Strategic Trading and Blockholder Ownership: Implications for IPO Design [*]

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Abstract

We study strategic trading by a blockholder who both monitors the firm and trades over time. When the blockholder has access to private information, the blockholder faces a less liquid market and delays selling his shares, particularly in high productivity periods. This increases the block size, the intensity of monitoring, and leads to greater firm value. Contrary to static settings, we show that asymmetric information is Pareto improving relative to the symmetric information case: asymmetric information not only increases the stock price and benefits small uninformed investors, but also benefits the blockholder.

As an application, we study the optimal IPO mechanism. We demonstrate that the possibility to trade after the IPO, significantly modifies the IPO design, sometimes exacerbating aftermarket volatility. Finally, we evaluate policy such as IPO lockup periods as a means to mitigate moral hazard and reinforce the blockholder's monitoring.

Keywords: Strategic Trading, Blockholder, Reputation, Activism.
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1   Introduction

Blockholders play a prominent role in capital markets (Holderness (2007)). They monitor firms and promote changes that affect firm productivity through various channels (e.g., negotiations with management, proxy fights, etc). These activities are costly to the blockholder and small shareholders free-ride on them. A blockholder thus faces a trade-off: he can mitigate free-riding and enhance his incentive to monitor the firm, by owning a large block, but by doing so, he compromises his own portfolio needs.[1]

We study strategic trading by a blockholder who is privately informed about his (time varying) ability to monitor the firm, and we investigate the impact of asymmetric information on the dynamics of blockholder ownership, firm productivity, and stock prices. Furthermore, we derive implications for optimal IPO design, thus making a contribution to the literature on mechanism design with aftermarket trading (Tirole, 2012; Philippon and Skreta, 2012).

Building on Admati et al. (1994) and DeMarzo and Urošević (2006) we consider a dynamic model of trading between a large investor (or blockholder) and a continuum of competitive investors.[2]In each period, the blockholder can both trade and exert effort to influence the firm's cash flows (i.e., monitor the firm). Crucially, the blockholder cannot commit to holding a large block, and trades over time based on his private information and portfolio preferences. The productivity of the blockholder's monitoring effort is private information and varies over time (it is a binary Markov chain). Thus, we depart from previous literature by considering a setting that combines moral hazard and asymmetric information. Thus, we also contribute to the literature that analyses the signaling role of retention and extend it to a dynamic environment (Leland and Pyle, 1977; Gale and Stiglitz, 1989; DeMarzo and Duffie, 1999).[3]

First, we study the equilibrium trading and pricing taking the blockholder's initial holdings as given. We show that, due to lack of commitment, the blockholder winds up selling his entire block, regardless of his ability to increase the firm's productivity via monitoring. That is, no matter how beneficial is the blockholder's monitoring, the blockholder sells his block over time because he does not fully internalize the erosion of value caused by weaker monitoring.

However, the information environment is a key determinant of the speed with which the block-

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[1]These trade-offs have been long identified by corporate governance scholars and practitioners at least going back to the work by Berle and Means (1932), Alchian and Demsetz (1972), and Jensen and Meckling (1976).

[2]The seminal papers on large shareholder monitoring are Huddart (1993) and Admati et al. (1994). The closest paper is DeMarzo and Urošević (2006) who extend the static models to a fully dynamic environment where blockholders can monitor and trade over time. Unlike DeMarzo and Urošević (2006), we consider a setting with asymmetric information.

[3]Gomes (2000) also studies a reputation game, with two types of manager/owners, who differ in terms of their cost of effort. In Gomes (2000) the manager effort is observable. Unlike in Gomes, we allow for hidden effort and time-varying private information.

holder sells his shares. Under asymmetric information, the blockholder's selling behavior is naturally affected by illiquidity. Since the market does not observe the blockholder's productivity, but learns about it from the blockholder order flow, the blockholder faces an illiquid market where his trading has price impact. As a consequence, when productivity is high, the blockholder slows down his selling speed and, when productivity drops, he accelerates selling, relative to the public information case.

We assume that the blockholder bears inventory holding costs, to capture the blockholder's liquidity and diversification needs. Two cases need to be distinguished depending on the magnitude of the inventory cost. First, when the blockholder's inventory costs are low, the blockholder's continuation payoff is convex in block size due to increasing returns to scale. In this context, the blockholder refrains from selling in the high productivity state and sells smoothly in the low state. Second, when inventory costs are large, the blockholder's continuation payoff is concave when the state is low; In this context, the blockholder sells his holdings immediately when the productivity drops, consistent with the Coase conjecture.

We study the welfare impact of asymmetric information. Contrary to static settings, we show that giving the blockholder access to private information is a Pareto improvement, relative to the public information case. On the one hand, asymmetric information benefits small uninformed investors because it delays the blockholder selling, thereby boosting monitoring, and ultimately increasing the firm's cash flows. Indeed, by reducing the liquidity faced by the blockholder, asymmetric information reduces the speed of selling, thus extending the blockholder's monitoring, particularly when it is most effective, namely in the high productivity state. This leads to a higher stock price. In turn, this increases the value of small investors' holdings.

On the other hand, the blockholder's payoff also increases when he has access to private information, unlike in a static setting where private information would typically force the blockholder to signal his type via inefficient retention (Leland and Pyle, 1977; Vanasco, 2017). Again, the nature of the equilibrium depends on the magnitud of the inventory cost. First, when inventory costs are small, asymmetric information has no impact on the blockholder's payoff. In other words, the blockholder obtains the same payoff when information is private versus public. The reason is that, due to lack of commitment, the blockholder neither can extract monopoly rents from trading nor he bears signaling costs —as in static setting— but winds up trading in a competitive fashion, regardless of whether he has access to private information or not.

Surprisingly, when the blockholder's inventory cost is large, the blockholder's payoff increases under asymmetric information, particularly in the high productivity state. In this context, it becomes too costly for the "low type" to imitate the high type by selling slowly, and as mentioned above, the low type sells immediately. In turn, this improves liquidity for the high type and allows

him to sell shares slowly without triggering a large price drop. Hence, when inventory costs are large, some of the gains from trade are exploited in the high state. More importantly, under asymmetric information, the high type can not sell too fast, because of his price impact, which acts as a commitment device that mitigates the forces of the Coase conjecture, and allows the blockholder to extract monopoly rents.

So, contrary to conventional wisdom, the blockholder's access to private information improves the welfare of uninformed investors without having an adverse effect on the blockholder's payoff, in stark contrast with a static setting (and a setting without monitoring). This result is reminiscent of the theory of the second best (Lipsey and Lancaster, 1956), whereby two frictions combined (lack of commitment and asymmetric information) lead to a more efficient outcome than a single friction (lack of commitment).

Our paper speaks to the literature on the role of liquidity on corporate governance. A key issue in this literature is that, a blockholder may have incentives to sell his shares ("cut and run") instead of bearing the cost of monitoring, particularly when the firm is under-performing. This has led some authors to conclude that market liquidity might be detrimental to corporate governance (Coffee, 1991; Kahn and Winton, 1998; Noe, 2002; Faure-Grimaud and Gromb, 2004).[4]

One counterargument is that liquidity might reduce free riding problem in takeovers (Grossman and Hart, 1980; Shleifer and Vishny, 1986). By facilitating the creation of a large block in the first place, liquidity can actually strengthen the firm's corporate governance and improve performance (Kyle and Vila, 1991; Maug, 1998; Back et al., 2018).[5] Another counterargument is that liquidity facilitates the use of "voice" as a governance mechanism (Hirschman, 1970).[6] Indeed, if manager's compensation is tied to the price of the company, so the manager is hurt by selling pressures that bring the price down, then investors can discipline the firm by threatening to sell their shares (Admati and Pfleiderer, 2009; Edmans, 2009).

Our results support the notion that iliquidity/adverse selection may have a positive effect because it mitigates the blockholder's lack of commitment to hold his shares and monitor the firm, particularly when this is most useful, that is, when productivity is high.[7]

While in the first part of this paper we assume that the initial block is exogenous, we later

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[4]This idea has been behind policy proposals attempting to reduce trading. For example, the European Union agreed to implement a transaction tax in September 2016.

[5]We depart from this literature by considering a dynamic signaling model – in which blockholdings are observable – rather than a microstructure model, where trading is unobservable and trading by the blockholder is obscured by the presence of noise traders.

[6]This literature is surveyed in Becht, Bolton, and Röell (2003) and Edmans and Holderness (2017).

[7]A related literature on loan sales and security design considers the impact that liquidity in secondary markets has on ex-ante screening of project quality. For example, Vanasco (2017) studies the role that adverse selection may play in fostering ex-ante screening to originate projects of high quality.

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endogenize the size of the initial block. Specifically, we consider the size of the initial block as the outcome of an optimal IPO, whereby an uninformed issuer sells the stock to both the blockholder and to a continuum of small uninformed investors, in anticipation of the fact that the investors will trade their shares in the aftermarket. In this way, we extend previous literature that study optimal IPO mechanism by analyzing the impact that secondary markets have on the optimal design (Benveniste and Wilhelm, 1990; Spatt and Srivastava, 1991; Stoughton and Zechner, 1998).

The possibility of post-IPO trading significantly alters the IPO design, and among other things, justifies commonly observed practices, such as IPO lockup periods and loyalty shares. We show that in equilibrium, and consistent with empirical evidence (see Brav and Gompers (2003)), the optimal IPO entails two stages. First, the entrepreneur conducts a pre-IPO placement where the blockholder receives a block at a discounted price. Second, the actual IPO takes place whereby the remaining shares are sold to small investors at a competitive price. In equilibrium, the entrepreneur offers a menu to the blockholder who in turn selects his preferred block/price pair based on his private information.

As mentioned above, the IPO design is crucially affected by post-IPO trading dynamics. In the absence of ex-post trading, the issuer screens the blockholder by allocating a larger block when the blockholder is more productive and vice-versa. Hence in equilibrium, the block size is informative about the blockholder productivity. But, under the possibility of post-IPO trading, a pooling allocation may emerge in equilibrium. The reason is as follows. Though incentive compatibility requires the allocation to be monotonic in productivity, a monotone allocation might not always be in the issuer's best interest because the valuation of the blockholder, relative to that of small investors, may decrease in blockholder productivity due to liquidity reasons. Indeed, in the high productivity state, the blockholder's valuation relative to the market is relatively low, at the margin, because of iliquidity. The blockholder must hold his shares to avoid a negative price impact, and small investors benefit from stronger monitoring, which pushes the price up. Hence, from the issuer's perspective, allocating a large block to the blockholder entails an opportunity cost. By contrast, in the low productivity state the blockholder and the small investors' valuations are identical at the margin, so the issuer faces no opportunity cost from selling to the blockholder. Thus, in the low productivity state, the issuer has an incentive to sell a large block to the blockholder to maximize his effort and thus increase the price paid by the small investors. This may lead to a pooling allocation, particularly when the issuer is pessimistic about productivity. Furthermore, insofar as the IPO allocation is uninformative about the state, this causes large volatility in the aftermarket (as documented empirically by Lowry et al. (2010); Brav and Gompers (2003)) as the blockholder information is revealed through his aftermarket trading.

To conclude, we note that our paper is related to the literature on durable good monopoly with

incomplete information, and the literature on bargaining with two sided asymmetric information (Cho, 1990; Ausubel and Deneckere, 1992). The closest paper in this literature is Ortner (2020) who consider a bargaining model with time varying cost. Because of the different focus and application, our model differs in a number of ways that affect the nature of the equilibrium. Unlike in the durable goods monopoly, we derive conditions under which the equilibrium entails trade by all types, and positive rents for blockholders. We show that the Coase conjecture holds (in the sense that the monopolist is unable to extract rents) when the blockholder's cost of holding large positions is small. But we also show that the Coase conjecture fails if this cost is sufficiently high. Perhaps surprisingly, a small increment in the cost of holding shares can lead to an increment in the blockholder's payoff. In durable good monopoly and bargaining models, incomplete information about costs generates an extreme form of inefficiency by eliminating trade completely. On the contrary, we show that asymmetric information increases welfare in our setting where blockholding has an effect on productivity. Not only overall welfare increases, but we also show that the equilibrium with asymmetric information Pareto dominates the equilibrium with symmetric information.

2 Setting

Following Admati et al. (1994) and DeMarzo and Urošević (2006) we study the behavior of a large investor (henceforth, blockholder) who can both trade a firm's stock and take costly actions (e.g., monitoring activities) to improve the firm's productivity. In addition to a large blockholder, there is a continuum of small investors who are price takers and cannot influence the firm's cash flows. In the baseline, we take the initial holdings of the blockholder x_0 as exogenous, but in Section 5 we model x_0 as arising from an optimal IPO mechanism.

Asset   Time t is continuous and the horizon is infinite. There is a single firm in unit supply with expected cash flows δ_t

$$\mathbb{E}_t[\delta_t]dt = (\mu + \theta_t a_t)dt,$$

where a_t is the blockholder's effort and $\theta_t \in \{\underline{\theta}, \bar{\theta}\}$ is his productivity. We make this multiplicative assumption, instead of an additive θ_t, to capture the fact that, empirically, there is wide variation in blockholder involvement, which suggests that blockholders vary in terms of productivity (see e.g., Hadlock and Schwartz-Ziv (2019)).

By blockholder productivity we mean the quality of the match between a firm and the blockholder. This quality is subject to uncertainty and variation over time, insofar as the blockholder's incentive and ability to monitor, as well as the intensity of agency frictions, varies due to random reasons (an alternative interpretation is that θ_t captures the blockholder's opportunity cost of

monitoring the firm, which depends on how busy the blockholder is at a given point in time).

The distinction between private information regarding the firm cash flows (μ) and private information regarding the blockholder's productivity (θ) is important when we consider its impact on the initial IPO allocation. Formally, if the blockholder were privately informed about μ instead of θ, then the efficient allocation at the time of the IPO would be independent of the blockholder's private information. The main impact of the IPO mechanism would be on the distribution of rents rather than economic efficiency.

We assume that the cash flows δ_t are publicly observable but the blockholder's effort a_t and productivity θ_t are not. The realized cash flows are paid to shareholders in each period, and as such we sometimes interpret δ_t as the firm's dividends. Conditional on θ_t, the firm's dividend δ_t is random. This assumption is important because otherwise the market would be able to infer the state θ_t from observing the dividend, and the blockholder's trading would be uninformative about the firm's fundamental. However, since the market is risk-neutral and the blockholder's preferences are specified directly over his holdings x, we do not need to specify the distribution of noise in the dividend process.

We refer to a_t as effort but interpret it broadly as any costly action that affects the firm's cash flows. We are agnostic as to the source of this externality. In the case of an external investor, one can think of a_t as the blockholder's monitoring — which disciplines managers and mitigates agency conflicts— or as the influence the blockholder exerts on the firm's management (as in Admati et al. (1994); Stoughton and Zechner (1998); DeMarzo and Urošević (2006)). Examples of a_t include public criticism of management or launching a proxy fight, advising management on strategy, figuring out how to vote on proxy contest launched by others or not taking private benefits for himself. In the case of a CEO or the founder of a company, a_t can represent effort or a reduction of private benefits that increases the productivity of the firm.

Productivity follows a two-state Markov-chain with switching intensity $\{\lambda_H, \lambda_L\}$, where λ_H is the switching intensity from $\underline{\theta}$ to $\bar{\theta}$, and λ_L is the switching intensity from $\bar{\theta}$ to $\underline{\theta}$.

Agents: Competitive investors, hereafter the market, are risk neutral and discount future cash flows at a discount rate r. The blockholder has quadratic flow preferences

$$u(x, a, \theta) = \mathbb{E}[\mu + \theta a]x - \frac{1}{2}\left(\phi^{-1}a^2 + \gamma x^2\right),$$

where $a^2/2\phi$ is the private cost of effort, and $1/\phi$ captures the severity of moral hazard. In practice this varies across blockholder types, and affects the blockholder's involvement.[8] Indeed, the empirical literature has documented that blockholder's involvement varies.[9] In our model, this pattern would be consistent with financial blockholders having a large cost of monitoring, or a small ϕ.

The parameter γ captures the cost of holding a stake x which we refer to as inventory cost. Although the holding cost cannot be directly linked to risk aversion, the presence of the inventory costs is meant to be a reduced-form way of capturing the financing cost of holding a large position in the firm.[10] The quadratic holding cost is popular among practitioners in financial institutions (Almgren and Chriss, 2001), and has also been used extensively in the dynamic trading literature (Vives, 2011; Du and Zhu, 2017; Duffie and Zhu, 2017). If γ is too large, then the blockholder sell his shares immediately, which is equivalent to a setting without a blockholder. Thus, to focus on the interesting case, we assume that γ isn't too large.

Information: The blockholder observes the dividend δ_t and the firm's productivity θ_t. On the other hand, the market only observes the dividend δ_t and the blockholder holdings x_t and trading flow q_t.

