“Implementation of the Dynamic Time-History Linear Seismic Analysis in MatLab for 3D frames”

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Overview

This software is the implementation of the Dynamic Time-History Seismic Analysis for 3D frames fully in MatLab. The functions involve the modal analysis and the method for the numerical solution of Second Order differential equations in time steps. Two structural examples are presented, both of which are analysed with damping effects and without them. An artificial acceleration history vector is given as entry data to assess the equivalent inertial forces in the different DOF of the structures at each time step from the modes and frequencies primarily obtained by solving the system $[K - \omega^2 M]\phi = 0$. The numerical solution is entirely linear, that is without considering non-linearities of the elements' materials.

Keywords: Dynamic Time-History, Seismic Analysis, Linear solution, 3D Frames

1 The Dynamic Non-Linear Time-History Seismic Analysis

This analysis is considered as the most sophisticated one of any other analysis given that the non-linear response of materials and/or geometrical non-linearities are considered here in every time-step of assessment so that a more accurate analysis of the structural response can be carried out for a given ground acceleration history (1). The equivalent inertial forces at any of the system DOF's are obtained by solving the classical eigenvalue and eigenvector problem without considering the damping matrix (2), even when damping effects are considered. Thus, the displacement response before such obtained equivalent inertial forces can be carried out through a quasi Non-linear Pushover analysis at each time-step so that the structure's stiffness degradation and damping degradation can be considered.

$$[M]\{\ddot{U}\} + [C]\{\dot{U}\} + [K]\{U\} = \{F(t)\} = \frac{\phi_i^T M[1]}{M*} a(t) M \phi_i \tag{1}$$

$$([K] - \omega^2 [M])[\phi] = \{0\} \tag{2}$$

1.1 Material Non-Linearities - Quasi Pushover analysis

By referring to executing a *quasi* Non-Linear Pushover analysis to consider the material non-linearities of the structure's response it means that no incremental load is computed but only those induced by the ground acceleration at a given time-step $F(t)$ so that the main purpose would be to identify if any plastic hinge formation takes place due to such external loads so that a new stiffness matrix and an equivalent internal force vector is obtain to consider such damage in the following time-steps and in the structural response in general. The process of this such Quasi Pushover analysis is described below in pseudo-code:

Algorithm 1.1: Pseudo-code for the Quasi Pushover to consider material non-linearities in the Newmark-β method.

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1. Input materials and geometry properties of the structure, E,A,G,J,Iz,Iy, coord-xyz-nodes
2. Input current plastic hinge state and Resistant Bending moments at each element's ends Mp
3. Perform a static linear analysis with the current plastic hinge state and check for new plastic hinge formations
4. Build a new global stiffness matrix and the corresponding equivalent internal force vector in case a new plastic hinge formation took place

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For the transformation process of equivalent non-plastic structures to plastic ones, the liberation of the corresponding DOFs and the proper distribution of internal loads must be done. For this purpose the following mechanisms of **Fig. 1**, **Fig. 2** and **Fig. 3** apply with their corresponding equivalent numerical matricial representations:

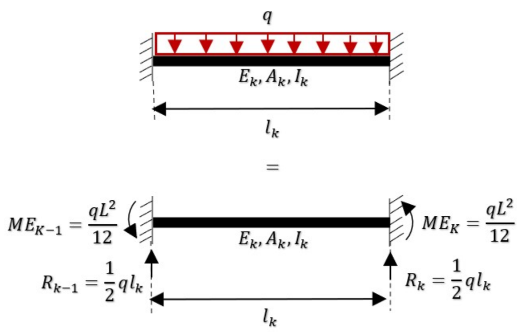

Figure 1. Equivalent structural system for a beam element without plastic hinge formations at the ends.

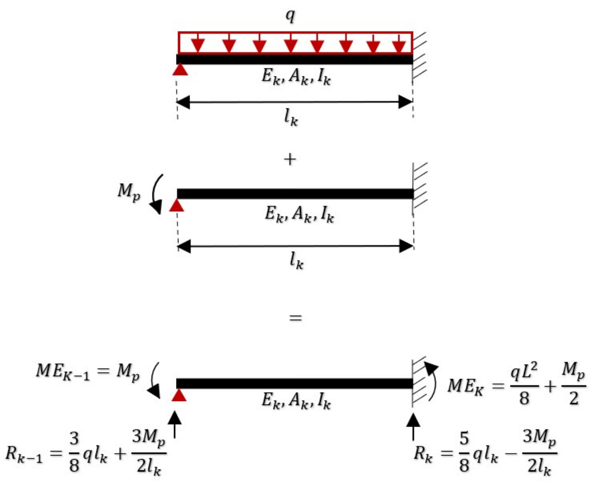

Figure 2. Equivalent structural for a beam element with a plastic hinge formation at one end.

