Laboratory #1: Heat Engine

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Abstract

This experiment investigates the Ericsson cycle using a piston-cylinder system to compare experimental efficiency with theoretical predictions. The Ericsson cycle is an ideal thermodynamic process that reaches Carnot efficiency in the ideal limit. A piston-cylinder setup was used to perform a simplified Ericsson cycle, where pressure, temperature, and volume were recorded at key states. The experimentally measured efficiencies averaged 0.0198, approximately 25% of the Carnot efficiency (0.0815), closely matching the theoretical efficiency of a non-regenerative Ericsson cycle (0.0213). Measurement uncertainty averaged 31.6%, primarily due to imprecise volume measurements. Suggestions are provided for experimental improvements to reduce uncertainty. Results validate theoretical expectations and highlight the practical challenges of the Ericsson cycle. Without regeneration, the Ericsson cycle remains less efficient than other common thermodynamic cycles which restricts its real-world uses.

Nomenclature

A = *area*
d = *diameter*
g = *gravitational acceleration*
H = *cylinder height*
L = *length*
m = *mass*
n = *number of molecules in system*
P = *pressure*
Q = *heat*
R = *specific gas constant*
T = *temperature*
V = *volume*
W = *work*
X,Y = *quantities used to compute efficiency*
η = efficiency
σ = uncertainty

Subscripts
1,2,3,4 = denotes quantity associated with specific state
cold = denotes cool-temperature quantity
hot = denotes hot-temperature quantity
piston = denotes quantity associated with piston

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I. Introduction

THE Ericsson cycle is an ideal, reversible, thermodynamic cycle. It was invented by John Ericsson in the late 1800s. Like a double-acting Stirling cycle or a Carnot cycle, the Ericsson cycle achieves the maximum theoretical efficiency in the ideal limit. Analyzing and quantifying heat engine cycles is important because it identifies areas for future development in heat engines. If a more efficient heat engine cycle could be developed and utilized widely, cars would be more efficient, ships would pollute less, and humanity would have a cheaper and better source of energy.

There are four processes in the ideal Ericsson cycle. The first is an isothermal compression from a cold uncompressed state to a cold compressed state. The second is isobaric heat addition, where the gas is maintained in a compressed state and heated to the hot compressed state. The third is an isothermal expansion, where the gas expands to the hot uncompressed state. The final process is an isobaric heat rejection, where the gas is cooled so it returns to the original cold uncompressed state. In this context, the terms 'hot' and 'cold' refer to the high and low temperature reservoirs or heat sources.

In the ideal limit, the Ericsson cycle operates with isothermal expansion and compression. This is the most efficient way to transfer heat. In addition, the Ericsson cycle uses regeneration. The heat is recycled between the hot and cold cycles, which minimizes energy loss.

However, the ideal limit is impossible to reach in a real Ericsson engine. True isothermal expansion and compression require infinite time. In addition, real heat exchangers are not perfectly efficient, so regeneration is not perfectly efficient. Therefore, it is impossible to build an ideal Ericsson engine, and it is unrealistic to build an efficient one.

To quantify the error between an ideal and practical Ericsson cycle, this laboratory experiment compares the ideal Ericsson cycle with an experimental version of the cycle performed in the laboratory. A simple piston-cylinder setup is used. The system undergoes isobaric compression by placing a fixed mass on top of the cylinder. Next, it undergoes a quasi-isothermal expansion by heating the air in the cylinder. The mass is removed, and the system undergoes isobaric expansion. Finally, a quasi-isothermal compression cools the gas to the starting temperature.

Based on collected pressure and temperature measurements, the efficiency of the laboratory Ericsson cycle is calculated. This is compared with the calculated efficiency for an ideal Ericsson cycle.

A simple uncertainty analysis is performed. Sources of uncertainty and error are identified and quantified. Conclusions are drawn about the performance of the experimental Ericsson cycle, and suggestions are made for improvements on accuracy in future experiments.

II. Methods

Theory of the Ericsson Cycle

As described in the Introduction, the Ericsson cycle consists of 4 steps:

1. Isothermal Compression
2. Isobaric Heat Addition
3. Isothermal Expansion
4. Isobaric Heat Rejection

Pressure-Volume and Temperature-Entropy diagrams help to characterize these cycles. The P-V and T-S diagrams for the Ericsson cycles are provided below:

