

Strategic Trading and Blockholder Dynamics^{*}

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Abstract

We study strategic trading by a blockholder who may both monitor a firm and trades its shares over time. We study the effects of the blockholder having access to private information. In the presence of private information, the blockholder faces an illiquid market. We show that private information leads to a larger block size in good states, but might lead to a lower block size in bad states, by increasing the blockholder's trading speed. Despite the heterogeneous impact on expected block size, and unlike in static settings, we show that private information leads to a Pareto improvement: it not only increases the stock price, and benefits small uninformed investors, but also benefits the blockholder, notwithstanding its negative impact on liquidity. We apply our model to study the optimal IPO mechanisms. We show that post-IPO trading significantly modifies the IPO design, sometimes exacerbating aftermarket volatility.

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1 Introduction

Blockholders play a prominent role in capital markets ([Holderness \(2007\)](#)). They monitor firms and promote changes that affect firm productivity through various channels (e.g., negotiations with management, proxy fights, etc). These activities are costly to the blockholder and small shareholders may free-ride on them. A blockholder thus faces a trade-off: he can mitigate free-riding, by owning a large block thereby enhancing his own incentive to monitor the firm, however by doing so, he compromises his own portfolio (diversification) needs.¹

We study strategic trading when a large blockholder is privately informed about his (time varying) ability to monitor the firm, and investigate the impact of asymmetric information on the dynamics of blockholder ownership, firm productivity, and stock prices.

Empirically, blockholder monitoring varies over time in ways that are hard to anticipate for market participants (see [Hadlock and Schwartz-Ziv \(2019\)](#)) as these decisions depend on blockholder incentives, which often are private information. For example, sometimes a blockholder's involvement may be boosted by the arrival of ideas and opportunities to improve firm management, or be hampered by a deterioration of the relation between the blockholder and the firm's manager. Sometimes the blockholder's involvement may change as a consequence of variation in the blockholder's liquidity needs. This uncertain blockholder involvement is not only empirically relevant but also has important consequences that have not been theoretically studied.

Building on [Admati et al. \(1994\)](#) and [DeMarzo and Urošević \(2006\)](#) we consider a dynamic model of trading between a large investor (or blockholder) and a continuum of competitive investors, but consider an environment with private information.² In each period, the blockholder can both trade and monitor the firm to influence the firm's cash flows. Crucially, the blockholder cannot commit to holding a large block, thus he trades over time based on his private information and portfolio preferences. The productivity of the blockholder's monitoring effort is private information and varies over time (it is a binary Markov chain).

¹These trade-offs have been long identified by corporate governance scholars and practitioners at least going back to the work by [Berle and Means \(1932\)](#), [Alchian and Demsetz \(1972\)](#), and [Jensen and Meckling \(1976\)](#).

²The seminal papers on large shareholder monitoring are [Huddart \(1993\)](#) and [Admati et al. \(1994\)](#). The closest paper is [DeMarzo and Urošević \(2006\)](#) who extend the static models to a fully dynamic environment where blockholders can monitor and trade over time. Unlike [DeMarzo and Urošević \(2006\)](#), we consider a setting with asymmetric information.

Thus, we depart from previous literature by considering a setting that combines moral hazard under asymmetric information. Thus, we also contribute to the literature studying the signaling role of ownership retention and extend it to a dynamic environment (Leland and Pyle, 1977; Gale and Stiglitz, 1989; DeMarzo and Duffie, 1999).³

We analyze the equilibrium taking the blockholder’s initial holdings as given. As in settings without asymmetric information, the blockholder sells his entire block over time – regardless of his ability to increase the firm value via monitoring – due to the lack of commitment (DeMarzo and Urošević, 2006). It would be natural to expect that asymmetric information would reduce the amount of trading. This is indeed the case when productivity is high – or alternative when the cost of holding a large position is low – asymmetric information reduces the speed of trading, which is consistent with previous literature looking at the impact of illiquidity. As a consequence, when productivity is high, the blockholder slows down his selling speed, relative to the symmetric information case. As he holds his shares for a longer period of time, his incentives to monitor the firm become stronger and the price goes up.

However, contrary to the intuition arising from static signaling models, asymmetric information might *increase* the trading speed when productivity is low. The seller accelerates selling, relative to the public information case. This happens as an equilibrium reaction to the price increment in the high productivity state. Otherwise, the price would go up in the low state – driven by stronger monitoring in the high state – which in turn would reinforce the blockholder incentive to sell quickly. It follows that asymmetric information might reduce expected holdings and increase the cross-sectional dispersion in blockholdings.

We assume that the blockholder bears inventory holding costs, to capture the blockholder’s liquidity and diversification needs. Two cases need to be distinguished depending on the magnitude of the inventory cost. First, when the blockholder’s inventory costs are low, the blockholder’s continuation payoff is convex in block size due to increasing returns to scale. This convexity arises due to the monitoring effort being proportional to the block size in equilibrium. In this context, the blockholder refrains from selling in the high productivity state and sells smoothly in the low state. Second, when inventory costs are large,

³Gomes (2000) also studies a reputation game, with two types of manager/owners, who differ in terms of their cost of effort. In Gomes (2000) the manager effort is observable. Unlike in Gomes, we allow for hidden effort and time-varying private information.

the blockholder’s continuation payoff becomes concave in the low state. In this context, the blockholder sells smoothly under high productivity, but as soon as the productivity drops, the blockholder immediately liquidates his holdings, consistent with the Coase conjecture.

We study the welfare impact of asymmetric information. Contrary to static settings, where signaling entails deadweight costs to the seller, in our dynamic setting private information yields a Pareto improvement, relative to the symmetric information case. On the one hand, asymmetric information benefits small uninformed investors because it leads to a larger block, thereby boosting monitoring, and ultimately increasing the firm’s dividends. As mentioned above, by reducing liquidity, asymmetric information reduces the speed of selling in high productivity states, thus extending the blockholder’s monitoring, particularly when it is most effective. This leads to a higher stock price. In turn, this increases the dividends earned by small investors.

On the other hand, the blockholder’s payoff weakly increases when he has access to private information, unlike in a static setting where private information would typically force the blockholder to signal his type via inefficient retention ([Leland and Pyle, 1977](#); [Vanasco, 2017](#)). Again, the size of the effect depends on the magnitude of the inventory cost. First, when inventory costs are small, asymmetric information has no impact on the blockholder’s payoff. In other words, the blockholder obtains the same payoff regardless of the information structure. The reason is that, due to lack of commitment, the blockholder neither can extract rents from trading nor he bears signaling costs—as in static setting—but trades in a competitive fashion, regardless of whether he has access to private information or not.

Surprisingly, when the blockholder’s inventory cost is large, the blockholder’s payoff increases under asymmetric information, particularly in the high productivity state. In this context, it becomes too costly for the “low type” to sell slowly, and as mentioned above, the low type sells immediately. Since the low type exits the market immediately, the high type faces a more liquid market thereafter, which allows him to sell without triggering a large price drop. Hence, when inventory costs are large, some of the gains from trade are exploited in the high state. More importantly, under asymmetric information, the high type refrains from selling too fast, because of his price impact, which acts as a commitment device that mitigates the forces of the Coase conjecture, and allows the blockholder to extract monopoly rents.

So, contrary to conventional wisdom, the blockholder’s access to private information improves the welfare of uninformed investors without having an adverse effect on the blockholder’s payoff, in stark contrast with a static setting (and a setting without monitoring). This result is reminiscent of the theory of the second best (Lipsey and Lancaster, 1956), whereby two frictions combined (lack of commitment and asymmetric information) lead to a more efficient outcome than a single friction (lack of commitment).

Our paper speaks to the literature on the role of liquidity on corporate governance. A key issue in this literature is that, a blockholder may have incentives to sell his shares (“cut and run”) instead of bearing the cost of monitoring, particularly when the firm is under-performing. This has led some authors to conclude that market liquidity might be detrimental to corporate governance (Coffee, 1991; Kahn and Winton, 1998; Noe, 2002; Faure-Grimaud and Gromb, 2004).⁴

One counterargument is that liquidity might reduce free riding problem in takeovers (Grossman and Hart, 1980; Shleifer and Vishny, 1986). By facilitating the creation of a large block in the first place, liquidity can actually strengthen the firm’s corporate governance and improve performance (Kyle and Vila, 1991; Maug, 1998; Back et al., 2018).⁵ Another counterargument is that liquidity facilitates the use of “voice” as a governance mechanism (Hirschman, 1970).⁶ Indeed, if manager’s compensation is tied to the price of the company, so the manager is hurt by selling forces that would bring the price down, then investors can discipline the firm by threatening to sell their shares (Admati and Pfleiderer, 2009; Edmans, 2009).

Our results support the notion that illiquidity/adverse selection may have a positive effect because it mitigates the blockholder’s lack of commitment to hold his shares and monitor the firm, particularly when this is most useful, that is, when productivity is high.⁷

While in the first part of this paper we assume that the initial block is exogenous, we

⁴This idea has been behind policy proposals attempting to reduce trading. For example, the European Union agreed to implement a transaction tax in September 2016.

⁵We depart from this literature by considering a dynamic signaling model – in which blockholdings are observable – rather than a microstructure model, where trading is unobservable and trading by the blockholder is obscured by the presence of noise traders.

⁶This literature is surveyed in Becht, Bolton, and Röell (2003) and Edmans and Holderness (2017).

⁷A related literature on loan sales and security design considers the impact that liquidity in secondary markets has on ex-ante screening of project quality. For example, Vanasco (2017) studies the role that adverse selection may play in fostering ex-ante screening to originate projects of high quality.

then endogenize the size of the initial block size. Specifically, we consider the size of the initial block as the outcome of an optimal IPO, whereby an uninformed issuer sells the stock to both the blockholder and to a continuum of small uninformed investors, in anticipation of the fact that the investors will trade their shares in the aftermarket. In this way, we extend previous literature that study optimal IPO mechanism by analyzing the impact that the post IPO trading has on the optimal IPO design (Benveniste and Wilhelm, 1990; Spatt and Srivastava, 1991; Stoughton and Zechner, 1998).

The possibility of post-IPO trading significantly alters the IPO design, and among other things, justifies commonly observed practices, such as IPO lockup periods and loyalty shares. We show that in equilibrium, and consistent with empirical evidence (see Brav and Gompers (2003)), the optimal IPO entails two stages. First, the entrepreneur conducts a pre-IPO placement where the blockholder receives a block at a discounted price. Second, the actual IPO takes place whereby the remaining shares are sold to small investors at a competitive price. In equilibrium, the entrepreneur offers a menu to the blockholder who in turn selects his preferred block/price pair based on his private information.

As mentioned above, the IPO design is crucially affected by post-IPO trading dynamics. In the absence of ex-post trading, the issuer screens the blockholder by allocating a larger block when the blockholder is more productive and vice-versa. Hence in equilibrium, the block size is informative about the blockholder productivity. But, under the possibility of post-IPO trading, a pooling allocation may emerge in equilibrium, particularly when the issuer is pessimistic about the blockholder productivity. Furthermore, insofar as the IPO allocation is uninformative about the state, this causes large volatility in the aftermarket (as documented empirically by Lowry et al. (2010); Brav and Gompers (2003)) as the blockholder information is revealed through his aftermarket trading.

To conclude, we note that our paper is related to the literature on durable good monopoly with incomplete information, and the literature on bargaining with two sided asymmetric information (Cho, 1990; Ausubel and Deneckere, 1992). The closest paper in this literature is Ortner (2020) who consider a bargaining model with time varying cost. Because of the different focus and application, our model differs in a number of ways. First, unlike this previous literature, we consider a setting with common values in which the seller's private information affects the buyer's valuation. Second, the buyer's valuation is directly affected by the blockholder trading strategy. These differences have a significant effect in the nature

of the equilibrium. In contrast to the literature looking at durable goods monopolies with two sided incomplete information, we derive conditions under which the equilibrium entails trade by all types, and positive rents for blockholders. We show that the Coase conjecture holds (in the sense that the monopolist is unable to extract rents) only when the blockholder’s cost of holding large positions is small. However, we show that the Coase conjecture fails if this cost is sufficiently high. Perhaps surprisingly, a small increment in the cost of holding shares can lead to an increment in the blockholder’s payoff. In durable good monopoly and bargaining models, incomplete information about costs generates an extreme form of inefficiency by eliminating trade completely. On the contrary, we show that asymmetric information increases welfare in our setting where blockholding has an effect on productivity. Not only overall welfare increases, but we also show that the equilibrium with asymmetric information Pareto dominates the equilibrium with symmetric information.

2 Setting

Following [Admati et al. \(1994\)](#) and [DeMarzo and Urošević \(2006\)](#) we study the behavior of a large investor (henceforth, blockholder) who can both trade a firm’s stock and take costly actions (e.g., monitoring activities) to improve the firm’s productivity. In addition to a large blockholder, there is a continuum of small investors who are price takers and cannot influence the firm’s cash flows. In the baseline, we take the initial holdings of the blockholder x_0 as exogenous, but in [Section 5](#) we model x_0 as arising from an optimal IPO mechanism.

Asset Time t is continuous and the horizon is infinite. There is a single firm in unit supply with expected cash flows δ_t

$$\mathbb{E}_t[\delta_t]dt = (\mu + \theta_t a_t)dt,$$

where a_t is the blockholder’s effort and $\theta_t \in \{\underline{\theta}, \bar{\theta}\}$ is his productivity. We make this multiplicative assumption, instead of an additive θ_t , to capture the fact that, empirically, there is wide variation in blockholder involvement, which suggests that blockholders vary in terms of productivity (see e.g., [Hadlock and Schwartz-Ziv \(2019\)](#)). To be specific, variation in θ may capture the fact that the arrival of ideas and projects to improve management is somewhat random. Alternatively, this may also capture the idea that the very relationship between

a blockholder and the firm management may also be subject to random shocks, that could affect the ability of the blockholder to boost the firm’s performance.

By blockholder productivity we mean the quality of the match between a firm and the blockholder. This quality is subject to uncertainty and variation over time, insofar as the blockholder’s incentive and ability to monitor, as well as the intensity of agency frictions, varies due to random reasons (an alternative interpretation is that θ_t captures the blockholder’s opportunity cost of monitoring the firm, which depends on how busy the blockholder is at a given point in time).

The distinction between private information regarding the firm cash flows (μ) and private information regarding the blockholder’s productivity (θ) is important when we consider its impact on the initial IPO allocation. Formally, if the blockholder were privately informed about μ instead of θ , then the efficient allocation at the time of the IPO would be independent of the blockholder’s private information. The main impact of the IPO mechanism would be on the distribution of rents rather than economic efficiency.

