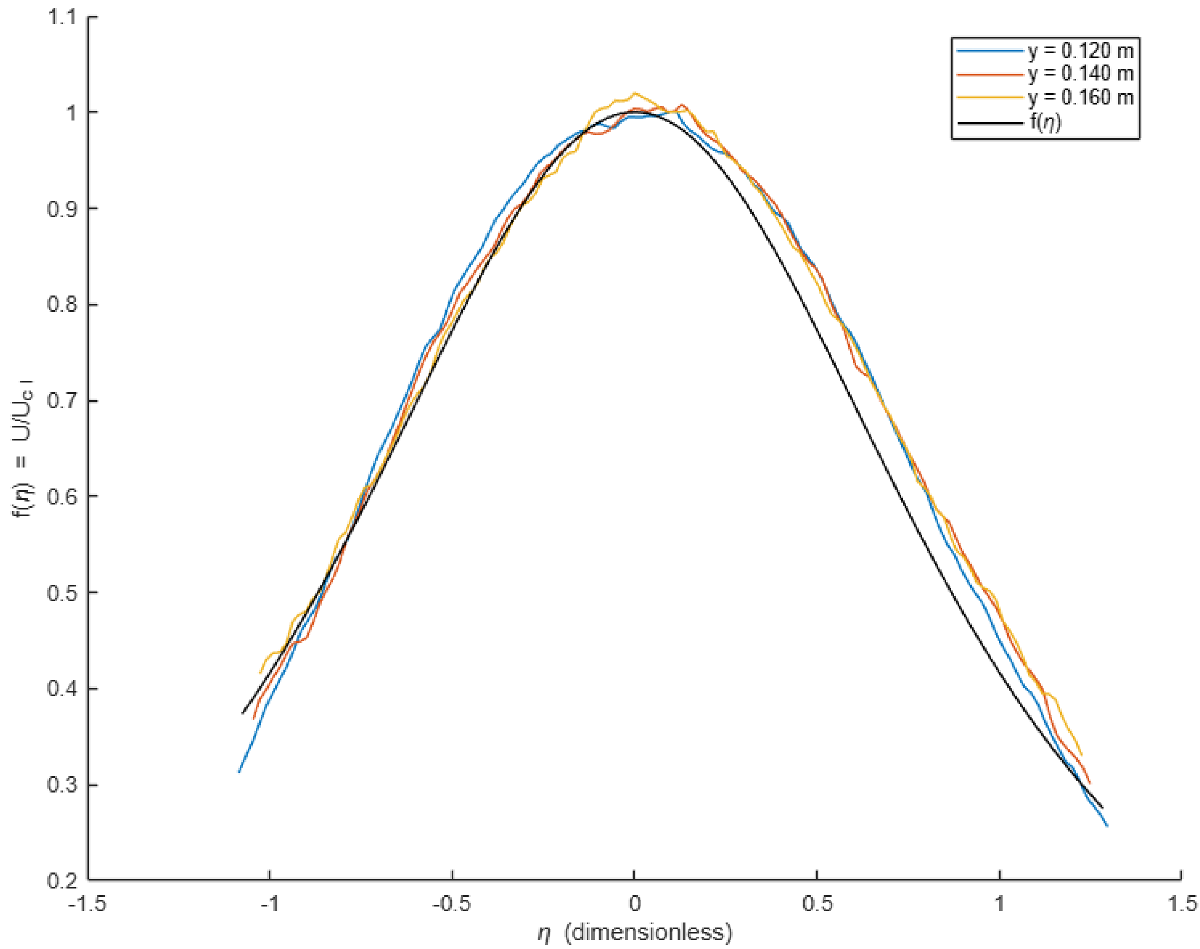

Figure 13: Plot of f(eta), U/U_centerline.

In Figure 13, the normalized velocity profiles at each downstream location are well-aligned. They follow a roughly parabolic shape. The analytical velocity profile, f(eta), is plotted in black. The analytical velocity profile lines up well with the measured profiles.

To determine the function g, relation 6.27 from the book is used. The functions g and f are related by:

$$g = -\nu_T f'$$

For an axisymmetric jet, the function f is:

$$f(\eta) = (1 + a\eta^2)^{-2}$$

which gives g as:

$$g(\eta) = 4\nu_T a\eta(1 + a\eta^2)^{-3}$$

where the value for a is 0.55.