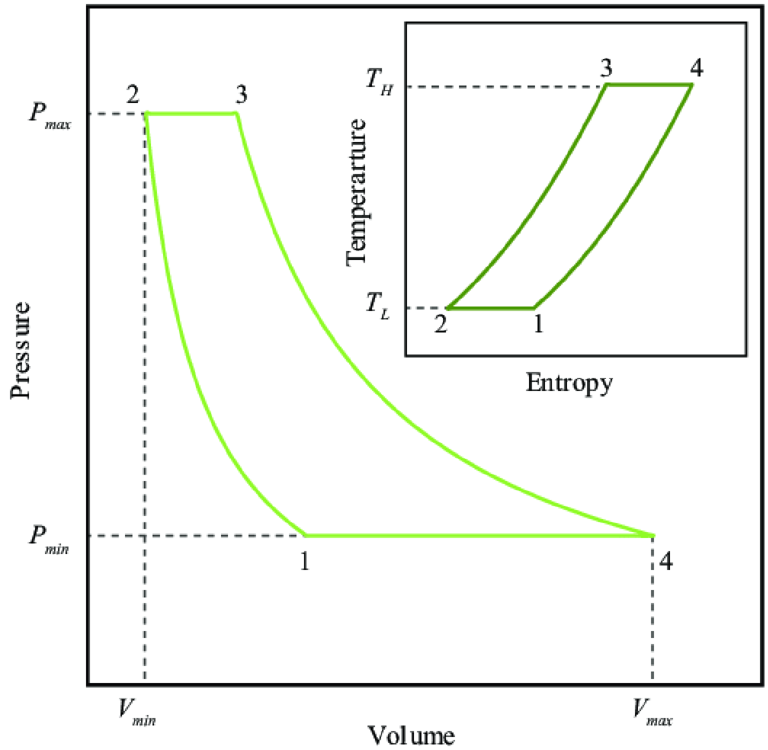

Figure 1: P-V, T-S diagram of Ericsson cycle [1]

In theory, all the heat absorbed from the high-temperature heat source is converted into work, and there are no viscous losses or internal or external efficiencies. This is clearly physically unrealistic, but .

The air in the Ericsson cycle is assumed to be ideal and calorically perfect. These assumptions hold for pressures and temperatures orders of magnitude on either side of room temperature and pressure (298K, 1atm). Therefore, the ideal gas law is used for analysis:

$$PV = nRT \tag{1}$$

Under the assumption that air is not transferred into or out of the system, n is constant. R is a gas property, so for a cycle where air is not transferred in or out of the system:

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} = const. \tag{2}$$

Given these equations, an analysis is performed based on the ideal Ericsson cycle, as shown in Fig. 1. At state one, the gas properties are:

$$V_1 = A_{piston}H_1 + V_{other} \tag{3}$$

$$P_1 = P_{atm} + \frac{g}{A_{piston}}(m_{piston}) \tag{4}$$

$$T_1 = T_{cold} \tag{5}$$

To move from state one to state two, mass is added to the piston. This performs an isothermal compression on the air. The added mass is 50g, as given in the laboratory manual. The conditions at the second state are then:

$$V_2 = A_{piston}H_2 + V_{other} \tag{6}$$

$$P_2 = P_{atm} + \frac{g}{A_{piston}}(m_{piston} + m_{weight}) \tag{7}$$

$$T_2 = T_{cold} \tag{8}$$

To move from state two to state three, the cylinder is immersed in the hot bath, which performs an isobaric heat addition. The same mass as state two is maintained. Therefore, the conditions at the third state are:

$$V_3 = A_{piston} H_3 + V_{other} \tag{9}$$

$$P_3 = P_2 = P_{atm} + \frac{g}{A_{piston}}\left(m_{piston} + m_{weight}\right) \tag{10}$$

$$T_3 = T_{hot} \tag{11}$$

To move from state three to state four, the weight is removed from the top of the piston. The cylinder is maintained in the hot bath. This process is therefore an isothermal expansion. At state four, the gas state is defined with:

$$V_4 = A_{piston} H_4 + V_{other} \tag{12}$$

$$P_4 = P_{atm} + \frac{g}{A_{piston}}\left(m_{piston}\right) \tag{13}$$

$$T_4 = T_{hot} \tag{14}$$

Finally, to move from state four back to state one, the cylinder is moved to the cold bath. This results in isobaric heat removal.

To analyze the efficiency of the cycle, the work out and heat in must be quantified. The work performed by a gas for any thermodynamic process is:

$$W = \int_{V_1}^{V_2} P\, dV \tag{15}$$

Evaluating this integral through each phase of the cycle results in an expression for the net work performed. Simplifications are possible based on the relationship between volume for each state, and the ideal gas law.

$$W_{net} = m_{air} R\left[T_{cold} \ln\left(\frac{V_2}{V_1}\right) + T_{hot} \ln\left(\frac{V_4}{V_3}\right)\right] = m_{air} R[T_{hot} - T_{cold}] \ln\left(\frac{V_1}{V_2}\right) \tag{16}$$

The heat added throughout the cycle comes from the cylinder immersed in the hot bath. This heat counteracts allows for isothermal work to be performed, as well as the gas temperature to rise from state two to state three. Therefore, the heat added can be quantified as:

$$Q_{in} = m_{air} c_p [T_{hot} - T_{cold}] + m_{air} R T_{hot} \ln\left(\frac{V_1}{V_2}\right) \tag{17}$$

In general, the thermal efficiency of a cycle is quantified as the ratio of the net work to the added heat. Thus, the efficiency of this cycle is:

$$\eta = \frac{R[T_{hot} - T_{cold}] \ln\left(\frac{V_1}{V_2}\right)}{c_p[T_{hot} - T_{cold}] + R T_{hot} \ln\left(\frac{V_1}{V_2}\right)} \tag{18}$$

The theoretical ideal efficiency, referred to as the Carnot efficiency, is given as a function of the hot and low temperatures:

$$\eta_{Carnot} = 1 - \frac{T_{cold}}{T_{hot}} \tag{19}$$

Where the temperatures are in absolute temperature. The efficiency of this experiment will be lower since there is no regeneration mechanism.