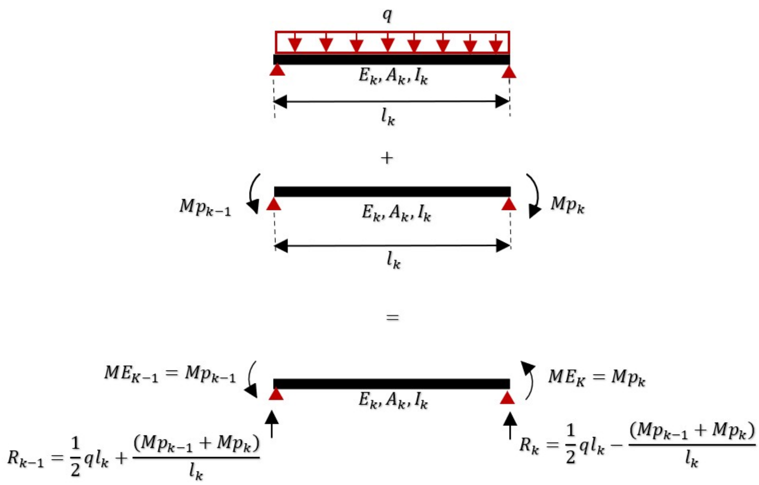

Figure 3. Equivalent structural system for a beam element with two plastic hinge formations at the ends.

It must be stressed that not only the stiffness matrix should be updated for the material non-linearity consideration but also the equivalent internal force system, so that the hypothesis of perfect $Elasto - Plastic$ behaviour is complied. This way, the force history vector $F(t)$ of inertial equivalent seismic forces may be updated at any given time-step. For 3D beams, however, one must be careful to include the previous equivalent force systems at the proper DOF's. The following drawing of **Fig. 4** illustrates the global and local axis system of reference for an element:

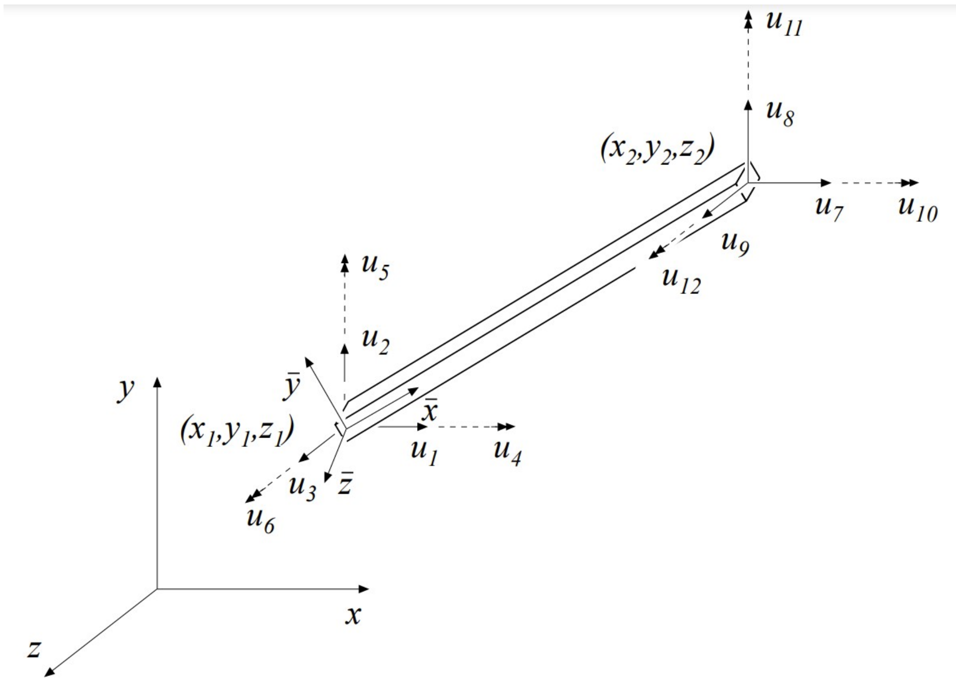

Figure 4. Global and local system of reference considered in this project for the beam elements. The negative direction of the global Z axis corresponds to the direction of gravity.

