HIT Processing Part 2

Question 4

a.

i.

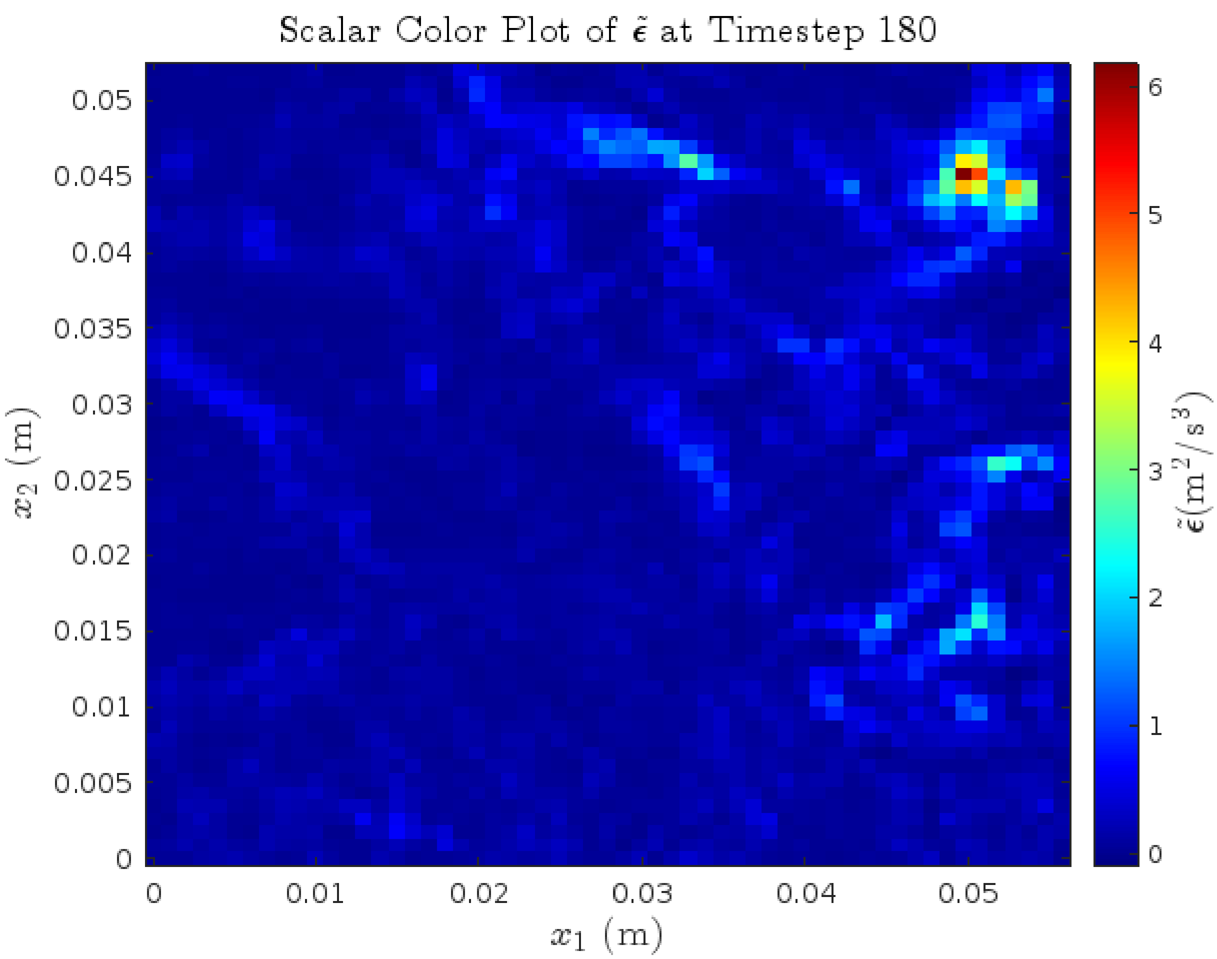

Figure 1: Instantaneous pseudo-dissipation at frame 180

The color plot of the instantaneous pseudo-dissipation clearly shows that it is contained in the small scales. It appears in highly localized, highly concentrated regions, with small tendrils of lower regions extending throughout the field. In magnitude, the instantaneous pseudo-dissipation is almost zero everywhere except the few localities where it is high.

ii.