In order to set up the experiment, the optimal hot bath temperature to provide a 40mm rise in the piston during the expansion must be calculated. Based on ideal gas theory, the temperature is calculated as:

$$T_{hot} = T_{cold}\left(\frac{A_{piston}\Delta h}{V_2} + 1\right) \tag{20}$$

Where the volume is yet to be determined.

The model assumes air behaves as an ideal gas. Ideal gas behavior is reasonable but introduces small errors at higher pressures and temperatures. The experiment simplifies the Ericsson cycle by neglecting regeneration and assuming near-isothermal processes. True isothermal expansion and compression are impossible in practice. The absence of regeneration reduces efficiency compared to the ideal case. Despite limitations, the model captures key trends and provides useful insights.

Experimental Setup

This section describes the components of the experimental setup. Equations used for analyzing and setting up the system are derived and explained. Pictures of components are included.

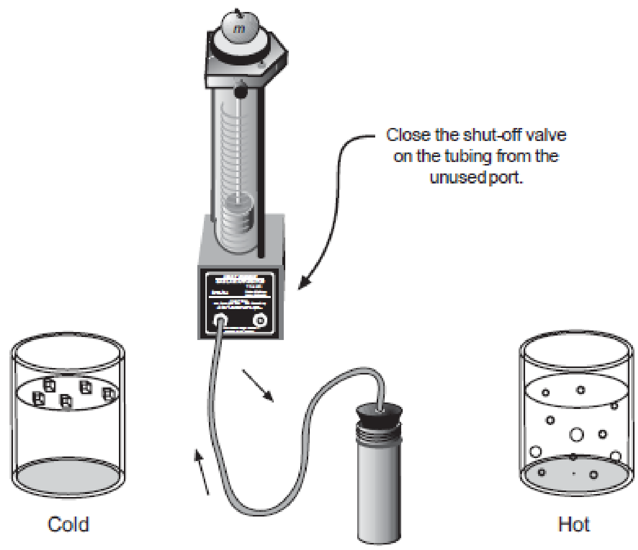

Figure 2: Experimental setup. A piston is connected by a hose to a cylinder, which is immersed in a hot or cold bath. [2]

A. **Piston-Cylinder System**

This system consists of a piston, tubing, and a watertight cylinder. The piston contains air and has a travel height of 100mm. The piston has a stand on top which allows for weights to be added for isobaric compression or expansion. The small, air-tight tubing connects the piston to the watertight cylinder. The cylinder is made of brass and allows for the air in the system to be heated or cooled without direct application of heating or cooling to the piston. In this experiment, the heating and cooling is performed by immersing the watertight cylinder in hot or cold baths. The air temperature in the entire system asymptotically approaches the temperature of the bath. Once the temperature has leveled out, it is assumed that the temperature in the piston and tube is the same as the bath temperature.

The volume of air within the system must be known. The volume inside the piston is quantified with the diameter and height. The diameter is 32.5mm +/- 0.1mm. This value is obtained directly from the manufacturer specifications. The height is measured during the experiment and has an uncertainty of +/-1mm. It is difficult to accurately read the piston height, so a conservative uncertainty of 1mm was used instead of 0.5mm. Based on this, the volume is given as:

$$V_{piston} = \frac{\pi}{4} d_{piston}^2 h_{piston} \tag{20}$$

The volume inside the tube is obtained by measuring the length and diameter of the tube. The diameter is measured with high-accuracy digital calipers. The air-containing diameter is measured as 4.07mm with an uncertainty of 0.01 mm. The length is measured with a measuring tape. The length is measured as 1070mm with an uncertainty of 1mm. The tube volume is then:

$$V_{tube} = \frac{\pi}{4} d_{tube}^2 L_{tube} = 13.92 cm^2 \tag{21}$$

The volume in the watertight cylinder was measured. The volume is composed of a cylinder, minus an internal plug cutout. Originally, dimensions were provided, but the provided uncertainties were very high. The external dimensions were remeasured. The internal dimensions were taken from the spec sheet, with uncertainties of 0.1in. The uncertainties for the external dimensions are +/- 0.1mm.

$$V_{cylinder} = \frac{\pi}{4} d_{cylinder}^2 h_{cylinder} - A_{plug} h_{plug} = 312.01 cm^3 \tag{22}$$

Together, the tube volume and cylinder volume are collectively referred to as V_{other}.

The air temperature in the piston is sensed with a 55000 series thermistor. This thermistor provides a 95% confidence interval uncertainty of +/-0.2 C/K in all measurements of temperature through operating ranges observed during the experiment. A picture of the piston and cylinder is below.