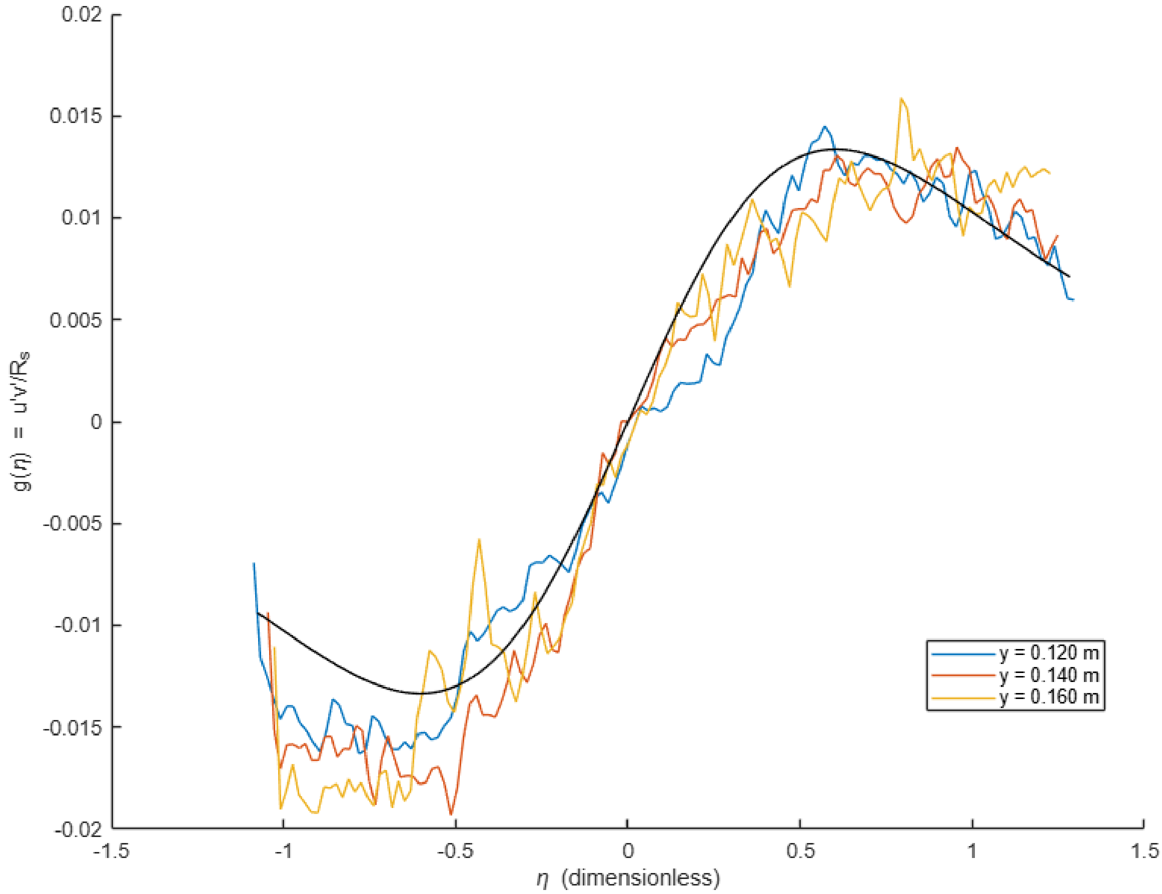

Figure 14: Plot of normalized Reynolds stress profiles. Curves are normalized with U_cl^2

Figure 14 demonstrates good agreement between the stress profiles for the three downstream locations. These profiles are more chaotic, because they involve the product of fluctuating components. The function g(eta) was derived above and is plotted in black. g(eta) depends on turbulent viscosity. It will be shown that the turbulent viscosity is non-uniform. To compare g to the measured profiles, it was scaled to be roughly the same magnitude.

b & c.

The turbulent viscosity was calculated based on eq. 6.27 and is displayed below:

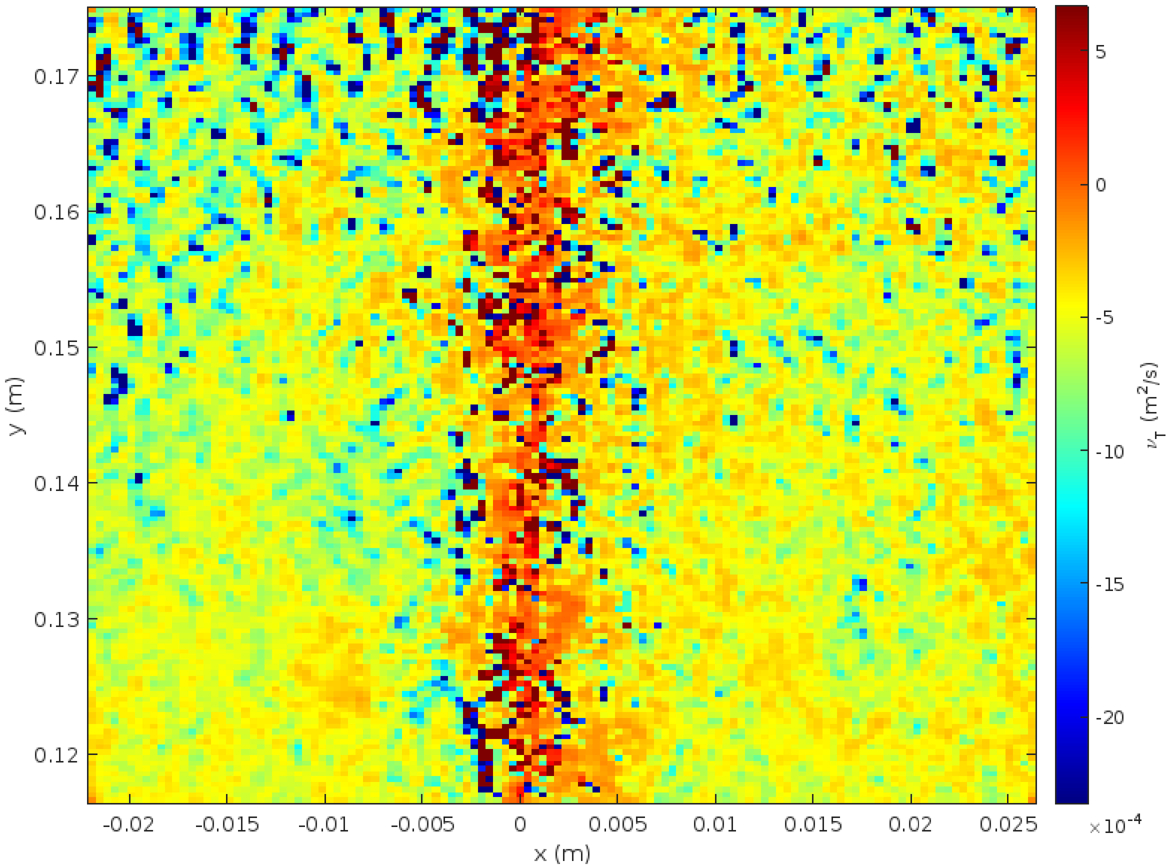

Figure 15: Turbulent viscosity throughout jet field

Visually, the turbulent viscosity is chaotic. There are many spots with highly negative values of turbulent viscosity. These are likely experimental noise, since it is physically unrealistic to have a negatively diffusive process. This could also signal that the eddy viscosity model is a poor model for some regions of this jet flow.

The turbulent viscosity is highest in the core of the jet. The magnitude decreases when moving away from the jet centerline. Ignoring the noise, the turbulent viscosity seems to be roughly constant in magnitude in the jet core region.

In the jet core, the magnitude of turbulent viscosity is on the order of 3-5*10^-4 m^2/s. This is approximately 20 times larger than the fluid viscosity (1.46*10^-5). Based on this data and analysis, the assumption that turbulent viscosity is much larger than fluid viscosity is appropriate in the jet core region, but not throughout the whole jet.