1.2 The consistent mass method

The construction of the mass matrix is done through a Finite Element method approach in which an element mass matrix Me as the one shown below is built for all elements and assembled into a global one as for the stiffness matrix.

$$\frac{\rho A L}{420}\begin{bmatrix}
140 & 0 & 0 & 0 & 0 & 0 & 70 & 0 & 0 & 0 & 0 & 0 \\
 & 156 & 0 & 0 & 0 & 22L & 0 & 54 & 0 & 0 & 0 & -13L \\
 & & 156 & 0 & -22L & 0 & 0 & 0 & 54 & 0 & 13L & 0 \\
 & & & 140r_x^2 & 0 & 0 & 0 & 0 & 0 & 70r_x^2 & 0 & 0 \\
 & & & & 4L^2 & 0 & 0 & 0 & -13L & 0 & -3L^2 & 0 \\
 & & & & & 4L^2 & 0 & 13L & 0 & 0 & 0 & -3L^2 \\
 & & & & & & 140 & 0 & 0 & 0 & 0 & 0 \\
 & & & & & & & 156 & 0 & 0 & 0 & -22L \\
 & & & & & & & & 156 & 0 & 22L & 0 \\
 & & & & & & & & & 140r_x^2 & 0 & 0 \\
 & & & & & & & & & & 4L^2 & 0 \\
\text{sym} & & & & & & & & & & & 4L^2
\end{bmatrix}$$

1.3 Structural Rayleigh damping

For MDOF the damping matrix C can be computed as a function of the mass matrix and the stiffness matrix by using the Rayleigh's coefficients α and β as in (3), where α and β are determined by solving the linear system of equations (4) from the selection of a pair of chosen modal damping factors ξ_1, ξ_2

and a corresponding pair of frequencies ω_1, ω_2:

$$C_e = \alpha \cdot M_e + \beta \cdot K_e \tag{3}$$

$$\begin{aligned} \xi_1 &= \frac{1}{2\omega_1}\alpha + \frac{\omega_1}{2}\beta \\ \xi_2 &= \frac{1}{2\omega_2}\alpha + \frac{\omega_2}{2}\beta \end{aligned} \tag{4}$$

Even though Rayleigh damping has the clear disadvantage of not allowing realistic damping for all modes of interest (in case there are far more than two in question) it provides a great solution approach when only a few modes are sought to be analysed.

2 Dynamic acceleration

When aiming at solving a dynamic system subject to an earthquake excitation the accelerations in time are required, for which purpose, a real or artificial accelerogram as the one shown in **Fig. 5** should be given as entry data.

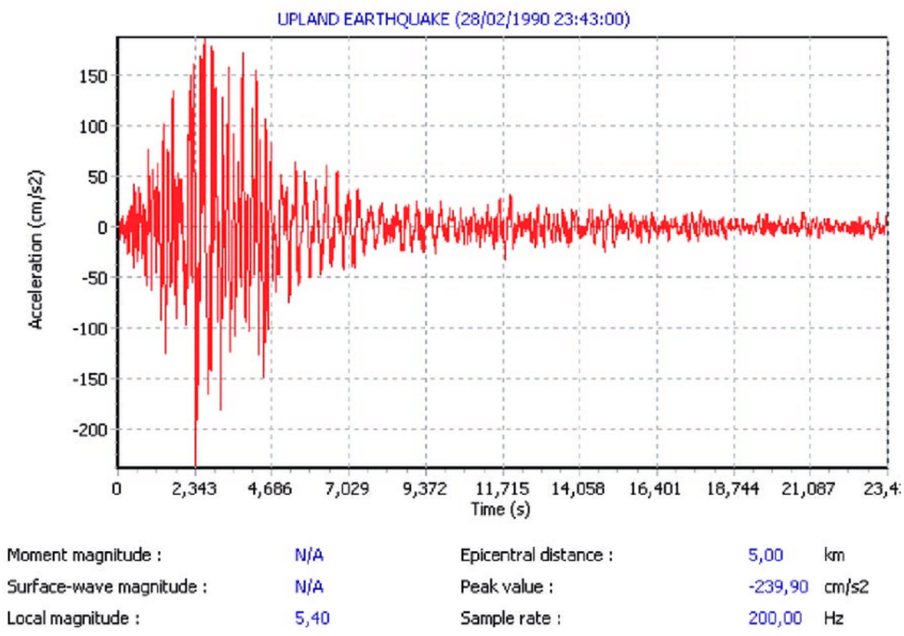

Figure 5. Example of an accelerogram for an earthquake.