Throughout the paper we denote by $\hat{\theta}_t$ the expected value of θ_t given the market's information. Also whenever needed, we let $\hat{\mathbb{E}}_t[\cdot]$ denote the expected value at time t given the market conjectured strategy $(\hat{a}, \hat{q})$ and state $(\hat{\theta}_t)_{t \geq 0}$, and $\mathbb{E}_t[\cdot]$ the expected value given the blockholder true strategy (a, q) and state $(\theta_t)_{t \geq 0}$.

Strategies: The blockholder chooses effort a_t and trading $q_t \equiv dx_t/dt$ given his private information at time t. Competitive investors choose a trading strategy adapted to the public information. We denote the blockholder holdings at time t by x_t. Since the firm is in unit supply the market clearing condition at time t is

$$x_t + y_t = 1.$$

We assume that short sales are not allowed, so at any time t, the blockholder holdings x_t must be between zero and one.

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[8]It is possible extend our results to more general utility functions $u(x, a, \theta)$ such that $u_{aa} < 0$, $u_{xx} < 0$, $u_{x\theta} > 0$, and $u_{a\theta} > 0$.

[9]For example, Hadlock and Schwartz-Ziv (2019) argue that "many of the data patterns can be interpreted as consistent with a governance role through monitoring by nonfinancial blocks, and through trading for financial blocks."

[10]The one exception where the holding cost can be directly linked to risk aversion is the case in which cash flows are normally distributed and traders have CARA preferences.

Because competitive investors are risk neutral, in equilibrium the stock price is given by

$$p_t = \hat{\mathbb{E}}_t \left[\int_t^\infty e^{-r(s-t)} (\mu + \hat{\theta}_t \hat{a}_t) dt \right].$$

Equilibrium definition We focus on separating equilibria in Markov strategies. In particular,

Definition 1. *A separating Markov equilibrium is given by a strategy $\langle a(x,\theta), q(x,\theta) \rangle$, a price price function $p(x,q)$ and market beliefs $\hat{\theta} = \hat{\theta}(q,x)$ such that*

$$(a_s, q_s)_{s \geq t} \in \arg \max_{(a_s, q_s)_{s \geq t}} \mathbb{E} \left[\int_t^\infty e^{-r(s-t)} \left(u(x_s, a_s, \theta_s) - q_s p(x_s, q_s) \right) dt \middle| \theta_t \right]$$

$$p(x,q) = \hat{\mathbb{E}} \left[\int_t^\infty e^{-r(s-t)} \left(\mu + \theta_s a(x_s, \theta_s) \right) dt \middle| \theta_t = \hat{\theta}(q,x) \right]$$

$$\hat{\theta}(q(x,\theta), x) = \theta.$$

The equilibrium definition is standard. The blockholder chooses effort a_t and trading q_t to maximize his discounted payoff. The market sets the price of the firm as the present value of its future dividends, using the blockholder's trading history to forecast future effort and cash flows. Conditional on trading, the dividend does not play an information role. Naturally, in a separating equilibrium the market beliefs are, at each point, consistent with the true state.

Some caveats are in order. As in static signaling models, dynamic signaling models suffer from equilibrium multiplicity. Thus the choice of a selection criteria is a key part of the analysis. Our model is closely related to models of durable goods monopoly with incomplete information about costs, and models of bargaining with one-sided offers and two-sided incomplete information (Ausubel and Deneckere (1992); Cho (1990); Ortner (2020)). It is well known in this literature that a large class of equilibria can be constructed using optimistic off-equilibrium beliefs that assign off-equilibrium beliefs to the weakest type. Moreover, in many of these models it is possible to artificially introduce commitment by using this kind of punishing beliefs. To discipline our model, it is thus natural to focus on equilibria that satisfy some stationarity properties so the equilibrium does not depend on the complete history of the game (this is in the same spirit as the restriction to Markov Perfect Equilibrium). In our context, this stationary property requires that the blockholder strategy only depends on its type and current holdings, and that the market pricing only depends on the blockholder's holdings and trading rate. Finally, this class of separating equilibria are attractive from an analytical point of view due to their simple structure, which renders tractable dynamic models with incomplete information.

3   Equilibrium Characterization

We start characterizing separating equilibria tanking the blockholder initial holdings, x_0, as given. Later on, we derive the equilibrium when the state θ_t is publicly observable and evaluate the impact of asymmetric information. By definition, in a separating equilibrium, the beliefs of the market $\hat{\theta}_t$ coincide with the true state θ_t at each point in time.

Denote the blockholder's continuation value by $V(x,\theta)$. Given an equilibrium price function $p(x,q)$, the continuation value satisfies the following HJB equation

$$
\begin{aligned}
rV(x,\bar{\theta}) = \max_{a,q} \ & (\mu + \bar{\theta}a)x - \frac{1}{2}\left(\phi^{-1}a^2 + \gamma x^2\right) + q\left(V_x(x,\bar{\theta}) - p(x,q)\right) \\
& + \lambda_L\left(V(x,\underline{\theta}) - V(x,\bar{\theta})\right) \\
rV(x,\underline{\theta}) = \max_{a,q} \ & (\mu + \underline{\theta}a)x - \frac{1}{2}\left(\phi^{-1}a^2 + \gamma x^2\right) + q\left(V_x(x,\underline{\theta}) - p(x,q)\right) \\
& + \lambda_H\left(V(x,\bar{\theta}) - V(x,\underline{\theta})\right).
\end{aligned}
\tag{1}
$$

The blockholder earns the dividend but bears both effort and inventory costs. In addition, the blockholder makes a profit whenever there is a gap between the price $p(x,\theta)$ and his own marginal valuation $V_x(x,\theta)$, and makes a capital gain whenever there is a productivity transition.

Inspecting the HJB equation above, we see that the effort decision is myopic, hence the blockholder chooses effort to maximize his flow payoff.[11] The optimal effort is thus given by

$$
a(x,\theta) = \phi\theta x.
\tag{2}
$$

From inspecting the HJB equation above, we see that the optimal trading strategy maximizes the blockholder's trading profits, as given by $q\left(V_x(x,\theta) - p(x,\theta)\right)$. It follows that his trading strategy $q(x,\theta)$ must satisfy the following incentive compatibility constraint

$$
\begin{aligned}
q(x,\bar{\theta})\left(V_x(x,\bar{\theta}) - p(x,\bar{\theta})\right) &\geq q(x,\underline{\theta})\left(V_x(x,\bar{\theta}) - p(x,\underline{\theta})\right) \\
q(x,\underline{\theta})\left(V_x(x,\underline{\theta}) - p(x,\underline{\theta})\right) &\geq q(x,\bar{\theta})\left(V_x(x,\underline{\theta}) - p(x,\bar{\theta})\right)
\end{aligned}
\tag{3}
$$

The one-shot deviation principle ensures that these conditions are sufficient to guarantee incentive compatibility. Upon adding both incentive compatibility constraints, we get the following inequality

$$
\left(q(x,\bar{\theta}) - q(x,\underline{\theta})\right)\left(V_x(x,\bar{\theta}) - V_x(x,\underline{\theta})\right) \geq 0.
\tag{4}
$$

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[11]Effort is myopic in a separating equilibrium. In a pooling equilibrium, this might not necessarily hold because effort would have a a long-term impact on the market beliefs.

Inequality (4) establishes a natural monotonicity property that must be satisfied by any separating equilibrium: if the marginal valuation of $\bar{\theta}$ is higher than that of $\underline{\theta}$, then $q(\bar{\theta})$ must also be higher than that of $q(\underline{\theta})$; put differently, a necessary condition for the trading strategy to be incentive compatible, the blockholder must buy more shares when his marginal valuation is higher.

In any equilibrium, the dividend — given by $\mu + \phi\theta^2 x$, — is higher for the high type. Thus, in the absence of dynamic trading, the single crossing condition would be immediately satisfied. However, in a dynamic game, this condition does not follow immediately from single crossing in flow payoffs but needs to hold for the continuation value.

Instead of imposing a restriction directly on beliefs to guarantee single crossing, we restrict attention to equilibria satisfying this monotonicity property. Similarly to Cho (1990), the single crossing condition in the dynamic trading game can be motivated by "Divinity." Using backward induction, and extending the ideas in Cho (1990) to our setting, it is possible to establish such a property by considering the limit of a discrete time model with a finite number of trading rounds, and applying a refinement such as Divinity to the static game in each trading round.[12]

Condition (M). *An equilibrium satisfies Monotonicity if $V_x(x, \bar{\theta}) > V_x(x, \underline{\theta})$ for all $x > 0$.*

Condition (M) together with the incentive compatibility condition (4) imply that in any separating equilibria we have $q(x, \bar{\theta}) > q(x, \underline{\theta})$ for all $x > 0$.

Next, we characterize the equilibrium price that is consistent with the monotonicity condition (M).

Lemma 1 (Equilibrium Price). *In any smooth trading equilibria satisfying condition (M), the equilibrium beliefs are*

$$\hat{\theta}(q, x) = \begin{cases} \underline{\theta} & \text{if } q < 0 \\ \bar{\theta} & \text{if } q > 0. \end{cases}$$

The price function is given by

$$p(x, q) = \begin{cases} p(x, \underline{\theta}) & \text{if } q < 0 \\ p(x, \bar{\theta}) & \text{if } q > 0, \end{cases}$$

where $p(x, \theta)$ satisfies the following equation:

$$rp(x, \bar{\theta}) = \mu + \bar{\theta}a(x, \bar{\theta}) + q(x, \bar{\theta})p_x(x, \bar{\theta}) + \lambda_L \left(p(x, \underline{\theta}) - p(x, \bar{\theta}) \right)$$
$$rp(x, \underline{\theta}) = \mu + \underline{\theta}a(x, \underline{\theta}) + q(x, \underline{\theta})p_x(x, \underline{\theta}) + \lambda_H \left(p(x, \bar{\theta}) - p(x, \underline{\theta}) \right). \tag{5}$$

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[12] See Fudenberg and Tirole (1991) for a formal definition of Divinity.

11

The intuition behind Lemma 1 is as follows. In a separating equilibrium, whenever the low type sells, it receives the same payoff it would get in a symmetric information model. The low type does not refrain from trading since his reputation is at the lowest level (this is the traditional result in static signaling model whereas the lowest type gets symmetric information outcome, although in our dynamic setting the rate of trading changes even for the low type).

Because the optimization problem in equation (1) is linear in the trading rate, in any equilibrium with smooth trading, the low type necessarily gets zero profits from trading, which reflects the forces behind the Coase conjecture. This zero-rent at the bottom result, means that any price that would induce the high type to sell his shares, would also induce the low type to sell his. So the high type cannot sell in equilibrium. We can use the same argument to show that the low type cannot buy. The logic behind this result is similar to the no-trade theorem in Ausubel and Deneckere (1992), who study a durable good monopoly with incomplete information. Lemma 1 relies crucially on the assumption that equilibrium entails smooth trading. Later on, we consider the case in which the equilibrium involves atomic trading.

Again, given that the maximization problem in the HJB equation (1) is linear in the trading rate q, in an equilibrium with smooth trading, when he trades the blockhoder must be indifferent whether to trade, and the price must satisfy $p(x, \theta) = V_x(x, \theta)$. Accordingly, an implication of Lemma 1 is that the equilibrium price is given by

$$
p(x, q) = \begin{cases} V_x(x, \underline{\theta}) & \text{if } q < 0 \\ V_x(x, \bar{\theta}) & \text{if } q > 0, \end{cases}
$$

therefore the blockolders's continuation value is calculated as if there was no trading on path, thus solving

$$
\begin{aligned}
rV(x, \bar{\theta}) &= \max_a \ (\mu + \bar{\theta}a)x - \frac{1}{2}\left(\phi^{-1}a^2 + \gamma x^2\right) - \lambda_L\left(V(x, \bar{\theta}) - V(x, \underline{\theta})\right) \\
rV(x, \underline{\theta}) &= \max_a \ (\mu + \underline{\theta}a)x - \frac{1}{2}\left(\phi^{-1}a^2 + \gamma x^2\right) - \lambda_H\left(V(x, \underline{\theta}) - V(x, \bar{\theta})\right).
\end{aligned}
\tag{6}
$$

Furthermore, given the underlying free-riding problem, it is optimal for the blockholder to eventually sells all his shares, $x = 0$. Consistent with this intuition, we prove that $q(x, \underline{\theta}) < 0$ for all $x > 0$. On the other hand, there cannot be an equilibrium in which $p(x, \bar{\theta}) = V_x(x, \bar{\theta})$ and $q(x, \underline{\theta}) > 0$ (this is verified later in section 3.2). Thus, it must be the case that $q(x, \bar{\theta}) = 0$.

Plugging this in the asset pricing equation (5), yields the following intermediate result:

$$p(x, \bar{\theta}) = \frac{\mu + \bar{\theta}a(x, \bar{\theta}) + \lambda_L V_x(x, \underline{\theta})}{r + \lambda_L} \tag{7}$$

$$q(x, \underline{\theta}) = -\frac{\mu + \underline{\theta}a(x, \underline{\theta}) + \lambda_H p(x, \bar{\theta}) - (r + \lambda_H)V_x(x, \underline{\theta})}{V_{xx}(x, \underline{\theta})}. \tag{8}$$

Equation (7) characterizes the price in the high state, which follows from plugging $q(x, \bar{\theta}) = 0$ in the equation for $p(x, \bar{\theta})$, while equation (8) provides the equilibrium trading rate $q(x, \underline{\theta})$ which follows from plugging $p(x, \underline{\theta}) = V_x(x, \underline{\theta})$ into the equation for $p(x, \underline{\theta})$ in (5).

To guarantee the existence of an equilibrium with smooth trading, inventory costs γ need not be too large. The next proposition characterizes the equilibrium under low inventory costs.

Proposition 1. *Assume that inventory costs γ satisfies*

$$\underline{\theta}^2 + \frac{\lambda_H}{r + \lambda_L + \lambda_H}(\bar{\theta}^2 - \underline{\theta}^2) \geq \frac{\gamma}{\phi}. \tag{9}$$

Then there is a unique equilibrium with smooth trading satisfying Condition (M). In this equilibrium the blockholder's payoff is

$$V(x, \theta) = \frac{\mu}{r}x + \frac{1}{2}C(\theta)x^2 \tag{10}$$

where $C(\theta) > 0$ is given by

$$\begin{aligned}
rC(\underline{\theta}) &= \phi\underline{\theta}^2 - \gamma + \phi\frac{\lambda_H}{r + \lambda_L + \lambda_H}(\bar{\theta}^2 - \underline{\theta}^2) \\
rC(\bar{\theta}) &= \phi\bar{\theta}^2 - \gamma - \phi\frac{\lambda_L}{r + \lambda_L + \lambda_H}(\bar{\theta}^2 - \underline{\theta}^2).
\end{aligned} \tag{11}$$

The stock price satisfies

$$p(x, q) = \begin{cases} \frac{\mu}{r} + C(\underline{\theta})x & \text{if } q < 0 \\ \frac{\mu}{r} + \frac{\phi\bar{\theta}^2 + \lambda_L C(\underline{\theta})}{r + \lambda_L}x & \text{if } q = 0 \\ \frac{\mu}{r} + C(\bar{\theta})x & \text{if } q > 0 \end{cases}$$

and the blockholder's trading rate follows

$$q(x, \theta) = \begin{cases} -\left(1 + \frac{\lambda_H}{r + \lambda_L}\right)\frac{\gamma}{C(\underline{\theta})}x & \text{if } \theta = \underline{\theta} \\ 0 & \text{if } \theta = \bar{\theta} \end{cases}.$$

In this model, lack of commitment leads the blockholder to sell his entire block over time,

until x_t reaches zero. This is clearly inefficient compared with the first-best benchmark.[13] The blockholder's selling behavior causes an externality: the firm's productivityg gradually deteriorates as the blockholder unwinds his holdings thereby weakening his own incentive monitor the firm.

Notice that, in the high state, the blockholder does not sell shares, even though his valuation is lower than that of small investors $(p(x, \bar{\theta}) > V_x(x, \bar{\theta}))$. The blockholder refrains from selling because of his price impact; if the blockholder were to sell, the market would interpret this as a negative signal of productivity and the stock price would drop drastically, even below the blockholder's valuation, in which case the blockholder would experience a capital loss. By contrast, in the low state, the blockholder sells gradually until his holdings are fully depleted, or until a positive shock creates incentives to pause this selling process for some time.[14]

Proposition (1) is predicated on the low type's continuation value being convex in x, which corresponds to condition (9) in the proposition. This convexity reflects the presence of increasing returns to scale in block size, x. Despite the convexity of inventory costs, there are increasing returns to scale because the cash flow depends on effort a, which in equilibrium is proportional to x. Yet the value function is convex in x as long as these returns of scale dominate the convexity of inventory costs.

To understand the equilibrium, notice that in a separating equilibrium, the low type's payoff is the same as that arising under symmetric information. As in the previous literature on the durable goods monopoly (Ausubel and Deneckere (1992)), lack of commitment prevents the blockholder from extracting rents from trading, which yields the continuation payoff $V(x, \underline{\theta})$ in the proposition. However, this does not imply that the blockholder trades immediately towards his long-term target. Contrary to the standard prediction of the Coase conjecture, the blockholder sells slowly.

