“Computation of Hysteresis Curves of 2D Frames through Cyclic Plastic Loading”

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Overview

This software is the implementation of Cyclic Plastic Loading for 2D Frames in MatLab for the computation of Hysteresis Curves. Three main Hysteresis models are included: the Clough’s model, the Nakeda’s model and the Sucuoglu’s Energy-Based model. The Static Non-Linear Pushover analysis is deployed for the computation of the Frames’ stiffness degradation before the applied incremental monotonic lateral loading per floor. The computation of a couple of Combined Cumulative Damage Indices are also included, although only for the Energy-Based Hysteresis Model (see [1]).

Keywords: Cyclic Plastic Loading, Pushover, 2D Frames, Teaching, Hysteresis Curves, Energy-Based Model, Combined Cumulative Damage Indices

1 Hysteresis Modelling

The devastating consequences over buildings and structures that natural and man-made hazards across the globe provoke in terms of human casualties and economic loss has led scientists and engineers throughout the years to develop more efficient and accurate models for the proper performance evaluation and structural dynamic response assessment of structures before such events. For this purpose, repeated cyclic loading assessment has become of utmost significance, in which not only a load-deformation relationship of a structural member in the plastic range can be obtained but also its capacity to dissipate energy (see [1]).

Nowadays, there exist many different hysteresis models developed by different erudites and can be classifiedmanly into Polygonal Hysteresis Models (PHM) and Smooth Hysteresis Models (SHM). In PHM, stiffness degradation is assessed at different yielding stages of a structure during the cyclic loading process, whereas in SHM continuous stiffness changes due to yielding and sharp changes during deteriorating and unloading are considered (see [1]). The hysteresis models hereby presented and develop belong to this the PHM group and are described next.

1.1 Bilinear Degrading Stiffness Model

This model, also referred to as the Clough’s model, in regard to the author is one of the frist hysteresis models developed and operates on a bilinear primary curve with ascending post yielding branches (strain hardening, stiffness degradation, etc) during load reversals as observed in **Fig. 1**. Here, the reloading branch projects towards the previous unloading point of the loading history.

The disadvantage or inconvenience with this model is that a substantial increase in the strength capacity of structure is verified with the increase in lateral displacement in relation to the real hysteresis curves (as shown in **Fig. 2** for a certain structural system) since it does not consider negative stiffness beyond the post-yielding stage (see [2]).

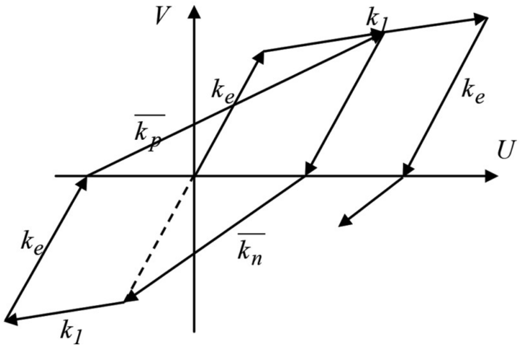

Figure 1. Load-Deflection diagram of bilinear degrading stiffness model developed by Clough.

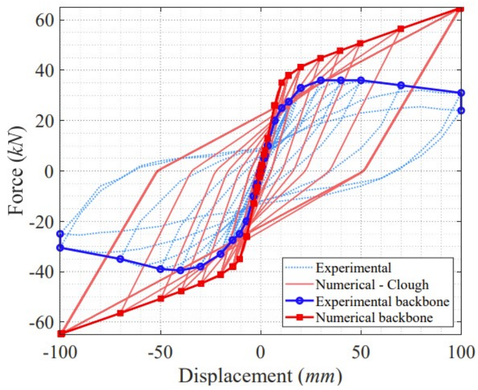

Figure 2. (Left) Load-Deflection diagram of bilinear degrading stiffness model developed by Clough, (Right) Comparison of the model with a real hysteresis curve.

