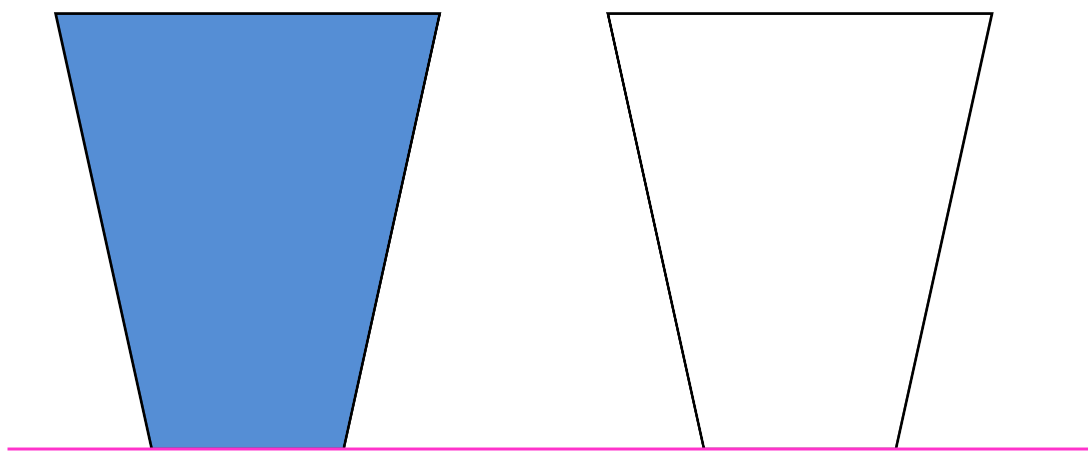
Introduction to Fuzzy Logic

Motivation

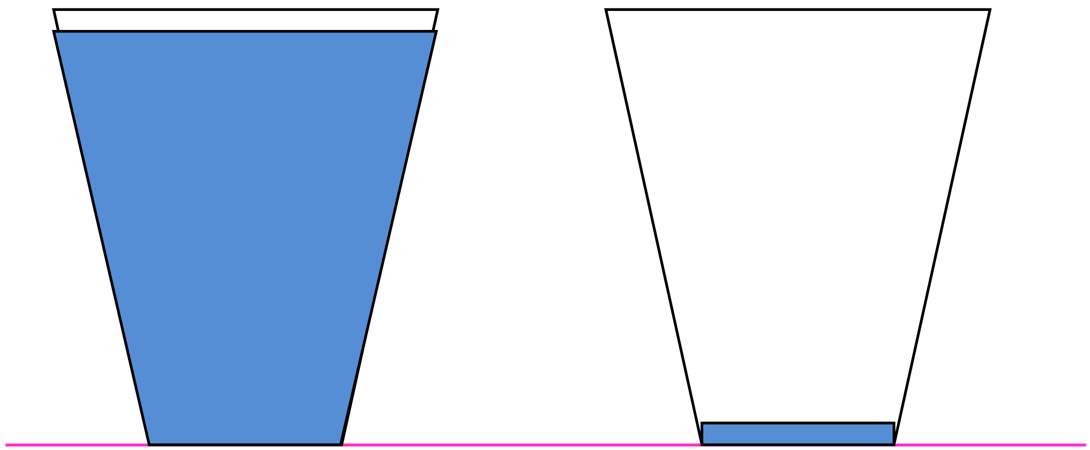
- The term “fuzzy logic” refers to a logic of approximation.
- Boolean logic assumes that every fact is either entirely true or false.
- Fuzzy logic allows for varying degrees of truth.
- Computers can apply this logic to represent vague and imprecise ideas, such as “hot”, “tall” or “old”.

Conception of Fuzzy Logic

- Many decision-making and problem-solving tasks are too complex to be defined precisely
- however, people succeed by using imprecise knowledge
- Fuzzy logic resembles human reasoning in its use of approximate information and uncertainty to generate decisions.



- Full glass
- Empty glass

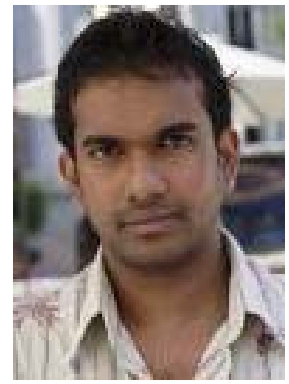


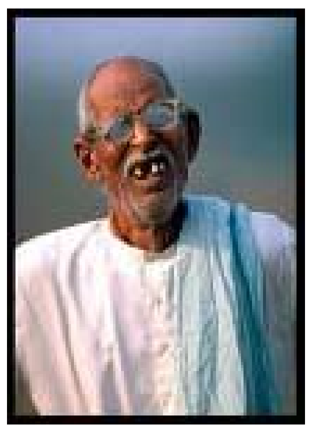
- Almost full glass
- Almost empty glass


fense

Spot the old person(s) on this slide



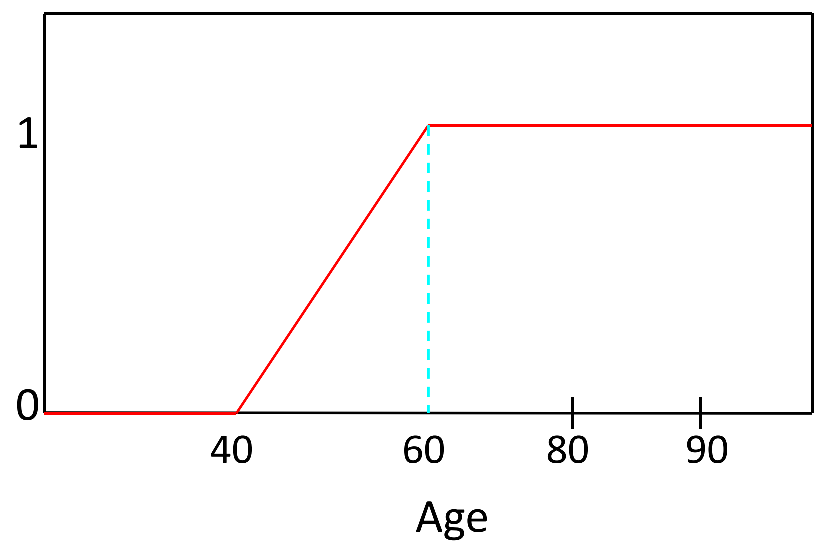





Fuzzy Logic

- An approach to uncertainty that combines real values [0…1] and logic operations
- Fuzzy logic is based on the ideas of fuzzy set theory and fuzzy set membership often found in natural (e.g., spoken) language.

Fuzzy Variable: Old



Every person is given a degree of membership between 0 and 1 to indicate how OLD the person is.

History

- Plato laid a foundation for what would become fuzzy logic, indicating that there was a third region (beyond True and False) where these opposites "tumbled about."
- The modern philosophers, Hegel, Marx, and Engels, echoed this sentiment.

History

- In the early 1900’s, Lukasiewicz described a three-valued logic. The third value can be translated as the term “possible,” and he assigned it a numeric value between True and False.

- Later, he explored four-valued logics, five-valued logics, and declared that in principle there was nothing to prevent the derivation of an infinite-valued logic.

History

- Lotfi Zadeh, at the University of California at Berkeley, first presented fuzzy logic in the mid-1960's.
- Zadeh developed fuzzy logic as a way of processing data. Instead of requiring a data element to be either a member or non-member of a set, he introduced the idea of partial set membership.
- In 1974 Mamdani and Assilian used fuzzy logic to regulate a steam engine.
- In 1985 researchers at Bell laboratories developed the first fuzzy logic chip.

