The Unfair Dice Problem

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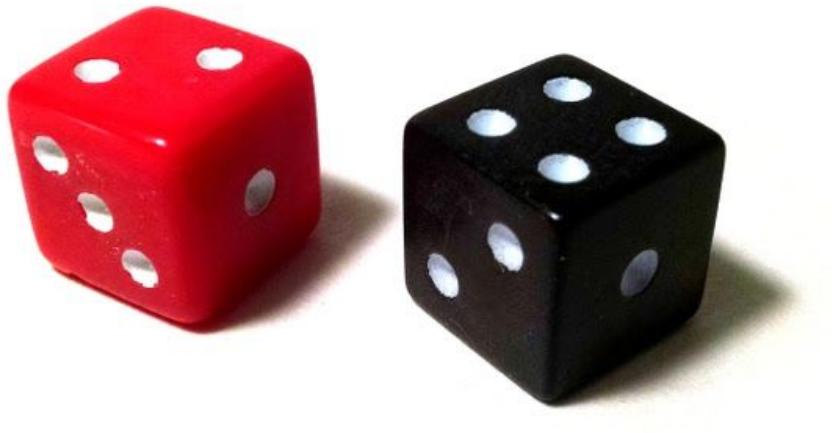
The Unfair Dice Problem is a free resource for teachers and students, and is part of the Callysto Project (www.callysto.ca), a federally-funded initiative to bring computational thinking and mathematical problem-solving into Grade 5-12 Canadian classrooms.



Laura developed the initial draft. She was curious about using Python and Jupyter notebooks to simulate a situation in which two parties compete for the same resources, where one of them has a fixed slight advantage over the other. Laura wanted to see how the outcome of each player differed, particularly when the players engaged in the game multiple times over a long period of time. Laura later shared this notebook with Michael and Richard, who provided valuable feedback and modified the game to incorporate it into Grades 5 - 12 classroom.

In this Callysto Notebook, we present The Unfair Dice Problem, a lesson that was first taught to Grades 5 - 12 pre-service teachers at UBC and Langara in Vancouver.

Hands-on activities:

1. Ask the class to form pairs
2. Each pair gets two dice of different colours
3. Ask each person in the pair to choose their die - they will need to take note of the outcome they get when they roll
4. Ask each pair to decide who will play “Alice” and who will play “Bob”
5. Explain the rules of the game:
    a. Both players roll the dice
    b. If Alice’s outcome is greater than or equal to Bob’s (that is if her outcome is larger or if there is a tie), then Alice takes one point (usually represented by candy, toothpicks, or other object) from Bob.
    c. Otherwise Bob takes one point from Alice.
6. Ask them to play 10 rounds.
7. After the 10 rounds are completed, ask them if they think the game is fair and why.
8. Ask players Alice to raise their hands if at the end of the 10 rounds, they have the highest number of points. Do the same for players Bob. Compare the number of players “Alice” who won against the number of players “Bob” who won.
9. Repeat the game, but this time have Bob take two points from Alice instead of one. The rest of the rules stay the same.
10. Ask the students to share whether they think the game is fair and why.
11. Ask players Alice to raise their hands if at the end of the 10 rounds, they have the highest number of points. Do the same for players Bob. Compare the number of players “Alice” who won against the number of players “Bob” who won.

Discussion

At the end of the activity, items which can be discussed with the students can be modified depending the topic you are interested in teaching and can include, but are not limited to:

