"Implementation of the Static Modal Analysis in MatLab for 3D frames"

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Overview

This software is the implementation of the Static Modal Analysis for 3D frames. The analysis is based on the *Consistent mass method* in which all possible modes of vibration of a plane frame are computed by solving the eigenvector and eigenvalue equation $det(K - \omega^2 M) = 0$ in which damping is neglected. The equivalent inertial forces are calculated for all the DOF with respect to one or more than one modes of vibration. When more than one modes of vibration are given to compute such equivalent forces, then the superposition of all such given modes of vibration is computed. A design spectrum of constant acceleration at the base of the structure is considered. For the computation of the Mass matrix not only the mass of each element is considered but also the external uniformly distributed loads in the vertical direction of the beam (if any).

Keywords: Static Modal Analysis, 3D Frames, Design spectrum, Constant acceleration, Consistent mass method

1 The Static Modal Analysis

Due to the ground's movement at the base of buildings that an earthquake can generate great inertial forces can be induced to them. The flexibility of a structure, in function of its mass and shape/topology makes a structure to vibrate in a certain frequency usually different than the soil's on which it is supported, so that the induced inertial forces acting on the structure depend directly on its dynamic properties.

According to the original MDOF dynamic system equation (1) where K is the stiffness matrix, M the mass matrix and C is the damping matrix, when performing a modal static analysis where no damping is considered for the structure the dynamic system equation is transformed to (2) whose solution is given by (3):

$$[M]\{\ddot{U}\} + [C]\{\dot{U}\} + [K]\{U\} = \{F(t)\} \tag{1}$$

$$[M]\{\ddot{U}\} + [K]\{U\} = \{0\} \tag{2}$$

$$\{U\} = \{\dot{U}\}cos(\omega t) \tag{3}$$

Thus, when derivating U with respect to ω and substituting in (2) the only unknown becomes ω

and could determined by solving the system (4), representing the structure frequencies for each mode of vibration:

$$det([K] - \omega^2[M]) = \{0\} \tag{4}$$

The previous system can be solved by assimilating the problem to a eigenvalue and eigenvector problem in the form $[A]\phi = \lambda\phi$ as (5), where ϕ would represent the modes of vibration of the structure for each frequency:

$$([K] - \omega^2[M])[\phi] = \{0\} \tag{5}$$

1.1 The consistent mass method

The construction of the mass matrix is done through a Finite Element method approach in which an element mass matrix Me as the one shown below is built for all elements and assembled into a global one as for the stiffness matrix, where $r_x^2 = (I_y + I_z)/A$, being I_y and I_z the element's cross-section inertia momentums in both x and z local axis.

$$\frac{\rho AL}{420}\begin{bmatrix} 140 & 0 & 0 & 0 & 0 & 0 & 70 & 0 & 0 & 0 & 0 & 0 \\ & 156 & 0 & 0 & 0 & 22L & 0 & 54 & 0 & 0 & 0 & -13L \\ & & 156 & 0 & -22L & 0 & 0 & 0 & 54 & 0 & 13L & 0 \\ & & & 140r_x^2 & 0 & 0 & 0 & 0 & 0 & 70r_x^2 & 0 & 0 \\ & & & & 4L^2 & 0 & 0 & 0 & -13L & 0 & -3L^2 & 0 \\ & & & & & 4L^2 & 0 & 13L & 0 & 0 & 0 & -3L^2 \\ & & & & & & 140 & 0 & 0 & 0 & 0 & 0 \\ & & & & & & & 156 & 0 & 0 & 0 & -22L \\ & & & & & & & & 156 & 0 & 22L & 0 \\ & & & & & & & & & 140r_x^2 & 0 & 0 \\ & & & & & & & & & & 4L^2 & 0 \\ \text{sym} & & & & & & & & & & & 4L^2 \end{bmatrix}$$

2 Design response spectrum

A response spectrum represents the max response of a SDOF oscillator of unitary mass subject to a given base movement measured by an accelerometer which is then graphed for various vibration period of the oscillator itself (as in **Fig. 1**). This such response can be expressed either by displacements, velocities or max accelerations. Given the long-time process of calculations required to obtain these such response spectrums it is common in structural engineering to simplify such response by a *pseudo-spectrum* so that an acceleration response would then be transformed to a *pseudo-acceleration* response.