History

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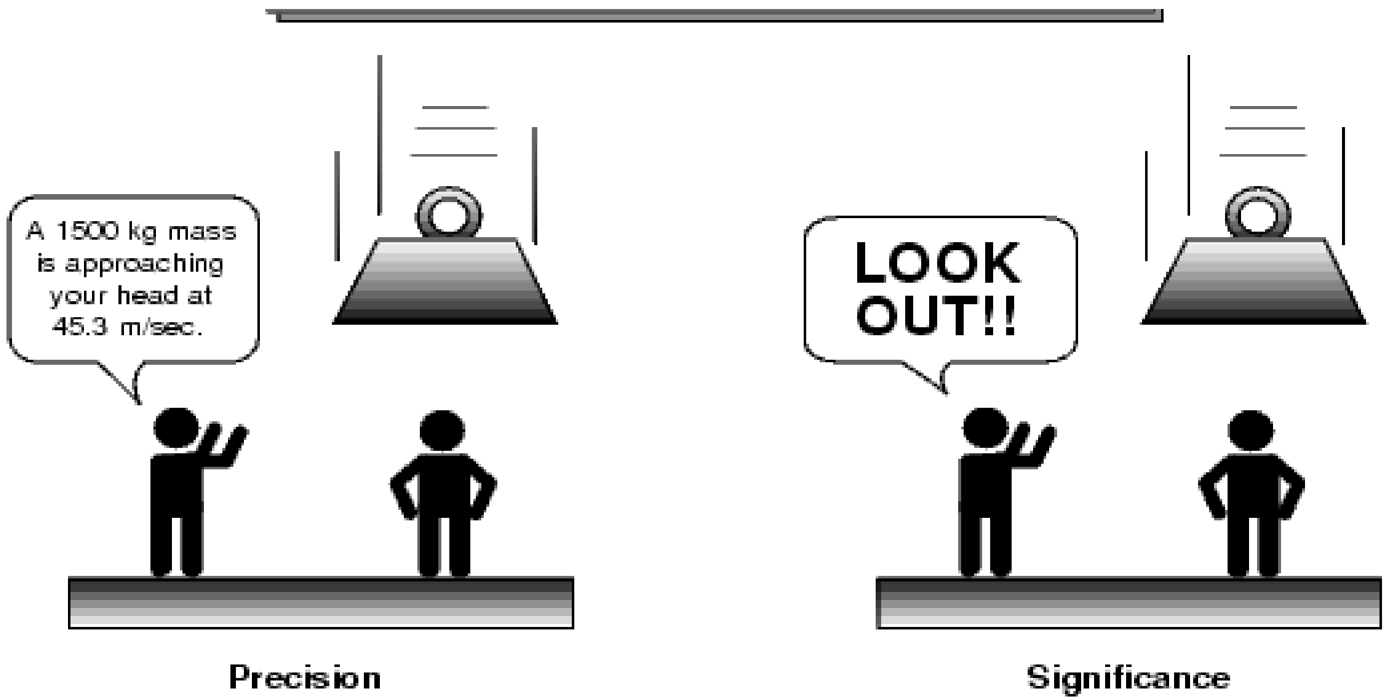
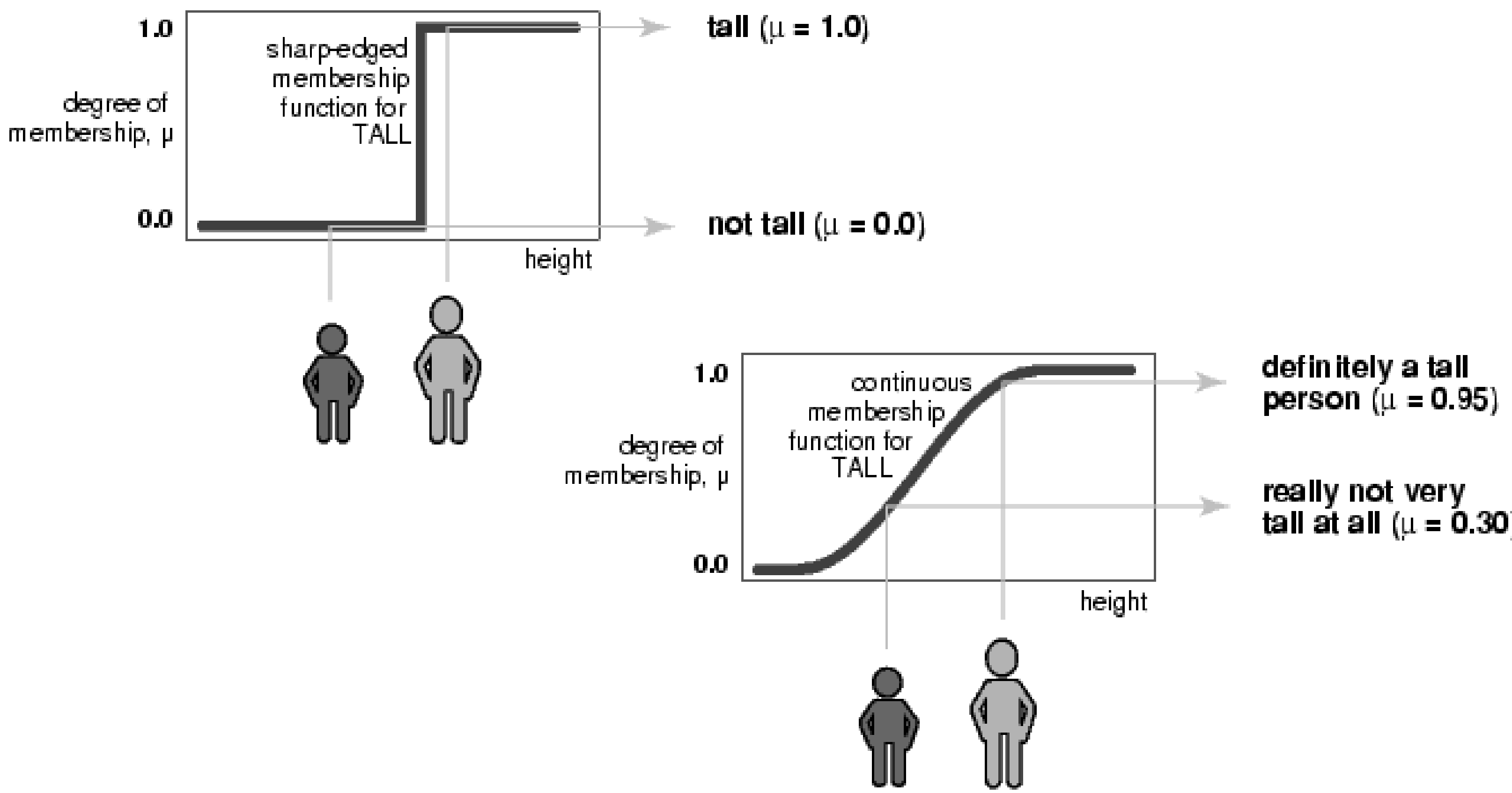
Precision and Significance



Eliminate the Vague?

- It might be argued that vagueness is an obstacle to clarity of meaning.
- But there does seem to be a loss of expressiveness when statements like, “Dan is balding” are eliminated from the language.
- This is what happens when natural language is translated into classic logic. The loss is not severe for accounting programs or computational mathematics programs, but will appear when the programming task turns to issues of queries and knowledge.


1.0
sharp-edged
membership
function for
TALL
degree of
membership, μ
0.0
height
tall (μ = 1.0)
not tall (μ = 0.0)
1.0
continuous
membership
function for
TALL
degree of
membership, μ
0.0
height
definitely a tall
person (μ = 0.95)
really not very
tall at all (μ = 0.30)

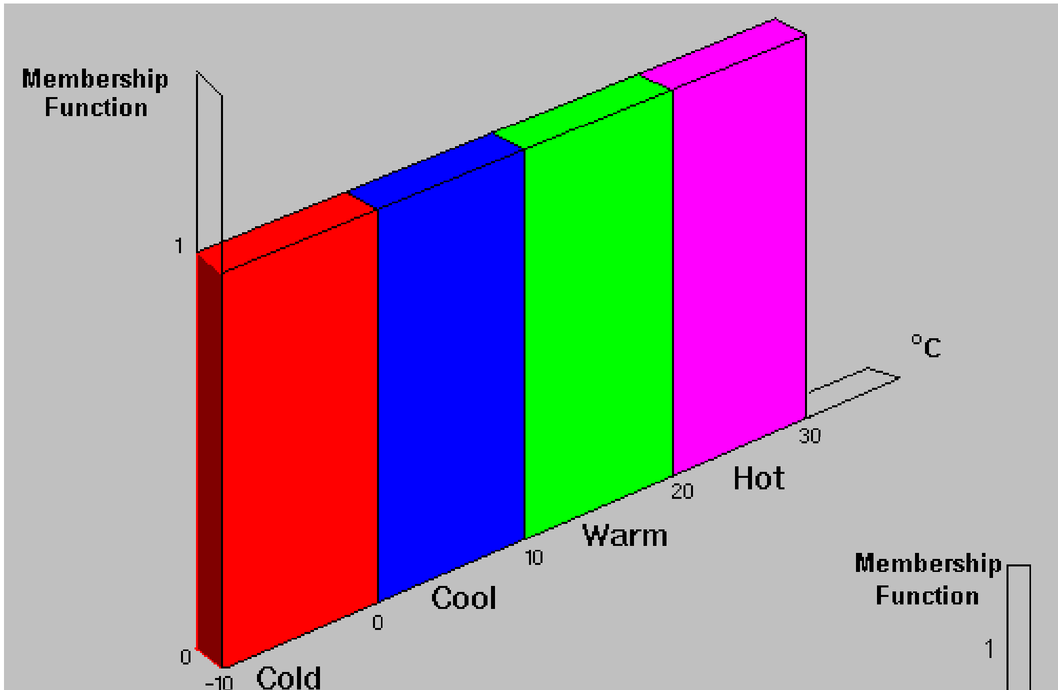
Experts are Vague

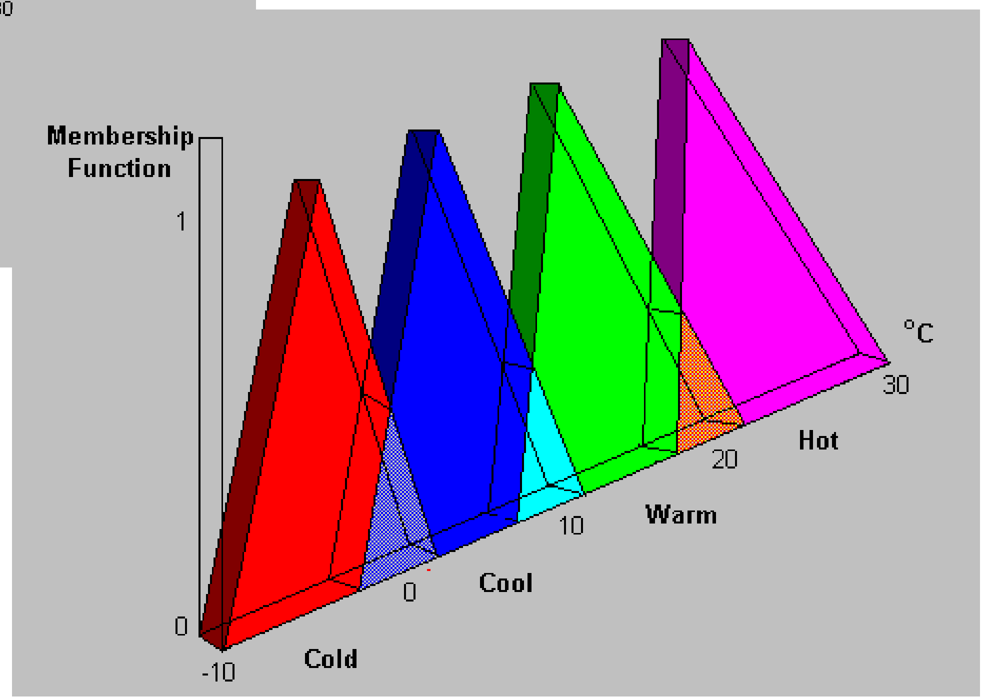
- To design an expert system a major task is to codify the expert's decision-making process.
- In a domain there may be precise, scientific tests and measurements that are used in a "fuzzy", intuitive manner to evaluate results, symptoms, relationships, causes, or remedies.
- While some of the decisions and calculations could be done using traditional logic, fuzzy systems afford a broader, richer field of data and manipulations than do more traditional methods.