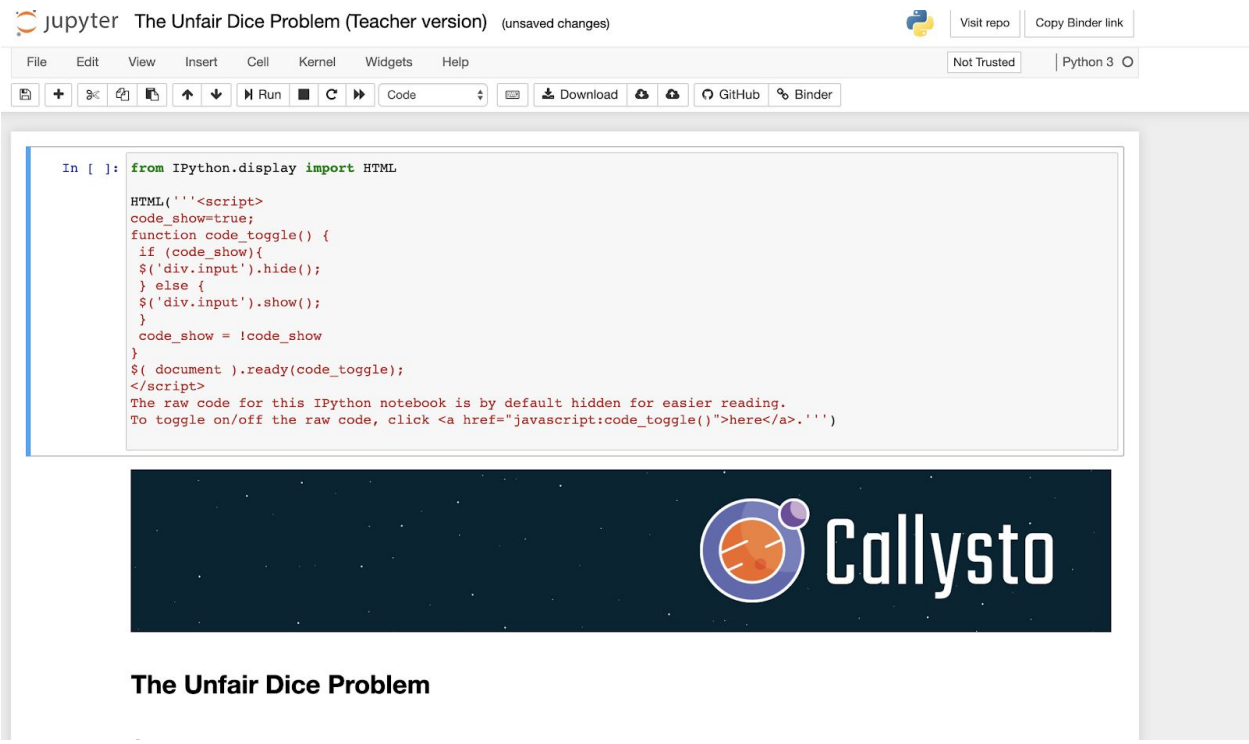
1. Sample space associated to the probability game
2. Expected per-round payoff for each player
3. Theoretical vs experimental probability

Computational Thinking and Simulation

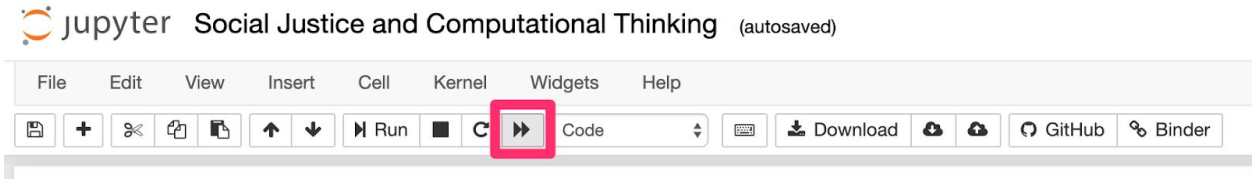
In the notebook, we provide an example of what discussing points 1 and 2 looks like.
To access this free Notebook, visit https://bit.ly/3alasVv using either Google Chrome, Firefox or Safari web browsers.

This link will take you to a screen that looks like this.

Students are to click on the Student Notebook, while this document lists the contents of the Teacher Notebook. When opening both notebooks, the top of the page looks as follows



To start the notebook, press the >> button. Confirm “Restart and Run All Cells”.



