Intermediary Financing without Commitment

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Abstract

Banks can reduce frictions in the credit market through monitoring. To be a credible monitor, a bank needs to retain a fraction of its loans; we study the credit market dynamics when it cannot commit to doing so. Loan prices drop in anticipation of loan sales and reductions in monitoring. With commitment, *intermediation* is irrelevant if the bank *certifies* it will monitor. Without commitment, a bank that only certifies sells its loans over time. By contrast, an intermediating bank that issues short-term deposits internalizes the externalities from monitoring, and therefore retains its loans. Although intermediation leads to more lending, an entrepreneur with high net worth may choose certification.

Keywords: commitment; certification; intermediation; trading; dynamic models

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1 Introduction

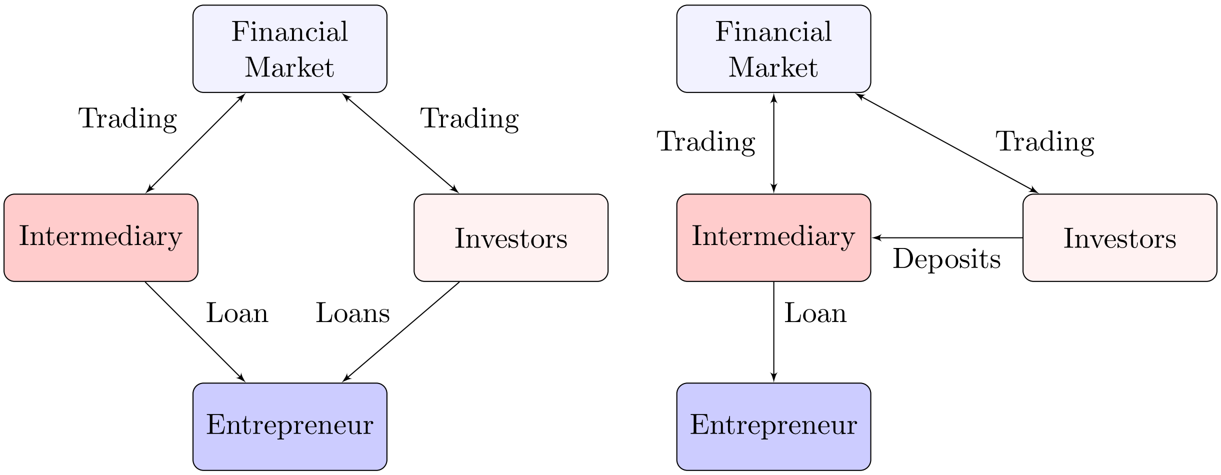
Financial intermediaries exist to reduce agency frictions in the credit market, and therefore benefit the real economy. To conduct valuable services such as screening and monitoring, the optimal arrangement typically involves banks retaining a fraction of loans as the skin in the game; otherwise, the incentives can be misaligned. In practice, banks' commitment to loan retentions is limited: 60% of the loans are sold within one month after origination, and nearly 90% are sold within one year (Drucker and Puri, 2009).[1] Moreover, evidence shows that securitization and the "originate-to-distribute" model have led to lax screening and monitoring (Keys et al., 2010, 2012), thereby reducing the loan quality.[2] This paper studies the equilibrium credit market dynamics when banks *cannot* commit to the loan retentions. By doing so, we develop a dynamic theory of intermediary financing without commitment.

We build on the classic model of Holmstrom and Tirole (1997) in which banks can monitor to increase an entrepreneur's borrowing capacity. Being a credible monitor requires the bank to retain a sufficient fraction of loans on its balance sheet. However, in the presence of the financial market, the bank has incentives to sell its loans, due to its high cost of capital. The more it sells, the less likely it will monitor, and the price of the loans will drop more as well. We study two types of implementation structures: certification and intermediation. As shown in Figure 1, in certification, both the bank and investors directly invest in the borrower's venture, and the role of the bank is to certify that it will monitor. In intermediation, investors deposit in the bank, which then invests a collection of its own funds and the deposits into the borrower's venture. Although the two structures are equivalent in the static framework with the bank's commitment to its retention, they lead to very different credit market dynamics without commitment. In certification, the lack of commitment induces the bank to sell its loans gradually, and the bank's monitoring intensity declines over time. In intermediation, the bank is able to issue short-term deposits, which helps it commit to the retention and the decision to monitor. As a result, an intermediating bank never sells its loans, and the entrepreneur is always able to borrow more. However, if both structures enable the entrepreneur to borrow enough to invest, she may end up choosing certification. In other words, the implementation structure that maximizes the initial borrowing may not be the one that maximizes the borrower's expected payoff.

More specifically, an entrepreneur is endowed with an investment opportunity, which requires a fixed-size of investment and pays off some final cash flows at a random time in the future. She has

[1] More recently, Blickle et al. (2020) show that in loan syndications, the lead bank sells its entire share for 12% of all loans and for 27% of all term loans shortly after loan origination.

[2] Meanwhile, Begley and Purnanandam (2017) show securitization, if well structured, does not necessarily lead to a reduction in loan quality.



(a) Certification

(b) Intermediation

Figure 1: Certification vs. Intermediation

limited personal wealth and needs to borrow to make up the investment shortfall. Due to moral hazard in effort choices, she can only pledge a fraction of the final output to outside creditors, including banks and investors. Banks have a higher cost of capital, but only they can monitor to reduce the entrepreneur's private benefits. Although monitoring increases the project's pledgeable income and enables the entrepreneur to borrow more, it also entails a physical cost. Therefore, a credible monitor needs to retain a sufficient fraction of loans as its skin in the game.

We depart from Holmstrom and Tirole (1997) by introducing a competitive financial market, in which the bank is allowed to trade its loans. Loans are rationally priced, and therefore the prices depend on the bank's incentives to monitor both contemporaneously and in the future. If the bank has sold a large fraction of the loans, it will monitor less often, and consequently the price of loans will fall substantially. This price impact deters the bank from selling the loans too fast and too aggressively.

Although certification and intermediation are equivalent in a two-period model under bank's commitment, they lead to different equilibrium dynamics in the absence of commitment. Specifically, the lack of commitment hurts a certifying bank as in the standard durable-goods monopoly problem (Coase, 1972; Fudenberg et al., 1985; Gul et al., 1986).[3] A similar result arises when a borrower cannot commit to its debt level (Bulow and Rogoff, 1989; Bizer and DeMarzo, 1992; Admati et al., 2018; DeMarzo and He, forthcoming), or when a large shareholder cannot commit

[3] In traditional durable-goods monopoly and bargaining models, all trade is immediate in the continuous-trading limit. However, Fuchs and Skrzypacz (2010) show the equilibrium can involve delay with smooth trading when competitive buyers arrive over time, or when the production technology exhibits increasing returns to scale.

to its future stake (DeMarzo and Urošević, 2006). Indeed, a certifying bank has incentives to sell, because its marginal valuation is below that of investors. After its initial loan sell, the bank has incentives to keep selling to exploit the remaining gains of trade. The price of the loans drops following the expectation that the bank will keep selling, reducing the likelihood of monitoring, which in turn reduces the bank's proceeds from selling. Hence, the bank trades off the immediate trading gains versus the drop in its future payoff, including the drop in the loan's valuation as well as the decline in the expected payments it can collect upon project maturity. Similar to the result in the Coase conjecture, the certifying bank does not benefit from its ability to trade loans at all.

By contrast, an intermediating bank does not have incentives to sell, due to its ability to issue short-term deposits. Indeed, when the bank has access to deposits as a source of cheap financing, selling loans will not only depress the valuation of the loan, but also increase the interest rate of deposits. The elevated deposit rate acts as a mechanism that deters the bank from selling. As a result, the bank finds it optimal to retain the entire loan on its balance sheet even though it has not committed to doing so. This result is related to the literature on the commitment role of short-term debt (Calomiris and Kahn, 1991; Diamond and Rajan, 2001); however, the channel is different. Whereas this literature emphasizes the demandable feature of debt and the externalities from creditor runs, our result does not rely on the first-come-first-serve constraint. Instead, our mechanism depends on the endogenous deposit rate as the discipline device. Deposits are modeled as pari-passu debt, so that if the bank fails, all depositors receive an equal amount.[4]

This distinction between certification and intermediation points out an important externality in bank monitoring. As in Diamond (1984), monitoring suffers from the free-rider problem in that investors enjoy the benefits but do not share the cost with the bank. Therefore, the equilibrium monitoring effort is too low. In certification, the bank reduces its probability of monitoring over time. In intermediation, however, deposits help the bank internalize the externalities. Indeed, because these deposits are fairly priced, the bank internalizes the cost of monitoring through the rate of deposits. Therefore, the bank and its depositors share both the benefits and costs of monitoring. Essentially, deposits create a market for the monitoring services the bank offers to be fairly priced.

Next, we turn to the entrepreneur's initial choice. We show that if the entrepreneur's net worth is high enough, she may end up choosing certification, even though she borrows less under certification. The reason is that intermediation can lead to too much monitoring (Pagano and Röell, 1998). Intuitively, the entrepreneur cares about not only the value of the project, but also her future

[4]The commitment problem is also mitigated when claims are collateralized. Rampini and Viswanathan (2019) emphasize the advantage of intermediaries in collateralizing claims. In their paper, certification and intermediation are still equivalent.

benefits, and the level of monitoring if the bank retains the entire loan may be excessive. Although bank monitoring introduces a positive externality to investors, it also imposes a negative externality on the entrepreneur by reducing her private benefits. This effect dominates if private benefits are sufficiently large, which is why the entrepreneur prefers the certification structure.

Finally, we explore the impact of common policies designed to address the bank commitment problem. In particular, we look at the impact of a lock-up period and minimum retention levels.[5] Although the qualitative nature of the equilibrium outcome remains unchanged, these policies allow the entrepreneur to borrow more upfront. Moreover, if an intermediating bank's deposits have long maturity or are subsidized by the government, the bank's commitment to retention is also impaired.

2 The Model

Our model builds on the fixed-size investment setup in Holmstrom and Tirole (1997). A key innovation is to introduce a competitive financial market in which the bank can trade its loans. As a result, the bank cannot commit to its loan retention, and thus the decisions to monitor in the future. The more it sells, the less likely it will monitor, and the loans will be valued lower as well. We explore how the bank's equilibrium trading behavior interacts with its monitoring decisions, in the dynamic context of certification and intermediation.

2.1 Agents and Technology

Time is continuous and goes to infinity: $t \in [0, \infty)$. There are three groups of agents: one entrepreneur (she) – the borrower; competitive intermediaries – banks; and investors. All agents are risk neutral and have limited liabilities. The entrepreneur starts out with cash level A, whereas banks and investors have deep pockets. We assume investors do not discount future cash flows, whereas the entrepreneur and intermediaries discount the future at a rate $\rho > 0$.

At time 0, the entrepreneur has access to a project that requires a fixed investment size $I > A$. Thus, she needs to borrow at least $I - A$. The project matures at a random time τ_ϕ, which arrives upon a Poisson event with intensity $\phi > 0$. Define $\Phi = \frac{\phi}{\rho+\phi}$ as the effective time discount the entrepreneur and banks apply to the project's final cash flows. At τ_ϕ, the project generates the final cash flows R in the case of success and 0 in the case of failure. The probability of success is p_H if the entrepreneur works at τ_ϕ, and $p_L = p_H - \Delta$ if she shirks. Two options of shirking are

[5]The Dodd-Frank Act imposes a minimum of 5% credit risk retention – known as the "risk-retention rule." The EU has had similar restrictions in place since 2019. However, the D.C. Circuit Court invalidated the risk-retention rule for collateralized loan obligations (CLOs) in 2018.

available: the high option brings private benefit B, which exceeds b, the private benefit associated with the low option.

2.2 Monitoring, Financial Structures, and Contracts

A competitive set of banks are present at $t = 0$, and the entrepreneur signs a contract with one of them. One can think about this bank as the relationship lender. Note we do not allow for multiple banking relationships to avoid duplication of monitoring efforts and the free-rider problem (Diamond, 1984). At τ_ϕ, the project matures, and the bank can monitor to eliminate the high shirking option. To do so, it needs to pay a private monitoring cost $\tilde{\kappa} > 0$, where $\tilde{\kappa} \in [0, \bar{\kappa}]$ has a distribution with $F(\cdot)$ and $f(\cdot)$ being the cumulative distribution function (CDF) and probability density function (PDF), respectively. Note we allow the cost $\tilde{\kappa}$ to vary stochastically. One can interpret the stochastic cost as variations in legal and enforcement costs or simply the fluctuations in the costs of hiring loan officers. The stochastic-cost assumption makes the bank's equilibrium monitoring decisions smooth and is a continuous function of its loan retention.[6] Figure 2 describes the timing. Note that for simplicity, we assume both the entrepreneur's effort and the bank's monitoring are needed only at τ_ϕ when the project matures. Introducing long-term effort and monitoring will complicate the model without bringing many new insights.

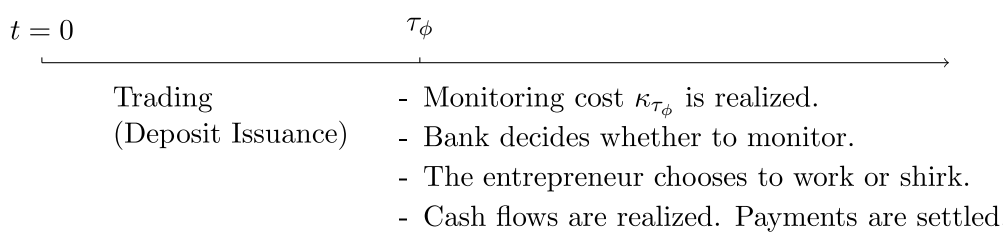


Figure 2: Timing

We study two types of financial structures: certification and intermediation. In certification, the bank puts its own funds in the entrepreneur's venture to ensure it will monitor, which then attracts investors to *directly* invest in the venture as well. One can think of this type of bank as

[6] Holmstrom and Tirole (1997) also introduce an extension with a continuous distribution of private benefits to model the continuous monitoring intensity. An alternative formulation is to follow subsection 9.2 in Tirole (2010) by assuming monitoring is only effective with some probability, and this probability varies continuously with the bank's monitoring effort (which is increasing in θ). This formulation will generate equivalent results as a stochastic monitoring cost.

either a venture capitalist or the lead investment bank in loan syndications. In certification, the entrepreneur directly signs the initial contract with the bank and investors. Under limited liability, no agent receives anything if the project fails. If the project succeeds, let R_f, R_m, and R_u be the scheduled payments to the entrepreneur, the bank, and investors, which sum up to R. We use $R_o = R_m + R_u$ to denote the total claims held by outside creditors, namely, the bank and investors. For the remainder of this paper, we also refer to R_o as *loans*.

In intermediation, investors do not directly invest in the entrepreneur's project. Instead, they make deposits in the bank, which in turn lends to the entrepreneur a collection of its own funds and these deposits. This type of bank resembles a commercial bank. In intermediation, the entrepreneur signs an initial contract with a bank and promises to repay $R_o = R - R_f$ if the project succeeds. The bank, in turn, offers deposit contracts $\{D_t, y_t\}$ to investors/depositors over time, where D_t is the amount of deposits and y_t is the associated interest payment (henceforth, the deposit rate). We model the deposit contract as debt with *instant* maturity. Whenever the bank fails to honor its deposit payments, the remaining loans are sold to repay its depositors. For simplicity, we assume no bankruptcy cost (or loss of charter value) is incurred if the bank fails, and a positive bankruptcy cost will change the results only quantitatively.

Given that the entrepreneur has (weakly) the highest cost of capital among all the agents, she should retain as little stake as possible. Therefore, in both certification and intermediation, it is optimal to let the entrepreneur retain $R_f = b/\Delta$, which guarantees she will work if the bank monitors. For the rest of this paper, we always set $R_o = R - b/\Delta$.

2.3 Trading and Pricing in the Financial Market

A competitive financial market opens in which loans can be traded.[7] We normalize the total share of loans outstanding to one and use θ_t to denote the bank's retention at time t. In our model, θ_t will be the payoff-relevant state variable. Before trading starts, the bank's initial retention is $\theta_{0-} = R_m/(R_m + R_u)$ in certification and $\theta_{0-} = 1$ in intermediation. Sometimes θ_t is also referred to as the bank's skin in the game, which is publicly observable. We consider trading strategies that admit both smooth and atomic trading, as well as mixed strategies over the time of atomic trades. A Markov trading strategy is defined as $(\theta_t)_{t\geq 0}$ being a Markov process.

The price of loans depends on whether the entrepreneur works or shirks, which in turn depends on the probability of bank monitoring. Conditional on the project maturing, let $p(\theta)$ be the

[7] We assume the entrepreneur's retention R_f is not tradable, or equivalently, the entrepreneur can commit to hold onto R_f on the balance sheet.

equilibrium probability that the bank with retention θ monitors; investors of the loan receive

$$d(\theta) = p(\theta) R_o \tag{1}$$

per share. Let $q(\theta)$ be the price of the loan per share when $\theta_t = \theta$. In a competitive financial market, the price is given by the expected present value of the asset:

$$q(\theta) = \mathbb{E}\left[d\left(\theta_{\tau_\phi}\right) \middle| \theta_t = \theta \right], \tag{2}$$

where the expectation operator is taken with respect to the equilibrium path of $\{\theta_s\}_{t \le s \le \tau_\phi}$.