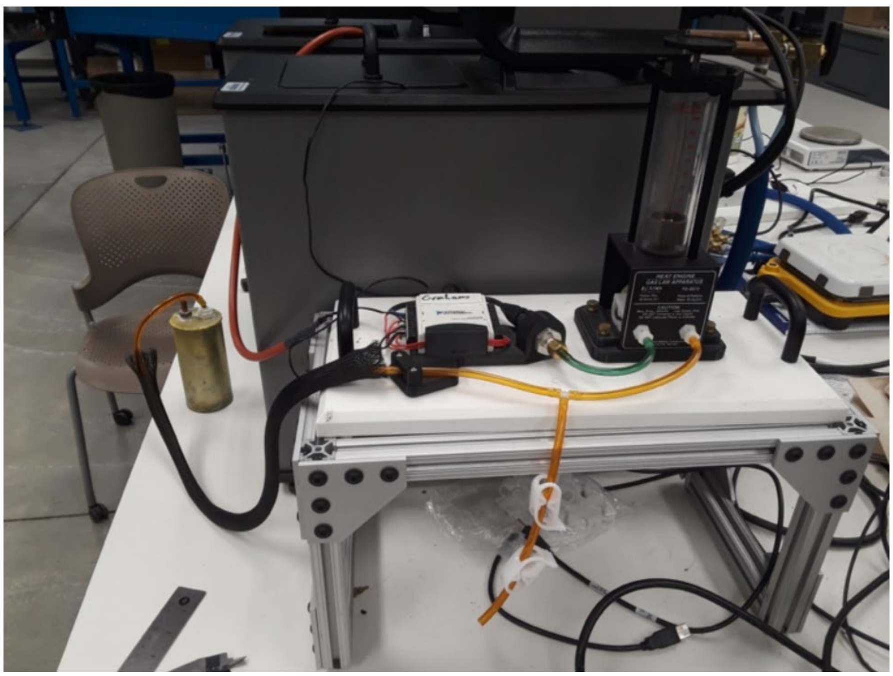

Figure 3: Piston and cylinder. Clamps are visible on the hose, which isolate the system from the atmosphere.

Fig. 3 illustrates a key mechanism. The clamps visible on the orange hose tighten and isolate the system. Leaks through these clamps were observed during the experiment and back-calibrated against.

B. Pressure Transducer

The system to measure pressure is composed of a National Instruments USB 6009 attached to a pressure gauge. The gauge is connected to the free end of a tube connected to the piston-cylinder system. The pressure gauge senses pressure changes as voltages, and these voltages are measured and read into LabView through the NI 6009. The pressure system must be calibrated for local atmospheric conditions.

An Excel spreadsheet is set up to calibrate the pressure transducer. Known weights are placed on the stand of the piston. These weights correlate directly with known pressure changes. The measured voltages and pressure changes are evaluated to decide on a linear fit for pressure, given voltage. The coefficients from the linear fit are fed into

LabView, so that LabView displays a pressure reading directly, instead of a voltage reading. For this experiment, the pressure fit was:

$$P = 8607V_{measured} - 4196\,V_{measured} \tag{23}$$

Where V is the voltage.

According to system specifications, the pressure transducer has an uncertainty of 2%. No confidence interval is specified so 95% is assumed. This assumption is valid, because most part specifications provide uncertainty at a 2% level.

C. Water Baths

Two water baths are used in this experiment. One is maintained at a ‘cold’ temperature of 20°. The other is maintained at a hot temperature. The hot temperature is calculated based on a desired piston rise of 40mm, as described in the section of this report defining the Ericsson system. The water baths have a very high thermal mass, compared to the thermal mass of the piston-cylinder system. Therefore, it is assumed that the water baths maintain constant temperature throughout the cycle. Experimentally, it was observed that the temperature reading in the water baths did not change.

When this laboratory experiment was performed, the thermistors described in the lab manual were inoperative. As a result, the only way to quantify the temperature of the water baths was from the readout screens of the equipment. The uncertainty of the internal temperature sensors in the water baths was determined to be the same as the precision of the readout, as +/- 0.1°C at a 95% confidence interval. A picture of the water baths and the cylinder which is to be immersed in them is below:

Fig. 4 Cold water bath display and watertight cylinder.

Although the display on the water bath in Fig. 4 indicates a higher sensitivity, the uncertainty of temperature is 0.1 °C.

III. Results

Equation 20 provides the method of calculating the temperature of the hot bath. A temperature of 46°C was calculated and used throughout the experiment. The cold bath is maintained at 20°C. This temperature is maintained across all five trials.

For each trial four states are recorded. The states are described in the Methods section. At each state, the temperature, pressure, and volume of the piston-cylinder system are measured. Because the piston-cylinder system has leaks, the leak rate is quantified and used to correct the data. A 50g weight is placed on the cylinder and the total air loss over 5 minutes is measured. This loss rate is used as the baseline loss. Because the air loss is driven by the pressure difference between the atmosphere and the piston, it is assumed that air is only lost when the weight is on the cylinder. This correction process is a response to the bias error induced by the air loss in the system.

