Bargaining in Securities*

Isaías N. Chaves† and Felipe Varas‡

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Abstract

Many corporate negotiations involve contingent payments or *securities*, yet the bargaining literature overwhelmingly focuses on pure cash transactions. We characterize equilibria in a continuous-time model of bargaining in securities. A privately informed buyer and a seller negotiate the terms of a joint project. The buyer's private information affects both his standalone value and the net returns from the project. The seller makes offers in a one-dimensional family of securities (e.g., equity splits). We show how outcomes change as the underlying security becomes more sensitive to the buyer's information, and we apply the framework to entrepreneurial finance and mergers and acquisitions under financial constraints.

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†Kellogg School of Management, Northwestern University, Evanston, IL 60208, U.S.A. Email: isaias.chaves@kellogg.northwestern.edu

‡Fuqua School of Business, Duke University, Durham, NC 27708, U.S.A. Email: felipe.varas@duke.edu

1 Introduction

Many corporate negotiations involve payments other than cash. In merger and acquisitions (M&A), acquirers often pay the target using shares of their own companies (Malmendier et al., 2016). Oil and gas lease agreements, which are typically negotiated between an individual land owner and a local oil and gas producer, tend to specify an upfront cash payment and a pre-specified royalty over future revenues.[1] Likewise, procurement contracts, many of which are arrived at via negotiation with supplies, specify some cost sharing rule. And in Chapter 11 bankruptcy procedures, claim holders bargain not only over cash payments, but over the terms of the restructuring plan such as the face value, maturity and seniority of new debt (White, 1989).

In short, negotiating parties frequently make offers in contingent payments or *securities*, and yet, the bargaining literature overwhelmingly focuses on pure cash transactions. We therefore lack an understanding how how negotiations change when—as is the case with contingent payments—the value of offers depends on the private information of parties.

This paper characterizes equilibria in a continuous-time model of bargaining in securities. We abstract somewhat from the institutional detail in order to isolate the impact of security payments on bargaining without commitment. In the model, a privately informed buyer and a seller negotiate over the terms of a joint project. The buyer has private information that affects both his assets in place (standalone value) and the net return of the project; the seller has a commonly known cost. The seller makes offers in a given one-dimensional family of securities (e.g., debt, indexed by the face value, or equity, indexed by the share being traded), so that the value of an accepted offer depends on the buyer's private information. The seller can revise her offers infinitely frequently, and both players discount the future at the same exponential rate. We focus on a tractable class of Markovian "skimming" equilibria, in which buyer types accept gradually in a given order.

The model can be interpreted in a few different ways. If one interprets the seller's cost as an investment required for the joint project, then the model is a monopolistic, dynamic version of Myers and Majluf (1984), with the seller as a financier and the buyer as an entrepreneur with private information about returns and assets in place. One can also interpret the model as an M&A negotiation between an acquirer and a target, in which one of the two parties has private information about the value of its assets in place and the value of

[1] Government oil lease agreements are usually auctioned but individual lease agreements are commonly settled by negotiation.

the potential synergies from the acquisition. It would be standard to think of the acquirer as the buyer and the target as the seller, but the reverse mapping is also possible: we stress that the labels are somewhat arbitrary when bargaining in securities, and the choice of labels will mostly depend on which party one things has the relevant private information.[2] If one is interested in adverse selection rather than moral hazard concerns, the model could also capture negotiations over executive compensation, which is mostly paid in non-cash securities.[3] The seller would then be the firm, and the buyer would be an executive with private information about her contribution to the firm and her outside option.

We provide two sets of results. First, we completely characterize bargaining dynamics, and second, we show how outcomes change as the underlying security becomes more sensitive to the buyer's information.

The bargaining dynamics depend on the buyer's marginal rate of substitution between delay (or time of trade) and offer.[4] Depending on both the primitives and the security family, that marginal rate of substitution may be increasing or decreasing in the buyer's type. When it is decreasing in type—so that high types are less willing to endure additional delays in exchange for receiving a better offer—trade is fully efficient, and there is no delay. In contrast, when that rate is increasing, there will be delay *provided adverse selection is sufficiently severe*. Delay takes on a very particular form. The game begins with a phase of gradual concessions, in which the seller's offer becomes more generous smoothly, and buyer types accept gradually in ascending order. Eventually, the negotiation reaches an impasse of random length, during which the seller intransigently refuses to improve his offers, even though they are continuously rejected. And finally, the impasse ends in a flash: the seller "submits," drops his ask discontinuously and trades immediately with all remaining types. Whenever there is delay, the seller breaks even on every offer that is accepted in equilibrium. The inefficiencies therefore involve not

[2]For example, if the parties are negotiating over an equity split, one can just as well think of the target (in layman's terms a "seller") as "buying" shares in a merged entity using its own equity as payment.

[3]Executives are not only compensated in cash but also with options, restricted stocks, performance based bonuses, etc. In fact, most of a CEO's compensation comes from these latter sources rather than from wages. For example, in 2020, only $3 million of Apple's CEO Tim Cook compensation was salary. Most of his compensation over the years has been in the form of stock grants, and other forms of non-equity compensation, such as bonuses based on performance.

[4]Note that, unlike cash bargaining models, "offer" here is <u>not</u> synonymous with "payment." For example, when bargaining over equity splits, the "offer" is the share of the gross returns being proposed; the "payment" is the value of that share.

only delay, but also cross subsidization: high types who trade at the final offer "subsidize" low types who trade at that offer, allowing them to receive more generous terms than they would have had they traded individually.

Second, focusing on scenarios with non-trivial delay, we show how equilibrium delay, ex-post payments, and ex-ante payoffs are affected by sensitivity of the security to the buyer's private information. We rely on DeMarzo et al. (2005)'s notion of *steepness* to partially order security families according to their informational sensitivity. For example, if there is a fixed royalty that must be paid to the seller, but the parties negotiate over an additional cash payment, then the higher the royalty rate, the steeper (more informationally sensitive) the overall "cash + royalty" security family will be.

For concreteness, we explain our general results through the lens of our main application to equity bargaining under financial constraints; however, we note that essentially all our results generalize to security bargaining examples such as the above "cash + royalty" one in which steepness is indexed by a one-dimensional parameter, and the bulk of them generalize even further to steepness comparisons that are not "parametrized."[5]

Our main application is to M&A negotiations under financial constraints. We present two formulations of equity bargaining under financial constraints for which the tightness of the buyer's pre-existing financial constraints indexes the steepness of the equity being offers. In the first formulation, the parties negotiate in equity, but the buyer has a *fixed* amount of cash that is added to the offer. Insofar as up-front cash is costly to procure, the amount of cash indexes both the tightness of the buyer's liquidity constraints and the informational sensitivity of the security offer: the lower the cash amount, the steeper the overall security is with respect to the buyer's information. In the second formulation, the parties negotiate in equity, but the buyer has pre-existing debt and maximizes the total value of debt and equity holders. The higher the leverage, the more financially constrained the buyer is, and the more informationally sensitive levered equity will be.

A common thread throughout our results is that, due to the cross-subsidization that takes place in equilibrium, tighter financial constraints (and steepness in general) affect payments and bargaining frictions in a predictable, monotone way for types on the high and low ends of the distribution, but may have non-monotonic effects in the middle of the distribution. Tighter financial constraints lower expected payments for high types, have no effect for low types,

[5]Equity offers, for instance, are more informationally sensitive than cash offers—the value of a cash offer to the seller does not depend on what the buyer knows about the project, but the value of an equity offer does—but there is no overarching parametrized security family that includes pure equity offers and pure cash offers as special cases.

but strictly *raise* payments for intermediate types. Regarding bargaining frictions, we measure them according the (type-specific) *certainty-equivalent delay*: the deterministic delay that causes the same expected discounting cost as the random equilibrium delay. As financial constraints tighter, certainty-equivalent delay grows for sufficiently low and sufficiently high types; for intermediate types, no such ranking is possible. We provide numerical examples where a "large" tightening of financial constraints will raise bargaining frictions for all types, but we also prove mathematically that a sufficiently small tightening must lower bargaining frictions for some intermediate types. Indeed we show that tightening financial constraints on the margin (i) raises bargaining frictions for almost all types and lowers expected payments for almost all types, but (ii) *lowers* financial frictions and *raises* expected payments for a small, but positive measure of types.

As financial constraints tighten, payments and bargaining frictions move in opposite directions for some types, so the net effect on the buyer's utility can be ambiguous. We prove that types below a threshold are always harmed by tighter constraints, including some types whose payments actually decrease. And we provide an easy-to-check sufficient condition under which tighter financial constraints harm all types, even those whose bargaining frictions decrease.

Using a normal-linear model parametrization of the levered-equity model, we show how the negotiation depends on the nature of the synergies or net returns generated by the project. In an M&A setting, merger synergies related to cost savings may be easier to estimate than merger synergies from market expansion or product market fit. A mean preserving-spread of future synergies raises the value of levered equity, but it also may dilute how tightly the buyer's signal covaries with the expected value of the project; it may therefore not be intuitively clear what the equilibrium effect on bargaining is. We vary the precision of the buyer's signal about net returns, and we prove that the net effect of raising the precision is equivalent to negotiating in a steeper security. Hence, as the buyer's information becomes more precise, high and low types endure worse bargaining frictions. Our theoretical results that types below a threshold are always harmed by the increased precision, but in the numerical examples we have computed, the harm seems to be uniform across all types. In other words, we show that *more precise information can be value-destroying*. (We note, in addition, that the intuition about "diluting the signal" is in fact incomplete, and in fact is false in absolute terms: the slope of expected value with respect to the buyer's signal may go up or down as the precision increases).

Our results on the effects of tighter financial frictions are mostly in line with empirical studies, but they point to some avenues for additional research and some challenges to inference. If the discount rate reflects an underlying exogenous probability of deal failure, then our measure of bargaining frictions maps one-to-one to deal failure probabilities. The results above then imply that tightening financial constraints on the margin will raise deal failure probabilities for most of the distribution (and large increases in tightness will often raise those probabilities for all types). Previous studies (Malmendier et al., 2016; Uysal, 2011) indeed have shown that tighter financial constraints are associated with higher probabilities of deal failure and lower M&A activity by firms. However, the specifications used in these studies, since they cannot distinguish acquirer types and force a constant marginal effect of financial constraints on outcomes, only capture a composite effect that averages both across constraint changes of various sizes and across types. As such, these studies may be conflating substantial heterogeneity in effects that is highlighted by our model.

We also show that data on delay, should it be available, can be a misleading guide to the underlying bargaining frictions. Detailed delay data would at best identify an *expected* delay curve across the distribution of types. Due to the randomness in equilibrium delay and the convexity of exponential discounting, expected delay will differ from certainty-equivalent delay, which is the true welfare-relevant measure of bargaining frictions and not directly observable. We provide numerical examples in which tighter financial constraints lower expected delay across all types, even though—in line with our theoretical results—they increase financial frictions for most types. In other examples, expected and certainty-equivalent delay move in the same direction for all types. This suggests a challenge for empirical studies of bargaining that try to use data on delay.

Section 1 reviews related literature. Section 2 presents the model setup and our continuous-time equilibrium notion, and Section 3 presents our equilibrium construction and our results on equilibrium uniqueness for general securities. Section 4 presents our comparative statics results for general steepness comparisons, both parametrized and not, focusing on the case with non-trivial delay, Section 5 applies the theoretical results to equity bargaining under financial constraints. Section 6 discusses some extensions and connections to the security auctions literature. All omitted proofs are in the appendix.

Related Literature This work relates to a literature on bargaining with asymmetric information and frequent offers, e.g., (Fudenberg et al., 1985; Gul et al., 1986; Fuchs and Skrzypacz, 2010). Deneckere and Liang (2006)'s key contribution characterized frequent offers dynamics in a bargaining model with interdependent values and a lemons problem. The bargaining in that model is in cash, time is discrete, and the type space is discrete. In the gap case, if efficiency is unattainable in a static model, there must be delay in the frequent offers limit; surprisingly, in equilibrium bursts of trade are followed by long quiet periods in which no serious offers are made. In contrast, in our paper the level of interdependence in values is endogenous to the security and the offers made. Our type space is continuous, and we formulate the bargaining problem directly in continuous time, which dramatically streamlines the analysis. Moreover, the explicit expressions for the speed of trade allow for clean comparative statics on the space of securities and a study of how the model primitives affect delay. Notably, the dynamics (even the direction of screening) in our model depend not only on the information primitives, but also on the security being used.

The continuous-time formulation that we use originated in Ortner (2017) and Daley and Green (2020), which are models with discrete types and driving Brownian process (changing costs in the former, news about the informed party's type in the latter). These were adapted to standard continuous-type Coasean bargaining (without a driving Brownian process) in Chaves (2019). The bargaining dynamics we find in the upward skimming case (smooth trade, followed by an atom of trade) are reminiscent of those in Daley and Green (2020), but their model does not generate an impasse phase. The forces leading to gradual trade are very different: without news, their model reduces to the frequent-offers limit of Deneckere and Liang (2006), and the equilibrium features two bursts of trade with a quiet period in between.

We also contribute to a nascent bargaining literature that considers bargaining over richer objects than cash. Strulovici (2017) considers a two-type Coasean bargaining model where parties negotiate over the terms of contracts, including, for instance, the quantity or quality of goods traded. He shows that agreement is efficient in the continuous-time limit for a broad class of contracting spaces. Hanazono and Watanabe (2018) consider the splitting of a stochastic pie in a common value setting: both players receive private noisy signals about the size of the pie, so their offers about how to divide the pie are a form of equity claims on a variable surplus. They characterize the conditions under which parties can efficiently aggregate their dispersed information in equilibrium.

Finally, there is a large literature in finance studying security design in static settings or settings with full commitment power. We briefly describe some points of connection. Since Myers and Majluf (1984), a key concern in corporate finance, and in particular in the security design literature, is to understand the impact of asymmetric information on financing. A theme of that literature is that using informationally sensitive securities (steep securities) is costly due to adverse selection. For example, the pecking order theory developed by Myers and Majluf (1984) states that companies choose informationally insensitive securities, such as debt, as their main source of financing, and only rely on informationally sensitive securities, such as equity, when other sources of financing are unavailable. DeMarzo and Duffie (1999) consider the implications that adverse selection has on security design: more informationally sensitive securities generate a more severe lemons problem and a less liquid market, while less informationally sensitive securities reduce the amount of capital that can be raised.

The literature on mergers and acquisitions has also studied the effects of different security choices. Fishman (1989) considers securities bidding in take-over competitions; he shows that flatter securities are more effective in equilibrium at preempting competing bids. Hansen (1987) shows how an uninformed acquirer can use stock offers to screen out low quality targets, and Rhodes-Kropf and Viswanathan (2004) consider equity auctions to explain the existence of merger waves driven by aggregate changes in valuation.

Our work also speaks to a growing literature on auctions with contingent payments that emphasizes the effects of informational sensitivity on equilibrium outcomes (Hansen, 1985; DeMarzo et al., 2005; Che and Kim, 2010). Our definition of informational sensitivity ("steepness") is taken from DeMarzo et al. (2005). Using Linkage-Principle techniques (Milgrom and Weber, 1982), they show, under a condition related to our "downward-skimming" case, that steeper securities increase revenue. We follow Che and Kim (2010)'s extension of DeMarzo et al. (2005) that allows bidders' private information to affect their standalone value; they show—similar to our "increasing marginal rate of substitution" case—this can cause severe adverse selection and equilibria in decreasing strategies. We discuss the how our results relate to theirs' Section 6. Outside of the security auction literature, Lam (2020) studies the impact of steepness in a directed search model. Workers with privately known productivity match with owners of heterogeneous assets of known quality. Lam (2020) characterizes the inefficiencies that arise as the market moves (exogenously) from cash transfers to output share (equity) payments; when asset owners are free to choose among securities, competition drives them to offer only cash payments.

We contribute to this prior literature by characterizing the interaction between the means of payment and the lack of commitment. In particular, we relate the informational sensitivity of securities to the amount of bargaining delay, and we show how changes in inefficient delay can overwhelm the Linkage Principle forces that are at play in the security auctions literature.

2 Setup

Players and Information A buyer (he) and a seller (she) negotiate over the terms of a joint project, the rights to which initially rest with the seller. (To fix ideas, one can think of the buyer as an acquirer and the seller as a target). The buyer has a assets-in-place with standalone value $\tilde{A}$, and the project generates a net return $\tilde{R}$. The value of the project is therefore $\tilde{V} := \tilde{R} + \tilde{A}$.

The buyer observes a private signal $\theta \sim U[0,1]$ that is informative about both his assets-in-place/standalone value and the net return of the project. In particular, we assume that $\tilde{V}|\theta$, $\tilde{R}|\theta$, and $\tilde{A}|\theta$ each have conditional densities $g_V(\cdot|\theta)$, $g_R(\cdot|\theta)$, and $g_A(\cdot|\theta)$ that have full support on $[0, \bar{V}]$ for each θ, are smooth in both arguments, and satisfy the monotone likelihood ratio property (MLRP). As in DeMarzo et al. (2005), we assume $g_V(v|\theta)$ satisfying strict MLRP, is twice-differentiable in both arguments, and $v g_V(v|\theta), v\left|\frac{\partial}{\partial\theta} g_V(v|\theta)\right|$, and $v\left|\frac{\partial^2}{\partial\theta^2} g_V(v|\theta)\right|$ are all integrable on $v > 0$.

Let $R(\theta)$ denote $\mathbb{E}[\tilde{R}|\theta]$ and similarly for $A(\theta)$ and $V(\theta)$. Throughout we maintain that $R'(\cdot)$ and $A'(\cdot)$ exist, are non-negative (which follows by MLRP), and bounded above, and that $V'(\cdot)$ is strictly positive.

Securities and Bargaining Protocol The seller makes offers to the buyer, who at each point in time chooses whether to accept or reject. This is as in standard models of Coasean bargaining. Unlike those models, the seller offers *contingent payments* belonging to a particular *ordered security family*. With minor modifications to the definitions in DeMarzo et al. (2005), an ordered family of securities is a function $S : [\underline{\alpha}, \bar{\alpha}] \times \mathbb{R}_+ \to \mathbb{R}_+$ that maps from offers and total values into payments, such that (i) $S(\alpha, \tilde{V})$ is weakly increasing in both arguments, and (ii) the expected payment conditional on type, denoted $\bar{S}(\alpha, \theta) := \mathbb{E}[S(\alpha, \tilde{V})|\theta]$, satisfies the following:

Assumption 1 (Non-degeneracy)**.**

1. $\bar{S}(\bar{\alpha}, \theta) \geq R(\theta) > \bar{S}(\underline{\alpha}, \theta)\, \forall\theta$.
2. $\bar{S}(\bar{\alpha}, \theta) \geq c > \bar{S}(\underline{\alpha}, \theta)\, \forall\theta$.
3. $\bar{S}_\alpha := \frac{\partial \bar{S}}{\partial \alpha}$ exists, is strictly positive, and smooth in α, θ.
4. $\bar{S}_\theta(\alpha, \theta) := \frac{\partial \bar{S}(\alpha,\theta)}{\partial \theta}$ exists, is strictly positive for $\alpha > \underline{\alpha}$, and is smooth in α, θ.

Conditions 1 and 2 ensure that the expected payment is sufficiently variable as a function of the offer: in a one-shot game, there exist sufficiently unfavorable offers that any player would definitely want to reject, and sufficiently favorable ones that any player would definitely accept. The assumption would be trivially satisfied if the parties were bargaining in an unrestricted amount of cash. Condition 3 is a non-degeneracy condition ensuring that higher offers lead to higher expected payments, type by type. Condition 4 says that higher types are strictly good news for the seller; in fact it follows from the assumptions on $g_V(\cdot|\theta)$ (Lemma 1 in DeMarzo et al. (2005)); but we include it here for ease of reference. In particular, it implies that

$$\text{For all } \alpha > \underline{\alpha}, \mathbb{E}[S(\alpha, \tilde{V})|\theta \in [k, k']] \text{ is strictly increasing in } k, k'.$$

The family S is fixed throughout the bargaining interaction; different offers by the seller therefore correspond to different indeces α and α'. Below, when we write "the seller makes an offer of α," and the security family is S, we mean that the seller asks for a contingent payment $S(\alpha, \tilde{V})$ in order to agree to the merger.

Payoffs The seller enjoys a flow payoff of rc before agreement is reached. For example, in the mergers and acquisitions example, c can represent either the target's cashflows or management's flow benefits of control. Hence, if the buyer with gross realized returns $\tilde{V}$ and type θ accepts a security with index α at time t, the seller receives ex post profits of

$$(1 - e^{-rt})c + e^{-rt} S(\alpha, \tilde{V}). \tag{1}$$

while the buyer earns (in expectation over $\tilde{V}$, conditional on knowing θ),

$$(1 - e^{-rt})A(\theta) + e^{-rt} \left(V(\theta) - \bar{S}(\alpha, \theta)\right) \tag{2}$$

We assume throughout that $R(0) \geq c$, i.e. there are gains from trade with every type of buyer. We distinguish between the *gap* ($R(0) > c$) and *no gap* ($R(0) = c$) cases (Fudenberg et al., 1985; Gul et al., 1986)

An important object for the analysis is $\alpha^f(\theta)$, the solution to

$$V(\theta) - \bar{S}(\alpha^f(\theta), \theta) = A(\theta). \tag{3}$$

This is the highest take-it-or-leave-it offer that type θ would consider accepting (as a mnemonic, the superscript on α^f stands for "final").

Discussion of the Model By changing the interpretation slightly, our setting can model different applications of security bargaining.

Entrepreneurial Finance: The seller is a financier, the cost c corresponds to the investment required for the project, $A(\theta)$ corresponds to the value of the entrepreneur's assets in place, and $R(\theta)$ corresponds to the present value of the cash flows generated by the new project. Myers and Majluf (1984) considers the static case with a competitive market of financiers.

Mergers and Acquisitions: In one version, the "buyer" is the acquirer and the "seller" is the target. $A(\theta)$ then corresponds to the current value of the acquirer, $R(\theta)$ corresponds to the synergies between the two firms, and c measures the current value of the target. In the above we took the acquirer as the privately informed "buyer" and the target as the uninformed "seller". However, these labels are somewhat arbitrary with security payments. When equity is used, the value of the transaction is shared in a linear way. Thus, if the relevant private information belongs to the target, one can alternatively interpret the "buyer" in the model as the target firm and the "seller" in the model as the acquiring firm.