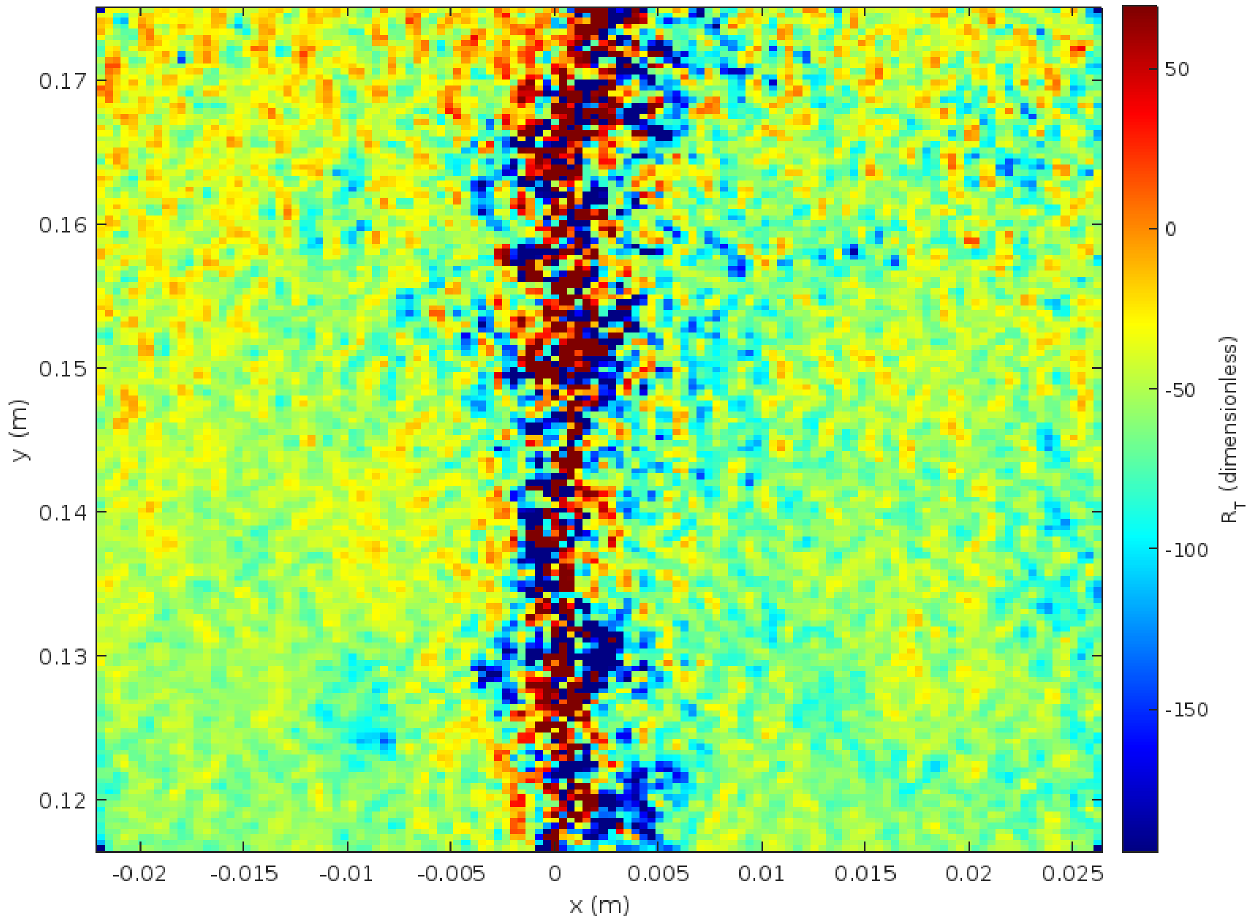

Figure 16: Turbulent Reynolds number R_T.

The turbulent Reynolds number can be calculated based on the turbulent viscosity, the jet half-width, and the centerline velocity. Thin shear layer theory predicts a constant turbulent Reynolds number of 40 for the axisymmetric jet. The distribution of R_T in this plot clearly shows that it is non-constant. However, near the centerline in regions where noise in the turbulent is low, the turbulent Reynolds number takes on values close to 40.

e.

The turbulent viscosity and turbulent Reynolds number exhibit some similarities and some departures from the expectations in the book. Based on thin shear layer analysis, the turbulent viscosity should be a function of downstream distance only. It appears to follow this trend only in the core region. The turbulent Reynolds number should be constant, but it varies chaotically. In the core region, it takes on a positive and quasi-constant value. To me, this indicates that the thin shear layer approximations are not as valid outside of the core region, or that the experimental data is not capturing all the predicted fluctuations.

The shapes of f and g are nearly identical between the expected and measured profiles. The magnitude of g does not match the measured profiles because of the chaotic and non-uniform turbulent viscosity. The magnitude of f matches the measured profiles.

Question 4.

The dissipation was calculated directly, using the spatial average of:

$$\epsilon(x,y) = \nu\left(-\left(\frac{\partial u_1'}{\partial x_1}\right)^2 + 2\left(\frac{\partial u_1'}{\partial x_2}\right)^2 + 2\left(\frac{\partial u_2'}{\partial x_1}\right)^2 + 8\left(\frac{\partial u_2'}{\partial x_2}\right)^2\right)$$

This method requires resolving gradients of fluctuating quantities for spatial scales on the order of the Kolmogorov length. This dataset does not capture those gradients, and as a result is likely to underpredict the dissipation. Based on the relation between dissipation calculated with this method and other methods in the HIT assignment, the dissipation calculated with this method was scaled by a factor of 2.

The Kolmogorov length and velocity scales can be calculated from this dissipation. The dissipation is a spatial variable, so the length and velocity scales are plotted below as scalars.