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[13]In the first best benchmark the allocation x_t is chosen at each point to maximize social welfare under symmetric information about θ_t, but subject to the blockholder choosing effort privately (or in a non-contractible manner) which, as leads to

$$a(x, \theta) = \phi \theta x. \tag{12}$$

In this context, the social welfare is given

$$W = \frac{\mu}{r} + \int_0^\infty e^{-rt} \, \mathbb{E} \left[\phi \theta_t^2 x_t - \frac{1}{2} \left(\phi \theta_t^2 + \gamma \right) x_t^2 \right] dt.$$

That is welfare consists of the discounted cash flows net of effort and inventory costs. The first best allocation thus maximizes the objective point-wise, and yields

$$x_t = \frac{\phi \theta_t^2}{\phi \theta_t^2 + \gamma}. \tag{13}$$

From a first-best perspective, the optimal block size is interior. It increases when productivity goes up, and decreases in the blockholder's inventory costs.

[14]The long-run holdings would be positive if the blockholder enjoyed private benefits, in which case the order flow would sometimes be positive, for instance when the initial holdings are smaller than the long run target.

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To understand why this happens, recall that in a competitive setting, the price equals marginal cost. In our setting, the marginal cost is represented by $V_x(x, \underline{\theta})$. Figure 1(a) illustrates that a competitive equilibrium would imply payoffs for the blockholder that are below the value of not trading at all. Of course, this is just a restatement of the well known result that the price cannot equal marginal cost in the presence of increasing returns to scale, because this would generate losses to the firm. On the other hand, the equilibrium must entail some trade. If there was no trade, the price would be above μ/r, generating incentives to trade. Thus, in equilibrium, the blockholder moves smoothly along the curve $V(x, \underline{\theta})$ trading at a price $V_x(x, \underline{\theta})$. Later we will see that – unlike in static signaling models – although the payoff of the low type is the same as that arising with symmetric information, the equilibrium trading strategy is not.

For the high type, the equilibrium trading is determined by the incentive compatibility constraint. As previously discussed, Lemma 1 shows that in equilibrium the high and low type cannot trade in the same direction at the same time. Any price that is high enough to induce the high type to sell, would attract the low type, who in equilibrium can't extract rents. Thus, the only possibility is that either the high type buy shares or does not not trade at all. Due to the presence of inventory costs, it is not possible that the high type buys in equilibrium (that is no longer the case once we consider possible private benefits of control later on) thus, in equilibrium, the high type does not trade.
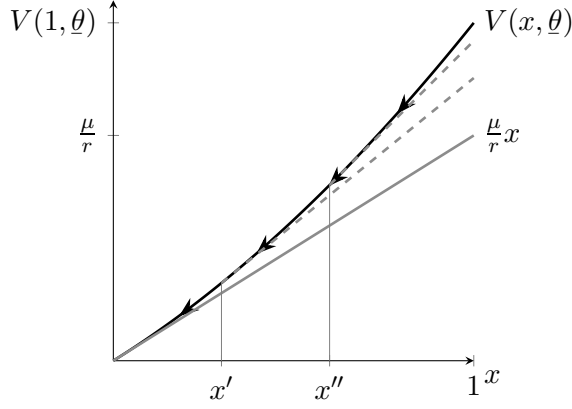
The previous argument does not apply when condition (9) is not satisfied, in which case the value function of the low type $\underline{\theta}$ is concave in x. Then, the low type sells immediately towards $x = 0$. Figure 1(b) illustrates that the blockholder is better off by liquidating his holdings immediately, regardless of his price impact. In equilibrium, the low type sells his entire block at a price μ/r. More interestingly, in this case, the argument in Lemma 1 no longer applies. The low type payoff is strictly higher than the payoff of not trading ($V(x, \underline{\theta})$), so the incentives of the low type to imitate the hight type are reduced and an equilibrium where the high type sells a positive amount becomes possible.

Next, we proceed to derive the equilibrium in this case. The high type has incentives to sell only if $p(x, q(x, \bar{\theta})) = p(x, \bar{\theta}) > V_x(x, \bar{\theta})$. In which case, the value function satisfies
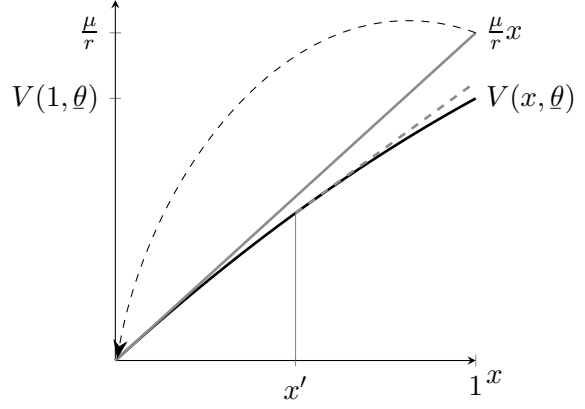
$$rV(x, \bar{\theta}) = \mu x + \frac{1}{2} \left(\phi \bar{\theta}^2 - \gamma \right) x^2 + q(x, \bar{\theta}) \left(V_x(x, \bar{\theta}) - p(x, \bar{\theta}) \right) + \lambda_L \left(V(x, \underline{\theta}) - V(x, \bar{\theta}) \right) \qquad (14)$$

On the other hand, the low type has no incentive to imitate the high type only if

$$rV(x, \underline{\theta}) \geq \mu x + \frac{1}{2} \left(\phi \underline{\theta}^2 - \gamma \right) x^2 + q(x, \bar{\theta}) \left(V_x(x, \underline{\theta}) - p(x, \bar{\theta}) \right) - \lambda_H \left(V(x, \underline{\theta}) - V(x, \bar{\theta}) \right) \qquad (15)$$

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(a) Convex value function and smooth trading   (b) Concave value function and atomic trading

Figure 1: Value function with and without smooth trading. In this figure, $V(x, \theta)$ indicates the value of holding to x forever. In (a), the value of staying put at x is above the value of liquidating. In equilibrium, the blockholder trades moving around the curve $V(x, \theta)$. In the (b), the blockholder is better off liquidating everything at the lowest possible price (μ/r), so the equilibrium entails immediate liquidation (there is an immediate jump to $x = 0$).

The left hand side of equation (15) corresponds to the equilibrium payoff of the low type, which has to be weakly higher than the right-hand side, which corresponds to the value of pooling with the high type for a short period of time. Substituting $V(x, \underline{\theta}) = p(0, \underline{\theta})x = (\mu/r)x$ in equation (15), we get the following incentive compatibility constraint for the low type

$$q(x, \bar{\theta}) \geq -\frac{\frac{1}{2}r\left(\gamma - \phi\underline{\theta}^2\right)x^2 - \lambda_H\left(\mu x - rV(x, \bar{\theta})\right)}{rp(x, \bar{\theta}) - \mu}. \tag{16}$$

In this case, there are multiple equilibria that differ based on the trading rate of the high type. Given $q(x, \bar{\theta})$, the price is given by the solution to the ODE

$$rp(x, \bar{\theta}) = \mu + \phi\bar{\theta}^2 x + q(x, \bar{\theta})p_x(x, \bar{\theta}) + \lambda_L\left(p(x, \underline{\theta}) - p(x, \bar{\theta})\right), \tag{17}$$

with initial condition $p(0, \bar{\theta}) = \mu/r$.

The least costly separating equilibrium maximizes the speed of trading of the high type, hence the incentive compatibility constraint (30) is binding. Substituting the trading rate in equations (14) and (17) we get an ordinary differential equation for $V(x, \bar{\theta})$ and $p(x, \bar{\theta})$.[15]

We can guess and verify that the equilibrium trading strategy is given by $q(x, \bar{\theta}) = -\bar{q}x$, the

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[15]Notice that this is the least costly separation equilibrium in the "stage game" with payoffs $q(V_x(x, \theta) - p(q, x))$. One can motivate the focus on the least costly separating equilibrium by following a similar reasoning to the one used in static signaling games applied to the stage game.

price is $p(x, \bar{\theta}) = \mu/r + \bar{p}x$, and the value function of the high type is $V(x, \bar{\theta}) = (\mu/r)x + \frac{1}{2}\bar{C}x^2$ for some constants $\bar{p}, \bar{C}$ that we provide next.

Proposition 2 (Large Inventory Costs). *Assume that the inventory cost γ satisfies*

$$\underline{\theta}^2 + \frac{\lambda_H}{r + \lambda_L + \lambda_H}(\bar{\theta}^2 - \underline{\theta}^2) < \frac{\gamma}{\phi} < 2\bar{\theta}^2 + \underline{\theta}^2. \tag{18}$$

Then the least costly separating equilibrium is as follows:

- *The low type value function is $V(x, \underline{\theta}) = (\mu/r)x$ and the high type value function is $V(x, \bar{\theta}) = (\mu/r)x + \frac{1}{2}C(\bar{\theta})x^2$, where the coefficient $C(\bar{\theta})$ corresponds to the unique positive root, $\bar{C}$, of the quadratic equation*

$$\lambda_H(r + \lambda_L + \lambda_H)\bar{C}^2 - \left[\lambda_H\left(\phi\left(2\underline{\theta}^2 + \bar{\theta}^2\right) - \gamma\right) - (r + \lambda_L)\left(\gamma + \phi\left(2\bar{\theta}^2 - \underline{\theta}^2\right)\right)\right]\bar{C}$$
$$- \phi\left(\bar{\theta}^2 - \underline{\theta}^2\right)\left(\phi\left(2\bar{\theta}^2 + \underline{\theta}^2\right) - \gamma\right) = 0.$$

- *The low type blockholder liquidates his holdings x_0 immediately at $t = 0$ and holds no shares thereafter.*

- *Type $\bar{\theta}$ trades at a rate*

$$q(x, \bar{\theta}) = -\frac{\left(\gamma + \lambda_H C(\bar{\theta}) - \phi\underline{\theta}^2\right)(r + \lambda_L)}{\phi(2\bar{\theta}^2 + \underline{\theta}^2) - \gamma - \lambda_H C(\bar{\theta})}x.$$

- *The equilibrium price is*

$$p(x, q) = \begin{cases} \frac{\mu}{r} & \text{if } q < q(x, \bar{\theta}) \\ \frac{\mu}{r} + \frac{\phi(2\bar{\theta}^2 + \underline{\theta}^2) - \gamma - \lambda_H \bar{C}}{2(r + \lambda_L)}x & \text{if } q \in [q(x, \bar{\theta}), 0] \\ \frac{\mu}{r} + C(\bar{\theta})x & \text{if } q > 0 \end{cases}$$

Several comments are in order. When γ is large, there are decreasing returns to scale in x for the low type, which leads to the classic form of the Coase conjecture whereby the blockholder sells immediately his shares at a competitive price (recall, we require $\frac{\gamma}{\phi} < 2\bar{\theta}^2 + \underline{\theta}^2$ so at least the high type does not sell immediately. Otherwise, the equilibrium would be trivial: both types would sell immediately, and the price would be $p(x, \theta) = \mu/r$).

Even though the firm cash flow is still convex in x, this convexity is offset by the convexity of

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inventory costs, leading to concave payoffs for the low type. Still, as long as

$$\frac{\gamma}{\phi} < 2\bar{\theta}^2 + \underline{\theta}^2$$

the payoff of the high type is convex, and he must sell slowly. His trading rate is limited by his price impact. That is even though $V_x(x,\theta) < p(x,\theta)$ the blockholder sells smoothly, reflecting a concern that the price would drop significantly if he traded faster because the market would interpret that as indicating low productivity.

It is worth noting that an increase in the inventory cost parameter γ has a large positive effect on the payoff of the high type, as we transition from an equilibrium with smooth-trading (as in Proposition 1) to an equilibrium with immediate trading by the low type (as in Proposition 2). This effect is illustrated in Figure 3. This positive effect arises because as the low type exits the market by selling his block, the high type is able to sell shares without triggering a strong drop in the stock price. On some level, the immediate exit of the low type, increases the liquidity facing the high type which explains why he benefits from greater inventory costs. On the other hand, this transition leads to a sharp decline in $p(x,\theta)$ caused by the higher speed of selling and the associated decrease in monitoring.

3.1 Private Benefits of Control

The baseline model predicts that the blockholder only sells shares in equilibrium but never buys. This can be generalized by assuming the blockholder enjoys a private benefit of control. We can capture this by adding a private benefit term bx to the payoffs of the blockholder, so his preferences are given by $bx+u(x,a,\theta)$. We assume that the marginal benefit of control is lower than the marginal inventory cost at $x = 1$, which requires that $b < \gamma$. The derivation of the equilibrium closely follows the one without private benefits. The only difference is that the block no longer converges to zero but to $x^\dagger = b/\gamma$. This is the ownership level that balances inventory cost and private benefit of controls.

Notice that the long-term ownership is still inefficient as it ignores the impact on incentives. Lemma 1 implies that the high and low type cannot both trade in the same direction at the same time. In equilibrium, whenever $x > x^\dagger$ we have that $q(x,\bar{\theta}) = 0$ and $q(x,\underline{\theta}) < 0$. Similarly, whenever $x < x^\dagger$, we have that $q(x,\bar{\theta}) > 0$ and $q(x,\underline{\theta}) = 0$. Given that the derivation is similar to the one without private benefits of control, we relegate the details of the derivation to the appendix. The next proposition provides the equilibrium in the presence of private benefits.

Proposition 3 (Private Benefits)**.** *Assume that inventory cost γ satisfies condition* (9)*. Then there is a unique equilibrium with smooth trading satisfying Condition (M). In this equilibrium the*

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blockholder's payoff is

$$V(x,\theta) = \frac{\mu + b}{r}x + \frac{1}{2}C(\theta)x^2 \tag{19}$$

where $C(\theta) > 0$ is given by equation (11) *. The stock price satisfies*[16]

$$p(x,q) = \begin{cases} \frac{\mu+b}{r} + C(\underline{\theta})x & \text{if } q < 0 \text{ and } x > b/\gamma \\ \frac{\mu}{r} + \frac{\lambda_L}{r+\lambda_L}\frac{b}{r} + \frac{\phi\bar{\theta}^2 + \lambda_L C(\underline{\theta})}{r+\lambda_L}x & \text{if } q = 0 \text{ and } x > b/\gamma \\ \frac{\mu}{r} + \frac{\lambda_H}{r+\lambda_H}\frac{b}{r} + \frac{\phi\underline{\theta}^2 + \lambda_H C(\bar{\theta})}{r+\lambda_H} & \text{if } q = 0 \text{ and } x < b/\gamma \\ \frac{\mu+b}{r} + C(\bar{\theta})x & \text{if } q > 0 \text{ and } x < b/\gamma \end{cases}$$

and the blockholder's trading rate follows[17]

$$q(x,\underline{\theta}) = -\left(1 + \frac{\lambda_H}{r+\lambda_L}\right)\frac{(b-\gamma x)^-}{C(\underline{\theta})} \tag{20}$$

$$q(x,\bar{\theta}) = \left(1 + \frac{\lambda_L}{r+\lambda_H}\right)\frac{(b-\gamma x)^+}{C(\bar{\theta})}. \tag{21}$$

The case with private benefits of control generalizes our model, and leads the blockholder to hold a positive block in the long-run. Also, in this generalized model, the blockholder buys shares if his initial holdings are lower than the long-run target. This generalization is relevant because it expands the scope of our analysis to applications where the blockholder gradually builds their block over time (as is the case of hedge fund activists).

3.2   Equilibrium with Observable Shocks

To understand the consequences of information asymmetry and iliquidity, here we consider a benchmark with symmetric information where the productivity shocks on θ_t are observable.

Consider the case when θ_t is publicly observed. The payoff of the blockholder continues to be characterized by equation (6). The trading rate $q(x,\theta)$ satisfies the following HJB equations:

$$rp(x,\bar{\theta}) = \mu + \bar{\theta}a(x,\bar{\theta}) + q(x,\bar{\theta})p_x(x,\bar{\theta}) + \lambda_L\left(p(x,\underline{\theta}) - p(x,\bar{\theta})\right)$$

$$rp(x,\underline{\theta}) = \mu + \underline{\theta}a(x,\underline{\theta}) + q(x,\underline{\theta})p_x(x,\underline{\theta}) + \lambda_H\left(p(x,\bar{\theta}) - p(x,\underline{\theta})\right)$$

As in the asymmetric information case, the blockholder trades at a competitive price and is always

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[16]Off-equilibrium, $p(x,q) = \frac{\mu+b}{r} + C(\bar{\theta})x$ if $q > 0$ and $x > b/\gamma$. Similarly, $p(x,q) = \frac{\mu+b}{r} + C(\underline{\theta})x$ if $q < 0$ and $x < b/\gamma$.