Executive Compensation: The seller is a firm, and the buyer is a prospective CEO. $R(\theta)$ corresponds to the value generated by the CEO, $A(\theta)$ corresponds to the CEO's outside option, while c corresponds to the value the firm can obtain from hiring an alternative CEO.[6]

Direction of Skimming With quasilinear payoffs (i.e., bargaining in cash), a now standard argument (Fudenberg et al., 1985) shows that equilibria satisfy a "skimming" property: if a type θ is indifferent between accepting and rejecting an offer p after history H_t, then all types $\theta' > \theta$ strictly prefer to accept p at H_t; beliefs after every history are therefore right-truncations of the prior. Intuitively, high types like cash just as much as low types, but they dislike delay relatively more. Analysis typically focuses on Markovian equilibria with the truncation point as a state variable.

The standard argument breaks down when bargaining in non-cash securities because the buyer's true type affects his expected payment. When bargaining in equity, for example, high buyer types dislike delay more, but they also dislike giving up their equity more. High types may therefore be *more* willing than low types to wait for better equity offers. For tractability, we will focus nonetheless on Markovian equilibria with a skimming structure—buyer types accept in a predetermined "order"—but what that order is, and whether it matches the natural order of types, depends on both the primitives and the security family.

Definition 1 (Upward vs Downward Skimming)**.** Let ι^S given by

[6]Contingent compensation can be used to provide incentives in the case of moral hazard, and to screen CEOs in the case of asymmetric information. Our model corresponds to the case of asymmetric information. We could extend our model to incorporate ex-post moral hazard. The main difference with our current setup is that the distributions $g_V(\cdot|\theta)$ and $g_R(\cdot|\theta)$ would also depend on α.

$$\iota^S(\theta,\alpha) := -\frac{R(\theta)-\bar{S}(\alpha,\theta)}{\bar{S}_\alpha(\alpha,\theta)} \tag{4}$$

Say the environment satisfies **upward skimming** if $\iota^S(\cdot,\alpha)$ is strictly increasing for every α. The environment satisfies **downward skimming** if $\iota^S(\cdot,\alpha)$ is strictly decreasing for every α. The environment satisfies the **skimming property** if it is either upward skimming or downward skimming.

To unpack the definition, note that $\iota^S(\theta,\alpha)$ is the marginal rate of substitution between delay (t) and offer (α) in the buyer's utility in (2). When ι^S is increasing in θ, the indifference curves of low types in (α,t) space cross the indifference curves of high types from below, and high types are more willing to trade off additional delays in order to get an improvement in the offer (and vice-versa when ι^S is decreasing in θ).

We use the classification in Definition 1 to guide our search for equilibria in tractable classes. When the environment is upward skimming, it will be fruitless to search for skimming equilibria where higher types accept first, so we in those environments we look for upward skimming Markov equilibria, where (i) the seller's beliefs about the buyer are *left*-truncations of the prior (*lower* types accept first), and (ii) the truncation point is the relevant state variable for continuation play. (Vice versa for downward skimming environments).[7]

For a quick example, adapted from Che and Kim (2010)'s work on security auctions, suppose that the seller makes offers in equity shares of the gross value of the project, i.e., $S(\alpha,\tilde{V}) = \alpha\tilde{V}$. Then $\iota^S(\theta,\alpha) = -(R(\theta)V(\theta)^{-1}-\alpha))$, and the environment is upward skimming iff R/V is everywhere decreasing, and downward-skimming if its everywhere increasing. Intuitively, if R/V is decreasing, then, as the buyer's type grows, his disagreement motive (i.e., the assets in place) grows proportionally faster than his agreement motive (i.e., the net surplus), so higher types will tend to trade later than low types.[8]

[7] See Lemma 2 in the appendix, which shows that, for deterministic offer paths, every selection of maximizers is non-decreasing in type. Note that this is weaker than the usual skimming result invoked in the literature on cash bargaining, so our focus on Markovian skimming equilibria is a stronger restriction than the analogous restriction in models with cash bargaining. Nevertheless, (i) the offer paths in the equilibria we construct, while stochastic, are such that Lemma 2 will suffice to verify incentive compatibility for the buyer; and (ii) the result is sufficient to rule out equilibria where, say, θ' accepts before $\theta'' < \theta'$ with certainty in an upward-skimming environment. We provide additional details in Remark 3 in the appendix.

[8] Che and Kim (2010) identified an analogous condition that governs whether second-price sealed bid equity auctions have equilibria in decreasing strategies. We postpone a fuller discussion of Che and Kim (2010) to section 6.

Severity of Adverse Selection Bargaining dynamics will depend crucially on whether or not efficiency is achievable in that static game, i.e., whether there exists on offer that all buyer types accept on which the seller can break even (Deneckere and Liang, 2006):

Definition 2 (Static Lemons Condition)**.**

1. In an *upward skimming environment*, say the **Static Lemons Condition** (SLC) holds iff
$$\mathbb{E}[S(\alpha^f(1), \tilde{V})] < c.$$
If SLC holds, let k^{SLC} be defined by
$$k^{SLC} = \inf\{k \leq 1 : \mathbb{E}\left[S(\alpha^f(1), \tilde{V})|\theta \in [k, 1]\right] \geq c\}.$$

2. In a *downward skimming environment*, say the Static Lemons Condition holds iff
$$\mathbb{E}[S(\alpha^f(0), \tilde{V})] < c.$$

Below we refer to k^{SLC} as the "critical type."

Equilibrium Notion For environments that satisfy the skimming property, the game has a natural state variable: the truncation of the seller's prior beliefs that yields her current posterior. For upward skimming environments, this is a *left* truncation: if the state at time t is $K_t = k$, then, given the history of offers and rejections, the seller believes $\theta > k$. For downward skimming environments, k is a *right* truncation, i.e., if $K_t = k$, the seller believes $\theta < k$. We focus on a tractable class of equilibria that are Markovian in the truncation (henceforth, "the cutoff"). To simplify the exposition, we describe the equilibrium notion for upward skimming environments, and later explain how the notion must be adapted for downward skimming.

We first give a brief verbal description of the equilibrium notion. Following recent formulations of Coasean bargaining in continuous time (see Ortner (2017), Daley and Green (2020), and, most relevant for the current setup, Chaves (2019)), the seller solves a Markovian optimal stopping-control problem, and the buyer solves a Markovian optimal stopping problem. Formally, the buyer's chooses a reservation offer strategy $\alpha(\cdot)$. On path, the seller chooses how fast to screen through buyer types, taking as given that to screen through types $\theta < k$, she must offer $\alpha(k)$. That is, the seller chooses paths of belief cutoffs $t \mapsto K_t$, which result in paths of offers $t \mapsto \alpha(K_t)$. We also give the seller an option to "give up on screening": she can make a pooling offer $\alpha(1)$ that would be accepted by all remaining types, thereby ending the game.

Given a law of motion for cutoffs, and that future offers are given by $\alpha(K_t)$, the buyer then solves an optimal stopping problem with state K_t. Finally (unlike Ortner (2017), Daley and Green (2020), and Chaves (2019)), the buyer's strategy $\alpha(\cdot)$ must sometimes be discontinuous in equilibrium. We therefore augment the seller's strategy space *off-path*: after the rejection of an off-path offer $\alpha' \notin \alpha([0,1])$, we give the seller the ability to randomize over offers in a way that depends on α'. In the lingo of the discrete time literature, our equilibrium will be "weak Markov" (see Fudenberg et al. (1985) and Gul et al. (1986) for the origins of this "Weak Markov" approach).

The technical detals are as follows. First, the set of all measurable non-decreasing paths is an unmanageably large strategy space for the seller. Using the approach in Chaves (2019), we impose some restrictions on seller strategies that make the analysis tractable while still allowing a rich set of dynamics.

Definition 3 (Seller Strategy Space)**.**

1. A *plan* of on-path offers by the seller consists of a non-decreasing cutoff path $t \mapsto K_t$ and a stopping time T at which to make a pooling offer $\alpha(1)$. We denote an entire cutoff path $(K_t)_{t\geq 0}$ by K. K is *admissible* if it has no singular-continuous parts. We allow for mixed strategies in the stopping time T, which are represented by a CDF $F = (F_t)_{t\geq 0}$. Thus a plan for the seller is given by a pair (K, F), and we denote by $\mathcal{A}_k^U$ the set of admissible plans (K, F) satisfying $K_{0^-} = k$, i.e., with initial value k, and generic element K^k. The stopping time T is *Markov* if its hazard measure $dF(t)/(1-F(t-))$ is a function of K_{t-}. At any point where F_t is absolutely continuous, we denote its hazard rate by the arrival rate λ_t. In this case, the Markov assumption amounts to $\lambda_t = \lambda(K_t)$, for some non-negative function $\lambda(\cdot)$.

2. Time intervals $[\underline{t}, \bar{t})$ where $dF_t = 0, \Delta K_{t^-} = 0$ are *smooth trade regions*, and $\dot{K}_t$ is the *trading speed*. A special case of a smooth trade region is a *quiet period*, i.e., an interval $[\underline{t}, \bar{t})$ with $\dot{K}_t = dF_t = \Delta K_{t^-} = 0$.

3. A plan for on-path offers is supplemented by a plan following off-path offers. For any off-equilibrium offer $\alpha' \notin \alpha([0,1])$ made at time t, we let $\sigma_t(\alpha') \in \Delta([0,1])$ be the randomized offer that "immediately" follows the rejection of α'.

The discrete-time "Weak Markov" equilibria in Fudenberg et al. (1985) and Gul et al. (1986) sustain Markovian behavior on path by prescribing randomization immediately following the rejection of an off-path offer. In continuous time, there is no "next" period immediately after a seller deviation. Hence, to capture this off-path randomization, in the third item we "stop the clock" after an off-equilibrium offer is made, and we allow the seller to immediately make a new offer when the off-path offer is rejected.[9]

Definition 4 (Buyer and Seller Problems)**.** At state k, a buyer type θ takes $\alpha(\cdot)$ and K as given, and solves

$$\sup_{\tau\in\mathcal{T}} \mathbb{E}\left[(1-e^{-r(\tau\wedge T)})A(\theta)+e^{-r(\tau\wedge T)}\left(V(\theta)-\bar{S}(\alpha(K_{\tau\wedge T}),\theta)\right)\right] \tag{5}$$

where by definition $K_T = 1$, and $\mathcal{T}$ is the set of stopping times adapted to the filtration generated by T. Meanwhile, the seller S takes $\alpha(\cdot)$ as given. Given any path Q_t and realization of the stopping time T, the seller payoff is

$$\begin{aligned}\Pi(Q,T) \equiv \int_0^T e^{-rt}\mathbb{E}\left[\bar{S}\left(\alpha(Q_t),v\right)\middle|\theta\in[Q_{t-},Q_t]\right]dQ_t + e^{-rT}\mathbb{E}\left[\bar{S}\left(\alpha^f(1),v\right)\middle|\theta\in[Q_{T-},1]\right] \\ +\left(1-(1-Q_T)e^{-rT}-\int_0^T e^{-rt}dQ_t\right)c,\end{aligned}$$

and, at each k, the seller strategy (Q,F) solves

$$\sup_{(Q,F)\in\mathcal{A}_k^U}\int_0^\infty \Pi(Q,T)dF(T). \tag{6}$$

We can now fully define a weak Markov equilibrium.

Definition 5 (Equilibrium)**.** A weak *Markov Equilibrium* of an upward-skimming game consists of a tuple

$$(\{K^k\}_{k\in[0,1]},F,\alpha(\cdot),\sigma(\cdot|\cdot,\cdot))$$

together with a value $J(\cdot)$ for the the seller and a value $B(\cdot,\cdot)$ for the buyer such that

1. For all $\theta\in[0,1]$, $k\in[0,1]$, accepting at $\tau^* = \inf\{t : \alpha(K_t^k)\le\alpha(\theta)\}$ solves the buyer's problem (5) and delivers value $B(\theta,k)$.
2. $\alpha(1)=\alpha^f(1)$, where α^f is defined in (3).
3. For all $k\in[0,1]$ and T in the support of F, K^k is an admissible path and T is a Markov stopping time that together solve (6) and deliver value $J(k)$.

[9] The idea of stopping the clock to allow for multiple sequential moves in a continuous time game has been used in bargaining models by Smith and Stacchetti (2002) and Fanning (2016). An alternative approach is to follow the formalization in Fudenberg and Tirole (1985) and to consider "intervals of consecutive atoms."

4. For any point of discontinuity of $\alpha(\cdot)$, k', and any off-equilibrium offer $\alpha' \in (\alpha(k'+), \alpha(k'-))$, $\sigma(\cdot|k', \alpha')$ maximizes[10]

$$\int_0^1 \Big\{(\alpha^{-1}(\tilde{\alpha}) - k')^+ \mathbb{E}\left[\bar{S}\left(\tilde{\alpha}, v\right) \Big| \theta \in [k', \alpha^{-1}(\tilde{\alpha}) \wedge k']\right] \\ + (1 - \alpha^{-1}(\tilde{\alpha})) J\left(\alpha^{-1}(\tilde{\alpha})\right)\Big\} \, d\sigma(\tilde{\alpha}|k', \alpha')$$

5. For any point of discontinuity of $\alpha(\cdot)$, k', and any off-equilibrium offer $\alpha' \in (\alpha(k'+), \alpha(k'-))$, $\sigma(\cdot|k', \alpha')$ satisfies

$$V(k') - \bar{S}(\alpha', k') \leq B(k', k') \int_0^{\alpha(k')} d\sigma(\tilde{\alpha}|k', \alpha') \\ + \int_{\alpha(k')}^1 \left(V(k') - \bar{S}(\tilde{\alpha}, k')\right) d\sigma(\tilde{\alpha}|k', \alpha')$$

Condition 2 is a natural refinement inspired by the corresponding discrete time game.[11] In a stationary equilibrium of the discrete time game, for any positive period length, the seller would never offer more than $\alpha^f(1)$ when her beliefs are concentrated at $\theta = 1$. (And $\alpha(1)$ can never be above $\alpha^f(1)$, since $\theta = 1$ would strictly prefer to reject, i.e. $\alpha(1)$ cannot be a reservation offer for $\theta = 1$.) Condition 4 and 5 say that, when the seller makes an off-path offer "by mistake", the buyer still accepts according the reservation offer curve $\alpha(\cdot)$, and after making the mistake, the seller randomizes in way that justifies the buyer's choice to accept according to $\alpha(\cdot)$ (Fudenberg et al., 1985; Gul et al., 1986).

Finally, to streamline the derivation of necessary conditions, we restrict our search to equilibria to an amenable subclass:

Definition 6 (Regularity)**.** A weak Markov Equilibrium is *regular* if

1. J is continuous and C^1 in the interior of smooth regions;
2. $\dot{K}_t$ is continuous in the interior of smooth trade regions.
3. Jump discontinuities in cutoff paths are *isolated*.

Below, we refer to regular weak Markov Equilibria as simply "equilibria."

[10] Here, $\alpha^{-1}(\cdot)$ represents the generalized inverse defined as $\alpha^{-1}(y) \equiv \sup\{x > 0 : \alpha(x) \geq y\}$.

[11] See, for example, the discussions in Ortner (2017) and Daley and Green (2020), who impose conditions similar to our Condition 1; Ortner (2017) shows that, absent this kind of refinement, continuous time equilibria can violate this natural discrete-time property.

Remark 1 (Modifications for Downward Skimming)**.** In downward skimming environments, regular weak Markov Equilibria are defined almost identically, with the following changes:

1. Admissible paths $t \mapsto K_t^k$ are non-*increasing* and satisfy $K_0^1 = 0$. The admissible set at state k is denoted $\mathcal{A}_k^D$.
2. Condition 2 in Definition 5 becomes $\alpha(0) = \alpha^f(0)$.
3. The seller's objective is now written as

$$\Pi(Q) = \int_0^\infty e^{-rt}\mathbb{E}\left[\bar{S}\left(\alpha(Q_t), v\right)\middle|\theta \in [Q_t, Q_{t-}]\right] d(1-Q_t) + \left(1 - \int_0^\infty e^{-rt} d(1-Q_t)\right) c \quad (7)$$

3 Dynamics for General Securities

Within our class of equilibria, we can fully characterize equilibrium dynamics. Here we provide an informal derivation of the equilibrium in an upward skimming case when adverse selection is sufficiently severe, relegating the full proof of necessary conditions and equilibrium verification to the appendix.

We construct an equilibrium where the game starts with smooth trade. By the usual Coasean logic, whenever the seller is trading smoothly, her payoff is pinned down at c: otherwise, she would have strict incentives to speed up trade. To wit, the HJB equation in the smooth trading region is given by

$$rJ(k) = \sup_{\dot{k}\geq 0} \left(\bar{S}(\alpha(k), k) - J(k)\right)\frac{\dot{k}}{1-k} + J'(k)\dot{k} + rc. \quad (8)$$

The choice variable $\dot{k}$ enters the HJB in an affine way. Hence, if trade is happening at a positive speed ($\dot{k} > 0$ is optimal), the coefficients on $\dot{k}$ must cancel. It follows that $J(k) = c$, and $\bar{S}(\alpha(k), k) = c$, i.e., the seller exactly breaks even, conditional on trading with type k.

The seller is therefore indifferent among speeds of trade in a smooth trading region, but and the speed of trade at state k is pinned down instead by the marginal buyer $\theta = k$ and his incentives to delay. Ignoring second-order effects, if the buyer $\theta = K_t$ waits an additional dt units of time before accepting, he suffers discounting costs

$$rdt\left(V(K_t) - \bar{S}(\alpha(K_t), K_t)\right).$$

However, while waiting he receives flow utility $rdtA(K_t)$, and the price he faces improves by

$$\alpha'(K_t)\bar{S}_\alpha(\alpha(K_t), K_t)\dot{K}_t dt.$$

Setting marginal costs equal to marginal benefits, the speed $\dot{K}_t$ must satisfy

$$\dot{K}_t = r\frac{R(K_t) - c}{-\alpha'(K_t)\bar{S}_\alpha(\alpha(K_t), K_t)}.$$

Consider the numerator and denominator in this last equation. The numerator represents the gains from trade with type K_t, while the denominator represents the (absolute value of) the slope of expected payments with respect to the state. Trade is faster (i.e., types are skimmed more quickly) when the gains of trade are larger, and they are slower when the equilibrium expected payment changes more quickly with respect to the state. Intuitively, when expected payments are more sensitive to the state, the buyer has a stronger incentive to "move the state along" by rejecting offers and misrepresenting his type. Incentive compatibility then requires that trade is slower.

We obtain a useful re-interpretation of the speed of trade by leveraging the seller's breaks-even condition $S(\alpha(K_t), K_t) = c$, which holds when trade is smooth. Totally differentiating on both sides with respect to K_t,

$$0 = \alpha'(K_t)\bar{S}_\alpha(\alpha(K_t), K_t) + \bar{S}_\theta(\alpha(K_t), K_t),$$

Plugging into our expression for the speed of trade, we have that, starting at a state k with smooth trade, K_t evolves according to the ODE

$$\dot{K}_t = r\frac{R(K_t) - c}{\bar{S}_\theta(\alpha(K_t), K_t)}, K_0 = k. \tag{9}$$

Hence—foreshadowing our steepness results—the speed of trade depends on the sensitivity of expected payments to the true type.

A natural guess is that, using (9), one can construct an equilibrium in smooth trade. However, smooth trading cannot persist indefinitely. If the seller were to continue screening more and more types starting from the bottom, eventually the state would reach *and cross* k^{SLC}. At that point, trading instantly with all remaining types at an offer they would all accept would become strictly more profitable than trading smoothly with the marginal type: for $k > k^{SLC}$, $\mathbb{E}[\bar{S}(\alpha^f(1), \theta)|\theta \in [k, 1]] > c$.

Our equilibrium construction therefore specifies smooth trade at $k < k^{SLC}$, with each type trading separately at different offers, and an atom of trade at $k > k^{SLC}$, with all remaining types $[k, 1]$ trading simultaneously at the pooling offer $\alpha^f(1)$. However, this implies implies that the offer must *drop discontinuously (become discontinuously more favorable for the buyer) at state*

k^{SLC}. Indeed, for a type slightly below k^{SLC}, the seller just breaks even conditional on trading *only* with that type. At the same time, the seller also breaks even when trading simultaneously with all types strictly above k^{SLC}, i.e., $\mathbb{E}[\bar{S}(\alpha^f(1), \theta)|\theta \in (k^{SLC}, 1]] = c$.[12] In order to make types just below k^{SLC} willing to trade at the higher offer, when types just above face a discontinuously lower one, the seller must delay offering $\alpha^f(1)$. In equilibrium, she delays just long enough to make $\theta = k^{SLC}$ indifferent between accepting $\alpha(k^{SLC})$ "now" and rejecting it in in hopes of receiving $\alpha^f(1)$ "later." The expected discount until $\alpha^f(1)$ is offered, denoted by D, must solve

$$\underbrace{V\left(k^{SLC}\right) - \bar{S}\left(\alpha(k^{SLC}), k^{SLC}\right)}_{\text{payoff from accepting } \alpha(k^{SLC})} = \underbrace{(1-D)X\left(k^{SLC}\right) + D\left(V\left(k^{SLC}\right) - \bar{S}\left(\alpha^f(1), k^{SLC}\right)\right)}_{\text{payoff from waiting for } \alpha^f(1)}, \tag{10}$$

which simplifies[13] to

$$c = (1-D)R(k^{SLC}) + D\bar{S}(\alpha^f(1), k^{SLC}). \tag{11}$$

Since the seller must use a Markov stopping time at k^{SLC}, she can implement this delay by postponing the final offer until the first tick of a Poisson clock with a rate λ given by $\lambda/(r+\lambda) = D$.

For these equilibrium dynamics, one can use standard mechanism design arguments (together with Lemma 2 in the appendix) to show that it is globally incentive-compatible for buyers to accept from lowest to highest according to $\alpha(\cdot)$. A verification approach shows that these screening dynamics are also optimal for the seller, given $\alpha(\cdot)$.

We have outlined the construction of an equilibrium, but in fact, in Theorem 1 we prove that these are the *only* possible equilibrium dynamics. Formally, we prove the following theorem, which also covers the remaining cases:

Theorem 1. *In skimming environments, there exists a (regular weak Markov) equilibrium.*

1. *In a downward-skimming environment, all equilibria have instant trade at an offer $\alpha^f(0)$.*

[12] On the one hand, since the seller trades smoothly for $k < k^{SLC}$, $\alpha(k^{SLC}-)$ must satisfy $\bar{S}(\alpha(k^{SLC}-), k^{SLC}) = c$. On the other, given that all types in $(k^{SLC}, 1]$ trade at the final offer $\alpha^f(1)$, $\mathbb{E}[\bar{S}(\alpha(k^{SLC}+), \theta)|\theta \in (k^{SLC}, 1]] = c$. It follows that $\alpha(k^{SLC}+) < \alpha(k^{SLC})$.