The probability of monitoring $p(\theta)$ will differ in certification and intermediation. Let κ be the realization of the stochastic monitoring cost $\tilde{\kappa}$. In certification, the bank with retention θ chooses to monitor if and only if

$$\kappa \le \kappa_c := \Delta R_o \theta, \tag{3}$$

where $\Delta := p_H - p_L$. In intermediation, a bank with retention θ and deposits D monitors if and only if

$$\kappa \le \kappa_i := \Delta\left(R_o \theta - D\right). \tag{4}$$

A comparison between (3) and (4) illustrates the static tradeoff between certification and intermediation. For the same retention θ, a higher D reduces an intermediating bank's incentive to monitor: this result is the standard debt-overhang effect.

Let $G(\theta)$ be the bank's instant trading gains. In the case of continuous trading, $dG(\theta) = -q(\theta)\dot{\theta}dt$. In the case of atomic trading, the bank's holding jumps to θ^+ and the associated trading gain is $dG(\theta) = q(\theta^+)(\theta - \theta^+)$. Note that trading is settled at price $q(\theta^+)$ to reflect the price impact.

2.4 Equilibrium Definition

2.4.1 Certification

If the project matures at time t, the bank's expected payoff is

$$\pi_c(\theta) = p_c(\theta) R_o \theta - \int_0^{\kappa_c} \kappa dF(\kappa), \tag{5}$$

where the project succeeds with probability

$$p_c(\theta) := p_L + F(\kappa_c)\Delta, \tag{6}$$

upon which the bank receives $R_o\theta$. The bank maximizes the sum of its payoff upon the project's maturation $e^{-\rho(\tau_\phi - t)}\pi_c\left(\theta_{\tau_\phi}\right)$ and the cumulative trading gains $\int_0^{\tau_\phi} e^{-\rho(s-t)} dG\left(\theta_s\right)$. Because τ_ϕ follows the exponential distribution, the maximization problem is equivalent to

$$\max_{\{\theta_t\}_{t\geq 0}} \mathbb{E}\left[\int_0^\infty e^{-(\rho+\phi)t}\Big(\phi\pi_c\left(\theta_t\right)dt + dG\left(\theta_t\right)\Big)\right], \tag{7}$$

where the expectation operator allows for mixed strategies in $\{\theta_t\}$.

We consider a Markov perfect equilibrium in which the state variable is the bank's retention θ, henceforth, the certification equilibrium.[8]

Definition 1. *A* ***certification equilibrium*** *is a Markov perfect equilibrium consisting of a price function* $q\colon [0,1] \to \mathbb{R}_+$ *and a trading strategy* $(\theta_t)_{t\geq 0}$ *that satisfy the following:*

1. *For all* $\theta_0 \in [0,1]$, $(\theta_t)_{t\geq 0}$ *is a Markov trading strategy with initial value* θ_0 *that maximizes* (7).

2. *For all* $\theta \in [0,1]$, *the price* $q(\theta)$ *satisfies the break-even condition* (2).

2.4.2 Intermediation

If the project matures at time t, the bank's expected payoff is

$$\hat{\pi}_i\left(\theta, D\right) = \hat{p}\left(\theta, D\right)\left(\theta R_o - D\right) - \int_0^{\kappa_i} \kappa dF\left(\kappa\right), \tag{8}$$

where the project succeeds with probability

$$\hat{p}_i\left(\theta, D\right) \coloneqq p_L + F\left(\kappa_i\right)\Delta, \tag{9}$$

at which time the bank's equity holder receives $\theta R_o - D$ after paying off its depositors. Besides trading gains, an intermediating bank also receives income from deposit issuance. In particular, let D_0 be the value of deposits issued at $t = 0$. The bank's net income from deposit issuance at time

[8] If $\{\theta_t\}$ is not restricted to the class of Markov processes, one may construct equilibria that are close to the commitment solution. In the context of a durable-goods monopoly, Ausubel and Deneckere (1989) show that in the no-gap case, non-Markov equilibria exist in which the seller can achieve payoffs close to the commitment solution. The logic behind the construction is similar to the one in the folk theorem for repeated games. The no-gap case corresponds to the version of our model under $p_L = 0$. At $\theta = 0$, the marginal valuation of investors coincides with the bank's.

t is $dD_t - y_t D_t dt$, where

$$y_t = \hat{y}(\theta, D) = \phi(1 - \hat{p}(\theta, D)) \tag{10}$$

compensates the default risk borne by depositors.[9] In intermediation, the bank trades loans and issues deposits to maximize the the expected payoff upon the project maturing, together with the net income from deposit issuance and trading gains; that is,

$$\max_{\{\theta_t,\, D_t\}} \mathbb{E}\left[\int_0^\infty e^{-(\rho+\phi)t}\Big(\phi\hat{\pi}_i(\theta_t, D_t)\,dt + dD_t - \hat{y}(\theta_t, D_t)\,D_t dt + dG(\theta_t)\Big)\right]. \tag{11}$$

The choice of D_t in (11) is restricted by the bank's limited liability, which imposes a borrowing constraint illustrated below in (12). Lemma 1 shows the choice of D_t is essentially a static decision. Therefore, we can use θ_t as the state variable and suppress the problem's dependence on D_t.

Lemma 1. *The maximization problem* (11) *is equivalent to solving*

$$\phi\pi_i(\theta) := \max_{D\le\Pi_i(\theta)}\left\{\phi\left[\hat{p}_i(\theta, D)\theta R_o - \int_0^{\kappa_i}\kappa dF(\kappa)\right] + \rho D\right\}, \tag{12}$$

and

$$\Pi_i(\theta_0) = E(\theta_0, D_0) + D_0 = \max_{(\theta_t)_{t\ge 0}} \int_0^\infty \mathbb{E}\left[e^{-(\rho+\phi)t}\Big(\phi\pi_i(\theta_t)dt + dG(\theta_t)\Big)\right]. \tag{13}$$

Lemma 1 implies we can solve deposit issuance and loan trading separately. From (12), the choice of deposit D only involves static tradeoffs. A higher D reduces the probability of monitoring $\hat{p}(\theta, D)$ (see equation (4)), thereby reducing the value of loans. On the other hand, ρD shows that deposits are a cheaper means of funding. Given that all the functions on the right-hand side of (12) are continuous, the optimal deposit issuance is also continuous, that is, does not admit jumps. Deposit issuance is bounded by the endogenous constraint $D \le \Pi_i(\theta)$, which arises from the bank's limited liability. Here, $\Pi_i(\theta)$ is the bank's value function given its retention θ, which implicitly assumes deposit issuance has been chosen at the optimal level. Therefore, this constraint involves a fixed point for the value function $\Pi_i(\theta)$. In (13), the left-hand side $\Pi_i(\theta_0)$ includes $E(\theta_0, D_0)$,

[9] Implicitly, the bank will honor its debt payments whenever $t < \tau_\phi$, that is, if the project does not mature. In the case of strategic default, depositors acquire the remaining loans and immediately resell them, which necessarily leads to zero payoff to the bank. Thus, strategic default never happens for $t < \tau_\phi$. Strategic defaults won't happen at $t = \tau_\phi$ either, under the assumption that the bank cannot renegotiate with its depositors or the loan investors. Diamond and Rajan (2001) show renegotiation with depositors will trigger runs. Renegotiation with dispersed public investors is also costly due to free-rider problems (Bolton and Scharfstein, 1996).

the value to the bank's equity holders, and D_0, the value to the depositors. For the remainder of this paper, we sometimes refer to Π_i, E, and D as the bank value, the equity value, and the deposit value, respectively. One implication of Lemma 2 is that even though the bank's equity holders decide its trading strategy, maximizing the bank's equity value $E(\theta_0, D_0)$ is equivalent to maximizing the total bank value $\Pi_i(\theta_0)$, because deposits D_0 are fairly priced at time 0.

We look for a Markov perfect equilibrium in state variable θ_t, henceforth, the intermediation equilibrium.

Definition 2. *An* ***intermediation equilibrium*** *is a Markov perfect equilibrium consisting on a price function $q\colon [0,1] \to \mathbb{R}_+$, a trading strategy $(\theta_t)_{t\geq 0}$, a deposit-issuance policy $D^*\colon [0,1] \to \mathbf{R}_+$, and the deposit-rate function $y\colon [0,1] \to \mathbf{R}_+$ that satisfy the following:*

1. *For all $\theta \in [0,1]$, the deposit-issuance policy $D^*(\theta)$ solves (12).*
2. *For all $\theta_0 \in [0,1]$, $(\theta_t)_{t\geq 0}$ is Markov trading strategy with initial value θ_0 that maximizes (13).*
3. *For all $\theta \in [0,1]$, the price $q(\theta)$ satisfies the break-even condition (2).*
4. *For all $\theta \in [0,1]$, the deposit rate $y(\theta) := \hat{y}(\theta, D^*(\theta))$ satisfies (10).*

2.5 Parametric Assumptions

To make the problem non-trivial, we impose the following parametric assumptions. The first assumption says the project's expected payoff is always higher if the entrepreneur works.

Assumption 1.

$$p_H R > p_L R + B.$$

Assumption 2 ensures the problem is not driven by some particular feature of the monitoring cost's distribution function. It is satisfied by most commonly-used distribution functions such as the uniform distribution.

Assumption 2.

$$\frac{f'}{f^2} < \frac{\Phi}{1-\Phi}\Delta, \qquad \forall \kappa \in [0, \bar{\kappa}].$$

Finally, we restrict the (expected) monitoring cost to be sufficiently low.

Assumption 3.

$$\Phi F\left(\Delta R-b\right)\left(\Delta R-b\right)-\int_{0}^{\Delta R-b}\kappa dF\left(\kappa\right)\geq\left(1-\Phi\right)p_{L}\left(\Delta R-b\right).$$

This assumption leads to the following result. If the bank always retains the entire loan (i.e., $\theta_t \equiv 1,\ \forall t \leq \tau_\phi$), the bank's payoff exceeds that if it immediately sells the entire loan and never monitors. If this assumption is violated, bank monitoring is never needed in equilibrium.

2.6 Two Benchmarks

We present two benchmark cases in this subsection.

2.6.1 Static Benchmark: Equilibrium without the Financial Market

We first solve the static model without the financial market. The goal of this subsection is to establish two equivalence results. First, the maximum amount that the entrepreneur is able to borrow is equivalent under certification and intermediation. Second, maximizing the initial borrowing amount is equivalent to maximizing the entrepreneur's expected payoff subject to lenders' break-even.

With slight abuse of notation, let Φ be the bank's one-period discount rate in this subsection. The optimization problem in certification is

$$L_c=\max_{\left\{R_f,R_m,R_u\right\}}p_c\left(\Phi R_m+R_u\right)-\Phi\int_0^{R_m\Delta}\kappa dF\left(\kappa\right)$$
$$s.t.\qquad R_m+R_u=R-\frac{b}{\Delta},$$

where L_c is the entrepreneur's maximum borrowing amount. The optimization problem in intermediation includes two steps. First, the bank chooses R_u,[10] taken $R_o = R_m + R_u$ as given. In the second step, the entrepreneur chooses R_o to maximize her borrowing amount L_i:

$$L_i=\max_{R_o}\left\{\max_{R_u}\ p_i\left(\Phi R_m+R_u\right)-\Phi\int_0^{R_m\Delta}\kappa dF\left(\kappa\right)\right\}.$$

Proposition 1 shows certification and intermediation are equivalent here.

[10]Note the deposit choice in the static model is $D = R_u$.

Proposition 1 (Static Benchmark)**.** *In the static model, $L_c = L_i$ so that the equilibrium outcomes in certification and intermediation are equivalent. Moreover, the optimal contract that leads to maximum borrowing also maximizes the borrower's expected payoff.*

The proof is straightforward and therefore omitted. We offer intuitions for the second equivalence results. Because lenders always break even, and the project's expected payoff on the equilibrium path is always $p_H R$, maximizing the borrowing amount is equivalent to maximizing the borrower's expected payoff. We show in subsection 3.3 that this result is no longer true in the dynamic setup without commitment.

2.6.2 The Constrained Social Planner's Problem

The social planner chooses the bank's retention $\{\theta_t\}_{t\geq 0}$ to maximize the aggregate social welfare, subject to the constraint that the bank decides whether to monitor given its retention. Under the commitment to $\{\theta_t\}_{t\geq 0}$, the choices of deposit D_t are redundant.

Proposition 2 (Constrained Social Planner's Problem)**.** *A social planner subject to constraint* (3) *chooses $\{\theta_t\}_{t\geq 0}$ to maximize the social welfare:*

$$W = \max_{(\theta_t)_{t\geq 0}} \int_0^\infty \phi e^{-\phi t} \Bigg\{ \left(1 - e^{-\rho t}\right)(1-\theta_t) d\left(\theta_t\right) + e^{-\rho t}\left[p(\theta_t)R + \left(1 - F(\Delta R_o \theta_t)\right) B - \int_0^{\Delta R_o \theta_t} \kappa dF\left(\kappa\right) \right] \Bigg\} dt. \tag{14}$$

The optimal retention always satisfies $\theta_t < 1$.

Due to differences in time discounting, the flow payoff in (14) is a weighted sum of the payoff to investors ($(1-\theta_t)d\left(\theta_t\right)$) and the payoff to the bank and the entrepreneur. The project succeeds with probability $p(\theta_t)R$, and the entrepreneur shirks to receive the high private value B with probability $(1 - F(\Delta R_o \theta_t))$. Note that in this case, the optimal retention θ_t is essentially a static choice that balances the benefits and costs of bank monitoring. Moreover, the difference in time discounting makes this tradeoff time-varying, which implies the optimal retention is in general time-varying as well.

Before concluding this section, note that given all parties are risk neutral and transfers can be made at the initial date, the constrained social planner's solution is identical to one in which the entrepreneur chooses $\{\theta_t\}_{t\geq 0}$ to maximize her payoff, subject to the lenders' participation constraints as well as the constraint that the bank decides whether to monitor.

3 Equilibrium Solutions

In this section, we solve the model. Subsections 3.1 and 3.2 respectively derive the certification and intermediation equilibrium. Subsection 3.3 presents a special case in which the monitoring cost $\tilde{\kappa}$ follows the uniform distribution and $p_L = 0$, where we obtain closed-form solutions in primitives. The entrepreneur's initial choices are therefore derived by comparing the two equilibria.

3.1 Certification Equilibrium

In general, the bank can trade loans smoothly or atomically. We show both types of trading can occur in equilibrium. Let $\Pi_c(\theta)$ be the bank's value function with retention θ.[11] In the smooth-trading region, $\Pi_c(\theta)$ satisfies the following Hamilton-Jacobi-Bellman (HJB) equation:

$$\rho\Pi_c(\theta) = \max_{\dot{\theta}} \ \phi\Big[\pi_c(\theta) - \Pi_c(\theta)\Big] + \dot{\theta}\Big[\Pi_c'(\theta) - q_c(\theta)\Big]. \tag{15}$$

Whereas the left-hand side stands for the bank's required return, the first term on the right-hand side represents the event of the project maturing, in which case the bank receives $\pi_c(\theta)$ defined in (5). The second term captures the overall benefit of trading, which includes the change to the bank's continuation value as well as the trading gain. A necessary condition for smooth trading is

$$\Pi_c'(\theta) = q_c(\theta), \tag{16}$$

so that the bank is indifferent between trading or not. In this case, the per-share trading gain $q_c(\theta)$ is offset by the drop in the bank's continuation value $\Pi_c'(\theta)$. Substituting the indifference condition (16) into (15), we immediately get that in the region of smooth trading,

$$\Pi_c(\theta) = \Phi\pi_c(\theta). \tag{17}$$

Note the bank value is equal to the payoff it receives conditional on the project maturing, times the bank's effective discount rate $\Phi = \frac{\phi}{\rho+\phi}$. Surprisingly, the bank does not benefit from its ability to trade these loans in the financial market at all; its payoff is identical to the one if it retains θ until the project finally matures at τ_ϕ. The observation that lack of commitment fully offsets the trading gains has already been noted in previous bargaining models (Fuchs and Skrzypacz, 2010) and in other corporate finance settings (DeMarzo and Urošević (2006) in the context of trading by a large shareholder, and DeMarzo and He (forthcoming) in the context of leverage dynamics).

[11] A certifying bank does not issue deposits, so there is no distinction between the bank value and the bank equity holder's value.