[17]Here, we are using the notation for the negative and positive parts of a function. For any function $f(x)$, $f(x)^+ \equiv \max\{f(x), 0\}$ and $f(x)^- \equiv \max\{-f(x), 0\}$.
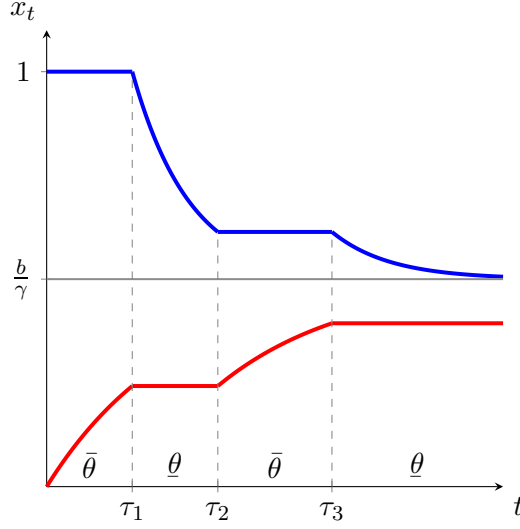
Figure 2: Ownership Sample Path with Private Benefits of Control

indifferent whether to trade, thus

$$p(x, \theta) = V_x(x, \theta) = \frac{\mu}{r} + C(\theta)x,$$

and $a(x, \theta) = \phi\theta x$. The lack of commitment of the blockholder, makes the blockholder unable to exploit his market power. Also, he is charged an implicit tax/subsidy whenever he trades because the market anticipates that any change in blockholder's ownership will affect the blockholder incentive to monitor the firm thereafter.

Substituting the above condition in the HJB for price, yields the equilibrium trading strategy under public information:

$$q(x, \bar{\theta}) = \left(rC(\bar{\theta}) + \lambda_L \left(C(\bar{\theta}) - C(\underline{\theta}) \right) - \phi\bar{\theta}^2 \right) \frac{x}{C(\bar{\theta})}$$

$$q(x, \underline{\theta}) = \left(rC(\underline{\theta}) - \lambda_H \left(C(\bar{\theta}) - C(\underline{\theta}) \right) - \phi\underline{\theta}^2 \right) \frac{x}{C(\underline{\theta})}$$

where $C(\bar{\theta})$ and $C(\underline{\theta})$ are the same as in the case with asymmetric information (see equation (11)).

Proposition 4. *(**Symmetric Information**) Suppose productivity shocks are observable. If condition 9 is satisfied, then the payoff is $V(x, \theta) = (\mu/r)x + \frac{1}{2}C(\theta)x^2$, where $C(\theta)$ is given in equation (11). The blockholder's trading rate is given by*

$$q^o(x, \theta) = -\frac{\gamma}{C(\theta)}x \tag{22}$$

and the stock price is $p^o(x, \theta) = \mu/r + C(\theta)x$. If 9 is violated and $\phi\bar{\theta}^2 > \gamma$, then $V(x, \underline{\theta}) = (\mu/r)x$ and $V(x, \bar{\theta}) = (\mu/r)x + \frac{1}{2}C(\bar{\theta})x^2$, where

$$C(\bar{\theta}) = \frac{\phi\bar{\theta}^2 - \gamma}{r + \lambda_L}.$$

The low type liquidates immediately at a price $p^o(x, \underline{\theta}) = \mu/r$, while the high type trades at a rate given by (22) and price $p^o(x, \bar{\theta}) = \mu/r + C(\bar{\theta})x$. If $\phi\bar{\theta}^2 < \gamma$, then both types liquidate their positions immediately at a price $p^o(x, \theta) = \mu/r$.

The blockholder sells until his holdings are fully depleted, $x = 0$. Naturally, the blockholder sells faster in the low state and when his block x is larger. A higher productivity, does not change the long-run target but it does slow-down the blockholder's selling towards his target.

Unlike under asymmetric information, the price does not depend on the order flow q which is no longer informationally relevant. However, similar to the asymmetric information case, the price depends on block size x which affects the blockholder's incentives to exert effort.

4 The Impact of Asymmetric Information

Having solved for the public information benchmark, we are equipped to analyze the impact of private information on trading, prices, and welfare. Our first result studies the impact on trading and prices.

Corollary 1 (Pricing and Trading).
If condition (9) satisfied, so the equilibrium entails smooth trading by both types, then:

- *Information asymmetry reduces the speed of trading in the high productivity state, and increases the speed of trading in the low productivity state.*

- *Asymmetric information increases the stock price in the high productivity state, and has no impact on the stock price in the low productivity state.*

On the other hand, if condition (18) is satisfied, then:

- *Asymmetric information increases the stock price in the high productivity state, and has no impact on the stock price in the low productivity state.*

- *Asymmetric information decreases the speed of trading in the high productivity state, and has no impact on the speed of trading in the low productivity state.*[18]

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[18]For the statement regarding the price, notice that in the low productivity state the price is μ/r with and without

Under asymmetric information, the blockholder's order flow has price impact because of signaling effects. Hence, it is natural to conjecture that asymmetric information would slow down trading as it often doe in dynamic signaling settings (see e.g., Daley and Green (2012); Admati and Perry (1987)), but this is true only in the high state (when inventory costs are low). By contrast, in the low state, the blockholder selling speed increases under asymmetric information. Otherwise, the firm cash flows would be more persistent in the low state, and the stock price would thus increase relative to the public information case, thereby creating a gap between the stock price and the blockholder's marginal valuation, which in turn would induce the blockholder to sell immediately to take advantage of that gap.

Despite this ambiguity in the effect of asymmetric information upon the selling speed, asymmetric information does increase the stock price unambiguously. The fact that the blockholder slows down selling in the high state, leads to more monitoring, more persistent cash flows, and ultimately to a higher stock price because monitoring is stronger when it it is most effective.

Next, we turn to the impact of asymmetric information on blockholder payoffs. The impact of asymmetric information on payoffs depends on the level of the inventory cost γ. On the one hand, when inventory costs are sufficiently low, the equilibrium payoffs with and without asymmetric information are the same. On the other hand, when inventory costs are high, asymmetric information increases the blockholder payoffs.
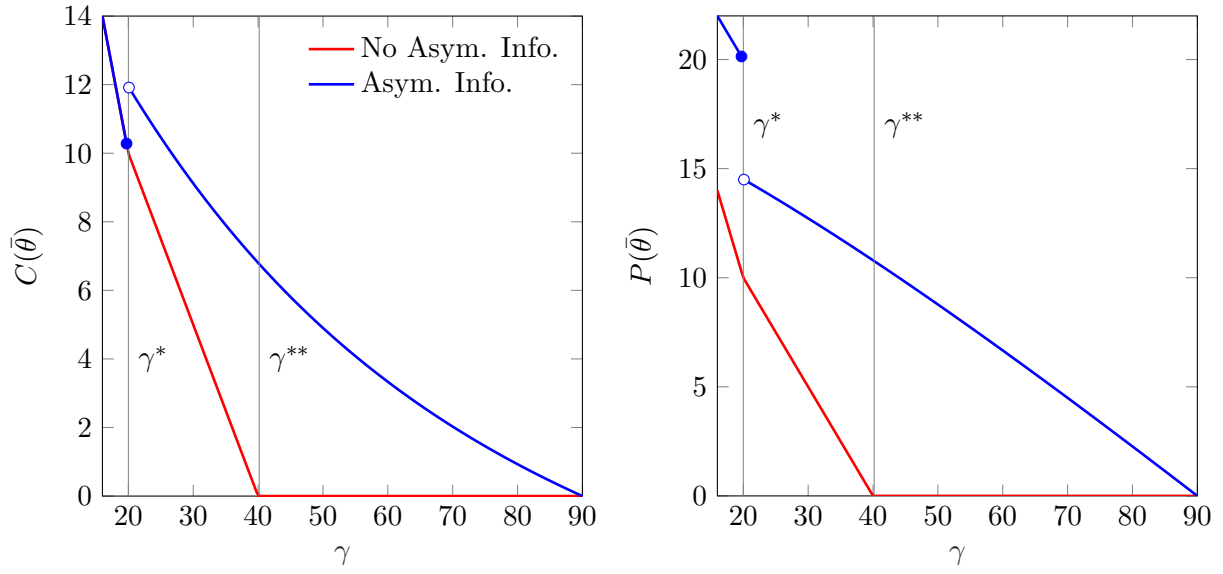
Corollary 2 (Blockholder's Payoff).

- *If condition (9) satisfied, then the blockholder payoff is the same with and without asymmetric information.*

- *On the other hand, if condition (18) is satisfied, the equilibrium blockholder payoff in the high state is higher with asymmetric information, while the equilibrium payoff in the low state is unaffected by asymmetric information.*[19]

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asymmetric information. With asymmetric information, the price in the high state is $p(x, \bar{\theta}) = (\mu/r)x + \bar{p}x$ for some coefficient $\bar{p} > C(\bar{\theta}) = \bar{C}$. The conclusion follows from the fact that $C(\bar{\theta})$ is higher in the case with asymmetric information. For the trading speed we have that, because the price is higher with asymmetric information in the high state, and it is unaffected in the low state, it follows from the asset pricing equation (17) that $q(x, \bar{\theta})p_x(x, \bar{\theta})$ is higher with asymmetric information. As $q(x, \bar{\theta}) < 0$, and $p_x(x, \bar{\theta})$ is positive and larger with asymmetric information, it must be the case that $q(x, \bar{\theta})$ is also higher with asymmetric information (that is, its absolute value is smaller).

[19]The fact that payoffs are the same in the smooth trading case follows directly from Propositions 1 and 4. In the case where condition (18) is satisfied, the equilibrium payoff of the high type is $V(x, \bar{\theta}) = (\mu/r)x + \frac{1}{2}\bar{C}x^2$, where $\bar{C} = (\phi\bar{\theta}^2 - \gamma + 2\bar{q}(\bar{p} - \bar{C}))/(r + \lambda_L)$, $\bar{q} > 0$ and $\bar{p} > \bar{C}$. Hence, $\bar{C} > (\phi\bar{\theta}^2 - \gamma)/(r + \lambda_L)$ so it follows from Propositions 2 and 4 that the continuation payoff with asymmetric information is higher than the payoff without asymmetric information.

(a) Coefficient $C(\bar{\theta})$ of blockholder's value function.

(b) Coefficient $P(\bar{\theta})$ of price function $p(x,\bar{\theta}) = \mu/r + P(\bar{\theta})x$.

Figure 3: Impact of asymmetric information on blockholder's payoff and prices. The blue line corresponds to the equilibrium with information asymmetries while the red line corresponds to the case with symmetric information. γ^* corresponds to inventory cost that makes condition (9) hold with equality, while $\gamma^{**} = \phi\bar{\theta}^2$. Parameters: $\mu = 1$, $r = 1$, $\phi = 10$, $\lambda_L = 1$, $\lambda_H = 1$, $\bar{\theta} = 2$, $\underline{\theta} = 1$.

This result shows that information asymmetry not only increases the stock price but also boosts the blockholder payoff, in stark contrast with static settings or settings without effort. This leads us to the following corollary.

Corollary 3 (Welfare). *Asymmetric information leads to a Pareto improvement, relative to the case with symmetric information. Small investors are always better-off when the blockholder is privately informed about θ_t. Similarly, the blockholder's payoff is weakly higher with asymmetric information.*

Again, two cases must be distinguished here. First, when inventory costs are low, the blockholder payoff is invariant to the information environment (i.e., private vs public information about θ). When inventory costs are large, the blockholder payoff is strictly higher under asymmetric information. This is in stark contrast with a static setting (Leland and Pyle (1977)) where signaling incentives would impose a deadweight cost on the sender, forcing him to hold his shares, to signal his productivity despite the existence of gains from trade. In a static setting, this signaling inefficiency operates in conjunction with the sender's market power. But in a dynamic setting the blockholder's size does not necessarily translates into market power, due to the Coase conjecture: the blockholder's lack of commitment means that no matter the information environment he always trades at a competitive price that equals his own marginal valuation. As a result, signaling incentives do not affect the blockholder's payoff (when γ is low).

Surprisingly, our analysis suggests that contrary to conventional wisdom, small uninformed investors also benefit from the blockholder having access to private information. As mentioned above, this is due to the blockholder holding his shares for longer, particularly when he is most productive, which increases the duration of the blockholder's effort, and boosts the firm's productivity and the expected cash flows. Therefore, the value of small investors holdings goes up when the blockholder has access to private information.

5 IPO Design

The previous analysis assumes that the blockholder's initial block x_0 is exogenous. However, in practice, x_0 may arise endogenously when an entrepreneur (henceforth, the issuer) sells the stock to investors via an Initial Public Offering (IPO). To account for this, and understand the optimal IPO design, here we endogenize x_0 as the outcome of an optimal IPO whereby the issuer sells the stock to two types of investors, a privately informed blockholder, and a continuum of uninformed investors. As the small investors, the entrepreneur is uncertain about θ_0 and his belief is $\Pr(\theta_0 = \bar{\theta}) = \pi$.

As usual, we assume that the issuer has commitment power as a mechanism designer. The IPO

is an optimal mechanism specifying i) the fraction of shares initially allocated to the blockholder $x_0(\theta)$, and ii) the transfer $T(\theta)$ that the blockholder must pay in exchange for the block.

Formally, the issuer solves the following program:

$$\max_{\{T(\theta),x_0(\theta)\}_{\theta\in\{\underline{\theta},\bar{\theta}\}}} E^\theta[T(\theta) + (1 - x_0(\theta))p(x_0(\theta),\theta)]$$

subject to

$$V(x_0(\theta),\theta) - T(\theta) \geq V(x_0(\theta'),\theta) - T(\theta'), \ \forall \theta, \theta' \in \{\underline{\theta},\bar{\theta}\}$$

$$V(x_0(\theta),\theta) - T(\theta) \geq V(0,\theta), \ \forall \theta \in \{\underline{\theta},\bar{\theta}\}$$

(23)

The issuer allocates a block $x_0(\theta)$ to the blockholder in exchange for a monetary transfer $T(\theta)$. The remainder $1 - x_0(\theta)$ is sold in the open market at a price $p(x_0,\theta)$ determined competitively (see equation 1).

The entrepreneur must satisfy participation constraints and incentive compatibility constraints that are type dependent as they depend on the subsequent dynamic trading payoffs. Consistent with practice, we can think of this mechanism as entailing two stages: a first stage, similar to a pre-IPO placement,[20] where the issuer negotiates privately with the blockholder and, a second stage, or the actual IPO, where small investors bid for the remainder shares $1 - x_0$.

Observe that implicitly we allow for price discrimination between the blockholder and the small investors, because the average price paid by the blockholder is $\frac{T(\theta)}{x_0(\theta)}$ may be different from the equilibrium $p(x,\theta)$. Arguably, this is the most realistic case, and qualitative predictions do not hinge on the possibility of discrimination[21].

5.1   A Static Benchmark

Before studying the case in which the blockholder can sell his shares after the IPO, we consider the case where this is not permitted. This represents a static situation where the owner can prevent the blockholder from "flipping" after the IPO and selling his shares. Specifically, suppose the seller offers a mechanism $(T(\theta), x_0(\theta))$ and enforces $x_t = x_0$ for all t. This benchmark is useful because it captures a static problem, and allows us to isolate the effect of dynamic trading on the IPO design.

First notice that in this case the continuation payoff of the blockholder continues to be given by $V(x,\theta)$ (see equation 19)). The reason again is related to the Coase conjecture: the possibility of

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[20]A pre-initial public offering placement is a private sale of large blocks of shares before a stock is listed on a public exchange. The buyers are typically private equity firms, hedge funds, and other institutions willing to buy large stakes in the firm. Due to the size of the investments being made and the risks involved, the buyers in a pre-IPO placement usually get a discount from the price stated in the prospective for the IPO.

[21]The analysis without price discrimination is available from the authors upon request.

trading does not provide any benefit to the blockholder in our dynamic setting as $q(x, \theta)(V_x(x, \theta) - p(x, \theta)) = 0$. In other words, the inability to trade post IPO does not affect the blockholder's payoffs. However, this restriction does affect the equilibrium market price $p(x, \theta)$ because it makes the blockholder's effort more persistent.

The price $p(x, \theta)$ still solves the HJB equation 5 but now subject to the restriction $q(x, \theta) = 0$. This yields the following prices:

$$p(x, \bar{\theta}) = \frac{\mu}{r} + \frac{(r + \lambda_H)\,\bar{\theta}^2 + \lambda_L \underline{\theta}^2}{r\,(r + \lambda_H + \lambda_L)} \phi x = \frac{\mu}{r} + \left(C(\bar{\theta}) + \frac{\gamma}{r}\right) x$$

$$p(x, \underline{\theta}) = \frac{\mu}{r} + \frac{\lambda_H \bar{\theta}^2 + (r + \lambda_L)\,\underline{\theta}^2}{r\,(r + \lambda_H + \lambda_L)} \phi x = \frac{\mu}{r} + \left(C(\underline{\theta}) + \frac{\gamma}{r}\right) x.$$

Naturally, the blockholder's inability to trade makes the price more sensitive to the block x, simply because the cash flows are now more persistent.

Armed with the equilibrium prices, we can consider the optimal IPO. This is a standard mechanism design problem with quasi-linear preferences, except that the valuations $V(x, \theta)$ are type dependent and endogenous (as in an auction with resale opportunities) being given by the HJB equations 1. The blockholder's valuation satisfy the standard single crossing condition $V_{x\theta}(x, \theta) > 0$, which implies that the optimal allocation $x_0(\theta)$ should be monotone in θ to be incentive compatible.