[13] The simplification uses $\bar{S}(\alpha(k^{SLC}), k^{SLC}) = c$ and $R(k^{SLC}) - \bar{S}(\alpha^f(k^{SLC}), k^{SLC}) = A(k^{SLC})$. Such a $D \in (0,1)$ always exists: since there are gains of trade and $R(\cdot)$ is strictly increasing, we always have that

$$\bar{S}(\alpha^f(k^{SLC}), k^{SLC}) = R(k^{SLC}) > c = \bar{S}(\alpha(k^{SLC}), k^{SLC}).$$

2. *In an upward-skimming environment, if SLC fails, all equilibria have instant trade at an offer of* $\alpha^f(1)$.

3. *In an upward-skimming environment, if SLC holds, and there is no gap, there is no trade in any equilibrium.*

4. *In an upward-skimming environment, if SLC holds and there is a gap, there is a unique on-path equilibrium triple* $\left(\{K^k\}_{k\in[0,1]}, F, \alpha(\cdot)\right)$*:*

   - *The buyer's acceptance strategy is given by* $\bar{S}(\alpha(k), k) = c$ *for* $k \leq k^{SLC}$*, and* $\alpha(k) = \alpha^f(1)$ *for* $k > k^{SLC}$.

   - *There is smooth trade for* $k \in [0, k^{SLC})$*. In the smooth trade region, the cutoff* K_t *is the unique solution to equation* (9).

   - *At state* k^{SCL}*, there is a temporary (random) breakdown in trade. The seller makes the final offer* $\alpha^f(1)$ *with a Poisson arrival intensity* $\lambda = rD/(1-D)$*, where* D *is defined by equation* (11).

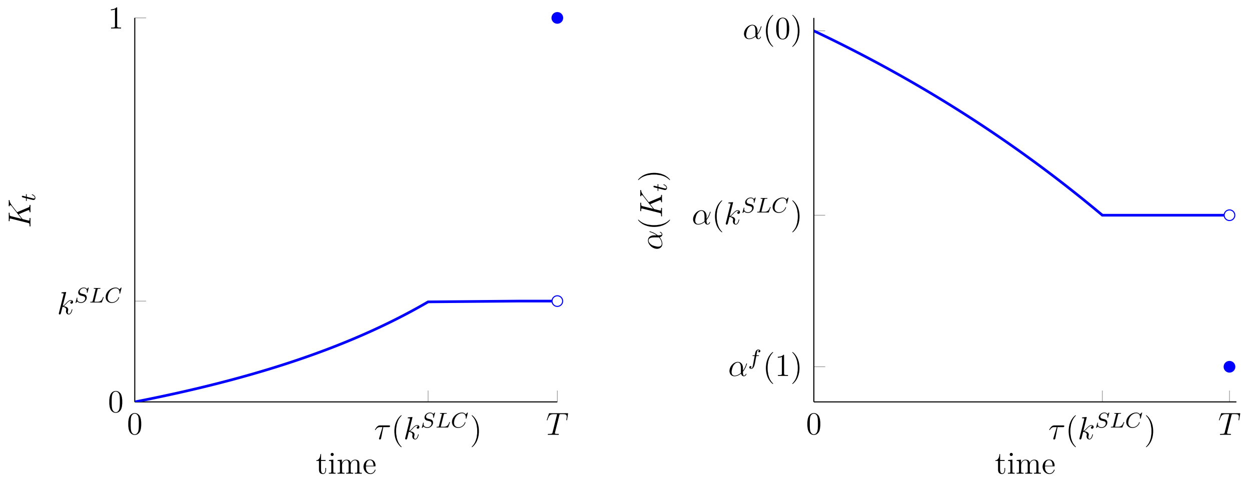
   - *For* $k > k^{SLC}$*, the seller immediately offers* $\alpha^f(1)$.

Figure 1 illustrates typical realized paths of outcomes for the case with non-trivial delay dynamics (Theorem 1.4).[14] The cutoff rises gradually from 0 until it reaches k^{SLC}, with the seller gradually dropping her offers from $\alpha(0)$ to $\alpha(k^{SLC})$. When the state arrives at k^{SLC}, the game reaches an impasse, with the cutoff frozen at k^{SLC} for a random amount of time $T - \tau(k^{SLC})$. During the impasse, the seller "stubbornly" refuses to move her offer from $\alpha(k^{SLC})$, until finally, at a random time, she concedes, dropping her offer to $\alpha^f(1)$. At that point, all remaining types $\theta \in (k^{SLC}, 1]$ accept suddenly, and the cutoff jumps to $k = 1$.

We remark on three distinctive features of the equilibrium.

First, it is instructive to connect our results to the classic model of equity financing in Myers and Majluf (1984). If we interpret the seller in our setting as the financier, and the cost c as the required funding, our model corresponds to a non-competitive, no commitment version of their model. To provide some contrast, let us consider a very simple version of their where there is a competitive market and the firm has to finance the project issuing new equity. If the firm issues new equity to finance the investment, its market value is $\overline{V} := \mathbb{E}\left[V(\theta) \mid \text{Issue}\right]$. Since the required investment is c, the zero profit condition for competitive investors implies that required amount of equity is $(1-\alpha)\overline{V} = c$. The manager (who acts on behalf of old shareholders and knows

[14]The graphs are actual model output, using the setup in Section 5 below.



(a) Equilibrium path of cutoff K_t

(b) Equilibrium path of offers $\alpha(K_t)$

Figure 1: Illustration of a realized equilibrium path in an upward skimming environment satisfying the SLC. $\tau(\theta)$ denotes the realized time at which state θ is reached on-path.

θ) invests only if

$$\frac{c}{\overline{V}} \geq \frac{A(\theta)}{V(\theta)} = 1 - \frac{R(\theta)}{V(\theta)},$$

If $R(\theta)/V(\theta)$ is decreasing—the upward-skimming case, where assets in place are the dominant source of private information—the firm invests if and only if $\theta \leq \hat{\theta}$, where $\hat{\theta}$ solves $c\,\mathbb{E}[V(\theta)|\theta \leq \hat{\theta}]^{-1} = 1 - R(\hat{\theta})V(\hat{\theta})^{-1}$. If instead $R(\theta)/V(\theta)$ is increasing—the downward-skimming case—the firm invests only if $\theta \geq \tilde{\theta}$, where $\tilde{\theta}$ solves $c\,\mathbb{E}[V(\theta)|\theta \geq \tilde{\theta}]^{-1} = 1 - R(\hat{\theta})V(\hat{\theta})^{-1}$. Hence, for a fixed probability of acceptance, inefficiencies are worse when the source of private information is assets in place, since the types who do not invest are precisely the high types who generate the greatest net returns. In our setting, this efficiency "wedge" between the upward- and downward-skimming cases becomes especially stark: when the dominant source of asymmetric information is the return of the project, inefficiencies are not only reduced but vanish.

Second, unlike cash bargaining, with contingent payments one must distinguish between the equilibrium *offer* $\alpha(\theta)$ that type θ accepts, and the expected equilibrium *payment* $\bar{S}(\alpha(\theta), \theta))$ that he faces. For types below k^{SLC} their offers and expected payments are linked by $\bar{S}(\alpha(\theta), \theta) = c$; while they all accept different offers $\alpha(\theta)$, they make the *exact same* expected payment c in equilibrium. Meanwhile, all types in $(k^{SLC}, 1]$ accept the same exact same offer $\alpha^f(1)$, but they all make different expected payments according to $\bar{S}(\alpha^f(1), \theta)$, which is strictly increasing in θ.

Third, since the equilibrium time at which any θ trades could be random, it is useful to have a one-dimensional summary of how much delay is experienced by type θ. Let $\tau(\theta)$ denote the (possibly random) time at which time θ trades. Then type θ's *certainty-equivalent delay*, $\tau^{CE}(\theta)$, is the (deterministic) delay that solves

$$\mathbb{E}[e^{-r\tau(\theta)}] = e^{-r\tau^{CE}(\theta)}.$$

Since $\tau^{CE}(\theta)$ varies one-to-one with expected discounting costs, it provides a welfare-relevant measure of bargaining frictions. Using Theorem 1, we obtain explicit expressions for τ^{CE} that come in handy below:

$$\tau^{CE}(\theta; L) = \begin{cases} \int_0^\theta \frac{\bar{S}_\theta(\alpha(s),s)}{r(R(s)-c)} ds, & \theta \leq k^{SLC}, \\ \int_0^{k^{SLC}} \frac{\bar{S}_\theta(\alpha(s),s)}{r(R(s)-c)} ds - \frac{\log D}{r}, & \theta > k^{SLC} \end{cases} \tag{12}$$

where D is given by (10).[15]

We illustrate the bargaining inefficiencies summarized in τ^{CE} through a convenient parametrization:

Example 1 (Uniform-Linear Primitives, Equity Bargaining)**.** Suppose the buyer and seller bargain in equity (i.e., the security family is $S(\alpha, \tilde{V}) = \alpha\tilde{V}, \alpha \in [0,1]$), and primitives are linear-uniform: the stand alone value is $A(\theta) = \chi\theta$, and the synergy/net return is $\tilde{R}|\theta \sim U[0, 2(c + \Delta + \beta\theta)]$. The expected net return is therefore $R(\theta) = c + \Delta + \beta\theta$. Let $\zeta := \chi + \beta$.

For this specification, we obtain closed form expressions that are easy to compute.In Figure 2, we plot τ^{CE} for parameters $c = 10, r = 1, \Delta = 1/5, \zeta = 2$, and several values of χ.[16] Raising χ while holding ζ fixed changes the content

[15]Types below k^{SLC} trade at a deterministic time governed by equation (9); the expression above follows from using the Inverse Function Theorem and integrating. Types above k^{SLC} experience an additional discounting cost D (after waiting for those under k^{SLC} to trade first).

[16]SLC holds for these parameters. Minor calculus yields $\iota^S_\theta(\theta,\alpha) \propto \chi(c + \Delta)$, so the environment is upward-skimming. The equilibrium in this case is

$$\tau^{CE}(\theta) = \begin{cases} \frac{c\zeta \log\left(\frac{\Delta(c+\Delta+\zeta\theta)}{(c+\Delta)(\Delta+\beta\theta)}\right)}{r(\Delta\chi-\beta c)} & \text{if } \theta \leq k^{SLC} \\ \frac{c\zeta \log\left(\frac{\Delta(c+\Delta+\zeta k^{SLC})}{(c+\Delta)(\Delta+\beta k^{SLC})}\right)}{r(\Delta\chi-\beta c)} - \frac{\log D}{r} & \text{if } \theta > k^{SLC} \end{cases}$$

$$D = \frac{(\chi - \beta)(c + \Delta) - \zeta\beta}{2\chi(c + \Delta)}$$

$$k^{SLC} = \frac{c(\chi - \beta - 2\Delta) - (\beta + \Delta)(\zeta + 2\Delta)}{\zeta(c + \Delta + \beta)}.$$

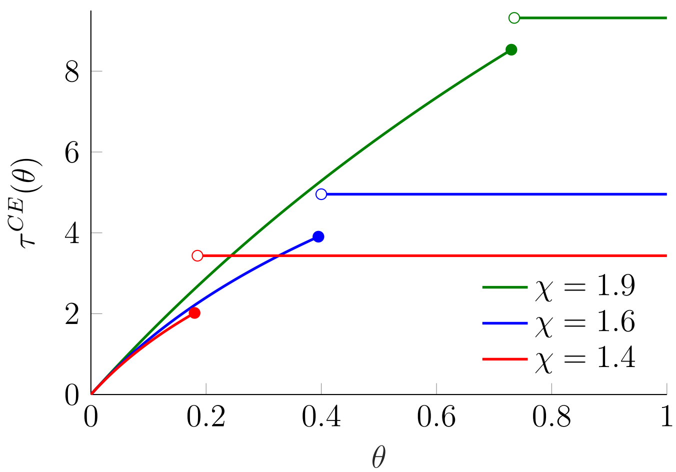


Figure 2: Bargaining frictions and source of private information. *Holding fixed the sensitivity of expected total value to private information, illustrates changes to* τ^{CE} *as the standalone value becomes more sensitive to private information. Parameters are* $c = 10, r = 1, \Delta = 1/5, \zeta = 2$

of the buyer's private information from being mostly about the gains from trade (the net return $\tilde{R}$) to being about his outside option (the assets in place $\tilde{A}$). Intuitively, this should make adverse selection more severe, since higher types become relatively more likely to reject. For any value of χ, τ^{CE} has an upward jump due to equilibrium impasse that high enough types get stuck in. As χ rises, fewer types reach the impasse, and bargaining frictions rise for sufficiently low and sufficiently high types, but they may drop for intermediate types.

Remark 2 (Downward Skimming Equilibria)**.** To understand why delay cannot be sustained in downward-skimming environments, notice that in any such case, the SLC will necessarily fail. Indeed, for any $k > 0$,

$$\mathbb{E}[S(\alpha^f(0), \tilde{V})|\theta \in [0, k]] > \mathbb{E}[S(\alpha^f(0), \tilde{V})|v = 0] = R(0) \geq c,$$

where the strict inequality uses the non-degeneracy conditions 1 and 3 in Assumption 1. In words, under downward skimming, pooling "favors the seller:" higher types accept any final offer that a lower type accepts, so if the seller can make money by trading with the lower type at a given offer, she makes money at that offer with any type that pools with that lowest one.

This rules out the possibility of any smooth trade or any quiet periods in a downward-skimming world, since the seller's payoff in such a case would be exactly c (formal details are in the appendix).

4 Means of Payment and Bargaining Dynamics

In this section, we study how bargaining dynamics change as as a function of the underlying security family used for bargaining. We focus on changes in the "informational sensitivity" or *steepness* of the security family. DeMarzo et al. (2005) define steepness as follows:

Definition 7 (Steepness)**.** Take two security families $S^1 : [\underline{\alpha}_1, \bar{\alpha}_1] \times \mathbb{R}_+ \to \mathbb{R}_+$,$S^2 : [\underline{\alpha}_2, \bar{\alpha}_2] \times \mathbb{R}_+ \to \mathbb{R}_+$. S^1 is steeper than S^2 if, for any feasible offers $\alpha_1 \in [\underline{\alpha}_1, \bar{\alpha}_1]$ and $\alpha_2 \in [\underline{\alpha}_2, \bar{\alpha}_2]$,

$$\bar{S}^1(\alpha_1, \theta) = \bar{S}^2(\alpha_2, \theta) \Rightarrow \bar{S}^1_\theta(\alpha_1, \theta) > \bar{S}^2_\theta(\alpha_2, \theta)$$

Steepness is a partial order on the space of ordered security families. As a simple example, the security family $S^1(\alpha, \tilde{V}) = \alpha\tilde{V}$, which describes equity indexed by the share of gross return, is is a steeper security family than $S^2(\alpha, \tilde{V}) = \min\{\alpha, \tilde{V}\}$, which describes debt indexed by its face value. Below, we often use the shorthand "steeper (flatter) security" to mean "steeper (flatter) security family."

The bulk of our comparative statics results apply to general steepness comparisons as the above example. However, stronger comparative statics results are possible for more structured comparisons where the two security families S^1 and S^2 are part of the same parametrized class:

Definition 8. An ordered security class of *parametrized steepness* is a function $S : [\underline{\alpha}, \overline{\alpha}] \times \mathbb{R}_+ \times [\underline{\gamma}, \overline{\gamma}] \to \mathbb{R}_+$ such that

1. for any γ, $S(\cdot, \cdot; \gamma)$ is an ordered security family satisfying Assumption 1.
2. for any $(\alpha, \theta) \in [\underline{\alpha}, \overline{\alpha}] \times [0, 1]$, $\bar{S}(\alpha, \theta; \gamma) := \mathbb{E}[S(\alpha, \tilde{V}; \gamma)|\theta]$ is a continuous function of γ; and
3. for any pair $\gamma', \gamma'' \in [\underline{\gamma}, \overline{\gamma}]$ with $\gamma' < \gamma''$, $S(\cdot, \cdot; \gamma'')$ is steeper than $S(\cdot, \cdot; \gamma')$.

Some examples of parametrized steepness classes include[17]

- Equity plus a fixed cash component γ: $S(\alpha, \tilde{V}; \gamma) = \left(\bar{L} - \gamma\right) + \alpha\tilde{V}$.

[17] It follows from Lemma 5 in DeMarzo et al. (2005) that higher γ's correspond to steeper securities in these examples.

- Levered equity, with face value of debt γ: $S(\alpha, \tilde{V}; \gamma) = \alpha \max\{\tilde{V} - \gamma, 0\}$.
- Cash plus royalty rate γ: $S(\alpha, \tilde{V}; \gamma) = \alpha + \gamma \tilde{V}$.

We are interested in comparing pairs of security families that both generate non-trivial delay dynamics.

Definition 9 (Delayed trade)**.** Given c and a joint distribution for $\tilde{R}$ and $\tilde{A}$, a security family $S(\cdot,\cdot)$ *generates delayed trade*, denoted $S \in \mathcal{D}_{\tilde{R},\tilde{A},c}$, if

1. There is upward-skimming under S: $\iota^S(\cdot, \alpha)$ is strictly increasing for every α.
2. The Static Lemons Condition holds: $\mathbb{E}[S(\alpha^f(1), \tilde{V})] < c$.
3. There are strict gains from trade: $R(0) > c$.

For two security families S^1 and S^2 in a parametrized steepness class $S(\cdot,\cdot;\cdot)$, with S^1 steeper, we write $[S_2, S_1] \in \mathcal{D}_{\tilde{R},\tilde{A},c}$ if

1. there exist steepness parameters $\gamma_1 > \gamma_2$ such that $S^1(\cdot,\cdot) = S(\cdot,\cdot;\gamma_1)$, $S^2(\cdot,\cdot) = S(\cdot,\cdot;\gamma_2)$; and
2. $S(\cdot,\cdot;\gamma) \in \mathcal{D}_{\tilde{R},\tilde{A},c}$ for all $\gamma \in [\gamma_2, \gamma_1]$.

For pairs of securities that generate delay, we characterize the effects of increasing steepness on trading dynamics, payments, and equilibrium utilities. First, regarding trading dynamics and delay, we show:

Proposition 1. *Take S^1 and S^2 in $\mathcal{D}_{\tilde{R},\tilde{A},c}$, with S^1 steeper. Let $k^{SLC}(S^i)$ and $\tau^{CE}(\theta; S^i)$ denote, respectively, the critical type and the (equilibrium) certainty-equivalent delay suffered by type θ under security S^i.*

1. *Under S^1 there is less pooling, and a slower gradual concessions phase:*
   (a) $k^{SLC}(S^1) > k^{SLC}(S^2)$.
   (b) $\tau^{CE}(\theta; S^1) > \tau^{CE}(\theta; S^2)$ *for* $\theta \in (0, k^{SLC}(S^2)]$.
2. *If, in addition, S^1 and S^2 belong to the same parametrized steepness class, with $[S_2, S_1] \in \mathcal{D}_{\tilde{R},\tilde{A},c}$ then types above $k^{SLC}(S^1)$ also suffer strictly higher certainty-equivalent delay under S^1:*
$$\tau^{CE}(\theta; S^1) > \tau^{CE}(\theta; S^2), \quad \textit{for all } \theta \in (0,1] \setminus (k^{SLC}(S^2), k^{SLC}(S^1)].$$

We postpone the proof of item 2, which depends on some technical lemmas, to the appendix. Meanwhile, items 1(a) and 1(b) follow directly from

Theorem 1 and the definition of steepness. Consider the size of the pooling region. The pooling region makes the seller on average break even with the final offer $\alpha^{f,i}(1)$. Since type-by-type expected payments $\bar{S}^i(\alpha^{f,i}(1), \theta)$ are higher under S^2—the payments of the flatter security cannot cross those of the steeper one from below—the seller must average over a strictly worse pool (include even lower types) to break even under S^2. Formally, let $\alpha^{f,i}(1)$ denote the final equilibrium offer under security S^i. By definition, the $\alpha^{f,i}$'s satisfy $\bar{S}^1(\alpha^{f,1}(1), 1) = \bar{S}^2(\alpha^{f,2}(1), 1) = R(1)$, which, by the greater steepness of S^1, implies that

$$\bar{S}^1(\alpha^{f,1}(1), \theta) < \bar{S}^2(\alpha^{f,2}(1), \theta) \text{ for all } \theta < 1. \tag{13}$$

Therefore, for all $k < 1$,

$$\overbrace{\frac{1}{1-k}\int_k^1 \bar{S}^1(\alpha^{f,1}(1), \theta) d\theta}^{\mathbb{E}[\bar{S}^1(\alpha^{f,1}(1),\theta)|\theta\in[k,1]]} \quad < \quad \overbrace{\frac{1}{1-k}\int_k^1 \bar{S}^2(\alpha^{f,2}(1), \theta) d\theta}^{\mathbb{E}[\bar{S}^2(\alpha^{f,2}(1),\theta)|\theta\in[k,1]]}, \tag{14}$$

and item 1(a) follows immediately.

Likewise, consider the gradual concessions phase. The buyer's incentive to reject an offer is that, by rejecting, he can affect the seller's beliefs about his type and can obtain a better price in the future. However, because of the Coasean force the seller's expected payment is constant in the state. Letting α^1 and α^2 denote the reservation offer curves for the bargaining games with securities S^1 and S^2, we have $\bar{S}^i(\alpha^i(k), k) = c$. *The change in price from a change in the seller's beliefs therefore exactly equals the change in price from a change in the buyer's type*: for $i = 1, 2$ and $\theta \leq k^{SLC}(S^2)$,

$$\bar{S}^i(\alpha(k), k) = c \Rightarrow \qquad \underbrace{-\overbrace{(\alpha^i)'(k)\bar{S}^i_\alpha(\alpha(k), k)}^{<0}}_{\text{Price improvement from changing seller's belief}} \qquad = \underbrace{\frac{\partial}{\partial\theta}\bar{S}^i(\alpha(k), \theta)\Big|_{\theta=k}}_{\text{sensitivity to } \theta \text{ at } \theta = k}$$

Hence, the more sensitive price is to the buyer's private information, the greater the price improvement that he expects from rejecting an offer, the greater his incentives to reject, and the slower trade must be. Formally, $\bar{S}^i(\alpha^i(k), k) = c$ implies $\bar{S}^1_\theta(\alpha^1(k), k) > \bar{S}^2_\theta(\alpha^2(k), k)$, since S^1 is steeper. Plugging this inequality into (12) yields $\tau^{CE}(\theta; S^1) > \tau^{CE}(\theta; S^2)$ for all $\theta \in (0, k^{SLC}(S^2))$.