Even though the bank's equilibrium payoff is identical to the one if it does not trade at all, it doesn't imply the bank will not trade loans on the equilibrium path. In fact, the price of loans following a no-trade strategy will be too high, which gives the bank strict incentives to sell. Indeed, the bank sells the loans for two reasons. First, due to the higher cost of capital, the bank's marginal valuation is below that of investors. Second, after the initial sell and the reduction of retention, the bank is willing to sell again because the price impact only accrues to a smaller number of shares. Now, we characterize the bank's equilibrium trading strategy, which comes from the determination of the equilibrium loan prices. Because investors do not discount future cash flows, $q_c(\theta)$ must satisfy the following asset-pricing equation whenever the bank trades smoothly:

$$0 = \phi \Big[d_c(\theta) - q_c(\theta) \Big] + \dot{\theta} q_c'(\theta), \tag{18}$$

where $\phi \Big[d_c(\theta) - q_c(\theta) \Big]$ resembles the dividend income and $\dot{\theta} q_c(\theta)$ the capital gain.[12] Combining (16), (17), and (18), and using the relation $d_c(\theta) = \pi_c'(\theta)$, one can derive the following dynamic trading strategies in equilibrium:

$$\dot{\theta} = -\phi \frac{(1-\Phi)\,\pi_c'(\theta)}{\Phi \pi_c''(\theta)} < 0. \tag{19}$$

Clearly, in the smooth-trading region, the bank sells loans over time and its retentions declines continuously, even though it is indifferent between selling the loan or not. Intuitively, the equilibrium loan price is forward-looking, and therefore takes into account the bank's decisions regarding future monitoring. To satisfy the bank's indifference condition, the equilibrium price of the loan cannot be too high, implying the probability of monitoring must decline over time. Therefore, the only trading strategy consistent with this price requires the bank to sell its loans over time.

So far, we have focused only on the case of smooth trading. Meanwhile, the bank also has the option to sell an atom of loans. In general, the bank can sell either a fraction or all the remaining loans. Lemma 3 in the appendix proves the bank will never sell a fraction. This result follows the intuition in standard Coasian dynamic models. Atomic trading arises whenever the bank has strict incentives to sell. If so, it prefers to sell as fast as possible. Therefore, we are left to check when the bank decides to sell off all its retention at a price $q_c(0)$, where $q_c(0) = p_L R_o$ is the per-share loan price without monitoring. Indeed, we show a unique θ_* exists such that $\theta_* q_c(0) = \Phi \pi_c(\theta_*)$, and $\theta_* q_c(0) < \Phi \pi_c(\theta_*)$ if and only if $\theta > \theta_*$. In other words, a unique cutoff θ_* exists below which the bank finds it optimal to sell off all the remaining loans.

[12] The terms without subscripts c have been defined in equations (1) and (2).

The final step in the equilibrium construction is to derive the trading strategy at $\theta = \theta_*$. First, note the bank cannot hold onto the loans forever, because the resulting loan price will be too high to induce the bank to sell. The bank cannot sell smoothly either; shortly afterwards, the bank will have strict incentives to sell. Suppose the bank sells off immediately after θ_t reaches θ_*. The price of the loan will then experience a deterministic downward jump, which is inconsistent with the asset-pricing equation (2). Mathematically, let $T_* \coloneqq \inf\{t > 0 : \theta_t = \theta_*\}$: whereas $q(\theta_{T_*-})$ satisfies (16), the price at time T_* would satisfy $q(\theta_{T_*}) = q(0) = p_L R_o$. Therefore, the equilibrium necessarily involves some delay before the entire selloff. The only (stationary) trading strategy at θ_* consistent with (2) is for the bank to adopt a mixed strategy[13]: the bank sells off all its remaining loans at some Poisson rate λ that satisfies

$$q_c(\theta_*) = \mathbb{E}\left[d_c(\theta_{\tau_\phi})|\theta_t = \theta_*\right] = \frac{\lambda}{\phi+\lambda}d_c(0) + \frac{\phi}{\phi+\lambda}d_c(\theta_*).$$

Simple derivation shows λ is determined by

$$p_L + \frac{\phi}{\lambda+\phi}F\left(\Delta R_o\theta_*\right)\Delta = \Phi\left[p_L + F\left(\Delta R_o\theta_*\right)\Delta\right]. \tag{20}$$

Proposition 3 summarizes the previous discussion and describes the equilibrium outcome. The formal proof requires verification that the bank's trading strategy is optimal, which is supplemented in the appendix using results from the theory of optimal control in stratified domains.[14]

Proposition 3 (Certification Equilibrium)**.** *A unique* ***certification equilibrium*** *exists. Given the bank's initial retention θ_0, the bank sells its loans smoothly at a rate given by equation (19) until T_*, after which it sells off its remaining loans at some Poisson rate λ that satisfies (20). The equilibrium loan price is*

$$q_c(\theta_t) = \begin{cases} \Phi\Big(p_L + F\left(\Delta R_o\theta_t\right)\Delta\Big)R_o & t < T_* \\ \Big(p_L + \frac{\phi}{\lambda+\phi}F\left(\Delta R_o\theta_*\right)\Delta\Big)\,R_o & T_* \le t < \tau_\lambda \\ p_L R_o & t \ge \tau_\lambda. \end{cases} \tag{21}$$

The contract that maximizes the initial borrowing amount has $\theta_0 = 1$, in which case the borrowing

[13]The delay can also be deterministic, but the equilibrium is no longer within the class of a Markov perfect equilibrium. The equilibrium would also depend on the time since the bank's retention reached θ_*. The price q_t would not be stationary, and it would depend on the trading history before time t.

[14]Due to the discontinuity in the price function $q_c(\theta)$, the HJB equation (15) is discontinuous at θ_*. This technical problem can be sidestepped using (discontinuous) viscosity solution methods.

amount is

$$L_c = \Phi \pi_c (1) . \tag{22}$$

A contract that has the entrepreneur borrow exclusively from the bank at $t = 0$ enables the most upfront borrowing, even though bank capital is costly. Intuitively, under this contract, the bank needs to spend the longest period to fully offload its loans. This result is in contrast to that in Holmstrom and Tirole (1997), where the entrepreneur prefers to use as little bank capital as possible, because bank capital is expensive. Our setup, however, contains a second channel whereby higher bank retention slows down the bank's selling process. The longer the bank takes to sell off the entire loan, the more likely the bank will monitor, and the value of the loan is also higher. As a result, the second channel necessarily dominates.

We end this subsection by describing the borrower's expected payoff V_c. Specifically, given that the entrepreneur retains R_f, her payoff is

$$V_c = \mathbb{E}\left[\int_0^\infty \phi e^{-(\rho+\phi)t} \left\{ \mathbb{1}_{\{\kappa \leq \kappa_c\}} p_H R_f + \mathbb{1}_{\{\kappa \leq \kappa_c\}} (p_L R_f + B) \right\} dt \right], \tag{23}$$

where κ is the realization of $\tilde{\kappa}$. The expectation operator is taken with respect to the bank's equilibrium trading strategy, which involves mixed strategies. Intuitively, if the realized monitoring cost is lower than the threshold κ_c defined in (3), the bank monitors, and the entrepreneur receives $p_H R_f$ in expectation. Otherwise, the bank chooses not to monitor, and the entrepreneur receives the expected return $p_L R_f$ together with the private benefits B.

3.2 Intermediation Equilibrium

The analysis of the intermediation equilibrium has two steps: deposit issuance and loan trading.

3.2.1 Deposit Issuance

Plugging (10) and (8) into (12) and dividing both sides by $\rho + \phi$, we can rewrite the bank's objective function without trading gains as follows:

$$\mathcal{V}(D, \theta) := \Phi \left[\hat{p}_i (\theta, D) \theta R_o - \int_0^{\kappa_i} \kappa dF(\kappa) \right] + (1 - \Phi) D. \tag{24}$$

This objective function includes two terms. The first term is the net payoff to the bank and its depositors: with probability $\hat{p}(\theta, D)$, the project succeeds so that they receive θR_o; $\int_0^{\kappa_i} \kappa dF(\kappa)$ is the expected monitoring cost. The second term in (24) is the value from issuing deposits.

An increase in D reduces the bank's monitoring incentive and therefore reduces the first term. Meanwhile, an increase in D also reduces the bank's funding cost and therefore increases the second term. The optimal D is chosen to balance the two effects, which after some simple derivation, solves

$$\phi\left[f(\kappa_i)\Delta^2\right]\tilde{D} = \rho \tag{25}$$

under Assumption 2. For any $\theta \in (0,1)$, we denote the solution by $\tilde{D}(\theta)$. Meanwhile, note the bank's equity holders' limited liability constraint requires that $E(\theta, D) = \Pi_i(\theta) - D \geq 0$, so that for any θ, $D \leq \Pi_i(\theta)$. This constraint implicitly defines a maximum value of deposit that the bank can issue for any choice of θ, which we denote as $D^{\max}(\theta)$.[15] Because $\Pi_i(\theta)$, the bank's maximal value given its retention θ has implicitly assumed that deposit issuance is optimally chosen, solving for $D^{\max}(\theta)$ is therefore equivalent to looking for a fixed-point in the constraint. To finish the derivation of optimal deposit choice $D^*(\theta)$, we need one more assumption.

Assumption 4. *The distribution function $f(\tilde{\kappa})$ satisfies*

$$\Delta f'(\kappa)\, D - f(\kappa) < 0, \qquad \forall D \leq D^{\max}(\theta),\ \forall\theta \in [0,1],\ \forall\kappa \in [0,\bar{\kappa}].$$

Assumption 4 guarantees that for any given θ, the second-order partial derivative of (24) satisfies $\frac{\partial^2 \mathcal{V}}{\partial D^2} < 0$. This condition is always satisfied if f is non-increasing, such as the uniform distribution. Under this assumption, we get the following result.

Lemma 2. *A unique $\theta_\dagger \in [0,1]$ exists such that*

$$D^*(\theta) = \begin{cases} \tilde{D}(\theta) & \text{if } \theta \geq \theta_\dagger \\ D^{\max}(\theta) & \text{if } \theta < \theta_\dagger. \end{cases} \tag{26}$$

3.2.2 Trading

Next, we turn to the maximization problem (13) and study how an intermediating bank trades its loans over time. Following similar steps in the certification equilibrium, the term $\dot{\theta}\left(\Pi_i'(\theta) - q_i(\theta)\right)$ must vanish in the smooth-trading region, so the bank's continuation value satisfies the HJB equation:

$$\rho\Pi_i(\theta) = \phi\left[\pi_i(\theta) - \Pi_i(\theta)\right], \tag{27}$$

[15] Note in the case the bank decides to sell off all its remaining loans, $D^{\max}(\theta)$ is defined as 0.

and the equilibrium price is determined by the indifference condition $\Pi_i'(\theta) = q_i(\theta)$. As in the certification case, given $q_i(\theta)$, the trading strategy follows from the asset-pricing equation

$$0 = \phi\Big[d_i(\theta) - q_i(\theta)\Big] + \dot{\theta} q_i'(\theta) \Longrightarrow \dot{\theta} = -\phi \frac{d_i(\theta) - q_i(\theta)}{q_i'(\theta)}. \tag{28}$$

The functions $d_i(\theta)$ and $\pi_i(\theta)$ are defined under the optimal deposit issuance $D^*(\theta)$. We can apply the envelope theorem in Milgrom and Segal (2002) to get

$$\pi_i'(\theta) = d_i(\theta) + \left[f(\kappa_i)\frac{\partial \kappa_i}{\partial \theta}\Delta\right]\tilde{D} + z(\theta) D^{\max\prime}(\theta), \tag{29}$$

where $z(\theta) \geq 0$ is the Lagrange multiplier of the constraint $D \leq D^{\max}(\theta)$. Using condition (25), the equilibrium trading rate is[16]

$$\dot{\theta} = \phi \frac{(1-\Phi)(1-p(\theta)) R_o + \Phi z(\theta) D^{\max\prime}(\theta)}{\Phi \pi_i''(\theta)} > 0. \tag{30}$$

A comparison between (30) and (19) highlights a crucial difference between the two implementation structures. In certification, the bank has incentives to sell because investors have a higher valuation of these loans. In intermediation, this incentive disappears. In fact, the bank has incentives to increase its retentions. This distinction arises because an intermediating bank issues deposits to finance its loans. To see this, compare $\pi_c'(\theta) = d_c(\theta)$ with (29), which clearly shows an increase in θ leads to two additional benefits in intermediation. The first benefit is characterized by the term $\left[f(\kappa_i)\frac{\partial \kappa_i}{\partial \theta}\Delta\right]\tilde{D}$. Intuitively, $f(\kappa_i)\frac{\partial \kappa_i}{\partial \theta}$ is the marginal effect of retention θ on the incremental probability of bank monitoring, so that $f(\kappa_i)\frac{\partial \kappa_i}{\partial \theta}\Delta$ is the equilibrium probability that deposits will be repaid. Given that deposits are valued fairly, more retention enables the bank to issue *cheaper* deposits. The second benefit is captured by the last term $z(\theta)D^{\max\prime}(\theta)$, which is only positive if $D^*(\theta) = D^{\max}(\theta)$. Intuitively, an increase in θ also relaxes the constraint on deposit issuance derived from the bank equity holders' limited liability protection. Therefore, more retention has an add-on benefit by further allowing the bank to issue *more* deposits. Note this second benefit disappears whenever the deposit-issuance constraint is slack, that is, $z(\theta) = 0$. Even though the bank has a higher cost of capital and deposits reduce the bank's incentive to monitor, the availability of cheap deposits as a source of financing offers the bank sufficient incentives to increase its retention. Recall that by definition, $\theta_0 = 1$ in intermediation. In equilibrium, the bank never sells its loans and $\theta_t \equiv 1$ for all $t \geq 0$. In other words, an intermediating bank retains the entire loan until the project matures.

[16] Assumption 2 guarantees $\pi_i''(\theta) > 0$.

The result that short-term deposits (or in general short-term debt) could help with the commitment problem has been introduced in Calomiris and Kahn (1991), Diamond and Rajan (2001), and Diamond (2004). Whereas the mechanism of these papers relies on the demandable feature of deposits and run externalities, the channel in our paper is different. In particular, the bank retention choice directly affects the cost (and also the amount) of deposits. The rates of deposits enable the bank to internalize the externalities, such that monitoring is "awarded" in the form of cheaper deposits. In this sense, our paper is related to Myers (1977), who suggests short-term debt as a possible solution to the debt-overhang problem.[17] Another difference is the nature of the commitment problem. Whereas these papers study the bank's commitment to enforce claims and collect payments, we focus on how the bank commits to retain its loans over time.

Finally, to complete the characterization of the equilibrium, we consider the case in which the bank trade loans atomically. Following the same logic as in the certification equilibrium, we can show a unique θ_* exists such that the bank sells off all the remaining loans at price $q(0) = p_L R_o$ if $\theta < \theta_*$. However, when $\theta = \theta_*$, unlike in the certification equilibrium, the mixed strategy is no longer needed. Instead, the bank chooses to buy the loan smoothly so that θ will increase above θ_*. Therefore, the price of the loan satisfies $q_i(\theta_*) = \Phi\pi'(\theta_*)$.[18] The following proposition summarizes the results.

Proposition 4. *A unique* ***intermediation equilibrium*** *exists in which the bank holds $\theta_t = 1$ until the project matures, and keeps a constant debt level $D_t = D^*(1)$. The equilibrium loan price is $q_i(1) = \Phi\pi_i'(1)$.*

Similar to certification, we use V_i to denote the entrepreneur's expected payoff in intermediation:

$$V_i = \mathbb{E}\left[\int_0^\infty \phi e^{-(\rho+\phi)t}\left\{\mathbb{1}_{\{\kappa\leq\kappa_i\}}p_H R_f + \mathbb{1}_{\{\kappa\leq\kappa_i\}}(p_L R_f + B)\right\}dt\right].$$

The expressions differs from (23) in that the threshold cost for monitoring is replaced by κ_i.

Discussion: Trading Dynamics in Certification and Intermediation. We have seen the equilibrium trading dynamics are remarkably different in certification and intermediation. We offer some further discussions regarding the fundamental mechanisms behind the difference. In general, monitoring has the property of a public good in the sense that investors can free ride the bank.

[17] One can think of monitoring as value-enhancing investment.

[18] Both in the case of certification and intermediation, the price function $q(\theta)$ is discontinuous at θ_*. However, whereas in certification the bank trades toward the discontinuity point (i.e., $\dot{\theta}(\theta_*+) < 0$), in intermediation, the bank trades away from the discontinuity point (i.e., $\dot{\theta}(\theta_*+) > 0$). The construction of the equilibrium (and the analysis of the bank's optimal control problem) is simpler in this latter case because the trajectory of θ_t does not "see" the discontinuity.

All creditors commonly share the benefits from monitoring, whereas the bank exclusively bears the cost. In other words, by monitoring the borrower, the bank brings positive externalities to investors and therefore benefits the entrepreneur indirectly. Therefore, the equilibrium monitoring effort is inefficiently low, and in a dynamic framework with certification, the bank reduces its probability of monitoring over time. Price impacts deter the bank from selling the loan too fast and too aggressively.

In intermediation, the role of deposits is to help the bank internalize the externality from monitoring. Indeed, these deposits are fairly priced, which reflects the probability of monitoring. The income that the bank receives from deposit insurance thus compensates its cost incurred during monitoring. In this case, the bank and its depositors share both the benefits and costs of monitoring. Essentially, deposits create a market for the services offered by the bank, that is, monitoring, to be fairly priced.

The above argument also explains the importance of short-term deposits to align the bank's incentive in monitoring. Indeed, in the case of instantly maturing deposits, the value and the issuance of deposits are continuously adjusted without any friction. If, instead, depositors have a positive maturity, the bank will not completely internalize the externalities from monitoring.