Introduce Fuzziness

- Fuzzy logic extends Boolean logic to handle the expression of vague concepts.
- To express imprecision quantitatively, a set membership function maps elements to real values between zero and one (inclusive). The value indicates the “degree” to which an element belongs to a set.
- A fuzzy logic representation for the “hotness” of a room, would assign100° F a membership value of one and 25° F a membership value of zero. 75° F would have a membership value between zero and one.

Bivalence and Fuzz





Fuzzy Is Not Probability

- Fuzzy systems and probability operate over the same numeric range.
- The probabilistic approach yields the natural-language statement, “There is an 80% chance that Dan is balding.”
- The fuzzy terminology corresponds to “Dan's degree of membership within the set of balding people is 0.80.”

Fuzzy Is Not Probability

- The probability view assumes that Dan is or is not balding (the Law of the Excluded Middle) and that we only have an 80% chance of knowing which set he is in.
- Fuzzy supposes that Dan is "more or less" balding, corresponding to the value of 0.80.
- Confidence factors also assume that Dan is or is not balding. The confidence factor simply indicates how confident, how sure, one is that he is in one or the other group.

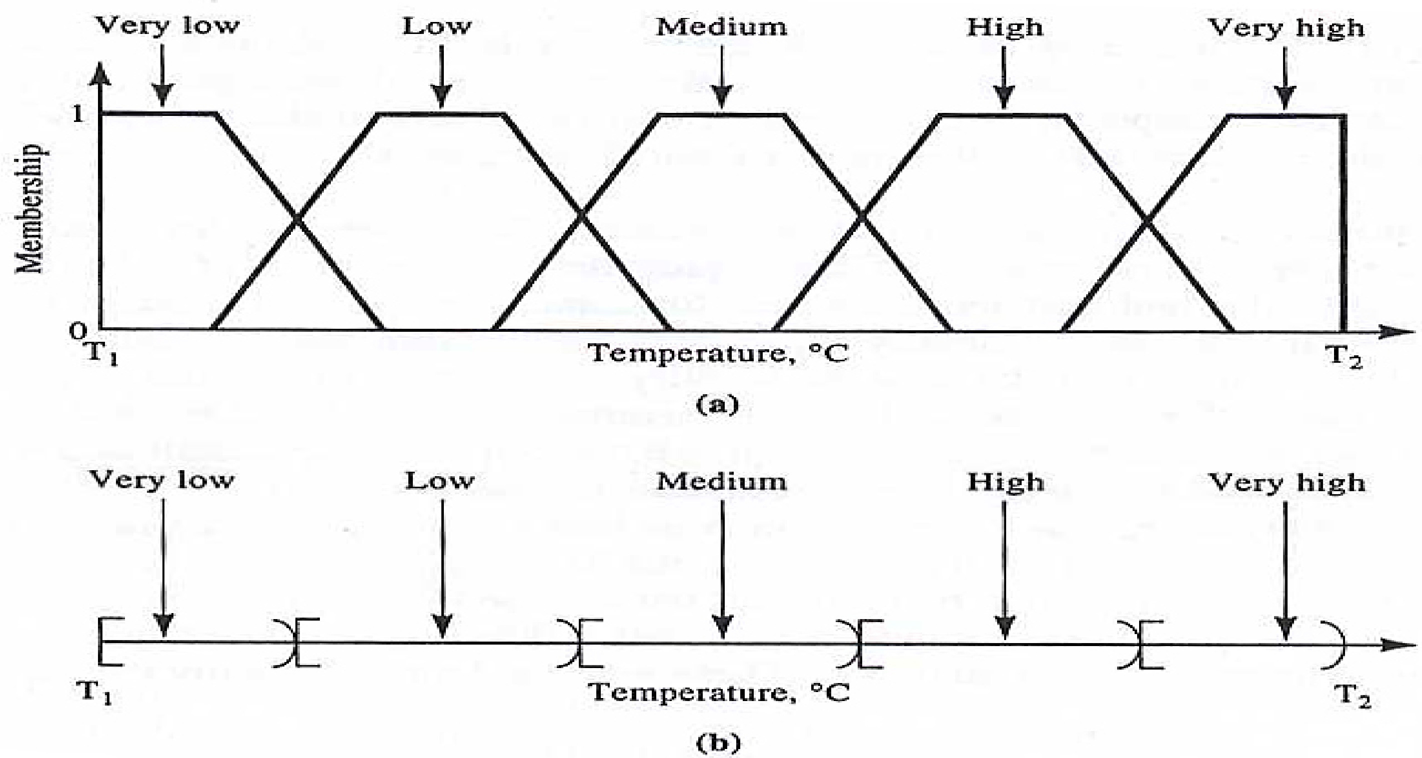
Introduction

- Application areas
  - Fuzzy Control
    - Subway trains
    - Cement kilns
    - Washing Machines
    - Fridges
    - Cameras
    - Boiler control

Sets and Fuzzy Sets

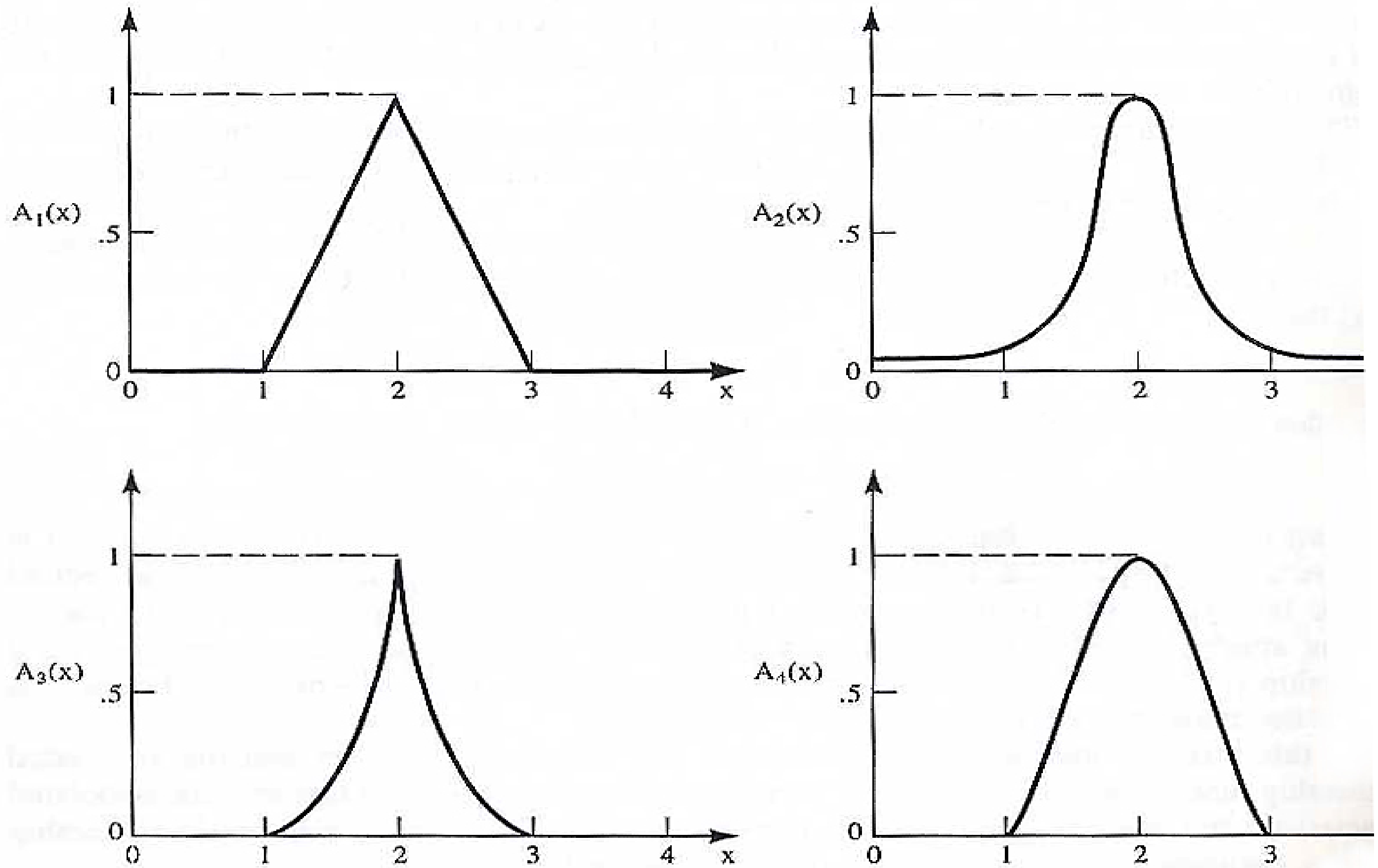
Fuzzy sets – admits gradation such as all tones between black and white. A fuzzy set has a graphical description that expresses how the transition from one to another takes place. This graphical description is called a membership function.