We also characterize the effect of information sensitivity on expected payments. The effect is always heterogeneous across types, raising equilibrium payments for some and lowering them for others:

Proposition 2 (Expected payments are "single-crossing")**.** *Take S^1 and S^2*

in $\mathcal{D}_{\tilde{R},\tilde{A},c}$, with S^1 steeper. Let $\pi_i(\theta) := \bar{S}^i(\alpha^i(\theta), \theta)$, θ's equilibrium expected payment under S^i.

1. *Under S^1, high types pay strictly less, low types pay more: there exists a unique $k^{cross} \in (k^{SLC}(S^2), 1)$ such that*

$$\begin{aligned} \pi_1(\theta) &= \pi_2(\theta), \quad \theta \in [0, k^{SLC}(S^2)]. \\ \pi_1(\theta) &> \pi_2(\theta), \quad \theta \in (k^{SLC}(S^2), k^{cross}). \\ \pi_1(\theta) &< \pi_2(\theta), \quad \theta \in (k^{cross}, 1). \end{aligned} \tag{15}$$

2. *Let k^* solve $\bar{S}^2(\alpha^{f,2}(1), k^*) = c$. Then $k^{cross} = \min\{k^*, k^{SLC}(S^1)\}$.*

Since the expected payment of the highest type is the same under both securities ($\bar{S}^i(\alpha^{f,i}(1), 1) = R(1)$), and high enough types accept $\alpha^{f,i}(1)$ in either case, they must pay less under the steeper security. At the same time, since flatter security has a larger pooling region, there are types at the bottom of the interval that would be separated under a steep security (and pay c), but get cross-subsidized by very high types when they face the flat security. They must therefore pay less strictly than c under the flat security. Finally, types at the bottom of the distribution are separated and pay c in either case.

The previous propositions deal with arbitrary increases in steepness. It will be useful for empirical applications to study the effect of "small" or local changes in steepness. Doing so uncovers important subtleties. On the one hand, for small increases in steepness, expected payments and bargaining frictions change "almost" monotonically (monotonically for almost all types). On the other, as we already saw in Proposition 2, the cross-subsidization across types that happens in equilibrium causes expected payments to behave non-monotonically with respect to steepness. For small increases in steepness, bargaining frictions may also have this non-monotonic behavior for a small segment of types.

Corollary 1 (Local Increases in Steepness)**.** *Consider a parametrized steepness class $S(\cdot,\cdot;\cdot), \gamma \in [\underline{\gamma}, \bar{\gamma}]$ For any $\gamma \in [\underline{\gamma}, \bar{\gamma})$ with $S(\cdot,\cdot;\gamma) \in \mathcal{D}_{\tilde{R},\tilde{A},c}$:*

1. *For any $\delta \in (0,1)$, there exists $\varepsilon > 0$ small enough that, for a measure $1-\delta$ of types*

   (a) *certainty-equivalent delay $\tau^{CE}(\cdot)$ is strictly higher under $S(\cdot,\cdot;\gamma+\varepsilon)$ than under $S(\cdot,\cdot;\gamma)$.*

   (b) *expected payments $\pi(\cdot)$ are strictly lower under $S(\cdot,\cdot;\gamma+\varepsilon)$ than under $S(\cdot,\cdot;\gamma)$.*

2. *There exists $\varepsilon > 0$ small enough that, for a positive measure of types,*

    (a) *$\pi(\cdot)$ is strictly higher under $S(\cdot,\cdot;\gamma+\varepsilon)$ than under $S(\cdot,\cdot;\gamma)$.*

    (b) *τ^{CE} is strictly* lower *under $S(\cdot,\cdot;\gamma+\varepsilon)$ than under $S(\cdot,\cdot;\gamma)$.*

Point 1 and Point 2(a) follow immediately from Propositions 1 and 2 using the continuity of the critical cutoff k^{SLC} with respect to the steepness parameter (Lemma 5 in the appendix). In contrast, Point 2(b) follows from *dis*continuity of τ^{CE}. As we increase γ slightly by $d\gamma$, types inside $[k^{SLC}(\gamma), k^{SLC}(\gamma+d\gamma))$ shift from trading in the final atom—after the impasse—to trading in the initial phase of smooth trading. This has two effects on the bargaining frictions they endure. First, τ^{CE} for these types drops discontinuously so as to lie along a smooth trading locus; second, the smooth trading locus itself rises—continuously—because of the slightly higher steepness. For a small enough change in steepness, the discontinuous drop must dominate, so bargaining frictions must drop for some intermediate types.

Notice that, in light of the corollary, the results in Proposition 1 are therefore "tight" in that there exist pairs of securities $S^1, S^2 \in \mathcal{D}_{\tilde{R},\tilde{A},c}$, with S^2 steeper, for which some intermediate types suffer lower bargaining frictions with the steeper security.

We discuss empirical implications of the corollary and Propositions 1 and 2 in the following section, where we connect steepness parameters to different proxies for the tightness of financial constraints. In the interim, we note that studies that try to calculate a single parameter summarizing the effect of higher steepness on returns on deal failure may conceal important heterogeneities across the distribution of firms.

Having ranked equilibrium delay and payments by type, we now rank, where possible, equilibrium utilities by type.

Proposition 3. *Take two securities S^1 and S^2 in $\mathcal{D}_{c,\tilde{R},\tilde{A}}$, with S^1 steeper.*

1. *Let k^{cross} be as in Proposition 2. Then there exists $k' > k^{cross}$ such all types $\theta \in [0, k')$ prefer the flatter security S^2, and strictly so for $\theta > 0$.*

2. *Suppose, in addition, that S^1 and S^2 belong to the same parametrized steepness class $S(\cdot,\cdot;\cdot)$, with $[S_2, S_1] \in \mathcal{D}_{\tilde{R},\tilde{A},c}$. If*

$$R(\theta) - \bar{S}(\alpha^f(1;\gamma), v; \gamma) \tag{16}$$

    is log-supermodular in $(\theta, -\gamma)$, then all types prefer S^2, strictly so for $\theta \in (0,1)$.

Said differently, types who pay less under the flatter security always prefer to bargain with it, no matter whether they suffer higher or lower delay under that security. For parametrized steepness comparisons, an easy-to-check sufficient condition ensures that *all* types prefer bargaining in the flatter security, even when equilibrium requires them to pay more or suffer more equilibrium delay.[18] Our sufficient condition is not stated in terms of the primitive security family, and it does not follows directly from steepness. However, the condition is easily verified for particular families of securities, as we do in section 5.

Let us explain the ways in which this ranking is and is not straightforward, to elucidate the economic content of Proposition 3. For low enough types (those who trade smoothly under both S^1 and S^2, i.e., $k < k^{SLC}(S^2)$), the payoff ranking is immediate: they trade strictly faster under the flatter security, and make the exact same equilibrium payment in both cases. By continuity of payoffs, it is unsurprising that this ranking extends somewhat beyond $k^{SLC}(S^2)$, even when we cannot rank the delay experienced by types just above $k^{SLC}(S^2)$.

For higher types, that continuity argument breaks down without additional structure or additional arguments. Even though steepness arguments suffice on their own to rank equilibrium payments, they do not (to our knowledge) suffice to rank delay for general comparisons. And even for those parametrized comparisons where we know that types above $k^{SLC}(S^1)$ trade faster with the flatter security, we face the problem that those same types also *pay* strictly more in that case, making the overall utility comparison ambiguous.

In light of the above, the added value of Proposition 3 is to provide a way to extend the continuity arguments that ranked payoffs for types just above $k^{SLC}(S^2)$ to much higher types. Using envelope-theorem arguments (Lemma 4 in the Appendix), we show that the payoff ranking always extends to all types who pay weakly less under the flatter security, i.e., $k \leq k^{cross}$—regardless of whether those types suffer more or less delay under the flat security. For parametrized steepness comparisons, the condition in (16) ensures that, if indirect utilities were ever to cross somewhere above k^{cross}, the utility for the flatter security would be crossing from below. Since types below k^{cross} must prefer the flatter security, this is impossible.

[18] Note that, under the conditions of Proposition 3, there are no types for whom the steeper security leads to strictly more certainty-equivalent delay *and* strictly higher payments. In that sense, there is a "delay-payment trade-off."

5 Mergers and Acquisitions with Financial Constrains

Here we show how we can apply our general results to the issue of mergers and acquisitions under financial constraints. We study this through two different models in which financially constrained acquirers negotiate over the equity split in the merged entity. First, we look at the case in which the acquirer has pre-existing debt that the merged company will have to assume; for any given equity split, the acquirer's leverage lowers the value of the equity, and it makes that equity more sensitive to the acquirer's private information. Second, we look at the case in which the acquirer has a limited amount of cash, fixed at the outset of the negotiation, that is added to the equity payment to the target. For any equity split, lowering the amount of cash also makes the total payment more sensitive, in relative terms, to the acquirer's private information. Insofar as cash is costly for firms—and costlier for the financially constrained ones—the amount of cash added to the equity payment parametrizes the acquirer's liquidity constraints. (For example, the acquirer might not have enough cash in hand to complete the transaction, or external financing might be prohibitively expensive).[19]

Using these examples, we first derive empirical implications for impact of financial constraints on bargaining frictions and acquisition returns. Through numerical examples, we also show how a reduced form analyses of the effects that financial constraints have on those quantities of interest can be very misleading. Second, we show how negotiation patterns depend on the nature of the synergies that the merger would create.

Formally, we structure our discussion around two security classes that capture these constraints:

Equity bids with limited up-front cash Offers belong to the parametrized steepness class $S_{liq}(\alpha, \tilde{V}; L) = \alpha\tilde{V} + L, \alpha \in [0, 1]$. That is, the target and acquirer negotiation over equity in the merged entity. This equity is "sweetened" by a *fixed* amount of cash $L < c$, which improves the offer

[19]Even if the company has sufficient cash, the opportunity cost of depleting its cash reserves may outweigh the efficiency benefits from negotiating in a less informationally sensitive security. Indeed, there is empirical evidence that financial constraints limit the use of cash. For example, Alshwer et al. (2011) finds that financially constrained acquirers rely more on stock as a method of payment than financially unconstrained ones. Other empirical studies have found that, even when acquirers have enough cash to complete a transaction, they tend to use stock as a means of payment if they are financially constrained.

but on its own is insufficient to persuade the target. We write S^L_{liq} as a shorthand for $S_{liq}(\cdot, \cdot; L)$.

Levered equity offers Offers belong to the parametrized steepness class $S_{lev}(\alpha, \tilde{V}; d) = \alpha(\tilde{V} - d)_+, \alpha \in [0, 1], d \geq 0$. That is, the target and acquirer negotiate over equity in the merged entity, which takes on the acquirer's debt. The correct specification depends on whether the acquirer is maximizing firm value or the value of the original equity holders. Here, to match the utility specification in (2), we assume that the acquirer maximizes *total firm value* and not just the value of equity holders. This would be the case if there are covenants that require approval from debt holders. [20] We write S^d_{lev} as a shorthand for $S_{lev}(\cdot, \cdot; d)$.

For both specifications, tighter financial constraints (lower L and higher D) correspond to steeper security families. Moreover, these examples have the useful property that, if a security causes delay, then every steeper security in the same class also causes delay. We can therefore apply Propositions 1 through 3 to study the effects of tightening financial constraints:

Lemma 1.

- *If $S^L_{liq} \in \mathcal{D}_{\tilde{R},\tilde{A},c}$, then $S^{L'}_{liq} \in \mathcal{D}_{\tilde{R},\tilde{A},c}$ for all $L' \leq L$. Moreover, if $S^0_{liq} \in \mathcal{D}_{\tilde{R},\tilde{A},c}$, then there exists $L^* < c$ such that $S^L_{liq} \in \mathcal{D}_{\tilde{R},\tilde{A},c}$ for all $L \leq L^*$.*
- *If $S^d_{lev} \in \mathcal{D}_{\tilde{R},\tilde{A},c}$, then $S^{d'}_{lev} \in \mathcal{D}_{\tilde{R},\tilde{A},c}$ for all $d' > d$. Thus, if $S^0_{lev} \in \mathcal{D}_{\tilde{R},\tilde{A},c}$, then $S^d \in \mathcal{D}_{\tilde{R},\tilde{A},c}$ for all $d \geq 0$.*

For concreteness, we often focus on the following convenient parametrization:

Example 2 (Normal-Linear Primitives)**.** The stand alone value is $A(\theta) = \chi\theta$, $\Delta, \chi, \beta > 0$, and synergy value, conditional on θ, is distributed $\tilde{R}|\theta \sim \mathcal{N}(c + \Delta + \beta v, \eta^{-2})$. η is therefore the *precision* of the buyer's signal about synergies. We denote $\zeta = \chi + \beta$.

Empirical implications for deal failure, acquisition returns, and premia With a slight reinterpretation of the discounting cost r, our model generates predictions for deal failures and M&A activity. If negotiations break down at a Poisson rate r, then $e^{-r\tau^{CE}(\theta)}$ is the probability of a negotiation failure for

[20]The case in which the acquirer maximizes the value of old equity holders can be reduced to the case of pure equity in Example 1 by redefining: $\bar{A}(\theta; d) \equiv \mathbb{E}[(\tilde{A} - d)^+|\theta]$ and $\bar{V}(\theta; d) \equiv \mathbb{E}[(\tilde{V} - d)^+|\theta]$.

an acquirer of type θ. Proposition 1 then predicts that a marginal tightening of financial constraints (a small increase in d or a small decrease in L) increases the probability of deal failure for all types outside a small intermediate region. This is broadly consistent with empirical studies that have looked at financial constraints and M&A activity, e.g. Malmendier et al. (2016) and Uysal (2011). The former study shows that successful acquisitions have a larger cash component, while the latter shows that over-levered firms (firms that are more levered than predicted by other covariates) are less likely to make an acquisition in the observation period. We note that, for larger changes in financial constraints (e.g., a large increase in d), there exists a possibly sizeable range of intermediate types for which Proposition 1 does not provide any concrete prediction on the change in deal failure probability. Indeed, as illustrated in the bottom right panel of Figure 3, the deal failure probability may even *decrease* for those intermediate types. The regressions in Malmendier et al. (2016) and Uysal (2011), which estimate an average effect (averaged over the unobservable type θ), may therefore conflate heterogenous effects that pull in opposite directions. Identifying those heterogeneous impacts is an interesting possibility for additional empirical research.

Our model also has implications for acquirers' ex post returns which equal $1 - \frac{\bar{S}_{liq}(\alpha(\theta),\theta;L)}{V(\theta)}$ in the liquidity constraints model and $1 - \frac{\bar{S}_{lev}(\alpha(\theta),\theta;d)}{V(\theta)}$ in the leverage model. Empirical work on M&A typically evaluates acquisitions using ex post returns, but our analysis also shows that we need to be careful when we evaluate the return that acquirers obtain from acquisitions, for two reasons. First, Proposition 2 implies that marginally tightening the acquirer's financial constraints simultaneously *strictly raises* returns for all types above a threshold and weakly lowers them for types inside below that threshold. Empirical studies that regress, say, leverage indicators on the realized return from acquisitions, estimate a single effect averaged over types and may therefore obscure the heterogenous impact of tighter constraints. Second, our results show that higher returns do not mean that the acquirer is better off in an ex ante sense–in fact, they might mean the opposite. Take, for instance, the liquidity constraints model with a linear uniform specification: $A(\theta) = \chi\theta$, $\tilde{R}|\theta \sim U[0, 2(c + \Delta + \beta\theta)]$. Proposition 3.2 then applies,[21] and all types of the buyer strictly benefit in ex ante terms from looser financial constraints, even though all types above a threshold pay more, and therefore face lower returns. So for example, even if we were to observe that more cash-heavy offers reduce realized returns, those offers may still be in the ex ante best interests of the

[21]Using $R(\theta) = c + \Delta + \beta\theta$, $V(\theta) = c + \Delta + \zeta\theta$, and $\alpha^f(1; L) = (R(1) - L)/V(1)$, one finds that $\frac{\partial}{\partial L} \log(R(\theta) - \bar{S}^L_{liq}(\alpha^f(1; L), \theta))$ is constant in θ.

acquiring firm's shareholders.

We close this subsection by describing some problems with reduced-form inference on bargaining frictions that are raised by our model. Recent studies on bargaining have measured inefficiencies by looking directly at realized delay. Since the time at which a negotiation starts is seldom observable by the researcher, but longer delay implies higher inefficiencies, these datasets are thought to provide rare direct evidence on bargaining frictions. We explain how this line of reasoning—longer expected delay, measurable in the reduced form, implies greater inefficiency— can be incorrect in our model.

With a sufficiently rich dataset on M&A negotiations that controls for the target value and industry characteristics, one can plausibly hold the fundamentals $(r, \tilde{R}, \tilde{A}, c)$ fixed while varying the tightness of financial constraints. Since $\tau(\theta)$, the possibly random time at which θ trades, is increasing in θ realization by realization, then by measuring the quantiles of realized delay while holding $(r, \tilde{R}, \tilde{A}, c)$, one can recover the *expected delay curve* directly from data:

$$\tau^{Exp}(\theta) = \mathbb{E}[\tau(\theta)] = \begin{cases} \tau^{CE}(\theta), & \theta < k^{SLC}, \\ \tau^{CE}(k^{SLC}) + \frac{1-D}{rD}, & \theta \geq k^{SLC}, \end{cases} \tag{17}$$

with d given by (10).

It would seem that a pointwise increase in $\tau^{Exp}(\theta)$ is strong reduced-form evidence in favor of rising bargaining inefficiencies. And yet, due to the convexity of exponential discounting and the randomness in equilibrium delay, the (observable) expected delay curve τ^{Exp} necessarily differs from the (unobservable) certainty-equivalent delay curve τ^{CE}, which is the true welfare-relevant measure of bargaining frictions. The contrast is stark: as we now show, τ^{Exp} and τ^{CE} can move in very different directions as financial constraints tighten, and τ^{CE} can rise uniformly even when τ^{CE} drops for the vast majority of types.

Consider the levered equity scenario with normal-linear primitives, [22] Figure 3 shows changes to τ^{Exp} and τ^{CE} as we move from the low leverage (loose financial constraints) case of $d_1 = 0$ to the high leverage case of $d_2 = 5$. The top row displays these changes for the parameter set $(c = 5, \Delta = 1/2, r = 1, \chi = 10, \beta = 1, \eta = 1)$. For these parameters, as when the buyer's financial constraints tighten, both certainty-equivalent delay (welfare-relevant bargain-

[22] This leads a smooth trade offer of $\alpha(\theta) = c/\mathbb{E}[(\tilde{V} - d)^+|\theta]$, a final offer of $\alpha^f(1) = \mathbb{E}[(\tilde{V} - d)^+|1]$, with an expected value of equity given by

$$\mathbb{E}[(\tilde{V} - d)^+|\theta] = \int_d^\infty \Phi\left(\eta(c + \Delta + \zeta\theta - z)\right) dz,$$

where Φ is the CDF of the standard normal distribution.

ing frictions) and expected (i.e., observable) delay rise uniformly for all buyer types.

The bottom row displays the effects of the same leverage change for a new parameter set ($c = 5, \Delta = 1/2, r = 1, \chi = 5, \beta = 1, \eta = 1/7$). Here, tightening the buyer's financial constraints raises bargaining frictions for buyer types in the top 90% percentile,[23] but lowers them for types below that threshold. In contrast to the top row, tighter financial constraints now uniformly *lower* expected (i.e., observable) delay for almost all types.[24] Put differently, a change that worsens bargaining frictions for the top 90% percentile of acquiring firms would show up in the data as "improving" observable delay for nearly all firms!

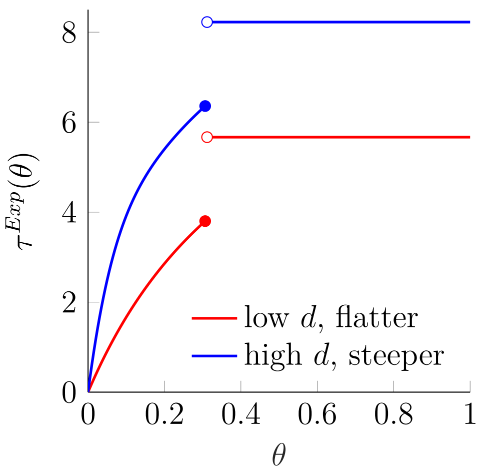
Remarkably, for either parameter sets used in Figure 3, all buyer types are strictly better off when they are less financially constrained, so that observable delay can vary independently of both bargaining frictions and buyer preferences.

The impact of rising uncertainty about synergies Returning the leverage model with normal-linear primitives, we examine how bargaining outcomes change as the acquirer's signal about potential synergies becomes more precise. Synergies comes either from cost reduction or from revenue improvements. Usually, it is easier to fulfill cost reduction coming from reducing some fixed costs, then to fulfill increments in revenue from expanding into new markets (Berk and DeMarzo, 2013). This means that it is natural to think that an acquirer that hopes to achieve certain cost efficiencies through a merger may have a more precise estimate of the potential synergies than one who hopes to exploit a particular kind of product market fit. Rising precision about synergies affects the negotiation through two different channels. On the one hand, for any offer α, it lowers the value of the levered equity: $\max\{\tilde{V} - d, 0\}$ is convex in $\tilde{V}$, and lowering η causes a mean-preserving spread of $\tilde{V}|\theta$. On the other, changes to η affect the slope of levered equity with respect to the buyer's private information; an initial intuition would suggest that, by raising the signal-to-noise ratio for the buyer's signal θ, a higher η would make levered more sensitive to θ. *A priori* it is not clear how what the equilibrium repercussions of these effects may be. We show that, even thought the slope of levered equity with respect to the buyer's type may go up or down in *absolute* terms, the net effect of increases precision is indeed a "heightened sensitivity": for a given debt level, raising the precision of the buyer's signal is equivalent to bargaining in a steeper security:

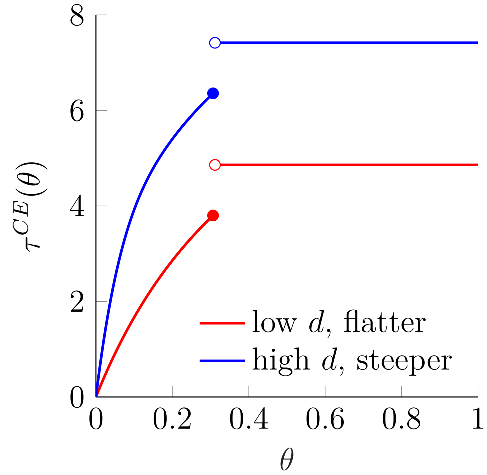
Proposition 4. *Consider a levered equity model with normal-linear primitives*

[23]When $d = 5$, the pooling threshold is $k^{SLC} = 0.1063$

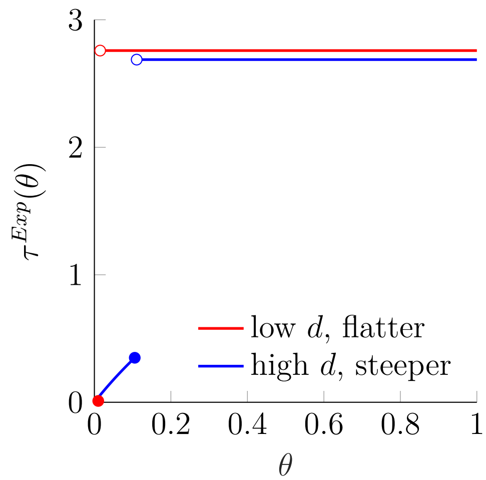
[24]Precisely, types below $\theta \sim 1.4 \times 10^{-2}$.