We assume that the cash flows δ_t are publicly observable but the blockholder’s effort a_t and productivity θ_t are not. The realized cash flows are paid to shareholders in each period, and as such we sometimes interpret δ_t as the firm’s dividends. Conditional on θ_t , the firm’s dividend δ_t is random. This assumption is important because otherwise the market would be able to infer the state θ_t from observing the dividend, and the blockholder’s trading would be uninformative about the firm’s fundamental. However, since the market is risk-neutral and the blockholder’s preferences are specified directly over his holdings x , we do not need to specify the distribution of noise in the dividend process.

We refer to a_t as effort but interpret it broadly as any costly action that affects the firm’s cash flows. We are agnostic as to the source of this externality. In the case of an external investor, one can think of a_t as the blockholder’s monitoring — which disciplines managers and mitigates agency conflicts— or as the influence the blockholder exerts on the firm’s management (as in [Admati et al. \(1994\)](#); [Stoughton and Zechner \(1998\)](#); [DeMarzo and Urošević \(2006\)](#)). Examples of a_t include public criticism of management or launching a proxy fight, advising management on strategy, figuring out how to vote on proxy contest launched by others or not taking private benefits for himself. In the case of a CEO or the founder of a company, a_t can represent effort or a reduction of private benefits that increases the productivity of the firm.

Productivity follows a two-state Markov-chain with switching intensity $\{\lambda_H, \lambda_L\}$, where λ_H is the switching intensity from $\underline{\theta}$ to $\bar{\theta}$, and λ_L is the switching intensity from $\bar{\theta}$ to $\underline{\theta}$.

Agents: Competitive investors, hereafter the market, are risk neutral and discount future cash flows at a discount rate r . The blockholder has quadratic flow preferences

$$u(x, a, \theta) = \mathbb{E}[\mu + \theta a]x - \frac{1}{2} (\phi^{-1}a^2 + \gamma x^2),$$

where $a^2/2\phi$ is the private cost of effort, and $1/\phi$ captures the severity of moral hazard. In practice this varies across blockholder types, and affects the blockholder's involvement. Indeed, the empirical literature has documented that blockholder's involvement varies.⁸ In our model, this pattern would be consistent with financial blockholders having a large cost of monitoring, or a small ϕ .

The parameter γ captures the cost of holding a stake x which we refer to as inventory cost. Although the holding cost cannot be directly linked to risk aversion, the presence of the inventory costs is meant to be a reduced-form way of capturing the financing cost of holding a large position in the firm.⁹ The quadratic holding cost is popular among practitioners in financial institutions (Almgren and Chriss, 2001), and has also been used extensively in the dynamic trading literature (Vives, 2011; Du and Zhu, 2017; Duffie and Zhu, 2017). If γ is too large, then the blockholder sell his shares immediately, which is equivalent to a setting without a blockholder. Thus, to focus on the interesting case, we assume that γ isn't too large.

Notice that it is possible extend our results to more general utility functions $u(x, a, \theta)$. The key assumption is that high type has a higher willingness to hold shares. This might be because the high type is more productive or because it has a lower holding cost. Later in the paper, in section 3.3, we show how our model can be interpreted as one where private information comes from shocks to the blockholder's liquidity rather than his productivity.

⁸For example, Hadlock and Schwartz-Ziv (2019) argue that “many of the data patterns can be interpreted as consistent with a governance role through monitoring by nonfinancial blocks, and through trading for financial blocks.”

⁹The one exception where the holding cost can be directly linked to risk aversion is the case in which cash flows are normally distributed and traders have CARA preferences.

Information: The blockholder observes the dividend δ_t and the firm's productivity θ_t . On the other hand, the market only observes the dividend δ_t and the blockholder holdings x_t and trading flow q_t .

Throughout the paper we denote by $\hat{\theta}_t$ the expected value of θ_t given the market's information. Also whenever needed, we let $\hat{\mathbb{E}}_t[\cdot]$ denote the expected value at time t given the market conjectured strategy (\hat{a}, \hat{q}) and state $(\hat{\theta}_t)_{t \geq 0}$, and $\mathbb{E}_t[\cdot]$ the expected value given the blockholder true strategy (a, q) and state $(\theta_t)_{t \geq 0}$.

Strategies: The blockholder chooses effort a_t and trading $q_t \equiv dx_t/dt$ given his private information at time t . Competitive investors choose a trading strategy adapted to the public information. We denote the blockholder holdings at time t by x_t . Since the firm is in unit supply the market clearing condition at time t is

$$x_t + y_t = 1.$$

We assume that short sales are not allowed, so at any time t , the blockholder holdings x_t must be between zero and one.

Because competitive investors are risk neutral, in equilibrium the stock price is given by

$$p_t = \hat{\mathbb{E}}_t \left[\int_t^\infty e^{-r(s-t)} (\mu + \hat{\theta}_s \hat{a}_s) ds \right].$$

Equilibrium definition We focus on separating equilibria in Markov strategies. In particular,

Definition 1. A separating Markov equilibrium is given by a strategy $\langle a(x, \theta), q(x, \theta) \rangle$, a price function $p(x, q)$ and market beliefs $\hat{\theta} = \hat{\theta}(q, x)$ such that

$$\begin{aligned} (a_s, q_s)_{s \geq t} &\in \arg \max_{(a_s, q_s)_{s \geq t}} \mathbb{E} \left[\int_t^\infty e^{-r(s-t)} (u(x_s, a_s, \theta_s) - q_s p(x_s, q_s)) ds \middle| \theta_t \right] \\ p(x, q) &= \hat{\mathbb{E}} \left[\int_t^\infty e^{-r(s-t)} (\mu + \theta_s a(x_s, \theta_s)) ds \middle| \theta_t = \hat{\theta}(q, x) \right] \\ \hat{\theta}(q(x, \theta), x) &= \theta. \end{aligned}$$

The equilibrium definition is standard. The blockholder chooses effort a_t and trading q_t to maximize his discounted payoff. The market sets the price of the firm as the present value of its future dividends, using the blockholder’s trading history to forecast future effort and cash flows. Conditional on trading, the dividend does not play an information role. Naturally, in a separating equilibrium the market beliefs are, at each point, consistent with the true state.

Some caveats are in order. As in static signaling models, dynamic signaling models suffer from equilibrium multiplicity. Thus the choice of a selection criteria is a key part of the analysis. Our model is closely related to models of durable goods monopoly with incomplete information about costs, and models of bargaining with one-sided offers and two-sided incomplete information (Ausubel and Deneckere (1992); Cho (1990); Ortner (2020)). It is well known in this literature that a large class of equilibria can be constructed using optimistic off-equilibrium beliefs that assign off-equilibrium beliefs to the weakest type. Moreover, in many of these models it is possible to artificially introduce commitment by using this kind of punishing beliefs. To discipline our model, it is thus natural to focus on equilibria that satisfy some stationarity properties so the equilibrium does not depend on the complete history of the game (this is in the same spirit as the restriction to Markov Perfect Equilibrium). In our context, this stationary property requires that the blockholder strategy only depends on its type and current holdings, and that the market pricing only depends on the blockholder’s holdings and trading rate. Finally, this class of separating equilibria are attractive from an analytical point of view due to their simple structure, which renders tractable the analysis of dynamic models with incomplete information.

3 Equilibrium Characterization

3.1 Constrained Planner’s Problem

Before we characterize the the equilibrium, we provide the solution of a planner who can control the block size over time but can not contract on the effort provided by the blockholder (thus the constrained qualification).

This “first-best” allocation is a useful benchmark. In this benchmark, the allocation x_t is chosen at each point to maximize social welfare under symmetric information about θ_t , but

subject to the blockholder choosing effort privately (or in a non-contractible manner) which leads to effort

$$a(x, \theta) = \phi \theta x. \quad (1)$$

In this context, social welfare is

$$W = \frac{\mu}{r} + \int_0^\infty e^{-rt} \mathbb{E} \left[\phi \theta_t^2 x_t - \frac{1}{2} (\phi \theta_t^2 + \gamma) x_t^2 \right] dt.$$

That is, welfare consists of the discounted cash flows net of effort and inventory costs. The first best allocation thus maximizes the objective point-wise, and yields

$$x_t = \frac{\phi \theta_t^2}{\phi \theta_t^2 + \gamma}. \quad (2)$$

In this benchmark, the optimal block size is interior. It increases when productivity goes up, and decreases in the blockholder's inventory costs.

3.2 Equilibrium with Unobservable Shocks

Next, we characterize separating equilibria taking the blockholder initial holdings, x_0 , as given. Later on, we derive the equilibrium when the state θ_t is publicly observable and evaluate the impact of asymmetric information. By definition, in a separating equilibrium, the beliefs of the market $\hat{\theta}_t$ coincide with the true state θ_t at each point in time.

Denote the blockholder's continuation value by $V(x, \theta)$. Given an equilibrium price function $p(x, q)$, the continuation value satisfies the following HJB equation

$$\begin{aligned} rV(x, \bar{\theta}) &= \max_{a, q} (\mu + \bar{\theta}a)x - \frac{1}{2} (\phi^{-1}a^2 + \gamma x^2) + q (V_x(x, \bar{\theta}) - p(x, q)) \\ &\quad + \lambda_L (V(x, \underline{\theta}) - V(x, \bar{\theta})) \\ rV(x, \underline{\theta}) &= \max_{a, q} (\mu + \underline{\theta}a)x - \frac{1}{2} (\phi^{-1}a^2 + \gamma x^2) + q (V_x(x, \underline{\theta}) - p(x, q)) \\ &\quad + \lambda_H (V(x, \bar{\theta}) - V(x, \underline{\theta})). \end{aligned} \quad (3)$$

The blockholder earns the dividend but bears effort and inventory costs. In addition, the blockholder makes a profit whenever there is a gap between the price $p(x, \theta)$ and his

own marginal valuation $V_x(x, \theta)$, and makes a capital gain whenever there is a productivity transition.

Inspecting the HJB equation above, we see that the effort decision is myopic, hence the blockholder chooses effort to maximize his flow payoff.¹⁰ The optimal effort is thus

$$a(x, \theta) = \phi \theta x. \quad (4)$$

From inspecting the HJB equation above, we see that the optimal trading strategy maximizes the blockholder's trading profits, as given by $q(V_x(x, \theta) - p(x, \theta))$. It follows that his trading strategy $q(x, \theta)$ must satisfy the following incentive compatibility constraint

$$\begin{aligned} q(x, \bar{\theta}) (V_x(x, \bar{\theta}) - p(x, \bar{\theta})) &\geq q(x, \underline{\theta}) (V_x(x, \bar{\theta}) - p(x, \underline{\theta})) \\ q(x, \underline{\theta}) (V_x(x, \underline{\theta}) - p(x, \underline{\theta})) &\geq q(x, \bar{\theta}) (V_x(x, \underline{\theta}) - p(x, \bar{\theta})) \end{aligned} \quad (5)$$

The one-shot deviation principle guarantees that these conditions are sufficient for incentive compatibility. Adding both incentive compatibility constraints, yields the following inequality

$$(q(x, \bar{\theta}) - q(x, \underline{\theta})) (V_x(x, \bar{\theta}) - V_x(x, \underline{\theta})) \geq 0. \quad (6)$$

Inequality (6) establishes a natural monotonicity property that must be satisfied by any separating equilibrium: if the marginal valuation of $\bar{\theta}$ is higher than that of $\underline{\theta}$, then $q(\bar{\theta})$ must also be higher than that of $q(\underline{\theta})$; put differently, a necessary condition for the trading strategy to be incentive compatible, the blockholder must buy more shares when his marginal valuation is higher.

In any equilibrium, the dividend — given by $\mu + \phi \theta^2 x$ — is higher for the high type. Thus, in the absence of dynamic trading, the single crossing condition would be immediately satisfied. However, in a dynamic game, this condition does not follow immediately from single crossing in flow payoffs but needs to hold for the continuation value.

Instead of imposing a restriction directly on beliefs to guarantee single crossing, we restrict attention to equilibria satisfying this monotonicity property. Similarly to [Cho \(1990\)](#), the single crossing condition in the dynamic trading game can be motivated by “Divinity.” Using backward induction, and extending the ideas in [Cho \(1990\)](#) to our setting, it is possible to

¹⁰Effort is myopic in a separating equilibrium. In a pooling equilibrium, this might not necessarily hold because effort would have a long-term impact on the market beliefs.

establish such a property by considering the limit of a discrete time model with a finite number of trading rounds, and applying a refinement such as Divinity to the static game in each trading round.¹¹

Condition 1. *We say that an equilibrium is monotonic if $V_x(x, \bar{\theta}) > V_x(x, \underline{\theta})$ for all $x > 0$.*

This condition together with the incentive compatibility condition (6) imply that in any separating equilibria we have $q(x, \bar{\theta}) > q(x, \underline{\theta})$ for all $x > 0$.

Next, we characterize the equilibrium price that is consistent with the monotonicity condition 1.

Lemma 1 (Equilibrium Price). *In any monotonic equilibrium with smooth trading, the equilibrium beliefs are*

$$\hat{\theta}(q, x) = \begin{cases} \underline{\theta} & \text{if } q < 0 \\ \bar{\theta} & \text{if } q > 0. \end{cases}$$

The price function is given by

$$p(x, q) = \begin{cases} p(x, \underline{\theta}) & \text{if } q < 0 \\ p(x, \bar{\theta}) & \text{if } q > 0, \end{cases}$$

where $p(x, \theta)$ satisfies the following equation:

$$\begin{aligned} rp(x, \bar{\theta}) &= \mu + \bar{\theta}a(x, \bar{\theta}) + q(x, \bar{\theta})p_x(x, \bar{\theta}) + \lambda_L (p(x, \underline{\theta}) - p(x, \bar{\theta})) \\ rp(x, \underline{\theta}) &= \mu + \underline{\theta}a(x, \underline{\theta}) + q(x, \underline{\theta})p_x(x, \underline{\theta}) + \lambda_H (p(x, \bar{\theta}) - p(x, \underline{\theta})). \end{aligned} \tag{7}$$

The intuition behind Lemma 1 is as follows. In a separating equilibrium, whenever the low type sells, he receives the same payoff as in a symmetric information model. The low type does not refrain from trading since his reputation is at the lowest level (this is the traditional result in static signaling model whereas the lowest type gets symmetric information outcome, although in our dynamic setting the rate of trading changes even for the low type).