The data for all five trials at each of the four states is presented below. The first state is achieved twice, at the start and the end. The second time the first state is achieved is referred to as the 'fifth state'.

Table 1 Raw data from five trials

Trial – State	Elapsed Time (s)	Piston Height [adjusted for loss] (mm)	Air Temperature (K)	Pressure Difference (Pa)	System Volume (cm3)	Total Volume Uncertainty (cm3)
1-1	0	39.5	293.45	412	131.0731	2.223733
1-2	27	36	293.21	997	119.4591	2.21969
1-3	170	99.95	317.55	974	331.6648	2.343939
1-4	192	101.95	317.65	430	338.3014	2.349326
1-5	330	37.45	293.02	403	124.2706	2.221322
2-1	0	31	292.75	408	102.8675	2.214537
2-2	25	28.875	292.65	1028	95.81612	2.212572
2-3	165	91.775	317.65	1013	304.5376	2.322737
2-4	190	93.775	317.75	430	311.1743	2.327799
2-5	360	28.775	292.93	392	95.48429	2.212483
3-1	0	22.5	292.83	390	74.66191	2.207491
3-2	33	20.655	292.81	984	68.53964	2.206251
3-3	150	83.25	317.75	1024	276.2491	2.302106
3-4	165	85.75	317.95	432	284.5448	2.307994
3-5	350	15.75	292.95	390	52.26334	2.203464
4-1	0	30.5	293.55	411	101.2084	2.214063
4-2	28	28.48	293.4	983	94.50539	2.212221
4-3	150	86.25	317.55	970	286.204	2.309188
4-4	166	88.25	317.65	432	292.8406	2.314018
4-5	309	23.75	293.35	392	78.8098	2.20839
5-1	0	18.5	293.15	392	61.38868	2.204936
5-2	27	15.945	292.95	1024	52.91041	2.203561
5-3	134	78.69	317.4	1044	261.1176	2.291725
5-4	164	81.69	317.65	432	271.0725	2.298502
5-5	307	16.69	293.15	390	55.38255	2.203941

The measurements of temperature, pressure, volume, and time provide the basis for further analysis. Specifically, the work done, heat in, and efficiency are desired. The calculation to obtain these quantities is described in Equation 16-18. The mass is calculated using Equation 1, the ideal gas law. Five efficiencies are obtained, one per trial. These are presented below:

Table 2 Net Work, Heat In, and Efficiency for five trials

Trial Number	Mass of Air in System (g)	Net Work Done (J)	Net Heat In (J)	Efficiency	Uncertainty of Efficiency	Percent Uncertainty
1	0.39933	0.276568	13.81234977	0.020023	0.004109	20.523
2	0.31335	0.166098	10.21710211	0.016257	0.005802	35.6921
3	0.22734	0.145192	7.714813193	0.01882	0.007437	39.51852
4	0.30828	0.157688	9.981278262	0.015798	0.005973	37.80621

5	0.18639	0.206787	7.394766011	0.027964	0.006843	24.47242
Average	0.286938	0.190467	9.824061869	0.019772	0.006033	31.60245

The above table contains some key trends. The average efficiency is 0.0198. The Carnot efficiency for the temperatures used in this experiment is defined by Equation 19, and it is 0.0815. The efficiency achieved in this experiment is only 25% of Carnot efficiency. However, this difference is expected. In the ideal limit, when it achieves Carnot efficiency, the Ericsson cycle includes a regenerative step to recover the heat. This regeneration was not included in the experiment, so much lower efficiencies are expected.

A comparison can also be made to the ideal full Ericsson cycle, without regeneration. Each state is calculated using the ideal gas law, and the hot and cold temperatures (Equations 1-16).

Table 3 States during Ideal Cycle Analyzed

State	Ideal Height (mm)	Ideal Air Temperature (K)	Ideal Pressure Difference (Pa)	Ideal Volume (cm3)
1	40	293	413	359.1130724
2	37.02529816	293	1004	356.6453287
3	75.17451831	319	1004	388.2930371
4	78.41318686	319	413	390.9797614
5	40	293	413	359.1130724

The values for height and volume in this table cannot readily be compared directly to the experimental trials, because the exact same starting height was not used. As a result, there is a different mass of air in the system. Nevertheless, efficiency can be calculated using Equation 18. The calculated efficiency for this ideal non-regenerative cycle is $\eta = 0.0212861$. The experimentally measured efficiencies are closer to the non-regenerative efficiency, which was determined assuming no losses and the ideal gas law. This is an indication that the experiment was performed correctly.

The uncertainty of efficiency is also presented in Table 2. All uncertainties are at a 95% confidence level. The process by which it is obtained is described in the Uncertainty section. On average, the uncertainty is 31.6%, with a maximum of 39.5%. These large uncertainties are not unexpected. The system changes are quite small from a macro perspective, and quantifying the changes requires precise measurements.