Fuzzy Sets (figure from Klir &Yuan)



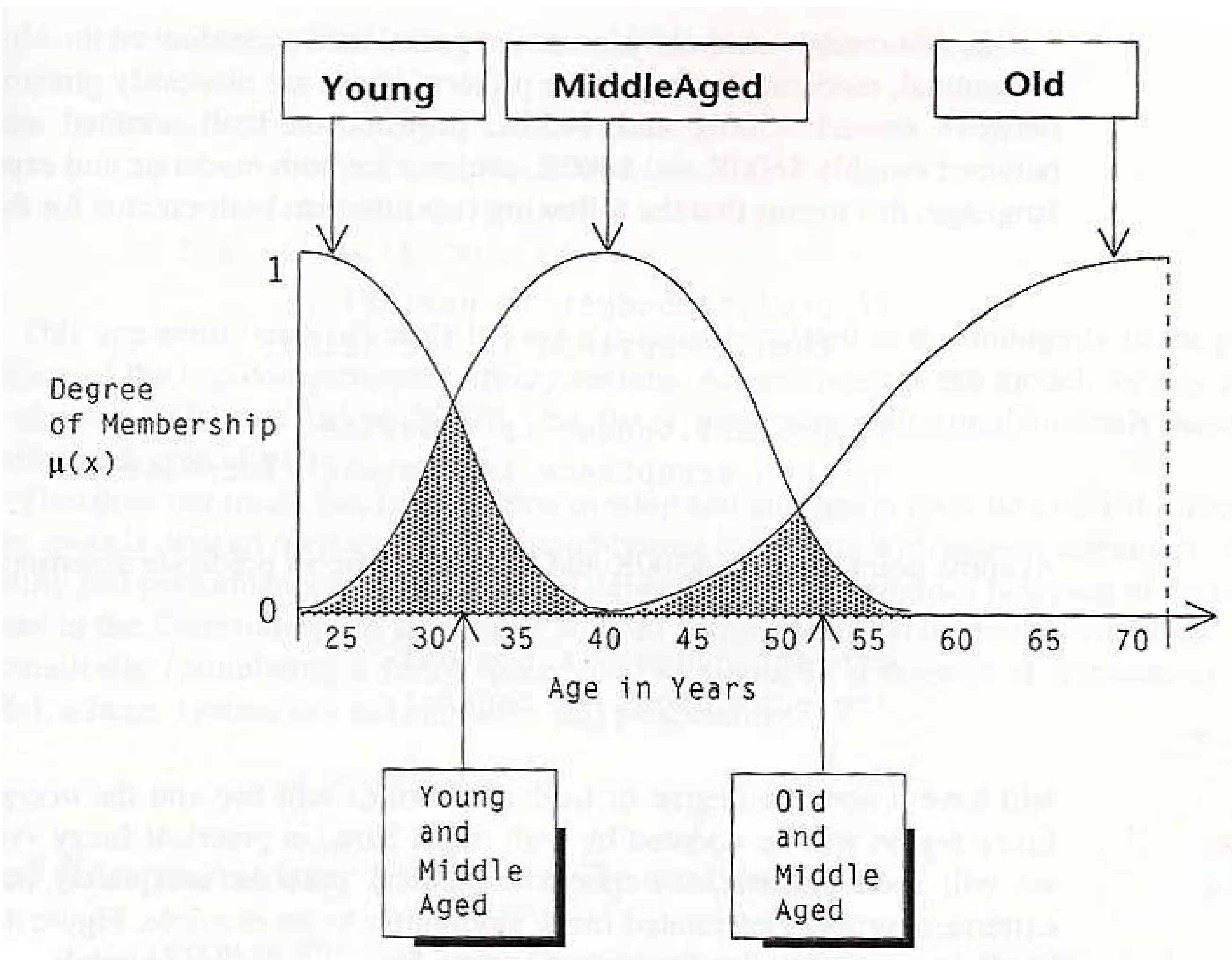
Temperature in the range $[T_1, T_2]$ conceived as: (a) a fuzzy variable; (b) a traditional (crisp) variable.

Membership functions
(figure from Klir &Yuan)



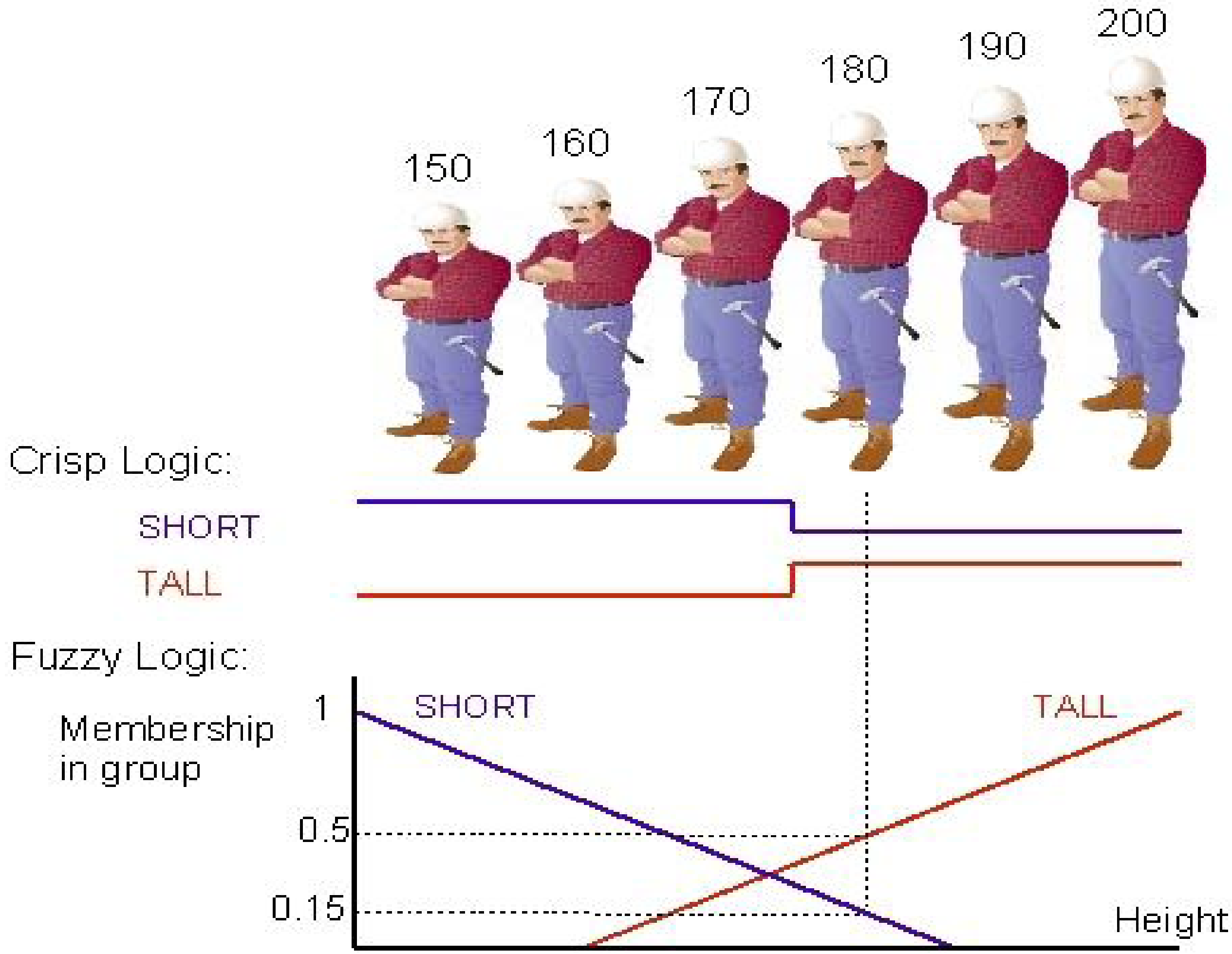
Examples of membership functions that may be used in different contexts for characterizing fuzzy sets of real numbers close to 2.

Fuzzy set (figure from Earl Cox)



Ambiguous Fuzzy Regions in a Shared Domain

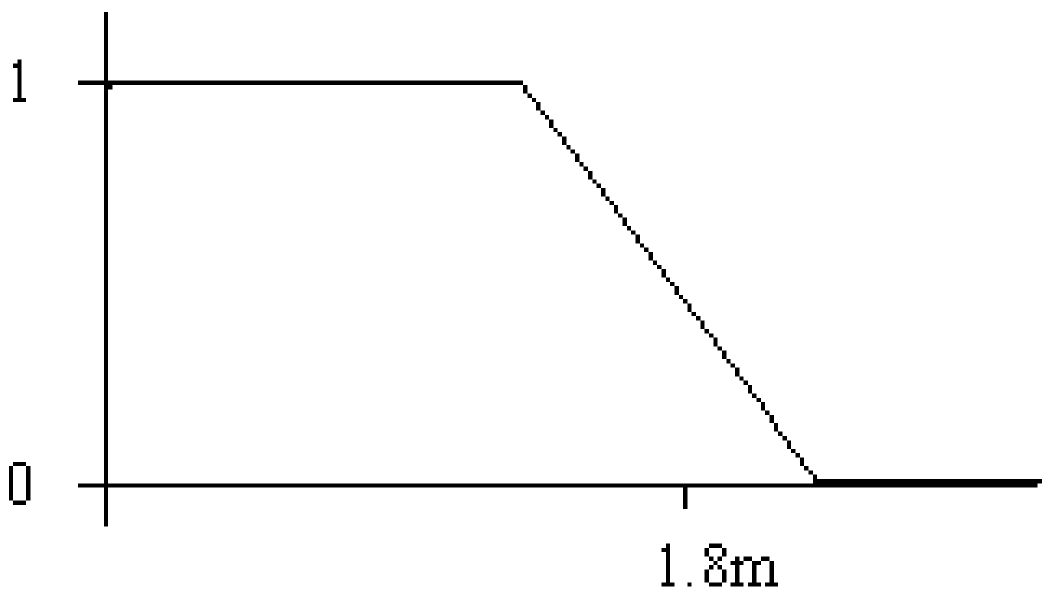
Crisp Logic vs Fuzzy Logic



- Crisp logic needs hard decisions
- Fuzzy Logic deals with “membership in group” functions