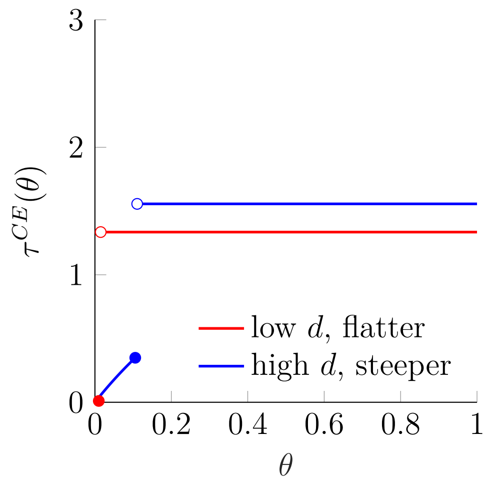
(a) Expected Delay



(b) Certainty-Equivalent Delay



(c) Expected Delay



(d) Certainty-Equivalent Delay

Figure 3: Discounting and observable delay are distinct. *Panels (a) and (b)*: parameters are $c = 5, \Delta = 1/2, \chi = 10, \beta = 1, r = 1, d_1 = 0, d_2 = 5,\ \eta = 1$. All types suffer higher expected delay when leverage is higher. *Panels (c) and (d)*: parameters are $c = 5, \Delta = 1/2, \chi = 5, \beta = 1, r = 1, d_1 = 0, d_2 = 5,\ \eta = 1/7$. Types in the pooling region suffer *higher* certainty equivalent delay when leverage is higher, and all types prefer having lower leverage. However, all but a vanishing fraction of types suffer *lower* expected delay when leverage is higher.

$\tilde{V}|\theta \sim \mathcal{N}(c+\Delta+\zeta\theta, \eta^{-2})$, so that η is the precision of $\tilde{V}|\theta$. Let $\bar{S}^d_{lev}(\alpha, \theta; \eta) := \mathbb{E}[\alpha \max\{\tilde{V}-d, 0\}|\theta]$ denote the expected payment as a function of offer, type, and precision.

1. *There exist $\bar{d} > \underline{d}$ such that*

$$\frac{\partial^2}{\partial\theta\partial\eta}\bar{S}^d_{lev}(\alpha, \theta; \eta) = \begin{cases} < 0, & d > \bar{d} \\ > 0, & d < \underline{d} \end{cases}$$

2. *For any $\eta_1 > \eta_2$, and α_1, α_2,*

$$\bar{S}^d_{lev}(\alpha_1, \theta; \eta_1) = \bar{S}^d_{lev}(\alpha_2, \theta; \eta_2) \Rightarrow \frac{\partial}{\partial\theta}\bar{S}^d_{lev}(\alpha_1, \theta; \eta_1) > \frac{\partial}{\partial\theta}\bar{S}^d_{lev}(\alpha_2, \theta; \eta_2) \tag{18}$$

3. *If there is non-trivial delay for some η (i.e., the environment is upward-skimming and SLC holds), there is non-trivial delay for every $\eta' > \eta$.*

4. *Let $h(\cdot)$ be the hazard rate of the standard normal distribution. A sufficient condition for the environment to be upward-skimming for all d is*[25]

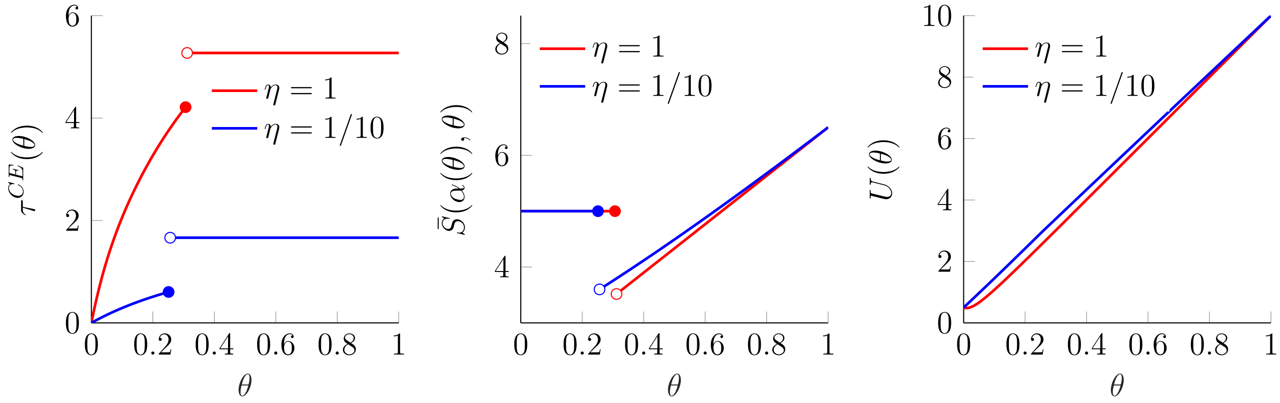
$$\frac{\beta}{\beta+\chi} < \frac{\eta(c+\Delta)}{\eta(c+\Delta)+\eta(\beta+\chi)+h(-\eta(c+\Delta))}.$$

Points 2 and 3 together imply that we can treat an increase in precision as though the parties were bargaining in a steeper security, and all the results from Section 4 apply unchanged.

Figure 4 shows the changes in certainty equivalent delay, expected payments $\bar{S}_{lev}(\alpha(\theta; d), v; d)$, and indirect buyer utilities as η increases from $\eta = 1/10$ to $\eta = 1$. The other parameters are held fixed at $c = 5, \Delta = 1/2, \beta = 1, \chi = 10, r = 1, d = 4$. With a higher precision about synergies, bargaining frictions—measured by τ^{CE}—rise for all types, and all types are made better off. As predicted by Propositions 1-3, (i) expected payments rise for types above a threshold and (weakly) drop for types under that threshold, (ii) there are fewer types are in the final pooling region, (iii) bargaining frictions rise for low enough and high enough types, and (iv) types below a threshold are harmed by the increase in precision For this large rise in the precision, we see moreover that bargaining frictions rise for *all* types, and all types other than the corners are made strictly worse off.

[25] An even cruder sufficient condition, of the kind that can be checked on a small napkin, is

$$\frac{\beta}{\beta+\chi} < \frac{\eta(c+\Delta)}{\eta(c+\Delta)+\eta(\beta+\chi)+2/\sqrt{2\pi}}. \tag{19}$$



(a) Certainty-Equivalent Delay (b) Ex-post Payment (c) Expected Payoff

Figure 4: Discounting, ex-post payments, and payoff. *Parameters are* $c = 5, \Delta = 1/2, \chi = 10, r = 1, d = 4$.

Put differently, increasing the precision of the buyer's signal *destroys value*. The seller's equilibrium payoff is unaffected (since the game starts with smooth trade, it equals c regardless of the precision level), so this destruction is all at the buyer's expense. The buyer, in particular, would not want to invest in a technology that improves its prediction about synergies. If, as in the cost reduction vs market examples above, we see different η's as modeling different possible mergers *ceteris paribus*, this suggest that, net of negotiation inefficiencies, the buyer has an incentive to focus mergers with more uncertain synergies.

6 Discussion

Connection to Security Auction Literature In the security auctions literature, Che and Kim (2010) first pointed out that having the value of assets-in-place $A(\theta)$ be increasing in θ could lead to decreasing bidding strategies, i.e., higher types bid less and are less likely to win the auction. They study the effect on revenue of moving from a flat security S^2 to a steeper one S^1, when both have equilibria in decreasing strategies.[26]

[26] Their results are more general, but we emphasize this aspect of them to streamline the comparison to ours. In fact, they show that, whenever S^2 has decreasing bidding strategies ("upward-skimming," in our terminology), S^1 also will. Moreover, they show that upward-skimming steeper securities yield lower revenue than flatter securities, regardless of whether the flatter ones are upward- or downward-skimming. In our setup, it is easy to show that

With the usual mapping between higher probability of winning and lower expected discounting costs, their comparative statics correspond to our comparison between S^1, $S^2 \in \mathcal{D}_{c,\tilde{R},\tilde{A}}$. We now contrast our results to theirs.

First, Che and Kim (2010) prove, both for first and second price auctions, that expected payments are higher type-by-type under S^2, the flatter security.[27] The difference between our results and theirs stems from the seller's extreme commitment problem in our model. Because of her extreme lack of commitment, *the seller makes exactly c on every trade*. Therefore,

- Low types ($\theta \in [0, k^{SLC}(S^2)]$) face an expected payment that is *constant* across securities, since they are separated under either security.
- Intermediate types ($\theta \in (k^{SLC}(S^2), k^{cross})$) pay strictly less under S^1, since types in that range are separated in the S^1 equilibrium, but they get cross-subsidized (by types above k^*) in the S^2 equilibrium.

A consequence of the revenue rankings in Che and Kim (2010) is that in their model, among two securities that induce decreasing strategies bidders weakly prefer the steeper one (strictly so for types below the highest). In our model, that preference can be reversed uniformly for all types—even for types who pay more under the flatter security—and in fact is always reversed for types under k^*. The reason is as follows: the allocation in Che and Kim (2010)'s comparison is the same across both securities, so payment rankings translate into utility rankings. Meanwhile, in our bargaining game, changing the security changes the amount of delay and the expected allocation; the impact of this allocation change on payoffs can overwhelm the impact of the higher expected payments under S^2.[28]

if SLC holds for S^2, then it must hold for S^1, but it does not follow that if S^2 is upward-skimming, S^1 will also be.

[27] We refer to the working paper version, which contains results on both auction formats (available on the authors' website here: `https://emu-perch-bjgm.squarespace.com/s/security-comment-1.pdf`). Proposition 3 in that version shows that payments are strictly higher *ex-post* in the second price auction. The proof of their Proposition 5 shows that interim utilities are strictly higher (except for $\theta = 1$) under S^1; since the allocations are the same in both cases (both security auctions have decreasing strategies), interim expected payments must be higher under S^2.

[28] To emphasize this point—that our departure from the results in Che and Kim (2010) and DeMarzo et al. (2005) is "allocation-driven," consider two upward-skimming securities that both fail SLC. Since all types trade instantly in either case, the allocation, meaning the expected discount until trade, is constant across securities. Then, given that all types pay $\bar{S}^i(\alpha^{f,i}(1), \theta)$ under S^i, it follows from (13) that flatter securities lead to higher revenue and make buyers worse off, as in Che and Kim (2010). Taking two downward-skimming securities, we get the opposite result, as in DeMarzo et al. (2005).

Increasing net surplus and cross-subsidization We assumed throughout that $R' \geq 0$. Here we describe the role of this assumption and how the equilibrium changes when it is relaxed. Note that there is nothing pathological about a strictly decreasing b.[29] For example, in the M&A setting, $R(\theta)$ is a measure of the synergies in a merger, which can be higher or lower for high types. Suppose that the buyer is acquiring the seller for access to a proprietary technology. θ measures how close the buyer is to the technological frontier. A higher θ would then increase the expected value of assets in place A and may even increase the total value V, but the marginal value of the seller's technology $V - A = b$ can be lower the closer the buyer is to the technological frontier.

So long as the non-degeneracy conditions in Assumption 1 hold, and the environment is upward-skimming, a b with decreasing portions does not change the equilibrium analysis, with one key exception: cross-subsidization across types may vanish. Even if $R(0) > c$—i.e., there is a gap at the bottom—one could have $R(1) = c$—i.e., there is no gap at the *top*. In such a case, per Theorem 1, we avoid a complete breakdown of trade, but there will be no final atom. Consider the SLC. Since $c = R(1) := \bar{S}(\alpha^f(1), 1)$, $\mathbb{E}[\bar{S}(\alpha^f(1), \theta)|\theta \in [k, 1]] < c$ for all k. In other words, no matter how many types the seller has screened, adverse selection is always severe enough that she prefers trading with the marginal type to trading with the remaining types. The ranking results in Section 4 also simplify considerably: if $R(0) > R(1) = c$, for any two upward-skimming securities ranked by steepness, it follows that all types trade faster with the flatter security, at the same price as with the steeper security, and (naturally) they are better off for it. The non-monotonicities caused by cross-subsidization therefore vanish.

We can contrast this $R(1) = c$ case to the examples in Section 5. There we showed that flatter securities could increase bargaining frictions, raising discounting costs for some types and even raising expected delay for practically all types. Altogether, the dynamics depend not only on whether there is a gap at the bottom ($R(0) > c$), but whether there is a gap at the top ($R(1) > c$).

References

Alshwer, Abdullah A, Valeriy Sibilkov, and Nataliya S Zaiats (2011), "Financial constraints and the method of payment in mergers and acquisitions." *Available at SSRN 1364455.*

[29]We are grateful to Brett Green for suggesting this possibility.

Berk, Jonathan and Peter DeMarzo (2013), *Corporate Finance*, 3rd edition. Pearson Series in Finance.

Chaves, Isaías N. (2019), "Privacy in Bargaining: The Case of Endogenous Entry." Working paper, Northwestern University, Kellogg School of Management.

Che, Yeon-Koo and Jinwoo Kim (2010), "Bidding with Securities: Comment." *American Economic Review*, 100, 1929–35.

Coddington, Earl A. and Norman Levinson (1955), *Theory of Ordinary Differential Equations*. McGraw-Hill Book Company, Inc.

Daley, Brendan and Brett Green (2020), "Bargaining and News." *American Economic Review*, 110, 428–74.

DeMarzo, Peter and Darrell Duffie (1999), "A Liquidity-based Model of Security Design." *Econometrica*, 67, 65–99.

DeMarzo, Peter M, Ilan Kremer, and Andrzej Skrzypacz (2005), "Bidding with Securities: Auctions and Security Design." *American Economic Review*, 95, 936–959.

Deneckere, Raymond and Meng-Yu Liang (2006), "Bargaining With Interdependent Values." *Econometrica*, 74, 1309–1364.

Edlin, Aaron S and Chris Shannon (1998), "Strict Monotonicity in Comparative Statics." *Journal of Economic Theory*, 81, 201–219.

Fanning, Jack (2016), "Mediation in reputational bargaining." Working Paper, Brown University.

Fishman, Michael J (1989), "Preemptive Bidding and the Role of the Medium of Exchange in Acquisitions." *Journal of Finance*, 44, 41–57.

Fuchs, William and Andrzej Skrzypacz (2010), "Bargaining With Arrival Of New Traders." *American Economic Review*, 100, 802–836.

Fudenberg, Drew, David K. Levine, and Jean Tirole (1985), "Infinite-Horizon Models of Bargaining With One-Sided Incomplete Information." In *Game-theoretic Models of Bargaining* (Alvin E. Roth, ed.), 73–98, Cambridge University Press, Cambridge, UK.

Fudenberg, Drew and Jean Tirole (1985), "Preemption and rent equalization in the adoption of new technology." *Review of Economic Studies*, 52, 383–401.

Gordon, Robert D (1941), "Values of Mills' Ratio of Area to Bounding Ordinate and of the Normal Probability Integral for Large Values of the Argument." *Annals of Mathematical Statistics*, 12, 364–366.

Gul, Faruk, Hugo Sonnenschein, and Robert Wilson (1986), "Foundations of dynamic Monopoly and the Coase Conjecture." *Journal of Economic Theory*, 39, 155–190.

Hanazono, Makoto and Yasutora Watanabe (2018), "Equity bargaining with common value." *Economic Theory*, 65, 251–292.

Hansen, Robert G (1985), "Auctions With Contingent Payments." *American Economic Review*, 75, 862–865.

Hansen, Robert G (1987), "A theory for the choice of exchange medium in mergers and acquisitions." *Journal of Business*, 75–95.

Lam, Wing Tung (2020), "Inefficient sorting under output sharing." *Journal of Economic Theory*, 105031.

Malmendier, Ulrike, Marcus M Opp, and Farzad Saidi (2016), "Target revaluation after failed takeover attempts: Cash versus stock." *Journal of Financial Economics*, 119, 92–106.

Milgrom, Paul (2004), *Putting Auction Theory to Work*. Cambridge University Press, Cambridge.

Milgrom, Paul and Chris Shannon (1994), "Monotone Comparative Statics." *Econometrica*, 62, 157–157.

Milgrom, Paul R. and Robert J. Weber (1982), "A Theory of Auctions and Competitive Bidding." *Econometrica*, 1089–1122.

Myers, Stewart C. and Nicholas S. Majluf (1984), "Corporate Financing and Investment Decisions When Firms Have Information That Investors do not Have." *Journal of Financial Economics*, 13, 187–221.

Ortner, Juan (2017), "Durable Goods Monopoly with Stochastic Costs." *Theoretical Economics*, 12, 817–861.

Rhodes-Kropf, Matthew and S. Viswanathan (2004), "Market valuation and merger waves." *Journal of Finance*, 59, 2685–2718.

Smith, Lones and Ennio Stacchetti (2002), "Aspirational bargaining." Working Paper, NYU.

Strulovici, Bruno (2017), "Contract Negotiation and the Coase Conjecture: A Strategic Foundation for Renegotiation-Proof Contracts." *Econometrica*, 85, 585–616.

Teschl, Gerald (2012), *Ordinary Differential Equations and Dynamical Systems*, volume 140. American Mathematical Society.

Uysal, Vahap B (2011), "Deviation From the Target Capital Structure and

Acquisition Choices." *Journal of Financial Economics*, 102, 602–620.

White, Michelle J (1989), "The corporate bankruptcy decision." *Journal of Economic Perspectives*, 3, 129–151.

Appendix

Preliminaries

We begin with a key technical lemma characterizing the direction of skimming as a function of the primitives and the security family:

Lemma 2 (Direction of skimming)**.** *Fix a security family S. Let ι^S be as in* (4)*, U be given by*

$$U(t, \alpha, \theta) := (1 - e^{-rt})A(\theta) + e^{-rt}(V(\theta) - \bar{S}(\alpha, \theta)),$$

and let $\alpha^f(\cdot)$ be as in (3)*.*

Fix an arbitrary (deterministic) sequence of offers $\{\tilde{\alpha}_t\}_{t\geq 0}$, and let $T(\theta) := \arg\max_{t\in\mathbb{R}_+\cup\{+\infty\}} U(t, \tilde{\alpha}_t, \theta)$.

1. *If $\iota^S(\cdot, \alpha)$ is strictly increasing for all α, every selection from $T(\theta)$ is non-decreasing,*[30] *and $\alpha^f(\cdot)$ is strictly decreasing.*
2. *If $\iota^S(\cdot, \alpha)$ is strictly decreasing for all α, every selection from $T(\theta)$ is non-increasing, and $\alpha^f(\cdot)$ is strictly increasing.*

Remark 3 (Relation to Usual Skimming Notions)**.** Lemma 2 is weaker than the usual skimming result invoked in the literature on cash bargaining, so our focus on Markovian skimming equilibria is a stronger restriction than the analogous restriction in models with cash bargaining. To highlight the differences, focus on the downward-skimming case. In the literature on Coasean bargaining with cash, if a type θ is indifferent between accepting and rejecting an offer p after a history H_t, then all types $\theta' > \theta$ strictly prefer to accept p at H_t regardless of continuation play after the rejection. In contrast, the present lemma covers only deterministic offer paths, and it allows for offer histories where both θ and θ' are indifferent between accepting and rejecting. Hence, the Lemma does not entirely rule out histories in which θ' accepts strictly earlier—for example, if θ and θ' are both indifferent and randomize over their acceptance decisions.

However, Lemma 2 shows, for example, that when $\iota^S_\theta(\theta, \alpha) > 0$ it is be fruitless to search for skimming equilibria where higher types accept first with certainty, and so it guides the search for tractable equilibria. And even though Lemma 2 only covers deterministic offer paths, the stochasticity of equilibrium offers is such that, with some additional arguments, we can use the lemma to verify incentive compatibility for the buyer.

[30] In the usual order on the extended real line.

Proof of Lemma 2. We prove the case of strictly increasing $\iota^S(\cdot, \alpha)$ (the argument for an decreasing $\iota^S(\cdot, \alpha)$ is symmetric). The statements on α^f follow by implicit differentiation: since α^f solves $R(\theta) = \bar{S}(\alpha^f(\theta), \theta)$,

$$\frac{\partial}{\partial v}\alpha^f(\theta) = \frac{R'(\theta) - \bar{S}_\theta(\alpha^f(\theta), \theta)}{\bar{S}_\alpha(\alpha^f(\theta), \theta)} - \overbrace{\left[R(\theta) - \bar{S}(\alpha^f(\theta), \theta)\right]}^{=0} \frac{\bar{S}_{\alpha\theta}(\alpha^f(\theta), \theta)}{\bar{S}_\alpha(\alpha^f(\theta), \theta)^2}$$
$$= -\frac{\partial}{\partial \theta}\iota^S(\theta, \alpha^f(\theta)) < 0 \quad (20)$$

We separate the statement on $T(\theta)$ into two claims:

1. First, we show that selections from $\arg\max_{t\in\mathbb{R}_+} U(t, \tilde{\alpha}_t, \theta)$ are non-decreasing.
2. Then we show that if $t = +\infty \in T(\underline{\theta})$, then for any $\bar{\theta} > \underline{\theta}$, $T(\bar{\theta}) = \{+\infty\}$. Formally, if $\sup_{t\in R_+} U(t, \tilde{\alpha}, \underline{\theta}) = A(\underline{\theta})$, then $\sup_{t\in R_+} U(t, \tilde{\alpha}, \bar{\theta}) = A(\bar{\theta})$ and $U(t, \tilde{\alpha}_t, \bar{\theta}) < A(\bar{\theta})$ for all $t \in \mathbb{R}_+$.

