Financial Derivative Securities Course Syllabus

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Shanghai University of International Business and Economics

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Course Descriptions

- This course provides an introduction to the fastest growing areas in derivative securities. It builds on previous training in investments and corporate finance to provide insights to the nature of financial derivatives and applications of such instruments used in an investments and corporate setting. This course is divided into two parts:
  (1) options
  (2) forwards, futures, and swaps
- Emphasis will be placed on derivatives on equity instruments (stocks and stock indices), currencies, commodities, and short and long term interest bearing instruments (corporate and Treasury bonds).

Course Objectives

- Understand the structural differences among options, forwards, futures, and swaps.
- Understand how the above derivative securities are traded in exchanges and/or over-the-counter markets.
- Understand how to price each of the above derivative securities with different pricing models and know model assumptions.
- Understand how to use the derivative securities for hedging and/or speculation purposes.

Course Materials

- *An Introduction to Derivatives and Risk Management,* Don Chance and Robert Brooks, Thomson South-Western,
  - 6th edition, 2004, ISBN: 032417800X
  - 7th edition, 2007, ISBN: 0324321392
  - 8th edition, 2010, ISBN: 0324601212
  - 9th edition, 2013, ISBN: 9781133190202 or 1133190197.
- Lecture notes have been made available to you.

Course Materials

- Chapter 1: Introduction: derivative markets, *Overview and role of derivative securities.*
- Chapter 2: Structure of options markets, *Option exchanges, trading process, quotation, and regulations.*
- Chapter 3: Principles of option pricing, *Option price boundary conditions and put-call parity relations.*
- Chapters 4 & 5: Option pricing models, *Binominal tree model and the Black-Scholes option price model.*
- Chapters 6 & 7: Option strategies, *Option spreads, straddles, straps, strips, and more*

- Chapter 8: Structure of forward and futures markets, *Futures exchanges, trading process, and quotations.*
- Chapter 9: Principles of forward/futures pricing, *Cost-of-carry model, no-arbitrage conditions.*
- Chapter 10 & 11: Futures strategies, *Long/short hedges, hedge ratios, commodity/bond/equity index hedges.*
- Chapter 12: Swaps: *Interest rate swaps, currency swaps, equity swaps, commodity swaps, and CDS.*

Course Evaluation

- Class attendance: 14%
- Class participation: 10%
- Final exam: 76%

Course Schedule

日期		上课时间	教室	内容
5月20日	星期五	5-8节	D104	Options
5月22日	星期日	1-4节	D202	Options
5月22日	星期日	5-8节	D202	Options
5月23日	星期一	5-8节	D204	Futures
5月23日	星期一	9-12节	D204	Futures
5月24日	星期二	1-4节	B401	Swaps
5月24日	星期二	9-12节	B303	Swaps
5月31日	星期二	9-12节	B303	复习
6月1日	星期三	9-12节	B203	考试

Reference Books

1. *Fundamentals of Futures and Options Markets*, John Hull, 8th ed., 2013
2. *Options, Futures, and Other Derivatives*, John Hull, 9th ed., 2014
   http://www-2.rotman.utoronto.ca/~hull/
3. *Derivative Markets*, Robert McDonald, 3rd ed., 2009
4. *Derivatives*, Fred Arditti, 1996
5. *Derivative Securities*, Robert Jarrow and Stuart Turnbull, 1999
   http://www.defaultrisk.com/rs_jarrow_robert.htm
6. *Derivative Markets*, Peter Ritchken, 1996

Reference Journals

1. *RISK*
2. *Journal of Derivatives*
3. *Derivatives Quarterly*
4. *Journal of Futures Markets*
5. *Review of Derivatives Research*
6. *Financial Analysts Journal*

Reference Websites

1. http://www.cboe.com/

   VIX: http://www.cboe.com/micro/volatility/introduction.aspx
2. http://www.cmegroup.com/
3. http://www.bloomberg.com/
4. http://finance.yahoo.com/
5. http://money.cnn.com/

About the Instructor

- **Tie Su,** Associate Professor, Department of Finance, University of Miami. His research interest focuses on investments, specifically in option pricing and market micro structure. His teaching portfolio consists of derivative securities, fixed income, corporate finance, investments, international finance, and wealth management. He has taught CFA review sessions since 1996 and has been a CFA Subject Matter Expert (SME) since 2003. For more information, visit http://moya.bus.miami.edu/~tsu/; http://www.linkedin.com/in/tiesu.

Introduction to Derivatives

1. Definition of derivatives
2. Discounted Cash Flow (DCF) valuation

Derivative security:

A contingent claim (financial contract) whose value is derived from an underlying primary asset.

Derivative security: options, forwards, futures, and swaps.

Derivative security:

Underlying asset:

1. Stocks, stock indices / bonds, interest rates / currencies
2. Commodities (gold, crude oil, natural gas, corn, orange juice)
3. Another derivative contract:
   1. Futures: futures options (options on futures)
   2. Swaps: swaptions
   3. Options: call on call, call on put, put on call, put on put
4. Other assets:
   1. Volatility: VIX derivatives: http://www.cboe.com/micro/volatility/introduction.aspx
   2. Weather: http://www.cmegroup.com/trading/weather/;
   3. Credit events: http://www.cmegroup.com/trading/cds/index.html;

Types of derivatives:

- **Options**: right, not obligation, to enter a transaction.
- **Forwards**: an obligation to transact, negotiated in the over-the-counter market.
- **Futures**: a standardized forward contract, traded on an exchange.
- **Swaps**: a portfolio of forward contracts.
- Other products of **financial engineering**:
  - Structured products: https://www.google.com/?gws_rd=ssl#q=structured+products

An example of a structured product:

- An equity-linked note: invest \$1 today and the future payoff is based on the next year's return on the S&P 500 Index, r_{sp500}.
  - \$1 if r_{sp500} is negative;
  - $\$1\times(1+0.6\times r_{sp500})$ if r_{sp500} is positive.

- **Equity-linked note** (ELN) is a debt instrument, usually a bond, that differs from a standard fixed-income security in that the final payout is based on the return of the underlying **equity**, which can be a single stock, basket of stocks, or an **equity** index. **Equity-linked notes** are a type of structured products.

Asset value and market price

- **Value** is an intrinsic nature of the asset. However, value is unobservable: intrinsic value, fundamental value.
- **Price** is from a transaction or negotiation. Price is directly observable.
- In an efficient market, we hope that price is an unbiased estimator of value.
- Economists determine price by the intersection of **supply and demand curves**.
- Financial economists use the **<u>Discounted Cash Flow (DCF)</u>** approach to determine value as **<u>the present value of expected future cash flows</u>**.

The DCF approach

$$\boxed{value = \sum_{t=0}^{\infty} \frac{E\left(cash\ flow_t\right)}{\left(1+r_t\right)^t}}$$

The Discounted Cash Flow (DCF) approach is the most important approach in asset valuation. It focuses on the size, timing, and risk of cash flows.

Options Markets

1. Terminology
2. Profit and payoff profiles

What is an option?

- The **right**, not obligation, to buy an underlying asset at a pre-specified price, on or before a pre-specified date is called an American style call option.

- The **right**, not obligation, to sell an underlying asset at a pre-specified price, on or before a pre-specified date is called an American style put option.