Example: Fuzzy Short

- Short(x) = {0 if x >= 1.9m ,
  - 1 if x <= 1.7m
  - else (1.9 - x) / 0.2 }



Membership function

- Crisp set representation
  - Characteristic function

$$f_A(x) : X \rightarrow 0,1$$

$$f_A(x) = \begin{cases} 1, & \text{if} \quad x \in A \\ 0 & \text{if} \quad x \notin A \end{cases}$$

- Fuzzy set representation
  - Membership function
  -

$$\mu_A(x) = 1$$ if x is totally in A

$$\mu_A(x) = 0$$ if x is not in A

$$0 < \mu_A(x) < 1$$ If x is partly in A

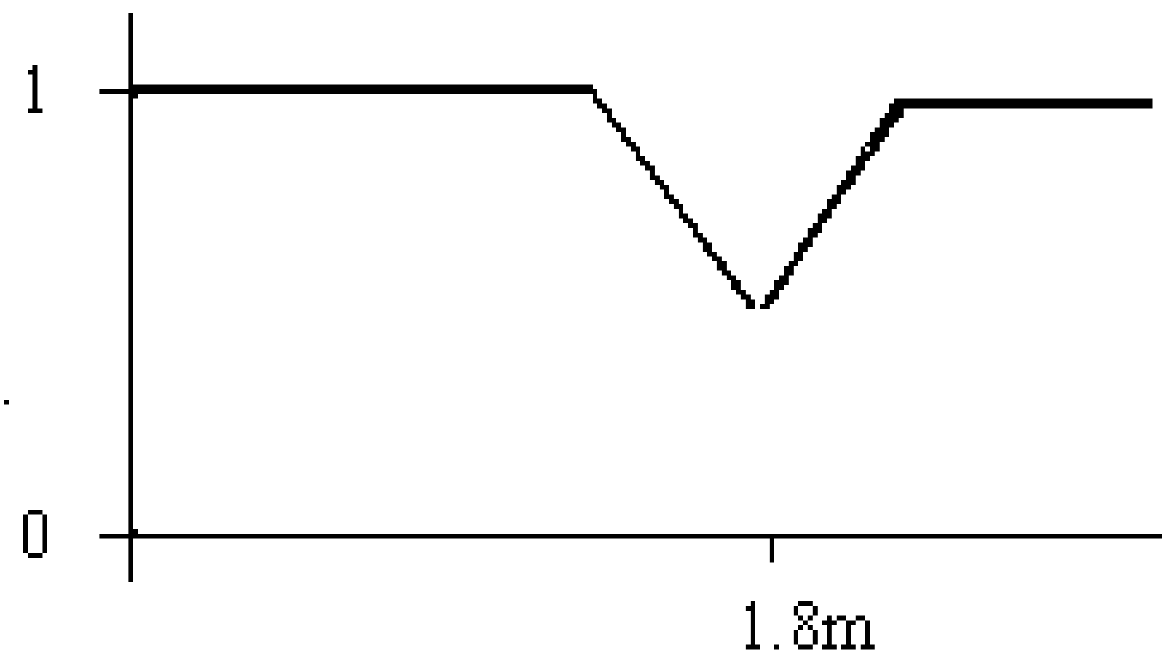
Fuzzy Set Operators

- Fuzzy Set:
  - Union
  - Intersection
  - Complement

- Many possible definitions
  - we introduce one possibility

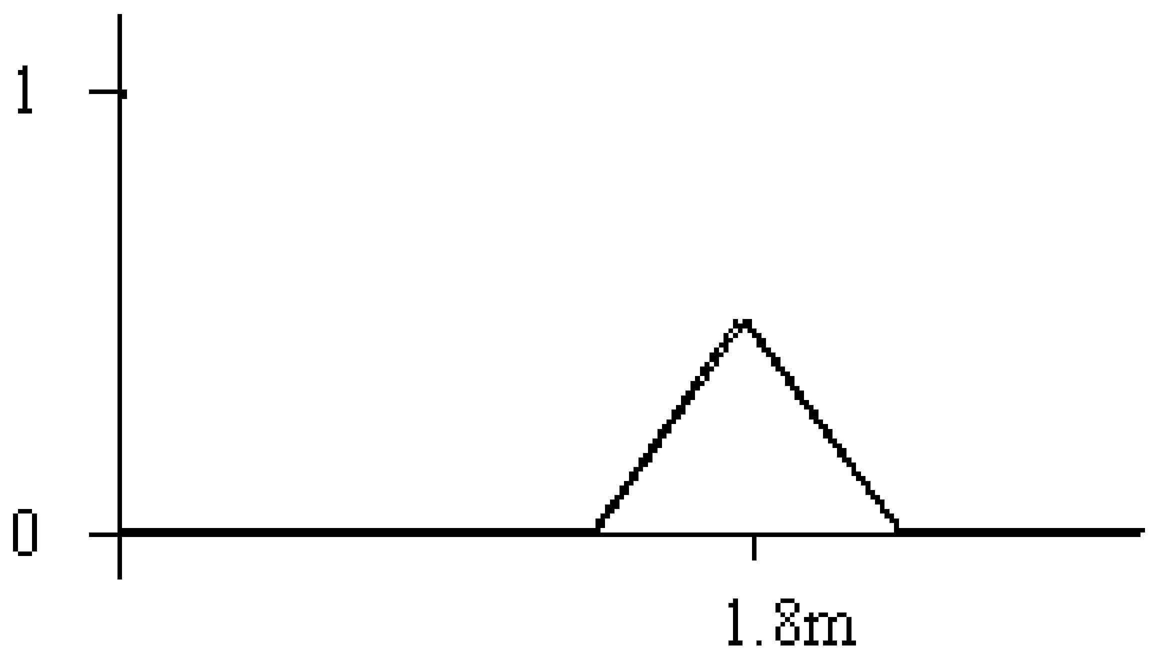
Fuzzy Set Union

- Union ($f_A(x)$ and $f_B(x)$) =
  - max ($f_A(x)$, $f_B(x)$)
- Union (Tall(x) and Short(x))



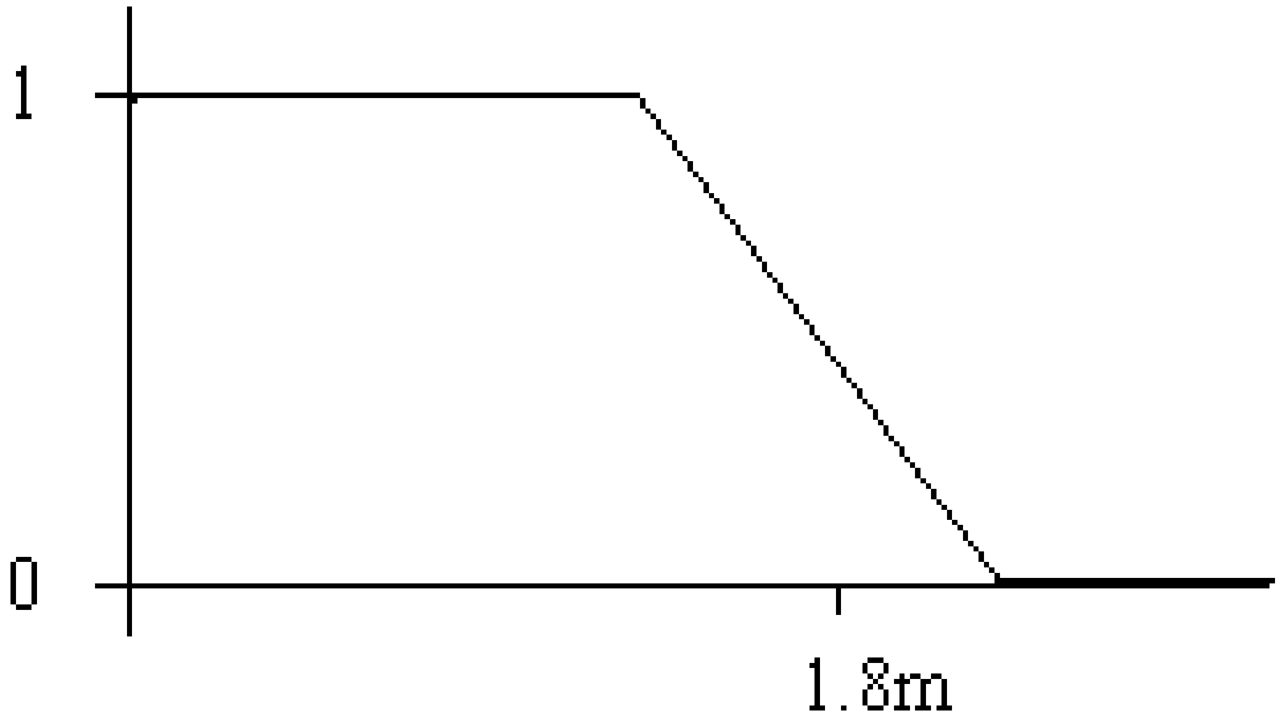
Fuzzy Set Intersection

- Intersection ($f_A(x)$ and $f_B(x)$) =
  - min ($f_A(x)$, $f_B(x)$)
- Intersection (Tall(x) and Short(x))



Fuzzy Set Complement

- Complement($f_A(x)$) = 1 - $f_A(x)$
- Not (Tall(x))