Because the optimization problem in equation (3) is linear in the trading rate, in any equilibrium with smooth trading, the low type necessarily gets zero profits from trading, which reflects the forces behind the Coase conjecture. This zero-rent at the bottom result,

¹¹See [Fudenberg and Tirole \(1991\)](#) for a formal definition of Divinity.

means that any price that would induce the high type to sell his shares, would also induce the low type to sell his. So the high type cannot sell in equilibrium. We can use the same argument to show that the low type cannot buy. The logic behind this result is similar to the no-trade theorem in [Ausubel and Deneckere \(1992\)](#), who study a durable good monopoly with incomplete information. Lemma 1 relies crucially on the assumption that equilibrium entails smooth trading. Later on, we consider the case in which the equilibrium involves atomic trading.

Again, given that the maximization problem in the HJB equation (3) is linear in the trading rate q , in an equilibrium with smooth trading, when he trades the blockholder must be indifferent whether to trade, and the price must satisfy $p(x, \theta) = V_x(x, \theta)$. Accordingly, an implication of Lemma 1 is that the equilibrium price is given by

$$p(x, q) = \begin{cases} V_x(x, \underline{\theta}) & \text{if } q < 0 \\ V_x(x, \bar{\theta}) & \text{if } q > 0, \end{cases}$$

therefore the blockholders's continuation value is calculated as if there was no trading on path, thus solving

$$\begin{aligned} rV(x, \bar{\theta}) &= \max_a (\mu + \bar{\theta}a)x - \frac{1}{2} (\phi^{-1}a^2 + \gamma x^2) - \lambda_L (V(x, \bar{\theta}) - V(x, \underline{\theta})) \\ rV(x, \underline{\theta}) &= \max_a (\mu + \underline{\theta}a)x - \frac{1}{2} (\phi^{-1}a^2 + \gamma x^2) - \lambda_H (V(x, \underline{\theta}) - V(x, \bar{\theta})). \end{aligned} \quad (8)$$

Furthermore, given the underlying free-riding problem, it is optimal for the blockholder to eventually sell all his shares, $x = 0$. Consistent with this intuition, we prove that $q(x, \underline{\theta}) < 0$ for all $x > 0$. On the other hand, there cannot be an equilibrium in which $p(x, \bar{\theta}) = V_x(x, \bar{\theta})$ and $q(x, \underline{\theta}) > 0$ (this is verified later in section 3.4). Thus, it must be the case that $q(x, \bar{\theta}) = 0$.

Plugging this in the asset pricing equation (7), yields the following intermediate result:

$$p(x, \bar{\theta}) = \frac{\mu + \bar{\theta}a(x, \bar{\theta}) + \lambda_L V_x(x, \underline{\theta})}{r + \lambda_L} \quad (9)$$

$$q(x, \underline{\theta}) = -\frac{\mu + \underline{\theta}a(x, \underline{\theta}) + \lambda_H p(x, \bar{\theta}) - (r + \lambda_H) V_x(x, \underline{\theta})}{V_{xx}(x, \underline{\theta})}. \quad (10)$$

Equation (9) characterizes the price in the high state, which follows from plugging $q(x, \bar{\theta}) = 0$ in the equation for $p(x, \bar{\theta})$, while equation (10) provides the equilibrium trading rate $q(x, \underline{\theta})$ which follows from plugging $p(x, \underline{\theta}) = V_x(x, \underline{\theta})$ into the equation for $p(x, \underline{\theta})$ in (7).

To guarantee the existence of an equilibrium with smooth trading, inventory costs γ need not be too large. The next proposition characterizes the equilibrium under low inventory costs.

Proposition 1. *Assume that inventory costs γ satisfies*

$$\underline{\theta}^2 + \frac{\lambda_H}{r + \lambda_L + \lambda_H}(\bar{\theta}^2 - \underline{\theta}^2) \geq \frac{\gamma}{\phi}. \quad (11)$$

Then there is a unique monotonic equilibrium with smooth trading. In this equilibrium the blockholder's payoff is

$$V(x, \theta) = \frac{\mu}{r}x + \frac{1}{2}C(\theta)x^2 \quad (12)$$

where $C(\theta) > 0$ is given by

$$\begin{aligned} rC(\underline{\theta}) &= \phi\underline{\theta}^2 - \gamma + \phi \frac{\lambda_H}{r + \lambda_L + \lambda_H}(\bar{\theta}^2 - \underline{\theta}^2) \\ rC(\bar{\theta}) &= \phi\bar{\theta}^2 - \gamma - \phi \frac{\lambda_L}{r + \lambda_L + \lambda_H}(\bar{\theta}^2 - \underline{\theta}^2). \end{aligned} \quad (13)$$

The stock price satisfies

$$p(x, q) = \begin{cases} \frac{\mu}{r} + C(\underline{\theta})x & \text{if } q < 0 \\ \frac{\mu}{r} + \frac{\phi\bar{\theta}^2 + \lambda_L C(\underline{\theta})}{r + \lambda_L}x & \text{if } q = 0 \\ \frac{\mu}{r} + C(\bar{\theta})x & \text{if } q > 0 \end{cases}$$

and the blockholder's trading rate follows

$$q(x, \theta) = \begin{cases} -\left(1 + \frac{\lambda_H}{r + \lambda_L}\right) \frac{\gamma}{C(\underline{\theta})}x & \text{if } \theta = \underline{\theta} \\ 0 & \text{if } \theta = \bar{\theta} \end{cases}.$$

In this model, lack of commitment leads the blockholder to sell his entire block over time, until x_t reaches zero. This is clearly inefficient compared with the first-best benchmark.

The blockholder's selling behavior causes an externality: the firm's productivity gradually deteriorates as the blockholder unwinds his holdings thereby weakening his own incentive to monitor the firm.

Notice that, in the high state, the blockholder does not sell shares, even though his valuation is lower than that of small investors ($p(x, \bar{\theta}) > V_x(x, \bar{\theta})$). The blockholder refrains from selling because of his price impact; if the blockholder were to sell, the market would interpret this as a negative signal of productivity and the stock price would drop drastically, even below the blockholder's valuation, in which case the blockholder would experience a capital loss. By contrast, in the low state, the blockholder sells gradually until his holdings are fully depleted, or until a positive shock creates incentives to pause this selling process for some time.¹²

Proposition (1) is predicated on the low type's continuation value being convex in x , which corresponds to condition (11) in the proposition. This convexity reflects the presence of increasing returns to scale in block size, x . Despite the convexity of inventory costs, there are increasing returns to scale because the cash flow depends on effort a , which in equilibrium is proportional to x . Yet the value function is convex in x as long as these returns of scale dominate the convexity of inventory costs.

To understand the equilibrium, notice that in a separating equilibrium, the low type's payoff is the same as that arising under symmetric information. As in the previous literature on durable goods monopoly, lack of commitment prevents the blockholder from extracting rents from trading, which yields the continuation payoff $V(x, \underline{\theta})$ in the proposition. However, this does not imply that the blockholder trades immediately towards his long-term target. Contrary to the standard prediction of the Coase conjecture, the blockholder sells slowly.

Why does this happen? Recall that in a competitive setting, the price equals marginal cost. In our setting, the marginal cost is represented by $V_x(x, \underline{\theta})$. Figure 1(a) illustrates that a competitive equilibrium would imply payoffs for the blockholder that are below the value of not trading at all. Of course, this is just a restatement of the well known result that the price cannot equal marginal cost in the presence of increasing returns to scale, because this would generate losses to the firm. On the other hand, the equilibrium must entail some trade. If

¹²The long-run holdings would be positive if the blockholder enjoyed private benefits, in which case the order flow would sometimes be positive, for instance when the initial holdings are smaller than the long run target.

there was no trade, the price would be above μ/r , generating incentives to trade. Thus, in equilibrium, the blockholder moves smoothly along the curve $V(x, \underline{\theta})$ trading at a price $V_x(x, \underline{\theta})$. Later we will see that – unlike in static signaling models – although the payoff of the low type is the same as that arising with symmetric information, the equilibrium trading strategy is not.

For the high type, the equilibrium trading is determined by the incentive compatibility constraint. As previously discussed, Lemma 1 shows that in equilibrium the high and low type cannot trade in the same direction at the same time. Any price that is high enough to induce the high type to sell, would attract the low type, who in equilibrium can't extract rents. Thus, the only possibility is that either the high type buy shares or does not trade at all. Due to the presence of inventory costs, it is not possible that the high type buys in equilibrium (that is no longer the case once we consider possible private benefits of control later on) thus, in equilibrium, the high type does not trade.

The previous argument does not apply when condition (11) is not satisfied, in which case the value function of the low type $\underline{\theta}$ is concave in x . Then, the low type sells immediately towards $x = 0$. Figure 1(b) illustrates that the blockholder is better off by liquidating his holdings immediately, regardless of his price impact. In equilibrium, the low type sells his entire block at a price μ/r . More interestingly, in this case, the argument in Lemma 1 no longer applies. The low type payoff is strictly higher than the payoff of not trading ($V(x, \underline{\theta})$), so the incentives of the low type to imitate the high type are reduced and an equilibrium where the high type sells a positive amount becomes possible.

Next, we proceed to derive the equilibrium in this case. The high type has incentives to sell only if $p(x, q(x, \bar{\theta})) = p(x, \bar{\theta}) > V_x(x, \bar{\theta})$. In which case, the value function satisfies

$$rV(x, \bar{\theta}) = \mu x + \frac{1}{2} (\phi \bar{\theta}^2 - \gamma) x^2 + q(x, \bar{\theta}) (V_x(x, \bar{\theta}) - p(x, \bar{\theta})) + \lambda_L (V(x, \underline{\theta}) - V(x, \bar{\theta})) \quad (14)$$

On the other hand, the low type has no incentive to imitate the high type only if

$$rV(x, \underline{\theta}) \geq \mu x + \frac{1}{2} (\phi \underline{\theta}^2 - \gamma) x^2 + q(x, \bar{\theta}) (V_x(x, \underline{\theta}) - p(x, \bar{\theta})) - \lambda_H (V(x, \underline{\theta}) - V(x, \bar{\theta})) \quad (15)$$

The left hand side of equation (15) corresponds to the equilibrium payoff of the low type, which has to be weakly higher than the right-hand side, which corresponds to the value of

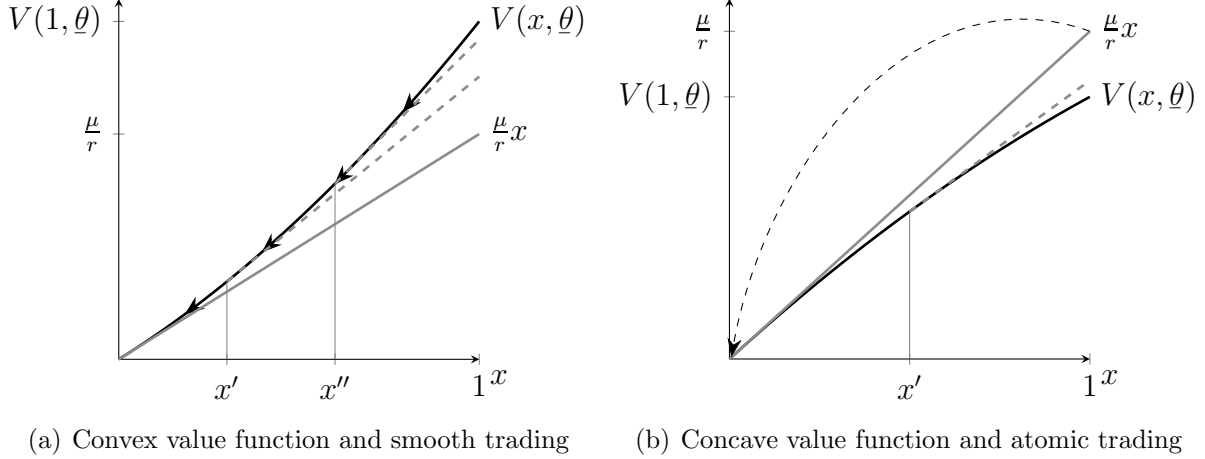


Figure 1: Value function with and without smooth trading. In this figure, $V(x, \underline{\theta})$ indicates the value of holding to x forever. In (a), the value of staying put at x is above the value of liquidating. In equilibrium, the blockholder trades moving around the curve $V(x, \underline{\theta})$. In the (b), the blockholder is better off liquidating everything at the lowest possible price (μ/r), so the equilibrium entails immediate liquidation (there is an immediate jump to $x = 0$).

pooling with the high type for a short period of time. Substituting $V(x, \underline{\theta}) = p(0, \underline{\theta})x = (\mu/r)x$ in equation (15), we get the following incentive compatibility constraint for the low type

$$q(x, \bar{\theta}) \geq -\frac{\frac{1}{2}r(\gamma - \phi\bar{\theta}^2)x^2 - \lambda_H(\mu x - rV(x, \bar{\theta}))}{rp(x, \bar{\theta}) - \mu}. \quad (16)$$

In this case, there are multiple equilibria that differ based on the trading rate of the high type. Given $q(x, \bar{\theta})$, the price is given by the solution to the ODE

$$rp(x, \bar{\theta}) = \mu + \phi\bar{\theta}^2x + q(x, \bar{\theta})p_x(x, \bar{\theta}) + \lambda_L(p(x, \underline{\theta}) - p(x, \bar{\theta})), \quad (17)$$

with initial condition $p(0, \bar{\theta}) = \mu/r$.

We consider the least costly separating equilibria. In a dynamic model, there are two different notions of a least costly separating equilibrium. One notion considers the least costly separating equilibrium from the perspective of time zero, while the other one considers each period individually taking the continuation value as given. The first notion introduces some implicit commitment power by relying on history dependent off-equilibrium beliefs as a means of punishing deviations (Kaya, 2009). The second approach avoid such a commitment

by ignoring previous deviations and only focusing on the current stage. Consistent with our focus on Markov equilibria, we follow the second approach, and consider separating equilibrium in the “stage game” with payoffs $q(V_x(x, \theta) - p(q, x))$. One can motivate the focus on the least costly separating equilibrium by following a similar reasoning to the one used in static signaling games applied to the stage game (see [Hennessy et al. \(2010\)](#) for a related approach).