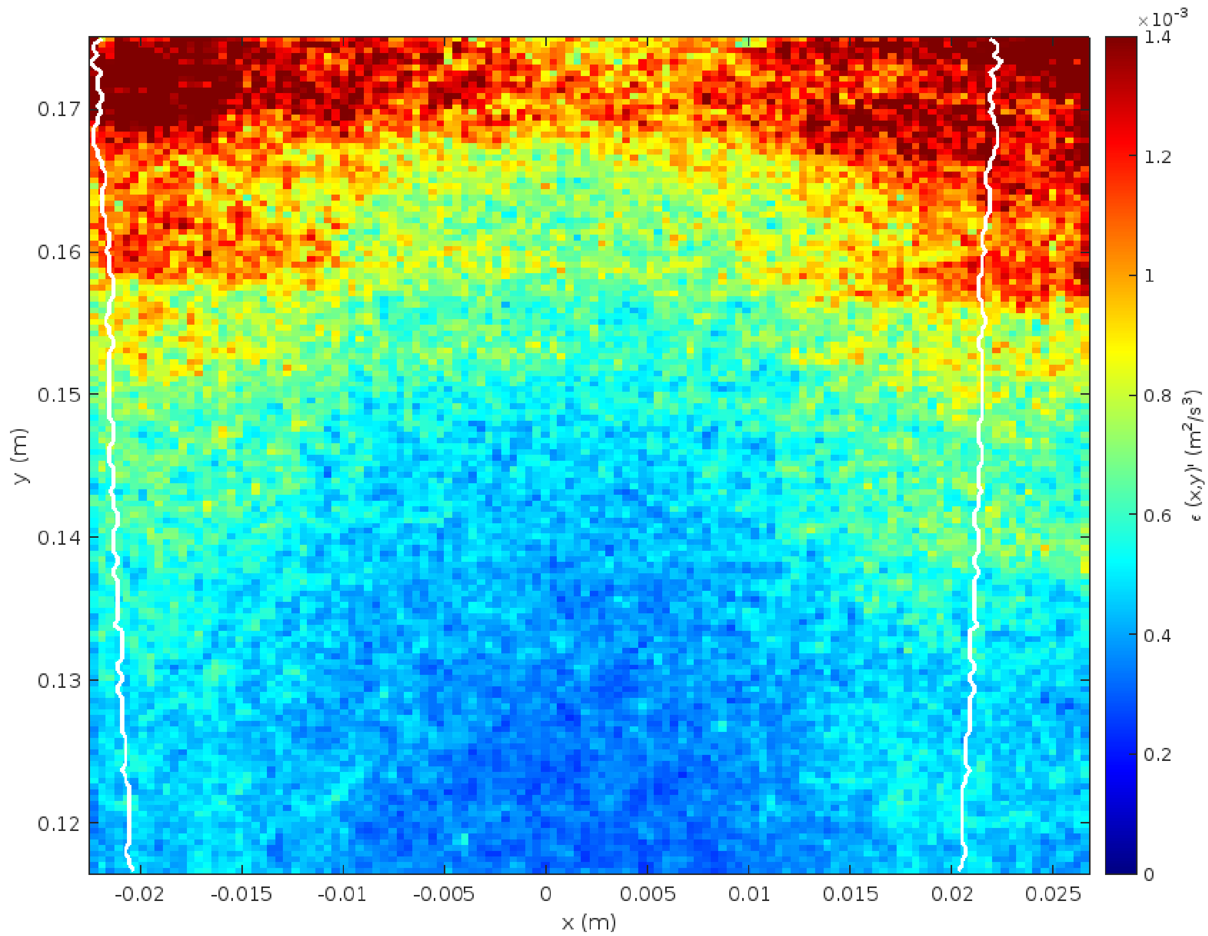

Figure 17: Dissipation throughout jet. The white lines indicate the jet boundaries, as measured by the half-width.

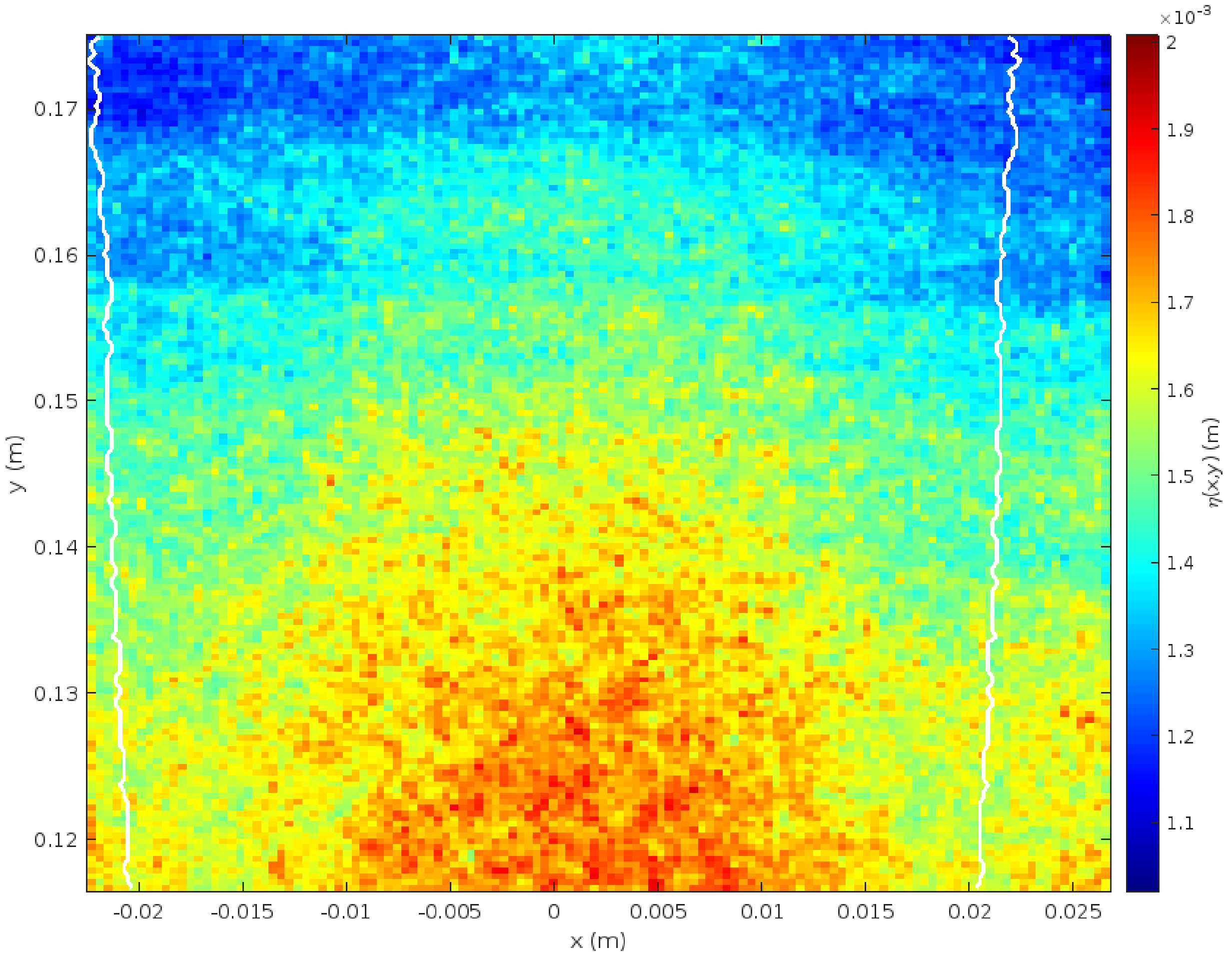

Figure 18: Kolmogorov length scale throughout the jet. The white lines indicate the jet boundaries, as measured by the half-width.