1.2 Trilinear Degrading Stiffness Model

This model, also referred to as the Takeda's model in regard to the author, is more adequate to represent structural cyclic behaviour than the Clough's model, specially for small lateral displacements, given that a trilinear backbone curve is obtained. This model also considers stiffness degradation in the unloading process and cracking mechanisms (for RC structures), however, much like the previous model, it tends to overestimate the post-yielding stiffness, as shown in **Fig. 3** (see [2]), although with a less pronounced increase:

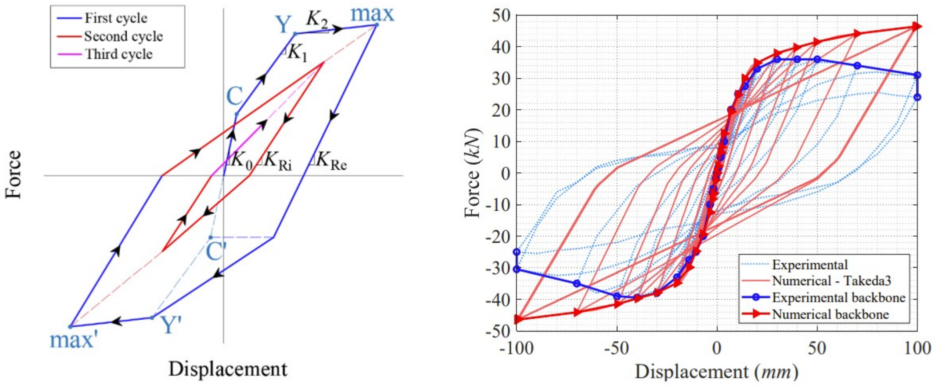

Figure 3. (Left) Load-Deflection diagram of trilinear degrading stiffness model developed by Takeda, (Right) Comparison of the model with a real hysteresis curve.

1.3 Energy-Based Hysteresis Model

This model also operates on a bilinear curve with elastic stiffness K_0. Here the reloading branch projects towards the post-yielding point of the previous unloading point (as shown in **Fig. 4**).

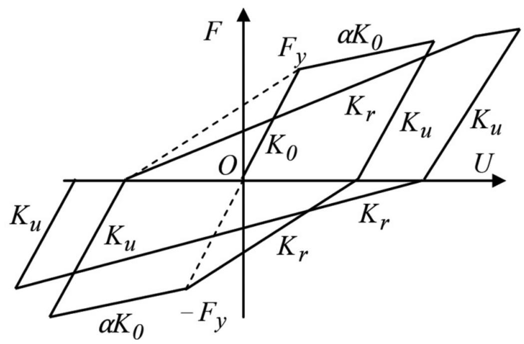

Figure 4. Load-Deflection diagram of Energy-Based Hysteresis model by Sucuoglu and Erberik.

To relate the loss in energy dissipation capacity in the displacement cycle with the reduced strength, the following deteriorating system of **Fig. 5** is taken as reference, so that the energy dissipated in the first cycle $E_{h,1}$ and the $n-th$ cycle $E_{h,n}$ can be computed as (1) and (2), respectively:

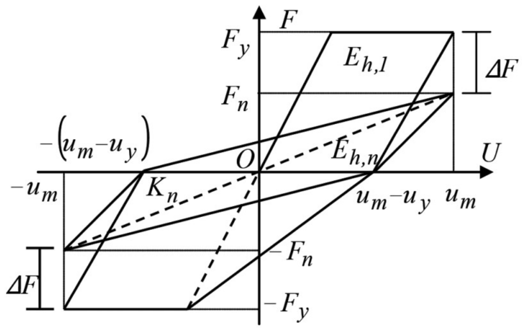

Figure 5. Relationship between reduced strength and dissipated energy of the Energy-Based hysteresis model.

$$E_{h,1} = 2.5F_y(u_m - u_y) \tag{1}$$

$$E_{h,n} = 2.5F_n(u_m - u_y) \tag{2}$$

2 Damage assessment

In order to properly assess the performance and safety of structure before certain loads and events accurate Damage Assessment processes must carried out. Structural damage can be defined as a degradation degree which may represent the load capacity of structures and their remaining capacity before reaching a failure limit. For this purpose, Damage Indices are computed, so that the performance of structures can be investigated and better decision-making processes can be executed. They can also provide information as to what level of retrofit or maintenance a structure needs or might need.