Claim 1: The key step is an argument by Milgrom and Shannon (1994) and Edlin and Shannon (1998). Edlin and Shannon (1998)'s Theorem 2 has additional conditions that are violated in our setting, but which are only necessary to derive their conclusions on strict comparative statics. For completeness, we reproduce here the part of the argument that suffices for our purposes:

Definition 10. For $U : \mathbb{R}_+ \times [\underline{\alpha}, \bar{\alpha}] \times [0, 1] \to \mathbb{R}$, U satisfies the strict Spence-Mirrlees condition in $((t, \alpha), \theta)$ if if U is C^1, $U_t/|U_\alpha|$ is strictly increasing in t, and $U_\alpha \neq 0$ and has a constant sign.

Lemma 3 (Adapted from Theorem 2 in Edlin and Shannon (1998))**.** *Assume $U : \mathbb{R}_+ \times [\underline{\alpha}, \bar{\alpha}] \times [0, 1]$ satisfies the strict Spence-Mirrlees condition and has path-connected indifference sets. Then every selection from $\arg\max_{t\in\mathbb{R}_+} U(t, \tilde{\alpha}_t, \theta)$ is non-decreasing.*

Proof. By Theorem 3 in Milgrom and Shannon (1994), U is strictly single crossing in $((t, \alpha); \theta)$, where $\mathbb{R}_+ \times [0, 1]$ is endowed with the lexicographic order. With that order on $\mathbb{R}_+ \times [\underline{\alpha}, \bar{\alpha}]$, U is quasisupermodular in (t, α) and the set $\{(t, \alpha) : \alpha = \hat{\alpha}_t\}$ is a sublattice of $\mathbb{R}_+ \times [0, 1]$. The result then follows by Theorem 4' in Milgrom and Shannon (1994). □

The Spence-Mirrlees condition follows from simple calculus: using $U_\alpha = -e^{-rt}\bar{S}_\alpha < 0$,

$$\frac{U_t}{|U_\alpha|} = -\frac{r(R(\theta) - \bar{S}(\alpha, \theta))}{\bar{S}_\alpha(\alpha, \theta)} = r\iota^S(\alpha, \theta). \quad (21)$$

so U satisfies the strict Spence-Mirrlees condition whenever $\iota^S(\cdot, \theta)$ is strictly increasing.

To show complete regularity of U, fix θ and $\underline{t} < \bar{t}$ and $\underline{\alpha}, \bar{\alpha}$ such that $U(\bar{t}, \bar{\alpha}, \theta) = U(\underline{t}, \underline{\alpha}, \theta) = \bar{u}$. We construct a continuous function $\tilde{\alpha} : [\underline{t}, \bar{t}] \to [0,1]$ satisfying $\tilde{\alpha}(\underline{t}) = \underline{\alpha}$, $\tilde{\alpha}(\bar{t}) = \bar{\alpha}$, and $U(t, \tilde{\alpha}(t), \theta) = \bar{u}$.

If $\bar{u} = A(\theta)$, then $\bar{\alpha} = \underline{\alpha} = \alpha^f(\theta)$ and $U(t, \alpha^f(\theta), \theta)$ is constant in t; setting a constant $\tilde{\alpha}(t) = \alpha^f(\theta)$ trivially suffices. Focus then on $\bar{u} > A(\theta)$; the proof for $\bar{u} < A(\theta)$ is symmetric. For all (t', α') such that $U(t', \alpha', \theta) = \bar{u}$, $R(\theta) - \bar{S}(\alpha', \theta) > 0$ and therefore $U_t(t', \alpha', \theta) = re^{-rt'}(R(\theta) - \bar{S}(\alpha', \theta)) > 0$. By the Implicit Function Theorem, since $U_\alpha < 0$, for any $t_0 \in [\underline{t}, \bar{t}]$, there exists some open neighborhood $\mathcal{O} \subset [\underline{t}, \bar{t}]$ with $t_0 \in \mathcal{O}$ and a $C^1(\mathcal{O})$ function $\tilde{\alpha} : \mathcal{O} \to [0,1]$ satisfying

$$\tilde{\alpha}'(t) = -\frac{U_t(t, \tilde{\alpha}(t), \theta)}{U_\alpha(t, \tilde{\alpha}(t), \theta)}, t \in \mathcal{O}, \quad U(t_0, \tilde{\alpha}(t_0), \theta) = \bar{u},$$

i.e., $\tilde{\alpha}$ is in fact a local solution to an initial value problem.[31]

We extend the solution to the IVP above to yield the desired function $\tilde{\alpha}$. Take the open domain $\mathcal{D} = (\underline{t}, \bar{t}) \times (\underline{\alpha}, \bar{\alpha})$. We only show how to extend $\tilde{\alpha}$ continuously rightward up to $\bar{t}$, since extending it leftward to $\underline{t}$ is done symmetrically. Since U is C^1, U_t is bounded above and below on $\mathcal{D}$, and $U_\alpha < 0$, $g(t, \alpha) := -U_t(t, \alpha, \theta)/U_\alpha(t, \alpha, \theta)$ is continuous and bounded on $\mathcal{D}$. By standard extension theorems (Lemma 2.14 in Teschl (2012) and Theorem 4.1 in Coddington and Levinson (1955)), either $\tilde{\alpha}$ can be extended rightwards inside $\mathcal{D}$ to all of $[t_0, \bar{t})$, or there exists some $t' \in (t_0, \bar{t}]$ such that $\tilde{\alpha}$ extends rightwards up to $[t, t')$ with $\tilde{\alpha}(t') = \bar{\alpha}$.[32] If $\tilde{\alpha}$ can be extended rightwards to all of $[t_0, \bar{t})$, then by the continuity of U, $\tilde{\alpha}(\bar{t}-) = \bar{\alpha}$, and we are done.

Suppose, then, that $\tilde{\alpha}$ cannot be extended rightwards to all of $[t_0, \bar{t})$, so there exists some $t' \leq \bar{t}$ with $\tilde{a}(t') = \bar{\alpha}$ as above. If $t' = \bar{t}$, we are done, so focus on the remaining case $t' < \bar{t}$. Since $U(t', \bar{\alpha}, \theta) = \bar{u} = U(\bar{t}, \bar{\alpha}, \theta)$, by Rolle's theorem there exists some $t'' \in (t', \bar{t})$ such that $U_t(t'', \bar{\alpha}, \theta) = 0$. That would require $\bar{\alpha} = \alpha^f(\theta)$, a contradiction to $\bar{u} > A(\theta)$.

Claim 2: If $\sup_{t \in R_+} U(t, \tilde{\alpha}, \underline{\theta}) = A(\underline{\theta})$, so that $t = +\infty$ achieves that supremum, it must be that, for all $t \in \mathbb{R}_+$, $R(\underline{\theta}) \leq \bar{S}(\tilde{\alpha}_t, \underline{\theta})$. But then, using $\bar{S}_\alpha < 0$, it follows that, for all $t \in \mathbb{R}_+$,

$$\tilde{\alpha}_t \geq \alpha^f(\underline{\theta}) > \alpha^f(\bar{\theta}).$$

where the strict inequality was shown in (20). Therefore, using $\bar{S}_\alpha < 0$ and

[31] Since $U_\alpha < 0$, one can solve for $\tilde{\alpha}(t_0)$ in $U(t_0, \tilde{\alpha}(t_0), \theta) = \bar{u}$.

[32] To be precise, $\tilde{\alpha}$ extends up to $[t, t')$ and $\tilde{\alpha}(t'-) = \bar{\alpha}$.

$R(\bar{\theta}) = \bar{S}(\alpha^f(\bar{\theta}), \theta)$, we have that, for all $t \in \mathbb{R}_+$,

$$R(\bar{\theta}) < \bar{S}(\tilde{\alpha}_t, \bar{\theta}) \Rightarrow U(t, \tilde{\alpha}_t, \bar{\theta}) < A(\bar{\theta}) \text{ and } \sup_{t \in R_+} U(t, \tilde{\alpha}, \bar{\theta}) = A(\bar{\theta})$$

We conclude that $\arg\max_{t \in \mathbb{R}_+ \cup \{+\infty\}} U(t, \tilde{\alpha}_t, \bar{\theta}) = \{+\infty\}$, as required.

□

A Proofs for Section 4

Throughout this section, S^1 and S^2 refer to two order securities in $\mathcal{D}_{\tilde{R},\tilde{A},c}$, with S^1 steeper. An outline for this section is as follows. The ranking of critical types $k^{SLC}(S^i)$ was proved in the text (Proposition 1.1). With that in hand, we prove Proposition 2, which ranks equilibrium expected payments by type. After deriving an envelope-like representation of equilibrium payoffs (Lemma 4), we prove Proposition 3, our ranking on equilibrium utilities by type. Finally, with the ranking on equilibrium utilities, we return to the issue of certainty-equivalent delay and prove Proposition 1.2, which gives ranks delay for types outside an intermediate region.

Proof of Proposition 2. The first line of (15) follows from smooth trading: $\pi_i(\theta) = \bar{S}^i(\alpha^i(\theta), \theta) = c$ on $[0, k^{SLC}(S^2))$. To show the latter two lines, for $\theta \in (k^{SLC}(S^1), 1)$, the equilibrium payment under S^i satisfies $\pi_i(\theta) = \bar{S}^i(\alpha^{f,i}(1), \theta)$; the inequality in (13) then becomes

$$\pi_1(\theta) < \pi_2(\theta), \theta \in \left(k^{SLC}(S^1), 1\right).$$

Meanwhile, by definition $k^{SLC}(S^2)$ must satisfy

$$\mathbb{E}[\bar{S}^2(\alpha^{f,2}(1), \theta) | \theta \in (k^{SLC}(S^2), 1]] = c,$$

so there exists a $k^* \in (k^{SLC}(S^2), 1)$ such that

$$\bar{S}^2(\alpha^{f,2}(1), \theta) \begin{cases} < c, \theta \in (k^{SLC}(S^2), k^*), \\ = c, \theta = k^* \\ > c, \theta \in (k^*, 1] \end{cases} \tag{22}$$

Since $\pi_1(\theta) = c$ for $\theta \leq k^{SLC}(S^1)$, if $k^* < k^{SLC}(S^1)$, then (13) and (22) yield $k^{cross} = k^*$. On the other hand, if $k^* \geq k^{SLC}(S^1)$, (13) and (22) yield $k^{cross} = k^{SLC}(S^1)$. □

The payoff ranking relies on the following convenient representation of equilibrium payoffs:

Lemma 4. *Let $U_i(\theta)$ be the equilibrium indirect utility under an upward-skimming security S^i, and let α^i be the associated equilibrium offer. Define*

$$\nu^i(\theta,\alpha) := \frac{R'(\theta) - \bar{S}^i_\theta(\alpha,\theta)}{R(\theta) - \bar{S}^i(\alpha,\theta)}$$

Then for $\theta < 1$,

$$U_i(\theta) = A(\theta) + (R(0) - c)\exp\left\{\int_0^v \nu^i(y, \alpha^i(y))dy\right\}. \tag{23}$$

Proof. Equilibrium payoffs are

$$U_i(\theta) = A(\theta) + e^{-r\tau_i^{CE}(\theta)}\left(R(\theta) - \bar{S}^i(\alpha^i(\theta),\theta)\right), \tag{24}$$

and, by the envelope theorem, their derivative is

$$U_i'(\theta) = A'(\theta) + e^{-r\tau_i^{CE}(\theta)}\left(R'(\theta) - \bar{S}^i_\theta(\alpha^i(\theta),\theta)\right) \tag{25}$$

almost everywhere. Using equation (24) to substitute $e^{-r\tau_i^{CE}(\theta)}$ into the envelope condition (25) we get[33]

$$\frac{U_i'(\theta) - A'(\theta)}{U_i(\theta) - A(\theta)} = \frac{\partial}{\partial v}\log\left(U_i(\theta) - A(\theta)\right) = \frac{R'(\theta) - \bar{S}^i_\theta(\alpha^i(\theta),\theta)}{R(\theta) - \bar{S}^i(\alpha^i(\theta),\theta)}$$

almost everywhere. Integrating with respect to θ yields (23). □

With (25), we can prove Proposition 3. We split the proof in two parts.

Proof of Proposition 3.1. Let $\tilde{\alpha}^i(\theta)$ denote the locus $\bar{S}^i(\tilde{\alpha}^i(\theta),\theta) = c$, and $\alpha^i(\theta)$ denote the equilibrium offer accepted by type θ under security i. Using Propositions 1 and 2, we have

$$\begin{aligned} &\alpha^1(\theta) = \tilde{\alpha}^1(\theta), \theta \in [0, k^{cross}] \\ &\alpha^2(\theta)\begin{cases} = \tilde{\alpha}^2(\theta), \theta \in [0, k^{SLC}(S^2)] \\ < \tilde{\alpha}^2(\theta), \theta \in (k^{SLC}(S^2), k^{cross}) \end{cases} \end{aligned} \tag{26}$$

In addition, let $U_i(\theta)$ denote θ's equilibrium expected utility when bargaining with security S^i.

Since S^1 is steeper,

$$\nu^1(\theta,\tilde{\alpha}^1(\theta)) = \frac{R'(\theta) - \bar{S}^1_\theta(\tilde{\alpha}^1(\theta),\theta)}{R(\theta) - \bar{S}^1(\tilde{\alpha}^1(\theta),\theta)} < \frac{R'(\theta) - \bar{S}^2_\theta(\tilde{\alpha}^2(\theta),\theta)}{R(\theta) - \bar{S}^2(\tilde{\alpha}^2(\theta),\theta)} = \nu^2(\theta,\tilde{\alpha}^2(\theta)).$$

Integrating with respect to θ for $\theta < k^{cross}$, and using (26), one obtains

[33] We know from Theorem 1 that $R(\theta) > \bar{S}^i(\alpha^i(\theta),\theta)$ for all $\theta < 1$.

$$
\begin{aligned}
U_1(\theta) = A(\theta) + (R(0) - c)e^{\int_0^v \nu^1(y,\alpha^1(y))dy} &= A(\theta) + (R(0) - c)e^{\int_0^v \nu^1(y,\tilde{\alpha}^1(y))dy} \\
&< A(\theta) + (R(0) - c)e^{\int_0^v \nu^2(y,\tilde{\alpha}^2(y))dy} \\
&< A(\theta) + (R(0) - c)e^{\int_0^v \nu^2(y,\alpha^2(y))dy} = U_2(\theta),
\end{aligned}
$$

where the second inequality uses

$$
\begin{aligned}
\frac{\partial}{\partial \alpha}\nu^2(\theta,\alpha) \propto \frac{\partial}{\partial \alpha}\left(R'(\theta) - \bar{S}^2_\theta(\alpha,\theta)\right)\left[R(\theta) - \bar{S}^2(\alpha,\theta)\right] + \bar{S}^2_\alpha(\alpha,\theta)\left[R'(\theta) - \bar{S}^2_\theta(\alpha,\theta)\right] \\
= -\frac{\partial}{\partial \theta}\iota^{S^2}(\theta,\alpha) < 0 \quad (27)
\end{aligned}
$$

□

To prove the second part of Proposition 3, we need a quick technical lemma:

Lemma 5. *Let $S(\cdot,\cdot,\gamma)$ be a security belonging to a parametrized steepness class. Let $k^{SLC}(\gamma)$ be the critical type under security $S(\cdot,\cdot;\gamma)$, and let $k^*(\gamma)$ solve $\bar{S}(\alpha^f(1;\gamma), k^*(\gamma);\gamma) = c$, where $\alpha^f(1;\gamma)$ is final offer under $S(\cdot,\cdot;\gamma)$. For $\hat{\gamma} > \gamma$ sufficiently close to γ, $k^*(\gamma) > k^{SLC}(\hat{\gamma})$.*

Proof. By definition, $\mathbb{E}[\bar{S}(\alpha^f(1;\gamma), v;\gamma)|\theta \in [k^{SLC}(\gamma), 1]] = c = \bar{S}(\alpha^f(1;\gamma), k^*(\gamma))$, so we must have $k^*(\gamma) > k^{SLC}(\gamma)$. The lemma then follows by the continuity of $k^{SLC}(\gamma)$, which we now show. Let $\alpha^f(1;\gamma)$ denote the final (pooling) offer under security $S(\cdot,\cdot;\gamma)$. By the continuity of $\bar{S}(\cdot, v;\cdot)$, $\alpha^f(1;\cdot)$ is continuous, using the inverse function theorem. The critical type $k^{SLC}(\gamma)$ solves

$$
\int_k^1 \bar{S}(\alpha^f(1;\gamma), v, \gamma)d\theta - (1-k)c = 0.
$$

with respect to k. By the continuity of $\bar{S}$ and $\alpha^f(1;\cdot)$, $k^{SLC}(\cdot)$ is also continuous. □

Proof of Proposition 3.2. By Lemma 5, $k^*(\gamma) > k^{SLC}(\hat{\gamma})$ for all $\hat{\gamma} > \gamma$ sufficiently close to γ, $k^*(\gamma) > k^{SLC}(\hat{\gamma})$. By Proposition 3.1, buyer types in $[0, k^*(\gamma)]$ must then prefer $S(\cdot,\cdot;\gamma)$ to $S(\cdot,\cdot;\hat{\gamma})$, strictly so for $\theta > 0$.

Using the payoff representation in Lemma 4, we can write the indirect utility for type $\theta \in (k^*(\gamma), 1)$ under security $S(\cdot,\cdot;\gamma^\dagger)$, $\gamma^\dagger \in \{\gamma, \hat{\gamma}\}$ as

$$
U(\theta;\gamma^\dagger) = A(\theta) + U(k^*(\gamma);\gamma^\dagger)\exp\left\{\int_{k^*(\gamma)}^v \nu(y, \alpha^f(1;\gamma^\dagger);\gamma^\dagger)dy\right\}. \qquad (28)
$$

It follows that from the assumption that $\nu(\theta, \alpha^f(1;\gamma);\gamma)$ is decreasing in γ, and from $U(k^*(\gamma);\gamma) > U(k^*(\gamma);\hat{\gamma})$, that

$$U(k^*(\gamma);\gamma)\exp\left\{\int_{k^*(\gamma)}^{v}\nu(y,\alpha^{\dagger}(1;\gamma);\gamma)dy\right\}\geq$$
$$U(k^*(\gamma);\hat{\gamma})\exp\left\{\int_{k^*(\gamma)}^{v}\nu(y,\alpha^{\dagger}(1;\hat{\gamma});\hat{\gamma})dy\right\}$$

Plugging this into the representation (28) above, $U(\theta,\gamma)\geq U(\theta,\hat{\gamma})$ for all $\theta\in[0,1]$, and strictly so for interior θ's. Since $\gamma\in[\gamma',\gamma'']$ was arbitrary, we conclude that for any $U(\theta,\gamma)$ is a weakly decreasing function of γ, and strictly decreasing for interior θ's. □

Proof of Proposition 1.2. Let $\tau^{CE}(1;\gamma)$ denote the certainty-equivalent delay for $\theta=1$ under security $S(\cdot,\cdot;\gamma)$; we show that, for all $\gamma\in[\gamma_2,\gamma_1]$, $\tau^{CE}(1;\gamma)$ is strictly increasing in γ, which implies the result.

By Lemma 5, for $\hat{\gamma}>\gamma$ sufficiently close to γ, $k^*(\gamma)>k^{SLC}(\hat{\gamma})$. By Proposition 3.1, for small enough $\xi>0$, buyer $k^{SLC}(\hat{\gamma})+\xi$ must then strictly prefer $S(\cdot,\cdot;\gamma)$ to $S(\cdot,\cdot;\hat{\gamma})$. And yet, by Proposition 2, in expectation $k^{SLC}(\hat{\gamma})+\xi$ pays strictly less under $S(\cdot,\cdot;\gamma)$ than under $S(\cdot,\cdot;\hat{\gamma})$. It follows that $k^{SLC}(\hat{\gamma})+\xi$ must suffer strictly higher discounting costs under $S(\cdot,\cdot;\hat{\gamma})$. Since $k^{SLC}(\hat{\gamma})+\xi$ is in the final trading atom under $S(\cdot,\cdot;\gamma)$ *and* under $S(\cdot,\cdot;\hat{\gamma})$, $\tau^{CE}(1;\hat{\gamma})>\tau^{CE}(1;\gamma)$, as required. □

B Proofs for Section 5

Proof of Lemma 1.

Cash plus Equity: A minor calculation yields $\iota^{S^L}(\theta,\alpha)=-[R(\theta)-L]V(\theta)^{-1}+\alpha$. $\iota^{S^0}(\theta,\alpha)$ is increasing in θ for every α if and only iff b/A is strictly decreasing. Hence, by continuity, there exists some L_a small enough that, for all $L\leq L_a$, $\iota^{S^L}(\theta,\alpha)$ remains strictly increasing in θ for all α. Let $\alpha^f(1;L)=\frac{R(1)-L}{V(1)}$ denote the final offer that makes $\theta=1$ just indifferent under S^L. Using $\alpha^f(1;0)=\frac{R(1)}{V(1)}$, SLC holds under S^0 (equity bargaining) iff $\mathbb{E}[\tilde{V}]\frac{R(1)}{V(1)}<c$, while SLC holds under S^L iff

$$L\leq\frac{c\left[1-\frac{R(1)}{c}\frac{\mathbb{E}[\tilde{V}]}{V(1)}\right]}{1-\frac{\mathbb{E}[\tilde{V}]}{V(1)}}$$

Therefore, whenever SLC holds under S^0, there exists $L_b\in(0,c)$ such that SLC holds under S^L for all $L\leq L_b$. Taking $L^*=\min\{L_a,L_b\}$ concludes the proof.