Basic terminology:

- Exercise price, strike price, striking price (**X**)

  Adjusted for stock splits, stock dividends, special cash dividends, but not adjusted for regular cash dividends. The U.S. exchange-traded options are "dividend unprotected".

- Expiration date (**T**), every Friday in the near term.

Option classes:

- Call options (C)
- Put options (P)

Option series:

- Different exercise prices (say 10 different exercise prices)
- Different expiration months (say 6 different time to maturities)

- One underlying asset derives ($10 \times 6 = 60$) calls and 60 puts.

Option style:

- **American**-style options: options that can be exercised at any time on or before maturity date. In the U.S., all stock options and S&P 100 Index (OEX) options are American-style options.

- **European**-style options: options that can be exercised on only the day of expiration, but not before expiration. In the U.S., all stock index (except OEX) options are European-style options.

- Buyers of an American-style option:
  - Continue to hold the long position in the option
  - Sell the option
  - Exercise the option
- Writers of an American-style option:
  - Continue to hold the short position in the option
  - Buy back the option
  - Fulfill the option's buyer's exercise of the option
- Buyers of a European-style option:
  - Continue to hold the long position in the option
  - Sell the option
  - Exercise the option <u>at option expiration</u>
- Writers of a European-style option:
  - Continue to hold the short position in the option
  - Buy back the option
  - Fulfill the option's buyer's exercise of the option <u>at option expiration</u>

Moneyness of options:

- In-the-money
  - Call option: $S > X$
  - Put option: $S < X$
- At-the-money
  - Call option: $S = X$
  - Put option: $S = X$
- Out-of-the-money
  - Call option: $S < X$
  - Put option: $S > X$

Intrinsic values of American-style options (IV):

- IV(call) = Max(S-X, 0)
- IV(put) = Max(X-S, 0)

Option premium: the value of the option

Option premium = intrinsic value (IV) + time value (TV)

- **Intrinsic** (parity, exercise) **value**: the value of the option if it were exercised right away.
- **Time** (speculative) **value**: the value of the ability to wait.
- Time value = option premium – intrinsic value
- **Early exercise premium** = option premium of an American-style option – option premium of an otherwise identical European-style option = $C^A - C^E$ or $P^A - P^E$. It measures the value of the ability to exercise the option prior to its maturity.

Apple Inc. (AAPL) stock price was at $98.53 on Monday, 1/11/2016, up $1.57 from the previous trading day.

CALL OPTIONS: Expire at close Friday, October 21, 2016

Strike	Contract Name	Last	Bid	Ask	Change	%Change	Volume	Open Interest	Implied Volatility
95.00	AAPL161021 C00095000	11.35	11.75	11.90	0.26	2.34%	69	186	29.50%
97.50	AAPL161021 C00097500	10.50	10.45	10.60	0.15	1.48%	91	484	29.24%
100.00	AAPL161021 C00100000	9.45	9.25	9.40	0.75	8.62%	217	14910	28.99%
105.00	AAPL161021 C00105000	7.15	7.15	7.30	0.40	5.93%	1616	1470	28.54%

http://finance.yahoo.com/q/op?s=AAPL&date=1477008000

Apple Inc. (AAPL) stock price was at $98.53 on Monday, 1/11/2016, up $1.57 from the previous trading day.

PUT OPTIONS: Expire at close Friday, October 21, 2016

Strike	Contract Name	Last	Bid	Ask	Change	%Change	Volume	Open Interest	Implied Volatility
95.00	AAPL161021 P00095000	9.25	8.85	9.00	-0.20	-2.12%	232	2625	31.40%
97.50	AAPL161021 P00097500	10.20	10.05	10.25	-0.50	-4.63%	111	755	31.24%
100.00	AAPL161021 P00100000	11.35	11.35	11.55	-0.50	-4.22%	531	1867	30.97%
105.00	AAPL161021 P00105000	14.82	14.25	14.40	0.07	0.47%	36	985	30.39%

http://finance.yahoo.com/q/op?s=AAPL&date=1477008000

Trading commission is $10+$2 per contract.

- How much do you collect when you write 5 put contracts?

- How much do you pay when you buy 10 call contracts?

Consider the AAPL October 2016 \$97.50 call and put options:

- Today = 1/11/2016
- Option expiration = 10/21/2016
- Stock price = S = \$98.53
- Option exercise price = X = \$97.50
- Option time to maturity = T = 284 days = 284/365 years
- Call option premium = C = \$10.50
- Put option premium = P = \$10.20
- Both call and put option are American-style options.
- AAPL is expected to pay a \$0.52 dividend on 2/7, 5/7, and 8/7/2016.

http://finance.yahoo.com/q/hp?s=AAPL&a=11&b=12&c=1980&d=00&e=11&f=2016&g=v

- Current six-month T-bill yields 0.42% per year

http://www.bloomberg.com/markets/rates-bonds/government-bonds/us/

What are the intrinsic value and time value of the call option?

- IV(Call) = max(0, S-X)

  = max(0, 98.53 – 97.50) = \$1.03 (in-the-money)

- TV(Call) = C – IV(Call) = 10.50 – 1.03 = \$9.47

  Does the call option have a large time/speculative premium?

  List all factors to support your answer.

  What does a call option buyer expect?

  List all factors that would make a call option buyer profitable.

  What does a call option writer/seller expect?

  List all factors that would make a call option writer profitable.

Factors that affect option time value:

1. Stock return volatility
2. Level of stock price
3. Option time to maturity

Call option buyer expects:

- Call option value to increase
  - Stock return volatility to increase
  - Stock price to increase

Call option writer/seller expects:

- Call option value to decrease
  - Stock return volatility to decrease
  - Stock price to decrease
  - Option time to maturity to decrease

What are the intrinsic value and time value of the put option?

- IV(Put) = max(0, X-S)

  = max(0, 97.50 – 98.53) = $0 (out-of-the-money)
- TV(Put) = P – IV(put) = 10.20 – 0 = $10.20

  Does the put option have a large time/speculative premium?

  List all factors to support your answer.

  What does a put option buyer expect?

  List all factors that would make a put option buyer profitable.

  What does a put option writer expect?

  List all factors that would make a put option writer profitable.

Put option buyer expects:

- Put option value to __________
  - Stock return volatility to __________
  - Stock price to __________

Put option writer/seller expects:

- Put option value to __________
  - Stock return volatility to __________
  - Stock price to __________
  - Option time to maturity to __________

Arbitrage-Driven Option Pricing Conditions

1. PCP: European-style options
2. Examples
3. Early exercise of American-style options

Assume that financial markets are **efficient** and **perfectly competitive**.

Efficient: no arbitrage opportunities. All securities are correctly priced.

Perfectly competitive: no transaction costs, no market frictions. All investors are price-takers, whose trades do not affect market prices. They share the same information set.

Based on these assumptions, we derive option **Put-Call Parity** (PCP).

Put-Call Parity relation for **European** style options: ★★★★

$$\boxed{S_0 + P = PV(X) + C}$$

What is the intuition? Why the above relation has to hold? What if it does not?