Fuzzy Logic Operators

- Fuzzy Logic:
  - NOT (A) = 1 - A
  - A AND B = min(A, B)
  - A OR B = max(A, B)

Fuzzy Logic NOT

A	NOT A
0	1
0.25	0.75
0.5	0.5
0.75	0.25
1	0

Fuzzy Logic AND

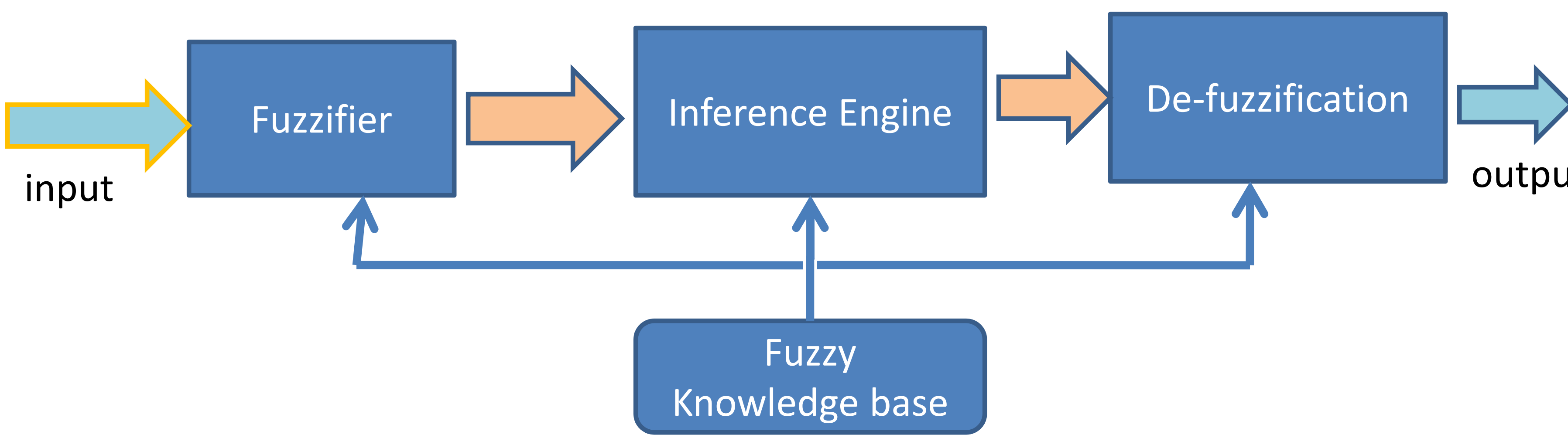
A AND B / B → A ↓	0	0.25	0.5	0.75	1.0
0	0	0	0	0	0
0.25	0	0.25	0.25	0.25	0.25
0.5	0	0.25	0.5	0.5	0.5
0.75	0	0.25	0.5	0.75	0.75
1	0	0.25	0.5	0.75	1

Fuzzy Logic OR

A OR B / B → A ↓	0	0.25	0.5	0.75	1.0
0	0	0.25	0.5	0.75	1.0
0.25	0.25	0.25	0.5	0.75	1.0
0.5	0.5	0.5	0.5	0.75	1.0
0.75	0.75	0.75	0.75	0.75	1.0
1	1.0	1.0	1.0	1.0	1.0

Fuzzy Inference Systems

- Fuzzy logical operations
- Fuzzy rules
- Fuzzification
- Implication
- Aggregation
- Defuzzification



Fuzzy rule as a relation

- if x is A then y is B

  “x is A”, “y is B” – fuzzy predicates A(x), B(y)

- if A(x) then B(y)

  can be represented as a relation

  R(x,y): A(x) ® B(y)

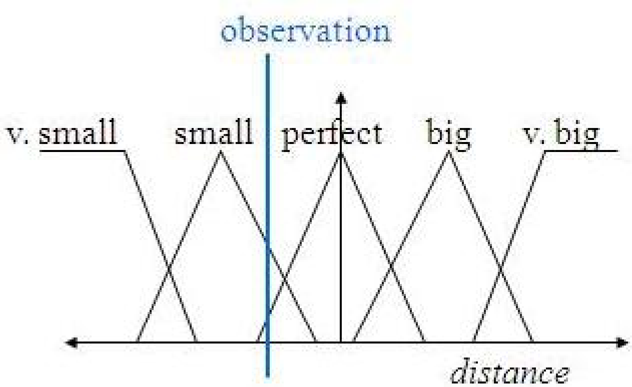
  where R(x,y) can be considered a fuzzy set with 2-dimentional membership function

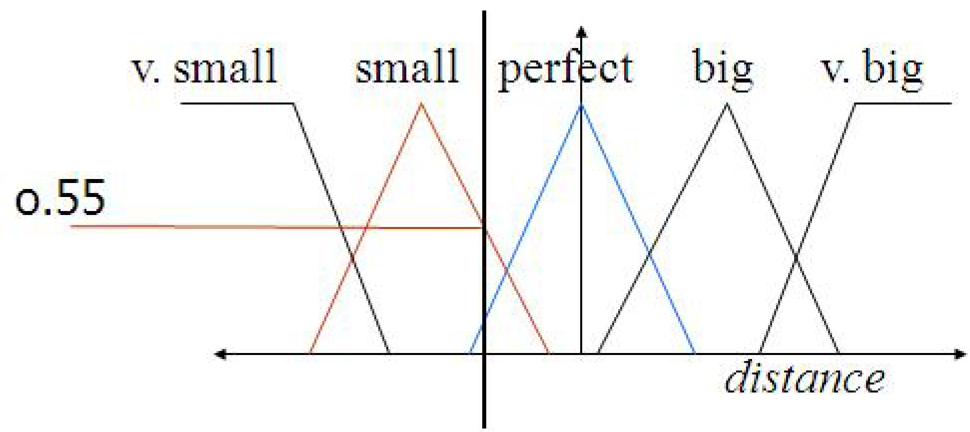
$$\mu_R(x, y) = f(\mu_A(x), \mu_B(y))$$

where f is fuzzy implication function

Fuzzifier

- Converts the crisp input to a linguistic variable using the membership functions stored in the fuzzy knowledge base.





IF distance is Small THEN Slow Down

Example for Fuzzy rules

- For a washing machine, the speed of rotation should be based on
- the quantity of clothes and
- the softness of the clothes

INPUTS:

Laundry Quantity (Fuzzy)

Laundry softness (Fuzzy)

OUTPUT:

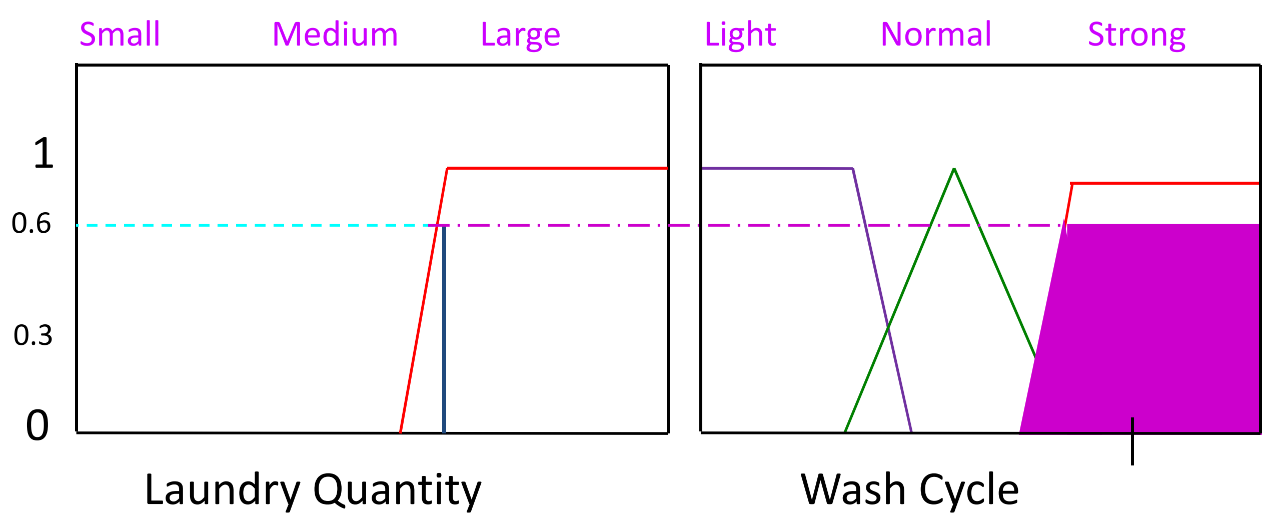
Washing cycle types (Fuzzy)

We need rules connecting these variables

Washing Machine - rule 1

If Laundry quantity is large *(Fuzzy)* then wash cycle is strong *(Fuzzy)*

Washing machine needs a NON-fuzzy information.



Step 4: Defuzzification

The last step in the fuzzy inference process is defuzzification.

Fuzziness helps us to evaluate the rules, but the final output of a fuzzy system has to be a crisp number.

The input for the defuzzification process is the aggregate output fuzzy set and the output is a single number.