The least costly separating equilibrium maximizes the speed of trading of the high type, hence the incentive compatibility constraint (25) is binding. Substituting the trading rate in equations (14) and (17) we get an ordinary differential equation for $V(x, \bar{\theta})$ and $p(x, \bar{\theta})$. We can guess and verify that the equilibrium trading strategy is given by $q(x, \bar{\theta}) = -q(\bar{\theta})x$, the price is $p(x, \bar{\theta}) = \mu/r + p(\bar{\theta})x$, and the value function of the high type is $V(x, \bar{\theta}) = (\mu/r)x + \frac{1}{2}C(\bar{\theta})x^2$ for some constants $p(\bar{\theta}), C(\bar{\theta})$ that we provide next.

Proposition 2 (Large Inventory Costs). *Assume that the inventory cost γ satisfies*

$$\underline{\theta}^2 + \frac{\lambda_H}{r + \lambda_L + \lambda_H}(\bar{\theta}^2 - \underline{\theta}^2) < \frac{\gamma}{\phi} < 2\bar{\theta}^2 + \underline{\theta}^2. \quad (18)$$

Then the least costly separating equilibrium is as follows:

- *The low type value function is $V(x, \underline{\theta}) = (\mu/r)x$ and the high type value function is $V(x, \bar{\theta}) = (\mu/r)x + \frac{1}{2}C(\bar{\theta})x^2$, where the coefficient $C(\bar{\theta})$ corresponds to the unique positive root of the quadratic equation*

$$\begin{aligned} \lambda_H(r + \lambda_L + \lambda_H)C(\bar{\theta})^2 - \left[\lambda_H(\phi(2\bar{\theta}^2 + \bar{\theta}^2) - \gamma) - (r + \lambda_L)(\gamma + \phi(2\bar{\theta}^2 - \underline{\theta}^2)) \right] C(\bar{\theta}) \\ - \phi(\bar{\theta}^2 - \underline{\theta}^2)(\phi(2\bar{\theta}^2 + \underline{\theta}^2) - \gamma) = 0. \end{aligned}$$

- *The low type blockholder liquidates his holdings x_0 immediately at $t = 0$ and holds no shares thereafter.*
- *Type $\bar{\theta}$ trades at a rate*

$$q(x, \bar{\theta}) = -\frac{(\gamma + \lambda_H C(\bar{\theta}) - \phi \underline{\theta}^2)(r + \lambda_L)}{\phi(2\bar{\theta}^2 + \underline{\theta}^2) - \gamma - \lambda_H C(\bar{\theta})} x.$$

- The equilibrium price is

$$p(x, q) = \begin{cases} \frac{\mu}{r} & \text{if } q < q(x, \bar{\theta}) \\ \frac{\mu}{r} + \frac{\phi(2\bar{\theta}^2 + \theta^2) - \gamma - \lambda_H C(\bar{\theta})}{2(r + \lambda_L)} x & \text{if } q \in [q(x, \bar{\theta}), 0] \\ \frac{\mu}{r} + C(\bar{\theta})x & \text{if } q > 0 \end{cases}$$

Several comments are in order. When γ is large, there are decreasing returns to scale in x for the low type, which leads to the classic form of the Coase conjecture whereby the blockholder sells immediately his shares at a competitive price (recall, we require $\frac{\gamma}{\phi} < 2\bar{\theta}^2 + \theta^2$ so at least the high type does not sell immediately. Otherwise, the equilibrium would be trivial: both types would sell immediately, and the price would be $p(x, \theta) = \mu/r$).

Even though the firm cash flow is still convex in x , this convexity is offset by the convexity of inventory costs, leading to concave payoffs for the low type. Still, as long as $\frac{\gamma}{\phi} < 2\bar{\theta}^2 + \theta^2$ the payoff of the high type is convex, and he must sell slowly. His trading rate is limited by his price impact. That is even though $V_x(x, \theta) < p(x, \theta)$ the blockholder sells smoothly, reflecting a concern that the price would drop significantly if he traded faster because the market would interpret that as indicating low productivity.

It is worth noting that an increase in the inventory cost parameter γ has a large positive effect on the payoff of the high type, as we transition from an equilibrium with smooth-trading (as in Proposition 1) to an equilibrium with immediate trading by the low type (as in Proposition 2). This effect is illustrated in Figure 3. This positive effect arises because as the low type exits the market by selling his block, the high type is able to sell shares without triggering a strong drop in the stock price. On some level, the immediate exit of the low type, increases the liquidity facing the high type which explains why he benefits from greater inventory costs. On the other hand, this transition leads to a sharp decline in $p(x, \theta)$ caused by the higher speed of selling and the associated decrease in monitoring.

3.3 Productivity Shocks vs Liquidity Shocks

In our model, the source of asymmetric information is the blockholder's productivity. In practice, the blockholder might be privately informed about other relevant variables. For example, it is quite natural to consider the case in which the blockholder source of private

information is the inventory cost γ . In this case, shocks to γ could be interpreted as privately observed liquidity shocks. Our model can be easily adapted to consider this case. After substituting the effort policy (4) in the HJB equation (3) we get

$$rV(x, \theta) = \max_q \mu x + \frac{1}{2} (\phi\theta^2 - \gamma) x^2 + q(V_x(x, \theta) - p(x, q)) + \lambda_{\theta'} (V(x, \theta') - V(x, \theta)).$$

The relevant source of asymmetric information is captured by the reduced-form parameter $\nu \equiv \phi\theta^2 - \gamma$. Thus, we can re-interpret a negative productivity shock in our model as a liquidity shock that increases the holding cost γ . This specification of liquidity shocks differs from the linear specification commonly used in dynamic trading models. Unlike in linear models, the mean reversion of holdings is state dependent. As we will see in Section 4, such nonlinearity has important implications for the dynamic of holdings.

3.4 Equilibrium with Observable Shocks

To understand the consequences of information asymmetry and illiquidity, here we consider a benchmark with symmetric information where the productivity shocks on θ_t are observable.

Consider the case when θ_t is publicly observed. The payoff of the blockholder continues to be characterized by equation (8). The trading rate $q(x, \theta)$ satisfies the following HJB equations:

$$\begin{aligned} rp(x, \bar{\theta}) &= \mu + \bar{\theta}a(x, \bar{\theta}) + q(x, \bar{\theta})p_x(x, \bar{\theta}) + \lambda_L (p(x, \underline{\theta}) - p(x, \bar{\theta})) \\ rp(x, \underline{\theta}) &= \mu + \underline{\theta}a(x, \underline{\theta}) + q(x, \underline{\theta})p_x(x, \underline{\theta}) + \lambda_H (p(x, \bar{\theta}) - p(x, \underline{\theta})). \end{aligned}$$

As in the asymmetric information case, the blockholder trades at a competitive price and is always indifferent whether to trade, thus

$$p(x, \theta) = V_x(x, \theta) = \frac{\mu}{r} + C(\theta)x,$$

and $a(x, \theta) = \phi\theta x$. The lack of commitment of the blockholder, makes the blockholder unable to exploit his market power. Also, he is charged an implicit tax/subsidy whenever he trades because the market anticipates that any change in blockholder's ownership will affect the blockholder incentive to monitor the firm thereafter.

Substituting the above condition in the HJB for price, yields the equilibrium trading strategy under public information:

$$\begin{aligned} q(x, \bar{\theta}) &= (rC(\bar{\theta}) + \lambda_L(C(\bar{\theta}) - C(\underline{\theta})) - \phi\bar{\theta}^2) \frac{x}{C(\bar{\theta})} \\ q(x, \underline{\theta}) &= (rC(\underline{\theta}) - \lambda_H(C(\bar{\theta}) - C(\underline{\theta})) - \phi\underline{\theta}^2) \frac{x}{C(\underline{\theta})} \end{aligned}$$

where $C(\bar{\theta})$ and $C(\underline{\theta})$ are the same as in the case with asymmetric information (see equation (13)).

Proposition 3. (*Symmetric Information*) Suppose productivity shocks are observable. If condition 11 is satisfied, then the payoff is $V(x, \theta) = (\mu/r)x + \frac{1}{2}C(\theta)x^2$, where $C(\theta)$ is given in equation (13). The blockholder's trading rate is given by

$$q^o(x, \theta) = -\frac{\gamma}{C(\theta)}x \tag{19}$$

and the stock price is $p^o(x, \theta) = \mu/r + C(\theta)x$. If 11 is violated and $\phi\bar{\theta}^2 > \gamma$, then $V(x, \underline{\theta}) = (\mu/r)x$ and $V(x, \bar{\theta}) = (\mu/r)x + \frac{1}{2}C(\bar{\theta})x^2$, where

$$C(\bar{\theta}) = \frac{\phi\bar{\theta}^2 - \gamma}{r + \lambda_L}.$$

The low type liquidates immediately at a price $p^o(x, \underline{\theta}) = \mu/r$, while the high type trades at a rate given by (19) and price $p^o(x, \bar{\theta}) = \mu/r + C(\bar{\theta})x$. If $\phi\bar{\theta}^2 < \gamma$, then both types liquidate their positions immediately at a price $p^o(x, \theta) = \mu/r$.

The blockholder sells until his holdings are fully depleted, $x = 0$. Naturally, the blockholder sells faster in the low state and when his block x is larger. A higher productivity, does not change the long-run target but it does slow-down the blockholder's selling towards his target.

Unlike under asymmetric information, the price does not depend on the order flow q which is no longer informationally relevant. However, similar to the asymmetric information case, the price depends on block size x which affects the blockholder's incentives to exert effort.

4 The Impact of Asymmetric Information

Having solved for the public information benchmark, we can now analyze the impact of private information on trading, prices, and welfare.

We start looking at the impact that asymmetric information has on the dynamics of ownership. The empirical literature finds that nonfinancial blockholders, such as individuals, corporations and strategic investors, tend to have larger and longer-lived block positions compared with generic financial blockholders, such as mutual funds ([Hadlock and Schwartz-Ziv, 2019](#)). Our model can shed light on this phenomenon. We argue that the size of blocks and the duration of them depends critically on the information environment that the blockholder faces as well as on individual blockholder characteristics (for example, inventory and effort costs). For example, [Hadlock and Schwartz-Ziv \(2019\)](#) find substantial differences in the median size of block positions, with a high of 13.0% for corporate blockholders and a low of 7.1% for generic financial blockholders. Moreover, blockholder positions tend to be moderately durable, with an implied expected durations around 4 years for nonfinancial blocks and around 3 years for financial blocks (notice that they find that the variation is richer when considered at a less aggregated level).

In the next corollary, we summarize the implications of the model regarding trading activity and share prices.

Corollary 1 (Pricing and Trading).

If condition [\(11\)](#) is satisfied, so the equilibrium entails smooth trading by both types, then:

- *Information asymmetry reduces the speed of trading in the high productivity state, and increases the speed of trading in the low productivity state.*
- *Asymmetric information increases the stock price in the high productivity state, and has no impact on the stock price in the low productivity state.*

On the other hand, if condition [\(18\)](#) is satisfied, then:

- *Asymmetric information increases the stock price in the high productivity state, and has no impact on the stock price in the low productivity state.*

- *Asymmetric information decreases the speed of trading in the high productivity state, and has no impact on the speed of trading in the low productivity state.*¹³

Under asymmetric information, the blockholder's order flow has price impact because of signaling effects. It would be natural to conjecture that asymmetric information slows down trading, and this would be consistent with previous work considering dynamic signaling models (Daley and Green, 2012; Admati and Perry, 1987). However, our analysis shows that this is not necessarily the case (when inventory costs are low). Perhaps surprisingly, in the low state, the blockholder selling speed actually increases under asymmetric information. Moreover, this result differs from what we would obtain in a static signaling setting such as Leland and Pyle (1977). The key aspect of our model behind this result is that types are not fully persistent, so changes in the incentives of the high type affect the incentives for the low type – let us stress that while the previous observation is immediate in a pooling equilibrium it is far from obvious in a separation equilibrium. Suppose that in equilibrium the trading strategy of the low type is unaffected by asymmetric information. If this were the case, the price in the low state would necessarily be higher with asymmetric information (because blockholdings would be larger for any realization of the shocks), and this would generate incentives for the low type to sell even faster, contradicting our initial assumption that the trading strategy is unaffected. Thus, in equilibrium, the low type must sell at a higher rate to offset the reduction in trading in the high state.

Given the heterogenous impact of asymmetric information – it reduces trading in the high state but increases it in the low state – its overall impact on expected blockholdings is not obvious. As the numerical example in Figure 2 illustrates, the expected block in the observable and unobservable case can cross if the initial state is low.¹⁴ As the figure shows, the expected block may be lower under asymmetric information when the initial state is low. In the low state, it is always the case that the expected block with asymmetric information

¹³For the statement regarding the price, notice that in the low productivity state the price is μ/r with and without asymmetric information. With asymmetric information, the price in the high state is $p(x, \bar{\theta}) = (\mu/r)x + p(\bar{\theta})x$ for some coefficient $p(\bar{\theta}) > C(\bar{\theta})$. The conclusion follows from the fact that $C(\bar{\theta})$ is higher in the case with asymmetric information. For the trading speed we have that, because the price is higher with asymmetric information in the high state, and it is unaffected in the low state, it follows from the asset pricing equation (17) that $q(x, \bar{\theta})p_x(x, \bar{\theta})$ is higher with asymmetric information. As $q(x, \bar{\theta}) < 0$, and $p_x(x, \bar{\theta})$ is positive and larger with asymmetric information, it must be the case that $q(x, \bar{\theta})$ is also higher with asymmetric information (that is, its absolute value is smaller).

¹⁴If we denote the drift of x_t in state θ by $\alpha(\theta) \equiv -q(x, \theta)/x$, we get that the expected blockholdings at

is below that with symmetric information for small t . The blockholder sells faster with asymmetric information and early transitions to the high state are unlikely. As times goes by, transitions to the high state become more likely and the expected trading rate decreases. This might lead that for large values of t , the expected block can be larger with asymmetric information. On the other hand, starting from the high state, the lower trading rate with asymmetric information dominates, so expected block-holdings are higher. So the positive impact of asymmetric information on the stock price established in Corollary 1, does not come merely from a uniform slow down in trading rates, but more precisely from the slow down in trading rates arising at high productivity levels.