The uncertainty section reveals that the largest numerical contribution to uncertainty is measurements of the volume. This is expected. Pressure and temperature were measured digitally, whereas volume was measured by eye. In particular, the measurement of the height of the piston was imprecise. To decrease the percent uncertainty, a digital method for volume measurement could be used. Alternatively, more precise calipers and measurement tools could be used. In particular, a better method for measuring the piston height would greatly decrease the uncertainty of the system.

Although the percent uncertainty is substantial, it is still well below 100%. The experiment was designed to minimize uncertainty where possible, but extremely precise techniques and equipment were not used. While the uncertainty limits the precision of the results, it does not render them meaningless. The trends observed are still valid, and the measured efficiency provides useful insight despite the margin of error. With improved measurement techniques, the uncertainty could be reduced further, but even in its current state, the data is informative.

IV. Uncertainty Calculations

Given an arbitrary function of many variables, the overall uncertainty of the function is given by:

$$\sigma_f = \sqrt{\sum_{i=1}^{n}\left(\frac{\partial f}{\partial x_i}\sigma_{x_i}\right)^2} \tag{22}$$

To obtain the overall uncertainty of the efficiency, multiple intermediate uncertainties will be determined.

The uncertainty of the volume of the cylinder, piston, and tube will be determined first. The uncertainty of a generic cylinder volume is given by:

$$\sigma_V = \sqrt{\left(\frac{\pi d h}{2}\sigma_d\right)^2 + \left(\frac{\pi d^2}{4}\sigma_h\right)^2} \tag{23}$$

Where for the cylinder in question, uncertainty due to the plug must also be included. The uncertainties of dimensions for each part are described in the methods section. The uncertainty of the tube section volume is 0.0696cm^3. The uncertainty of the watertight cylinder is 1.3cm^3. The uncertainty of the piston depends on the piston height and is calculated as shown above. To obtain the total volume uncertainty, the contribution from the piston, tube, and cylinder are summed.

The uncertainty of the pressure is obtained simply as 2% of the current pressure. The uncertainty of the thermistor-measured temperatures is +/- 0.2K, whereas the uncertainty of the bath temperatures is +/- 0.1K.

The full equation for efficiency is:

$$\eta = \frac{R[T_{hot} - T_{cold}]l\,n\left(\frac{V_1}{V_2}\right)}{c_p[T_{hot} - T_{cold}] + RT_{hot}l\,n\left(\frac{V_1}{V_2}\right)} \tag{18}$$

Partial derivatives with respect to each variable must be taken. With all uncertainties known, the overall uncertainty can be computed. The equation for overall uncertainty of the efficiency is

$$\sigma_\eta = \sqrt{\left(\frac{\partial\eta}{\partial T_{\text{hot}}}\sigma_{T_{\text{hot}}}\right)^2 + \left(\frac{\partial\eta}{\partial T_{\text{cold}}}\sigma_{T_{\text{cold}}}\right)^2 + \left(\frac{\partial\eta}{\partial V_1}\sigma_{V_1}\right)^2 + \left(\frac{\partial\eta}{\partial V_2}\sigma_{V_2}\right)^2} \tag{24}$$

The partial derivatives which make up this uncertainty are:

$$\frac{\partial\eta}{\partial T_{\text{hot}}} = \frac{Y(R\ln(V_1/V_2)) - X\left(c_p + R\ln(V_1/V_2)\right)}{Y^2} \tag{25}$$

$$\frac{\partial\eta}{\partial T_{\text{cold}}} = \frac{Y(-R\ln(V_1/V_2)) - X\left(-c_p\right)}{Y^2} \tag{26}$$

$$\frac{\partial\eta}{\partial V_1} = \frac{Y\left(R(T_{\text{hot}} - T_{\text{cold}})\frac{1}{V_1}\right) - X\left(RT_{\text{hot}}\frac{1}{V_1}\right)}{Y^2} \tag{27}$$

$$\frac{\partial \eta}{\partial V_2} = \frac{Y\left(-R(T_{\text{hot}} - T_{\text{cold}})\frac{1}{V_2}\right) - X\left(-RT_{\text{hot}}\frac{1}{V_2}\right)}{Y^2} \tag{28}$$

In the above equations, the quantities X and Y are defined for convenience and are:

$$X = R(T_{\text{hot}} - T_{\text{cold}})\ln\left(\frac{V_1}{V_2}\right) \tag{29}$$

$$Y = c_p(T_{\text{hot}} - T_{\text{cold}}) + RT_{\text{hot}}\ln\left(\frac{V_1}{V_2}\right) \tag{30}$$