3.3 A Special Case: Uniform Distribution

We present a special case of the model in which the monitoring cost $\tilde{\kappa}$ follows the uniform distribution. Although this assumption simplifies the analysis and the expressions, it also helps us focus on the insights that do not rely on the particular features of the distribution function. Under the uniform distribution, the marginal effect of retention θ_t on the probability of monitoring is always a constant. Moreover, we assume the probability of success is $p_L = 0$ if the entrepreneur shirks. This assumption naturally leads to a result that the bank will never sell off all its loans atomically, because the resulting price will be zero.[19] As a result, $\theta_* = 0$ and $T_* \to \infty$ in the certification equilibrium.

Assumption 5. *$f(\kappa) = \frac{1}{\bar{\kappa}}$ for $\kappa \in [0, \bar{\kappa}]$, and $p_L = 0$.*

The constrained optimal solution turns out very straightforward under Assumption 5.

Corollary 1. *Under Assumption 5, the constrained social planner's solution in Proposition 2 becomes*

$$\theta_t = \frac{1}{2 - e^{-\rho t}} \left[\left(1 - e^{-\rho t}\right) + e^{-\rho t} \frac{\Delta R - B}{\Delta R - b} \right].$$

[19] In the durable-goods monopoly literature, this is referred to as the "no-gap" case.

Moreover, $\theta_0 = \frac{\Delta R - B}{\Delta R - b}$ *and* $\lim_{t\to\infty} \theta_t = \frac{1}{2}$.

3.3.1 Certification Equilibrium

We first describe the certification equilibrium. Simple calculation shows $p_c(\theta) = (\kappa_c/\bar{\kappa})\Delta = \Delta^2 R_o \theta/\bar{\kappa}$. Equation (5), the bank's expected payoff when the project matures, becomes $\pi_c(\theta) = \frac{1}{2\bar{\kappa}}(\Delta R_o \theta)^2$. The evolution of θ_t in (19) follows $\dot{\theta} = -\rho\theta$, which implies $\theta_t = \theta_0 e^{-\rho t}$.

Corollary 2. *In the certification equilibrium, given any* θ_0*, the firm is able to borrow*

$$L_c(\theta_0) = \Pi_c(\theta_0) + (1-\theta_0)\, q_c(\theta_0) = \frac{\Phi}{2\bar{\kappa}}(\Delta R_o \theta_0)^2 + \frac{\Phi}{\bar{\kappa}}(\Delta R_o)^2 \theta_0 (1-\theta_0),$$

and the entrepreneur's payoff is

$$V_c(\theta_0) = \Phi B - \frac{\phi}{2\rho+\phi}\frac{\Delta R_o \theta_0}{\bar{\kappa}}(B-b).$$

Let $\theta_0^* = 1 - \frac{\rho+\phi}{2\rho+\phi}\frac{B-b}{\Delta R_o}$.

1. *If* $I - A \le L_c(\theta_0^*)$*, the entrepreneur chooses* $\theta_0 = \theta_0^*$.
2. *If* $I - A \in (L_c(\theta_0^*), L_c(1)]$*, the entrepreneur chooses* $\theta_0 = \theta_0^{\min}$*, where* $\theta_0^{\min}$ *is the unique solution to* $L_c(\theta_0^{\min}) = I - A$.
3. *If* $I - A > L_c(1)$*, the entrepreneur is unable to borrow enough.*

The expressions behind $L_c(\theta_0)$ and $V_c(\theta_0)$ have straightforward interpretations. Specifically, $L_c(\theta_0)$ includes $\Pi_c(\theta_0)$, the contribution from the bank, and $(1-\theta_0)\, q_c(\theta_0)$, the amount lent by investors. The entrepreneur's payoff is also clear. Whereas ΦB captures the discounted payoff if she always shirks and obtains the higher private benefit, the second term takes into account bank monitoring, which eliminates the high private benefit B. Instead, under $p_L = 0$ and $R_f = b/\Delta$, the manager effectively receives an expected payoff b. $\Delta R_o \theta_0/\bar{\kappa}$ captures the probability of monitoring under $\theta = \theta_0$, and the term $\frac{\phi}{2\rho+\phi}$ is the effective time discount, which includes random project maturing $(\rho+\phi)$, as well as the bank's equilibrium trading strategy so that its retention θ_t declines exponentially at a rate ρ. Clearly, a higher initial retention θ_0 increases bank monitoring and the amount of total lending $L_c(\theta_0)$; it also reduces the entrepreneur's expected payoff $V_c(\theta_0)$. Although the borrowing amount $L_c(\theta_0)$ is maximized at $\theta_0 = 1$, the retention that maximizes the firm's overall payoff $V_c(\theta_0) + L_c(\theta_0)$ has an interior solution $\theta_0^* \in (0,1)$. If the borrowing constraint is slack so that $L_c(\theta_0^*) > I - A$, the entrepreneur chooses θ_0^*. Otherwise, she chooses the minimum θ_0 that enables her to borrow just enough to invest.

3.3.2 Intermediation Equilibrium

Next, we describe the intermediation equilibrium under Assumption 5. As described in Proposition 4, $\theta_t \equiv \theta_0 = 1$ always holds. Simple calculations show $\tilde{D}(\theta) = \frac{\rho\bar{\kappa}}{\phi\Delta^2}$ is a constant, whereas

$$D^{\max}(\theta) = -\frac{\bar{\kappa}}{\Delta^2} + \sqrt{\left(\frac{\bar{\kappa}}{\Delta^2}\right)^2 + (R_o\theta)^2}.$$

At $\theta = 1$, the optimal choice of deposit is an interior solution if and only if $\tilde{D}(1) < D^{\max}(1)$, which requires Φ to be sufficiently high or, equivalently, the bank's cost of capital ρ to be sufficiently low. Otherwise, the bank's cost of capital is too high compared to that of investors. As a result, the bank will issue deposits up to the level constrained by its limited liability.

Corollary 3. *In the intermediation equilibrium, the firm is able to borrow*

$$L_i(1) = \Pi_i(1) = \Phi\pi_i(1),$$

where $\pi_i(1) = \frac{\Delta^2\left[R_o^2 - (D^*(1))^2\right]}{2\bar{\kappa}} + \frac{\rho}{\phi}D^*(1)$ *and* $D^*(1) = \min\left\{\frac{\rho\bar{\kappa}}{\phi\Delta^2}, -\frac{\bar{\kappa}}{\Delta^2} + \sqrt{\left(\frac{\bar{\kappa}}{\Delta^2}\right)^2 + (R_o)^2}\right\}$. *The entrepreneur's payoff is*

$$V_i = \Phi B - \Phi\frac{\Delta(R_o - D^*(1))}{\bar{\kappa}}(B - b).$$

1. *If* $\Phi \geq \underline{\Phi} := \sqrt{\frac{(\bar{\kappa}/\Delta^2)^2}{(\bar{\kappa}/\Delta^2)^2 + (R_o)^2}}$, *the bank's deposit choice satisfies* $D^*(1) = \tilde{D}(1) = \frac{\rho\bar{\kappa}}{\phi\Delta^2}$.

2. *Otherwise, the bank's deposit choice satisfies* $D^*(1) = D^{\max}(1) = -\frac{\bar{\kappa}}{\Delta^2} + \sqrt{\left(\frac{\bar{\kappa}}{\Delta^2}\right)^2 + (R_o)^2}$.

Similar to V_c, the entrepreneur's payoff V_i also has an intuitive interpretation. Whereas the first term considers the discounted payoff if she always shirks, the second term takes into account bank monitoring, which happens with conditional probability $\frac{\Delta R_o - D^*(1)}{\bar{\kappa}}$. Indeed, this expression is obtained by evaluating (4) at $\theta = 1$ and $D = D^*(1)$.

3.3.3 Initial Choice

We study the borrower's initial choice between certification and intermediation. Note the entrepreneur is always able to borrow more in intermediation, regardless of the deposit-issuance constraint. Therefore, if $I - A \in (L_c(1), L_i(1))$, the entrepreneur is only able to borrow through intermediation at $t = 0$. The most interesting comparison is under the case $I - A < L_c(\theta^*)$, so that the borrowing constraint is slack in both certification and intermediation. Proposition 5 shows certification could dominate intermediation.

Proposition 5. *Under Assumption 5 and $L_c(\theta_0^*) > I - A$, a threshold B^* exists such that the entrepreneur chooses certification if and only if $B > B^*$.*

We explain the mechanism behind Proposition 5. Given $L_i(1) > L_c(\theta_0)$,[20] the entrepreneur may prefer certification only if $V_c(\theta_0^*) > V_i$, that is, if the entrepreneur obtains a higher expected payoff under certification. Simple calculation shows

$$V_c - V_i = \frac{\phi\Delta}{\bar{\kappa}}(B - b)\left(\frac{R_o - D^*(1)}{\rho + \phi} - \frac{R_o\theta_0}{2\rho + \phi}\right).$$

The term $\frac{R_o\theta_0}{2\rho+\phi}$ captures the fact that in certification, the bank starts with θ_0 but sells at rate ρ over time. By contrast, $\frac{R_o - D^*(1)}{\rho+\phi}$ shows that an intermediating bank always retains $\theta = 1$, and the benefit from issuing deposits $D^*(1)$ will ultimately accrue to the entrepreneur. Clearly, $V_c > V_i$ if and only if

$$\frac{R_o - D^*(1)}{\rho + \phi} > \frac{R_o\theta_0}{2\rho + \phi},$$

which is necessarily the case if ρ, θ_0, and $D^*(1)$ are small. Conditional on $V_c > V_i$, a higher B leads to a larger difference in the entrepreneur's payoff. If the difference becomes sufficiently large, it can offset the difference in the amount of borrowing $(L_i(1) - L_c(\theta_0))$. In this case, the entrepreneur ends up choosing certification.

This result is reminiscent of the literature on the potential of overmonitoring (Pagano and Röell, 1998).[21] From the perspective of the entrepreneur, she cares not only about the market value of the project, but also her future benefits as the manager. Therefore, a level of monitoring that maximizes the firm value may be excessive. Although bank monitoring introduces a positive externality to investors, it also imposes a negative externality on the entrepreneur by restricting her from choosing the high private benefit B.

Our result implies more financially-constrained entrepreneurs borrows from intermediating banks, such as commercial and community banks, whereas entrepreneurs with more net worth and who potentially enjoy higher private benefits (or control rents) tend to borrow from certifying banks, such as venture capitalists (VCs). To the best of our knowledge, this result has not been covered by previous theoretical papers. De Bettignies and Brander (2007) argues the critical distinction between VCs and banks is that only VCs provide value-adding services. Winton and Yerramilli (2008) argues that compared with banks, VCs are more capable monitors but have higher costs of capital. Similarly, Ueda (2004) assumes VCs can screen borrowers better than banks before the investment is made. Her model predicts entrepreneurs will approach VCs when they require

[20]The proof follows because $L_i(1) \geq L_c(1) > L_c(\theta_0)$ and $D^*(1) \leq \tilde{D}(1) = \frac{\rho\bar{\kappa}}{\phi\Delta^2}$.

[21]See section 9.2.2 in Tirole (2010) for a summary of this literature.

large investment amounts. Landier (2005) introduces two-side hold-up problems and the stigma associated with a failed entrepreneur. He argues that if the entrepreneur is held up by investors, bank financing is more preferable. By contrast, VC financing is more preferable if investors can be held up by the entrepreneur.

Remark 1. *One may wonder whether the entrepreneur may benefit by simultaneously borrowing from both an intermediating bank and investors. The answer is yes. Under some conditions, the constrained-optimal allocation features the bank's retention θ_t increasing over time. An example is illustrated in Corollary 1, if $\theta_0 = \frac{\Delta R - B}{\Delta R - b} < \frac{1}{2}$. This solution can be better approximated by combining intermediation and direct borrowing from outside investors. Over time, the intermediating bank buys the loan from outside investors. This arrangement can potentially lead to a higher entrepreneur's payoff than both certification and intermediation.*

Numerical Example We illustrate the comparison using the following numerical example. Let $\phi = 1$, $b = 0$, $R = 2$, $p_H = 0.8$, $p_L = 0$, $\rho = 0.01$, and κ follow the uniform distribution on $[0, 2]$. Under given parameters, we can easily calculate that $\tilde{D}(1) = 0.0312$, $D^{\max} = 0.3122$, and $\kappa_i = 1.5750$. We also include the option of directly borrowing from investors, in which case the payoffs are $V_d = \Phi B$, $L_d = p_H R - B$, and $W_d = p_H R - (1 - \Phi) B$.

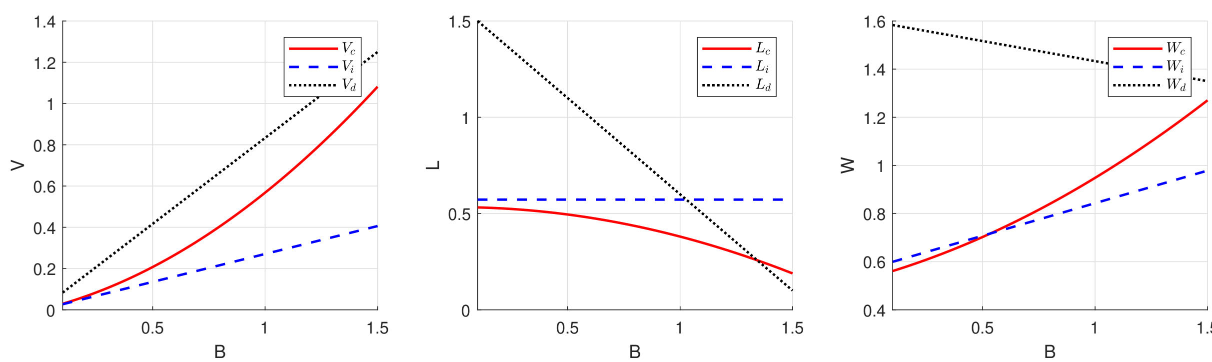


Figure 3: Valuation under Certification, Intermediation, and Direct Lending

Figure 3 illustrates the payoffs when the private benefit B varies. The red (solid), blue (dashed), and black (dotted) lines, respectively, stand for certification, intermediation, and directly lending. The left panel describes the entrepreneur's payoff, which increases with B in all three cases. The entrepreneur always obtains the highest payoff in direct lending. This result illustrates the negative externalities of bank monitoring on the entrepreneur. Moreover, her payoff is convex in B in certification but linear in intermediation. The middle panel shows the maximum amount of borrowing, which is independent of B in intermediation but decreases in the other two cases, because $\theta_t \equiv 1$ in intermediation but θ_t declines in certification. Finally, the right panel compares the overall payoff. Clearly, certification has a higher overall payoff than intermediation once B becomes sufficiently high.

4 Extensions

4.1 Certification: a Lockup Period and Minimum Retention

Lockup Period

We introduce a lockup arrangement that allows the bank to commit to θ_0 for a period $[0, t_\ell]$. Due to the stationary environment, the subgame starting from t_ℓ and the associated equilibrium are unchanged from those in 3.1. Between $[0, t_\ell]$, no trading occurs, and the bank's flow payoff is $\phi\pi_c(\theta_0)$. Let L_ℓ be the total lending at $t = 0$. Under Assumption 5, it is easily shown that

$$L_\ell = \Pi_c(\theta_0) + (1 - \theta_0)\left[d_c(\theta_0)\left(1 - e^{-\phi t_\ell}\right) + e^{-\phi t_\ell} q_c(\theta_0)\right].$$

Again, this expression confirms the earlier result that due to the lack of commitment, the certifying bank does not benefit from its ability to trade loans at all: it is able to lend exactly $\Pi_c(\theta_0)$ regardless of t_ℓ. Meanwhile, investors are willing to lend more as the lockup period t_L becomes longer, because $q_c(\theta_0) = \Phi d_c(\theta_0)$. Therefore, $\frac{\partial L_\ell}{\partial t_\ell} > 0$ so that the lockup period increases the total amount of lending: the incremental lending comes from investors' willingness to lend due to the bank's improved commitment.

Moreover, we use V_ℓ to denote the entrepreneur's overall payoff. Under Assumption 5,

$$V_\ell = \Phi\left[\frac{\Delta R_o \theta_0}{\bar{\kappa}} b + \left(1 - \frac{\Delta R_o \theta_0}{\bar{\kappa}}\right) B\right]\left(1 - e^{-(\rho+\phi)t_\ell}\right) + e^{-(\rho+\phi)t_\ell} V_c(\theta_0).$$

Intuitively, during the lockup period $[0, t_\ell]$, the entrepreneur is monitored and receives b with probability $\Delta R_o \theta_0 / \bar{\kappa}$; otherwise, she receives B. It is easily derived that $\frac{\partial V_\ell}{\partial t_\ell} < 0$, so that a longer lockup period leads to a lower payoff to the entrepreneur. Intuitively, the entrepreneur receives b with monitoring but B without monitoring. Although a longer lockup period increases monitoring, it also reduces the entrepreneur's equilibrium payoff.

Let $W_\ell = V_\ell + L_\ell$ be the aggregate social welfare, which is also the objective function that the entrepreneur tries to maximize at $t = 0$. Under Assumption 5, a longer lock-up period always increases the aggregate social welfare.