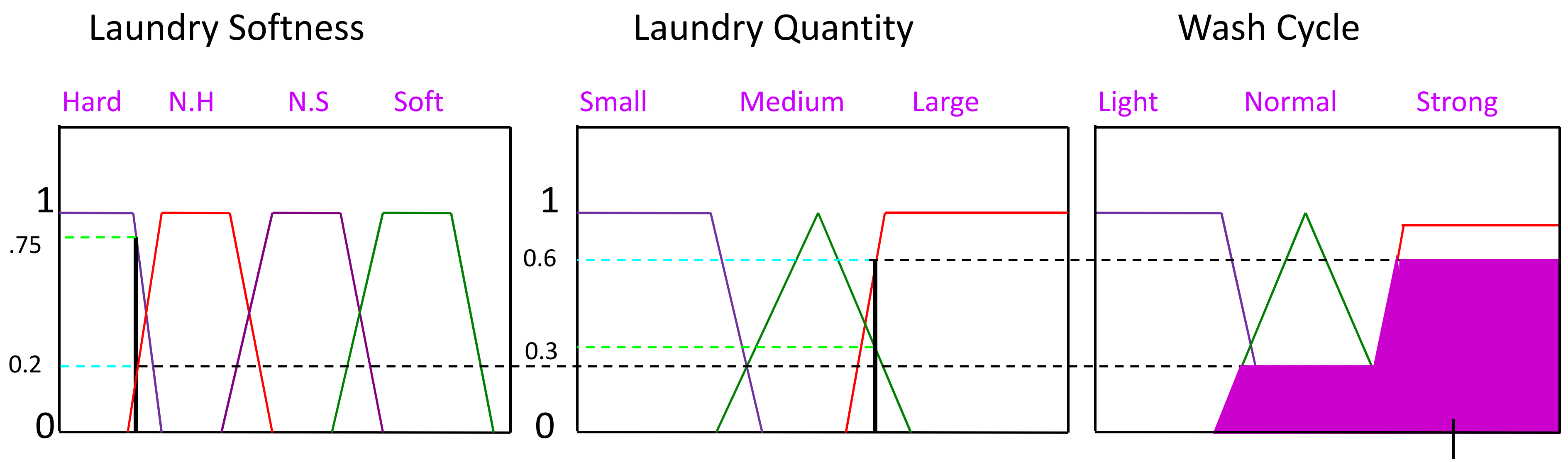
- There are several defuzzification methods, but probably the most popular one is the **centroid technique**.
- It finds the point where a vertical line would slice the aggregate set into two equal masses.
- Mathematically this **centre of gravity (COG)** can be expressed as:

$$\boxed{COG = \frac{\int_a^b \mu_A(x)\, x\, dx}{\int_a^b \mu_A(x)\, dx}}$$

Washing Machine – 2 rules

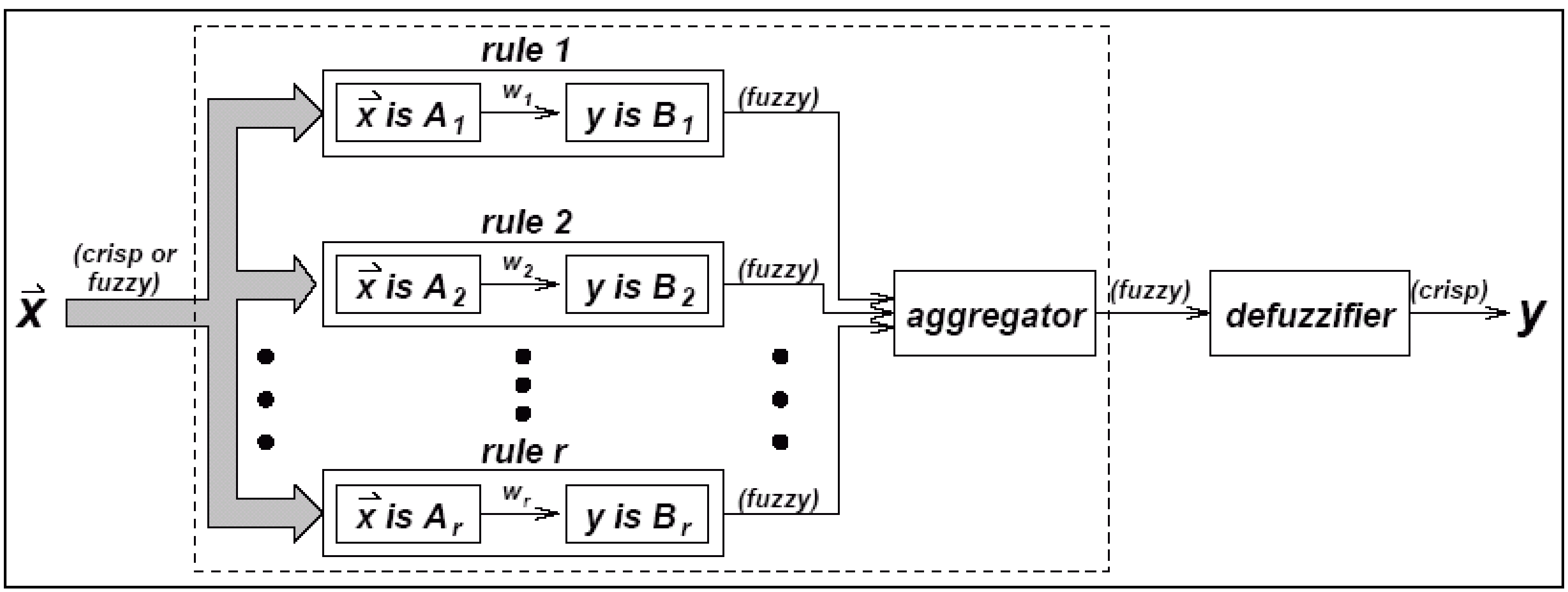
Rule 1: If Laundry quantity is LARGE and Laundry softness is HARD then wash cycle is strong.

Rule 2: If Laundry quantity is MEDIUM and Laundry softness is NOT SO HARD then wash cycle is normal.



Inference Engine

- Using If-Then type fuzzy rules converts the fuzzy input to the fuzzy output.



Fuzzy Relations

Fuzzy Relations

- Generalizes classical relation into one that allows partial membership
  - Describes a relationship that holds between two or more objects
    - Example: a fuzzy relation "Friend" describe the degree of friendship between two person (in contrast to either being friend or not being friend in classical relation!)

Fuzzy Relations

- A fuzzy relation $\tilde{R}$ is a mapping from the Cartesian space X x Y to the interval [0,1], where the strength of the mapping is expressed by the membership function of the relation $\mu_{\tilde{R}}(x,y)$
- The "strength" of the relation between ordered pairs of the two universes is measured with a membership function expressing various "degree" of strength [0,1]

Fuzzy Cartesian Product

Let

$\tilde{A}$ be a fuzzy set on universe X, and

$\tilde{B}$ be a fuzzy set on universe Y, then

$$\tilde{A} \times \tilde{B} = \tilde{R} \subset X \times Y$$

Where the fuzzy relation R has membership function

$$\mu_{\tilde{R}}(x, y) = \mu_{\tilde{A} x \tilde{B}}(x, y) = \min(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(y))$$

Fuzzy Cartesian Product: Example

Let

$\tilde{A}$ defined on a universe of three discrete temperatures, X = {x_1,x_2,x_3}, and

$\tilde{B}$ defined on a universe of two discrete pressures, Y = {y_1,y_2}

Fuzzy set $\tilde{A}$ represents the "ambient" temperature and

Fuzzy set $\tilde{B}$ the "near optimum" pressure for a certain heat exchanger, and the Cartesian product might represent the conditions (temperature-pressure pairs) of the exchanger that are associated with "efficient" operations. For example, let

$$\left.\begin{aligned} \tilde{A} &= \frac{0.2}{x_1} + \frac{0.5}{x_2} + \frac{1}{x_3} \\ &\textit{and} \\ \tilde{B} &= \frac{0.3}{y_1} + \frac{0.9}{y_2} \end{aligned}\right\} \quad \tilde{A} \times \tilde{B} = \tilde{R} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{bmatrix} 0.2 & 0.2 \\ 0.3 & 0.5 \\ 0.3 & 0.9 \end{bmatrix} \end{array}$$

Fuzzy Composition

Suppose

$\tilde{R}$ is a fuzzy relation on the Cartesian space X x Y,

$\tilde{S}$ is a fuzzy relation on the Cartesian space Y x Z, and

$\tilde{T}$ is a fuzzy relation on the Cartesian space X x Z; then fuzzy max-min and fuzzy max-product composition are defined as

$$\tilde{T} = \tilde{R} \circ \tilde{S}$$

$$\max - \min$$

$$\mu_{\tilde{T}}(x,z) = \bigvee_{y \in Y} (\mu_{\tilde{R}}(x,y) \wedge \mu_{\tilde{S}}(y,z))$$

$$\max - product$$

$$\mu_{\tilde{T}}(x,z) = \bigvee_{y \in Y} (\mu_{\tilde{R}}(x,y) \bullet \mu_{\tilde{S}}(y,z))$$

Fuzzy Composition: Example (max-min)

$$X = \{x_1, x_2\}, \quad Y = \{y_1, y_2\}, \text{ and } \quad Z = \{z_1, z_2, z_3\}$$

Consider the following fuzzy relations:

$$\tilde{R} = \begin{array}{c c} & \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \end{array} & \begin{bmatrix} 0.7 & 0.5 \\ 0.8 & 0.4 \end{bmatrix} \end{array} \quad \text{and} \quad \tilde{S} = \begin{array}{c c} & \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{array}{c} y_1 \\ y_2 \end{array} & \begin{bmatrix} 0.9 & 0.6 & 0.5 \\ 0.1 & 0.7 & 0.5 \end{bmatrix} \end{array}$$

Using max-min composition,

$$\left.\begin{aligned} \mu_{\tilde{T}}(x_1, z_1) &= \bigvee_{y \in Y} (\mu_{\tilde{R}}(x_1, y) \wedge \mu_{\tilde{S}}(y, z_1)) \\ &= \max[\min(0.7, 0.9), \min(0.5, 0.1)] \\ &= 0.7 \end{aligned}\right\} \quad \tilde{T} = \begin{array}{c c} & \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \end{array} & \begin{bmatrix} 0.7 & 0.6 & 0.5 \\ 0.8 & 0.6 & 0.4 \end{bmatrix} \end{array}$$