From such accelerations in time the vector $f(t)$ in (1) corresponding to the DOF forces in time could be obtained by solving the eigenvalue/eigenvector problem for the modes of vibration in function of such accelerations.

Note that the direction of the acceleration at any time will be given by the chosen mode of vibration.

3 Numerical evaluation of dynamic response

When considering coupled equations of motion in which damping is integrated then one must opt to apply other procedures different than *mode-superposition* methods to solve them, such as **direct integration** methods that aim mainly at the solution of second-order differential equations (see [2]).

3.1 Beta-Newmark method

The Newmark-β method initially introduced as the *Constant average acceleration method* is presented in a form appropriate for direct integration of linear second-order systems, such as those whose damping matrix cannot be diagonalized by the system of normal modes. This method in contrast with the *Central difference method* - which is probably the most known and fundamental algorithm for the approximate numerical solution second-order differential ordinary equations - provides **unconditionally stable** solutions, that is, that it is not entirely required to have qualitative knowledge a priori of the frequencies presented in the system's response to select a proper time step [2].

The basis of the Newmark-β is the assumption that the system displacement vector u_{n+i} and its derivative $\dot{u}_{n+1}$ can be written in the form (5), which can be viewed as a type of generalized Taylor series approximation. Thus, the Newmark-β method is said to be an implicit method with the discrete equation of motion being enforced at the time step t_{n+1} as (6):

$$\begin{aligned} \dot{u}_{n+1} &= \dot{u}_n + ((1-\gamma)\ddot{u}_n + \gamma\ddot{u}_{n+1})h \\ u_{n+1} &= u_n + \dot{u}_n h + (1-2\beta)\frac{h^2}{2}\ddot{u}_n + \beta h^2 \ddot{u}_{n+1} \end{aligned} \tag{5}$$

$$M\ddot{u}_{n+1} + C\dot{u}_{n+1} + Ku_{n+1} = p(t_{n+1}) \tag{6}$$

3.1.1 The Non-Linear Beta-Newmark method

One approach to compute the Non-Linear Newmark-β method is to consider the formation of plastic hinges at the ends of the structure's elements through the Static Non-Linear Pushover method, for instance, so that the stiffness matrix and therefore, the damping matrix may be modified at a given time-step of analysis due to the stiffness degradation that was caused by the plastic hinges formations. The following algorithm describes this such method in detail:

3.1.2 Algorithm for the Non-Linear Beta-Newmark method

Algorithm 3.1: Pseudo-code for the Non-Linear Newmark-β method for 2nd Order Ordinary Differential Equations.

1. Input M,K and C (if any)
2. Input initial conditions u_0, v_0 and compute the initial acceleration a_0 as:
   $a_0 = M^{-1}(p_0 - Cv_0 - Ku_0)$
3. Set the time step length h
4. Initialize loop iteration:

   For $n = 1$ **To** $n = n - steps$

   4.1 Compute the right hand side term RHS as:
   $RHS_n = -Ku_n - (C + hK)\dot{u}_n - [h(1-\gamma)C + \frac{h^2}{2}(1-2\beta)K]\ddot{u}_n$

   4.2 Solve for the second derivatives at the next time step:
   $[M + \gamma C + \beta h^2 K]\ddot{u}_{n+1} = RHS_n$

   4.3 Evaluate the set of displacements and velocities for the next time step:
   $\dot{u}_{n+1} = \dot{u}_n + ((1-\gamma)\ddot{u}_n + \gamma\ddot{u}_{n+1})h$
   $u_{n+1} = u_n + \dot{u}_n h + (1-2\beta)\frac{h^2}{2}\ddot{u}_n + \beta h^2 \ddot{u}_{n+1}$

   4.4 Assess if one or more plastic hinge formation took place by executing the Quasi Pushover method. If so, update K

   4.5 Update the damping matrix C (with the Rayleigh coefficients, for instance) as:
   $C = \alpha_1 M + \alpha_2 K$

   End For

References

[1] Comisión Federal de Electricidad CFE-2015, Manual de Diseño de Obras Civiles, Capítulo C.1.3 Diseño por sismo, México, 2015. Spanish version.

[2] Fundamentals of Structural Dynamics, Roy R. Craig Jr., Andrew J. Kurdila, 2nd Ed., John Wiley & Sons, Inc., 2006