- Portfolio 1: $S_0 + P$ insurance put
- Portfolio 2: $PV(X) + C$ fiduciary call

at options' expiry	Portfolio 1: S_0+P	Portfolio 2: PV(X)+C
$S_T \geq X$	$S_T + 0$	$X + (S_T - X)$
$S_T < X$	$S_T + (X - S_T)$	$X + 0$

$$PV(X) = X \times (1 + r_a)^{-T} \text{ or } PV(X) = X \times e^{-r_c \times T}$$

- r_a is annualized annually compounded rate. It is also called the effective annual rate (EAR), or annual percentage yield (APY).
- r_c is annualized continuously compounded rate.
- Unless otherwise clearly stated, a rate is a r_a, an EAR or APY.
- Not to complicate the issue, a bond yield, called a bond equivalent yield (BEY), is always quoted as a semi-annual effective rate × 2. It's an annual percentage rate (APR), not an APY.
- X = \$100, T = 0.6 years:

$$r_a = 5\%, PV(X) = 100 \times (1.05)^{-0.6} = \$97.1150$$
$$r_c = 5\%, PV(X) = 100 \times e^{-0.05 \times 0.6} = \$97.0446$$

Example 1: How to use the PCP

Assume current stock price of the XYZ Company is \$50. A put and a call option (both European style options) on the XYZ Company share the same exercise price of \$47, and the same expiration date, which is in exactly three months. Risk-free rate of interest is 10% per year. The market price of the call option is \$4.50. What is the fair market value of the put option?

$$P = PV(X) + C - S_0$$

Solution:

$$P = PV(X) + C - S_0 = \frac{47}{1.10^{0.25}} + 4.50 - 50 = \$0.39$$

Trader's view of the PCP: an arbitrage relation

In the previous problem, a trader finds that the market price of the put is \$1.00, well above the synthetic put price of \$0.39. The trader arbitrages the price difference:

$$P = PV(X) + C - S_0$$

- Sell the relatively over-priced market put, P
- Buy the relatively underpriced synthetic put, $PV(X) + C - S_0$

  to offset her short put position. After all, she wants to arbitrage, and does not want to take a risky (short put) position in the XYZ Company securities.

Today's position: $-P + [PV(X) + C - S_0]$

- Note: security position and cash flow are of opposite sign.
  - write a put: +1.00
  - buy a bond with face value of X $-47/(1.10)^{3/12} = -45.89$
  - buy a call -4.50
  - short sell a share of stock +50
- Net cash flow: +\$0.61

Maturity date's net position: flat.

	–P	+PV(X)	+C	–S	Sum
$S_T \geq X$	0	X	$S_T - X$	$-S_T$	0.00
$S_T < X$	$-(X - S_T)$	X	0	$-S_T$	0.00

If this profit is too good to be true, it probably is.

Other considerations:

- Cost of information
- Brokers' commissions
- Four bid-ask spreads
- Borrowing and lending at the risk-free rate
- Short selling restrictions
- Margins
- Dividends and distributions
- Taxes and other factors
- Price impact

Synthetic positions and power of arbitrage.

- How to create synthetic positions based on the PCP?
- How many times does the PCP hold at any time in markets?
- Applications:
  - Produce a risk-free asset when there are no Treasury securities.
  - Produce a risk-free asset when Treasury securities are subject to default.
  - Produce a stock when the underlying stock is non-marketable.
    - Trading halts
    - Selling restrictions (lockups)
  - Enhance market efficiency

What's the valuation difference between in C^A and C^E? Here is an important conclusion: If the underlying asset does not pay a dividend before the option's expiration, $C^A = C^E$.

- $C^A = IV(C^A) + TV(C^A)$
- $TV(C^A) \geq 0$
- $C^A \geq IV(C^A)$
- C^A is never less than its intrinsic value $IV(C^A)$.
- An option trader will not early exercise C^A.
- The early exercise feature of C^A will never be used and has NO value!
- $C^A = C^E$; $EEP(C^A) = 0$.
- A rational trader would sell the option and not exercise the option.
- An option is worth more ALIVE than DEAD.

Other arbitrage-driven distribution-free option pricing conditions:

	European Call	American Call	European Put	American Put
Intrinsic Value	$\max(0, S-X)$	$\max(0, S-X)$	$\max(0, X-S)$	$\max(0, X-S)$
Upper Bound	S	S	$PV(X)$	X
Lower Bound	$\max(0, S-PV(X))$	$\max(0, S-PV(X))$	$\max(0, PV(X)-S)$	$\max(0, X-S)$
As X increases	$\Downarrow$	$\Downarrow$	$\Uparrow$	$\Uparrow$
As S increases	$\Uparrow$	$\Uparrow$	$\Downarrow$	$\Downarrow$
As σ increases	$\Uparrow$	$\Uparrow$	$\Uparrow$	$\Uparrow$
As dividend δ increases	$\Downarrow$	$\Downarrow$	$\Uparrow$	$\Uparrow$
As T increases	$\Uparrow$ **if no dividends**	$\Uparrow$	**ambiguous**	$\Uparrow$
As r_f increases	$\Uparrow$	$\Uparrow$	$\Downarrow$	$\Downarrow$

Example 2: European-style put-call parity.

- S = \$100
- X = \$100
- Risk free bond price = \$0.96 per \$1.00 face value
- European-style put = \$3
- European-style call = \$8
- The put, call, and risk free bond share the same time to maturity.

How can you execute an arbitrage?

Example 3a: Time to maturity and European-style put options.

- S = \$0
- X = \$100
- European-style put option 1: T = 1 day
- European-style put option 2: T = 1 year

Q: Which option has higher value?

Example 3a: Time to maturity and **European**-style put options.

- $S = \$0$
- $X = \$100$
- European-style put option 1: $T = 1$ day
- European-style put option 2: $T = 1$ year

Q: Which option has higher value?

A: The one with shorter time to maturity!

Example 3b: **Early exercise** of American-style put options.

- $S = \$0$
- $X = \$100$
- American-style put option: $T = 1$ year

Q: Should you early exercise the American-style put or should you hold it?

Example 3b: **Early exercise** of American-style put options.

- S = $0
- X = $100
- American-style put option: T = 1 year

Q: Should you early exercise the American-style put or should you hold it?

A: Early exercise it. At 4% interest rate, $100 earns 1.1 cents of interest per day. Early exercise the American-style put today!

Conclusion: Decision to early exercise an American-style put option is based on the tradeoff between loss of **opportunity cost** (time value of money) on the intrinsic value of the put (no early exercise) and loss of **time value** of the put (early exercise).

Tradeoff: choose the smaller of the two:

loss of interest on IV(P^A) if not exercise P^A.

loss of TV(P^A) if exercise P^A.

Example 4a: Time to maturity and European-style put options.

- $S = \$100$
- $X = \$1,000,000$
- European-style put option: $T = 2,000$ years

Q: Your broker offers you this option at \$2 per option. Is it a good deal?

Example 4a: Time to maturity and European-style put options.

- $S = \$100$
- $X = \$1,000,000$
- European-style put option: $T = 2,000$ years

Q: Your broker offers you this option at \$2 per option. Is it a good deal?

A: It's a terrible deal. Don't buy any. If you could, sell/write it!