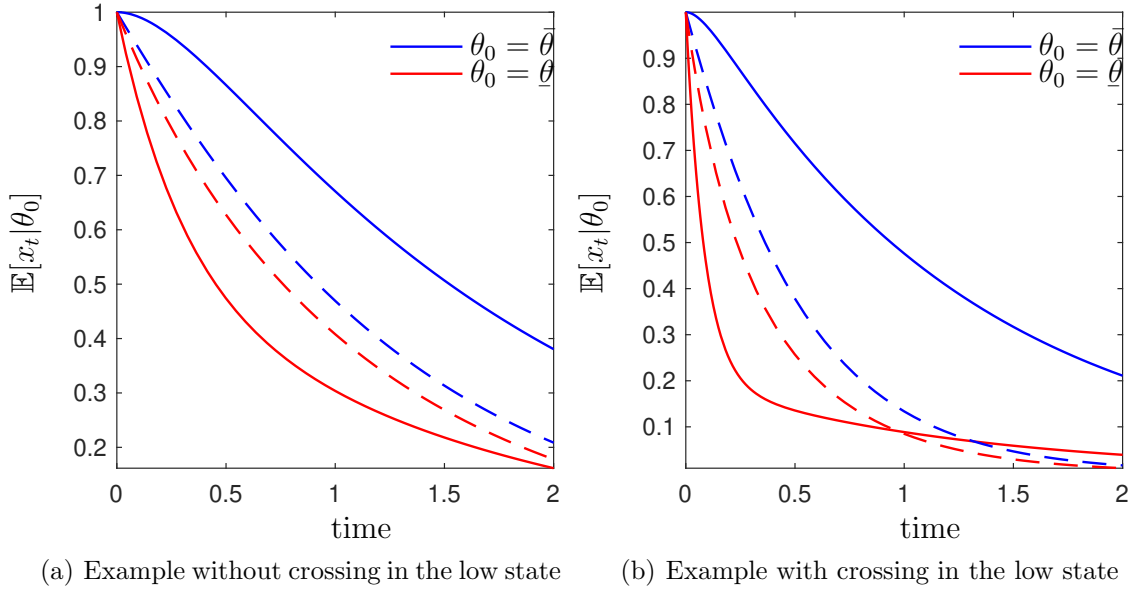


Figure 2: Expected path of holdings conditional on θ_0 and $x_0 = 1$. The solid line indicates the expected path with asymmetric information, while the dotted line indicates the expected path when θ_t is observable. Parameters: $\mu = 1$, $r = 0.05$, $\lambda_L = 1$, $\bar{\theta} = 1.25$, $\underline{\theta} = 0.75$. In figure (a), $\gamma = 10$ and $\lambda_H = 1$. In figure (b), $\gamma = 10$ and $\lambda_H = 1$

time t conditional on θ_0 is given by

$$\begin{pmatrix} \mathbb{E}[x_t | \bar{\theta}] \\ \mathbb{E}[x_t | \underline{\theta}] \end{pmatrix} = e^{-At} \begin{pmatrix} 1 \\ 1 \end{pmatrix} x_0,$$

where e^{-At} is the matrix exponential of

$$A = \begin{pmatrix} \alpha(\bar{\theta}) + \lambda_L & -\lambda_L \\ -\lambda_H & \alpha(\underline{\theta}) + \lambda_H \end{pmatrix}.$$

Given the ambiguity of the the effect of asymmetric information on trading, we would expect its impact on price to be ambiguous as well. However, we can show that the lower trading rate in the high state dominates, so asymmetric information does increase the stock price. The fact that the blockholder slows down selling in the high state, leads to more monitoring, more persistent cash flows, and ultimately to a higher stock price because monitoring is stronger when it is most effective.

Next, we turn our attention to the impact of asymmetric information on blockholder payoffs. We have seen that by reducing trading in high productivity states, asymmetric information leads to an increase in prices (in the most productive states), and so benefits shareholders. Now, because retention is costly, it is not clear that this would benefit the blockholder. In our model, the impact of asymmetric information on payoffs depends on the level of the inventory cost γ . On the one hand, when inventory costs are sufficiently low, the equilibrium payoffs with and without asymmetric information are the same. On the other hand, when inventory costs are high, asymmetric information increases the blockholder payoffs of the high type (and as usual, does not affect the payoff of the low type).

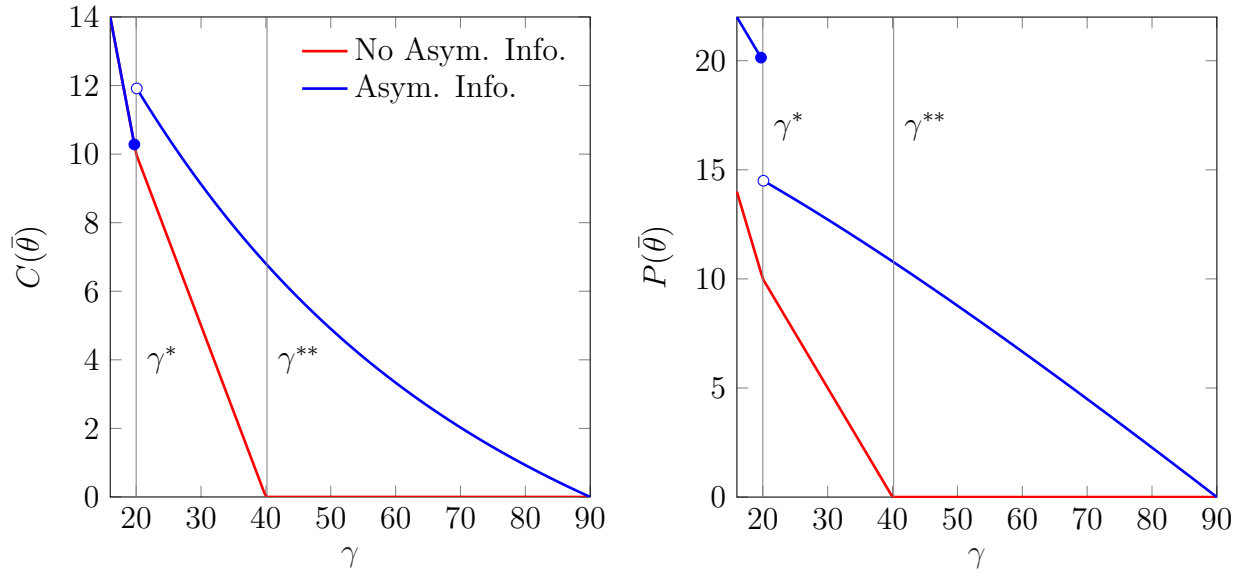
Corollary 2 (Blockholder's Payoff).

- *If condition (11) is satisfied, then the blockholder payoff is the same with and without asymmetric information.*
- *On the other hand, if condition (18) is satisfied, the equilibrium blockholder payoff in the high state is higher with asymmetric information, while the equilibrium payoff in the low state is unaffected by asymmetric information.*¹⁵

This result shows that information asymmetry not only increases the stock price but also boosts the blockholder payoff, in stark contrast with static settings or settings without effort. This leads us to the following corollary.

Corollary 3 (Welfare). *Asymmetric information leads to a Pareto improvement, relative to the case with symmetric information. Small investors are always better-off when the*

¹⁵The fact that payoffs are the same in the smooth trading case follows directly from Propositions 1 and 3. In the case where condition (18) is satisfied, the equilibrium payoff of the high type is $V(x, \bar{\theta}) = (\mu/r)x + \frac{1}{2}\bar{C}x^2$, where $\bar{C} = (\phi\bar{\theta}^2 - \gamma + 2\bar{q}(p(\bar{\theta}) - C(\bar{\theta}))) / (r + \lambda_L)$, $\bar{q} > 0$ and $p(\bar{\theta}) > C(\bar{\theta})$. Hence, $C(\bar{\theta}) > (\phi\bar{\theta}^2 - \gamma) / (r + \lambda_L)$ so it follows from Propositions 2 and 3 that the continuation payoff with asymmetric information is higher than the payoff without asymmetric information.



(a) Coefficient $C(\bar{\theta})$ of blockholder's value function. (b) Coefficient $P(\bar{\theta})$ of price function $p(x, \bar{\theta}) = \mu/r + P(\bar{\theta})x$.

Figure 3: Impact of asymmetric information on blockholder's payoff and prices. The blue line corresponds to the equilibrium with information asymmetries while the red line corresponds to the case with symmetric information. γ^* corresponds to inventory cost that makes condition (11) hold with equality, while $\gamma^{**} = \phi\bar{\theta}^2$. Parameters: $\mu = 1$, $r = 1$, $\phi = 10$, $\lambda_L = 1$, $\lambda_H = 1$, $\bar{\theta} = 2$, $\underline{\theta} = 1$.

blockholder is privately informed about θ_t . Similarly, the blockholder's payoff is weakly higher with asymmetric information.

Again, two cases must be distinguished here. First, when inventory costs are low, the blockholder payoff is invariant to the information environment (i.e., private vs public information about θ). When inventory costs are large, the blockholder payoff is strictly higher under asymmetric information. This is in stark contrast with a static setting (Leland and Pyle (1977)) where signaling incentives would impose a deadweight cost on the sender, forcing him to hold his shares, to signal his productivity despite the existence of gains from trade. In a static setting, this signaling inefficiency operates in conjunction with the sender's market power. But in a dynamic setting the blockholder's size does not necessarily translates into market power, due to the Coase conjecture: the blockholder's lack of commitment means that no matter the information environment he always trades at a competitive price that equals his own marginal valuation. As a result, signaling incentives do not affect the blockholder's payoff (when γ is low).

Surprisingly, our analysis suggests that contrary to conventional wisdom, small uninformed investors also benefit from the blockholder having access to private information. As mentioned above, this is due to the blockholder holding his shares for longer, particularly when he is most productive, which increases the duration of the blockholder's effort, and boosts the firm's productivity and the expected cash flows. Therefore, the value of small investors holdings goes up when the blockholder has access to private information.

5 Application: Initial IPO Allocation

The previous analysis assumes that the blockholder's initial block x_0 is exogenous. However, in practice, x_0 may arise endogenously when an entrepreneur (henceforth, the issuer) sells the stock to investors via an Initial Public Offering (IPO). As an application of our model, we consider the initial allocation of shares in an IPO, and study the impact that trading in the secondary market has on the optimal IPO design. In particular, we endogenize the initial allocation x_0 as the outcome of an optimal IPO whereby the issuer sells the stock to two types of investors, a privately informed blockholder, and a continuum of uninformed investors. As the small investors, the entrepreneur is uncertain about θ_0 and his belief is $\Pr(\theta_0 = \bar{\theta}) = \pi$. This corresponds to the standard screening problem by a monopolist study by [Mussa and Rosen \(1978\)](#)

We assume that the issuer has commitment power as a mechanism designer. The IPO is an optimal mechanism specifying *i*) the fraction of shares initially allocated to the blockholder $x_0(\theta)$, and *ii*) the transfer $T(\theta)$ that the blockholder must pay in exchange for the block.

Formally, the issuer solves the following program:

$$\begin{aligned} & \max_{\{T(\theta), x_0(\theta)\}_{\theta \in \{\underline{\theta}, \bar{\theta}\}}} E^\theta[T(\theta) + (1 - x_0(\theta))p(x_0(\theta), \theta)] \\ & \text{subject to} \\ & V(x_0(\theta), \theta) - T(\theta) \geq V(x_0(\theta'), \theta) - T(\theta'), \quad \forall \theta, \theta' \in \{\underline{\theta}, \bar{\theta}\} \\ & V(x_0(\theta), \theta) - T(\theta) \geq V(0, \theta), \quad \forall \theta \in \{\underline{\theta}, \bar{\theta}\} \end{aligned} \tag{20}$$

The issuer allocates a block $x_0(\theta)$ to the blockholder in exchange for a monetary transfer $T(\theta)$. The remainder $1 - x_0(\theta)$ is sold in the open market at a price $p(x_0, \theta)$ determined competitively (see equation 1).

The entrepreneur must satisfy participation constraints and incentive compatibility constraints that are type dependent as they depend on the subsequent dynamic trading payoffs. Consistent with practice, we can think of this mechanism as entailing two stages: a first stage, similar to a pre-IPO placement,¹⁶ where the issuer negotiates privately with the blockholder

¹⁶A pre-initial public offering placement is a private sale of large blocks of shares before a stock is listed on a public exchange. The buyers are typically private equity firms, hedge funds, and other institutions willing to buy large stakes in the firm. Due to the size of the investments being made and the risks involved, the buyers in a pre-IPO placement usually get a discount from the price stated in the prospectus for the IPO.

and, a second stage, or the actual IPO, where small investors bid for the remainder shares $1 - x_0$.

Observe that implicitly we allow for price discrimination between the blockholder and the small investors, because the average price paid by the blockholder is $\frac{T(\theta)}{x_0(\theta)}$ may be different from the equilibrium $p(x, \theta)$. Arguably, this is the most realistic case, and qualitative predictions do not hinge on the possibility of discrimination¹⁷.

5.1 Allocation without Ex-Post Trading

Before studying the case in which the blockholder can sell his shares after the IPO, we consider the case where this is not permitted. This represents a static situation where the owner can prevent the blockholder from “flipping” after the IPO and selling his shares. Specifically, suppose the seller offers a mechanism $(T(\theta), x_0(\theta))$ and enforces $x_t = x_0$ for all t . This benchmark is useful because it captures a static problem, and allows us to isolate the effect of dynamic trading on the IPO design.

The expected payoff of the blockholder given an allocation x is $V(x, \theta)$ (see equation 12)). Notice that it follows from our previous analysis that the blockholder’s value $V(x, \theta)$ is unaffected by existence of a secondary market. This follows because due to lack of commitment the possibility of trading does not provide any benefit to the blockholder. In other words, the inability to trade post IPO does not affect the blockholder’s payoffs. However, this does not mean that the existence of an aftermarket is irrelevant. The restriction does affect the equilibrium market price $p(x, \theta)$ because it makes the blockholder’s effort more persistent.

The price $p(x, \theta)$ can be obtained from the asset pricing equation (7) by imposing the restriction $q(x, \theta) = 0$, which yields:

$$\begin{aligned} p(x, \bar{\theta}) &= \frac{\mu}{r} + \frac{(r + \lambda_H) \bar{\theta}^2 + \lambda_L \underline{\theta}^2}{r(r + \lambda_H + \lambda_L)} \phi x = \frac{\mu}{r} + \left(C(\bar{\theta}) + \frac{\gamma}{r} \right) x \\ p(x, \underline{\theta}) &= \frac{\mu}{r} + \frac{\lambda_H \bar{\theta}^2 + (r + \lambda_L) \underline{\theta}^2}{r(r + \lambda_H + \lambda_L)} \phi x = \frac{\mu}{r} + \left(C(\underline{\theta}) + \frac{\gamma}{r} \right) x. \end{aligned}$$

Naturally, the blockholder’s inability to trade makes the price more sensitive to the block x , simply because the cash flows are now more persistent.