Thus, with a pseudo response spectrum acceleration Sa that may indicate the max impose accelerated movement at the base of a structure the equivalent inertial forces of the structure's DOF can be computed as (6), where $M* = \phi_i^T M\phi_i$, being ϕ the eigenvectors and M the mass matrix:

$$f_{max} = -\frac{\phi_i^T M[\delta]}{M*}Sa \tag{6}$$

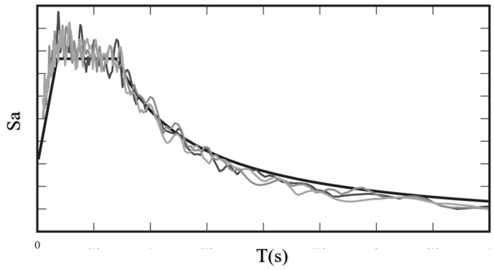

Figure 1. Typical pseudo-spectrum of acceleration.

2.1 Design Sprectra of the Eurocode EC8

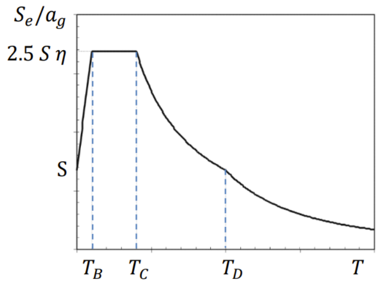

Figure 2. Elastic design spectrum from the EC8.

where S_e is the elastic design acceleration, η is the damping coefficient given by (7), S is the soil coefficient in accordance to the soil type (see **Table 3** and **Table 4**) depending on the value of the damping coefficient, a_g is the design ground acceleration, which is calculated as (8) where a_{gR} is the reference ground acceleration depending on the Seismic Hazard Zone (see **Table 5**) and γ_1 is the *importance factor* goes in accordance to **Table 6**:

$$\eta = \sqrt{\frac{10}{5+\xi}} \tag{7}$$

When $\eta \geq 0.55$:

soil type	S	T_B (s)	T_C (s)	T_D (s)
A	1.0	0.15	0.4	2.5
B	1.2	0.15	0.5	2.5
C	1.15	0.20	0.6	2.5
D	1.35	0.20	0.8	2.5
E	1.4	0.15	0.5	2.5

Figure 3. Seismic spectra from the EC8 for $\eta \geq 0.55$.

When $\eta \leq 0.55$:

ground type	S	T_B (s)	T_C (s)	T_D (s)
A	1.0	0.05	0.25	1.2
B	1.35	0.05	0.25	1.2
C	1.5	0.10	0.25	1.2
D	1.8	0.10	0.30	1.2
E	1.6	0.05	0.25	1.2

Figure 4. Seismic spectra from the EC8 for $\eta \leq 0.55$.

$$a_g = \gamma_1 a_{gR} \tag{8}$$

Seismic Hazard Zone	a_{gR} (g)
Z1	0.16
Z2	0.24
Z3	0.36

Figure 5. Reference ground acceleration according to the seismic zone.

bridge importance	γ_I
Less than average	0.85
Average	1.00
Greater than average	1.30

Figure 6. Seismic spectra from the EC8 for $\eta \leq 0.55$.

so that, depending on the period of the structure of interest, the elastic acceleration S_e is computed as (9):

$$S_e(T) = \begin{cases} a_g \cdot S \cdot [1 + \frac{T}{T_B}(2.5\eta - 1)], & 0 \leq T \leq T_B \\ a_g \cdot S \cdot 2.5\eta, & T_B \leq T \leq T_C \\ a_g \cdot S \cdot 2.5\eta(\frac{T_C}{T}), & T_C \leq T \leq T_D \\ a_g \cdot S \cdot 2.5\eta(\frac{T_C T_D}{T^2}), & T_C \leq T \leq T_D \end{cases} \tag{9}$$

References

[1] Comisión Federal de Electricidad CFE-2015, Manual de Diseño de Obras Civiles, Capítulo C.1.3 Diseño por sismo, México, 2015. Spanish version.