Note that PV(\$1,000,000, 2%, 2000 years) << \$0.01

FV(\$2, 2%, 2000 years) = $\$3.17 \times 10^{17}$ >> \$1 million

Example 4b: **Early exercise** of American-style put options.

- S = $100
- X = $1,000,000
- American-style put option: T = 2,000 years

What is the value of this American-style put?

Example 4b: **Early exercise** of American-style put options.

- S = $100
- X = $1,000,000
- American-style put option: T = 2,000 years

What is the value of this American-style put?

The value of the put is its intrinsic value, or $999,900. When an American option should be early exercised, its time value becomes zero.

At 4% interest rate, the opportunity cost (interest lost) on the intrinsic value ($999,900) is $109.58 per day, much higher than the remaining time value of the put option. The put should be exercised right away!

Example 5a: **Early exercise** of American-style call options and dividend payments.

- S = $100 (end of day price)
- X = $95
- dividend = $15
- ex-dividend date = tomorrow
- American style call option: T = 1 day

  What is the value of this American-style call?

Example 5a: **Early exercise** of American-style call options and dividend payments.

- S = $100 (end of day price)
- X = $95
- dividend = $15
- ex-dividend date = tomorrow
- American style call option: T = 1 day

  What is the value of this American-style call?

  a) $4.75

  b) $5.00

  c) $5.25

  Would you early exercise the call, or hold it till maturity?

Example 5a: **Early exercise** of American-style call options and dividend payments.

- S = \$100 (end of day price)
- X = \$95
- dividend = \$15
- ex-dividend date = tomorrow
- American style call option: T = 1 day

What is the value of this American-style call?

~~a) \$4.75~~: American option cannot be sold below its IV.

b) \$5.00

~~c) \$5.25~~: Everyone would write the option and profit.

Would you early exercise the call, or hold it till maturity?

Early exercise the call. When an American option should be early exercised, its time value becomes zero.

Example 5b: **Early exercise** of American-style call options and dividend payments.

- If the dividend is \$0.15, instead of \$15, then should you early exercise?
- S = \$100 (end of day price)
- X = \$95
- dividend = \$0.15
- ex-dividend date = tomorrow
- American style call option: T = 1 day

What is the value of this American-style call?

a) \$4.75

b) \$5.00

c) \$5.25

Would you early exercise the call, or hold it till maturity?

Example 5b: **Early exercise** of American-style call options and dividend payments.

- If the dividend is \$0.15, instead of \$15, then should you early exercise?
- S = \$100 (end of day price)
- X = \$95
- dividend = \$0.15
- ex-dividend date = tomorrow
- American style call option: T = 1 day

What is the value of this American-style call?

~~a) \$4.75~~

b) \$5.00

c) \$5.25

Would you early exercise the call, or hold it till maturity?

Conclusion: Decision to early exercise an American-style call option is based on the tradeoff between **reduction in intrinsic value** (dividend payment, not early exercise) and loss of **time value** (early exercise).

Tradeoff: choose the smaller of the two:

reduction of IV(C^A) due to a cash dividend payment if not exercise C^A.

loss of TV(C^A) if exercise C^A.

Option Pricing Models

1. Derivation of one-period binomial tree
2. Two-period binomial tree
3. The BS model: assumptions, derivation, and applications

The one-period (two-state) binomial tree:

<u>Assumptions</u>:

- a stock whose price is S
- a European style option (call or put) on the stock whose current price is O
- in one period,
  - stock price moves up by a proportion of u, or
  - stock price moves down by a proportion of d.
  - option matures.
  - If the stock price moves up (S_u), then the option price will be O_u
  - If the stock price moves down (S_d), then the option price will be O_d
- Markets are perfectly competitive, zero market friction.
  - All market participants can borrow and lend at the risk-free interest rate r.
  - No transaction cost.

The key to the model derivation is a **hedged portfolio**. A hedged portfolio is a riskless position, whose future value is insensitive to price changes in the underlying security. We construct a hedged portfolio using a combination of *h* shares of stock, and a short position in the option. The fraction of shares of stock, *h*, is called the hedge ratio.

- Today's position: $\boxed{hS - O}$
- End of period value: $hS(1+u) - O_u$ if stock price goes up

  $hS(1+d) - O_d$ if stock price goes down

To make our portfolio a hedged portfolio, we must equate the end of period values of the up and down states, *i.e.*:

$$hS(1+u) - O_u = hS(1+d) - O_d$$

- So far, we do not know what the **hedge ratio** *h* is. In the above equation, everything except the hedge ratio *h* is a known variable. Solve for *h*:

$$h = \frac{O_u - O_d}{S_u - S_d} = \frac{O_u - O_d}{S(u-d)}$$

Because we have a hedged portfolio, we must earn the risk-free rate:

$$\boxed{(hS - O)(1 + r) = hS(1 + u) - O_u = hS(1 + d) - O_d}$$

- Plug in the expression h in the above equation and solve for the option price **O**. Homework: Show that one-period (two-state) binomial model price is:

$$\boxed{\begin{gathered} O = \frac{p \times O_u + (1 - p) \times O_d}{1 + r} \\ p = \frac{r - d}{u - d} \end{gathered}}$$

Example 1:

Stocks of *Binomial Corporation* are currently at \$20 a share. In one year, the stock price will be either \$22 or \$19. Value a European call option to buy stock for \$21 in exactly one year. The risk-free rate is 4%.

Example 1: (cont.)

This problem fits in our one-period binomial option pricing setup.

S = \$20

u = 10%

d = –5%

r = 4%

X = \$21

T = 1

S_u = S(1+u) = \$22

S_d = S(1+d) = \$19

O_u = Max[0, 22 – 21] = \$1

O_d = Max[0, 19 – 21] = \$0

Example 1: (cont.)

Calculate call price:

$$p = \frac{r - d}{u - d} = \frac{4\% - (-5\%)}{10\% - (-5\%)} = 0.6$$

$$O = \frac{p \times O_u + (1 - p) \times O_d}{1 + r} = \frac{0.6 \times 1 + (1 - 0.6) \times 0}{1.04} = \$0.5769$$

➔ Using the one-period binomial model, a European call on *Binomial Corporation* with exercise price \$21 should be selling at \$0.5769 per option.

Example 1: (cont.)

Form a hedged portfolio:

$$h = \frac{O_u - O_d}{S_u - S_d} = \frac{1 - 0}{22 - 19} = \frac{1}{3}$$

How to interpret this number? (We use one third of shares and one short option to form a hedged portfolio.)

- Check hedging effect:

$hS_u - O_u = (1/3)(22) - 1 = 19/3$

$hS_d - O_d = (1/3)(19) - 0 = 19/3$

Example 1: (cont.)

An alternative way to solve for option price:

$$\left[\left(\frac{1}{3}\right)(20) - O\right](1.04) = \left(\frac{1}{3}\right)(22) - 1 = \left(\frac{1}{3}\right)(19) - 0 = \frac{19}{3}$$

- Solve for O:

$$O = \$0.5769$$

Example 1: (cont.)

Interpretation of p and O:

p: "risk neutral" **probability** of making an up-move;

O: **present value of expected future cash flows. DCF!**

$$O = PV[E(O_T)] = \frac{p \times O_u + (1-p) \times O_d}{1+r}$$

$$p = \frac{r-d}{u-d}$$

Aside: The price of a security is the present value of expected future cash flows: DCF!