Corollary 4 (Optimal Lockup Period)**.** *For $\theta_0 = \theta_0^*$ as in Corollary 2, the aggregate social welfare attains the maximum as $t_\ell \to \infty$.*

Minimum Retention

According to section 941 of the Dodd-Frank Act, securitizers are required to retain no less than 5% of the credit risks associated with any securitization to perform intermediation services. This rule is commonly known as risk retention. In February 2018, the circuit court exempted CLO funds, We evaluate such a policy by imposing a minimum retention requirement on the bank.[22]

Suppose the bank must hold at least a fraction $\underline{\theta}$ of the loans on its balance sheet. The equilibrium is qualitatively similar to the one in Proposition 3: some threshold $\tilde{\theta}$ exists such that smooth trading occurs for $\theta \in (\tilde{\theta}, 1]$, and an atom $\tilde{\theta}$ exists where the holdings jump to $\underline{\theta}$. For the same reasons that liquidation has to be random in the certification case, the jump from $\tilde{\theta}$ to $\underline{\theta}$ must also happen with a random delay.

The construction of the equilibrium is similar to the one without minimum retention. The only difference is that we need to consider the incentives to jump to the constant $\underline{\theta}$ rather than the incentives to sell off the entire loan (i.e., to jump to $\theta = 0$). Because the construction, and the intuition behind the equilibrium conditions, is similar, we omit it for conciseness.

4.2 Intermediation: Alternative Bank Liability Structures

Long-term Deposits/Outside Equity

We solve the intermediation equilibrium in which the bank can only issue long-term deposits at $t = 0$, that is, deposits that only mature with the project at τ_ϕ. Given that the final cash flow has a binary outcome of either R or 0, long-term deposits are identical to outside equity. This extension highlights the importance of short-term deposits in commitment. Specifically, we show that under long-term deposits, the equilibrium is identical to the certification equilibrium, in which the bank trades loans over time.

Let $D_0 = D$ be the amount of long-term deposits that the bank issues at $t = 0$. Note that after $t = 0$, the bank can no longer raise any deposits. The definitions for κ_i, $\hat{p}(\theta, D)$, and $\hat{\pi}_i(\theta, D)$ follow from those in section 2. Because $D_0 = D$ is only chosen at $t = 0$, we suppress these functions' dependence on D, and therefore refer to $\kappa_i(\theta)$, $p(\theta)$, and $\pi_i(\theta)$, respectively. The bank's

[22] Securitization is identical to loan sales in our setup, given the binary outcome of the final cash flows.

maximization problem becomes

$$\max_{\{\theta_t,\, D_0\}} \, (1 - y_0)\, D_0 + \mathbb{E}\left[\int_0^\infty e^{-(\rho+\phi)t} \Big(\phi \pi_i (\theta_t)\, dt + dG(\theta_t) \Big) \right],$$

where y_0 compensates the depositors' credit risk. Note that after $t = 0$, the bank only chooses its trading strategy, and the solution method as well as the equilibrium outcomes naturally follow the certification equilibrium in subsection 3.1. In fact, they two are identical because depositors are effectively investors who directly invest in the venture.[23]

Deposit Subsidies

Next, we introduce an extension where the deposit rates are partially subsidized by the government. One can think about this subsidy as either deposit insurance or the implicit guarantee from a government bailout.[24] We show that once the subsidy becomes sufficiently high, an intermediating bank can no longer commit to its retention but instead sells loans over time, just as a certifying bank. This exercise highlights the importance of the deposit rate in aligning the bank's incentives to commit to its retentions.

Specifically, we assume the bank only needs to pay a fraction ξ of the deposit rate so that equation (10) becomes

$$y_t = \phi\xi \left(1 - \hat{p}(\theta, D)\right),$$

where $\xi \in (0, 1)$. The analysis follows that in subsection 3.2, in which deposit issuance and trading are solved sequentially.

Proposition 6. *A $\xi_\dagger$ exists such that in the intermediation equilibrium, $\dot{\theta} < 0$ if $\xi < \xi_\dagger$ for θ sufficiently large.*

Intuitively, the bank no longer has the incentives to retain its loans if the deposit is mostly subsidized by the government and when θ_t is still high such that the deposit-issuance constraint is slack. If θ_t is low, this result is no longer true, because the deposit-issuance constraint binds; that

[23] A subtle difference is that long-term depositors cannot trade their deposits, whereas investors in certification can sell the loans. However, given that in the certification equilibrium, the bank sells the loan (and, equivalently, investors buy the loan) over time, this difference does not affect the result. In other words, in the equilibrium with long-term deposits, depositors buy loans in the secondary market after $t = 0$.

[24] In the U.S., deposit insurance takes the form of a maximum guaranteed amount that has been \$250,000 since 2010. There is a one-to-one mapping between the maximum insurance amount and the parameter ξ introduced later on. To see this, note one can think about the deposit rate as $y_t = 0$ for deposits below $\$250,000$ but following (10) for deposits above the limit. Our parameter ξ captures the fraction of deposits that are above the limit.

is, $D^*(\theta) = D^{\max}(\theta)$. In this case, the bank will have incentives to retain loans. The overall effect in this case combined the two.

Bail-in vs. Bailout

Let us modify the model by assuming if the project fails and generates nothing, the bank is able to pay the depositors up to $X > 0$. One can think about X as the level of the bank's risk-absorbing equity or the liquidity required to put aside in case of bank failure.[25] The bank's incentive compatibility constraint in monitoring becomes $\kappa_i = \Delta(R_o\theta + X - D)$. Moreover, it is never optimal for the bank to issue risk-less deposits. In other words, the endogenous choice of deposits always satisfies $D > X$, with an interest rate $\hat{y}(\theta, D) = \phi\big(1 - \hat{p}(\theta, D - X)\big)\left(1 - \frac{X}{D}\right)$.[26] The equilibrium in this case is qualitatively unchanged from the intermediation equilibrium described in Proposition 4, instead of the one highlighted in Proposition 6.

The difference between X and ξ can be interpreted as the bail-ins vs. bailouts. Once again, this difference highlights the importance of monitoring externalities. When deposits are subsidized, only a fraction of the externalities is internalized, so the bank's incentives to monitor are also reduced. In the extreme case where the deposit rate is independent of the bank's retention (and therefore monitoring), the results go back to the case of certification. By contrast, if a bail-in occurs and the bank compensates the depositors' part of the losses (D/X) in the case of a bank failure, the monitoring externalities are still internalized.

5 Conclusion and Implications

This paper presents a dynamic theory of intermediary financing when banks cannot commit to its retentions. We show two equivalence results fail in the dynamic framework without commitment. First, the implementation of the financial structure leads to different credit market dynamics. Whereas a certifying bank sells its loans over time, an intermediating bank retains its loans through issuing short-term deposits. As a result, certification in general leads to a lower borrowing amount than intermediation. Second, maximizing the borrowing amount is different from maximizing the entrepreneur's expected payoff. In particular, if the entrepreneur can borrow enough under both structures, she may prefer certification to intermediation for a higher expected payoff.

[25] One example is the liquidity-coverage ratio (LCR).

[26] Following the same step as in the derivation of equation (24), we can write the bank's payoff function in terms of net debt $\tilde{D} \equiv D - X$:

$$\mathcal{V}(\tilde{D}, \theta) := X + \Phi\left[\hat{p}_i(\theta, \tilde{D})\theta R_o - \int_0^{\tilde{\kappa}_i} \kappa dF(\kappa)\right] + (1 - \Phi)\,\tilde{D}.$$

In the paper, we only allow for one bank to monitor after $t = 0$. If we allow the entrepreneur to sign contracts with multiple banks that can all monitor after time 0, two additional effects will arise. First, if the monitoring costs are *i.i.d.* across banks, introducing multiple banks naturally leads to a diversification effect, which increases the chances of monitoring. However, a pecuniary externality exists when one bank trades its loans over time in the sense that it only has a partial price impact. As a result, banks will sell faster. The overall effect is therefore unclear.

For tractability reasons, we have assumed the entrepreneur is able to commit to her retention over time. Relaxing this assumption and studying the interaction between the commitment problems between the borrower and the bank can be an interesting extension in the future. In particular, we can study the role of many sophisticated contracts such as covenants in preventing the commitment issues. Another interesting extension is along the lines of Gorton and Pennacchi (1990). One can imagine information on the entrepreneur's asset payoffs arrives gradually so that uncertainty is resolved gradually. In this case, although instantly maturing deposits (or, in general, short-term debt) improve the commitment problem and eliminate the dilution effect (DeMarzo and He, forthcoming), they also expose the borrowers to rollover risks (Diamond, 1991). Studying how a carefully designed security could trade off between commitment and risk hedging would be interesting.

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Appendix

A Certification

Lemma 3. *The bank with retention θ never sells a fraction of the loans.*

Proof. Suppose the bank with retention θ sells $\theta^+ - \theta$, where $\theta^+ > 0$, and that after this it continuous trading smoothly. Multiple jumps are ruled out without loss of generality. In this case, the overall trading gains are $dG(\theta) + \Pi_c(\theta^+) - \Pi_c(\theta) = (\theta - \theta^+)\, q_c(\theta^+) + \Pi_c(\theta^+) - \Pi_c(\theta)$, where $dG(\theta)$ is the instant trading gain and $\Pi(\theta^+) - \Pi(\theta)$ are the gains (negative loss) in its continuation value. Block trading is suboptimal as long as

$$\theta = \arg\max_{\theta^+} \Big\{ \Pi_c(\theta^+) + (\theta - \theta^+) q_c(\theta^+) \Big\}. \tag{31}$$

It is easy to verify that the first order condition is always satisfied at $\theta^+ = \theta$, thus it suffice to show that the second order condition for global optimality is satisfied. $\square$

Verification of Optimality Trading Strategy

In this section, we complete the characterization of the equilibrium by verifying that the equilibrium trading strategy maximizes the bank's payoff given the price function $q(\theta)$. Because the payoff in a mixed strategy equilibrium is given by the payoff of any pure strategy in its support, we can restrict attention to pure strategies in the verification of optimality. A trading strategy for the bank is given by right continuous function with left limits. A trading strategy is admissible if it can be decomposed as

$$\theta_t = \int_0^t \dot{\theta}^c_t dt + \sum_{k \geq 0} (\theta^d_{t_k} - \theta^d_{t_k-}),$$

for some bounded function $\dot{\theta}^c_t$. We denote the set of admissible trading strategies by Θ. The bank's optimization problem is to choose $\theta \in \Theta$ to maximize its payoff

$$\Pi^*(\theta_0) = \sup_{\theta \in \Theta} \int_0^\infty e^{-(\rho+\phi)t} \big(\phi \pi(\theta_t) - \dot{\theta}^c_t q(\theta_t)\big) dt - \sum_{k \geq 0} e^{-(\rho+\phi)t_k} q(\theta_{t_k})(\theta^d_{t_k} - \theta^d_{t_k-}). \tag{32}$$

Due to the discontinuity in the price price function $q(\theta)$, the Hamilton-Jacobi-Bellman (HJB) equation is discontinuous at θ_*, so we need to resort to the theory of viscosity solutions for the analysis of the bank's problem. Our problem is a particular case of the general class of optimal control problems in stratified domains studied by Barles et al. (2018). Our proof relies on their characterization of the value function using viscosity solution methods. The analysis in Barles et al. (2018) does not consider the case in which the trajectory of the state variable can be discontinuous (impulse control). However, as we show below, we can approximate a trading $\theta_t \in \Theta$ by an absolutely continuous trading strategy with derivative $|\dot{\theta}_t| \leq N$ for some N large enough (the approximation is in the sense that it yields a similar payoff). Thus, we can consider a sequence of optimization problems

$$\Pi_N^*(\theta_0) = \sup_{|\dot{\theta}_t|\le N} \int_0^\infty e^{-(\rho+\phi)t}\big(\phi\pi(\theta_t) - \dot{\theta}_t q(\theta_t)\big)dt, \tag{33}$$

and verify that, for any $\theta \in [0,1]$, $\Pi_N^*(\theta) \to \Pi(\theta)$, where

$$\Pi(\theta) = \begin{cases} \Phi\pi(\theta) & \text{if } \theta \in [\theta_*, 1] \\ q(0)\theta & \text{if } \theta \in [0, \theta_*) \end{cases}.$$

The following Lemma establishes that we can indeed consider the limit of bounded absolutely continuous strategies.

Lemma 4. *For any $\theta_0 \in [0,1]$, $\lim_{N\to\infty} \Pi_N^*(\theta_0) = \Pi^*(\theta_0)$.*

Proof. Let $\theta_t^{\epsilon *}$ be and ϵ-optimal policy (at this point in the proof we have not established existence of an optimal policy). For any $k \ge 0$, let $\Delta_k \equiv \inf\{\Delta > 0 : \theta^{\epsilon *}_{t_k-\Delta} + \text{sgn}(\theta^{\epsilon *}_{t_k} - \theta^{\epsilon *}_{t_k-})N\Delta = \theta^{\epsilon *}_{t_k}\}$ (we can find Δ_k if N is large enough as $|\dot{\theta}_t^{\epsilon c *}| \le M$ for some finite M). Consider the policy $\hat{\theta}_t^N = \theta_t^{\epsilon *}$ if $t \notin \cup_{k\ge 0}(t_k - \Delta_k, t_k)$, and $\hat{\theta}_t^N = \theta^{\epsilon *}_{t_k-\Delta_k} + \text{sgn}(\theta^{\epsilon *}_{t_k} - \theta^{\epsilon *}_{t_k-})N(t - t_k + \Delta_k)$ if $t \in \cup_{k\ge 0}(t_k - \Delta_k, t_k)$. The difference between the payoff of $\theta_t^{\epsilon *}$ and $\hat{\theta}_t^N$ is

$$\begin{aligned} \Pi^{\epsilon *}(\theta_0) - \hat{\Pi}_N &= \sum_{k\ge 0} \bigg\{ \int_{t_k-\Delta_k}^{t_k} e^{-(\rho+\phi)t}\big(\phi\pi(\theta_t^{\epsilon *}) - \dot{\theta}_t^{\epsilon c *} q(\theta_t^{\epsilon *}) - \phi\pi(\hat{\theta}_t^N)\big)dt \\ &\quad + \int_{t_k-\Delta_k}^{t_k} e^{-(\rho+\phi)t}\text{sgn}(\theta^{\epsilon *}_{t_k} - \theta^{\epsilon *}_{t_k-})Nq(\hat{\theta}_t^N)dt - e^{-(\rho+\phi)t_k} q(\theta^{\epsilon *}_{t_k})(\theta^{\epsilon d *}_{t_k} - \theta^{\epsilon d *}_{t_k-}) \bigg\} - \epsilon \\ &= \sum_{k\ge 0} \bigg\{ \int_{t_k-\Delta_k}^{t_k} e^{-(\rho+\phi)t}\big(\phi\pi(\theta_t^{*}) - \dot{\theta}_t^{\epsilon c *} q(\theta_t^{*}) - \phi\pi(\hat{\theta}_t^N)\big)dt \\ &\quad + \frac{\theta^{\epsilon *}_{t_k} - \theta^{\epsilon *}_{t_k-\Delta_k}}{\Delta_k} \int_{t_k-\Delta_k}^{t_k} e^{-(\rho+\phi)t} q(\hat{\theta}_t^N)dt - e^{-(\rho+\phi)t_k} q(\theta^{\epsilon *}_{t_k})(\theta^{\epsilon d *}_{t_k} - \theta^{\epsilon d *}_{t_k-}) \bigg\} - \epsilon. \end{aligned}$$

For all $k \ge 0$, we have that $\Delta_k \downarrow 0$ as $N \to \infty$. It follows that

$$\lim_{\Delta_k \downarrow 0} \frac{1}{\Delta_k} \int_{t_k-\Delta_k}^{t_k} e^{-(\rho+\phi)t} q(\hat{\theta}_t^N)dt = \begin{cases} e^{-(\rho+\phi)t_k} q(\theta^{\epsilon *}_{t_k}-) & \text{if } \theta^{\epsilon *}_{t_k} > \theta^{\epsilon *}_{t_k-} \\ e^{-(\rho+\phi)t_k} q(\theta^{\epsilon *}_{t_k}+) & \text{if } \theta^{\epsilon *}_{t_k} < \theta^{\epsilon *}_{t_k-}. \end{cases}$$

The price function is right continuous so $q(\theta^{\epsilon *}_{t_k}+) = q(\theta^{\epsilon *}_{t_k})$. We can conclude that

$$\lim_{N\to\infty} (\Pi^{\epsilon *}(\theta_0) - \hat{\Pi}_N(\theta_0)) = \sum_{k\ge 0} e^{-(\rho+\phi)t_k}\big(q(0) - q(\theta_*)\big)(\theta^*_{t_k} - \theta^*_{t_k-})^+ \mathbf{1}_{\{\theta^*_{t_k} = \theta_*\}} - \epsilon \le 0.$$

Because this holds for any $\epsilon > 0$, we can conclude that $\lim_{N\to\infty}(\Pi^*(\theta_0) - \hat{\Pi}_N(\theta_0)) \le 0$, and given that $\Pi^*(\theta_0) \ge \hat{\Pi}_N(\theta_0)$, we get $\lim_{N\to\infty} \hat{\Pi}_N(\theta_0) = \Pi^*(\theta_0)$. For N large enough, the policy $\hat{\theta}_t^N$ satisfies $|\dot{\hat{\theta}}_t^N| \le N$

(this can be guarantee because for any ϵ there is M such that $|\dot{\theta}_t^{\epsilon c*}| \leq M$), so its payoff, $\hat{\Pi}_N(\theta_0)$ provides a lower bound to $\Pi_N^*(\theta_0)$, which means that $\lim_{N\to\infty} \Pi_N^*(\theta_0) = \Pi^*(\theta_0)$. $\square$

This shows that the value function converges (pointwise) to the one in the equilibrium under consideration. Hence, we can verify the optimality of the bank's strategy by analyzing the control problem (33). For future reference, recall that the price function in the control problem (33) is given by

$$q(\theta) = \begin{cases} \Phi\pi'(\theta) & \text{if } \theta \geq \theta_* \\ p_L R(1-\alpha) & \text{if } \theta < \theta_* \end{cases}, \tag{34}$$

where the threshold θ_* is given by $\Phi\pi(\theta_*) = q(0)\theta_*$. Notice that we are not computing the equilibrium in a model in which the bank is restricted to use absolutely continuous trading strategies with bounded derivative $\dot{\theta}_t$, but rather considering the equilibrium price function in the general case, and then considering a sequence of auxiliary optimization problems to construct the value function. Because the expected payoff of the candidate equilibrium strategy is equal to the value function, it is necessarily optimal.