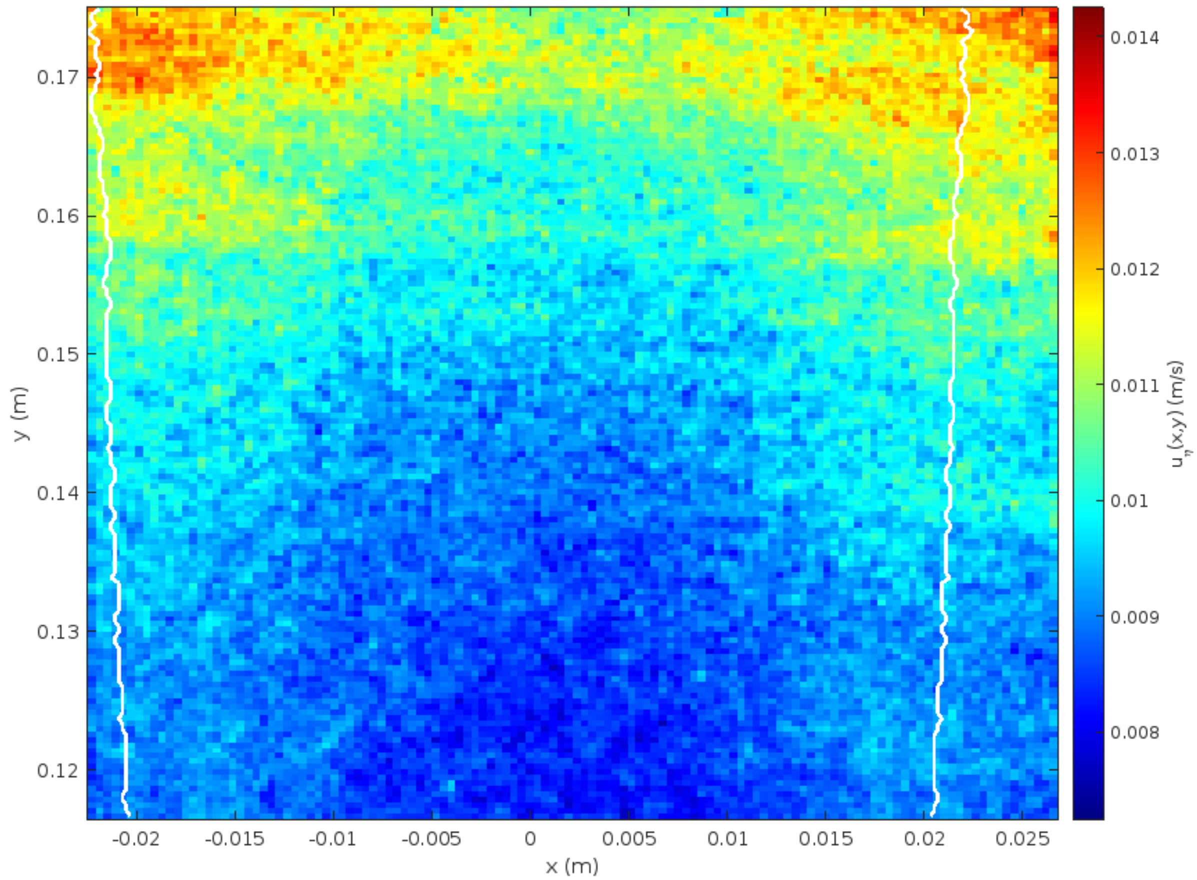

Figure 19: Kolmogorov velocity scale throughout the jet. The white lines indicate the jet boundaries, as measured by the half-width.

The dissipation and Kolmogorov scales can be averaged to obtain a single value. They were all averaged inside the region contained by one half-width from the centerline. These regions are bounded by the white lines in Figures 17, 18, and 19. The resulting values are: ε=6.6654*10^-4 m^2/s^3, η= 0.0015m, u_ η = 0.0098 m/s.

Question 5.

The advection term can be expanded, keeping only the terms in the 1 and 2 directions, as:

$$\frac{\overline{u_j}}{2}\frac{\partial}{\partial x_j}\left(\overline{u_i'u_i'}\right) = \frac{\overline{u_1}}{2}\frac{\partial}{\partial x_1}\left(\overline{u_1'u_1'}\right) + \frac{\overline{u_2}}{2}\frac{\partial}{\partial x_2}\left(\overline{u_1'u_1'}\right) + \frac{\overline{u_1}}{2}\frac{\partial}{\partial x_1}\left(\overline{u_2'u_2'}\right) + \frac{\overline{u_2}}{2}\frac{\partial}{\partial x_2}\left(\overline{u_2'u_2'}\right)$$

The production term can be expressed as:

$$-\overline{u_i'u_j'}\frac{\partial\overline{u_j}}{\partial x_i} = -\overline{u_1'u_1'}\frac{\partial\overline{u_1}}{\partial x_1} - \overline{u_1'u_2'}\frac{\partial\overline{u_2}}{\partial x_1} - \overline{u_2'u_1'}\frac{\partial\overline{u_1}}{\partial x_2} - \overline{u_2'u_2'}\frac{\partial\overline{u_2}}{\partial x_2}$$

The triple product term in the transport can be expressed as:

$$\frac{\partial}{\partial x_j}\left(-\frac{1}{2}\overline{u_i'u_i'u_j'}\right)=\frac{\partial}{\partial x_1}\left(-\frac{1}{2}\overline{u_1'u_1'u_1'}\right)+\frac{\partial}{\partial x_2}\left(-\frac{1}{2}\overline{u_1'u_1'u_2'}\right)+\frac{\partial}{\partial x_1}\left(-\frac{1}{2}\overline{u_2'u_2'u_1'}\right)+\frac{\partial}{\partial x_2}\left(-\frac{1}{2}\overline{u_2'u_2'u_2'}\right)$$

The viscous transport term can be expressed as:

$$\nu\frac{\partial}{\partial x_j}\left(\overline{u_i'\frac{\partial u_i'}{\partial x_j}}\right)=\nu\left[\frac{\partial}{\partial x_1}\left(\overline{u_1'\frac{\partial u_1'}{\partial x_1}}\right)+\frac{\partial}{\partial x_2}\left(\overline{u_1'\frac{\partial u_1'}{\partial x_1}}\right)+\frac{\partial}{\partial x_1}\left(\overline{u_1'\frac{\partial u_2'}{\partial x_1}}\right)+\frac{\partial}{\partial x_2}\left(\overline{u_1'\frac{\partial u_2'}{\partial x_1}}\right)\right]$$

The pressure term is assumed to be negligible, and the dissipation was previously calculated.