You may decide you want to share a picture of the sample space and expected payoff equations, found in both versions.

	1	2	3	4	5	6
1	0	1	2	3	4	5
2	-1	0	1	2	3	4
3	-2	-1	0	1	2	3
4	-3	-2	-1	0	1	2
5	-4	-3	-2	-1	0	1
6	-5	-4	-3	-2	-1	0

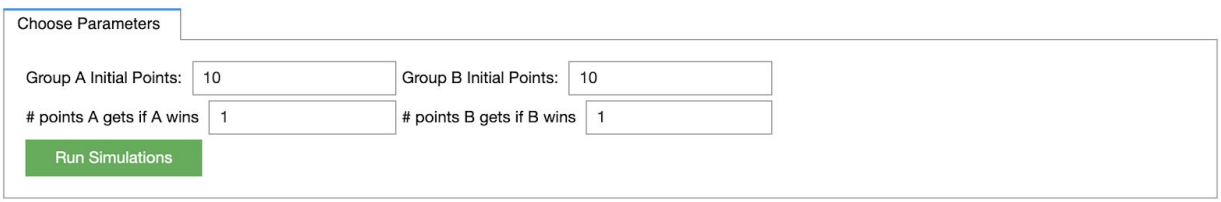
Ask the students to use the diagram below to determine what the probability each player will win is.
The probability Alice will win is

$$P(A) = \frac{21}{36}$$

while the probability Bob will win is

$$P(B) = \frac{15}{36}$$

Once you have explored that, you can ask the students how to test experimentally how the players do over time, and invite them to play with the simulation. Ask the students to change the number of points, and investigate how it changes the game.



An interesting exercise is to change the number of points to make the game as fair as possible. This happens when both players win approximately 50% of the time - the simulation will provide this feedback. This happens exactly when the expected per round payoff equation is equal to zero.

While the students play with the notebook, and once they have found a solution, ask them to take note of:

a. Group A initial points in your solution.
b. Group B initial points in your solution.
c. Number of points Group A gets if they win.
d. Number of points Group B gets if they win.
e. The average number of rounds it took for the game to end. (The game ends when one of the two players does not have any points left).
f. If your solution involves changing a different aspect of the game other than the number of points, describe what you changed.

They can enter their solutions and save them in the student version. Create room for discussion and have the students share their solutions with the class and how they found them. Some students may attempt to solve the equation

P(A)x Number of points Alice gets - P(B)x Number of points Bob gets = 0

Where:
P(A) = probability Alice will win and P(B) = probability Bob will win.

Other students might find it while tinkering with the simulation tool. There are multiple solutions, some of which can be encapsulated in the following table:

Initial Points A	Initial Points B	Points A gets if A wins	Points B gets if B wins
100	100	5 (or 10, 15, 20, 25,...)	7 (resp 14, 21, 28, 35,...)
10*	10*	1 (or 2,3,4,5,...)	1.4 (resp 2.8, 4.2, 5.6, 7,...)

* The number of initial points can change and will not impact the expected payoff result, but it will impact how long it takes for the game to end.

Food for thought

Invite the students to discuss with their partner situations in life in which some benefit more than others, and to come up with potential solutions to balance things out. Ask the students to share and brainstorm with the class. You may emphasize that while unfair situations in life can be common, they can be thought of as an opportunity for learning:

1. Individuals who benefit from a situation more than others can see it as an opportunity to acknowledge the role of luck in their lives, and to be open to learning what can be done to improve the situation.
2. Individuals who are on the losing end can feel hurt and hopeless upon learning this, however having knowledge of the unfairness of a situation can remove the burden associated with blaming loss to oneself. It can also be empowering to ask oneself what can be done with that knowledge.
3. Both parties can work together to improve the situation, where there is an understanding that perfect fairness cannot be attained, but that the effort to balance things out as much as possible is a worthy endeavour.