- Bonds: PV($coupon_1$) + PV($coupon_2$) + … + PV($coupon_n$) + PV(par value)
- Stocks: PV($dividend_1$) + PV($dividend_2$) + … + PV($dividend_n$) + …
- Options: C = PV(E(max(0, S_T – X)), P = PV(E(max(0,X – S_T)).

Binomial Option Pricing with Excel

Example 2:

Stocks of the *Binomial Too Corporation* are currently \$100 per share. The stock price will either increase by 10% per year or decrease by 5% per year, independently for the next two years. The risk-free rate is 4%. Value European and American call and put options on the stock with two years of life and an exercise price of \$112.

This problem fits in our two-period binomial option pricing setup.

$S = \$100$

$u = 10\%$

$d = -5\%$

$r = 4\%$

$X = \$112$

$T = 2$

$S_u = S(1+u) = \$110$

$S_d = S(1+d) = \$95$

$S_{uu} = S(1+u)^2 = \$121$

$S_{ud} = S_{du} = S(1+u)(1+d) = \$104.5$

$S_{dd} = S(1+d)^2 = \$90.25$

Grow the stock tree forward. Solve the option tree backward.

	period 0	period 1	period 2		IV(call)	IV(put)
			$121.00		$9.00	$0.00
		$110.00				
stock	$100.00		$104.50		$0.00	$7.50
		$95.00				
			$90.25		$0.00	$21.75

Grow the stock tree forward. Solve the option tree backward.

	period 0	period 1	period 2
			$0.00
European		$2.88	
Put Option	$6.55		$7.50
		$12.69	
			$21.75

	period 0	period 1	period 2
			$0.00
American		$2.88	
Put Option	$12.00		$7.50
		$17.00	
			$21.75

Example: European put option

OR compute put price using the "**step-by-step**" formulas:

$$O_u = \frac{0.6 \times 0 + (1 - 0.6) \times 7.50}{1.04} = \$2.88$$

$$O_d = \frac{0.6 \times 7.50 + (1 - 0.6) \times 21.75}{1.04} = \$12.69$$

$$O = \frac{0.6 \times 2.88 + (1 - 0.6) \times 12.69}{1.04} = \$6.55$$

The "step-by-step" formula is good for both European and American options.

Example: American put option

$$O_u = max\left(max(0, 112 - 110), \frac{0.6 \times 0 + (1 - 0.6) \times 7.50}{1.04}\right) = \$2.88$$

$$O_d = max\left(\boxed{max(0, 112 - 95)}, \boxed{\frac{0.6 \times 7.50 + (1 - 0.6) \times 21.75}{1.04}}\right)$$
$$= \$17.00$$

$$O = max\left(max(0, 112 - 100), \frac{0.6 \times 2.88 + (1 - 0.6) \times 17.00}{1.04}\right) = \$12.00$$

Note that the option exercise price X=$112.

Grow the stock tree forward. Solve the option tree backward.

	period 0	period 1	period 2
			\$9.00
European		\$5.19	
Call Option	\$3.00		\$0.00
		\$0.00	
			\$0.00

	period 0	period 1	period 2
			\$9.00
American		\$5.19	
Call Option	\$3.00		\$0.00
		\$0.00	
			\$0.00

Example: European call option

OR compute call price using the "**step-by-step**" formulas:

$$O_u = \frac{0.6 \times 9.00 + (1 - 0.6) \times 0}{1.04} = \$5.19$$

$$O_d = \frac{0.6 \times 0 + (1 - 0.6) \times 0}{1.04} = \$0$$

$$O = \frac{0.6 \times 5.19 + (1 - 0.6) \times 0}{1.04} = \$3.00$$

The "step-by-step" formula is good for both European and American options.

Example: American call option

$$O_u = max\left(max(0, 110 - 112), \frac{0.6 \times 9.00 + (1 - 0.6) \times 0}{1.04}\right) = \$5.19$$

$$O_d = max\left(\boxed{max(0, 95 - 112)}, \boxed{\frac{0.6 \times 0 + (1 - 0.6) \times 0}{1.04}}\right) = \$0$$

$$O = max\left(max(0, 100 - 112), \frac{0.6 \times 5.19 + (1 - 0.6) \times 0}{1.04}\right) = \$3.00$$

Note that the option exercise price X=$112.

Example: Early Exercise Premium

The early exercise premium (EEP) of an American option is defined to be the difference between the American option value and European option value. It's the premium for the ability to early exercise the American option.

$\boxed{\begin{array}{l} EEP(C^A) = 3.00 - 3.00 = \$0 \\ EEP(P^A) = 12.00 - 6.55 = \$5.45 \end{array}}$

In the absence of cash dividends in the underlying stock before the option's expiration, an American call should not be early exercised. An American put may be early exercised.

When u and d are unknown, use annualized stock return standard deviation σ:

$$u = e^{\frac{\sigma\sqrt{T}}{N}} - 1\,;\; d = e^{-\frac{\sigma\sqrt{T}}{N}} - 1$$
$$(1+u)(1+d) = 1$$
$$r_{periodic} = e^{\frac{r_{annual}\times T}{N}} - 1$$

where

T is the option time to maturity in years;

N is the number of steps in the binomial tree;

e: base of natural logarithm (ln(x)). In calculator e^x. In Excel "EXP(x)".

r_{annual}: the continuously compounded risk free rate.

$r_{periodic}$: the effective periodic risk free rate.

When u and d are unknown:

Example: The annualized stock return volatility (standard deviation) is 35%. We intend to price a one-month option using a 400-period binomial tree. Note that there are 22 trading days in a month and 6.5 trading hours each day. So each time period is $\frac{22\times6.5}{400} = 0.3575$ trading hours, or 21.45 minutes.

$$u = e^{\frac{0.35\sqrt{1/12}}{400}} - 1 = 0.0002526226\ (check)$$
$$d = \frac{1}{1+u} - 1 = -0.0002525588\ (check)$$

When u and d are unknown:

Example: Annualized stock return volatility (standard deviation) is 35%. We intend to price a one-month option using a 400-period binomial tree. Note that there are 22 trading days in a month and 6.5 trading hours each day. So each time period is $\frac{22\times6.5}{400} = 0.3575$ trading hours, or 21.45 minutes.

Under these parameters, in one month, the max/min stock prices are, assuming the current stock price of $50:

$$S_{max} = S \times (1+u)^{400} = 50e^{\frac{0.35\sqrt{1/12}}{400}\times400} = \$55.32$$

$$S_{min} = S \times (1+d)^{400} = 50e^{-\frac{0.35\sqrt{1/12}}{400}\times400} = \$45.20$$

Average price difference between adjacent leafs: $\frac{55.32-45.20}{400} =$ 0.0253.

The Black-Scholes (1973) option pricing model:

Assumptions:

- Stock price follows a **log-normal** distribution, or log of stock price follows a normal distribution.
- Security trading is **continuous**. **No jumps** in stock prices.
- Stock return volatility is **constant** throughout the life of the option.
- ~~No **dividends** on the underlying security during the remaining life of the option.~~
- The option is **European** style.
- Financial markets are **efficient**. No arbitrage opportunities.
- Financial markets are **perfectly competitive**:
  - No transactions costs or taxes. All securities are perfectly divisible.
  - Investors can borrow and lend at the same risk-free rate of interest.