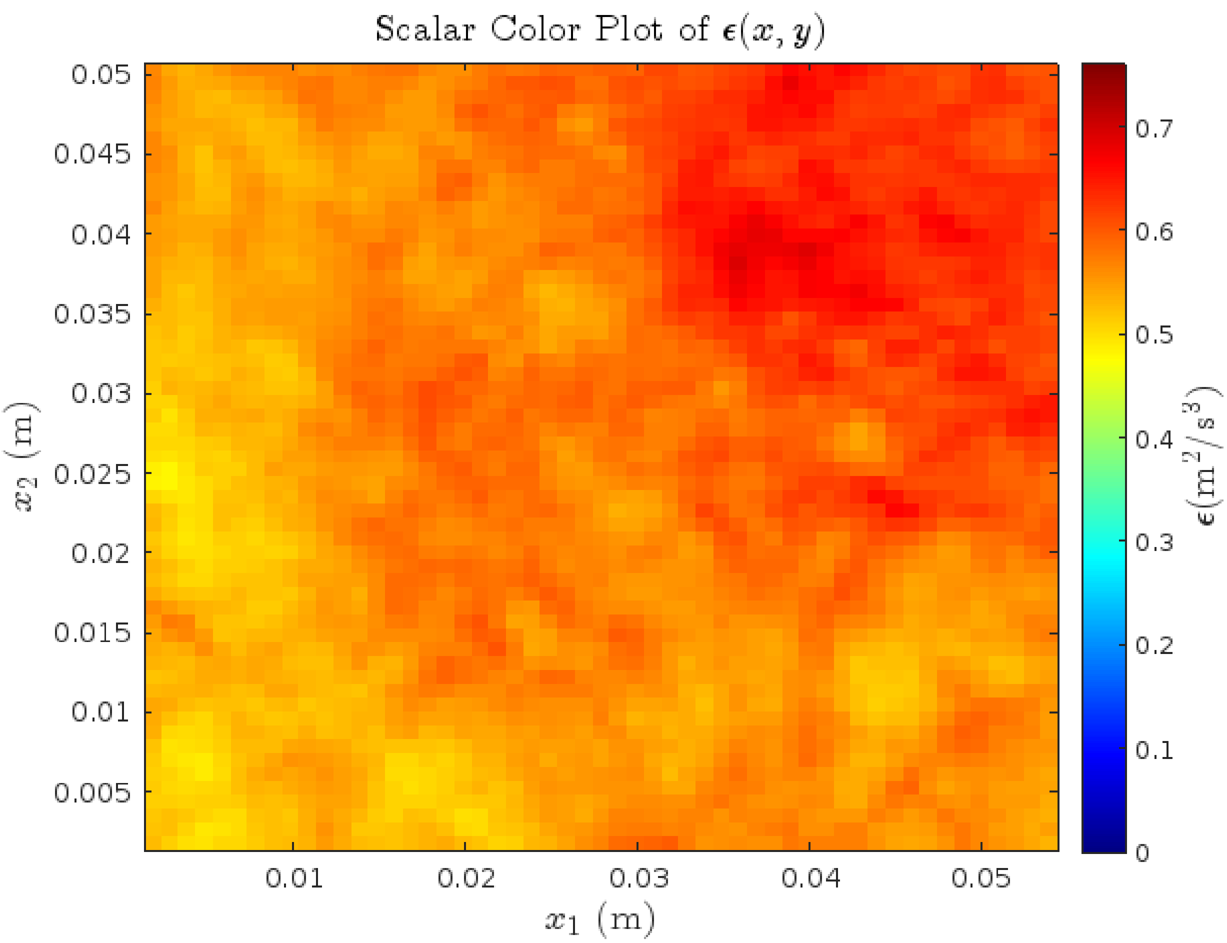

Figure 2: Time-averaged dissipation

iii.

The time and space-averaged pseudo-dissipation value is $\epsilon = 0.5532\,\frac{\mathrm{m}^2}{\mathrm{s}^3}$.

b.

i.