To solve this problem, we assume and verify that the low type's participation constraint and the high type's incentive compatibility constraints bind. The objective is concave in $x_0(\theta)$, and the optimal allocation is characterized by the first order conditions. The following proposition characterizes the solution.

Proposition 5 (Static IPO Allocation). *The optimal allocation in a static setting without post-IPO trading satisfies:*

$$x(\underline{\theta}) = \frac{C(\underline{\theta}) + \frac{\gamma}{r}}{C(\underline{\theta}) + \frac{2\gamma}{r} + \frac{\pi}{1-\pi}\left(C(\bar{\theta}) - C(\underline{\theta})\right)}$$

$$x\left(\bar{\theta}\right) = \frac{C(\bar{\theta}) + \frac{\gamma}{r}}{C(\bar{\theta}) + \frac{2\gamma}{r}}.$$

The solution has a familiar flavor. There is no-distortion at the top: the allocation of the high type $\bar{\theta}$ coincides with that arising under symmetric information, and maximizes the social surplus $(V(x, \theta) + (1-x)p(x, \theta))$. By contrast, the allocation of the low type $\underline{\theta}$ is distorted downwards, as a means to reduce the information rent earned by $\bar{\theta}$, which is proportional to $x_0(\bar{\theta})$. The size of the distortion to $x_0(\underline{\theta})$ is proportional to the probability of the high type π.

From a social point of view, the IPO is inefficient because the blockholder's ownership is smaller

than that required to maximize the social surplus, given the tension between efficiency and rent extraction. We can now proceed to analyze the dynamic case with ex-post trading. First we consider the symmetric information environment.

5.2 Optimal IPO with ex-post trading: Symmetric Information

We return to the baseline setting with post-IPO trading. First, we consider the solution with symmetric information. When the productivity θ_t is observable, the IPO mechanism does not need to satisfy incentive compatibility constraints. Naturally, the blockholder participation constraint is binding, so for each θ, the allocation solves

$$\max_{x_0(\theta)} V(x_0(\theta), \theta) + \big(1 - x_0(\theta)\big)p(x_0(\theta), \theta).$$

Taking the first order condition yields

$$V_x(x_0(\theta), \theta) - p(x_0(\theta), \theta) + \big(1 - x_0(\theta)\big)p_x(x_0(\theta), \theta) = 0.$$

From inspection of the first order conditions two effects become apparent. First, an additional share sold to the blockholder allows the seller to increase the fee $T(\theta)$ by an amount equal to the blockholder's marginal valuation $V_x(x, \theta)$, but it causes an opportunity cost, as it reduces the revenues coming from small investors by $p(x_0, \theta)$. Second, an additional share sold to the blockholder increases the blockholder's monitoring effort, thus increasing the stock price. Under symmetric information, the first effect is zero, because, the blockholder always faces a competitive price post-IPO such that $p = V_x$. Hence, the only relevant effect is the monitoring effect, which is always positive. In summary, under symmetric information the first order condition boils down to

$$\big(1 - x_0(\theta)\big)p_x(x_0(\theta), \theta) = 0,$$

which yields as a solution $x_0(\theta) = 1$. We conclude that symmetric information leads to a very large block.

The average price paid by the blockholder is $\hat{p}(x_0(\theta)) = V(1, \theta)$ while the price immediately after the IPO in the secondary market is $p(1, \theta) = V_x(1, \theta)$. Because $V(x, \theta)$ is convex in x, this implies that the blockholder pays a lower price, $\hat{p}(x_0(\theta), \theta) < p(x_0(\theta), \theta)$.

Proposition 6. *With symmetric information and post IPO trading the optimal mechanism assigns full ownership to the blockholder, $x_0(\theta) = 1$ regardless of productivity θ_0. The blockholder pays a lower price than the small investors.*

Consistent with empirical evidence and previous results (see Stoughton and Zechner (1998); Liu and Xiong (2010)) the blockholder pays a lower price than the small investors.[22] This is natural given that the blockholder exerts a positive externality on the firm's productivity. This does not imply the issuer is "leaving money on the table" as is often argued in the popular press.[23]

Next, we consider the case with asymmetric information.

5.3 Optimal IPO with ex-post Trading: Asymmetric Information

In this section, we study the optimal IPO mechanism under asymmetric information and post-IPO trading. In addition to participation constraints, the allocation must satisfy incentive compatibility constraints. These incentive constraints are shaped in equilibrium by the blockholder's own post-IPO trading incentives. As we shall see, unlike in static models, the combination of post IPO trading and information asymmetry may force the owner to offer a pooling allocation, although as we have seen, this would never happens in a static setting.

The design program is characterized by the problem in (23) but now the blockholder's payoff corresponds to the continuation payoff in Proposition (1). The following proposition describes the optimal mechanism.

Proposition 7. *Suppose that the inventory cost satisfies condition* (9)*. The optimal allocation at time zero* $(x_0(\theta), T(\theta))$ *is as follows:*

1. *If*

$$\frac{\pi}{1-\pi} \geq \frac{\gamma C(\underline{\theta})}{\left(\phi\bar{\theta}^2 + \lambda_L C(\underline{\theta})\right)\left(C(\bar{\theta}) - C(\underline{\theta})\right)} \tag{24}$$

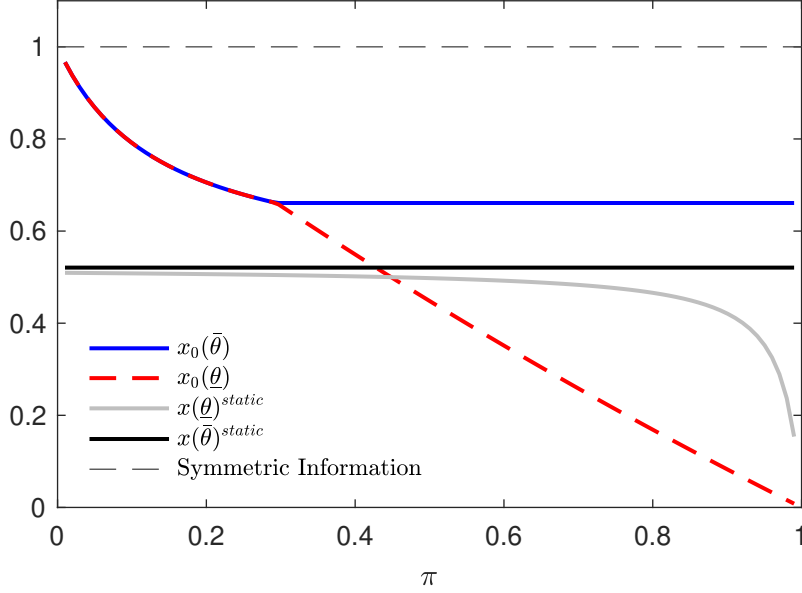   *then the allocation at time zero is*

$$x_0(\underline{\theta}) = \frac{C(\underline{\theta})}{C(\underline{\theta}) + \frac{\pi}{1-\pi}\left(C(\bar{\theta}) - C(\underline{\theta})\right)}$$

$$x_0(\bar{\theta}) = \frac{\phi\bar{\theta}^2 + \lambda_L C(\underline{\theta})}{\phi\bar{\theta}^2 + \lambda_L C(\underline{\theta}) + \gamma},$$

Figure 4: Optimal IPO allocation. $x_0(\theta)$ indicates the optimal allocation in the case with asymmetric information (both at time zero and afterwards), while $x(\theta)^{static}$ corresponds to the allocation in the absence of trading by the blockholders after the IPO. Parameters: $\mu = 1$, $\gamma = 10$, $\phi = 10$, $\lambda_H = 1$, $\lambda_L = 1$, $\bar{\theta} = 1.25$, $\underline{\theta} = 0.75$.

and the blockholder payment is

$$T(\underline{\theta}) = V(x_0(\underline{\theta}), \underline{\theta})$$
$$T(\bar{\theta}) = V(x_0(\bar{\theta}), \bar{\theta}) + V(x_0(\underline{\theta}), \underline{\theta}) - V(x_0(\underline{\theta}), \bar{\theta}).$$

2. *If condition* (24) *is not satisfied, then the optimal allocation at time zero is pooling, that is, $x_0(\bar{\theta}) = x_0(\underline{\theta}) = x_0$, and the blockholder payment is $T(\bar{\theta}) = T(\underline{\theta}) = V(x_0, \underline{\theta})$, where x_0 is*

$$x_0 = \min\left\{ \frac{\pi\left(\phi\bar{\theta}^2 + \lambda_L C(\underline{\theta})\right) + (1-\pi)(r+\lambda_L)C(\underline{\theta})}{2\pi\left(\phi\bar{\theta}^2 + \lambda_L C(\underline{\theta})\right) + (1-2\pi)(r+\lambda_L)C(\underline{\theta})}, 1 \right\}.$$

This result shows that the possibility of post IPO trading critically affects the allocation of the IPO. In the absence of post-IPO trading, the mechanism always features a separating menu, with the "no distortion at the top" property. The high type receives a larger allocation and a lower price per-share. The lower type's allocation is smaller and distorted downwards, relative to the symmetric information case, in order to reduce the informational rents earned by the high type. Since in the static setting, the blockholder cannot sell, the monotonicity IC requirement is never violated in the relaxed program (i.e, the program that ignores the monotonicity constraint) and

the allocation is always separating.

But with post-IPO trading, a monotone allocation may not be optimal, yet it is still required for incentive compatibility. The issue arises because the high type faces a particularly illiquid aftermarket in which selling is too costly (given his price impact) and where in equilibrium the price $p(x, \bar{\theta})$ is larger than his own marginal valuation $V_x(x, \bar{\theta})$. Hence, from the seller's perspective it is relatively costly to allocate a large x_0 to the high type, because by doing so, the seller incurs an opportunity cost, as he cannot sell these shares to small investors, who are willing to pay a higher price. This effect moderates the seller's incentive to allocate a very large block to the high type. However, this incentive is not present for the low type. The low type faces a competitive price ex-post, whereby $p(x, \theta) = V_x(x, \theta)$. In other words, on the margin, the willingness to pay by the market and by the blockholder are the same when productivity is low $\theta = \underline{\theta}$. Hence, in this case case, the issuer has an incentive to assign a large block to the blockholder to stimulate his monitoring effort and thus cash in the effect on the firm's cash flows (We can see this by looking at the maximization problem when we omit the incentive compatibility constraint.)

As a thought experiment, consider the case in which θ_0 is known, but after time zero its evolution is privately observed by the blockholder. In this case, we would expect that the optimal allocation assigns more shares to the high type. This would be the case in the static model without post-IPO trading; however, it is not the case in the presence of post-IPO trading (and asymmetric information in the secondary market). This is illustrated in Figure 5. The solid black line indicates the participation constraint while the thin solid line indicates the iso-profit curve. The unconstrained optimum is indicated by points A and B. Because the low type sells his shares very quickly after the IPO, it is optimal to allocate the maximum possible of shares to the blockholder (A). However, because the high type does not sell immediately, there is a tradeoff between inventory cost and productivity, so the number of shares assigned is strictly lower than the maximum (B).

Of course, under asymmetric information there is also a countervailing incentive to reduce the allocation of the low type $x(\underline{\theta})$ as a means to reduce the information rent earned by the high type. In the right panel of Figure 5, when the probability of the high type is large, the information rent effect dominates, and we get the traditional non-distortion at the top result. The optimal allocation for the high type, indicated by point C, is "efficient" and the high type's information rent is given by the vertical difference between B and C. Similarly the low type allocation, indicated by point D, is such the participation constraint of the low type is binding and the hight type is indifferent between C and D. Crucially, in this case point D is to the left of point C so the monotonicity constraint is satisfied. In sum, when π is large, minimizing the information rents of the high type becomes the seller's dominant concern. The allocation of the low type is strongly distorted downwards and monotonicity ceases to be an issue so a standard separating equilibrium emerges.

The situation is different when the probability of the high type is low π, which is illustrated in the right panel in Figure 5. Since the information rent is unlikely to be paid, the distortion is not helpful, so the seller has an incentive to allocate a large block to the low type $\underline{\theta}$ and the monotonicity constraint is violated (D is to the right of C). If the seller were to offer the menu C and D, the low type blockholder would be better off choosing C rather than D (the allocation C is inside the shaded area indicating the allocations that are preferred to D). As a consequence the seller finds it optimal to offer the pooling allocation E which is "inefficient" for both types.[24]

The previous distinction between the pooling and the separating cases have empirical implications for the volatility of prices following the IPO. In the case of a pooling allocation, the allocation is uninformative, and the IPO price is given by $p_0 = \pi p(x_0, \bar{\theta}) + (1-\pi) p(x_0, \underline{\theta})$. However, immediately after the IPO, the price jumps up or down depending on the trading behavior of the blockholder. This generates significant price volatility in the secondary market following the IPO. This contrast with the static case in which the allocation is separating. The IPO price is $p_0 = p(x_0(\theta), \theta)$ in this case, so there is no price volatility in the secondary market following the IPO (trading by the low type blockholder is anticipated by the market). Thus, the previous analysis predicts that the volatility of prices following the IPO should be higher when π is low.

(a) Separating allocation for high π (b) Pooling allocation for low π
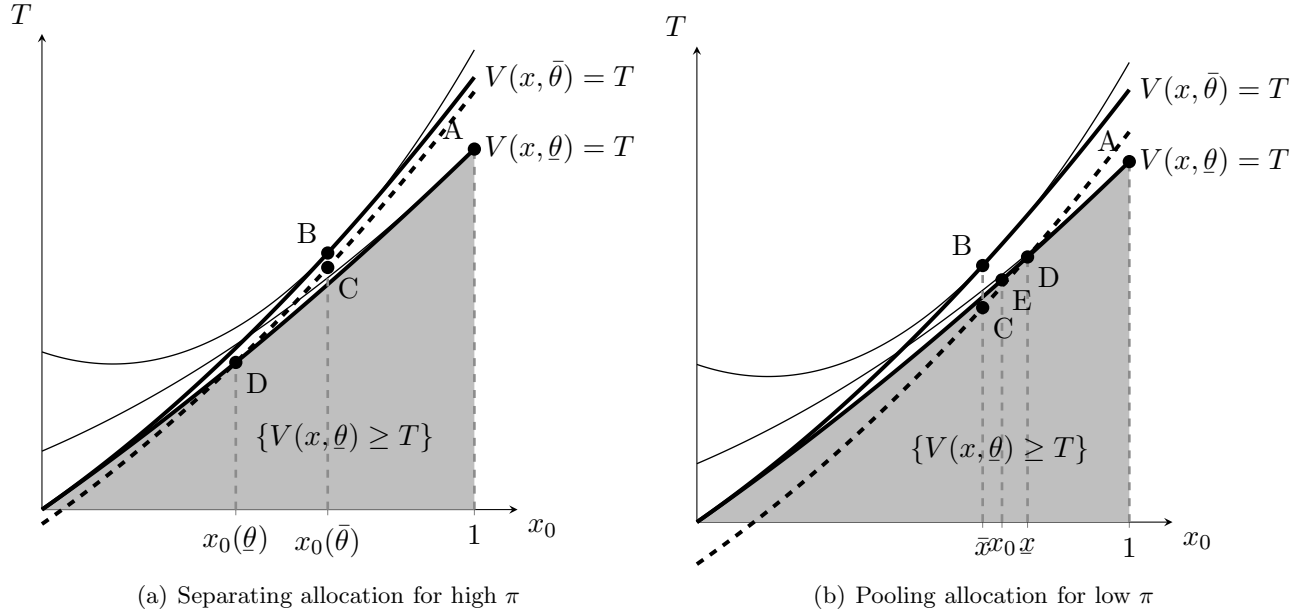
Figure 5: Optimal IPO allocation: optimistic vs pessimistic beliefs.

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[24]In principle, when the monotonicity constraint is violated, it could be optimal to use a random mechanism (so the monotonicity constrain only holds in expectation). We show in the appendix that it is not the case, and that the optimal mechanism is always deterministic.
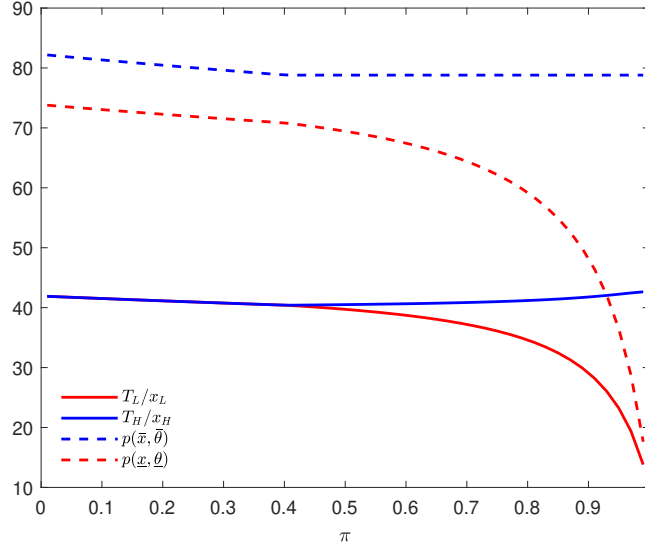
Figure 6: Mean price paid by blockholder vs small investors. Parameters: $\mu = 1$, $\gamma = 4$, $\phi = 10$, $\lambda_H = 1$, $\lambda_L = 1$, $\bar{\theta} = 1.25$, $\underline{\theta} = 0.75$.