Levered Equity: Next, we consider the situation in which the acquirer means of payment is equity over the merged entity, and the acquirer has outstanding debt. The question then is how the acquirers leverage impact the negotiation. We can derive the offer in the smooth trading region $\alpha(\theta)$ and the final offer $\alpha^f(1)$, which are given by

$$\alpha(\theta) = \frac{c}{\mathbb{E}[(Z-d)^+|\theta]}$$
$$\alpha^f(1) = \frac{R(1)}{\mathbb{E}[(Z-d)^+|1]}.$$

Using integration by parts we can write

$$\mathbb{E}[(Z-d)^+|\theta] = \int_d^\infty (1-G(x|\theta))dx.$$

With some abuse of notation we drop the subscript in $g_V(z|\theta)$. Denoting the cumulative density function of V conditional on $\theta \in [k,1]$ by $\bar{G}(z|k)$, which is given

$$\bar{G}(z|k) := \frac{1}{1-k}\int_k^1 G(z|\theta)d\theta].$$

We can directly characterize the conditions in terms of the cumulative density function $G(z|\theta)$. The threshold k^{SLC} is given by

$$k^{SLC} = \inf\left\{k \le 1 : \frac{\int_d^\infty \big(1-\bar{G}(z|k^{SLC})\big)dz}{\int_d^\infty \big(1-G(z|1)\big)dz} \ge \frac{c}{R(1)}\right\}.$$

The environment is upward skimming if

$$-\frac{\int_d^\infty G_\theta(z|\theta)dz}{\int_d^\infty \big(1-G(z|\theta)\big)dz} > \frac{R'(\theta)}{R(\theta)} \tag{29}$$

First, we verify that if the environment is upward skimming for d, it is also upward skimming for any $d' > d$. The left hand side of the upward skimming condition (29) can be written as

$$-\frac{\int_d^\infty G_\theta(x|\theta)dx}{\int_d^\infty (1-G(x|\theta))dx} = \frac{\partial}{\partial v}\log\int_d^\infty (1-G(x|\theta))dx.$$

After changing the order of differentiation, we get

$$-\frac{\partial}{\partial d}\frac{\int_d^\infty G_\theta(x|\theta)dx}{\int_d^\infty (1-G(x|\theta))dx} = -\frac{\partial}{\partial v}\frac{1-G(d|\theta)}{\int_d^\infty (1-G(x|\theta))dx} > 0, \tag{30}$$

where the inequality follows as, for all $z > \theta$, the ratio $\frac{1-G(z|\theta)}{1-G(d|\theta)}$ is increasing in θ by the monotone likelihood ratio property.

Next, we need to verify that if the lemons condition is satisfied for d, then it is also satisfied for $d' > d$. Next, we verify that if the lemon's condition is satisfied for d, then it is also satisfied for $d > d'$. The lemons condition can be written as

$$\frac{\int_d^\infty \big(1-\bar{G}(z|0)\big)dz}{\int_d^\infty \big(1-G(z|1)\big)dz} < \frac{c}{R(1)} \qquad \Longleftrightarrow$$
$$0 < R(1)\int_d^\infty \big(1-\bar{G}(z|0)\big)dz - c\int_d^\infty \big(1-G(z|1)\big)dz.$$

Letting

$$\psi(d) := R(1)\int_d^\infty \big(1-\bar{G}(z|0)\big)dz - c\int_d^\infty \big(1-G(z|1)\big)dz$$

we get

$$\psi'(d) = (1-G(d|1))\left[R(1)\left(\frac{1-\bar{G}(d|0)}{1-G(d|1)}\right) - c\right]$$

Notice that

$$R(1)\left(\frac{1-\bar{G}(\underline{z}|0)}{1-G(\underline{z}|1)}\right) - c = R(1) - c > 0;$$

thus, it is enough to show that $\frac{1-\bar{G}(d|0)}{1-G(d|1)}$ is increasing in d. Differentiating with respect to d we get

$$\frac{\partial}{\partial d}\frac{1-\bar{G}(d|0)}{1-G(d|1)} = \frac{1-\bar{G}(d|0)}{1-G(d|1)}\left[\frac{g(d|1)}{1-G(d|1)} - \frac{\bar{g}(d|0)}{1-\bar{G}(d|0)}\right],$$

which is positive as $G(z|1)$ dominates $\bar{G}(z|0)$ in the hazard rate order.

□

Proof of Proposition 4. Let $\hat{V}^d(\theta;\eta)$ denote the value of leveraged equity in footnote 22 for a given precision η. With this notation, $\bar{S}^d_{lev}(\alpha,\theta;\eta) = \alpha\hat{V}^d(\theta;\eta)$. Using the identity $\phi(x)x = -\phi'(x)$, we have

$$\hat{V}^d_\eta(\theta;\eta) = -\frac{1}{\eta}\int_d^\infty \phi'(\eta(c+\Delta+\zeta\theta-z))dz = -\frac{1}{\eta^2}\phi(\eta(c+\Delta+\zeta\theta-d))$$

where the second equality uses a change of variables and $\lim_{x\to-\infty}\phi(x) = 0$. Therefore,

$$\hat{V}^d_{\eta\theta} = = -\frac{\zeta}{\eta}\phi'(\eta(c+\Delta+\zeta\theta-d)).$$

The density ϕ is strictly single-peaked around 0, so for sufficiently large d, $\frac{\partial^2}{\partial\theta\partial\eta}\bar{S}^d_{lev}(\alpha,\theta) = \alpha\hat{V}^d_{\eta\theta} < 0$ for all θ, and the reverse holds for sufficiently small d. This proves the first statement.

To prove the second statement, we prove the stronger claim

$$\frac{\hat{V}^d_\theta}{\hat{V}^d} \text{ is strictly increasing in } \eta. \tag{$\star$}$$

which implies the result. Indeed, if the claim is true, then whenever $\bar{S}^d_{lev}(\alpha_1, \theta; \eta_1) = \bar{S}^d_{lev}(\alpha_2, \theta; \eta_2)$,

$$\frac{\hat{V}^d_\theta(\theta;\eta_1)}{\hat{V}^d(\theta;\eta_1)}\bar{S}^d_{lev}(\alpha_1,\theta;\eta_1) > \frac{\hat{V}^d_\theta(\theta;\eta_2)}{\hat{V}^d(\theta;\eta_2)}\bar{S}^d_{lev}(\alpha_1,\theta;\eta_1) = \frac{\hat{V}^d_\theta(\theta;\eta_2)}{\hat{V}^d(\theta;\eta_2)}\bar{S}^d_{lev}(\alpha_2,\theta;\eta_2)$$

The left hand side equals $\frac{\partial}{\partial\theta}\bar{S}^d_{lev}(\alpha_1,\theta;\eta_1)$ and the right hand side equals $\frac{\partial}{\partial\theta}\bar{S}^d_{lev}(\alpha_2,\theta;\eta_2)$, so (18) follows.

To prove ($\star$), we sign

$$\hat{V}^d_{\theta\eta}\hat{V}^d - \hat{V}^d_\theta\hat{V}^d_\eta. \tag{31}$$

which is proportional to $\frac{\partial}{\partial\eta}\frac{\hat{V}^d_\theta}{\hat{V}^d}$ by the quotient rule. First, simplify notation by labeling $\Gamma := \eta(c + \Delta + \zeta\theta - d)$ and $\mu := (c + \Delta + \zeta\theta)$. Note that μ is the mean of $\tilde{V}|\theta$. With this notation, $\hat{V}^d_\eta = -\phi(\Gamma)\eta^{-2}$, $\hat{V}^d_{\eta\theta} = \zeta\phi'(\Gamma)\eta^{-1}$. We can directly calculate that

$$\hat{V}^d_\theta(\theta;\eta) = \eta\zeta\int_d^\infty \phi(\eta(c+\Delta+\zeta\theta-z))dz = \zeta\Phi(\Gamma)$$

Next, using standard formulas for the moments of censored random variables and letting $h(\cdot) = \phi(\cdot)/(1-\Phi(\cdot))$ denote the inverse Mills ratio, we calculate $\hat{V}^d$:

$$\begin{aligned}
\hat{V}^d &= -d + \mathbb{E}[\max\{\tilde{V}, d\}|\theta] \\
&= -d + [1-\Phi(\eta(d-\mu))]\left[\mu + \frac{1}{\eta}h(\eta(d-\mu))\right] + \Phi(\eta(d-\mu))d \\
&= \frac{1}{\eta}[1-\Phi(-\Gamma)]\,[\Gamma + h(-\Gamma)] \qquad (32)
\end{aligned}$$

Plugging the expressions for $\hat{V}^d_{\eta\theta}$, $\hat{V}^d$, $\hat{V}^d_\eta$ and $\hat{V}^d_\theta$ into (31), we obtain a quantity proportional to

$$\begin{aligned}
-\phi'(\Gamma)\Phi(\Gamma)[\Gamma + h(-\Gamma)] + \Phi(\Gamma)\phi(\Gamma) &\propto -\phi'(\Gamma))[\Gamma + h(-\Gamma)] + \phi(\Gamma) \qquad (33)\\
&\propto \Gamma[\Gamma + h(-\Gamma)] + 1, \qquad (34)
\end{aligned}$$

where the last line uses the identify $\phi'(x) = -x\phi(x)$. If $\Gamma \geq 0$, we are done. Otherwise, if $\Gamma < 0$, we apply the following classic bound on the inverse Mills ratio (see Gordon (1941)):

$$h(x) < x + \frac{1}{x}, x > 0$$

to obtain

$$\Gamma\left[\Gamma + h(-\Gamma)\right] + 1 > \Gamma\left[\Gamma + \left(-\Gamma + \frac{1}{-\Gamma}\right)\right] + 1 = 0,$$

which proves that (31) is strictly positive.

To show that upward-skimming is preserved as η rises, note that, similar to an environment with unlevered equity, the environment is upward-skimming if

$$\frac{\hat{V}^d_\theta(\theta;\eta)}{\hat{V}^d(\theta;\eta)} > \frac{R'(\theta)}{R(\theta)}.$$

By ($\star$), it follows that if the environment is upward-skimming for η, it is upward-skimming for any $\eta' > \eta$.

The preservation of SLC as η rises also follows from ($\star$). Indeed, the SLC holds if

$$R(1)\int_0^1 \frac{\hat{V}^d(\theta;\eta)}{\hat{V}^d(1;\eta)} d\theta < c \tag{35}$$

where we have used the fact that the final offer is $R(1)/\hat{V}^d(1;\eta)$. From ($\star$) one has

$$0 < \frac{\partial}{\partial\eta}\frac{\hat{V}^d_\theta(\theta;\eta)}{\hat{V}^d(\theta;\eta)} = \frac{\partial^2}{\partial\eta\partial\theta}\log\hat{V}^d(\theta;\eta) = \frac{\partial}{\partial\theta}\frac{\hat{V}^d_\eta(\theta;\eta)}{\hat{V}^d(\theta;\eta)}.$$

Hence, for all $\theta \in [0,1]$,

$$\frac{\hat{V}^d_\eta(\theta;\eta)}{\hat{V}^d(\theta;\eta)} \le \frac{\hat{V}^d_\eta(1;\eta)}{\hat{V}^d(1;\eta)} \Rightarrow \frac{\partial}{\partial\eta}\frac{\hat{V}^d(\theta;\eta)}{\hat{V}^d(1;\eta)} \le 0.$$

The integrand in (35) therefore decreases as η increases, which concludes the proof.

Finally, the environment is upward-skimming if (omitting some arguments to reduce clutter)

$$\frac{\beta}{c+\Delta+\beta\theta} < \frac{\hat{V}^d_\theta(\theta)}{\hat{V}^d(\theta)} = \frac{\zeta\eta}{\eta(c+\Delta+\zeta\theta-d)+h(-\eta(c+\Delta+\zeta\theta-d))} \quad \forall\theta. \tag{36}$$

where we have substituted our previous expressions for $\hat{V}^d_\theta$ and $\hat{V}^d$. From Lemma 1, it suffices to satisfy (36) for $d = 0$, so we will have upward skimming if

$$\frac{\beta}{\beta+\chi} < \frac{\eta(c+\Delta+\beta\theta)}{\eta(c+\Delta+\zeta\theta)+h(-\eta(c+\Delta+\zeta\theta))}$$

Using the monotone hazard rate property of the normal distribution, a crude lower bound for the right hand side is

$$\frac{\eta(c+\Delta)}{\eta(c+\Delta)+\eta(\beta+\chi)+h(-\eta(c+\Delta))}.$$

Using, $h(0)=2/\sqrt{2\pi}$, an even cruder bound is

$$\frac{\eta(c+\Delta)}{\eta(c+\Delta)+\eta(\beta+\chi)+2/\sqrt{2\pi}}.$$

□

C Necessary Conditions

Proof of Theorem 1, Necessary Conditions.

Downward Skimming The proof for the downward skimming case proceeds by contradiction. We assume the existence of an equilibrium with smooth screening and show that the seller has incentive to accelerate trade as much as possible, which contradicts the optimality of smooth screening.

Since higher types trade first, the seller's beliefs are right-truncations of the prior, and the truncation cutoff k is the Markov state controlled by the seller. Let k' be a state in the interior of a smooth trade region. Then, for some $\varepsilon > 0$, the seller's HJB at all $k \in (k'-\varepsilon, k'+\varepsilon)$ is

$$rJ(k) = \sup_{\dot{k}\geq 0}\Big(\underbrace{\bar{S}(\alpha(k),k)}_{\mathbb{E}[S(\alpha(k),V)|k]} - J(k)\Big)\frac{|\dot{k}|}{k} - J'(k)|\dot{k}| + rc.$$

For $|\dot{k}| < \infty$ to be indeed optimal for the seller, it must be that for all $k \in (k'-\varepsilon, k+\varepsilon)$,

$$\mathbb{E}[S(\alpha(k),V)|k] \leq J(k) + J'(k)k \Rightarrow J(k) = c. \tag{37}$$

Consider first the case in which $|\dot{k}| \in (0,\infty)$ is optimal. Then the inequality above is an equality, and $\mathbb{E}[S(\alpha(k'),V)|k'] = J(k') = c$. In particular, $\alpha(\cdot)$ is C^1 in the region $(k'-\varepsilon, k'+\varepsilon)$. Differentiating both sides with respect to k',

$$\alpha'(k)\bar{S}_\alpha(\alpha(k),k) = -\bar{S}_\theta(\alpha(k),k) \tag{38}$$

Meanwhile, for buyer K_t in the interior of a smooth trade region to buy at t and not mimic any type in that region, the following local incentive constraint is necessary:

$$r(V(K_t) - \bar{S}(\alpha(K_t),K_t)) = rA(K_t) - \dot{K}_t\alpha'(K_t)\bar{S}_\alpha(\alpha(K_t),K_t).$$

Plugging in (38) and the seller's indifference condition (37), we obtain

$$\dot{K}_t = \frac{r(R(K_t) - c)}{\bar{S}_\theta(\alpha(K_t), K_t)}. \tag{39}$$

In particular, $\dot{K}_t \geq 0$, contradicting either forward skimming or the optimality of $|\dot{K}_t| \in (0, \infty)$ at time t for the seller.

If $\dot{k} = 0$ is (strictly) optimal for the seller on an interval of time with positive measure, by a simple dynamic programming argument, indefinitely ceasing trade is also optimal and $J(k) = c$. For no trade to be optimal, the reservation offer $\hat{\alpha}(k)$ of buyer k must satisfy $V(k) - \bar{S}(\hat{\alpha}(k), k) = A(k)$. In other words, $\hat{\alpha} = \alpha^f$, where the latter is defined in Definition 2. By the arguments in Lemma 1, $\hat{\alpha}(\cdot)$ must be weakly increasing. The seller can therefore offer $\alpha^f(0)$ to trade with all remaining types $[0, k]$ at prices of $\hat{\alpha}(0)$ (create an atom of trade of size k to obtain

$$\mathbb{E}[\bar{S}(\alpha^f(0), \theta)|\theta \leq k] > \bar{S}(\alpha^f(0), 0) = R(0) \geq c,$$

where the strict inequality follows from the nondegeneracy conditions in Assumption 1 and $\alpha^f(0) \neq \underline{\alpha}$.[34]

Altogether, in a downward skimming environment, there can be no smooth trade and no quiet periods. Given that jumps are well-separated in an regular equilibrium, this implies that the equilibrium must consist of a single atom of measure 1, i.e., instant trade.

Upward Skimming Higher types trade *later*, so the seller now controls the *left* truncation of her posterior beliefs as a Markov state.

As before, we first identify implications of smooth trading, which then guide the analysis of all possible dynamics. The HJB for k in the interior of a smooth trade region (in cutoff space) is now

$$rJ(k) = \sup_{\dot{k} \geq 0} \left(\bar{S}(\alpha(k), k) - J(k)\right) \frac{\dot{k}}{1 - k} + J'(k)\dot{k} + rc. \tag{40}$$

Therefore $J(k) = c$. If in addition, $\dot{k} \neq 0$ at such a state, $\bar{S}(\alpha(k), k) = c$.

Second, we show that if there is smooth trade, it happens only on a set of states $[0, k^{smooth})$, i.e., the games starts with smooth trading and ends with a jump. Suppose that, on some continuation game, there were a jump from k to $k' > k$. Since there are countably many jumps, and jumps are isolated,[35] smooth trade must recommence at k'. In particular, $J(k') = c$ and

[34] $\alpha^f(0)$ satisfies $\bar{S}(\alpha^f(0), 0) = R(0) \geq c$, so it must be greater than $\underline{\alpha}$.

[35] If jumps discontinuities were not isolated, then a jump from k to k^1 would be followed almost immediately by a jump from k^1 to k^2. However, this gives the types in $(k^1, k^2]$ no

$\mathbb{E}[S(\alpha(k'), \tilde{V})|v = k'] \leq c$. The seller's payoff from jumping to k' is therefore

$$\left(\frac{k' - k}{1 - k}\right) \mathbb{E}[\bar{S}(\alpha(k'), \theta)|\theta \in [k, k')] + \left(\frac{1 - k'}{1 - k}\right) c.$$

The seller can always freeze trade and ensure a payoff of c, so for such a jump to be optimal,

$$c \leq \mathbb{E}[\bar{S}(\alpha(k'), \theta)|\theta \in [k, k')] \leq \mathbb{E}[\bar{S}(\alpha(k'), \theta)|v = k'] \leq c.$$

If the second inequality were an equality, then $\alpha(k') = \underline{\alpha}$, by the nondegeneracy condition in Assumption 1. But that would contradict the last inequality, since $\bar{S}(\underline{\alpha}, k') < c$. The second inequality must therefore be strict, which is a contradiction. Therefore, the set of smooth trade states must be an interval $[0, k^{smooth})$.

Third, we show that $k^{smooth} = k^{SLC}$. By Condition 2 in the equilibrium definition, buyer $\theta = 1$'s equilibrium reservation price $\alpha(1)$ must be *equal* to $\alpha^f(1)$, the highest take-it-or-leave-it offer that he would acccept. Hence, if $k^{smooth} < k^{SLC}$, so that continuation play prescribes a jump before the state reaches k^{SLC}, then it is weakly optimal for the seller to jump directly to $k = 1$ at k^{smooth}. Her payoffs at k^{smooth} are therefore $\mathbb{E}[S(\alpha(1), \tilde{V})|\theta \in [k^{smooth}, 1]] < c$, by definition of k^{SLC}. This violates the seller's individual rationality, so we must have $k^{smooth} \geq k^{SLC}$. However, if $k^{smooth} > k^{SLC}$, then for any state $k \in (k^{SLC}, k^{smooth})$, the seller can jump the state to $\theta = 1$ with an offer of $\alpha^f(1)$; this gives her a payoff of $\mathbb{E}[S(\alpha^f(1), \tilde{V})|\theta \in [k, 1]] > c$, a profitable deviation. In particular, $k^{smooth} = k^{SLC}$ implies that, if the SLC fails, all equilibria have instant trade.

Fourth, we show that, for $k \in (0, k^{SLC})$, the optimal $\dot{k}$ in (40) must be strictly positive, i.e., there are no quiet periods. Suppose otherwise. By a typical dynamic programming argument, if starting at state $k \in (0, k^{SLC})$ we have $\dot{k} = 0$, then the continuation value of the marginal buyer equals $A(k)$, and $\alpha(k) = \alpha^f(k)$ as defined in Lemma 2. But then $\bar{S}(\alpha(k), k) = R(k) > c$, so (40) fails, a contradiction.

Finally, we show that if the SLC holds, but there is no gap, trade breaks down. Since the SLC holds, $k^{SLC} > 0$. By the previous point, The states $[k^{SLC}, 1]$ must be reached via smooth trade. By identical arguments to those in the downward skimming case, if trade is smooth at a positive speed, the speed of trade must satisfy (39). In particular, for a small enough but positive $\varepsilon > 0$, the unique smooth trade cutoff path that is locally incentive compatible

incentive to reject the offer that led to state k^1. Buyer optimality would therefore require all of $[k, k^2]$ to accept instantly. Hence, the distance between two jumps must be bounded away from zero.

for the buyer and the seller on $t \in [0, \varepsilon)$ solves the initial value problem

$$\dot{K}_t = \frac{r(R(K_t) - c)}{\bar{S}_\theta(\alpha(K_t), K_t)}, \quad K_t = 0,$$

i.e, $K_t = 0$ on $[0, \varepsilon)$.[36]

At $t = \varepsilon$, the situation replicates itself, so $k^{SLC} > 0$ is never reached. Again by previous arguments, the only remaining non-trivial trading dynamic involves instant trade. However, since the SLC holds, this is strictly suboptimal for the seller. $\square$

D Equilibrium Verification

Proof of Theorem 1: Equilibrium Verification. We present details for the case with non-trivial delayed trade, $S \in \mathcal{D}_{c,\tilde{R},\tilde{A}}$; the remaining cases are similar, but much simpler.

Verification: Seller's On-Path Strategies Now, we verify that the seller's choice of $\{K^k\}_{k \in [0,1]}$ and F are optimal, given the buyer's strategy

$$\alpha(k) = \begin{cases} \alpha^f(1) & \text{if } k \in (k^{SLC}, 1] \\ \bar{S}^{-1}(c, k) & \text{if } k \in [0, k^{SLC}), \end{cases} \tag{41}$$

where the inverse $\bar{S}^{-1}(c, k)$ is defined by $\bar{S}\big(\bar{S}^{-1}(c, k), k\big) = c$. From the previous, given $\alpha(k)$ in equation (41), seller's the continuation value is

$$J(k) = \begin{cases} c & \text{if } k \in [0, k^{SLC}] \\ \mathbb{E}[S(\alpha^f(1), \tilde{V}) | \theta \in [k, 1]] & \text{if } k \in (k^{SLC}, 1] \end{cases} \tag{42}$$

Notice that $J(\cdot)$ has a kink at k^{SLC} as

$$J'(k^{SLC}-) = 0 < J'(k^{SLC}+) = \frac{\partial}{\partial k} \mathbb{E}[S(\alpha^f(1), \tilde{V}) | \theta \in [k, 1]]\big|_{k=k^{SLC}}$$

The value function fails to be differentiable at at k^{SLC} due to the discontinuity in $\alpha(\cdot)$, Moreover, this implies that the HJB equation is discontinuous at this point. To avoid the technical complications associated to working with discontinuous HJB equations, and the theory of viscosity solutions, we take advantage that admissible cutoff polices are non-decreasing, and we split the verification of the optimal policies in two steps: First starting at $k_0 \in (k^{SLC}, 1]$, and then starting $k_0 \in [0, k^{SLC}]$.