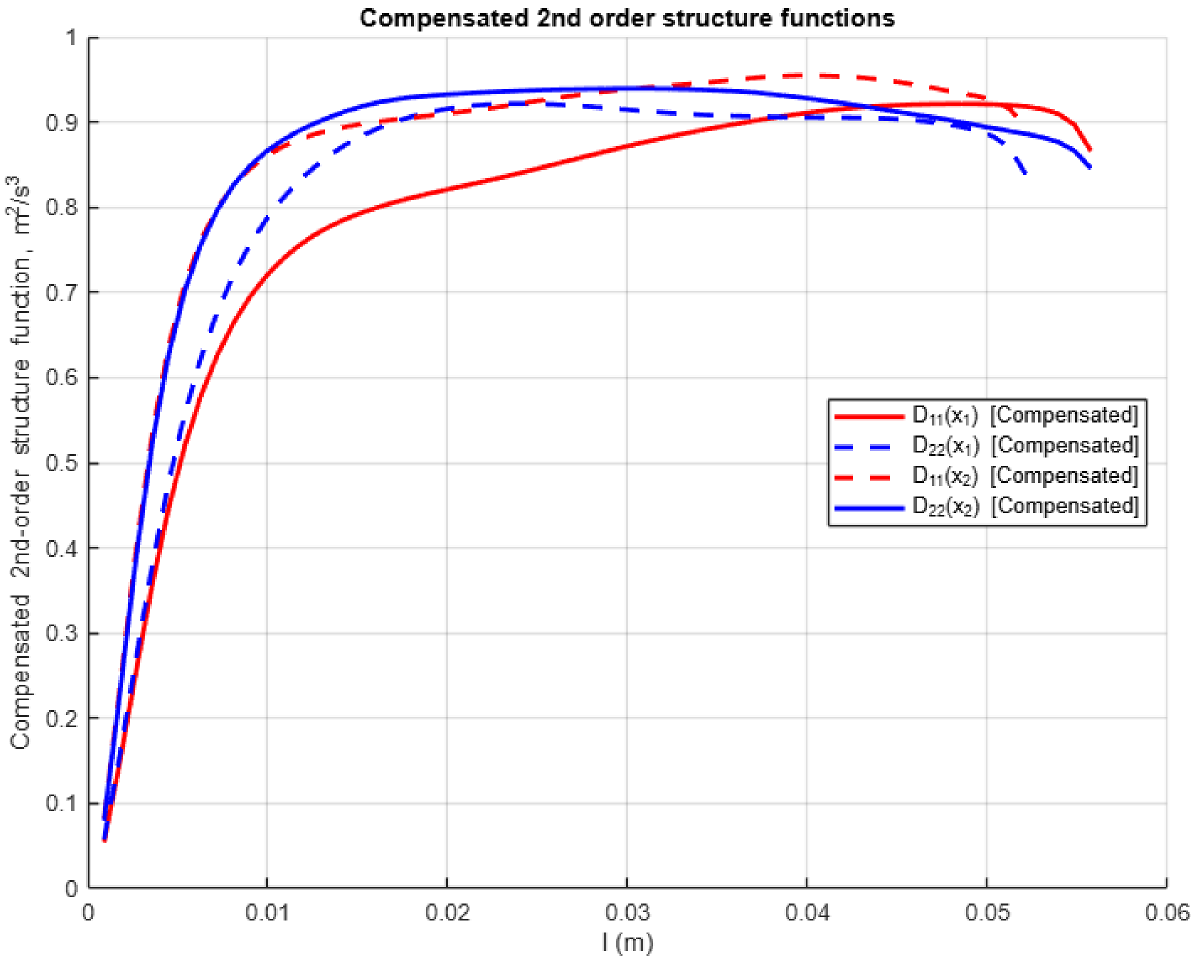

Figure 3: Four Compensated Structure Functions

ii.

Based on averaging the plateaus in all 4 compensated structure functions from l=0.015m to l=0.045m, a value for dissipation is obtained as $\epsilon = 0.9071 \frac{\mathrm{m}^2}{\mathrm{s}^3}$.

c.