¹⁷The analysis without price discrimination is available from the authors upon request.

Given the equilibrium prices, we can consider the optimal IPO. This is a standard mechanism design problem with quasi-linear preferences, except that the valuations $V(x, \theta)$ are type dependent and endogenous (as in an auction with resale opportunities). The blockholder's valuation satisfy the standard single crossing condition $V_{x\theta}(x, \theta) > 0$, which implies that the optimal allocation $x_0(\theta)$ should be monotone in θ to be incentive compatible.

To solve this problem, we assume and verify that the low type's participation constraint and the high type's incentive compatibility constraints bind. The objective is concave in $x_0(\theta)$, and the optimal allocation is characterized by the first order conditions. The following proposition characterizes the solution.

Proposition 4 (Static IPO Allocation). *The optimal allocation in a static setting without post-IPO trading satisfies:*

$$\begin{aligned} x(\underline{\theta}) &= \frac{C(\underline{\theta}) + \frac{\gamma}{r}}{C(\underline{\theta}) + \frac{2\gamma}{r} + \frac{\pi}{1-\pi} (C(\bar{\theta}) - C(\underline{\theta}))} \\ x(\bar{\theta}) &= \frac{C(\bar{\theta}) + \frac{\gamma}{r}}{C(\bar{\theta}) + \frac{2\gamma}{r}}. \end{aligned}$$

The solution has a familiar flavor. There is no-distortion at the top: the allocation of the high type $\bar{\theta}$ coincides with that arising under symmetric information, and maximizes the social surplus, $V(x, \theta) + (1 - x)p(x, \theta)$. By contrast, the allocation of the low type $\underline{\theta}$ is distorted downwards, as a means to reduce the information rent earned by $\bar{\theta}$, which is proportional to $x_0(\bar{\theta})$. The size of the distortion to $x_0(\underline{\theta})$ is proportional to the probability of the high type π .

From a social point of view, the IPO is inefficient because the blockholder's ownership is smaller than that required to maximize the social surplus, given the tension between efficiency and rent extraction. We can now proceed to analyze the dynamic case with ex-post trading.

5.2 Optimal IPO with Ex-Post Trading

Having identified the allocation in the absence of trading, we can now study the optimal IPO mechanism under asymmetric information and post-IPO trading. In this case incentive compatibility constraints are shaped in equilibrium by the blockholder's own post-IPO trading incentives. As we shall see, unlike in static models, the combination of post IPO trading

and information asymmetry may force the owner to offer a pooling allocation, although as we have seen, this would never happens in a static setting.

The design program is characterized by the problem in (20) but now the blockholder's payoff corresponds to the continuation payoff in Proposition 1. The following proposition describes the optimal mechanism.

Proposition 5. *Suppose that the inventory cost satisfies condition (11). The optimal allocation at time zero $(x_0(\theta), T(\theta))$ is as follows:*

1. *If*

$$\frac{\pi}{1-\pi} \geq \frac{\gamma C(\underline{\theta})}{(\phi \bar{\theta}^2 + \lambda_L C(\underline{\theta})) (C(\bar{\theta}) - C(\underline{\theta}))} \quad (21)$$

then the allocation at time zero is

$$\begin{aligned} x_0(\underline{\theta}) &= \frac{C(\underline{\theta})}{C(\underline{\theta}) + \frac{\pi}{1-\pi} (C(\bar{\theta}) - C(\underline{\theta}))} \\ x_0(\bar{\theta}) &= \frac{\phi \bar{\theta}^2 + \lambda_L C(\underline{\theta})}{\phi \bar{\theta}^2 + \lambda_L C(\underline{\theta}) + \gamma}, \end{aligned}$$

and the blockholder payment is

$$\begin{aligned} T(\underline{\theta}) &= V(x_0(\underline{\theta}), \underline{\theta}) \\ T(\bar{\theta}) &= V(x_0(\bar{\theta}), \bar{\theta}) + V(x_0(\underline{\theta}), \underline{\theta}) - V(x_0(\underline{\theta}), \bar{\theta}). \end{aligned}$$

2. *If condition (21) is not satisfied, then the optimal allocation at time zero is pooling, that is, $x_0(\bar{\theta}) = x_0(\underline{\theta}) = x_0$, and the blockholder payment is $T(\bar{\theta}) = T(\underline{\theta}) = V(x_0, \underline{\theta})$, where x_0 is*

$$x_0 = \min \left\{ \frac{\pi(\phi \bar{\theta}^2 + \lambda_L C(\underline{\theta})) + (1-\pi)(r + \lambda_L)C(\underline{\theta})}{2\pi(\phi \bar{\theta}^2 + \lambda_L C(\underline{\theta})) + (1-2\pi)(r + \lambda_L)C(\underline{\theta})}, 1 \right\}.$$

This result shows that the possibility of post IPO trading critically affects the allocation of the IPO. In the absence of post-IPO trading, the mechanism always features a separating menu, with the “no distortion at the top” property. The high type receives a larger allocation and a lower price per-share. The lower type's allocation is smaller and distorted downwards,

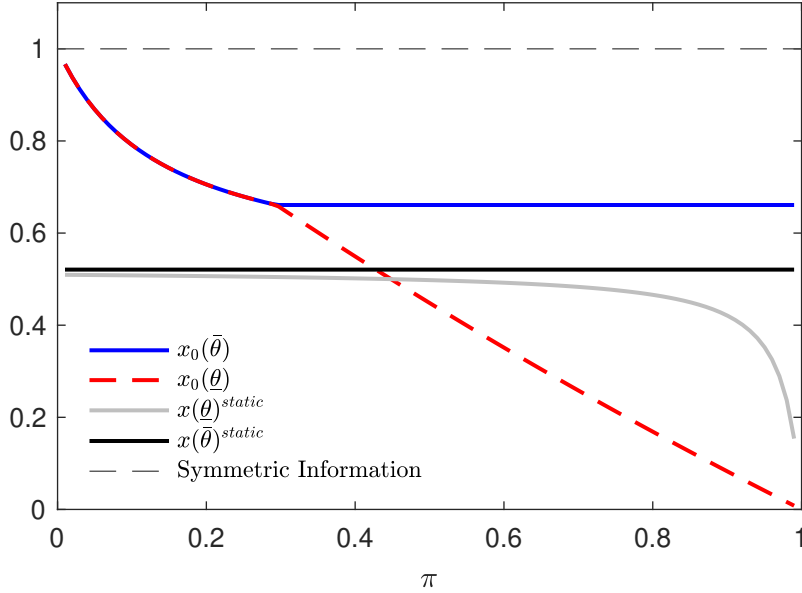


Figure 4: Optimal IPO allocation. $x_0(\theta)$ indicates the optimal allocation in the case with asymmetric information (both at time zero and afterwards), while $x(\theta)^{static}$ corresponds to the allocation in the absence of trading by the blockholders after the IPO. Parameters: $\mu = 1$, $\gamma = 10$, $\phi = 10$, $\lambda_H = 1$, $\lambda_L = 1$, $\bar{\theta} = 1.25$, $\underline{\theta} = 0.75$.

relative to the symmetric information case, in order to reduce the informational rents earned by the high type. Since in the static setting, the blockholder cannot sell, the monotonicity IC requirement is never violated in the relaxed program (i.e, the program that ignores the monotonicity constraint) and the allocation is always separating.

But with post-IPO trading, a monotone allocation may not be optimal, yet it is still required for incentive compatibility. The issue arises because the high type faces a particularly illiquid aftermarket in which selling is too costly (given his price impact) and where in equilibrium the price $p(x, \bar{\theta})$ is larger than his own marginal valuation $V_x(x, \bar{\theta})$. Hence, from the seller's perspective it is relatively costly to allocate a large x_0 to the high type, because by doing so, the seller incurs an opportunity cost, as he cannot sell these shares to small investors, who are willing to pay a higher price. This effect moderates the seller's incentive to allocate a very large block to the high type. However, this incentive is not present for the low type. The low type faces a competitive price ex-post, whereby $p(x, \theta) = V_x(x, \theta)$. In other words, on the margin, the willingness to pay by the market and by the blockholder are the same when productivity is low $\theta = \underline{\theta}$. Hence, in this case case, the issuer has an incentive to assign a large block to the blockholder to stimulate his monitoring effort and thus cash in

the effect on the firm's cash flows (We can see this by looking at the maximization problem when we omit the incentive compatibility constraint.)

To provide some intuition, consider the following thought experiment. Suppose that θ_0 is known, but that after time zero its evolution is privately observed by the blockholder. The traditional intuition would tell that the optimal allocation would assign more shares to the high type. This would obviously be the case in the static model without post-IPO trading. However, this is not the case in the presence of post-IPO trading with asymmetric information in the secondary market. This is illustrated in Figure 5. The solid black line indicates the participation constraint while the thin solid line indicates the iso-profit curve. The unconstrained optimum is indicated by points A and B . Because the low type sells his shares very quickly after the IPO, it is optimal to allocate the maximum possible of shares to the blockholder (A). However, because the high type does not sell immediately, there is a tradeoff between inventory cost and productivity, so the number of shares assigned is strictly lower than the maximum (B).

Of course, this is not the end of the story. Asymmetric information introduces a countervailing incentive to reduce the allocation of the low type $x(\underline{\theta})$ as a means to reduce the information rent earned by the high type. In the right panel of Figure 5, when the probability of the high type is large, the information rent effect dominates, and we get the traditional non-distortion at the top result. The optimal allocation for the high type, indicated by point C , is “efficient” and the high type's information rent is given by the vertical difference between B and C . Similarly the low type allocation, indicated by point D , is such the participation constraint of the low type is binding and the high type is indifferent between C and D . Crucially, in this case point D is to the left of point C so the monotonicity constraint is satisfied. In sum, when π is large, minimizing the information rents of the high type becomes the seller's dominant concern. The allocation of the low type is strongly distorted downwards and monotonicity ceases to be an issue so a standard separating equilibrium emerges.

The situation is different when the probability of the high type is low π , which is illustrated in the right panel in Figure 5. Since the information rent is unlikely to be paid, the distortion is not helpful, so the seller has an incentive to allocate a large block to the low type $\underline{\theta}$ and the monotonicity constraint is violated (D is to the right of C). If the seller were to offer the menu C and D , the low type blockholder would be better off choosing C rather than D (the allocation C is inside the shaded area indicating the allocations that are

preferred to D). As a consequence the seller finds it optimal to offer the pooling allocation E which is “inefficient” for both types.¹⁸

The previous distinction between the pooling and the separating cases have empirical implications for the volatility of prices following the IPO. First, in the situations in which the optimal allocation is pooling, we have that the pre-IPO allocation is less informative about the price of the shares in the secondary market. The IPO price, given by $p_0 = \pi p(x_0, \bar{\theta}) + (1 - \pi)p(x_0, \underline{\theta})$, immediately adjust once the blockholders starts to trade. Immediately after the IPO, the price jumps up or down depending on the trading behavior of the blockholder – notice that in practice there will be some delay before this adjustment occurs due to the presence of lock up periods. This generates significant price volatility in the secondary market following the IPO (or following the expiration of lock up periods when these are in place). This contrast with the static case in which the allocation is separating. The IPO price is $p_0 = p(x_0(\theta), \theta)$ in this case, so there is no price volatility in the secondary market following the IPO (trading by the low type blockholder is anticipated by the market). Thus, the previous analysis predicts that the volatility of prices following the IPO should be higher when π is low.

¹⁸In principle, when the monotonicity constraint is violated, it could be optimal to use a random mechanism (so the monotonicity constrain only holds in expectation). We show in the appendix that it is not the case, and that the optimal mechanism is always deterministic.

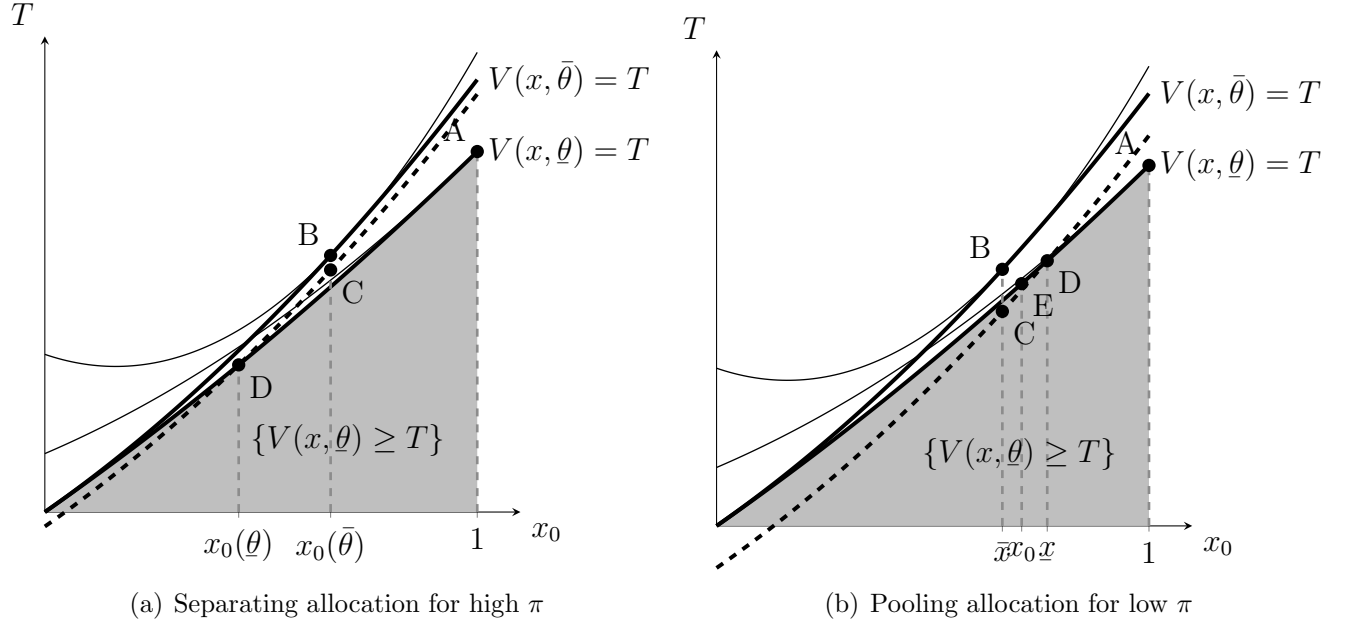


Figure 5: Optimal IPO allocation: optimistic vs pessimistic beliefs.