[36]The right hand side is C^1, given the assumptions on primitives and the expression for $\alpha(\cdot)$.

Verification for $k_0 \in [k^{SLC}, 1]$. Let's ignore the fact that for $\alpha(K_t) = \alpha^f(1)$, all types accept the offer, and consider a relaxed formulation in which the seller is allowed to smoothly screen on $k_0 \in [k^{SLC}, 1]$. To simplify notation, we consider F which are absolutely continuous and let $\Lambda_t = \int_0^t \lambda(s)ds$. The seller's value function is

$$J(k_0) = \sup_{Q,\lambda} \int_0^\infty e^{-rt-\Lambda_t}\Big(\mathbb{E}\left[\bar{S}\left(\alpha(Q_t), v\right) \Big| \theta \in [Q_{t-}, Q_t]\right] \frac{dQ_t}{1-k_0}$$
$$+\lambda_t \mathbb{E}\left[\bar{S}\left(\alpha^f(1), v\right) \Big| \theta \in [Q_{t-}, 1]\right] \Big) + \left(1 - \int_0^\infty e^{-rt-\Lambda_t}\left(\frac{1-Q_t}{1-k_0}\lambda_t dt + \frac{dQ_t}{1-k_0}\right)\right) c.$$

Rather than working with the value function $J(\cdot)$, it is convenient to work with the equivalent value function $\bar{J}(k) \equiv (1-k)J(k)$, so

$$\bar{J}(k_0) = \sup_{Q,\lambda} \int_0^\infty e^{-rt-\Lambda_t}\Big(\mathbb{E}\left[\bar{S}\left(\alpha(Q_t), v\right) \Big| \theta \in [Q_{t-}, Q_t]\right] dQ_t$$
$$+\lambda_t (1-k_0) \mathbb{E}\left[\bar{S}\left(\alpha^f(1), v\right) \Big| \theta \in [Q_{t-}, 1]\right] \Big) + \left(1 - k_0 - \int_0^\infty e^{-rt-\Lambda_t}\big(\lambda_t(1-Q_t)dt + dQ_t\big)\right) c.$$

We conjecture, and then verify, that the value function $\bar{J}(\cdot)$ satisfies the quasi-variational inequality

$$0 = \max\Big\{ \sup_{\dot{k}\geq 0, \lambda \geq 0} \big(\bar{S}(\alpha(k), k) + \bar{J}'(k)\big)\dot{k} + \lambda\left(\int_k^1 \bar{S}\left(\alpha^f(1), v\right) d\theta] - \bar{J}(k)\right)$$
$$+ r(1-k)c - r\bar{J}(k)\,,\, \mathcal{M}\bar{J}(k) - \bar{J}(k)\Big\}, \quad (43)$$

where the operator $\mathcal{M}$ is defined by

$$\mathcal{M}\bar{J}(k) = \max_{k' \in [k,1]} \left\{(k'-k)\,\mathbb{E}[\bar{S}(\alpha(k'), \theta)|\theta \in [k, k']] + \bar{J}(k')\right\}$$

First we verify that $\bar{J}(k) = (1-k)J(k)$, where $J(k)$ is as in (42) satisfies this quasi-variational inequality. First, it is immediate to verify that $\bar{J}(k) = \mathcal{M}\bar{J}(k)$, so the second term of (43) is satisfied. For the first term, notice that

$$\big(\bar{S}(\alpha(k), k) + \bar{J}'(k)\big)\dot{k} + \lambda\left(\int_k^1 \bar{S}\left(\alpha^f(1), v\right) d\theta] - \bar{J}(k)\right) + r(1-k)c - r\bar{J}(k) \leq$$
$$\big(\bar{S}(\alpha(k), k) + \bar{J}'(k)\big)\dot{k} = 0,$$

where we have used that $\bar{J}'(k) = -\bar{S}(\alpha^f(1), k)$. From here on, the verification is standard. Consider an arbitrary admissible policy Q_t. Using the change of value formula, we get that

$$e^{-rt-\Lambda_t}\bar{J}(Q_t) = J(k_0)+\int_0^t e^{-rs-\Lambda_s}\left(\dot{q}_s\bar{J}'(Q_{s-}) + \lambda_s\left(\int_k^1 \bar{S}\left(\alpha^f(1),v\right)d\theta] - \bar{J}(Q_{s-})\right)\right.$$
$$\left.-r\bar{J}(Q_{s-})\right)ds + \sum_{s<t} e^{-rs-\Lambda_s}\left(\bar{J}(Q_{s-}+\Delta Q^d_{s-}) - \bar{J}(Q_{s-})\right)$$

From the quasi-variational inequality (43) we get that

$$\bar{J}(Q_s) - \bar{J}(Q_{s-}) \leq (Q_s - Q_{s-})\,\mathbb{E}[\bar{S}(\alpha(Q_s),\theta)|\theta \in [Q_{s-},Q_s]]$$

and the term in the integral is less than

$$-r(1-Q_s)c - \dot{q}_s\bar{S}(\alpha(Q_{s-}),Q_{s-}) - \lambda_s\int_k^1 \bar{S}\left(\alpha^f(1),v\right)d\theta].$$

It follows that

$$\begin{aligned}\bar{J}(k_0) \geq& \int_0^t e^{-rs-\Lambda_s}\left(r(1-Q_{s-})c + \dot{q}_s\bar{S}(\alpha(Q_{s-}),Q_{s-})\right.\\ &\left.+\lambda_s\int_k^1 \bar{S}\left(\alpha^f(1),v\right)d\theta]\right)ds\\ &\sum_{s<t} e^{-rs-\Lambda_s}\left(Q^d_s - Q^d_{s-}\right)\mathbb{E}[\bar{S}(\alpha(Q_s),\theta)|\theta\in[Q_{s-},Q_s]] + e^{-rt-\Lambda_t}\bar{J}(Q_t)\\ =& \int_0^t e^{-rt-\Lambda_t}\mathbb{E}\left[S\left(\alpha(Q_s),\tilde{V}\right)\middle|\theta\in[Q_{s-},Q_s]\right]dQ_s\\ &+\left(1-k_0-e^{-rt-\Lambda_t}(1-Q_t) - \int_0^t e^{-rs-\Lambda_s}\left((1-Q_s)\lambda_s + dQ_s\right)\right)c\\ &+e^{-rt-\Lambda_t}\left(\bar{J}(Q_t)-(1-Q_t)c\right),\end{aligned}$$

where the equality

$$1-k_0-e^{-rt-\Lambda_t}(1-Q_t)-\int_0^t e^{-rs-\Lambda_s}\left((1-Q_s)\lambda_s+dQ_s\right) = \int_0^t e^{-rs-\Lambda_s}r(1-Q_{s-})ds,$$

follows by integration by parts. Taking the limit when $t\to\infty$, we get that $\bar{J}(k_0)$ is an upper bound on the payoff that the seller can attain starting at any $k_0 \geq k^{SLC}$. Finally, because all the inequalities hold with equality in the case of equation for our conjecture policy K, it follows that K is optimal starting at $k_0 \in [k^{SLC},1]$.

Verification for $k_0 \in [0,k^{SLC})$. Using the previous characterization of the value function $\bar{J}(\cdot)$ on $[k^{SLC},1]$, by the principle of dynamic programming, we can state the optimization problem on $[0,k^{SLC})$, as

$$\bar{J}(k_0) = \sup_{Q} \int_0^{\tau(Q)} e^{-rt-\Lambda_t} \mathbb{E}\left[\bar{S}\left(\alpha(Q_t), v\right) \middle| \theta \in [Q_{t-}, Q_t]\right] dQ_t$$

$$+ \left(1 - k_0 - \int_0^{\tau(Q)} e^{-rt-\Lambda_t} \left(\lambda_t(1-Q_t) + dQ_t\right)\right) c + e^{-r\tau(Q)} \left(\bar{J}(Q_{\tau(Q)}) - (1-Q_{\tau(Q)})c\right).$$

where $\tau(Q) = \inf\{t > 0 : Q_t \geq k^{SLC}\}$. Notice that the factor $(1 - Q_{\tau(Q)})c$ is added to account for the constant $(1-k)c$ in the expected payoff. Once again, we conjecture that the value function $\bar{J}(\cdot)$ satisfies the quasi-variational inequality (43).

First, we can verify that $\bar{J}(\cdot)$ defined by (42) (multiplied by $1-k$) satisfies equation (43) on $[0, k^{SLC})$. By construction, $\bar{S}(\alpha(k), k) = \bar{J}'(k) = -c$. Also,

$$\mathcal{M}\bar{J}(k) - \bar{J}(k) = \max_{k' \in [k,1]} \left\{(k'-k)\,\mathbb{E}[S(\alpha(k'), \tilde{V})|\theta \in [k,k']] + \bar{J}(k')\right\} - (1-k)c < 0,$$

so $\max_{\lambda \geq 0}\{\lambda(\int_k^1 \bar{S}\left(\alpha^f(1), v\right) d\theta] - \bar{J}(k))\} = 0$. Thus, the first term of the variational inequality is equal to zero, and because $\mathcal{M}\bar{J}(k) - \bar{J}(k)) \leq 0$, the second term also satisfies the required inequality. It follows then that $\bar{J}(k) = (1-k)c$ is a solution of (43) on $[0, k^{SLC})$. Consider an arbitrary policy Q, so, once again, using the change of value formula we get that

$$\mathbb{E}^Q\left[e^{-rt\wedge\tau(Q)}\bar{J}(Q_{t\wedge\tau(Q)})\right] = J(k_0) + \int_0^{t\wedge\tau(Q)} e^{-rs}\left(\dot{q}_s\bar{J}'(Q_{s-}) + \lambda_s\left(\bar{J}(Q_{s-} + \Delta Q^s_{s-})\right.\right.$$
$$\left.\left.-\bar{J}(Q_{s-})\right) - r\bar{J}(Q_{s-})\right) ds + \sum_{s<t\wedge\tau(Q)} e^{-rs}\left(\bar{J}(Q_{s-} + \Delta Q^d_{s-}) - \bar{J}(Q_{s-})\right)$$

Following the same steps that we did before, we get

$$\begin{aligned}
\bar{J}(k_0) \geq & \int_0^{t\wedge\tau(Q)} e^{-rs-\Lambda_s}\left(r(1-Q_{s-})c + \dot{q}_s\bar{S}(\alpha(Q_{s-}), Q_{s-})\right. \\
& \left.+\lambda_s\left(Q^s_s - Q^s_{s-}\right)\mathbb{E}[\bar{S}(\alpha(Q_s), \theta)|\theta \in [Q_{s-}, Q_s]]\right) ds \\
& \sum_{s<t\wedge\tau(Q)} e^{-rs-\Lambda_s}\left(Q^d_s - Q^d_{s-}\right)\mathbb{E}[\bar{S}(\alpha(Q_s), \theta)|\theta \in [Q_{s-}, Q_s]] + e^{-rt\wedge\tau(Q)-\Lambda_{t\wedge\tau(Q)}}\bar{J}(Q_{t\wedge\tau(Q)}) \\
= & \int_0^{t\wedge\tau(Q)} e^{-rs-\Lambda_s}\mathbb{E}\left[\bar{S}\left(\alpha(Q_s), v\right)\middle|\theta \in [Q_{s-}, Q_s]\right] dQ_s + (1-k_0)c \\
& -\int_0^{t\wedge\tau(Q)} e^{-rs-\Lambda_s}\left((1-Q_s)\lambda_s + dQ_s\right)c + e^{-rt\wedge\tau(Q)-\Lambda_{t\wedge\tau(Q)}}\left(\bar{J}(Q_{t\wedge\tau(Q)}) - (1-Q_{t\wedge\tau(Q)})c\right).
\end{aligned}$$

Taking the limit as $t \to \infty$ we get that $t \wedge \tau(Q) \to \tau(Q)$. It follows that $\bar{J}(k_0)$ is and upper bound on the seller's expected payoff. Finally, because in the case of the policy K all the inequalities hold with equality, we get that the value of the policy K is given by $\bar{J}(k_0)$, so K is optimal on $[0, k^{SLC})$.

Verification: Seller's Off-Path Strategy Finally, we characterize the off-equilibrium seller's offer $\sigma(\cdot|k', \alpha')$, where $\sigma(\cdot|k', \alpha')$ has to maximize

$$\int_0^1 \left\{ (\alpha^{-1}(\tilde{\alpha}) - k')^+ \mathbb{E}\left[S\left(\tilde{\alpha}, \tilde{V}\right) \middle| \theta \in [k', \alpha^{-1}(\tilde{\alpha}) \wedge k'] \right] \right. \\ \left. + (1 - \alpha^{-1}(\tilde{\alpha})) J\left(\alpha^{-1}(\tilde{\alpha})\right) \right\} d\sigma(\tilde{\alpha}|k', \alpha').$$

We consider an off-equilibrium offer with two mass points, given by

$$\sigma(\alpha|k', \alpha') = \begin{cases} \alpha(k') & \text{w.p. } p(k', \alpha') \\ \alpha^f(1) & \text{w.p. } 1 - p(k', \alpha'), \end{cases}$$

If $k' < k^{SLC}$, then, conditional on rejection of α', the cut-off is $\alpha^{-1}(\alpha') = k^{SLC}$. In this case, $\bar{S}(\alpha(k^{SLC}), k^{SLC}) = \mathbb{E}\left[S(\alpha^f(1), \tilde{V})|\theta \in [k^{SLC}, 1]\right] = c = J(k^{SLC})$, and this payoff is higher than any other serious offer. Thus, any probability $p(k', \alpha') \in [0, 1]$ is optimal, and in particular $p(k', \alpha')$ solving

$$\bar{S}(\alpha', k^{SLC}) = p(k^{SLC}, \alpha')\bar{S}(\alpha^f(1), k^{SLC}) + (1 - p(k^{SLC}, \alpha'))\bar{S}(\alpha(k^{SLC}), k^{SLC}).$$

If $k > k^{SLC}$, then the optimal offer is $p(k', \alpha') = 1$, as any other offer that is accepted with positive probability yields $\mathbb{E}\left[S(\alpha^f(1), \tilde{V})|\theta \in [k', k]\right] < \mathbb{E}\left[S(\alpha^f(1), \tilde{V})|\theta \in [k', 1]\right] = J(k')$.

Verification: Buyer's On-Path Strategy The proof use a direct mechanism representation of the continuation play together with the characterization in Lemma 2. We cannot apply Lemma 2 directly because the characterization only applies to a deterministic path of cut-offs, and the path cut-off is stochastic in our equilibrium construction (it jumps to $K_T = 1$ at time T). The first step then is to establish that, given the seller strategy, the buyer acceptance strategy is incentive compatible only if it incentive compatible for a deterministic path with the same delay for the pooling offer $\alpha^f(1)$. Let $\tau(k) = \inf\{t > 0 : K_t \geq k\}$, let $\alpha(k) \equiv \alpha(K_{\tau(k)})$, and $y(k) = 1 - \mathbb{E}[e^{-r\tau(k)}]$. Notice that, given the seller strategy K we have that $\alpha(k)$ is a deterministic function of k, so the only random variable is $\tau(k)$. Thus, we can write the buyer's problem as

$$
\begin{aligned}
B(\theta,k) &= \max_{k'\in[k,1]} \mathbb{E}^{K^k}[(1-e^{-r\tau(k')})A(\theta) + e^{-r\tau(k')}\left(V(\theta)-\bar{S}(\alpha(K_{\tau(k')}),\theta)\right)] \\
&= \max_{k'\in[k,1]} y(k')A(\theta) + (1-y(k'))\left(V(\theta)-\bar{S}(\alpha(k'),\theta)\right) \\
&= \max_{k'\in[k,1]} U(y(k'),\alpha(k'),\theta).
\end{aligned}
$$

It follows that it is without loss of generality to consider the incentive compatibility condition for a deterministic mechanism inducing the same $y(k)$ as K^k. By the arguments in Lemma 2, we know, for increasing $\iota^S(\alpha,\theta)$, $U(y,\alpha,\theta)$ satisfies strict single crossing differences in $((y,\alpha),\theta)$, where (y,α) is ordered lexicographically. Hence, for any $y \mapsto \tilde{\alpha}(y)$, $U(y,\tilde{\alpha}(y),\theta)$ has strict single-crossing differences in (y,θ).

We have shown that $y(\theta)$ is non-decreasing. If we prove that $U(y,\tilde{\alpha}(y),\theta)$ satisfies *smooth* single crossing differences, taking $\tilde{\alpha}(y)$ to be the candidate equilibrium mapping between choice of (1 minus) expected delay and equilibrium offer, and if the following envelope condition is satisfied

$$
U(y(\theta),\alpha(\theta),\theta) = U(y(0),\alpha(0),0) + \int_0^v U_\theta(y(s),\alpha(s),s)ds, \tag{44}
$$

then by Theorem 4.2 in Milgrom (2004), the buyer acceptance strategy $\alpha(\theta)$ will incentive compatible To check smooth single-crossing differences, take (y,θ) such that $\frac{d}{dy}U(y,\tilde{\alpha}(y),\theta)=0$. Taking the derivative, we have

$$
\bar{S}_\alpha(\tilde{\alpha}(y),\theta)\left[\iota^S(\tilde{\alpha}(y),\theta)-\tilde{\alpha}'(y)\right]=0. \tag{45}
$$

By assumption, $\tilde{S}_\alpha>0$, so if the above display is 0, $\iota^S(\tilde{\alpha}(y),\theta)=\tilde{\alpha}'(y)$. Then whenever the derivative exists,

$$
\frac{\partial}{\partial v}\frac{d}{dy}U(y,\tilde{\alpha}(y),\theta) = \bar{S}_\alpha(\tilde{\alpha}(y),\theta)\left[\frac{\partial}{\partial v}\iota^S(\tilde{\alpha}(y),\theta)\right] > 0,
$$

since the environment is upward skimming.

Now we show the relevant envelope condition. By definition, we have that for any θ and any (y,α)

$$
U_\theta(y,\alpha,\theta) = yX'(\theta) + (1-y)\left(V'(\theta)-\bar{S}_\theta(\alpha,\theta)\right)
$$

For any $\theta\in[0,k^{SLC}]$ we have

$$
\begin{aligned}
U(y(\theta),\alpha(\theta),\theta) = U(y(0),\alpha(0),0) + \int_0^\theta & \left(U_\theta(y(s),\alpha(s),s)\right. \\
& \left. + U_y(y(s),\alpha(s),s)y'(s) + U_\alpha(y(s),\alpha(s),s)\alpha'(s)\right) ds,
\end{aligned}
$$

where

$$
\begin{aligned}
U_y(\cdot)y'(s) + U_\alpha(\cdot)\alpha'(s) &= \left(A(s) - V(s) + \bar{S}(\alpha(s), s)\right) y'(s) - (1 - y(s))\bar{S}_\alpha(\alpha(s), s)\alpha'(s) \\
&= -\left(R(s) - \bar{S}(\alpha(s), s)\right) y'(s) - (1 - y(s))\bar{S}_\alpha(\alpha(s), s)\alpha'(s).
\end{aligned}
$$

From the local IC constraint way have that

$$
r\left(R(K_t) - \bar{S}(\alpha(K_t), K_t)\right) = -\dot{K}_t\alpha'(K_t)\bar{S}_\alpha(\alpha(K_t), K_t).
$$

By definition, on $[0, k^{SLC})$, $y'(k) = re^{-r\tau(k)}\tau'(k)$ and $\alpha'(k) = \alpha'(K_{\tau(k)})\dot{K}_{\tau(k)}\tau'(k)$. Hence, multiplying both sides of the local incentive compatibility constraint by $e^{-r\tau(k)}\tau'(k)$, and using the definition $K_{\tau(k)} = k$, we get

$$
\left(R(k) - \bar{S}(\alpha(k), k)\right) y'(k) = -(1 - y(k))\alpha'(k)\bar{S}_\alpha(\alpha(k), k),
$$

so $U_y(\cdot)y'(s) + U_\alpha(\cdot)\alpha'(s)$, and we obtain equation (44). Next, we verify he envelope representation (44) for $k \in (k^{SLC}, 1]$. Because $\alpha(k)$ and $y(k)$ are constant on $(k^{SLC}, 1]$ and $U_y(\cdot)y'(s) + U_\alpha(\cdot)\alpha'(s) = 0$ on $\theta \in [0, k^{SLC}]$ we have that

$$
\begin{aligned}
U(y(\theta), \alpha(\theta), \theta) = U(y(0), \alpha(0), 0) &+ \int_0^v U_\theta(y(s), \alpha(s), s)ds \\
&+ U(y(k^{SLC}+), \alpha(k^{SLC}+), k^{SLC}) - U(y(k^{SLC}), \alpha(k^{SLC}), k^{SLC}).
\end{aligned}
$$

By construction, the delay D in equation (11) is such

$$
U(y(k^{SLC}+), \alpha(k^{SLC}+), k^{SLC}) = U(y(k^{SLC}), \alpha(k^{SLC}), k^{SLC}),
$$

so the expected payoff $U(y(\theta), \alpha(\theta), \theta)$ satisfies the envelope condition (44).

Verification: Buyer's Off-Path Strategy The only step left is to verify the optimality of the reservation price strategy $\alpha(k)$ following an off-equilibrium offer $\alpha' \notin \alpha([0, 1])$. By construction, the $\sigma(\alpha|k', \alpha')$ is such the type k^{SLC} buyer is indifferent between accepting α' and reject it. Thus, we only need to verify that types above k^{SLC} are better off rejecting it. By construction

$$
\bar{S}(\alpha', k^{SLC}) = p(k^{SLC}, \alpha')\bar{S}(\alpha^f(1), k^{SLC}) + (1 - p(k^{SLC}, \alpha'))\bar{S}(\alpha(k^{SLC}), k^{SLC}).
$$

Let $p' \equiv p(k^{SLC}, \alpha')$, because $\bar{S}(\alpha', \theta)$ is increasing in θ, we have that

$$
\begin{aligned}
V(\theta) - \bar{S}(\alpha', \theta) &< V(\theta) - \bar{S}(\alpha', k^{SLC}) \\
&= p'\left(V(\theta) - \bar{S}(\alpha^f(1), k^{SLC})\right) + (1 - p')\left(V(\theta) - \bar{S}(\alpha(k^{SLC}), k^{SLC})\right) \\
&< p'\left(V(\theta) - \bar{S}(\alpha^f(1), k^{SLC})\right) + (1 - p')B(\theta, k^{SLC}),
\end{aligned}
$$

which means that types $\theta > k^{SLC}$ are strictly better off rejecting α'. A similar calculation shows that types $\theta < k^{SLC}$ are strictly better off accepting α'.

$\square$