Incorporating X and Y back into the partial derivatives yields:

$$\frac{\partial \eta}{\partial T_{\text{hot}}} = \frac{\left[c_p(T_{\text{hot}} - T_{\text{cold}}) + RT_{\text{hot}}\ln\left(\frac{V_1}{V_2}\right)\right] R\ln\left(\frac{V_1}{V_2}\right) - R(T_{\text{hot}} - T_{\text{cold}})\ln\left(\frac{V_1}{V_2}\right)\left(c_p + R\ln\left(\frac{V_1}{V_2}\right)\right)}{\left[c_p(T_{\text{hot}} - T_{\text{cold}}) + RT_{\text{hot}}\ln\left(\frac{V_1}{V_2}\right)\right]^2} \tag{31}$$

$$\frac{\partial \eta}{\partial T_{\text{cold}}} = \frac{\left[c_p(T_{\text{hot}} - T_{\text{cold}}) + RT_{\text{hot}}\ln\left(\frac{V_1}{V_2}\right)\right]\left(-R\ln\left(\frac{V_1}{V_2}\right)\right) - R(T_{\text{hot}} - T_{\text{cold}})\ln\left(\frac{V_1}{V_2}\right)(-c_p)}{\left[c_p(T_{\text{hot}} - T_{\text{cold}}) + RT_{\text{hot}}\ln\left(\frac{V_1}{V_2}\right)\right]^2} \tag{32}$$

$$\frac{\partial \eta}{\partial V_1} = \frac{\left[c_p(T_{\text{hot}} - T_{\text{cold}}) + RT_{\text{hot}}\ln\left(\frac{V_1}{V_2}\right)\right]\left(R(T_{\text{hot}} - T_{\text{cold}})\frac{1}{V_1}\right) - R(T_{\text{hot}} - T_{\text{cold}})\ln\left(\frac{V_1}{V_2}\right)\left(RT_{\text{hot}}\frac{1}{V_1}\right)}{\left[c_p(T_{\text{hot}} - T_{\text{cold}}) + RT_{\text{hot}}\ln\left(\frac{V_1}{V_2}\right)\right]^2} \tag{33}$$

Where $\frac{\partial \eta}{\partial V_2}$ is identical to $\frac{\partial \eta}{\partial V_1}$ except that V_1 is replaced with V_2.

This overall formula is difficult to evaluate due to its size. A Python script was used to compute the uncertainty of the efficiency. The script is included in the appendix. The results are different for every run, but trends were identified.

The inclusion of various terms in the uncertainty equation lends itself to a sensitivity analysis. This analysis looks at which components contribute most to the uncertainty. To perform the analysis, the magnitude of each term in the overall uncertainty equation is compared. As an example, the first run had uncertainty contributions of:

Table 4 Uncertainty Sensitivity Analysis

Quantity	Uncertainty Associated with Quantity
T_{hot}	2.922e-10
T_{cold}	3.463e-10
V_1	7.682e-06
V_2	9.199e-06

The table demonstrates that the volume is the largest contribution to uncertainty. The compressed volume contributes to the uncertainty more than the uncompressed volume.

In the results section, the relative uncertainty is expressed as a percentage. This percentage is obtained as:

$$\boldsymbol{\sigma}_{\eta,re} = \frac{\sigma_\eta}{\eta}\cdot 100 \tag{34}$$

V. Conclusion

In conclusion, the experimentally measured efficiency of the non-regenerative Ericsson cycle averaged 0.0198, which is about 25% of the Carnot efficiency. This aligns closely with the theoretical efficiency of an ideal non-regenerative cycle (0.0213), suggesting that the experiment was performed correctly. The observed efficiency gap is expected due to the absence of regeneration, which is a key component in improving the performance of the Ericsson cycle. While the experiment confirmed the expected trends in work, heat transfer, and efficiency, the results indicate that even under ideal conditions, the non-regenerative Ericsson cycle falls well below the efficiency levels of practical heat engines.

This experiment demonstrated the viability of the Ericsson cycle. However, its utility is in question. Typical heat engines can achieve thermal efficiencies on the order of 0.4–0.6. The most efficient engines typically achieve 50% of their Carnot efficiency. Improvements to this experimental setup, including the addition of regeneration, would increase efficiency as a percentage of the Carnot efficiency. To be a commercially useful heat engine process, the real efficiency of the Ericsson cycle would have to be similar to Otto, Diesel, and Brayton cycles.

One of the biggest limitations of this experiment was uncertainty, which averaged 31.6%, with volume measurements contributing the most. Volume was determined by measuring the height of the piston manually. More precise measurement tools, such as digital sensors for displacement or laser-based height measurement, could significantly reduce uncertainty. Additionally, automating data collection for piston height would remove observer bias and provide more reliable measurements. Reducing uncertainty in volume measurements would improve confidence in the calculated efficiency and allow for more precise comparisons with theoretical values.

In a real-world context, understanding and improving the Ericsson cycle has implications for energy efficiency in thermal systems. While the cycle in its non-regenerative form is not competitive with existing heat engine cycles, modifications such as regeneration could make it more viable for specialized applications. The ability to recover waste heat and improve efficiency could have applications in power generation, refrigeration, and industrial heat recovery. While this experiment does not directly demonstrate a commercially viable process, it highlights key areas for improvement that could contribute to more efficient and sustainable energy systems in the future.