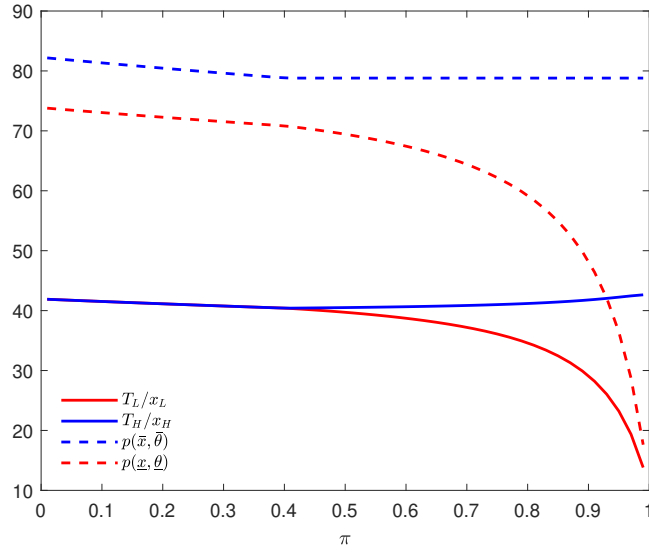


Figure 6: Mean price paid by blockholder vs small investors. Parameters: $\mu = 1$, $\gamma = 4$, $\phi = 10$, $\lambda_H = 1$, $\lambda_L = 1$, $\bar{\theta} = 1.25$, $\underline{\theta} = 0.75$.

6 Conclusions

This paper studies the monitoring and trading behavior of a blockholder with access to private information. After characterizing the dynamics for a given initial block, we endogenize the initial block size as the outcome of an optimal IPO.

Our analysis shows that when a blockholder has access to private information welfare improves. Asymmetric information leads the blockholder to hold their shares for longer, and monitor the firm more intensively, thus increasing the stock price. The lack of liquidity arising under asymmetric information, mitigates the blockholder’s commitment problem, and allows him to sell slowly and extract some monopoly rents, contrary to the symmetric information case.

We also study the optimal IPO allocation, and show that the blockholder post IPO trading, significantly modifies the IPO properties, and leads under some conditions to allocations that are insensitive to blockholder productivity and significant post-IPO volatility.

This paper has implications for the long-standing debate on the role of liquidity for blockholder activism (see e.g., [Norli et al. \(2014\)](#); [Edmans \(2009\)](#)). Our results suggest that information asymmetry reduces the liquidity facing the blockholder, but has desirable social effects, insofar as it leads to larger blocks and stronger monitoring (a similar point is made by [Vanasco \(2017\)](#)).

Our model has a number of limitations. For example, we assume that the blockholder’s order flow is perfectly observable and we focus on separating equilibria. Perfect observability of the order flow is a simplification, because in practice these data is available to investors with some delay. This assumption also leads to an equilibrium where cash flows are uninformative, conditional on the order flow. Extending our model to allow for noise trading, to obscure the blockholder’s order flow, is an interesting (and challenging) extension that we hope future research will address.

The observability of blockholder trading is also a policy question. The SEC requires that a blockholder discloses their stakes within 10 days of the purchase of more than 5% of the shares of a public company. This regulation, and the socially optimal level of the disclosure threshold is subject of an intense policy debate.¹⁹ Evaluating the effects of this regulation

¹⁹For example, The Economist notes that “Wachtell, Lipton, Rosen & Katz, the law firm that invented the poison pill, has also been seeking to make things harder still for activists by proposing a rule that anyone building a stake of 5% or more in a firm must disclose it within one day, not ten as now. So far the Securities

is also an open theory question.

We have ruled out the possibility that the blockholder takes value-destroying actions, as is often argued in the popular press. Indeed, critics often warn that blockholder activists exacerbate firms' short-termist tendencies (See e.g., "Let's do it my Way", *The Economist*, May 13, 2013). This possibility could be incorporated in our model by allowing the blockholder effort to have, at the same time, a negative impact on the firm's cash flows and a positive effect on the blockholder's payoff.

Finally, as another interesting extension one could consider the possibility of competition (or cooperation) among multiple blockholders with heterogenous beliefs to gain control of the firm and influence its corporate strategy (see e.g., [Hadlock and Schwartz-Ziv \(2019\)](#)).

and Exchange Commission is showing little interest. Indeed, its chairman, Mary Jo White, has argued that activists attempts to jog boards are not always a bad thing." See *Nasty Medicine*, *The Economist*, Jul 5th, 2014.

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Appendix

A Trading with Asymmetric Information

Proof of Lemma 1

Proof. First, we consider the trading problem of the high type

$$\max_q q(V_x(x, \bar{\theta}) - p(x, q)). \quad (22)$$

First, we establish that in any equilibria with smooth trading (that is, atomless) it must be that $p(x, q(x, \bar{\theta})) \geq V_x(x, \bar{\theta})$. Suppose that $V_x(x, \bar{\theta}) > p(x, q(x, \bar{\theta})) = p(x, \bar{\theta})$, then, as

$$p(x, q) = \Pr(\theta = \bar{\theta}|x, q)p(x, \bar{\theta}) + \Pr(\theta = \underline{\theta}|x, q)p(x, \underline{\theta}) \leq p(x, \bar{\theta}), \quad (23)$$

we have that for any $q > 0$, $q(V_x(x, \bar{\theta}) - p(x, q)) \geq q(V_x(x, \bar{\theta}) - p(x, \bar{\theta})) > 0$, and this implies that $\sup_{q \geq 0} \{q(V_x(x, \bar{\theta}) - p(x, q))\} = \infty$. Thus, in any atomless equilibria we must have that $V_x(x, \bar{\theta}) \leq p(x, \bar{\theta})$. By a similar argument, considering $q < 0$, we can conclude that $V_x(x, \underline{\theta}) \geq p(x, \underline{\theta})$. Thus, in any atomless equilibrium satisfying monotonicity, and for any $x > 0$, we have

$$p(x, \bar{\theta}) \geq V_x(x, \bar{\theta}) > V_x(x, \underline{\theta}) \geq p(x, \underline{\theta}) \quad (24)$$

Moreover, incentive compatibility requires that if $q(x, \bar{\theta}) > 0$ then $p(x, q(x, \bar{\theta})) = V_x(x, \bar{\theta})$ and $p(x, q) = p(x, \bar{\theta})$ for all $q > 0$. This follows from inequality (24) and the observation that, as the blockholder can always choose not to trade, it must be that $\max_{q \geq 0} \{q(V_x(x, \bar{\theta}) - p(x, q))\} \geq 0$. But then $q(x, \bar{\theta})$ is a solution of (22) only if $p(x, q) \geq p(x, \bar{\theta})$, so inequality (23) implies that $p(x, q) = p(x, \bar{\theta})$ for all $q > 0$. By a similar argument for the low type and $q < 0$, we can show that if $q(x, \underline{\theta}) < 0$, then $p(x, q) = p(x, \underline{\theta})$ for all $q < 0$.

The only remaining step in the proof is to show that, in any atomless equilibria satisfying condition 1, we have $q(x, \underline{\theta}) \leq 0 \leq q(x, \bar{\theta})$. First, we consider the case $q(x, \underline{\theta}) > 0$. Suppose that in equilibrium, for some $x > 0$ we have $q(x, \underline{\theta}) > 0$. Monotonicity together with the incentive compatibility constraint (6) require that $q(x, \bar{\theta}) > q(x, \underline{\theta})$. Using the inequality (24) we conclude that $q(x, \bar{\theta})(V_x(x, \bar{\theta}) - p(x, \bar{\theta})) = 0$. Thus, incentive compatibility requires that $q(x, \underline{\theta})(V_x(x, \bar{\theta}) - p(x, \underline{\theta})) \leq 0$, which implies that $V_x(x, \bar{\theta}) \leq p(x, \underline{\theta})$. However, this last

inequality contradicts inequality (24), so we can conclude that in a monotonic equilibrium $q(x, \underline{\theta}) \leq 0$. Using a similar argument, we can verify that in any monotonic equilibrium $q(x, \bar{\theta}) \geq 0$. Finally, suppose that for some $q' \in (0, q(x, \bar{\theta}))$ we have that the market off-equilibrium belief is $\beta = \Pr(\theta = \bar{\theta} | x, q') < 1$, then the equilibrium price must be $p(x, q') = \beta p(x, \bar{\theta}) + (1 - \beta)p(x, \underline{\theta}) < p(x, \bar{\theta}) = V_x(x, \bar{\theta})$. But then, the blockholder has a profitable deviation from $q(x, \bar{\theta})$ to q' . Thus, in equilibrium for any $q' \in (q(x, \bar{\theta}), 0)$ it must be the case that $\hat{\theta}(q, x) = \bar{\theta}$. Similarly, suppose that for some $q'' \in (0, q(x, \underline{\theta}))$, we have $\beta = \Pr(\theta = \bar{\theta} | x, q'') > 0$, then the equilibrium price must be $p(x, q'') = \beta p(x, \bar{\theta}) + (1 - \beta)p(x, \underline{\theta}) > p(x, \underline{\theta}) = V_x(x, \underline{\theta})$, which means that the low type blockholder has a profitable deviation from $q(x, \underline{\theta})$ to q'' . Thus, in equilibrium for any $q'' \in (q(x, \underline{\theta}), 0)$ it must be the case that $\hat{\theta}(q, x) = \underline{\theta}$. \square

Proof of Proposition 1

Proof. We conjecture a solution for $V(x, \theta)$ of the form $V(x, \theta) = B(\theta)x + \frac{1}{2}C(\theta)x^2$. Under this conjecture the optimal effort is

$$a(\theta, x) = \phi\theta x.$$

Substituting the HJB equation we get

$$\begin{aligned} rV(x, \bar{\theta}) &= \mu x + \frac{1}{2}\phi\bar{\theta}^2 x^2 - \frac{1}{2}\gamma x^2 - \lambda_L (V(x, \bar{\theta}) - V(x, \underline{\theta})) \\ rV(x, \underline{\theta}) &= \mu x + \frac{1}{2}\phi\underline{\theta}^2 x^2 - \frac{1}{2}\gamma x^2 - \lambda_H (V(x, \underline{\theta}) - V(x, \bar{\theta})) \end{aligned}$$

Substituting our conjecture

$$\begin{aligned} rB(\bar{\theta}) &= \mu + \lambda_L (B(\bar{\theta}) - B(\underline{\theta})) \\ rC(\bar{\theta}) &= \phi\bar{\theta}^2 - \gamma - \lambda_L (C(\bar{\theta}) - C(\underline{\theta})) \\ rB(\underline{\theta}) &= \mu + \lambda_H (B(\underline{\theta}) - B(\bar{\theta})) \\ rC(\underline{\theta}) &= \phi\underline{\theta}^2 - \gamma - \lambda_H (C(\underline{\theta}) - C(\bar{\theta})) \end{aligned}$$

From here we can solve for the coefficients $(B(\theta), C(\theta))$ to get

$$\begin{aligned} B(\bar{\theta}) &= B(\underline{\theta}) = \frac{\mu}{r} \\ rC(\underline{\theta}) &= \phi\underline{\theta}^2 - \gamma + \phi \frac{\lambda_H}{r + \lambda_L + \lambda_H} (\bar{\theta}^2 - \underline{\theta}^2) \\ rC(\bar{\theta}) &= \phi\bar{\theta}^2 - \gamma - \phi \frac{\lambda_L}{r + \lambda_L + \lambda_H} (\bar{\theta}^2 - \underline{\theta}^2) \end{aligned}$$

The value function $V(x, \theta) = (\mu/r)x + C(\theta)x^2/2$ is convex in x for all θ only if

$$\underline{\theta}^2 + \frac{\lambda_H}{r + \lambda_L + \lambda_H} (\bar{\theta}^2 - \underline{\theta}^2) \geq \frac{\gamma}{\phi}$$

Finally, substituting in equations (9) and (10) we get

$$\begin{aligned} p(x, \bar{\theta}) &= \frac{\mu}{r} + \frac{(\phi\bar{\theta}^2 + \lambda_L C(\underline{\theta}))x}{r + \lambda_L} \\ q(x, \underline{\theta}) &= - \left(1 + \frac{\lambda_H}{r + \lambda_L} \right) \frac{\gamma}{C(\underline{\theta})} x. \end{aligned}$$

□

Proof of Proposition 2

Proof. We conjecture a solution of the form

$$\begin{aligned} q(x, \bar{\theta}) &= -\bar{q}x \\ V(x, \bar{\theta}) &= \frac{\mu}{r}x + \frac{1}{2}\bar{C}x^2 \\ p(x, \bar{\theta}) &= \frac{\mu}{r} + \bar{p}x, \end{aligned}$$

for coefficients $(\bar{q}, \bar{C}, \bar{p})$ to be determined. Substituting in the IC constraint we get

$$q(x, \bar{\theta}) = -\frac{(\gamma - \phi\underline{\theta}^2) + \lambda_H\bar{C}}{2\bar{p}}x, \tag{25}$$

so

$$\bar{q} = \frac{\gamma - \phi\bar{\theta}^2 + \lambda_H\bar{C}}{2\bar{p}}.$$

The next step is to solve for \bar{p} . Substituting our conjecture in the equation for the price we get

$$\bar{p} = \frac{\phi\bar{\theta}^2}{r + \lambda_L + \bar{q}}$$

Finally, we use the HJB equation of the high type to get an equation for \bar{C} .

$$(r + \lambda_L + 2\bar{q})\bar{C} = (\phi\bar{\theta}^2 - \gamma) + 2\bar{q}\bar{p}$$

Substituting \bar{q} we get

$$\bar{C} = \frac{\phi(\bar{\theta}^2 - \underline{\theta}^2)}{r + \lambda_L - \lambda_H + 2\bar{q}}.$$

Summarizing, we get that $(\bar{q}, \bar{p}, \bar{C})$ solves

$$\begin{aligned}\bar{q} &= \frac{\gamma - \phi\bar{\theta}^2 + \lambda_H\bar{C}}{2\bar{p}} \\ \bar{p} &= \frac{\phi\bar{\theta}^2}{r + \lambda_L + \bar{q}} \\ \bar{C} &= \frac{\phi(\bar{\theta}^2 - \underline{\theta}^2)}{r + \lambda_L - \lambda_H + 2\bar{q}}.\end{aligned}$$