The Black-Scholes (1973) option pricing model:

1997 Nobel Prize winner: Black-Scholes-Merton option pricing model.

$$\boxed{\begin{aligned} C &= SN(d_1) - Xe^{-rT}N(d_2) \\ P &= Xe^{-rT}N(-d_2) - SN(-d_1) \end{aligned}}$$

$$d_1 = \frac{ln\left(\frac{S}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}};\ d_2 = d_1 - \sigma\sqrt{T}$$

$$N(-x) = 1 - N(x)$$

The Black-Scholes (1973) option pricing model: (cont.)

Relaxing the no-dividend assumption: use dividend-adjusted stock price, *S'*, instead of *S* in the original Black-Scholes formula.

- On a stock index or currency that pays a continuously compounded dividend yield of δ:

$$S' = S \times e^{-\delta T}$$

- On a share of stock that pays discrete dividends:

$$S' = S - \frac{div_1}{(1+r)^{t_1}} - \frac{div_2}{(1+r)^{t_2}}$$

Example 3: Compute the value of index options.

- The SPX index level = S = 1403.17
- exercise price = X = 1350
- time to option maturity = T = 38 days = 38/365 years
- continuously compounded 38-day Treasury Bill rate = r = 5.31%
- **Estimated** continuously-compounded dividend yield on the index = δ = 0.70% per year
- **Estimated** index return standard deviation = σ = 25.86% per year
- Use your self-designed BS price calculator (spread sheet) to compute prices.

- observed market call option price = C = \$81.00
- observed market put option price = P = \$18.00

Seven-step process to compute BS option prices:

1. compute S':
   $S' = Se^{-\delta T} = 1403.17 \times e^{-0.007 \times 38/365} = 1402.15$
2. compute d_1:
   $$d_1 = \frac{\ln(1402.15/1350) + (0.0531 + 0.2586^2/2)(38/365)}{0.2586\sqrt{38/365}} = \frac{0.0469}{0.0834} = 0.5622 \approx 0.56$$
3. compute d_2:
   $d_2 = 0.5622 - 0.2586\sqrt{38/365} = 0.4788 \approx 0.48$
4. look up $N(d_1)$:
   N(0.56) = 0.7123
5. look up $N(d_2)$:
   N(0.48) = 0.6844
6. compute the Black-Scholes call option price:
   C = 1402.15×(0.7123) - 1350×[exp(-0.0531×38/365)]×(0.6844)
   = 998.75 – 918.85 = **$79.91**
7. compute the Black-Scholes put option price:
   P = 1350×[exp(-0.0531×38/365)]×(1 - 0.6844) - 1402.15×(1 - 0.7123)
   = 423.71 – 403.40 = **$20.31**

Standard Normal Probabilities

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177

$$N(-x) = 1 - N(x)$$
$$N(-1.20) = 1 - N(1.20) = 1 - 0.8849 = 0.1151$$

Compute Black-Scholes option prices: (cont.)

Interpretation of $N(d_1)$, $N(d_2)$, C, and P:

- **$N(d_1)$**: the hedge ratio of a call option, "*h*", number of shares needed to hedge one short call option.
- **$N(d_2)$**: "risk neutral" probability of a call option finishing in-the-money.
  = risk neutral probability of $(S_T > X)$;

$$C = PV[E(call\ option\ payoff\ at\ expiration)] = e^{-rT}\int_0^{+\infty} max(S_T - X, 0) f(S_T) dS_T$$

$$P = PV[E(put\ option\ payoff\ at\ expiration)] = e^{-rT}\int_0^{+\infty} max(X - S_T, 0) f(S_T) dS_T$$

- hS – O: $N(d_1)S - C$; $[N(d_1) - 1]S - P$
- We need $N(d_1) = 0.7123$ shares of stock to hedge one short call option.
- We need $N(d_1) - 1 = 0.7123 - 1 = -0.2877$ shares of stock to hedge one short put option. Alternatively, 0.2877 shares of stock hedges one long put option.

- The probability that the call option finishes in-the-money is $N(d_2) = 0.6844$.
- The probability that the put option finishes in-the-money is $1 - N(d_2) = 1 - 0.6844 = 0.3156$.

Example 4: Dividend-adjusted Black-Scholes option prices – Must-do homework

The Dividend Adjusted Black-Scholes formula needs six inputs:

- The S&P 500 Index level = S = 1530.63
- exercise price = X = 1600
- time to option maturity = T = 85 days = 85/365 years
- continuously compounded 85-day risk-free rate = r = 3.63%
- **estimated** continuously-compounded dividend yield on the index = δ = 0.8% per year
- **estimated** index return standard deviation = σ = 16.97%
- observed market call option price = C = $27.00
- observed market put option price = P = $79.00

Example 4: (cont.)

Use your self-designed BS option price calculator (spread sheet), or seven-step process to compute BS option prices:

1. compute S':
2. compute d_1:
3. compute d_2:
4. look up $N(d_1)$:
5. look up $N(d_2)$:
6. compute the Black-Scholes call option price:
7. compute the Black-Scholes put option price:

Example 4 solution:

Seven-step process to compute BS option prices:

1. compute S':

   $S' = Se^{-\delta T} = 1530.63 \times e^{-0.0080 \times 85/365} = 1527.78$
2. compute d_1:

   $$d_1 = \frac{\ln(1527.78/1600) + (0.0363 + 0.1697^2/2)(85/365)}{0.1697\sqrt{85/365}} = \frac{-0.0344}{0.0819} = -0.4198 \approx -0.42$$
3. compute d_2:

   $d_2 = -0.4198 - 0.1697\sqrt{85/365} = -0.5017 \approx -0.50$
4. look up $N(d_1)$:

   N(-0.42) = 1 – N(0.42) = 1 – 0.6628 = 0.3372
5. look up $N(d_2)$:

   N(-0.50) = 1 – N(0.50) = 1 – 0.6915 = 0.3085
6. compute the Black-Scholes call option price:

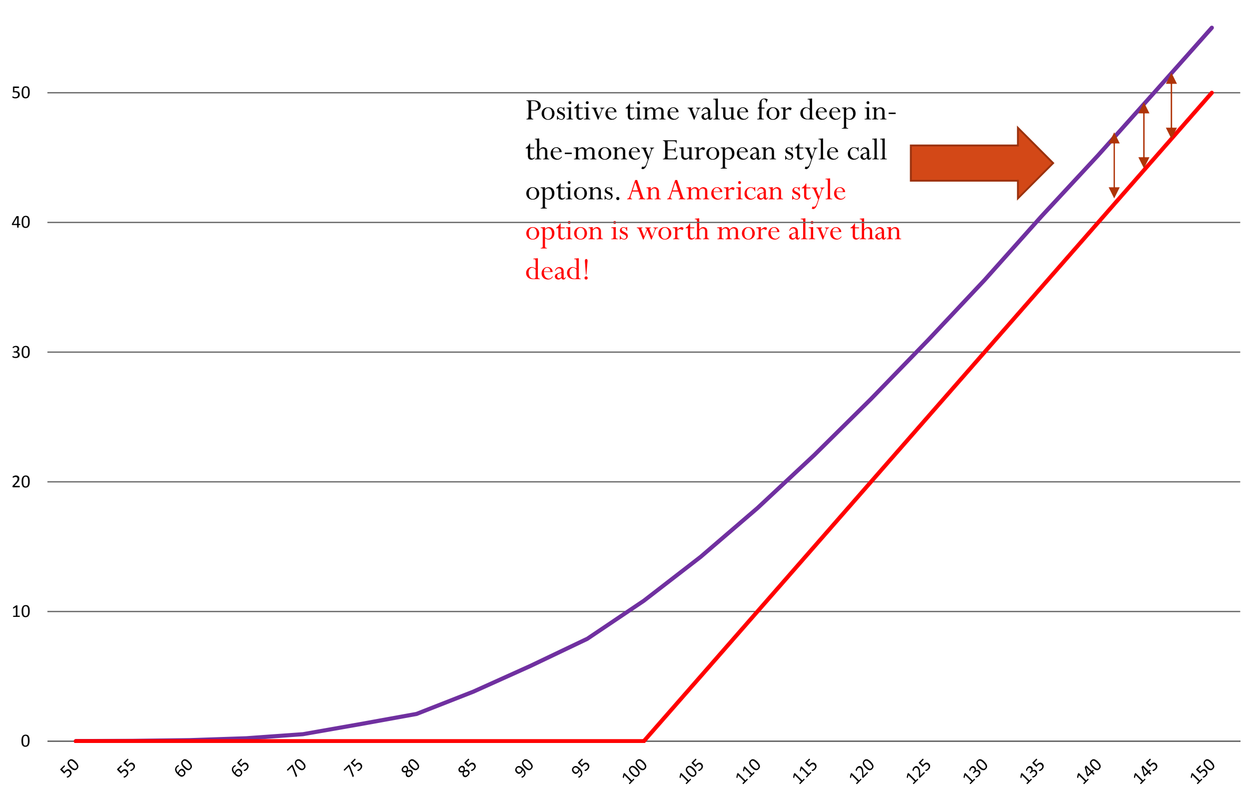
   C = 1527.78×0.3372 – 1600×[exp(-0.0363×85/365)]×0.3085

   = 515.17 – 489.44 = **$25.72**
7. compute the Black-Scholes put option price:

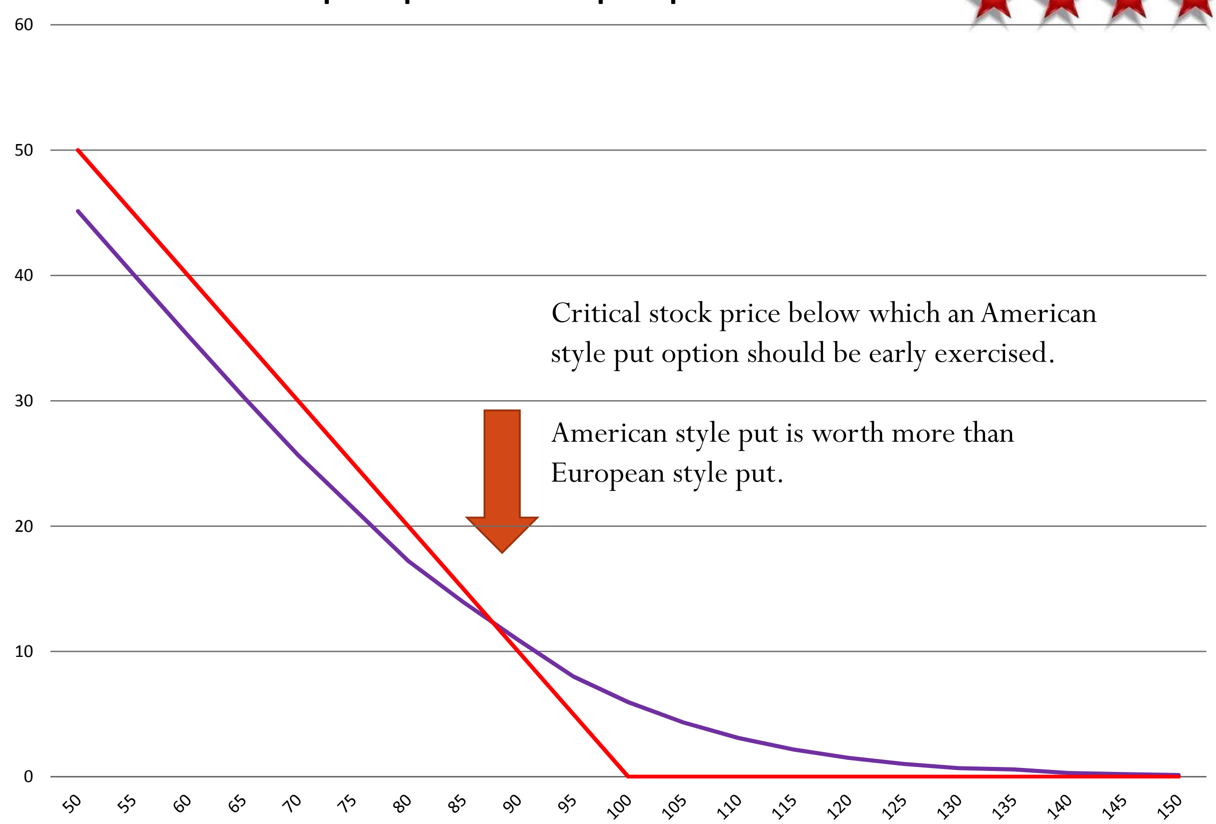
   P = 1600×[exp(-0.0363×85/365)]×(1-0.3085) – 1527.78×(1-0.3372)

   = 1097.09 – 1012.61 = **$84.47**

Call option premium and call option intrinsic value



Put option premium and put option intrinsic value



Black-Scholes Option Pricing with Excel

The Black-Scholes model.

Variable	Notation	Solution without Rounding	Solution with Rounding
stock	S	$2,116.00	$2,116.00
div. yield	delta	1.80%	1.80%
exercise	X	$2,050.00	$2,050.00
interest	r	1.00%	1.00%
time	t	0.25	0.25
St.dev.	sigma	0.15	0.15
adj. stock	S'	$2,106.50	$2,106.50
d1	d1	0.4333	0.4333
d2	d2	0.3583	0.3583
N(d1)	N(d1)	0.6676	0.6664
N(d2)	N(d2)	0.6400	0.6406
call	**C**	**$97.70**	**$93.82**
put	**P**	**$36.08**	**$32.20**
C+bond	C+PV(X)	$2,142.58	$2,138.70
P+stock	P+S'	$2,142.58	$2,138.70

1. Input variables are in red.
2. All other variables are computed using a formula.
3. To keep your answers exact, but may be inconsistent with your hand calculations: do not do any rounding in d1, d2, N(d1), and N(d2).
4. To keep your answers consistent with your hand calculations:
    1. Round d1 and d2 to two decimal places.
    2. Round N(d1) and N(d2) to four decimal places.