The Hamilton-Jacobi-Bellman equation (HJB) for the optimization problem (33) is

$$(\rho+\phi)\Pi_N(\theta) - H(\theta, \Pi_N'(\theta)) = 0, \tag{35}$$

where H

$$H(\theta, \Pi_N') \equiv \phi\pi(\theta) + \max_{|\dot{\theta}|\leq N} \left\{ \dot{\theta}\big(\Pi_N' - q(\theta)\big) \right\}. \tag{36}$$

We guess and verify that, for N large enough, the solution (in the viscosity sense) of the previous equation is

$$\Pi_N(\theta) = \begin{cases} \Phi\pi(\theta) & \text{if } \theta \in [\theta_*, 1] \\ e^{-\frac{\rho+\phi}{N}(\theta_*-\theta)}\Phi\pi(\theta_*) + \frac{(\rho+\phi)}{N}\int_\theta^{\theta_*} e^{-\frac{\rho+\phi}{N}(y-\theta)}\left(\Phi\pi(y) - \frac{N}{\rho+\phi}q(0)\right)dy & \text{if } \theta \in [\tilde{\theta}_N, \theta_*) \\ \frac{N}{\rho+\phi}\left(1 - e^{-\frac{(\rho+\phi)}{N}\theta}\right)q(0) + \frac{(\rho+\phi)}{N}\int_0^\theta e^{-\frac{(\rho+\phi)}{N}(\theta-y)}\Phi\pi(y)dy & \text{if } \theta \in [0, \tilde{\theta}_N), \end{cases} \tag{37}$$

where $\tilde{\theta}_N$ is the unique solution on $[0,1]$ to the equation

$$\frac{N}{\rho+\phi}\left(1 - e^{-\frac{(\rho+\phi)}{N}\tilde{\theta}_N}\right)q(0) + \frac{(\rho+\phi)}{N}\int_0^{\tilde{\theta}_N} e^{-\frac{(\rho+\phi)}{N}(\tilde{\theta}_N-y)}\Phi\pi(y)dy =$$
$$e^{-\frac{\rho+\phi}{N}(\theta_*-\tilde{\theta}_N)}\Phi\pi(\theta_*) + \frac{(\rho+\phi)}{N}\int_{\tilde{\theta}_N}^{\theta_*} e^{-\frac{\rho+\phi}{N}(y-\tilde{\theta}_N)}\left(\Phi\pi(y) - \frac{N}{\rho+\phi}q(0)\right)dy \tag{38}$$

A.1 Auxiliary Lemmas

Before proceeding with the verification theorem, we provide several Lemmas providing properties of our candidate value function $\Pi_N(\theta)$ that will be later used in the verification

Lemma 5. *If $\Phi\pi_c(1) > p_L R_o > 0$, then there exists a unique $\theta_* \in (0,1)$ solving the equation*

$$\theta_* q_c(0) = \Phi\pi_c(\theta_*) \tag{39}$$

If $p_L = 0$, then $\theta_* = 0$ is the unique solution to (39) on $[0,1]$.

Proof. As $\Phi\pi_c(0) = 0$, equation (39) is trivially satisfied at $\theta_* = 0$, we want to show that if $\Phi\pi_c(1) > p_L R_o = q_c(0)$, then there is a non trivial solution $\theta_* > 0$ that also satisfies equation (39). First, if $\Phi\pi_c(1) > p_L R_o$, then the right hand side of equation (39) is strictly larger than its left hand side evaluated at $\theta_* = 1$. Second, as $\Phi\pi_c'(0) < q_c(0)$ it follows that for ε small enough $\varepsilon q_c(0) > \Phi\pi_c(\varepsilon)$. Thus, it follows from continuity that a non trivial solution exists on $(0,1)$. Uniqueness follows because

$$\begin{aligned} q_c(0) - \Phi\pi_c'(\theta_*) &= \frac{\Phi\pi_c(\theta_*)}{\theta_*} - \Phi p_c(\theta_*) R_o \\ &= \Phi\left[p_c(\theta_*) R_o - \frac{1}{\theta_*}\int_0^{\kappa_c(\theta_*)} \kappa dF(\kappa)\right] - \Phi p_c(\theta_*) R_o < 0, \end{aligned}$$

so the function $\theta q_c(0) - \Phi\pi_c(\theta)$ single crosses 0 from above, which implies $\theta q_c(0) > \Phi\pi_c(\theta)$ on $\theta \in (0,\theta_*)$ and $\theta q_c(0) < \Phi\pi_c(\theta)$ on $\theta \in (\theta_*, 1]$. Finally, if $p_L = 0$, then $\Phi\pi_c'(0) = q_c(0) = 0$. It follows then from the convexity of $\pi_c(\theta)$ that $\theta_* = 0$ is a global maximum of $\theta q_c(0) - \Phi\pi_c(\theta)$, which means that $\theta q_c(0) < \Phi\pi_c(\theta)$ for all $\theta > 0$. $\square$

Lemma 6. *There is a unique solution $\tilde{\theta}_N \in (0,\theta_*)$ to equation (38).*

Proof. First, we show existence. Given the definition of θ_* and the convexity of $\pi(\theta)$ we have that $\Phi\pi(\theta) < \theta q(0)$ for all $\theta < \theta_*$. Hence,

$$\frac{N}{\rho+\phi}\left(1 - e^{-\frac{(\rho+\phi)}{N}\theta_*}\right) q(0) + \frac{(\rho+\phi)}{N}\int_0^{\theta_*} e^{-\frac{(\rho+\phi)}{N}(\theta - y)}\Phi\pi(y)dy < \Phi\pi(\theta_*).$$

We also have that

$$\begin{aligned} e^{-\frac{\rho+\phi}{N}\theta_*}\Phi\pi(\theta_*) + \frac{(\rho+\phi)}{N}\int_0^{\theta_*} e^{-\frac{\rho+\phi}{N}y}\left(\Phi\pi(y) - \frac{N}{\rho+\phi}q(0)\right)dy &\leq \Phi\pi(\theta_*) - \frac{N}{(\rho+\phi)}\left(1 - e^{-\frac{\rho+\phi}{N}\theta_*}\right) q(0) \\ &\leq \Phi\pi(\theta_*) - \theta_* q(0) = 0. \end{aligned}$$

The existence of a solution follows from the intermediate value theorem. To show uniqueness we consider the derivative of the difference between the left and the right hand sides of equation (38) evaluated at θ, which we denote by $G'(\theta)$.

$$G'(\theta) = e^{-\frac{(\rho+\phi)}{N}\theta} q(0) + \frac{(\rho+\phi)}{N}\Phi\pi(\theta) - \frac{(\rho+\phi)^2}{N^2}\int_0^\theta e^{-\frac{(\rho+\phi)}{N}(\theta-y)}\Phi\pi(y)dy$$
$$- \frac{(\rho+\phi)}{N} e^{-\frac{\rho+\phi}{N}(\theta_*-\theta)}\Phi\pi(\theta_*) + \frac{(\rho+\phi)}{N}\left(\Phi\pi(\theta) - \frac{N}{\rho+\phi}q(0)\right)$$
$$- \frac{(\rho+\phi)^2}{N^2}\int_\theta^{\theta_*} e^{-\frac{\rho+\phi}{N}(y-\theta)}\left(\Phi\pi(y) - \frac{N}{\rho+\phi}q(0)\right)dy$$

From here we get that

$$\begin{aligned}
G'(\theta_*) &= -\left(1 - e^{-\frac{(\rho+\phi)}{N}\theta_*}\right) q(0) - \frac{(\rho+\phi)^2}{N^2}\int_0^{\theta_*} e^{-\frac{(\rho+\phi)}{N}(\theta_*-y)}\Phi\pi(y)dy + \frac{(\rho+\phi)}{N}\Phi\pi(\theta_*) \\
&\leq \frac{(\rho+\phi)}{N}\left(\Phi\pi(\theta_*) - \theta_* q(0)\right) = 0 \\
G'(0) &= -\frac{(\rho+\phi)}{N} e^{-\frac{\rho+\phi}{N}\theta_*}\Phi\pi(\theta_*) - \frac{(\rho+\phi)^2}{N^2}\int_0^{\theta_*} e^{-\frac{\rho+\phi}{N}y}\left(\Phi\pi(y) - \frac{N}{\rho+\phi}q(0)\right)dy \\
&\geq -\frac{(\rho+\phi)}{N}\Phi\pi(\theta_*) + \left(1 - e^{-\frac{\rho+\phi}{N}\theta_*}\right) q(0) \\
&\geq \frac{(\rho+\phi)}{N}\left(\theta_* q(0) - \Phi\pi(\theta_*)\right) = 0
\end{aligned}$$

Moreover, we get that, for any $\theta \in (0, \theta_*)$, $G(\theta) = 0$ implies

$$\begin{aligned}
G'(\theta) &= \frac{2(\rho+\phi)}{N}\left[\Phi\pi(\theta) - \left(\frac{N}{\rho+\phi}\left(1 - e^{-\frac{(\rho+\phi)}{N}\theta}\right) q(0) + \frac{(\rho+\phi)}{N}\int_0^\theta e^{-\frac{(\rho+\phi)}{N}(\theta-y)}\Phi\pi(y)dy\right)\right] \\
&\leq \frac{2(\rho+\phi)}{N}\left[\Phi\pi(\theta) - \theta q(0)\right] < 0.
\end{aligned}$$

It follows that $G(\theta)$ single crosses 0, so there is a unique solution to the equation $G(\theta) = 0$. ☐

Lemma 7. *There is $\tilde{N}$ such that, for all $N > \tilde{N}$, $\Pi'_N(\theta) < q(0)$ on $(0, \tilde{\theta}_N)$ and $\Pi'_N(\theta) > q(0)$ on $(\tilde{\theta}_N, \theta_*)$.*

Proof. First, we very that $\Pi'_N(\theta) < q(0)$ on $(0, \tilde{\theta}_N)$. The derivative of $\Pi_N(\theta) - \theta q(0)$ on $(0, \tilde{\theta}_N)$ is given by

$$\begin{aligned}
\Pi'_N(\theta) - q(0) &= \frac{\rho+\phi}{N}\Phi\pi(\theta) - \left(1 - e^{-\frac{(\rho+\phi)}{N}\theta}\right) q(0) - \frac{(\rho+\phi)^2}{N^2}\int_0^\theta e^{-\frac{(\rho+\phi)}{N}(\theta-y)}\Phi\pi(y)dy \\
&\leq \frac{\rho+\phi}{N}\Phi\pi(\theta) - \left(1 - e^{-\frac{(\rho+\phi)}{N}\theta}\right) q(0) \leq \frac{\rho+\phi}{N}\left(\Phi\pi(\theta) - \theta q(0)\right) < 0.
\end{aligned}$$

The derivative of $\Pi_N(\theta) - \theta q(0)$ on $(\tilde{\theta}_N, \theta_*)$ is given by

$$\Pi'_N(\theta) - q(0) = \frac{\rho+\phi}{N}\left[e^{-\frac{\rho+\phi}{N}(\theta_*-\theta)}\Phi\pi(\theta_*) - \Phi\pi(\theta) + \frac{(\rho+\phi)}{N}\int_\theta^{\theta_*} e^{-\frac{\rho+\phi}{N}(y-\theta)}\left(\Phi\pi(y) - \frac{N}{\rho+\phi}q(0)\right)dy\right]$$

Differentiating the HJB equation we get that

$$\Pi_N''(\theta) = \frac{(\rho+\phi)}{N}\left(\Pi_N'(\theta) - \Phi\pi'(\theta)\right)$$
$$\Pi_N'''(\theta) = \frac{(\rho+\phi)}{N}\left(\Pi_N''(\theta) - \Phi\pi''(\theta)\right).$$

From here we get that $\Pi_N''(\theta) = 0 \implies \Pi_N'''(\theta) < 0$, so we the function $\Pi_N'(\theta)$ is quasi-concave on $(\tilde{\theta}_N, \theta_*)$. Moreover, $\Pi_N'(\theta_*-) = q(0)$, and

$$\Pi_N'(\tilde{\theta}_N+) = q(0) + \frac{(\rho+\phi)}{N}\left(\Pi_N(\tilde{\theta}_N+) - \Phi\pi(\tilde{\theta}_N)\right) > q(0),$$

so we can conclude that $\Pi_N'(\theta) > q(0)$ on $(\tilde{\theta}_N, \theta_*)$ as long as $\Pi_N(\tilde{\theta}_N+) > \Phi\pi(\tilde{\theta}_N)$, which follows from

$$\begin{aligned}
\Pi_N(\tilde{\theta}_N+) - \Phi\pi(\tilde{\theta}_N) &= \Pi_N(\tilde{\theta}_N-) - \Phi\pi(\tilde{\theta}_N) \\
&= \frac{N}{\rho+\phi}\left(1 - e^{-\frac{(\rho+\phi)}{N}\tilde{\theta}_N}\right)q(0) - \Phi\pi(\tilde{\theta}_N) + \frac{(\rho+\phi)}{N}\int_0^{\tilde{\theta}_N} e^{-\frac{(\rho+\phi)}{N}(\tilde{\theta}_N - y)}\Phi\pi(y)dy \\
&\geq \frac{N}{\rho+\phi}\left(1 - e^{-\frac{(\rho+\phi)}{N}\tilde{\theta}_N}\right)q(0) - \Phi\pi(\tilde{\theta}_N) \\
&= \tilde{\theta}_N q_0 - \frac{(\rho+\phi)\tilde{\theta}_N^2}{N} - \Phi\pi(\tilde{\theta}_N) + O\big(1/N^2\big).
\end{aligned}$$

$\tilde{\theta}_N q_0 > \Phi\pi(\tilde{\theta}_N)$ because θq_0 single crosses $\Phi\pi(\theta)$ at $\theta_* \geq \tilde{\theta}_N$. Hence, there is $\tilde{N}$ such that, for all $N \geq \tilde{N}$, we have $\Pi_N(\tilde{\theta}_N+) > \Phi\pi(\tilde{\theta}_N)$. □

Lemma 8. *Let*

$$\Pi(\theta) = \begin{cases} \Phi\pi(\theta) & \text{if } \theta \in [\theta_*, 1] \\ q(0)\theta & \text{if } \theta \in [0, \theta_*) \end{cases}.$$

Then, for any $\theta \in [0,1]$

$$\lim_{N\to 0} \Pi_N(\theta) = \Pi(\theta).$$

Proof. For all $\theta \geq \theta_*$, $\Pi_N(\theta) = \Pi(\theta)$, and, for any $\theta < \theta_*$, $\lim_{N\to\infty} \Pi_N(\theta) = \theta q(0) = \Pi(\theta)$ by L'Hopital's rule. □

A.2 Verification of Optimality

We start providing the necessary definitions from the theory of viscosity solutions, together with the relevant results from the theory of optimal control in stratified domains in Barles et al. (2018). We make some changes in notation to make it consistent with our setting, and to translate their minimization problem into a maximization one. While Barles et al. (2018) considers the state space to be the complete real line, the state space in our case is $[0,1]$. However, we can extend the state space by letting the payoff on the complement of $[0,1]$ to be sufficiently low. This can be achieved by adding a penalization term, and setting the flow payoff equal to $\phi\pi(1) - \dot{\theta}q(1) - k|\theta - 1|$ for $\theta > 1$, and $\phi\pi(0) - \dot{\theta}q(0) - k|\theta|$ for $\theta < 0$. By choosing k large enough, we can ensure that the optimal solution never exits the interval $[0, 1]$. Due to the discontinuity in the Hamiltonian at θ_*, a viscosity solution might fail to be unique. In order to fully characterize the value function we need to specify its behavior at θ_*. This is done in Barles et al. (2018) by considering the concept of *Flux-limited* sub- and supersolutions. Letting $\Omega_0 = (-\infty, \theta_*)$ and $\Omega_1 = (\theta_*, \infty)$, we consider the equation

$$\begin{cases} (\rho + \phi)\Pi - H_0(\theta, \Pi) = 0 & \text{in } \Omega_0 \\ (\rho + \phi)\Pi - H_1(\theta, \Pi) = 0 & \text{in } \Omega_1 \\ (\rho + \phi)\Pi - \phi\pi(\theta_*) = 0 & \text{in } \{\theta_*\}, \end{cases} \tag{40}$$

where

$$H_0(\theta, \Pi') = \phi\pi(\theta) + k\min\{0, \theta\} + \max_{|\dot{\theta}| \le N} \Big\{\dot{\theta}\big(\Pi' - q(0)\big)\Big\}$$
$$H_1(\theta, \Pi') = \phi\pi(\theta) - k\max\{0, \theta - 1\} + \max_{|\dot{\theta}| \le N} \Big\{\dot{\theta}\big(\Pi' - q(\theta)\big)\Big\}$$

In $\Omega_0 \cup \Omega_1$, the definitions are just classical viscosity sub- and supersolution, which we provide next for completeness.