Moreover, the following conditions need to be satisfied: $\bar{C} < \bar{p}$, $\bar{C} > 0$, and $\bar{q} > 0$. Solving for \bar{p} and \bar{q} we get

$$\begin{aligned}\bar{q} &= \frac{(\gamma + \lambda_H\bar{C} - \phi\bar{\theta}^2)(r + \lambda_L)}{\phi(2\bar{\theta}^2 + \underline{\theta}^2) - \gamma - \lambda_H\bar{C}} \\ \bar{p} &= \frac{\phi(2\bar{\theta}^2 + \underline{\theta}^2) - \gamma - \lambda_H\bar{C}}{2(r + \lambda_L)}\end{aligned}$$

Substituting \bar{q} in the equation for \bar{C} , we get the following

$$\begin{aligned}\lambda_H(r + \lambda_L + \lambda_H)\bar{C}^2 - \left[\lambda_H(\phi(2\bar{\theta}^2 + \underline{\theta}^2) - \gamma) - (r + \lambda_L)(\gamma + \phi(2\bar{\theta}^2 - \underline{\theta}^2)) \right] \bar{C} \\ - \phi(\bar{\theta}^2 - \underline{\theta}^2)(\phi(2\bar{\theta}^2 + \underline{\theta}^2) - \gamma) = 0\end{aligned}$$

If $\phi(2\bar{\theta}^2 + \underline{\theta}^2) > \gamma$, then there exist a unique one positive root to the previous equation. The constants \bar{q} and \bar{p} are positive only if $\lambda_H \bar{C} < \phi(2\bar{\theta}^2 + \underline{\theta}^2) - \gamma$ (whenever the condition for smooth trading by the low type is violated, we have $\gamma/\phi > \underline{\theta}^2$). Evaluating the quadratic equation at $\bar{C} = (\phi(2\bar{\theta}^2 + \underline{\theta}^2) - \gamma)/\lambda_H$, we get that the left hand side is proportional to $\phi(2\bar{\theta}^2 + \underline{\theta}^2) - \gamma > 0$. Hence, $\bar{C} < (\phi(2\bar{\theta}^2 + \underline{\theta}^2) - \gamma)/\lambda_H$ so \bar{q} and \bar{p} are positive. Finally, the condition $\bar{p} > \bar{C}$ requires that

$$\bar{C} < \frac{\phi(2\bar{\theta}^2 + \underline{\theta}^2) - \gamma}{2(r + \lambda_L) + \lambda_H}$$

Substituting the right hand side in the quadratic equation for \bar{C} , we find that the previous condition is satisfied only if

$$(\phi(2\bar{\theta}^2 + \underline{\theta}^2) - \gamma) \times ((r + \lambda_L)(\phi\underline{\theta}^2 + \gamma) - \lambda_H(\phi\bar{\theta}^2 - \gamma)) > 0.$$

It is straightforward to verify that the second term is positive whenever condition (11) is violated; hence, the previous inequality is satisfied as long as

$$\frac{\gamma}{\phi} < 2\bar{\theta}^2 + \underline{\theta}^2.$$

□

B Optimal IPO Allocation

Proof of Proposition 5

Proof. Suppose that the low type's participation constraint and the high type incentive compatibility are binding, then we can write the previous problem as

$$\begin{aligned} \max_{\{x_0(\theta)\}_{\theta \in \{\underline{\theta}, \bar{\theta}\}}} & V(x_0(\underline{\theta}), \underline{\theta}) + \pi \left[V(x_0(\bar{\theta}), \bar{\theta}) + (1 - x_0(\bar{\theta}))p(x_0(\bar{\theta}), \bar{\theta}) \right] \\ & + (1 - \pi)(1 - x_0(\underline{\theta}))p(x_0(\underline{\theta}), \underline{\theta}) - \pi V(x_0(\underline{\theta}), \bar{\theta}) \end{aligned}$$

First, we consider the first order condition for $x_0(\bar{\theta})$

$$V_x(x_0(\bar{\theta}), \bar{\theta}) - p(x_0(\bar{\theta}), \bar{\theta}) + (1 - x_0(\bar{\theta}))p_x(x_0(\bar{\theta}), \bar{\theta}) = 0.$$

The second order condition is

$$\begin{aligned} V_{xx}(x_0(\bar{\theta}), \bar{\theta}) - 2p_x(x_0(\bar{\theta}), \bar{\theta}) &= C(\bar{\theta}) - 2 \frac{(\phi\bar{\theta}^2 + \lambda_L C(\underline{\theta}))}{r + \lambda_L} \\ &= - \frac{\phi\bar{\theta}^2 + \lambda_L C(\underline{\theta}) + \gamma}{r + \lambda_L} < 0. \end{aligned}$$

Substituting $V_x(x, \bar{\theta})$ and $p(x, \bar{\theta})$ we get

$$\begin{aligned} x_0(\bar{\theta}) &= \frac{\phi\bar{\theta}^2 + \lambda_L C(\underline{\theta})}{2\phi\bar{\theta}^2 + 2\lambda_L C(\underline{\theta}) - (r + \lambda_L)C(\bar{\theta})} \\ &= \frac{\phi\bar{\theta}^2 + \lambda_L C(\underline{\theta})}{2\phi\bar{\theta}^2 + \lambda_L C(\underline{\theta}) - \lambda_L (C(\bar{\theta}) - C(\underline{\theta})) - rC(\bar{\theta})} \\ &= \frac{\phi\bar{\theta}^2 + \lambda_L C(\underline{\theta})}{\phi\bar{\theta}^2 + \lambda_L C(\underline{\theta}) + \gamma} \in (0, 1), \end{aligned}$$

Next, we consider the first order condition for $x_0(\underline{\theta})$.

$$V_x(x_0(\underline{\theta}), \underline{\theta}) - p(x_0(\underline{\theta}), \underline{\theta}) + (1 - \pi)(1 - x_0(\underline{\theta}))p_x(x_0(\underline{\theta}), \underline{\theta}) - \pi(V_x(x_0(\underline{\theta}), \bar{\theta}) - p(x_0(\underline{\theta}), \underline{\theta})) = 0,$$

while the second order condition is

$$\begin{aligned} V_{xx}(\underline{\theta}) - 2(1 - \pi)p_x(x_0(\underline{\theta}), \underline{\theta}) - \pi V_{xx}(x_0(\underline{\theta}), \bar{\theta}) &= \\ -(1 - \pi) \left[C(\underline{\theta}) + \frac{\pi}{1 - \pi} (C(\bar{\theta}) - C(\underline{\theta})) \right] &< 0 \end{aligned}$$

Substituting $p(x_0(\underline{\theta}), \underline{\theta}) = V_x(x_0(\underline{\theta}), \underline{\theta})$ and $p_x(x_0(\underline{\theta}), \underline{\theta}) = V_{xx}(x_0(\underline{\theta}), \underline{\theta})$ we get

$$(1 - x_0(\underline{\theta}))V_{xx}(x_0(\underline{\theta}), \underline{\theta}) = \frac{\pi}{1 - \pi} (V_x(x_0(\underline{\theta}), \bar{\theta}) - V_x(x_0(\underline{\theta}), \underline{\theta})) = 0.$$

Substituting $V_x(x, \theta) = C(\theta)x$, we get

$$x_0(\underline{\theta}) = \frac{C(\underline{\theta})}{C(\underline{\theta}) + \frac{\pi}{1 - \pi} (C(\bar{\theta}) - C(\underline{\theta}))}.$$

Finally, we need to verify the monotonicity constraint $x_0(\bar{\theta}) > x_0(\underline{\theta})$:

$$\frac{\phi\bar{\theta}^2 + \lambda_L C(\underline{\theta})}{\phi\bar{\theta}^2 + \lambda_L C(\underline{\theta}) + \gamma} \geq \frac{C(\underline{\theta})}{C(\underline{\theta}) + \frac{\pi}{1-\pi} (C(\bar{\theta}) - C(\underline{\theta}))}$$

which requires that

$$\frac{\pi}{1-\pi} \geq \frac{\gamma C(\underline{\theta})}{(\phi\bar{\theta}^2 + \lambda_L C(\underline{\theta})) (C(\bar{\theta}) - C(\underline{\theta}))}$$

If this condition is not satisfied, then we get that $x_0(\theta) = x_0$ and $T(\theta) = T$ where

$$\max_{T, x_0} T + \pi(1 - x_0)p(x_0, \bar{\theta}) + (1 - \pi)(1 - x_0)p(x_0, \underline{\theta})$$

subject to

$$V(x_0, \underline{\theta}) - T \geq V(0, \underline{\theta}).$$

From here we get, $T = V(x_0, \underline{\theta})$ so we can write

$$\max_{x_0} V(x_0, \underline{\theta}) + (1 - x_0) [\pi p(x_0, \bar{\theta}) + (1 - \pi)p(x_0, \underline{\theta})]$$

The first order condition is

$$V_x(x_0, \underline{\theta}) - [\pi p(x_0, \bar{\theta}) + (1 - \pi)p(x_0, \underline{\theta})] + (1 - x_0) [\pi p_x(x_0, \bar{\theta}) + (1 - \pi)p_x(x_0, \underline{\theta})] = 0$$

substituting $p(x_0, \underline{\theta})$ we get

$$\begin{aligned} 0 &= -\pi [p(x_0, \bar{\theta}) - p(x_0, \underline{\theta})] + (1 - x_0) [\pi p_x(x_0, \bar{\theta}) + (1 - \pi)p_x(x_0, \underline{\theta})] \\ &= -\pi \left[\frac{(\phi\bar{\theta}^2 + \lambda_L C(\underline{\theta}))}{r + \lambda_L} - C(\underline{\theta}) \right] x_0 + (1 - x_0) \left[\pi \left(\frac{(\phi\bar{\theta}^2 + \lambda_L C(\underline{\theta}))}{r + \lambda_L} \right) + (1 - \pi)C(\underline{\theta}) \right] \\ &= -2\pi \left[\frac{(\phi\bar{\theta}^2 + \lambda_L C(\underline{\theta}))}{r + \lambda_L} - C(\underline{\theta}) \right] x_0 + \pi \left[\frac{(\phi\bar{\theta}^2 + \lambda_L C(\underline{\theta}))}{r + \lambda_L} - C(\underline{\theta}) \right] + (1 - x_0)C(\underline{\theta}). \end{aligned}$$

The second order condition is

$$-2\pi p_x(x_0, \bar{\theta}) - (1 - 2\pi)p_x(x_0, \underline{\theta}) = - \left[C(\underline{\theta}) + 2\pi \frac{\phi\bar{\theta}^2 - rC(\underline{\theta})}{r + \lambda_L} \right] x_0 < 0$$

From here we get that

$$x_0 = \min \left\{ \frac{\pi(\phi\bar{\theta}^2 + \lambda_L C(\underline{\theta})) + (1 - \pi)(r + \lambda_L)C(\underline{\theta})}{2\pi(\phi\bar{\theta}^2 + \lambda_L C(\underline{\theta})) + (1 - 2\pi)(r + \lambda_L)C(\underline{\theta})}, 1 \right\}.$$

Finally, we rule out the optimality of stochastic mechanism when the monotonicity constraint is violated. Let $G(x_0|\theta)$ be the distribution of x_0 conditional on the report θ . Then, we have

$$\begin{aligned} \max_{\{G(x_0|\theta)\}_{\theta \in \{\underline{\theta}, \bar{\theta}\}}} \int_0^1 & \left(V(x_0, \underline{\theta}) + (1 - \pi)(1 - x_0)p(x_0, \underline{\theta}) - \pi V(x_0, \bar{\theta}) \right) dG(x_0, \underline{\theta}) \\ & + \pi \int_0^1 \left(V(x_0, \bar{\theta}) + (1 - x_0)p(x_0, \bar{\theta}) \right) dG(x_0, \bar{\theta}) \end{aligned}$$

subject to

$$\int_0^1 (V(x_0, \bar{\theta}) - V(x_0, \underline{\theta})) dG(x_0|\bar{\theta}) \geq \int_0^1 (V(x_0, \bar{\theta}) - V(x_0, \underline{\theta})) dG(x_0|\underline{\theta}).$$

The previous constraint reduces to

$$\int_0^1 x_0^2 dG(x_0|\bar{\theta}) \geq \int_0^1 x_0^2 dG(x_0|\underline{\theta}).$$

Let's define

$$\begin{aligned} \underline{U}(x_0, \eta) &\equiv V(x_0, \underline{\theta}) + (1 - \pi)(1 - x_0)p(x_0, \underline{\theta}) - \pi V(x_0, \bar{\theta}) - \eta x_0^2 \\ \bar{U}(x_0, \eta) &\equiv V(x_0, \bar{\theta}) + (1 - x_0)p(x_0, \bar{\theta}) + \eta x_0^2. \end{aligned}$$

Then, the problem can be written as

$$\min_{\eta \geq 0} \max_{\{G(x_0|\theta)\}_{\theta \in \{\underline{\theta}, \bar{\theta}\}}} \int_0^1 \underline{U}(x_0, \eta) dG(x_0, \underline{\theta}) + \int_0^1 \bar{U}(x_0, \eta) dG(x_0, \bar{\theta})$$

Notice that $\text{supp } G(x, \underline{\theta}) \subset \arg \max_{x_0} \underline{U}(x_0, \eta)$ and $\text{supp } G(x, \bar{\theta}) \subset \arg \max_{x_0} \bar{U}(x_0, \eta)$. So, if $\underline{U}(x_0, \eta)$ is concave, then the allocation is deterministic. It follows from analysis of the

second order conditions for the optimal deterministic allocation that, for any $\eta \geq 0$, the function $\bar{U}(x_0, \eta)$ is concave in x_0 . Hence, if a random allocation is optimal, it must be for a high report. The second order derivative of $\bar{U}(x_0, \eta)$ is

$$-\frac{\phi\bar{\theta}^2 + \lambda_L C(\bar{\theta}) + \gamma}{r + \lambda_L} + 2\eta,$$

so the function is convex only if

$$\eta > \frac{\phi\bar{\theta}^2 + \lambda_L C(\bar{\theta}) + \gamma}{2(r + \lambda_L)}.$$

If this were the case, the maximum would be an extreme point, so it would belong to $\{0, 1\}$.

We have that

$$\begin{aligned}\bar{U}(0, \eta) &= p(0, \bar{\theta}) = \frac{\mu}{r} \\ \bar{U}(1, \eta) &= V(1, \bar{\theta}) + \eta = \frac{\mu}{r} + C(\bar{\theta}) + \eta.\end{aligned}$$

Thus, $\bar{U}(1, \eta) > \bar{U}(0, \eta)$. But this means that a random allocation cannot be optimal. \square