Fuzzy Composition: Example (max-Prod)

$$X = \{x_1, x_2\}, \quad Y = \{y_1, y_2\}, \text{ and } \quad Z = \{z_1, z_2, z_3\}$$

Consider the following fuzzy relations:

$$\tilde{R} = \begin{array}{c} \\ x_1 \\ x_2 \end{array} \begin{array}{c} \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{bmatrix} 0.7 & 0.5 \\ 0.8 & 0.4 \end{bmatrix} \end{array} \quad \text{and} \quad \tilde{S} = \begin{array}{c} \\ y_1 \\ y_2 \end{array} \begin{array}{c} \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{bmatrix} 0.9 & 0.6 & 0.5 \\ 0.1 & 0.7 & 0.5 \end{bmatrix} \end{array}$$

Using max-product composition,

$$\begin{aligned} \mu_{\tilde{T}}(x_2, z_2) &= \bigvee_{y \in Y} (\mu_{\tilde{R}}(x_2, y) \bullet \mu_{\tilde{S}}(y, z_2)) \\ &= \max[(0.8, 0.6), (0.4, 0.7)] \\ &= 0.48 \end{aligned}$$

$$\tilde{T} = \begin{array}{c} \\ x_1 \\ x_2 \end{array} \begin{array}{c} \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{bmatrix} .63 & .42 & .25 \\ .72 & .48 & .20 \end{bmatrix} \end{array}$$

Application: Computer Engineering

Problem: In computer engineering, different logic families are often compared on the basis of their power-delay product. Consider the fuzzy set $\tilde{F}$ of logic families, the fuzzy set $\tilde{D}$ of delay times(ns), and the fuzzy set $\tilde{P}$ of power dissipations (mw).

If $\tilde{F}$ = {NMOS,CMOS,TTL,ECL,JJ},

$\tilde{D}$ = {0.1,1,10,100},

$\tilde{P}$ = {0.01,0.1,1,10,100}

Suppose $\tilde{R}_1 = \tilde{D} \times \tilde{F}$ and $\tilde{R}_2 = \tilde{F} \times \tilde{P}$

$$\tilde{R}_1 = \begin{array}{c} \\ 0.1 \\ 1 \\ 10 \\ 100 \end{array} \begin{array}{c} \begin{array}{ccccc} N & C & T & E & J \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & .6 & 1 \\ 0 & .1 & .5 & 1 & 0 \\ .4 & 1 & 1 & 0 & 0 \\ 1 & .2 & 0 & 0 & 0 \end{bmatrix} \end{array} \quad \text{and} \quad \tilde{R}_2 = \begin{array}{c} \\ N \\ C \\ T \\ E \\ J \end{array} \begin{array}{c} \begin{array}{ccccc} .01 & .1 & 1 & 10 & 100 \end{array} \\ \begin{bmatrix} 0 & .4 & 1 & .3 & 0 \\ .2 & 1 & 0 & 0 & 0 \\ 0 & 0 & .7 & 1 & 0 \\ 0 & 0 & 0 & 1 & .5 \\ 1 & .1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Application: Computer Engineering (Cont)

We can use max-min composition to obtain a relation between delay times and power dissipation: i.e., we can compute or

$$\tilde{R}_3 = \tilde{R}_1 \circ \tilde{R}_2 \qquad \mu_{\tilde{R}_3} = \vee(\mu_{\tilde{R}_1} \wedge \mu_{\tilde{R}_2})$$

$$\tilde{R}_3 = \begin{array}{c} \\ 0.1 \\ 1 \\ 10 \\ 100 \end{array} \begin{array}{c} \begin{array}{ccccc} .01 & .1 & 1 & 10 & 100 \end{array} \\ \begin{bmatrix} 1 & .1 & 0 & .6 & .5 \\ .1 & .1 & .5 & 1 & .5 \\ .2 & 1 & .7 & 1 & 0 \\ .2 & .4 & 1 & .3 & 0 \end{bmatrix} \end{array}$$

Application: Fuzzy Relation: Petite

Application: Fuzzy Relation Petite

Fuzzy Relation Petite defines the degree by which a person with a specific height and weight is considered petite.

Suppose the range of the height and the weight of interest to us are

{5’, 5’1”, 5’2”, 5’3”, 5’4”,5’5”,5’6”},

and

{90, 95,100, 105, 110, 115, 120, 125} (in lb).

Application: Fuzzy Relation Petite

We can express the fuzzy relation in a matrix form as shown below:

$$\tilde{P} = \begin{array}{c} \\ 5' \\ 5'1'' \\ 5'2'' \\ 5'3'' \\ 5'4'' \\ 5'5'' \\ 5'6'' \end{array} \begin{array}{c} \begin{array}{cccccccc} 90 & 95 & 100 & 105 & 110 & 115 & 120 & 125 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & .5 & .2 \\ 1 & 1 & 1 & 1 & 1 & .9 & .3 & .1 \\ 1 & 1 & 1 & 1 & 1 & .7 & .1 & 0 \\ 1 & 1 & 1 & 1 & .5 & .3 & 0 & 0 \\ .8 & .6 & .4 & .2 & 0 & 0 & 0 & 0 \\ .6 & .4 & .2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Application: Fuzzy Relation Petite

$$
\tilde{P} = \begin{array}{c} \\ 5' \\ 5'1'' \\ 5'2'' \\ 5'3'' \\ 5'4'' \\ 5'5'' \\ 5'6'' \end{array}
\begin{array}{c} \begin{array}{cccccccc} 90 & 95 & 100 & 105 & 110 & 115 & 120 & 125 \end{array} \\
\begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & .5 & .2 \\
1 & 1 & 1 & 1 & 1 & .9 & .3 & .1 \\
1 & 1 & 1 & 1 & 1 & .7 & .1 & 0 \\
1 & 1 & 1 & 1 & .5 & .3 & 0 & 0 \\
.8 & .6 & .4 & .2 & 0 & 0 & 0 & 0 \\
.6 & .4 & .2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \end{array}
$$

Once we define the petite fuzzy relation, we can answer two kinds of questions:

- What is the degree that a female with a specific height and a specific weight is considered to be petite?
- What is the possibility that a petite person has a specific pair of height and weight measures? (fuzzy relation becomes a possibility distribution)

Application: Fuzzy Relation Petite

Given a two-dimensional fuzzy relation and the possible values of one variable, infer the possible values of the other variable using similar fuzzy composition as described earlier.

Definition: Let X and Y be the universes of discourse for variables x and y, respectively, and x_i and y_j be elements of X and Y.

Let R be a fuzzy relation that maps X x Y to [0,1]

and the possibility distribution of X is known to be $\Pi_x(x_i)$.

Application: Fuzzy Relation Petite

The compositional rule of inference infers the possibility distribution of Y as follows:

max-min composition: $$\Pi_Y(y_j) = \max_{x_i}(\min(\Pi_X(x_i), \Pi_R(x_i, y_j)))$$

max-product composition: $$\Pi_Y(y_j) = \max_{x_i}(\Pi_X(x_i) \times \Pi_R(x_i, y_j))$$

Application: Fuzzy Relation Petite

Problem: We may wish to know the possible weight of a petite female who is 5'4".

Using max-min compositional, we can find the weight possibility distribution of a petite person 5'4" tall:

$$\tilde{P} = \begin{array}{c} \\ 5' \\ 5'1'' \\ 5'2'' \\ 5'3'' \\ 5'4'' \\ 5'5'' \\ 5'6'' \end{array} \begin{array}{c} \begin{array}{cccccccc} 90 & 95 & 100 & 105 & 110 & 115 & 120 & 125 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & .5 & .2 \\ 1 & 1 & 1 & 1 & 1 & .9 & .3 & .1 \\ 1 & 1 & 1 & 1 & 1 & .7 & .1 & 0 \\ 1 & 1 & 1 & 1 & .5 & .3 & 0 & 0 \\ .8 & .6 & .4 & .2 & 0 & 0 & 0 & 0 \\ .6 & .4 & .2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Fuzzy Logic:Intelligence, Control, and Information, J. Yen and R. Langari, PrenticeHall

Application: Fuzzy Relation Petite

Problem: We may wish to know the possible weight of a petite female who is about 5'4".

Assume About 5'4" is defined as

About-5'4" = {0/5', 0/5'1", 0.4/5'2", 0.8/5'3", 1/5'4", 0.8/5'5", 0.4/5'6"}

Using max-min compositional, we can find the weight possibility distribution of a petite person about 5'4" tall:

$$\tilde{P} = \begin{array}{c} \\ 5' \\ 5'1" \\ 5'2" \\ 5'3" \\ 5'4" \\ 5'5" \\ 5'6" \end{array} \begin{array}{c} \begin{array}{cccccccc} 90 & 95 & 100 & 105 & 110 & 115 & 120 & 125 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & .5 & .2 \\ 1 & 1 & 1 & 1 & 1 & .9 & .3 & .1 \\ 1 & 1 & 1 & 1 & 1 & .7 & .1 & 0 \\ 1 & 1 & 1 & 1 & .5 & .3 & 0 & 0 \\ .8 & .6 & .4 & .2 & 0 & 0 & 0 & 0 \\ .6 & .4 & .2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$\Pi_{weight}(90) = (0 \wedge 1) \vee (0 \wedge 1) \vee (.4 \wedge 1) \vee (.8 \wedge 1) \vee (1 \wedge .8) \vee (.8 \wedge .6) \vee (.4 \wedge 0)$$
$$= 0.8$$

Similarly, we can compute the possibility degree for other weights. The final result is

$$\Pi_{weight} = \{0.8/90, 0.8/95, 0.8/100, 0.8/105, 0.5/110, 0.4/115, 0.1/120, 0/125\}$$