The classification of Damage Indices has been done in four groups based on resistance demands (in the linear and non-linear stages), ductility requirements, energy dissipation and reduction of stiffness. Such Damage Indices usually have values between 0 and 1, where 0 would indicate that there is no damage in the structure and the structural behaviour remains in the elastic range, whereas the unit value would represent the failure state or collapse of the structure. In general, Damage Indices have been divided into local damage indices and global damage indices, used to assess the damage in individual elements or in whole structural systems, respectively. The local damage indices can also be divided into two categories: cumulative and non-cumulative.

3 Cumulative Damage Indices for Cyclic Loading

These sets of Damage Indices depend on the number of loading and unloading cycles imposed to the structure. In general terms, it has been concluded throughout the years that the degree of the damage for a structural element is not only dependent on the maximum displacement recorded under a hazard but also on the number of load cycles and hysteric energy absorbed.

When only cumulative displacement is considered the Low-Cycle Fatigue Damage Index DI_f can be used (3), where n refers to the total number of plastic cycles, μ_u is the maximum allowable ductility, μ_i is the cyclic ductility corresponding to the generic plastic displacement, Δmax_i is the so far maximum displacement at the i cycle, Δy is the deformation at the elastic limit.

$$DI_f = \sum_{i=1}^{n} (\frac{\mu_i - 1}{\mu_u - 1})^b = \sum_{i=1}^{n} (\frac{\Delta max_i - \Delta y}{\Delta x_u - \Delta y})^b \tag{3}$$

On the other hand, when only plastic energy dissipated is considered, then the Energy-Based Damage Index DI_{EB} proposed by Gosain can be used (4) where F_i and F_y represent the failure force and the yield force, Δ_i is the failure displacement and Δ_y represent the yield displacement. This such formula can only be used for $F_i \leq 0.75F_y$.

$$DI_{EB} = \sum_{i=1}^{n} (\frac{F_i \Delta_i}{F_y \Delta_y}) \tag{4}$$

Now, when both energy-dissipation and cumulative displacement are used, then a version of the Park-Ang Damage Index can be used. This index developed in 1985 became one of the broadest damage indices used worldwide and up to this point several versions of it have branched, each focusing on solving a certain disadvantage of the original version. In this regard, Wang proposed the following DI (5):

$$DI_{w,pa} = (1 - \beta) \frac{\Delta_m - \Delta_y}{\Delta_u - \Delta_y} + \beta \frac{\sum_i^n \beta_i E_i}{F_y(\Delta_u - \Delta_y)} \tag{5}$$

Here, β_i represents the energy weight factor relevant to the loading histories (6) (where c is a constant with values from 1 to 2) and β is a factor that considers the cyclic loading effect. According to several experimental tests, several values of β range from 0.05 to 0.24 for reinforced concrete and 0.025 to 0.23 for steel structures. E_i can be determined on each cycle as (1) and (2), accordingly.

$$\beta_i = (\frac{E_i}{E_t - \sum_{j=1}^{i} E_j})^c \tag{6}$$

Kumar and Usami proposed another version of the Park-Ang DI, where N_1 is the number of half cycles producing Δmax_i for the first time and n is the number of half cycles. The other parameters have the same meaning as for the previous indices.

$$DI_{ku} = (1 - \beta) \sum_{j=1}^{N_1} (\frac{\Delta_m - \Delta_y}{\Delta_u - \Delta_y})^c + \beta \sum_{i}^{n} (\frac{E_i}{F_y(\Delta_u - \Delta_y)})^c \tag{7}$$

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