6   The Role of Lockup Periods

As previously discussed the blockholder exerts a positive externality on the firm via monitoring. This induces the issuer to sell shares to the blockholder at a discount. Unfortunately, the blockholder's lack of commitment, and his tendency to sell over time, makes this positive externality short-lived.

To deal with this issue in practice, firms rely on lockup periods, whereby the blockholder is not allowed to sell his shares for some time after the IPO. A lockup period typically specifies that the blockholder can't directly or indirectly, sell, offer, contract to sell, make any short sale, pledge or otherwise dispose of any shares of Common Stock or any securities convertible into or exercisable for or any rights to purchase or acquire Common Stock for a period of 180 days following the commencement of the public offering of the Stock by the Underwriters.[25] In principle, lockup periods can be beneficial to align incentives in the presence of asymmetric information and moral hazard.[26]

Here we consider the effect of a fixed lockup period τ on the optimal IPO design. That is we assume that the blockholder can't sell his shares before time τ. Our focus is to analyze the effect of lockups, taking them as given. In our model, the issuer's revenue is increasing in the lock-up

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[25]See Field and Hanka (2001); Bradley et al. (2001); Aggarwal et al. (2002).

[26]For example Brav and Gompers (2003) examine the determinants of lockup periods and find that lock up periods are mostly used to deal with moral hazard. In particular, they find that lockups are longer when moral hazard is stronger.

period, so the optimal lockup period would always be $\tau = \infty$. Of course, such a lockup period would be too costly to implement in practice, insofar as early investors would not be willing to participate. So instead of directly modeling the frictions that lead to limited lockups we take them as given.

First, we derive the payoff for the blockholder and the price at time zero. Deriving the payoff of the blockholder is easy in the case in which condition (9) is satisfied, so the equilibrium entails smooth trading by the low type. In equilibrium, the blockholder's continuation payoff is the same as that obtained in the absence of trade, so we find that the blockholder is unaffected by the lockup period (this is not true if condition (18) is not satisfied). Indeed, we have that

$$
\begin{aligned}
V(x_0, \theta_0 | \tau) &= \mathbb{E}\left[\int_0^\tau e^{-rt} \left(\mu x_0 + \frac{1}{2}\left(\phi\theta_t^2 - \gamma\right)x_0^2 \right) dt + e^{-r\tau}V(x_0, \theta_\tau)\Big|\theta_0 \right] \\
&= \frac{\mu}{r}x_0 + \frac{1}{2}C(\theta_0)x_0^2.
\end{aligned}
$$

On the other hand, the lock up period does have an impact on stock prices. Since the blockholder is not allowed to trade, the market anticipates a reduction in moral hazard and higher cash flows due to more monitoring.

Let $\pi(t, \theta_0)$ be the probability that $\theta_t = \bar{\theta}$, which is given by $\pi(t, \underline{\theta}) = \bar{\pi}\left(1 - e^{-(\lambda_H + \lambda_L)t}\right)$ and $\pi(t, \bar{\theta}) = \bar{\pi} + (1 - \bar{\pi})e^{-(\lambda_H + \lambda_L)t}$, where $\bar{\pi} \equiv \lambda_H/(\lambda_H + \lambda_L)$ is the stationary distribution (Karlin and Taylor, 1981). Thus, with lock up period τ, the price at time zero is

$$
\begin{aligned}
p(x_0, \theta_0 | \tau) &= \mathbb{E}\left[\int_0^\tau e^{-rt} \left(\mu + \phi\theta_t^2 x_0 \right) dt + e^{-r\tau}p(x_0, \theta_\tau)\Big|\theta_0 \right] \\
&= \frac{\mu}{r} + P(\theta_0, \tau)x_0
\end{aligned}
$$

where[27]
$$
P(\theta, \tau) = C(\theta) + \frac{\gamma}{r}\left(1 - e^{-r\tau}\right) + e^{-r\tau}\pi(\tau, \theta)\left(\frac{\phi\bar{\theta}^2 + \lambda_L C(\underline{\theta})}{r + \lambda_L} - C(\bar{\theta}) \right).
$$

The derivation of the optimal allocation at time zero follows the one in Section 5.3, with the only

---

[27]Notice that
$$
V_x(x_0, \theta_0 | \tau) = \mathbb{E}\left[\int_0^\tau e^{-rt} \left(\mu + \left(\phi\theta_t^2 - \gamma\right)x_0 \right) dt + e^{-r\tau}V_x(x_0, \theta_\tau)\Big|\theta_0 \right],
$$

so we have that
$$
\begin{aligned}
p(x_0, \theta_0 | \tau) - V_x(x_0, \theta_0 | \tau) &= \mathbb{E}\left[\int_0^\tau e^{-rt}\gamma x_0 dt + e^{-r\tau}\left(p(x_0, \theta_\tau) - V_x(x_0, \theta_\tau)\right)\Big|\theta_0 \right] \\
&= \frac{\gamma}{r}(1 - e^{-r\tau})x_0 + e^{-r\tau}\pi(\tau, \theta_0)\left(\frac{\phi\bar{\theta}^2 + \lambda_L C(\underline{\theta})}{r + \lambda_L} - C(\bar{\theta}) \right)x_0.
\end{aligned}
$$

difference that we substitute $p(x,\theta)$ by $p(x,\theta|\tau)$. The following proposition presents the optimal time zero allocation.

Proposition 8. *Suppose that the inventory cost satisfies condition* (9). *The optimal allocation mechanism at time zero* $(x_0(\theta), T(\theta))$ *is as follows:*

1. *If*

$$\frac{\pi}{1-\pi} \geq \frac{\pi(\tau,\bar{\theta})\left(\frac{P(\theta,\tau)}{P(\bar{\theta},\tau)} - \frac{\pi(\tau,\theta)}{\pi(\tau,\bar{\theta})}\right)e^{-r\tau}\left(\frac{\phi\bar{\theta}^2 + \lambda_L C(\theta)}{r+\lambda_L} - C(\bar{\theta})\right) - \left(1 - \frac{P(\theta,\tau)}{P(\bar{\theta},\tau)}\right)\frac{\gamma}{r}\left(1 - e^{-r\tau}\right)}{C(\bar{\theta}) - C(\theta)} \quad (25)$$

then the allocation at time zero is

$$\begin{aligned}
x_0(\theta) &= \frac{P(\theta,\tau)}{2P(\theta,\tau) - C(\theta) + \frac{\pi}{1-\pi}\left(C(\bar{\theta}) - C(\theta)\right)} \\
x_0(\bar{\theta}) &= \frac{P(\bar{\theta},\tau)}{2P(\bar{\theta},\tau) - C(\bar{\theta})},
\end{aligned} \quad (26)$$

and the blockholder payment is

$$\begin{aligned}
T(\theta) &= V(x_0(\theta), \theta|\tau) \\
T(\bar{\theta}) &= V(x_0(\bar{\theta}), \bar{\theta}|\tau) + V(x_0(\theta), \theta|\tau) - V(x_0(\theta), \bar{\theta}|\tau).
\end{aligned}$$

2. *If condition* (25) *is not satisfied, then the optimal allocation at time zero is pooling, that is,* $x_0(\bar{\theta}) = x_0(\theta) = x_0$, *and the blockholder payment is* $T(\bar{\theta}) = T(\theta) = V(x_0, \theta|\tau)$, *where* x_0 *is*

$$x_0 = \min\left\{\frac{\pi P(\bar{\theta},\tau) + (1-\pi)P(\theta,\tau)}{2\left[\pi P(\bar{\theta},\tau) + (1-\pi)P(\theta,\tau)\right] - C(\theta)}, 1\right\}.$$

Figure 7 presents a numerical example with comparative statics with respect to the lockup period. This figure shows that two different patterns can arise depending on the parameters (in particular, depending if the allocation for the low type without lockup is below or above the one in the static model). In figure 7(a), the allocation is monotonically decreasing in the length of the lockup period. In the absence of a lockup period, it is optimal to increase the initial allocation to counteract the blockholder's tendency to sell over time. This effect is reduced as the length of the lockup period is increased. Moreover, in the absence of a lockup period the optimal allocation is pooling, but as the length of the lockup period increases it becomes optimal to screen the blockholder. This is natural since the optimal allocation is separating in the absence of aftermarket trading.

On the other hand, figure 7(b) presents a case in which the allocation of the low type is increasing in the length of the lockup, and the allocation of the hight type is non-monotonic. In the absence of a lockup period, the low type is allocated very few shares (the inventory cost γ is high that we are close to the point where condition (9) is violated so the low type is not assigned any shares). This happens because the value of allocating shares to the low type is too low because he sells his shares too quickly. This problem becomes less severe in the presence of the lockup period, which allows to increase its shares allocation. The high type allocation is driven by two effects. On the one hand, as in figure 7(a), the presence of the lockup period reduces the need to allocate as many shares. On the other hand, as the allocation of shares for the low type increases, the incentive compatibility constraint requires to increase the number of shares for the high type as well. The latter effect dominates for short lockup periods, while the former one dominates for longer lockups. Overall, the high type share allocation is non-monotonic in the length of the lockup period.

The previous analysis also has implications for asset prices. First, by reducing the moral hazard problem in the after market, the introduction of lockup periods increases prices in the IPO. Secondly, by reducing trading in the aftermarket, the introduction of the lockup period reduces volatility in the aftermarket immediately following the IPO. This happens because the lack of trading by the blockholder reduces the informativeness of trading following the IPO. Finally, our analysis has implication for the reaction of prices at the time of the lockup expiration. Before the the expiration of the lockup, the price of the share is given by

$$p(t, \theta_0) = \big(\pi(t, \theta_0)P(\bar{\theta}, \tau - t) + (1 - \pi(t, \theta_0))P(\underline{\theta}, \tau - t)\big)x_0(\theta_0).$$

In particular, just before the expiration of the lockup period, the price is $p(\tau-, \theta_0) = \pi(\tau, \theta_0)p(x_0, \bar{\theta}) + (1 - \pi(\tau, \theta_0))p(x_0, \bar{\theta})$. Immediately, after the lockup expiration, upon observing the trading behavior of the blockholder, the price adjust to $p(x_0, \theta_\tau)$. Thus, in the absence of trading by the blockholder we should observe an increment in price while in the presence of trading we should observe a price decrease. Overall, the model predicts that we should see an increment in volatility. Brav and Gompers (2003) finds that prices tend to drop at the time of lockup expiration, and this price reduction is accompanied by a permanent increase in trading volume. Our results are partially consistent with their evidence. On the one hand, we predict an increment in volume and volatility. On the other hand, in terms of price impact, we predict a price decline if the blockholder sells at the lockup expiration, and price increment if the blockholder does not sell. Thus, in our model, the reaction of prices depends on the trading behavior of the blockholder, so we should observe a negative correlation between blocktrading and prices at the time of expiration. That being said, while our model predicts that the average abnormal return at the time of expiration should be zero,

the evidence in Brav and Gompers (2003) indicates that the reaction is on average negative.

(a) Initial allocation decreasing case   (b) Initial allocation non-monotonic case
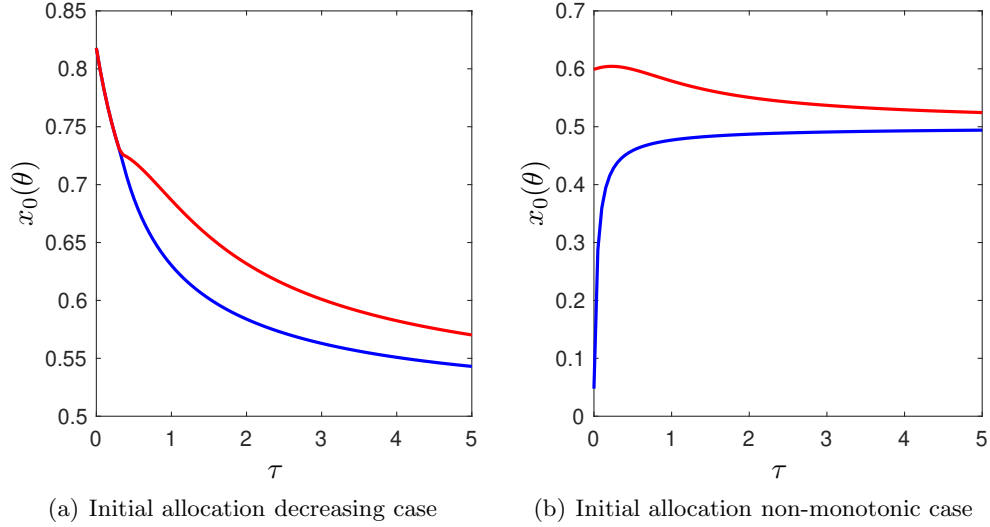
Figure 7: Example fo initial allocation as a function of lock up period τ. The red line indicates the allocation given $\theta_0 = \bar{\theta}$ while the blue line indicates the allocation given $\theta_0 = \underline{\theta}$. In the left panel, the allocation is decreasing in τ. In the right panel, the allocation is increasing for $\underline{\theta}$ and non-monotonic for $\bar{\theta}$. Paremeters: $\mu = 1$, $r = 0.05$, $\phi = 10$, $\lambda_L = 1$, $\lambda_H = 1$, $\bar{\theta} = 1.25$, $\underline{\theta} = 0.75$, $\pi_0 = 0.2$. The left panel has $\gamma = 10$ while the right panel has $\gamma = 10.5$.

7   Conclusions

This paper studies the monitoring and trading behavior of a blockholder with access to private information. After characterizing the dynamics for a given initial block, we endogenize the initial block size as the outcome of an optimal IPO.

Our analysis shows that when a blockholder has access to private information welfare improves. Asymmetric information leads the blockholder to hold their shares for longer, and monitor the firm more intensively, thus increasing the stock price. The lack of liquidity arising under asymmetric information, mitigates the blockholder's commitment problem, and allows him to sell slowly and extract some monopoly rents, contrary to the symmetric information case.

We also study the optimal IPO allocation, and show that the blockholder post IPO trading, significantly modifies the IPO properties, and leads under some conditions to allocations that are insensitive to blockholder productivity and significant post-IPO volatility.

This paper has implications for the long-standing debate on the role of liquidity for blockholder activism (see e.g., Norli et al. (2014); Edmans (2009)). Our results suggest that information asymmetry reduces the liquidity facing the blockholder, but has desirable social effects, insofar as it

leads to larger blocks and stronger monitoring (a similar point is made by Vanasco (2017)).

Our model has a number of limitations. For example, we assume that the blockholder's order flow is perfectly observable and we focus on separating equilibria. Perfect observability of the order flow is a simplification, because in practice these data is available to investors with some delay. This assumption also leads to an equilibrium where cash flows are uninformative, conditional on the order flow. Extending our model to allow for noise trading, to obscure the blockholder's order flow, is an interesting (and challenging) extension that we hope future research will address.

The observability of blockholder trading is also a policy question. The SEC requires that a blockholder discloses their stakes within 10 days of the purchase of more than 5% of the shares of a public company. This regulation, and the socially optimal level of the disclosure threshold is subject of an intense policy debate.[28] Evaluating the effects of this regulation is also an open theory question.

We have ruled out the possibility that the blockholder takes value-destroying actions, as is often argued in the popular press. Indeed, critics often warn that blockholder activists exacerbate firms' short-termist tendencies (See e.g., "Let's do it my Way", The Economist, May 13, 2013). This possibility could be incorporated in our model by allowing the blockholder effort to have, at the same time, a negative impact on the firm's cash flows and a positive effect on the blockholder's payoff.

Our analysis of the effect of lockup periods is restrictive. We consider lockup periods that are type independent, but one could consider the lockup period as an additional aspect of the optimal IPO. In this context, lockup periods would likely be type-dependent. This could be implemented for example, via a menu with price discounts for those blockholders who are willing to accept a longer lockup period. More generally, one could consider a dynamic mechanism whereby a blockholder is given extra shares as a reward for holding his shares sufficiently long. This is akin to the notion of loyalty shares whereby long-term blockholders are rewarded with special benefits such as preferential voting rights (see Bolton and Samama (2013)).

Finally, as another interesting extension one could consider the possibility of competition (or cooperation) among multiple blockholders with heterogenous beliefs to gain control of the firm and influence its corporate strategy (see e.g, Hadlock and Schwartz-Ziv (2019)).

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[28]For example, The Economist notes that "Wachtell, Lipton, Rosen & Katz, the law firm that invented the poison pill, has also been seeking to make things harder still for activists by proposing a rule that anyone building a stake of 5% or more in a firm must disclose it within one day, not ten as now. So far the Securities and Exchange Commission is showing little interest. Indeed, its chairman, Mary Jo White, has argued that activists attempts to jog boards are not always a bad thing." See Nasty Medicine, The Economist, Jul 5th, 2014.