Estimating dissipation based on the rms velocity and Longitudinal length scale gives $\epsilon = 0.9169 \frac{\mathrm{m}^2}{\mathrm{s}^3}$.

d.

An approximation of the production term can be used to estimate the dissipation. The space-averaged production is assumed to be equal to the dissipation, which yields a value of $\epsilon = 1.1412 \frac{\mathrm{m}^2}{\mathrm{s}^3}$.

e.

Method	Dissipation estimate (m^2/s^3)
Direct Calculation	0.5532
Compensated Structure Functions	0.9071
Production via RMS Velocity	0.9169
Production Direct Calculation	1.1412

Table 1: Comparison of dissipation estimates between methods

There are no underlying assumptions in the approximation via the first method, direct calculation, since it is the true definition of dissipation. However, it requires resolving gradients of fluctuating quantities. This requires resolution at extremely fine scales, which doesn't exist in this dataset. On top of that, at small scales measurements are more sensitive.

In the second method, the assumptions are made that Kolmogorov's scaling arguments hold. Under these arguments the structure function in the inertial subrange follows a power law scaling and depends on dissipation, and the compensated structure function corrects for the power law scaling to estimate dissipation as the plateau value in the inertial subrange. Because no gradients need to be resolved in this method, it is less sensitive.

In the third method, the assumption is made that production is equal to dissipation (ie neglecting transport of TKE). A dimensional argument can be used to derive a form for dissipation based on flow statistics. Empirical measurements in the past yield the constant for the dimensional argument. This method relies on statistics and once again, no gradients are resolved.

In the final method, the assumption is again made that production is equal to dissipation. However, production is calculated directly. In this method, mean flow gradients are required, but fluctuating gradients are not. This method is therefore less sensitive and does not require measurements at as small a scale as the first method.

In my opinion, of all the methods, the first is the least accurate. The others are roughly clustered together. I chose the second method, dissipation calculated via compensated structure functions. It does not require the resolution of any gradients, which is a sensitive process both in measurement and calculation.

Question 5.

a.

The calculated Kolmogorov scales are:

$$\eta = 0.00024202\mathrm{m}$$

$$\tau = 0.0040\mathrm{s}$$

$$u_\eta = 0.0603\frac{\mathrm{m}}{\mathrm{s}}$$

$$a_\eta = 15.0367\frac{\mathrm{m}}{\mathrm{s}^2}$$

b.

i. Calculating the Taylor microscale based on rms velocity and dissipation yields: $\lambda = 0.01003\text{m}$

ii. The Taylor microscale can also be calculated by fitting a parabolic function $\rho = 1 - \frac{l^2}{\lambda^2}$ to the first few points of the normalized transverse velocity correlations. Averaging the values obtained from the ρ_{11}^2 and ρ_{22}^1 correlations gives $\lambda = 0.00998\text{m}$.

iii. The values for the Taylor microscales are almost the same. This is either a good indication that the estimate of 0.01m for the Taylor microscale is correct, or a suspicious coincidence.

c.

η is the Kolmogorov length, which is on the order of the smallest lengths in turbulence. At and around the Kolmogorov scale, viscous effects dominate inertial effects, and turbulent eddies decay into heat.

λ is the Taylor microscale, which is larger than the Kolmogorov length but smaller than the integral length scales. It can roughly divide the turbulent cascade into the inertial range, where viscous effects are not important, and the subinertial range, where viscous effects begin to dominate.

L_{LL} is the longitudinal integral length scale. This relates to the largest scales in the flow. It is a statistical measure, but can be roughly related to the size of the largest eddies in the flow.

d.

The integral length scale Reynolds number was calculated using the characteristic velocity, U_T. The calculated value is $Re_L = 4460$. The Taylor microscale Reynolds number was calculated using the RMS average velocity. The resulting value is $Re_\lambda = 443.6$.