Appendix

A. Python Uncertainty Calculations

The source code to compute the uncertainty of efficiency is presented below:

```
import numpy as np

def uncertainty_eta(R, cp, Th, Tc, V1, V2, sigma_Th, sigma_Tc, sigma_V1, sigma_V2):
    # Compute X and Y
    ln_V = np.log(V1 / V2)
    X = R * (Th - Tc) * ln_V
    Y = cp * (Th - Tc) + R * Th * ln_V

    # Compute partial derivatives
    d_eta_dTh = (Y * R * ln_V - X * (cp + R * ln_V)) / Y**2
    d_eta_dTc = (Y * (-R * ln_V) - X * (-cp)) / Y**2
    d_eta_dV1 = (Y * (R * (Th - Tc) / V1) - X * (R * Th / V1)) / Y**2
    d_eta_dV2 = (Y * (-R * (Th - Tc) / V2) - X * (-R * Th / V2)) / Y**2

    print((d_eta_dTh * sigma_Th) ** 2 )
    print((d_eta_dTc * sigma_Tc) ** 2 )
    print((d_eta_dV1 * sigma_V1) ** 2 )
    print((d_eta_dV2 * sigma_V2) ** 2)

    # Compute overall uncertainty using propagation of errors
    sigma_eta = np.sqrt(
        (d_eta_dTh * sigma_Th) ** 2 +
        (d_eta_dTc * sigma_Tc) ** 2 +
        (d_eta_dV1 * sigma_V1) ** 2 +
        (d_eta_dV2 * sigma_V2) ** 2
    )

    return X/Y, sigma_eta

R = 287.101  # Specific gas constant for air (J/kg·K)
cp = 1005   # Specific heat capacity of air at constant pressure (J/kg·K)
Th = 273.15 + 46    # Hot temperature in K
Tc = 273.15 + 20    # Cold temperature in K
V1 = 61.38868395   # Initial volume in cm³
V2 = 52.91040895    # Final volume in cm³

# Uncertainties
sigma_Th = 0.2  # Uncertainty in Th (K)
sigma_Tc = 0.2  # Uncertainty in Tc (K)
```

```python
sigma_V1 = 2.223733448 # Uncertainty in V1 (cm³)
sigma_V2 = 2.219690495 # Uncertainty in V2 (cm³)

# Compute uncertainty in eta
eta, sigma_eta = uncertainty_eta(R, cp, Th, Tc, V1, V2, sigma_Th, sigma_Tc, sigma_V1,
sigma_V2)

print(f'Efficiency: {eta}')
print(f"Uncertainty in η: {sigma_eta}")
```

B. Full Presentation of Raw Data

	Time(s)	Height adjusted for loss (mm)	Pressure Difference (Pa)	Temp(K)	Volume (cm3)	Total volume(cm3)	Uncertainty of piston	Total volume uncertainty
Round 1	0	39.5	412	293.45	131.0731	457.0031	0.853733	2.223733
	27	36	997	293.21	119.4591	445.3891	0.84969	2.21969
	170	99.95	974	317.55	331.6648	657.5948	0.973939	2.343939
	192	101.95	430	317.65	338.3014	664.2314	0.979326	2.349326
	330	37.45	403	293.02	124.2706	450.2006	0.851322	2.221322
Round 2	0	31	408	292.75	102.8675	428.7975	0.844537	2.214537
	25	28.875	1028	292.65	95.81612	421.7461	0.842572	2.212572
	165	91.775	1013	317.65	304.5376	630.4676	0.952737	2.322737
	190	93.775	430	317.75	311.1743	637.1043	0.957799	2.327799
	360	28.775	392	292.93	95.48429	421.4143	0.842483	2.212483
Round 3	0	22.5	390	292.83	74.66191	400.5919	0.837491	2.207491
	33	20.655	984	292.81	68.53964	394.4696	0.836251	2.206251
	150	83.25	1024	317.75	276.2491	602.1791	0.932106	2.302106
	165	85.75	432	317.95	284.5448	610.4748	0.937994	2.307994
	350	15.75	390	292.95	52.26334	378.1933	0.833464	2.203464
Round 4	0	30.5	411	293.55	101.2084	427.1384	0.844063	2.214063
	28	28.48	983	293.4	94.50539	420.4354	0.842221	2.212221
	150	86.25	970	317.55	286.204	612.134	0.939188	2.309188
	166	88.25	432	317.65	292.8406	618.7706	0.944018	2.314018
	309	23.75	392	293.35	78.8098	404.7398	0.83839	2.20839
Round 5	0	18.5	392	293.15	61.38868	387.3187	0.834936	2.204936
	27	15.945	1024	292.95	52.91041	378.8404	0.833561	2.203561
	134	78.69	1044	317.4	261.1176	587.0476	0.921725	2.291725
	164	81.69	432	317.65	271.0725	597.0025	0.928502	2.298502
	307	16.69	390	293.15	55.38255	381.3125	0.833941	2.203941

References

[1] Costa, R. F., and Macdonald, B. D., "Comparison of the Net Work Output between Stirling and Ericsson Cycles," *Energies*, Vol. 11, No. 3, 2018, p. 670. https://doi.org/10.3390/en11030670.
[2] USU MAE Dept. "Heat Engine Lab Overview", *MAE 4400 Canvas.* 2025

AI Disclosure

Artificial Intelligence was used to edit this lab report and construct sections of the conclusion and abstract.