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Appendix

A   Trading with Asymmetric Information

Proof of Lemma 1

Proof. First, we consider the trading problem of the high type

$$\max_q \ q(V_x(x,\bar{\theta}) - p(x,q)). \tag{27}$$

First, we establish than in any equilibria with smooth trading (that is, atomless) it must be that $p(x,q(x,\bar{\theta})) \geq V_x(x,\bar{\theta})$. Suppose that $V_x(x,\bar{\theta}) > p(x,q(x,\bar{\theta})) = p(x,\bar{\theta})$, then, as

$$p(x,q) = \Pr(\theta = \bar{\theta}|x,q)p(x,\bar{\theta}) + \Pr(\theta = \underline{\theta}|x,q)p(x,\underline{\theta}) \leq p(x,\bar{\theta}), \tag{28}$$

we have that for any $q > 0$, $q(V_x(x,\bar{\theta}) - p(x,q)) \geq q(V_x(x,\bar{\theta}) - p(x,\bar{\theta})) > 0$, and this implies that $\sup_{q\geq 0}\{q(V_x(x,\bar{\theta}) - p(x,q))\} = \infty$. Thus, in any atomless equilibria we must have that $V_x(x,\bar{\theta}) \leq p(x,\bar{\theta})$. By a similar argument, considering $q < 0$, we can conclude that $V_x(x,\underline{\theta}) \geq p(x,\underline{\theta})$. Thus, in any atomless equilibrium satisfying monotonicity, and for any $x > 0$, we have

$$p(x,\bar{\theta}) \geq V_x(x,\bar{\theta}) > V_x(x,\underline{\theta}) \geq p(x,\underline{\theta}) \tag{29}$$

Moreover, incentive compatibility requires that if $q(x,\bar{\theta}) > 0$ then $p(x,q(x,\bar{\theta})) = V_x(x,\bar{\theta})$ and $p(x,q) = p(x,\bar{\theta})$ for all $q > 0$. This follows from inequality (29) and the observation that, as the blockholder can always choose not to trade, it must be that $\max_{q\geq 0}\{q(V_x(x,\bar{\theta}) - p(x,q))\} \geq 0$. But then $q(x,\bar{\theta})$ is a solution of (27) only if $p(x,q) \geq p(x,\bar{\theta})$, so inequality (28) implies that $p(x,q) = p(x,\bar{\theta})$ for all $q > 0$. By a similar argument for the low type and $q < 0$, we can show that if $q(x,\underline{\theta}) < 0$, then $p(x,q) = p(x,\underline{\theta})$ for all $q < 0$.

The only remaining step in the proof is to show that, in any atomless equilibria satisfying condition (M), we have $q(x,\underline{\theta}) \leq 0 \leq q(x,\bar{\theta})$. First, we consider the case $q(x,\underline{\theta}) > 0$. Suppose that in equilibrium, for some $x > 0$ we have $q(x,\underline{\theta}) > 0$. Monotonicity together with the incentive compatibility constraint (4) require that $q(x,\bar{\theta}) > q(x,\underline{\theta})$. Using the inequality (29) we conclude that $q(x,\bar{\theta})(V_x(x,\bar{\theta}) - p(x,\bar{\theta})) = 0$. Thus, incentive compatibility requires that $q(x,\underline{\theta})(V_x(x,\bar{\theta}) - p(x,\underline{\theta})) \leq 0$, which implies that $V_x(x,\bar{\theta}) \leq p(x,\underline{\theta})$. However, this last inequality contradicts inequality (29), so we can conclude that in a monotonic equilibrium $q(x,\underline{\theta}) \leq 0$. Using a similar argument, we can verify that in any monotonic equilibrium $q(x,\bar{\theta}) \geq 0$. Finally, suppose that for some $q' \in (0, q(x,\bar{\theta}))$ we have that the market off-equilibrium belief is

$\beta = \Pr(\theta = \bar{\theta}|x, q') < 1$, then the equilibrium price must be $p(x, q') = \beta p(x, \bar{\theta}) + (1 - \beta)p(x, \underline{\theta}) < p(x, \bar{\theta}) = V_x(x, \bar{\theta})$. But then, the blockholder has a profitable deviation from $q(x, \bar{\theta})$ to q'. Thus, in equilibrium for any $q' \in (q(x, \bar{\theta}), 0)$ it must be the case that $\hat{\theta}(q, x) = \bar{\theta}$. Similarly, suppose that for some $q'' \in (0, q(x, \underline{\theta}))$, we have $\beta = \Pr(\theta = \bar{\theta}|x, q'') > 0$, then the equilibrium price must be $p(x, q'') = \beta p(x, \bar{\theta}) + (1 - \beta)p(x, \underline{\theta}) > p(x, \underline{\theta}) = V_x(x, \underline{\theta})$, which means that the low type blockholder has a profitable deviation from $q(x, \underline{\theta})$ to q''. Thus, in equilibrium for any $q'' \in (q(x, \underline{\theta}), 0)$ it must be the case that $\hat{\theta}(q, x) = \underline{\theta}$. $\square$

Proof of Proposition 1

Proof. We conjecture a solution for $V(x, \theta)$ of the form $V(x, \theta) = B(\theta)x + \frac{1}{2}C(\theta)x^2$. Under this conjecture the optimal effort is

$$a(\theta, x) = \phi\theta x.$$

Substituting the HJB equation we get

$$rV(x, \bar{\theta}) = \mu x + \frac{1}{2}\phi\bar{\theta}^2 x^2 - \frac{1}{2}\gamma x^2 - \lambda_L \left(V(x, \bar{\theta}) - V(x, \underline{\theta}) \right)$$
$$rV(x, \underline{\theta}) = \mu x + \frac{1}{2}\phi\underline{\theta}^2 x^2 - \frac{1}{2}\gamma x^2 - \lambda_H \left(V(x, \underline{\theta}) - V(x, \bar{\theta}) \right)$$

Substituting our conjecture

$$rB(\bar{\theta}) = \mu + \lambda_L \left(B(\bar{\theta}) - B(\underline{\theta}) \right)$$
$$rC(\bar{\theta}) = \phi\bar{\theta}^2 - \gamma - \lambda_L \left(C(\bar{\theta}) - C(\underline{\theta}) \right)$$
$$rB(\underline{\theta}) = \mu + \lambda_H \left(B(\underline{\theta}) - B(\bar{\theta}) \right)$$
$$rC(\underline{\theta}) = \phi\underline{\theta}^2 - \gamma - \lambda_H \left(C(\underline{\theta}) - C(\bar{\theta}) \right)$$

From here we can solve for the coefficients $(B(\theta), C(\theta))$ to get

$$B(\bar{\theta}) = B(\underline{\theta}) = \frac{\mu}{r}$$
$$rC(\underline{\theta}) = \phi\underline{\theta}^2 - \gamma + \phi\frac{\lambda_H}{r + \lambda_L + \lambda_H}(\bar{\theta}^2 - \underline{\theta}^2)$$
$$rC(\bar{\theta}) = \phi\bar{\theta}^2 - \gamma - \phi\frac{\lambda_L}{r + \lambda_L + \lambda_H}(\bar{\theta}^2 - \underline{\theta}^2)$$

The value function $V(x,\theta) = (\mu/r)x + C(\theta)x^2/2$ is convex in x for all θ only if

$$\underline{\theta}^2 + \frac{\lambda_H}{r + \lambda_L + \lambda_H}(\bar{\theta}^2 - \underline{\theta}^2) \geq \frac{\gamma}{\phi}$$

Finally, substituting in equations (7) and (8) we get

$$p(x,\bar{\theta}) = \frac{\mu}{r} + \frac{\left(\phi\bar{\theta}^2 + \lambda_L C(\underline{\theta})\right)x}{r + \lambda_L}$$

$$q(x,\underline{\theta}) = -\left(1 + \frac{\lambda_H}{r + \lambda_L}\right)\frac{\gamma}{C(\underline{\theta})}x.$$

$\square$

Proof of Proposition 3

Proof. In the presence of private benefits, the HJB equation becomes

$$rV(x,\bar{\theta}) = (\mu + b)x + \frac{1}{2}\phi\bar{\theta}^2 x^2 - \frac{1}{2}\gamma x^2 - \lambda_L\left(V(x,\bar{\theta}) - V(x,\underline{\theta})\right)$$

$$rV(x,\underline{\theta}) = (\mu + b)x + \frac{1}{2}\phi\underline{\theta}^2 x^2 - \frac{1}{2}\gamma x^2 - \lambda_H\left(V(x,\underline{\theta}) - V(x,\bar{\theta})\right)$$

As before, we conjecture a solution for $V(x,\theta)$ of the form $V(x,\theta) = B(\theta)x + \frac{1}{2}C(\theta)x^2$. Substituting our conjecture in the HJB equation

$$rB(\bar{\theta}) = \mu + b + \lambda_L\left(B(\bar{\theta}) - B(\underline{\theta})\right)$$
$$rC(\bar{\theta}) = \phi\bar{\theta}^2 - \gamma - \lambda_L\left(C(\bar{\theta}) - C(\underline{\theta})\right)$$
$$rB(\underline{\theta}) = \mu + b + \lambda_H\left(B(\underline{\theta}) - B(\bar{\theta})\right)$$
$$rC(\underline{\theta}) = \phi\underline{\theta}^2 - \gamma - \lambda_H\left(C(\underline{\theta}) - C(\bar{\theta})\right)$$

The private benefit only appears on the equations determining the coefficients of the linear term $B(\theta)$, which is now given by $B(\theta) = (\mu + b)/r$. The coefficients $C(\theta)$ are the same ones as in Proposition 1.

In the region in which the low type sells, and the high type does not trade, we have that $p(x,\underline{\theta}) = V_x(x,\underline{\theta})$, which means that

$$rp(x,\bar{\theta}) = \mu + \bar{\theta}a(x,\bar{\theta}) + \lambda_L\left(V_x(x,\underline{\theta}) - p(x,\bar{\theta})\right)$$
$$rV_x(x,\underline{\theta}) = \mu + \underline{\theta}a(x,\underline{\theta}) + q(x,\underline{\theta})V_{xx}(x,\underline{\theta}) + \lambda_H\left(p(x,\bar{\theta}) - V_x(x,\underline{\theta})\right),$$

A3

so we have that

$$p(x, \bar{\theta}) = \frac{\mu + \bar{\theta}a(x, \bar{\theta}) + \lambda_L V_x(x, \underline{\theta})}{r + \lambda_L}$$

$$q(x, \underline{\theta}) = -\frac{\mu + \underline{\theta}a(x, \underline{\theta}) + \lambda_H p(x, \bar{\theta}) - (r + \lambda_H)V_x(x, \underline{\theta})}{V_{xx}(x, \underline{\theta})}.$$

Substituting $V_x(x, \underline{\theta}) = (\mu + b)/r + C(\underline{\theta})x$ we get

$$q(x, \underline{\theta}) = \left(1 + \frac{\lambda_H}{r + \lambda_L}\right)\frac{b - \gamma x}{C(\underline{\theta})}.$$

From here we get that $q(x, \underline{\theta}) = 0$ if $x = b/\gamma$. Similarly, in the case of the high type, whenever the high type trades and the low type doesn't we have

$$rp(x, \underline{\theta}) = \mu + \underline{\theta}a(x, \underline{\theta}) + \lambda_H \left(V_x(x, \bar{\theta}) - p(x, \underline{\theta})\right)$$

$$rV_x(x, \bar{\theta}) = \mu + \bar{\theta}a(x, \bar{\theta}) + q(x, \bar{\theta})V_{xx}(x, \bar{\theta}) + \lambda_L \left(p(x, \underline{\theta}) - V_x(x, \bar{\theta})\right),$$

so

$$p(x, \bar{\theta}) = \frac{\mu + \underline{\theta}a(x, \underline{\theta}) + \lambda_H V_x(x, \bar{\theta})}{r + \lambda_H}$$

$$q(x, \bar{\theta}) = -\frac{\mu + \bar{\theta}a(x, \bar{\theta}) + \lambda_L p(x, \underline{\theta}) - (r + \lambda_L)V_x(x, \bar{\theta})}{V_{xx}(x, \bar{\theta})}.$$

Substituting $V_x(x, \bar{\theta}) = (\mu + b)/r + C(\bar{\theta})x$, we get

$$p(x, \underline{\theta}) = \frac{\mu + \underline{\theta}a(x, \underline{\theta}) + \lambda_H V_x(x, \bar{\theta})}{r + \lambda_H}$$

$$q(x, \bar{\theta}) = \left(1 + \frac{\lambda_L}{r + \lambda_H}\right)\frac{b - \gamma x}{C(\bar{\theta})}.$$

so $q(x, \bar{\theta}) = 0$ if $x = b/\gamma$.

$\square$

A4

Proof of Proposition 2

Proof. We conjecture a solution of the form

$$q(x, \bar{\theta}) = -\bar{q}x$$
$$V(x, \bar{\theta}) = \frac{\mu}{r}x + \frac{1}{2}\bar{C}x^2$$
$$p(x, \bar{\theta}) = \frac{\mu}{r} + \bar{p}x,$$

for coefficients $(\bar{q}, \bar{C}, \bar{p})$ to be determined. Substituting in the IC constraint we get

$$q(x, \bar{\theta}) = -\frac{\left(\gamma - \phi\underline{\theta}^2\right) + \lambda_H \bar{C}}{2\bar{p}}x, \tag{30}$$

so

$$\bar{q} = \frac{\gamma - \phi\underline{\theta}^2 + \lambda_H \bar{C}}{2\bar{p}}.$$

The next step is to solve for $\bar{p}$. Substituting our conjecture in the equation for the price we get

$$\bar{p} = \frac{\phi\bar{\theta}^2}{r + \lambda_L + \bar{q}}$$

Finally, we use the HJB equation of the high type to get an equation for $\bar{C}$.

$$(r + \lambda_L + 2\bar{q})\bar{C} = \left(\phi\bar{\theta}^2 - \gamma\right) + 2\bar{q}\bar{p}$$

Substituting $\bar{q}$ we get

$$\bar{C} = \frac{\phi\left(\bar{\theta}^2 - \underline{\theta}^2\right)}{r + \lambda_L - \lambda_H + 2\bar{q}}.$$

Summarizing, we get that $(\bar{q}, \bar{p}, \bar{C})$ solves

$$\bar{q} = \frac{\gamma - \phi\underline{\theta}^2 + \lambda_H \bar{C}}{2\bar{p}}$$
$$\bar{p} = \frac{\phi\bar{\theta}^2}{r + \lambda_L + \bar{q}}$$
$$\bar{C} = \frac{\phi\left(\bar{\theta}^2 - \underline{\theta}^2\right)}{r + \lambda_L - \lambda_H + 2\bar{q}}.$$

A5

Moreover, the following conditions need to be satisfied: $\bar{C} < \bar{p}$, $\bar{C} > 0$, and $\bar{q} > 0$. Solving for $\bar{p}$ and $\bar{q}$ we get

$$\bar{q} = \frac{\left(\gamma + \lambda_H \bar{C} - \phi \underline{\theta}^2\right)(r + \lambda_L)}{\phi(2\bar{\theta}^2 + \underline{\theta}^2) - \gamma - \lambda_H \bar{C}}$$

$$\bar{p} = \frac{\phi(2\bar{\theta}^2 + \underline{\theta}^2) - \gamma - \lambda_H \bar{C}}{2(r + \lambda_L)}$$

Substituting $\bar{q}$ in the equation for $\bar{C}$, we get the following

$$\lambda_H(r + \lambda_L + \lambda_H)\bar{C}^2 - \left[\lambda_H\left(\phi(2\underline{\theta}^2 + \bar{\theta}^2) - \gamma\right) - (r + \lambda_L)\left(\gamma + \phi(2\bar{\theta}^2 - \underline{\theta}^2)\right)\right]\bar{C}$$
$$- \phi\left(\bar{\theta}^2 - \underline{\theta}^2\right)\left(\phi(2\bar{\theta}^2 + \underline{\theta}^2) - \gamma\right) = 0$$

If $\phi(2\bar{\theta}^2 + \underline{\theta}^2) > \gamma$, then there exist a unique one positive root to the previous equation. The constants $\bar{q}$ and $\bar{p}$ are positive only if $\lambda_H \bar{C} < \phi(2\bar{\theta}^2 + \underline{\theta}^2) - \gamma$ (whenever the condition for smooth trading by the low type is violated, we have $\gamma/\phi > \underline{\theta}^2$). Evaluating the quadratic equation at $\bar{C} = (\phi(2\bar{\theta}^2 + \underline{\theta}^2) - \gamma)/\lambda_H$, we get that the left hand side is proportional to $\phi(2\bar{\theta}^2 + \underline{\theta}^2) - \gamma > 0$. Hence, $\bar{C} < (\phi(2\bar{\theta}^2 + \underline{\theta}^2) - \gamma)/\lambda_H$ so $\bar{q}$ and $\bar{p}$ are positive. Finally, the condition $\bar{p} > \bar{C}$ requires that

$$\bar{C} < \frac{\phi\left(2\bar{\theta}^2 + \underline{\theta}^2\right) - \gamma}{2(r + \lambda_L) + \lambda_H}$$