Definition 3 (Bardi and Capuzzo-Dolcetta (2008), Definition 1.1)**.** *A function $u \in C(\mathbb{R})$ is a viscosity subsolution of* (35) *if, for any $\varphi \in C^1(\mathbb{R})$,*

$$(\rho + \phi)u(\theta_0) - H(\theta_0, \varphi'(\theta_0)) \le 0, \tag{41}$$

at any local maximum point $\theta_0 \in \mathbb{R}$ of $u - \varphi$. Similarly, $u \in C(\mathbb{R})$ is a viscosity supersolution of (35) *if, for any $\varphi \in C^1(\mathbb{R})$,*

$$(\rho + \phi)u(\theta_1) - H(\theta_1, \varphi'(\theta_1)) \ge 0, \tag{42}$$

at any local minimum point $\theta_1 \in \mathbb{R}$ of $u - \varphi$. Finally, u is a viscosity solution of (35) *if it is simultaneously a viscosity sub- and supersolution.*

Before providing the definition of sub- and supersolution on $\{\theta_*\}$, we introduce the following space $\Im$ of real valued test functions: $\varphi \in \Im$ if $\varphi \in C(\mathbb{R})$ and there exist $\varphi_0 \in C^1(\overline{\Omega}_0)$ and $\varphi_1 \in C^1(\overline{\Omega}_1)$ such that $\varphi = \varphi_0$ in $\overline{\Omega}_0$, and $\varphi = \varphi_1$ in $\overline{\Omega}_1$. Next, we introduce two Hamiltonians that are needed to define a flux-limited sub- and supersolution at $\{\theta_*\}$.

$$H_1^+(\theta_*, \Pi') \equiv \phi\pi(\theta) + \sup_{0 < \dot{\theta} \le N} \Big\{\dot{\theta}\big(\Pi' - q(\theta_*)\big)\Big\}$$
$$H_0^-(\theta_*, \Pi') \equiv \phi\pi(\theta) + \sup_{0 > \dot{\theta} \ge -N} \Big\{\dot{\theta}\big(\Pi' - q(0)\big)\Big\}.$$

Definition 4 (Barles et al. (2018), Definition 2.1)**.** *An upper semi-continuous, bounded function $u : \mathbb{R} \to \mathbb{R}$ is a flux-limited subsolution on $\{\theta_*\}$ if for any test function $\varphi \in \Im$ such that $u - \varphi$ has a local maximum at*

θ_*, *we have*

$$(\rho+\phi)u(\theta_*) - \max\{\phi\pi(\theta_*), H_0^-(\theta_*, \varphi_0'(\theta_*)), H_1^+(\theta_*, \varphi_1'(\theta_*))\} \leq 0. \tag{43}$$

A lower semi-continuous, bounded function $v : \mathbb{R} \to \mathbb{R}$ *is a flux-limited supersolution on* $\{\theta_*\}$ *if for any test function* $\varphi \in \Im$ *such that* $u - \varphi$ *has a local minimum at* θ_**, we have*

$$(\rho+\phi)v(\theta_*) - \max\{\phi\pi(\theta_*), H_0^-(\theta_*, \varphi_0'(\theta_*)), H_1^+(\theta_*, \varphi_1'(\theta_*))\} \geq 0. \tag{44}$$

The Hamiltonians H_0^- and H_1^+ are needed to express the optimality conditions at the discontinuity θ_*. H_1^+ consider controls that starting at θ_* take θ_t towards the interior of $[\theta_*, 1]$, and H_0^- considers controls that starting at θ_* take θ_t towards the interior of $[0, \theta_*]$. The use of the Hamiltonians H_0^- and H_1^+ at $\{\theta_*\}$, instead of H_0 and H_1, distinguishes *flux-limited* viscosity solutions from the traditional (discontinuous) viscosity solutions.

We consider the following control problem, equivalent to the one defined in (33),

$$\begin{aligned}
\tilde{\Pi}_N^*(\theta_0) &= \sup_{|\theta_t| \leq N} \int_0^\infty e^{-(\rho+\phi)t} \Big(\phi\tilde{\pi}(\theta_t) - \theta_t \tilde{q}(\theta_t)\mathbf{1}_{\{\theta_t \neq \theta_*\}} - k\big(\max\{0, \theta_t - 1\} - \min\{0, \theta_t\}\big)\Big) dt \\
\tilde{\pi}(\theta) &= \pi(0)\mathbf{1}_{\{\theta<0\}} + \pi(\theta)\mathbf{1}_{\{\theta\in[0,1]\}} + \pi(1)\mathbf{1}_{\{\theta>1\}} \\
\tilde{q}(\theta) &= q(0)\mathbf{1}_{\{\theta<0\}} + q(\theta)\mathbf{1}_{\{\theta\in[0,1]\}} + q(1)\mathbf{1}_{\{\theta>1\}}.
\end{aligned}$$

The following Theorem characterizes the value function $\tilde{\Pi}_N^*$ in terms of flux-limited viscosity solutions.

Theorem 1 (Barles et al. (2018), Theorem 2.9)**.** *The value function* $\tilde{\Pi}_N^*$ *is the unique flux-limited viscosity solution to equation* (40).

We can now proceed to apply Theorem 1 to verify that Π_N defined in (37) is the value function of the control problem (33).

Verification Lemmas 6 and 7 imply that Π_N is a classical solution on $\Omega \setminus \{\tilde{\theta}_N, \theta_*\}$ so we only need to verify the conditions for a viscosity solution on $\{\tilde{\theta}_N, \theta_*\}$. Π_N defined in (37) is a classical solution on $(\theta_*, 1)$. At $\theta = \theta_*$, Π_N has a convex kink so we only need to verify the supersolution property. That is, that for any $\varphi'(\theta_*)$ in the subdifferential of $\Pi_N(\theta)$ at θ_*, which is $[\Pi_N'(\theta_*-), \Pi_N'(\theta_*+)]$, inequality (44) is satisfied. $H_1^+\big(\theta_*, \varphi'(\theta_*)\big)$ is nondecreasing in $\varphi'(\theta_*)$ and $H_0^-\big(\theta_*, \varphi'(\theta_*)\big)$ is nonincreasing in $\varphi'(\theta_*)$; thus, the supersolution property follows from

$$\begin{aligned}
(\rho+\phi)\Pi_N(\theta_*) - H_1^+\big(\theta_*, \Pi_N'(\theta_*+)\big) &= (\rho+\phi)\Pi_N(\theta_*) - \phi\pi(\theta_*) = 0 \\
(\rho+\phi)\Pi_N(\theta_*) - H_0^-\big(\theta_*, \Pi_N'(\theta_*-)\big) &= (\rho+\phi)\Pi_N(\theta_*) - \phi\pi(\theta_*) = 0.
\end{aligned}$$

As $\Pi_N'(\tilde{\theta}_N-) < q(0) < \Pi_N'(\tilde{\theta}_N+)$, $\Pi_N(\theta)$ has a convex kink at $\tilde{\theta}_N$, we only need to verify the property for a supersolution. Thus, we need to verify that for any $\varphi'(\tilde{\theta}_N) \in [\Pi_N'(\tilde{\theta}_N-), \Pi_N'(\tilde{\theta}_N+)]$, inequality (42) is satisfied. This amounts to verify that

$$(\rho+\phi)\Pi_N(\tilde{\theta}_N)-\phi\pi(\tilde{\theta}_N)\geq N\max\left\{\left|\Pi_N'(\tilde{\theta}_N-)-q(0)\right|,\left|\Pi_N'(\tilde{\theta}_N+)-q(0)\right|\right\}$$

By definition of $\tilde{\theta}_N$, we have

$$(\rho+\phi)\Pi_N(\tilde{\theta}_N)-\phi\pi(\tilde{\theta}_N)=N\big(\Pi_N'(\tilde{\theta}_N+)-q(0)\big)=N\big(q(0)-\Pi_N'(\tilde{\theta}_N-)\big),$$

so it follows that

$$(\rho+\phi)\Pi_N(\tilde{\theta}_N)-\phi\pi(\tilde{\theta}_N)=N\big|\Pi_N'(\tilde{\theta}_N-)-q(0)\big|=N\big|\Pi_N'(\tilde{\theta}_N+)-q(0)\big|.$$

Finally, at $\theta=1$, by choosing k large enough, we have that the solution of the HJB equation on $\{\theta>1\}$ entails $\dot{\theta}(\theta)=-N$. Moreover, $\Pi'(1-)=q(1)$ implies that the value function is differentiable at $\theta=1$ (in the extended problem) and that $\dot{\theta}(1)\leq 0$ is optimal. Thus, the state constraint is satisfied at $\theta=1$. A similar argument applies at $\theta=0$. Thus, we can conclude that $\Pi_N(\theta)$ is a flux-limited viscosity solution, so, by Theorem 1, it is the value function of the optimal control problem.

Finally, uniqueness in Proposition 3 follows from the unique solution to the HJB and the boundary θ_*.

Proof of Corollary 2

Proof. Let us define

$$v(\theta)=F(\kappa_c)\,p_H\,(R-R_o)+(1-F(\kappa_c))\,B \tag{45}$$

as the borrower's expected payoff if the asset matures. Therefore, the borrower's expected payoff at $t=0$ is

$$V_c(\theta_0)=\int_0^\infty e^{-(\rho+\phi)t}\phi v(\theta_t)\,dt=\Phi B+\frac{\phi\,(b-B)}{\bar{\kappa}}\frac{\Delta R_o\theta_0}{2\rho+\phi}, \tag{46}$$

where we have substituted $R_o=R-\frac{b}{\Delta}$ and therefore $p_H\,(R-R_o)=b$. Finally, the borrower's problem at time 0 is

$$W_c=\max_{\{\theta_0,R_o\}}\quad V_c(\theta_0)+L_c(\theta_0) \tag{47}$$

$$s.t.\quad L_c(\theta_0)\geq I-A. \tag{48}$$

It is easily verified that, $L_c(\theta_0)$ is maximized at $\theta_0=1$. However, $V_c(\theta_0)+L_c(\theta_0)$ is maximized at

$$\theta_0=1-\frac{\rho+\phi}{2\rho+\phi}\frac{B-b}{\Delta R_o}. \tag{49}$$

□

B Intermediation

Proof of Lemma 1

Proof. Using integration by parts and a Transversality condition $\lim_{t\to\infty} e^{-(\rho+\phi)t}D_t = 0$, we have

$$\int_0^\infty e^{-(\rho+\phi)s} dD_s = \int_0^\infty e^{-(\rho+\phi)s}(\rho+\phi)D_s ds - D_0.$$

(11) can be rewritten as

$$\max_{(D_t,\theta_t)_{t\geq 0}} \mathbb{E}\left[\int_0^\infty e^{-(\rho+\phi)s}\left[\left(\phi\hat{\pi}(\theta_s, D_s) - y\left(\theta_s, D_s\right) D_s + (\rho+\phi)D_s\right) ds + dG\left(\theta_s\right)\right]\right] - D_0$$

The optimization in the lemma follows directly from the definition of $\phi\pi(\theta)$ ☐

Proof of Lemma 2

Proof. The bank's optimal deposit is given by

$$D(\theta) = \arg\max_D \mathcal{V}(D,\theta) \tag{50}$$

$$\Phi\pi_i(\theta) = \max_D \mathcal{V}(D,\theta). \tag{51}$$

The first order condition follows from (25), and the second order condition is

$$\mathcal{V}_{DD}(D,\theta) \propto \Delta f'\left(\kappa_i(\theta, D)\right) D - f\left(\kappa_i(\theta, D)\right) < 0. \tag{52}$$

☐

Unique $\theta_\dagger$

We show that there is a unique solution to the equation

$$\Phi\pi_i'(\theta) = q\left(0\right)\theta. \tag{53}$$

The existence proof naturally follows. The uniqueness follows if we take the *F.O.C.* of the two sides

$$\Phi\pi_i'(\theta) - q\left(0\right) = \Phi d_i(\theta) + \left(1-\Phi\right) R_o - p_L R_o = \Phi R_o\left(p\left(\theta\right) - p_L\right) + \left(1-\Phi\right) R_o\left(1-p_L\right) > 0. \tag{54}$$

Proof of Lemma 2

Proof. We need to show that $\mathcal{V}(\tilde{D}(\theta),\theta) - p_L R_o \theta$ is monotonic in θ. Using the envelope theorem, we have that

$$\frac{d\mathcal{V}(\tilde{D}(\theta),\theta)}{d\theta} - p_L R_o = \left[\Phi\Delta^2 f(\kappa_i)\tilde{D}(\theta) + \Phi\Delta F(\kappa_i) - (1-\Phi)p_L\right]R_o.$$

If the solution to the first order condition $\tilde{D}(\theta)$ is interior (that is, less than $D(\theta)^\dagger$) we have

$$\Phi\Delta^2 f\left(\kappa_i(\theta,\tilde{D}(\theta))\right)\tilde{D}(\theta) = 1-\Phi,$$

substituting in the previous equation, we get

$$\frac{d\mathcal{V}(\tilde{D}(\theta),\theta)}{d\theta} - p_L R_o = \left[(1-\Phi)(1-p_L) + \Phi\Delta F(\kappa_i)\right]R_o > 0.$$

□

Proof of Corollary 3

Proof. In the uniform case with $p_L = 0$ we have that $D^{\max}(\theta)$ solves

$$\frac{\Delta^2}{2\bar{\kappa}}\left[(R_o\theta)^2 - D^2\right] = D.$$

The previous equation has two roots, and the positive root is

$$D^{\max}(\theta) = -\frac{\bar{\kappa}}{\Delta^2} + \sqrt{\left(\frac{\bar{\kappa}}{\Delta^2}\right)^2 + (R_o\theta)^2}.$$

Next, let us look for conditions where

$$\tilde{D}(1) = \frac{\rho\bar{\kappa}^2}{\phi\Delta^2} \leq D^{\max}(1) = -\frac{\bar{\kappa}}{\Delta^2} + \sqrt{\left(\frac{\bar{\kappa}}{\Delta^2}\right)^2 + R_o^2}.$$

Simple derivation shows this is satisfied if and only if

$$\Phi > \sqrt{\frac{(\bar{\kappa}/\Delta^2)^2}{R_o^2 + (\bar{\kappa}/\Delta^2)^2}},$$

which holds if and only if ρ is sufficiently low. If $\tilde{D}(1) < D^{\max}(1)$, then we can plug in $D^*(1)$ and get $\pi_i(1) = \frac{\Delta^2}{2\bar{\kappa}}\left(R_o - \frac{\rho\bar{\kappa}}{\phi\Delta^2}\right)^2 + \frac{\rho+\phi\frac{\Delta^2}{\bar{\kappa}}\left(R_o - \frac{\rho\bar{\kappa}}{\phi\Delta^2}\right)}{\phi}\frac{\rho\bar{\kappa}}{\phi\Delta^2}$.