The range of length scales is calculated as 458.9. In comparison, the ratio between L_{LL} and η is 606.4. These numbers are not exactly the same, but they are order of magnitude accurate (and a lot of these scaling arguments are made on an order of magnitude basis). There are many intermediate steps taken to get to the Kolmogorov scale, the longitudinal length scale, the Reynolds numbers, etc. The quasi-agreement of these ranges supports the quality of the data as well as the calculations.

e.

The spatial and temporal resolutions are 0.000899m and 0.000196s, respectively. In comparison, the Kolmogorov length scale is 0.00024202m. As a result, the dataset does not capture fluctuations or motions at the Kolmogorov scale. The Kolmogorov temporal scale is $\tau = 0.0040s$, which is larger than the temporal resolution of the data.

The window size is 0.0557m by 0.0521m. The integral length scale is 0.1468. Fluctuations at the integral length scale cannot be captured as a result. The timespan of the data is 10.1292s, in comparison with an integral timescale of 0.33s. Many integral timescales are captured in the data.

Overall, both the Kolmogorov length and integral length scales are not resolved in the spatial data. The Kolmogorov and integral timescales are resolved in the temporal data. This dataset better reflects the temporal scales than the spatial scales.

Question 6

a.

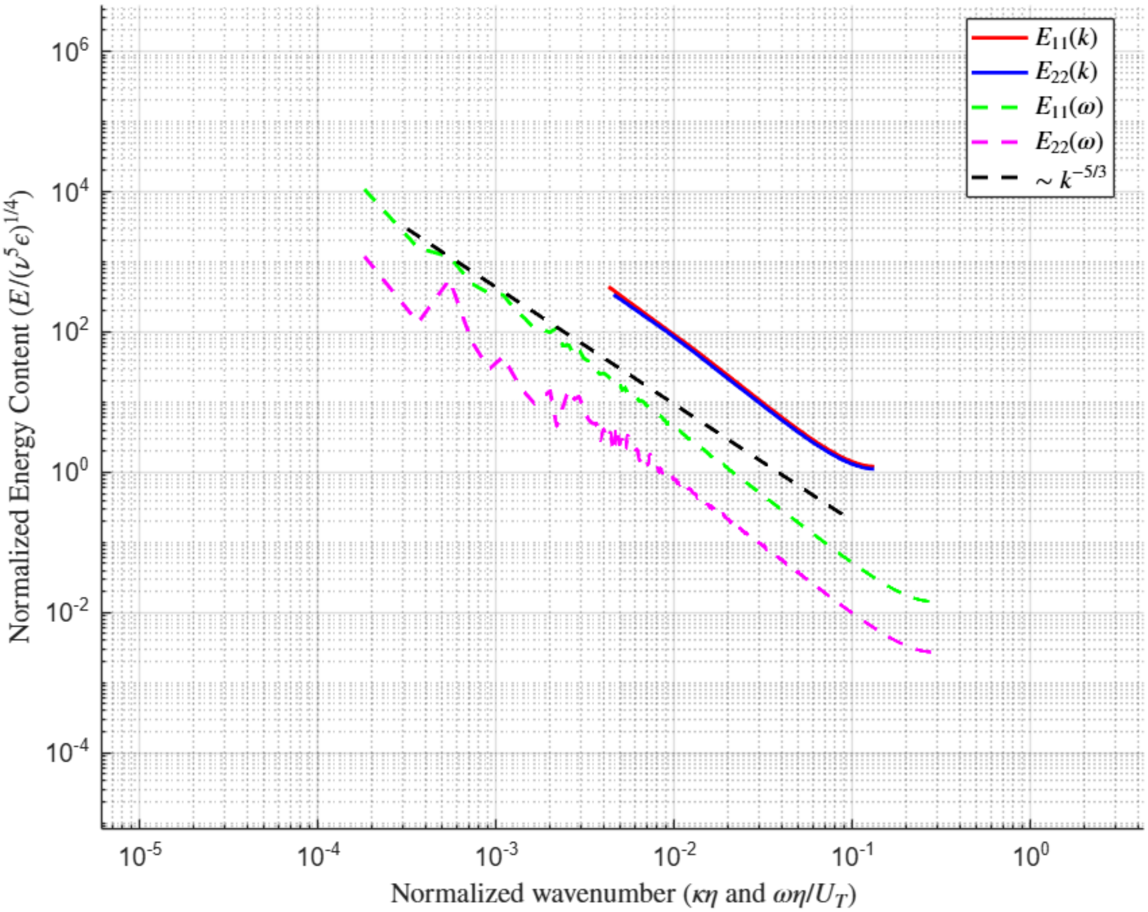

Figure 4: Normalized spatial and temporal energy spectra

b.