The dissipation term was previously calculated. The transport terms were summed, neglecting pressure, and all the terms are averaged in the y-direction and plotted:

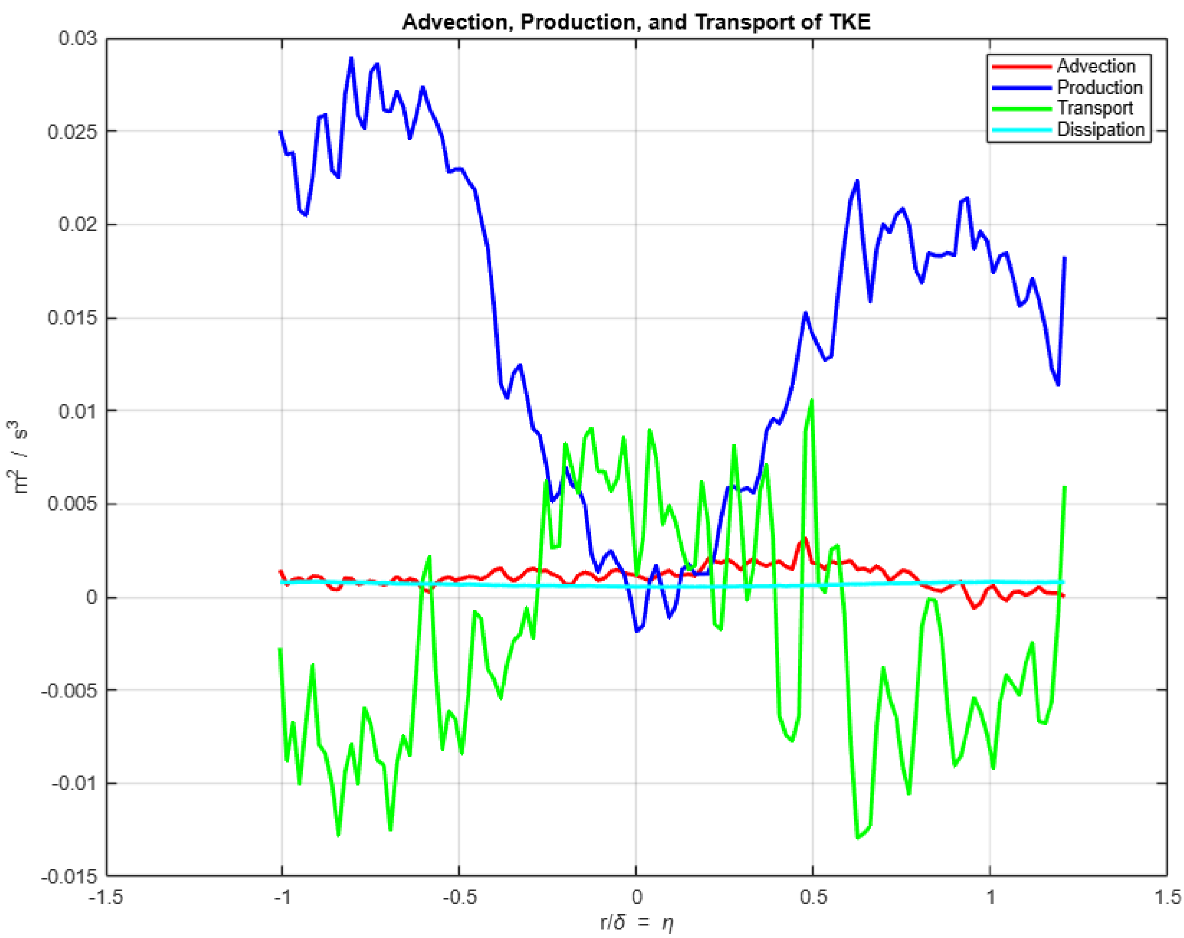

Figure 20: Terms in TKE equation for jet flow

The most noticeable difference between this plot and Fig. 6-5 in the book is that Figure 20 is much more chaotic. The data is very noisy, especially the transport term. The transport term is especially noisy because it involves a derivative of a fluctuating triple product.

Another difference is that production increases with radial distance, which is opposite the behavior seen in Fig. 6-5 in the book. I think my calculation of the production term was incorrect for that reason.

In terms of magnitude, advection is much lower compared to the other terms in Fig. 20 compared to Fig. 6-5. There are also no large variations in dissipation as observed in Fig. 6-5.

Overall, the two curves are not very similar. I believe this is due to noise in the data, as well as incorrect/poor techniques used to estimate these terms.