Substituting the right hand side in the quadratic equation for $\bar{C}$, we find that the previous condition is satisfied only if

$$\left(\phi\left(2\bar{\theta}^2 + \underline{\theta}^2\right) - \gamma\right) \times \left((r + \lambda_L)\left(\phi\underline{\theta}^2 + \gamma\right) - \lambda_H\left(\phi\bar{\theta}^2 - \gamma\right)\right) > 0.$$

It is straightforward to verify that the second term is positive whenever condition (9) is violated; hence, the previous inequality is satisfied as long as

$$\frac{\gamma}{\phi} < 2\bar{\theta}^2 + \underline{\theta}^2.$$

$\square$

B   Optimal IPO Allocation

Proof of Proposition 7

Proof. Suppose that the low type's participation constraint and the high type incentive compatibility are binding, then we can write the previous problem as

$$\max_{\{x_0(\theta)\}_{\theta \in \{\underline{\theta}, \bar{\theta}\}}} V(x_0(\underline{\theta}), \underline{\theta}) + \pi \Big[V(x_0(\bar{\theta}), \bar{\theta}) + \big(1 - x_0(\bar{\theta})\big) p(x_0(\bar{\theta}), \bar{\theta}) \Big]$$

$$+ (1 - \pi)\big(1 - x_0(\underline{\theta})\big) p(x_0(\underline{\theta}), \underline{\theta}) - \pi V(x_0(\underline{\theta}), \bar{\theta})$$

First, we consider the first order condition for $x_0(\bar{\theta})$

$$V_x(x_0(\bar{\theta}), \bar{\theta}) - p(x_0(\bar{\theta}), \bar{\theta}) + \big(1 - x_0(\bar{\theta})\big) p_x(x_0(\bar{\theta}), \bar{\theta}) = 0.$$

The second order condition is

$$V_{xx}(x_0(\bar{\theta}), \bar{\theta}) - 2 p_x(x_0(\bar{\theta}), \bar{\theta}) = C(\bar{\theta}) - 2 \frac{\big(\phi \bar{\theta}^2 + \lambda_L C(\underline{\theta})\big)}{r + \lambda_L}$$

$$= -\frac{\phi \bar{\theta}^2 + \lambda_L C(\underline{\theta}) + \gamma}{r + \lambda_L} < 0.$$

Substituting $V_x(x, \bar{\theta})$ and $p(x, \bar{\theta})$ we get

$$x_0(\bar{\theta}) = \frac{\phi \bar{\theta}^2 + \lambda_L C(\underline{\theta})}{2 \phi \bar{\theta}^2 + 2 \lambda_L C(\underline{\theta}) - (r + \lambda_L) C(\bar{\theta})}$$

$$= \frac{\phi \bar{\theta}^2 + \lambda_L C(\underline{\theta})}{2 \phi \bar{\theta}^2 + \lambda_L C(\underline{\theta}) - \lambda_L \big(C(\bar{\theta}) - C(\underline{\theta})\big) - r C(\bar{\theta})}$$

$$= \frac{\phi \bar{\theta}^2 + \lambda_L C(\underline{\theta})}{\phi \bar{\theta}^2 + \lambda_L C(\underline{\theta}) + \gamma} \in (0, 1),$$

Next, we consider the first order condition for $x_0(\underline{\theta})$.

$$V_x(x_0(\underline{\theta}), \underline{\theta}) - p(x_0(\underline{\theta}), \underline{\theta}) + (1 - \pi)\big(1 - x_0(\underline{\theta})\big) p_x(x_0(\underline{\theta}), \underline{\theta}) - \pi \big(V_x(x_0(\underline{\theta}), \bar{\theta}) - p(x_0(\underline{\theta}), \underline{\theta})\big) = 0,$$

while the second order condition is

$$V_{xx}(\underline{\theta}) - 2(1-\pi)p_x(x_0(\underline{\theta}),\underline{\theta}) - \pi V_{xx}(x_0(\underline{\theta}),\bar{\theta}) =$$

$$-(1-\pi)\left[C(\underline{\theta}) + \frac{\pi}{1-\pi}\left(C(\bar{\theta}) - C(\underline{\theta})\right)\right] < 0$$

Substituting $p(x_0(\underline{\theta}),\underline{\theta}) = V_x(x_0(\underline{\theta}),\underline{\theta})$ and $p_x(x_0(\underline{\theta}),\underline{\theta}) = V_{xx}(x_0(\underline{\theta}),\underline{\theta})$ we get

$$\left(1 - x_0(\underline{\theta})\right)V_{xx}(x_0(\underline{\theta}),\underline{\theta}) = \frac{\pi}{1-\pi}\left(V_x(x_0(\underline{\theta}),\bar{\theta}) - V_x(x_0(\underline{\theta}),\underline{\theta})\right) = 0.$$

Substituting $V_x(x,\theta) = C(\theta)x$, we get

$$x_0(\underline{\theta}) = \frac{C(\underline{\theta})}{C(\underline{\theta}) + \frac{\pi}{1-\pi}\left(C(\bar{\theta}) - C(\underline{\theta})\right)}.$$

Finally, we need to verify the monotonicity constraint $x_0(\bar{\theta}) > x_0(\underline{\theta})$:

$$\frac{\phi\bar{\theta}^2 + \lambda_L C(\underline{\theta})}{\phi\bar{\theta}^2 + \lambda_L C(\underline{\theta}) + \gamma} \geq \frac{C(\underline{\theta})}{C(\underline{\theta}) + \frac{\pi}{1-\pi}\left(C(\bar{\theta}) - C(\underline{\theta})\right)}$$

which requires that

$$\frac{\pi}{1-\pi} \geq \frac{\gamma C(\underline{\theta})}{\left(\phi\bar{\theta}^2 + \lambda_L C(\underline{\theta})\right)\left(C(\bar{\theta}) - C(\underline{\theta})\right)}$$

If this condition is not satisfied, then we get that $x_0(\theta) = x_0$ and $T(\theta) = T$ where

$$\max_{T,x_0} T + \pi\left(1-x_0\right)p(x_0,\bar{\theta}) + (1-\pi)\left(1-x_0\right)p(x_0,\bar{\theta})$$

subject to

$$V(x_0,\underline{\theta}) - T \geq V(0,\underline{\theta}).$$

From here we get, $T = V(x_0,\underline{\theta})$ so we can write

$$\max_{x_0} V(x_0,\underline{\theta}) + \left(1-x_0\right)\left[\pi p(x_0,\bar{\theta}) + (1-\pi)p(x_0,\underline{\theta})\right]$$

The first order condition is

$$V_x(x_0,\underline{\theta}) - \left[\pi p(x_0,\bar{\theta}) + (1-\pi)p(x_0,\underline{\theta})\right] + \left(1-x_0\right)\left[\pi p_x(x_0,\bar{\theta}) + (1-\pi)p_x(x_0,\underline{\theta})\right] = 0$$

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substituting $p(x_0, \underline{\theta})$ we get

$$0 = -\pi \left[p(x_0, \bar{\theta}) - p(x_0, \underline{\theta}) \right] + (1 - x_0) \left[\pi p_x(x_0, \bar{\theta}) + (1 - \pi)p_x(x_0, \underline{\theta}) \right]$$

$$= -\pi \left[\frac{(\phi\bar{\theta}^2 + \lambda_L C(\underline{\theta}))}{r + \lambda_L} - C(\underline{\theta}) \right] x_0 + (1 - x_0) \left[\pi \left(\frac{(\phi\bar{\theta}^2 + \lambda_L C(\underline{\theta}))}{r + \lambda_L} \right) + (1 - \pi)C(\underline{\theta}) \right]$$

$$= -2\pi \left[\frac{(\phi\bar{\theta}^2 + \lambda_L C(\underline{\theta}))}{r + \lambda_L} - C(\underline{\theta}) \right] x_0 + \pi \left[\frac{(\phi\bar{\theta}^2 + \lambda_L C(\underline{\theta}))}{r + \lambda_L} - C(\underline{\theta}) \right] + (1 - x_0)C(\underline{\theta}).$$

The second order condition is

$$-2\pi p_x(x_0, \bar{\theta}) - (1 - 2\pi)p_x(x_0, \underline{\theta}) = - \left[C(\underline{\theta}) + 2\pi \frac{\phi\bar{\theta}^2 - rC(\underline{\theta})}{r + \lambda_L} \right] x_0 < 0$$

From here we get that

$$x_0 = \min \left\{ \frac{\pi(\phi\bar{\theta}^2 + \lambda_L C(\underline{\theta})) + (1 - \pi)(r + \lambda_L)C(\underline{\theta})}{2\pi(\phi\bar{\theta}^2 + \lambda_L C(\underline{\theta})) + (1 - 2\pi)(r + \lambda_L)C(\underline{\theta})}, 1 \right\}.$$

Finally, we rule out the optimality of stochastic mechanism when the monotonicity constraint is violated. Let $G(x_0|\theta)$ be the distribution of x_0 conditional on the report θ. Then, we have

$$\max_{\{G(x_0|\theta)\}_{\theta \in \{\underline{\theta}, \bar{\theta}\}}} \int_0^1 \left(V(x_0, \underline{\theta}) + (1 - \pi)(1 - x_0)p(x_0, \underline{\theta}) - \pi V(x_0, \bar{\theta}) \right) dG(x_0, \underline{\theta})$$

$$+ \pi \int_0^1 \left(V(x_0, \bar{\theta}) + (1 - x_0)p(x_0, \bar{\theta}) \right) dG(x_0, \bar{\theta})$$

subject to

$$\int_0^1 \left(V(x_0, \bar{\theta}) - V(x_0, \underline{\theta}) \right) dG(x_0|\bar{\theta}) \geq \int_0^1 \left(V(x_0, \bar{\theta}) - V(x_0, \underline{\theta}) \right) dG(x_0|\underline{\theta}).$$

The previous constraint reduces to

$$\int_0^1 x_0^2 dG(x_0|\bar{\theta}) \geq \int_0^1 x_0^2 dG(x_0|\underline{\theta}).$$

A9

Let's define

$$\underline{U}(x_0, \eta) \equiv V(x_0, \underline{\theta}) + (1 - \pi)(1 - x_0)p(x_0, \underline{\theta}) - \pi V(x_0, \bar{\theta}) - \eta x_0^2$$
$$\bar{U}(x_0, \eta) \equiv V(x_0, \bar{\theta}) + (1 - x_0)p(x_0, \bar{\theta}) + \eta x_0^2.$$

Then, the problem can be written as

$$\min_{\eta \geq 0} \max_{\{G(x_0|\theta)\}_{\theta \in \{\underline{\theta}, \bar{\theta}\}}} \int_0^1 \underline{U}(x_0, \eta) dG(x_0, \underline{\theta}) + \int_0^1 \bar{U}(x_0, \eta) dG(x_0, \bar{\theta})$$

Notice that $\operatorname{supp} G(x, \underline{\theta}) \subset \arg\max_{x_0} \underline{U}(x_0, \eta)$ and $\operatorname{supp} G(x, \bar{\theta}) \subset \arg\max_{x_0} \bar{U}(x_0, \eta)$. So, if $\underline{U}(x_0, \eta)$ is concave, then the allocation is deterministic. It follows from analysis of the second order conditions for the optimal deterministic allocation that, for any $\eta \geq 0$, the function $\underline{U}(x_0, \eta)$ is concave in x_0. Hence, if a random allocation is optimal, it must be for a high report. The second order derivative of $\bar{U}(x_0, \eta)$ is

$$-\frac{\phi\bar{\theta}^2 + \lambda_L C(\underline{\theta}) + \gamma}{r + \lambda_L} + 2\eta,$$

so the function is convex only if

$$\eta > \frac{\phi\bar{\theta}^2 + \lambda_L C(\underline{\theta}) + \gamma}{2(r + \lambda_L)}.$$

If this were the case, the maximum would be an extreme point, so it would belong to $\{0, 1\}$. We have that

$$\bar{U}(0, \eta) = p(0, \bar{\theta}) = \frac{\mu}{r}$$
$$\bar{U}(1, \eta) = V(1, \bar{\theta}) + \eta = \frac{\mu}{r} + C(\bar{\theta}) + \eta.$$

Thus, $\bar{U}(1, \eta) > \bar{U}(0, \eta)$. But this means that a random allocation cannot be optimal. $\qquad\square$

Proof of Proposition 8

Proof. If the low type's participation constraint and the high type incentive compatibility are binding, then we can write the previous problem as

$$\max_{\{x_0(\theta)\}_{\theta \in \{\underline{\theta}, \bar{\theta}\}}} V(x_0(\underline{\theta}), \underline{\theta}|\tau) + \pi \Big[V(x_0(\bar{\theta}), \bar{\theta}|\tau) + (1 - x_0(\bar{\theta}))p(x_0(\bar{\theta}), \bar{\theta}|\tau) \Big]$$

$$+ (1 - \pi)(1 - x_0(\underline{\theta}))p(x_0(\underline{\theta}), \underline{\theta}|\tau) - \pi V(x_0(\underline{\theta}), \bar{\theta}|\tau)$$

First, we consider the first order condition for $x_0(\bar{\theta})$

$$V_x(x_0(\bar{\theta}), \bar{\theta}|\tau) - p(x_0(\bar{\theta}), \bar{\theta}|\tau) + \left(1 - x_0(\bar{\theta})\right)p_x(x_0(\bar{\theta}), \bar{\theta}|\tau) = 0.$$

Substituting $V_x(x, \bar{\theta}|\tau)$ and $p(x, \bar{\theta}|\tau)$ we get

$$x_0(\bar{\theta}) = \frac{P(\bar{\theta}, \tau)}{2P(\bar{\theta}, \tau) - C(\bar{\theta})}$$

$$= \frac{P(\bar{\theta}, \tau)}{P(\bar{\theta}, \tau) + \frac{\gamma}{r}\left(1 - e^{-r\tau}\right) + e^{-r\tau}\pi(\tau, \bar{\theta})\left(\frac{\phi\bar{\theta}^2 + \lambda_L C(\underline{\theta})}{r + \lambda_L} - C(\bar{\theta})\right)}$$

$$P(\theta_0, \tau) = C(\theta_0) + \frac{\gamma}{r}\left(1 - e^{-r\tau}\right) + e^{-r\tau}\pi(\tau, \theta_0)\left(\frac{\phi\bar{\theta}^2 + \lambda_L C(\underline{\theta})}{r + \lambda_L} - C(\bar{\theta})\right).$$

Next, we consider the first order condition for $x_0(\underline{\theta})$.

$$V_x(x_0(\underline{\theta}), \underline{\theta}|\tau) - p(x_0(\underline{\theta}), \underline{\theta}|\tau) + (1-\pi)\left(1 - x_0(\underline{\theta})\right)p_x(x_0(\underline{\theta}), \underline{\theta}|\tau) - \pi\left(V_x(x_0(\underline{\theta}), \bar{\theta}|\tau) - p(x_0(\underline{\theta}), \underline{\theta}|\tau)\right) = 0,$$

which yields

$$x_0(\underline{\theta}) = \frac{P(\underline{\theta}, \tau)}{P(\underline{\theta}, \tau) + \frac{\gamma}{r}\left(1 - e^{-r\tau}\right) + e^{-r\tau}\pi(\tau, \underline{\theta})\left(\frac{\phi\bar{\theta}^2 + \lambda_L C(\underline{\theta})}{r + \lambda_L} - C(\bar{\theta})\right) + \frac{\pi}{1-\pi}\left(C(\bar{\theta}) - C(\underline{\theta})\right)}$$

The monotonicity condition is satisfied if

$$\frac{\pi}{1 - \pi} \geq \frac{\pi(\tau, \bar{\theta})\left(\frac{P(\underline{\theta}, \tau)}{P(\bar{\theta}, \tau)} - \frac{\pi(\tau, \underline{\theta})}{\pi(\tau, \bar{\theta})}\right)e^{-r\tau}\left(\frac{\phi\bar{\theta}^2 + \lambda_L C(\underline{\theta})}{r + \lambda_L} - C(\bar{\theta})\right) - \left(1 - \frac{P(\underline{\theta}, \tau)}{P(\bar{\theta}, \tau)}\right)\frac{\gamma}{r}\left(1 - e^{-r\tau}\right)}{C(\bar{\theta}) - C(\underline{\theta})}$$

Finally, if the monotonicity constraint is violated, we have that

$$\max_{x_0} \, V(x_0, \underline{\theta}|\tau) + \left(1 - x_0\right)\left[\pi p(x_0, \bar{\theta}|\tau) + (1 - \pi)p(x_0, \underline{\theta}|\tau)\right].$$

Solving for the first order condition, we get

$$x_0 = \frac{\pi P(\bar{\theta}, \tau) + (1 - \pi)P(\underline{\theta}, \tau)}{2\left[\pi P(\bar{\theta}, \tau) + (1 - \pi)P(\underline{\theta}, \tau)\right] - C(\underline{\theta})}.$$

$\square$