The temporal data was also collected across roughly 55,000 timesteps. Only the first 15,000 were used to create this spectrum, and every 5th timestep was sampled. A non-uniform sampling focusing on the first part of the dataset is inappropriate, because numerical Fourier transforms are not well-defined for non-uniform datasets.

The normalization of the energy spectrum is based on a dimensional argument. In order to have good agreement with other data, including the data from Castro & Vanderwel, a correct estimate of dissipation is required. In magnitude, the energy content of this processed data is roughly 10 times lower (for the spatial spectra) and 100 times lower (for the temporal spectra) than the Castro & Vanderwel data. One possible reason for the difference between spatial and temporal is that the temporal data also requires the estimation of an integral velocity scale, in addition to the Kolmogorov length. Despite differences in magnitude, trends can be compared.

The biggest similarity between these spectra and Castro & Vanderwel's is the roughly -5/3 power law scaling. Both the temporal and spatial spectra exhibit close to -5/3 scaling (-5.5/3 matches better).

The range of the spectra calculated in this assignment are smaller than the data presented by Castro & Vanderwel, because of experimental limitations. At the inertial largest scales (smallest wavenumbers), the energy is roughly constant. As discussed in Question 5, the largest scales were not represented in this data. At the smallest scales (largest wavenumbers), an energy drop off is observed. Energy is dissipated into heat, and the -5/3 scaling drops off. This drop off is not visible in the energy spectra, because the smallest scales were not resolved. The flattening of both the temporal and spatial lines near the small scales in this data is due to experimental and calculation artifacts and is not physical.