Code

```
close all;
clear;
clc;

load('data_full.mat')
nu = 1.46*10^-5;
%% Part A %%
% Calculate fluctuating velocity components
t_rand = 180;
u1_bar = mean(u1,3);
u2_bar = mean(u2,3);
u1_prime = u1 - u1_bar;
u2_prime = u2 - u2_bar;
w3 = zeros(size(u1_prime(:,:,t_rand)));
w3(2:end-1,2:end-1) = ((u2(3:end,2:end-1,t_rand) - u2(1:end-2,2:end-
1,t_rand)) / (2 * (x1(2) - x1(1)))) - ...
                      ((u1(2:end-1,3:end,t_rand) - u1(2:end-1,1:end-
2,t_rand)) / (2 * (x2(2) - x2(1))));

%% Part A - Plot Fluctuations and Vorticity %%
figure
imagesc(x1, x2, u1_prime(:,:,t_rand))
set(gca, 'ydir', 'normal')
xlabel('x (m)')
ylabel('y (m)')
colormap(jet)
cb = colorbar;
ylabel(cb, "U1' (m/s)")

figure
imagesc(x1, x2, u2_prime(:,:,t_rand))
set(gca, 'ydir', 'normal')
xlabel('x (m)')
ylabel('y (m)')
colormap(jet)
cb = colorbar;
ylabel(cb, "U2' (m/s)")

figure
imagesc(x1, x2, w3)
set(gca, 'ydir', 'normal')
xlabel('x (m)')
ylabel('y (m)')
colormap(jet)
cb = colorbar;
ylabel(cb, "wz (1/s)")

%% Part B - Compute and Plot RMS, Mean, and Quiver %%
ubar_mag = sqrt(u1_bar.^2 + 2 * u2_bar.^2);
```

```
u1_rms = sqrt(mean(u1_prime.^2, 3));
u2_rms = sqrt(mean(u2_prime.^2, 3));
u_rms = (u1_rms + 2 * u2_rms) / 3;

figure
step=5;
quiver(x1(1:step:end), x2(1:step:end), u1_bar(1:step:end, 1:step:end),
u2_bar(1:step:end, 1:step:end), 2)
xlabel('x (m)')
ylabel('y (m)')
axis tight

figure
imagesc(x1, x2, ubar_mag)
set(gca, 'ydir', 'normal')
xlabel('x (m)')
ylabel('y (m)')
colormap(jet)
cb = colorbar;
ylabel(cb, "mag(u bar) (m/s)")

figure
imagesc(x1, x2, u1_rms)
set(gca, 'ydir', 'normal')
xlabel('x (m)')
ylabel('y (m)')
colormap(jet)
cb = colorbar;
ylabel(cb, "rms(u1) (m/s)")

figure
imagesc(x1, x2, u2_rms)
set(gca, 'ydir', 'normal')
xlabel('x (m)')
ylabel('y (m)')
colormap(jet)
cb = colorbar;
ylabel(cb, "rms(u2) (m/s)")

figure
imagesc(x1, x2, u_rms)
set(gca, 'ydir', 'normal')
xlabel('x (m)')
ylabel('y (m)')
colormap(jet)
cb = colorbar;
ylabel(cb, "rms(u) (m/s)")

% %% Compute Averages %%
% u1_rms_avg = mean(u1_rms, 'all');
% u2_rms_avg = mean(u2_rms, 'all');
% u_rms_avg = mean(u_rms, 'all');
```

```
% u_rms_ratio = u1_rms_avg / u2_rms_avg;
% k = 0.5 * (mean(u1_prime.^2, 3) + 2 * mean(u2_prime.^2, 3));
% k_avg = mean(k, 'all');

%% Part C - Spatially Averaged Kinetic Energy %%
k_spatial = 0.5 * (mean(u1_prime.^2, [1,2]) + 2 * mean(u2_prime.^2,
[1,2]));

figure
plot(1:length(k_spatial), squeeze(k_spatial));
axis([0, length(k_spatial), 0, max(k_spatial)]);
xlabel('Index Number')
ylabel('Spatially Averaged Kinetic Energy (J/kg)')
```

```
close all;

u_cl = u2_bar(:, round(size(u2_bar, 2) / 2));
a_values = [0.25, 0.5, 1, 2];
scaling_factors = u_cl(5) ./ (x2(5).^(-a_values));

figure
hold on
plot(x2, u_cl, 'k', 'LineWidth', 2)
for i = 1:length(a_values)
    plot(x2, scaling_factors(i) * x2.^(-a_values(i)))
end
hold off
xlabel('y (m)')
ylabel('Velocity (m/s)')
legend(['u_{cl}', arrayfun(@(a) ['a = ' num2str(a)], a_values,
'UniformOutput', false)])

% Go through every y-index
% Look for all the x-indices where u2>0.5*ucl
delta = zeros(size(u_cl));
for i = 1:size(x2,1)
    right_idx = find(u2_bar(i, :) > 0.5 * u_cl(i), 1, 'last');   %
Last point greater than 0.5*u_cl
    left_idx = find(u2_bar(i, :) < 0.5 * u_cl(i), 1, 'first'); % First
point less than 0.5*u_cl
    if ~isempty(left_idx) && ~isempty(right_idx)
        delta(i) = 0.5*( x1(1, right_idx) - x1(1, left_idx));
    end
end
b_values = [0.05, 0.10, 0.25];
scaling_factors = delta(5) ./ (x2(5).^(b_values));
figure
hold on
plot(x2, delta, 'b', 'LineWidth', 2)
for i = 1:length(b_values)
    plot(x2, scaling_factors(i) * x2.^(b_values(i)))
end
hold off
xlabel('y (m)')
ylabel('Half width (m)')
legend(['\delta', arrayfun(@(b) ['b = ' num2str(b)], b_values,
'UniformOutput', false)])


Re_jet = u_cl.*delta/nu;
figure
plot(x2, Re_jet, 'k', 'LineWidth', 2)
xlabel('y (m)')
ylabel('Jet Reynolds Number (dimensionless)')
axis([min(x2)-0.01, max(x2)+0.01, 0, max(Re_jet)*1.2]);
```

```
close all;
%%% Part A %%%
downstream_indices = [10,61,112];
RS = mean(u1_prime.*u2_prime,3);

% Plots for f, g
a = 0.55;
g_scaling = 280e-4;
index = 20;  % Replace with the desired index
eta = x1 / delta(index);
f_eta = (1 + a * eta.^2).^(-2);
f_prime = -4 * a * eta .* (1 + a * eta.^2).^(-3);
g_eta = -g_scaling * f_prime;

figure;
hold on
for d=downstream_indices
    plot(x1/delta(d),u2_bar(d,:)/u_cl(d))
end
plot(x1 / delta(index), f_eta, 'k-', 'LineWidth', 1, 'DisplayName',
'f(\eta)');
hold off
legend([arrayfun(@(d) sprintf('y = %.3f m', x2(d)),
downstream_indices, 'UniformOutput', false), {'f(\eta)'}], 'Location',
'best');
xlabel('\eta (dimensionless)')
ylabel('f(\eta) = U/U_{c l}')

figure;
hold on
for d=downstream_indices
    plot(x1/delta(d),-RS(d,:)/(u_cl(d).^2))
end
plot(x1 / delta(index), g_eta/(u_cl(112).^2), 'k-', 'LineWidth', 1,
'DisplayName', 'g(\eta)');
hold off