According to (4), $\kappa_i = \Delta\left(R_o - \frac{\rho\bar{\kappa}}{\phi\Delta^2}\right)$ and $p_i(1) = \frac{\kappa_i}{\bar{\kappa}}\Delta = \frac{\Delta^2}{\bar{\kappa}}\left(R_o - \frac{\rho\bar{\kappa}}{\phi\Delta^2}\right)$. Consequently, $\hat{\pi}_i(1, D^*)$ and

$\pi_i(1)$ defined in (8) and (12) become

$$\hat{\pi}_i(1, D^*) = \frac{\Delta^2}{2\bar{\kappa}}\left(R_o - \frac{\rho\bar{\kappa}}{\phi\Delta^2}\right)^2 \tag{55}$$

$$\pi_i(1) = \frac{\Delta^2}{2\bar{\kappa}}\left(R_o - \frac{\rho\bar{\kappa}}{\phi\Delta^2}\right)^2 + \frac{\rho + \phi\frac{\Delta^2}{\bar{\kappa}}\left(R_o - \frac{\rho\bar{\kappa}}{\phi\Delta^2}\right)}{\phi}\frac{\rho\bar{\kappa}}{\phi\Delta^2}. \tag{56}$$

As in the certification case, we can similarly define

$$v(1) = F(\kappa_i)\, p_H(R - R_o) + (1 - F(\kappa_i))\, B = B + \frac{\Delta\left(R_o - \frac{\rho\bar{\kappa}}{\phi\Delta^2}\right)}{\bar{\kappa}}(b - B) \tag{57}$$

$$V_i(1) = \int_0^\infty e^{-(\rho+\phi)t}\phi v(\theta_t)\, dt = \Phi v(1) \tag{58}$$

$$L_i(1) = \Pi_i(1) = \Phi\pi_i(1). \tag{59}$$

Finally, the borrower's payoff at time 0 is

$$W_i = V_i(1) + L_i(1). \tag{60}$$

□

Proof of Proposition 5

Proof. In the proposition, $L_c(\theta_0^*) > I - A$ guarantees that the borrowing constraints are slack in both certification and intermediation. Under certification

$$W_c = \Phi B + \frac{\phi(b - B)}{\bar{\kappa}}\frac{\Delta R_o}{2\rho + \phi}\theta_0 + \frac{\Phi}{\bar{\kappa}}(\Delta R_o)^2\theta_0\left(1 - \frac{\theta_0}{2}\right),$$

where θ_0 is evaluated at

$$\theta_0^* = 1 - \frac{\rho + \phi}{2\rho + \phi}\frac{B - b}{\Delta R_o} = 1 - \frac{1}{2 - \Phi}\frac{B - b}{\Delta R_o},$$

Under intermediation,

$$W_i = \Phi\left[B + \frac{\Delta\left(R_o - \frac{\rho\bar{\kappa}}{\phi\Delta^2}\right)}{\bar{\kappa}}(b - B)\right] + \frac{\Phi}{2\bar{\kappa}}\Delta^2\left(R_o - \frac{\rho\bar{\kappa}}{\phi\Delta^2}\right)^2 + \frac{\rho + \phi\frac{\Delta^2}{\bar{\kappa}}\left(R_o - \frac{\rho\bar{\kappa}}{\phi\Delta^2}\right)}{\rho + \phi}\frac{\rho\bar{\kappa}}{\phi\Delta^2}.$$

Certification dominates intermediation if $W_c > W_i$, letting $\Delta W \equiv W_c - W_i$,

$$\begin{aligned}
\Delta W =& \Phi\left[\frac{1}{2-\Phi}\frac{(b-B)}{\bar{\kappa}}\left(\Delta R_o - \frac{1}{2-\Phi}(B-b)\right) + \frac{1}{2\bar{\kappa}}\left(\Delta R_o - \frac{1}{2-\Phi}(B-b)\right)\left(\Delta R_o + \frac{1}{2-\Phi}(B-b)\right)\right. \\
&\left. - \frac{(b-B)}{\bar{\kappa}}\left(\Delta R_o - \frac{\rho\bar{\kappa}}{\phi\Delta}\right) - \frac{1}{2\bar{\kappa}}\left(\Delta R_o - \frac{\rho\bar{\kappa}}{\phi\Delta}\right)^2 - \left(\frac{1-\Phi}{\Phi} + \frac{\Delta}{\bar{\kappa}}\left(\Delta R_o - \frac{\rho\bar{\kappa}}{\phi\Delta}\right)\right)\frac{\rho\bar{\kappa}}{\phi\Delta^2}\right] \\
=& \frac{\Phi}{2\bar{\kappa}}\left[\left(\frac{B-b}{2-\Phi}\right)^2 + 2\left(\frac{1-\Phi}{2-\Phi}\Delta R_o - \frac{\rho\kappa}{\phi\Delta}\right)(B-b) - \left(\frac{\rho\bar{\kappa}}{\phi\Delta}\right)^2\right]
\end{aligned}$$

The previous equation is negative for B on $(\underline{B}, \bar{B})$, where

$$\begin{aligned}
\underline{B} &= b - (2-\Phi)^2\left[\sqrt{\left(\frac{1-\Phi}{2-\Phi}\Delta R_o + \frac{\rho\kappa}{\phi\Delta}\right)^2 + \left(\frac{\rho\bar{\kappa}}{\phi\Delta(2-\Phi)}\right)^2} - \left(\frac{1-\Phi}{2-\Phi}\Delta R_o - \frac{\rho\kappa}{\phi\Delta}\right)\right] \\
\bar{B} &= b + (2-\Phi)^2\left[\sqrt{\left(\frac{1-\Phi}{2-\Phi}\Delta R_o - \frac{\rho\kappa}{\phi\Delta}\right)^2 + \left(\frac{\rho\bar{\kappa}}{\phi\Delta(2-\Phi)}\right)^2} - \left(\frac{1-\Phi}{2-\Phi}\Delta R_o - \frac{\rho\kappa}{\phi\Delta}\right)\right]
\end{aligned}$$

Because, $\underline{B} < b$, we only need to consider the upper bound, so, after substituting R_o, we get that the expression is negative as long as B satisfies

$$b < B < b + (2-\Phi)^2\left[\sqrt{\left(\frac{1-\Phi}{2-\Phi}(\Delta R - b) - \frac{\rho\kappa}{\phi\Delta}\right)^2 + \left(\frac{\rho\bar{\kappa}}{\phi\Delta(2-\Phi)}\right)^2} - \left(\frac{1-\Phi}{2-\Phi}(\Delta R - b) - \frac{\rho\kappa}{\phi\Delta}\right)\right].$$

Thus, we can conclude that certification dominates if

$$B > b + (2-\Phi)^2\left[\sqrt{\left(\frac{1-\Phi}{2-\Phi}(\Delta R - b) - \frac{\rho\kappa}{\phi\Delta}\right)^2 + \left(\frac{\rho\bar{\kappa}}{\phi\Delta(2-\Phi)}\right)^2} - \left(\frac{1-\Phi}{2-\Phi}(\Delta R - b) - \frac{\rho\kappa}{\phi\Delta}\right)\right].$$

We need to compare now W_c and W_d.

$$W_c - W_d = B + \frac{\phi\,(b-B)}{\bar{\kappa}}\frac{\Delta R_o}{2\rho + \phi}\theta_0 + \frac{\Phi}{\bar{\kappa}}\left(\Delta R_o\right)^2\theta_0\left(1 - \frac{\theta_0}{2}\right) - p_H R$$

□

Proof of Proposition 2 and Corollary 1

Proof. The expression for the welfare function is straightforward. The optimal retention satisfies the first order condition

$$\left(1 - e^{-\rho t}\right)\left[(1-\theta_t) f\left(\kappa_c\right)\left(\Delta R_o\right)^2 - p\left(\theta_t\right) R_o\right] + e^{-\rho t} f\left(\kappa_c\right)\left(\Delta R - B - \kappa_c\right)\Delta R_o = 0.$$

Plugging in $\theta_t = 1$, we can show that the FOC satisfies

$$\left(1 - e^{-\rho t}\right)\left[-p\left(1\right)R_o\right] + e^{-\rho t} f\left(\kappa_c\right)\left(b - B\right)\Delta R_o < 0.$$

The solution under Assumption 5 is straightforward, as well as the verification that the second-order condition is satisfifed. $\square$

Proof of Corollary 4

Proof. For any θ_0, it is easily verified that

$$\frac{\partial W_\ell}{\partial t_\ell} = 0 \Longrightarrow \frac{\partial^2 W_\ell}{\partial t_\ell^2} > 0,$$

which means that $W_\ell(t_\ell)$ is quasi-convex, so it is maximized on $\{0, \infty\}$. Moreover, it is easily verified that

$$\lim_{t_\ell \to \infty} W_\ell(t_\ell) - W_\ell(0) = \left(1 - \theta_0^*\right)\left(1 - \Phi\right)\frac{\left(\Delta R_o\right)^2 \theta_0^*}{\bar{\kappa}} - \Phi \frac{\rho}{2\rho + \phi}\frac{\Delta R_o \theta_0^*}{\bar{\kappa}}\left(B - b\right) > 0,$$

so $\arg\max_{\{t_\ell \geq 0\}} W_\ell(t_\ell) = \infty$. $\square$

Proof of Proposition 6

Proof. We can similarly define $\mathcal{V}$, the bank's objective function without trading gains as

$$\mathcal{V}(D, \theta) := \Phi\left[\hat{p}_i\left(\theta, D\right)\theta R_o - \int_0^{\kappa_i} \kappa dF\left(\kappa\right)\right] + \left(1 - \Phi\right)D + \Phi\left(1 - \hat{p}_i\left(\theta, D\right)\right)\left(1 - \xi\right)D,$$

where the new term $\Phi\left(1 - \hat{p}_i\left(\theta, D\right)\right)\left(1 - \xi\right)D$ stands for the benefit of the government subsidy.

If θ is sufficiently large such that the deposit issuance constraint is slack, simple derivation shows that the optimal deposit issuance satisfies

$$\tilde{D}\left(\theta\right) = \frac{\left(1 - \Phi\right) - \Phi\left(1 - p(\theta)\right)\left(1 - \xi\right)}{\Phi\left[1 - \left(1 - p(\theta)\right)\left(1 - \xi\right)\right] f\left(\kappa_i\right)\Delta^2}.$$

Recall that in Lemma 2, the constraint is slack whenever θ is sufficiently high.

In the region that the deposit-issuance constraint is slack, the HJB equation implies

$$\dot{\theta} = \phi \frac{R_o \xi \frac{(1-\Phi) - \Phi(1-p(\theta))(1-\xi)}{1-(1-p(\theta))(1-\xi)} - \left(1 - \Phi\right)p(\theta)R_o}{\Phi \pi_i''\left(\theta\right)}.$$

Clearly, when $\xi = 1$, we get the results in subsection 3.2 that $\dot{\theta} = \phi \frac{R_o(1-\Phi)(1-p(\theta))}{\Phi \pi_i''(\theta)} > 0$. Moreover, when $\xi = 0$ so that the entire deposit rate is subsidized by the government, $\dot{\theta} = \phi \frac{-(1-\Phi)p(\theta)R_o}{\Phi \pi_i''(\theta)} < 0$, implying the bank sells loans over time. In general, there exists a $\xi_\dagger$ and $\dot{\theta} < 0$ if $\xi < \xi_\dagger$. $\square$

Proof of Proposition 2

Proof. Given any $\{\theta_t\}_{t\geq 0}$, the entrepreneur's payoff is $\int_0^\infty e^{-(\rho+\phi)t}\phi v(\theta_t)dt$, whereas the bank receives $\int_0^\infty e^{-(\rho+\phi)t}\phi\pi(\theta_t)dt$ and investors receive $\int_0^\infty e^{-\phi t}\phi(1-\theta_t)d(\theta_t)dt$.

$$\begin{aligned} v(\theta) &= p(\theta)(R-R_o) + (1-F(\kappa_c))B \\ \pi(\theta) &= p(\theta)R_o\theta - \int_0^{\kappa_c}\kappa dF(\kappa) \\ d(\theta) &= p(\theta)R_o. \end{aligned}$$

From here, we get that the planners problem is

$$\begin{aligned} W &= \max_{(\theta_t)_{t\geq 0}} \int_0^\infty \phi e^{-\phi t}\Big[(1-\theta_t)d(\theta_t) + e^{-\rho t}\left(v_c(\theta)+\pi_c(\theta)\right)\Big]dt \\ &= \max_{(\theta_t)_{t\geq 0}} \int_0^\infty \phi e^{-\phi t}\left\{\left(1-e^{-\rho t}\right)\left[(1-\theta_t)p(\theta_t)R_o\right] + e^{-\rho t}\left[p(\theta_t)R + (1-F(\kappa_c))B - \int_0^{\kappa_c}\kappa dF(\kappa)\right]\right\}dt. \end{aligned}$$

We can maximize the previous expression pointwise to get the first order condition

$$(1-\theta_t)f(\kappa_c)\Delta R_o\left(1-e^{-\rho t}\right)\frac{\partial\kappa_c}{\partial\theta} - p(\theta_t)\left(1-e^{-\rho t}\right)R_o + e^{-\rho t}f(\kappa_c)(\Delta R - B - \kappa_c)\frac{\partial\kappa_c}{\partial\theta} = 0$$

Substituting $\kappa_c = \Delta R_o\theta$ we get

$$(1-\theta_t)f(\kappa_c)\left(1-e^{-\rho t}\right)(\Delta R_o)^2 - (p_L + F(\kappa_c)\Delta)\left(1-e^{-\rho t}\right)R_o + e^{-\rho t}f(\kappa_c)(\Delta R - B + \kappa_c)\Delta R_o = 0$$

Under Assumption 5, this simplifies to

$$\theta_t = \frac{\Delta R - b - e^{-\rho t}(B-b)}{(\Delta R - b)(2-e^{-\rho t})}.$$

Differentiating we get

$$\dot{\theta}_t = \frac{\rho e^{\rho t}(2B - b - \Delta R)}{(\Delta R - b)(2-e^{\rho t})^2}.$$

Therefore,

$$\dot{\theta}_t < 0 \Leftrightarrow \frac{\Delta R - B}{B - b} > 1.$$

At time zero we have

$$\theta_0 = \frac{\Delta R - B}{\Delta R - b},$$

and as $t\to\infty$

$$\lim_{t\to\infty}\theta_t = \frac{\Delta R - b}{2(\Delta R - b)} = \frac{1}{2}.$$

□

C Minimum Retention

Next, we construct the equilibrium in the case of certification with minimum retention. First, let's consider the price of the loan once the bank hits the minimum retention level $\underline{\theta}$. Because $\theta = \underline{\theta}$ is an absorbing state, the price of the loan satisfies $q_c(\underline{\theta}) = d_c(\underline{\theta}) = \pi'_c(\underline{\theta})$. For any $\theta > \underline{\theta}$ with smooth trading, the HJB is (15) the loan price satisfies equation (18) are unchanged. Therefore, in any region with smooth-trading, the bank's value function, the equilibrium trading strategy, and the price of loans is unchanged. It remains to check if at some θ, the bank has incentives to trade atomically to $\theta^+ = \underline{\theta}$. The payoff of jumping from θ to $\underline{\theta}$ is $\Phi\pi_c(\underline{\theta}) + q_c(\underline{\theta})(\theta - \underline{\theta})$ while the payoff in the smooth trading region is $\Pi_c(\theta) = \Phi\pi_c(\theta)$. If $\Phi\pi_c(\underline{\theta}) + q_c(\underline{\theta})(1 - \underline{\theta}) > \Phi\pi_c(1)$, the certifying bank always sells immediately to $\theta^+ = \underline{\theta}$. Otherwise, as in the proof of the case without minimum requirements, there is a unique $\tilde{\theta}$ satisfying

$$\Phi\pi_c(\tilde{\theta}) = \Phi\pi_c(\underline{\theta}) + q(\underline{\theta})(\tilde{\theta} - \underline{\theta}) = \Phi\pi_c(\underline{\theta}) + \pi'_c(\underline{\theta})(\tilde{\theta} - \underline{\theta}). \tag{61}$$

By no-arbitrage, the price function must be upper semi-continuous (that is $q(\tilde{\theta}) = q(\tilde{\theta}+)$), which means that $q(\tilde{\theta}) = \Phi\pi'_c(\tilde{\theta})$.[27] At the same time, no-arbitrage also requires to be equal to the expected dividend $\mathbb{E}\left[d_c(\theta_{\tau_\phi})|\theta_t = \tilde{\theta}\right]$. If the bank stops trading at $\tilde{\theta}$, and remains there for an exponential time with mean arrival rate λ, at which time sells $\tilde{\theta} - \underline{\theta}$, the expected dividend is

$$\mathbb{E}\left[d_c(\theta_{\tau_\phi})|\theta_t = \tilde{\theta}\right] = \frac{\lambda}{\phi + \lambda} d_c(\underline{\theta}) + \frac{\phi}{\phi + \lambda} d_c(\tilde{\theta}).$$

Combining the previous conditions, we get that λ is implicitly given bu

$$\Phi\pi'_c(\tilde{\theta}) = \frac{\lambda}{\phi + \lambda}\pi'_c(\underline{\theta}) + \frac{\phi}{\phi + \lambda}\pi'_c(\tilde{\theta}) \tag{62}$$

The next proposition, summarize the previous discussion and describe the equilibrium in the certification case with minimum retention requirements.

Proposition 7 (Equilibrium with Minimum Retention)**.** *There is a unique* ***Certification Equilibrium with Minimum Retention*** . *Given the bank's initial retention θ_0, the bank sells its loans smoothly at a rate given by equation (19) until $\tilde{T} = \min\{t > 0 : \theta_t = \tilde{\theta}\}$, were $\tilde{\theta}$ is the unique solution to equation (61). After time $\tilde{T}$, the bank holds $\theta_t = \tilde{\theta}$ until an exponentially distributed random time τ_λ, at which time it sells off $\tilde{\theta} - \underline{\theta}$. The exponential time τ_λ has a mean arrival rate $\lambda \mathbf{1}_{\{\theta_t = \tilde{\theta}\}}$, where λ satisfies (62). After time τ_λ, the bank holds $\underline{\theta}$ until the projects maturity. The equilibrium loan price is*

$$q_c(\theta_t) = \begin{cases} \Phi\Big(p_L + F\left(\Delta R_o \theta_t\right)\Delta\Big) R_o & t < \tau_\lambda \\ \left(p_L + \frac{\phi}{\lambda + \phi} F\left(\Delta R_o \tilde{\theta}\right)\Delta\right) R_o & T_* \le t < \tau_\lambda \\ p_L + F\left(\Delta R_o \underline{\theta}\right)\Delta\Big) R_o & t \ge \tau_\lambda \end{cases} \tag{63}$$

[27] Otherwise, there would be a deterministic downward jump in the price at the time θ_t reaches $\tilde{\theta}$ which would be inconsistent with no-arbitrage.

We omit a formal proof of this proposition as it follows the steps of the proof of Proposition 3.