Code

Question 4

```
close all;
clc;

nu = 1.46*10^(-5); %m^2/s
dx = x1(2);
dy = x2(2);

%%% Part A %%%
du1p_dx1 = (u1_prime(2:end-1,3:end,:,:) - u1_prime(2:end-1,1:end-
2,:,:))/(2*dx);
du1p_dx2 = (u1_prime(3:end,2:end-1,:,:) - u1_prime(1:end-2,2:end-
1,:,:))/(2*dy);
du2p_dx1 = (u2_prime(2:end-1,3:end,:,:) - u2_prime(2:end-1,1:end-
2,:,:))/(2*dx);
du2p_dx2 = (u2_prime(3:end,2:end-1,:,:) - u2_prime(1:end-2,2:end-
1,:,:))/(2*dy);
pseudo_diss_a = nu * (-(du1p_dx1.^2) + 2*(du1p_dx2.^2) +
2*(du2p_dx1.^2)+8*(du2p_dx2.^2));
diss_a_spatial = mean(pseudo_diss_a,4);
diss_a = mean(diss_a_spatial, 'all');

%%% Calculate Part B %%%
C2 = 2.0;
D11_x1_compensated = (D11_x1/C2).^1.5 ./ x1(2:end);
D22_x2_compensated = (D22_x2/C2).^1.5 ./ x2(2:end)';
D11_x2_compensated = (D11_x2*0.75/C2).^1.5 ./ x2(2:end)';
D22_x1_compensated = (D22_x1*0.75/C2).^1.5 ./ x1(2:end);

x1_indices = find(x1(1:end-1) >= 0.012 & x1(1:end-1) <= 0.056);
x2_indices = find(x2(1:end-1) >= 0.012 & x2(1:end-1) <= 0.056);

diss_b = mean([mean(D11_x1_compensated(x1_indices)), ...
                      mean(D22_x2_compensated(x2_indices)), ...
                      mean(D11_x2_compensated(x2_indices)), ...
                      mean(D22_x1_compensated(x1_indices))]);

%%% Calculate Part C %%%
C_eps = 0.5;
diss_c = C_eps*u_rms_avg^3 / L_LL;

%%% Calculate Part D %%%
du1b_dx1 = (u1_bar(2:end-1,3:end,:,:) - u1_bar(2:end-1,1:end-
2,:,:))/(2*dx);
du1b_dx2 = (u1_bar(3:end,2:end-1,:,:) - u1_bar(1:end-2,2:end-
1,:,:))/(2*dy);
du2b_dx1 = (u2_bar(2:end-1,3:end,:,:) - u2_bar(2:end-1,1:end-
2,:,:))/(2*dx);
```

```
du2b_dx2 = (u2_bar(3:end,2:end-1,:,:) - u2_bar(1:end-2,2:end-
1,:,:))/(2*dy);
u1_prime_trimmed = u1_prime(2:end-1, 2:end-1, :, :);
u2_prime_trimmed = u2_prime(2:end-1, 2:end-1, :, :);
P = -mean(u1_prime_trimmed.^2,4).*du1b_dx1 ...
    - 2*mean(u1_prime_trimmed.*u2_prime_trimmed,4).*(du1b_dx2 +
du2b_dx1) ...
    - 2*mean(u2_prime_trimmed.^2,4).*du2b_dx2;
diss_d = mean(P,'all');

%%% Plotting %%%
figure;
imagesc(x1, x2, pseudo_diss_a(:,:,t_rand)');
set(gca, 'YDir', 'normal');
colormap jet; colorbar;

xlabel('$x_1$ (m)', 'Interpreter', 'latex', 'FontSize', 14);
ylabel('$x_2$ (m)', 'Interpreter', 'latex', 'FontSize', 14);
ylabel(colorbar, '$\tilde{\epsilon} $(m$^2$/s$^3$)', 'Interpreter',
'latex', 'FontSize', 14);
title('Scalar Color Plot of $\tilde{\epsilon}$ at Timestep 180',
'Interpreter', 'latex', 'FontSize', 14);

figure;
imagesc(x1(3:end-2), x2(3:end-2), diss_a_spatial(3:end-2, 3:end-2)');
set(gca, 'YDir', 'normal');
colormap jet; colorbar;
clim([0, 1.1*max(diss_a_spatial(:))]);
xlabel('$x_1$ (m)', 'Interpreter', 'latex', 'FontSize', 14);
ylabel('$x_2$ (m)', 'Interpreter', 'latex', 'FontSize', 14);
ylabel(colorbar, '$\epsilon $(m$^2$/s$^3$)', 'Interpreter', 'latex',
'FontSize', 14);
title('Scalar Color Plot of $\epsilon (x,y)$', 'Interpreter', 'latex',
'FontSize', 14);

% Should compensated structure functions be log-scaled?
figure;
hold on;
plot(x1(2:end), D11_x1_compensated, 'r-', 'LineWidth', 2); % Red solid
line
plot(x2(2:end), D22_x2_compensated, 'b--', 'LineWidth', 2); % Blue
dashed line
plot(x2(2:end), D11_x2_compensated, 'r--', 'LineWidth', 2); % Red
dashed line
plot(x1(2:end), D22_x1_compensated, 'b-', 'LineWidth', 2); % Blue
solid line

hold off;
xlabel('l (m)');
ylabel('Compensated 2nd-order structure function, m^{2}/s^{3}');
```

```
legend({'D_{11}(x_1) [Compensated]', 'D_{22}(x_1) [Compensated]',
'D_{11}(x_2) [Compensated]', 'D_{22}(x_2) [Compensated]'}, 'Location',
'Best');
title('Compensated 2nd order structure functions');
grid on;
% set(gca, 'XScale', 'log', 'YScale', 'log');

% Choose dissipation
diss = diss_b;
```