legend(arrayfun(@(d) sprintf('y = %.3f m', x2(d)), downstream_indices,
'UniformOutput', false), 'Location', 'best')
xlabel('\eta (dimensionless)')
ylabel('g(\eta) = u''v''/R_s');

%%%% Part B %%%
dx = x1(2)-x1(1);
dU_dr = (u2_bar(:,3:end) - u2_bar(:,1:end-2))/(2*dx);
% dU_dr = -abs(dU_dr); % Axisymmetric about centerline - velocity
should monotonically decrease
nu_T = -RS(:,2:end-1)./dU_dr;

figure
imagesc(x1(2:end-1), x2, nu_T)
set(gca, 'ydir', 'normal')
xlabel('x (m)')
```

```
ylabel('y (m)')
clims = prctile(nu_T(:), [2, 98]);
clim(clims)
colormap(jet)
cb = colorbar;
ylabel(cb, "\nu_T (m^2/s)")

R_T = u_cl.*delta./nu_T;
figure
imagesc(x1(2:end-1), x2, R_T)
set(gca, 'ydir', 'normal')
xlabel('x (m)')
ylabel('y (m)')
clims = prctile(R_T(:), [2, 98]);
clim(clims)
colormap(jet)
cb = colorbar;
ylabel(cb, "R_T (dimensionless)")
```

```
close all;
%%% Part A %%%
downstream_indices = [10,61,112];
RS = mean(u1_prime.*u2_prime,3);

% Plots for f, g
a = 0.55;
g_scaling = 280e-4;
index = 20;  % Replace with the desired index
eta = x1 / delta(index);
f_eta = (1 + a * eta.^2).^(-2);
f_prime = -4 * a * eta .* (1 + a * eta.^2).^(-3);
g_eta = -g_scaling * f_prime;

figure;
hold on
for d=downstream_indices
    plot(x1/delta(d),u2_bar(d,:)/u_cl(d))
end
plot(x1 / delta(index), f_eta, 'k-', 'LineWidth', 1, 'DisplayName',
'f(\eta)');
hold off
legend([arrayfun(@(d) sprintf('y = %.3f m', x2(d)),
downstream_indices, 'UniformOutput', false), {'f(\eta)'}], 'Location',
'best');
xlabel('\eta (dimensionless)')
ylabel('f(\eta) = U/U_{c l}')

figure;
hold on
for d=downstream_indices
    plot(x1/delta(d),-RS(d,:)/(u_cl(d).^2))
end
plot(x1 / delta(index), g_eta/(u_cl(112).^2), 'k-', 'LineWidth', 1,
'DisplayName', 'g(\eta)');
hold off
legend(arrayfun(@(d) sprintf('y = %.3f m', x2(d)), downstream_indices,
'UniformOutput', false), 'Location', 'best')
xlabel('\eta (dimensionless)')
ylabel('g(\eta) = u''v''/R_s');

%%%% Part B %%%
dx = x1(2)-x1(1);
dU_dr = (u2_bar(:,3:end) - u2_bar(:,1:end-2))/(2*dx);
% dU_dr = -abs(dU_dr); % Axisymmetric about centerline - velocity
should monotonically decrease
nu_T = -RS(:,2:end-1)./dU_dr;

figure
imagesc(x1(2:end-1), x2, nu_T)
set(gca, 'ydir', 'normal')
xlabel('x (m)')
```

```
ylabel('y (m)')
clims = prctile(nu_T(:), [2, 98]);
clim(clims)
colormap(jet)
cb = colorbar;
ylabel(cb, "\nu_T (m^2/s)")

R_T = u_cl.*delta./nu_T;
figure
imagesc(x1(2:end-1), x2, R_T)
set(gca, 'ydir', 'normal')
xlabel('x (m)')
ylabel('y (m)')
clims = prctile(R_T(:), [2, 98]);
clim(clims)
colormap(jet)
cb = colorbar;
ylabel(cb, "R_T (dimensionless)")
```

```
close all;

d_dx1t = @(arr) (arr(2:end-1,3:end,:)-arr(2:end-1,1:end-2,:))/(2*dx);
d_dx1 = @(arr) (arr(2:end-1,3:end)-arr(2:end-1,1:end-2))/(2*dx);
d_dx2t = @(arr) (arr(3:end,2:end-1,:)-arr(1:end-2,2:end-1,:))/(2*dy);
d_dx2 = @(arr) (arr(3:end,2:end-1)-arr(1:end-2,2:end-1))/(2*dy);
u1_bars = u1_bar(2:end-1,2:end-1);
u2_bars = u2_bar(2:end-1,2:end-1);
u1_primes = u1_prime(2:end-1,2:end-1,:);
u2_primes = u2_prime(2:end-1,2:end-1,:);

advection = u1_bars/2 .* (d_dx1(mean(u1_prime.*u1_prime,3)) +
d_dx1(mean(u2_prime.*u2_prime,3))) + ...
    u2_bars/2 .* (d_dx2(mean(u1_prime.*u1_prime,3)) +
d_dx2(mean(u2_prime.*u2_prime,3)));

production = -1*(mean(u1_primes.^2,3).*d_dx1(u1_bar) +
mean(u1_primes.*u2_primes,3).*(d_dx1(u2_bar)+d_dx2(u1_bar)) + ...
    mean(u2_primes.^2,3).*d_dx2(u2_bar));

transport_triple = -1./2. * (d_dx1(mean(u1_prime.^3,3)) +
d_dx1(mean(u2_prime.*u2_prime.*u1_prime,3))) + ...
    -1./2. * (d_dx2(mean(u2_prime.^3,3)) +
d_dx2(mean(u1_prime.*u1_prime.*u2_prime,3)));

transport_viscous = nu * ( ...
    d_dx1(mean(u1_primes .* d_dx1t(u1_prime), 3)) +
d_dx1(mean(u2_primes .* d_dx1t(u1_prime), 3)) + ...
    d_dx2(mean(u1_primes .* d_dx2t(u1_prime), 3)) +
d_dx2(mean(u2_primes .* d_dx2t(u1_prime), 3)));

advection = advection(2:end-1,2:end-1);
production = production(2:end-1,2:end-1);
transport = transport_triple(2:end-1,2:end-1) + transport_viscous;
diss = diss_spatial(2:end-1,2:end-1);

y_ind = 10;
figure;
% plot(x1(3:end-2), advection(y_ind, :), 'r-', 'LineWidth', 2,
'DisplayName', 'Advection');
% hold on;
% plot(x1(3:end-2), production(y_ind, :), 'b-', 'LineWidth', 2,
'DisplayName', 'Production');
% plot(x1(3:end-2), transport(y_ind, :), 'g-', 'LineWidth', 2,
'DisplayName', 'Transport');
plot(x1(3:end-2)/mean(delta), mean(advection,1), 'r-', 'LineWidth', 2,
'DisplayName', 'Advection');
hold on;
plot(x1(3:end-2)/mean(delta), mean(production,1), 'b-', 'LineWidth',
2, 'DisplayName', 'Production');
plot(x1(3:end-2)/mean(delta), mean(transport,1), 'g-', 'LineWidth', 2,
'DisplayName', 'Transport');
```

```
plot(x1(3:end-2)/mean(delta), mean(diss,1), 'c-', 'LineWidth', 2,
'DisplayName', 'Dissipation');

xlabel('r/\delta = \eta');
ylabel('m^2 / s^3');
title('Advection, Production, and Transport of TKE');
legend show;
grid on;
hold off;
```