Option Trading Strategies

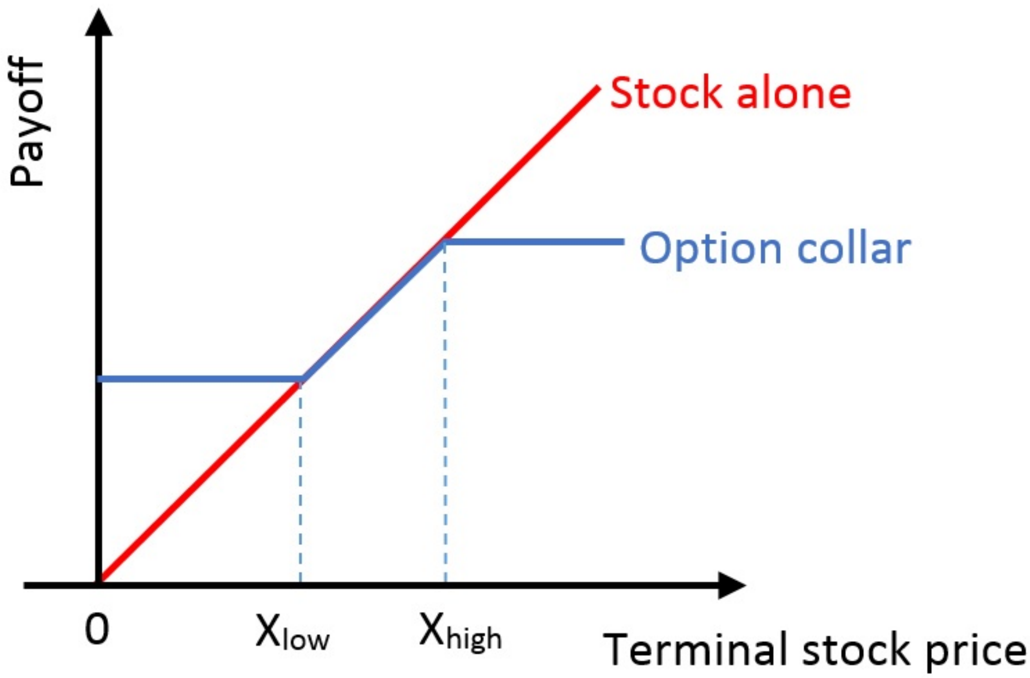
1. Collars
2. Arbitrage with box spreads.

Collar: buy a put and write a call with the same maturity and different exercise prices.

Example: –C(X=38, T=December) + P(X=20, T=December)

Combined with a long position in the underlying security, this strategy produces a ceiling and a floor for the underlying, *i.e.*, the portfolio value will not exceed the high exercise price (ceiling) and will not fall below the low exercise price (floor). If a trader chooses the high and low exercise prices so that the put premium and call premium exactly offset each other, then there is no net cash flow to set up the collar. Such a strategy is called a "zero-cost collar."

Option collar



Box Spread: This strategy implements a riskless portfolio, thus earns risk-free rate.

Example: [C(X=90) – C(X=100)] – [P(X=90) – P(X=100)], with the same maturity.

Box Spread: (cont.)

- Payoff of a box spread:

  $S' + P = PV(X) + C$ (European style put-call parity)

  $S' + P_1 = PV(X_1) + C_1$ (pick any exercise price X_1)

  $S' + P_2 = PV(X_2) + C_2$ (pick any exercise price X_2)

  $P_1 - P_2 = PV(X_1) - PV(X_2) + (C_1 - C_2)$ (Difference equation)

  $PV(X_2 - X_1) = (C_1 - C_2) - (P_1 - P_2)$

Box Spread: (cont.)

- By holding four positions today $[(C_1 - C_2) - (P_1 - P_2)]$, the future payoff will be $X_2 - X_1$:

Range	Box Spread Payoff
$S < X_1$	$[0 - 0] - [(X_1 - S) - (X_2 - S)] = X_2 - X_1$
$X_1 \le S \le X_2$	$[(S - X_1) - 0] - [0 - (X_2 - S)] = X_2 - X_1$
$X_2 < S$	$[(S - X_1) - (S - X_2)] - [0 - 0] = X_2 - X_1$

- A box spread constitutes a synthetic T-bill.
- Because a box spread is a **risk-free** option strategy, the rate of return of this investment should be the risk-free rate of interest. Otherwise an arbitrage opportunity exists:

$$\boxed{[(C_1-C_2) - (P_1-P_2)]\times(1+r_f)^T = X_2-X_1}$$

Example: **arbitrage** using box spreads.

- Index level = 1415.22
- Options' time to maturity = T = 39 days = 39/365 years
- Annualized 39-day risk-free interest rate = 5.34%
- All options are European-style.

Option Price	Call	Put
X_1 = \$1325	\$111.125	\$10.25
X_2 = \$1400	\$54.75	\$25
X_3 = \$1425	\$37.75	\$35

Example: (cont.)

Implement a box spread, using X_1 and X_3 options:

- buy 1325 call, sell 1425 call
- sell 1325 put, buy 1425 put

- cash flow today:
- –\$111.125 + \$37.75 + \$10.25 – \$35 = –\$98.125
- cash flow on the maturity date:
- 1425 – 1325 = +\$100.000
- What is the rate of return on this investment?
- $(100.000/98.125)^{365/39} - 1 = 19.38\%$
- Is the box over valued or under valued?

Example: (cont.)

To **arbitrage**:

- Borrow \$98.125 at the risk-free rate of 5.34% for 39 days.
- Buy the box spread, costing \$98.125.
- Net cash flow today is zero.
- 39 days later, the box spread expires to yield \$100 (risk-free, regardless the index level.)
- Pay off the risk free loan $\$98.125 \times 1.0534^{39/365} = \$98.6720$.
- Pocket \$100 – \$98.6720 = \$1.3280 at t=T. **Party**!

Example: **arbitrage** using box spreads.

- Index level = 1415.22
- Options' time to maturity = T = 39 days = 39/365 years
- Annualized 39-day risk-free interest rate = 5.34%
- All options are European-style

Option Price	Call	Put
X_1 = \$1325	\$111.125	\$10.25
X_2 = \$1400	\$54.75	\$25
X_3 = \$1425	\$37.75	\$35

Example: (cont.)

Implement a box spread, using X_1 and X_2 options:

From previous pages, we know that the following condition has to hold:

$$PV(X_2 - X_1) = (C_1 - C_2) - (P_1 - P_2)$$

So we will check that:

$X_1 = 1325$, $X_2 = 1400$,

$PV(1400 - 1325) = 75 \times 1.0534^{-39/365} = \$74.5843$

$(C_1 - C_2) - (P_1 - P_2) = (111.125 - 54.75) - (10.25 - 25) = \$71.125$

Example: (cont.)

Clearly the price of the T-bill ($74.5843) is greater than the price of the box ($71.125).

Consequently, a potential arbitrage is possible. To arbitrage, buy low and sell high:

1. Borrow a $75 face-value T-bill and sell it for $74.5843.
2. Buy the box spread, costing $71.125.
3. Net cash flow today is $3.4593. **Party**!
4. 39 days later, the box spread expires to yield $75 (risk-free, regardless the index level.)
5. Pay off the liability from the T-bill of $75.
6. Net cash flow is zero at t=T.