Question 5

```
close all;

%%% Part A %%%
eta = (nu^3 / diss)^0.25;
tau = sqrt(nu/diss);
u_eta = (nu*diss)^0.25;
a_eta = (diss^3 /nu)^0.25;

%%% Part B %%%
taylor_a = sqrt(15*nu*u_rms_avg^2/diss);
N = 3;
k1 = sum((1 - rho11_x2(1:N)) .* x2(1:N)'.^2) / sum(x2(1:N)'.^4);
k2 = sum((1 - rho22_x1(1:N)) .* x1(1:N).^2) / sum(x1(1:N).^4);
taylor_b1 = sqrt(1/k1);
taylor_b2 = sqrt(1/k2);


%%% Part d %%%
Re_L = U_T * L_LL / nu;
Re_lambda = u_rms_avg*taylor_a/nu;
scale_range = C_eps^0.25*Re_L^0.75;
```

Question 6

```
load('question2.mat')
load('question2_t.mat')
close all;

%%% Part A %%%
% Spatial FFTs
FS_x1 = 1/x1(2);
FS_x2 = 1/x2(2);
E11_k = fft(R11_x1);
E11_k = abs(E11_k/size(x1,2)).^2;
E11_k = E11_k(1:ceil(size(x1,2)/2));
f_E11_k = FS_x1*(0:size(x1,2)/2)/size(x1,2);
f_E11_k_norm = f_E11_k * eta;
E11_k_norm = E11_k / (nu^5 * diss)^0.25;
```

```
E22_k = fft(R22_x2);
E22_k = abs(E22_k/size(x2,1)).^2;
E22_k = E22_k(1:ceil(size(x2,1)/2));
f_E22_k = FS_x2*(0:size(x2,1)/2)/size(x2,1);
E22_k_norm = E22_k / (nu^5 * diss)^0.25;
f_E22_k_norm = f_E22_k * eta;

% Temporal FFTs
t_sampled = t(t_indices);
t_sampled = squeeze(t_sampled(1,1,1,:));
% dt_sampled = mean(diff(t_sampled));
% dt_sampled = t(2);
FS_t = 1/t(6);
E11_w = fft(R11_t);
E11_w = abs(E11_w/length(R11_t)).^2;
E11_w = E11_w(1:ceil(length(R11_t)/2));
f_E11_w = FS_t * (0:ceil(length(R11_t)/2)-1) / length(R11_t);
f_E11_w_norm = f_E11_w * eta / U_T;
E11_w_norm = E11_w / (nu^5 * diss)^0.25;

FS_t = 1/t(6);
E22_w = fft(R22_t);
E22_w = abs(E22_w/length(R22_t)).^2;
E22_w = E22_w(1:ceil(length(R22_t)/2));
f_E22_w = FS_t * (0:ceil(length(R22_t)/2)-1) / length(R22_t);
f_E22_w_norm = f_E22_w * eta / U_T;
E22_w_norm = E22_w / (nu^5 * diss)^0.25;

figure;
hold on;
plot(f_E11_k_norm, E11_k_norm, 'r-', 'LineWidth', 2); % Red solid line
plot(f_E22_k_norm, E22_k_norm, 'b-', 'LineWidth', 2); % Blue solid
line
plot(f_E11_w_norm, E11_w_norm, 'g--', 'LineWidth', 2); % Green dashed
line
plot(f_E22_w_norm, E22_w_norm, 'm--', 'LineWidth', 2); % Magenta
dashed line
% Add k^(-5/3) trendline
x_range = logspace(-3.5, -1, 100); % Define x-range for trendline
y_ref = (0.1*x_range.^(-5.5/3)) * max(E11_k_norm)/max(x_range.^(-5/3))
* 0.1; % Scale for visibility
plot(x_range, y_ref, 'k--', 'LineWidth', 2); % Black dashed trendline

hold off;

xlabel('Normalized wavenumber ($\kappa \eta$ and $\omega \eta /
U_T$)', 'Interpreter', 'latex', 'FontSize', 14);
ylabel('Normalized Energy Content ($E / (\nu^5 \epsilon)^{1/4}$)',
'Interpreter', 'latex', 'FontSize', 14);
legend({'$E_{11}(k)$', '$E_{22}(k)$', '$E_{11}(\omega)$',
'$E_{22}(\omega)$', '$\sim k^{-5/3}$'}, ...
```

```
    'Interpreter', 'latex', 'FontSize', 14, 'Location', 'Best');
grid on;
set(gca, 'XScale', 'log', 'YScale', 'log', 'FontSize', 14);
```