FUZZY LOGIC SYSTEMS: ORIGIN, CONCEPTS, AND TRENDS

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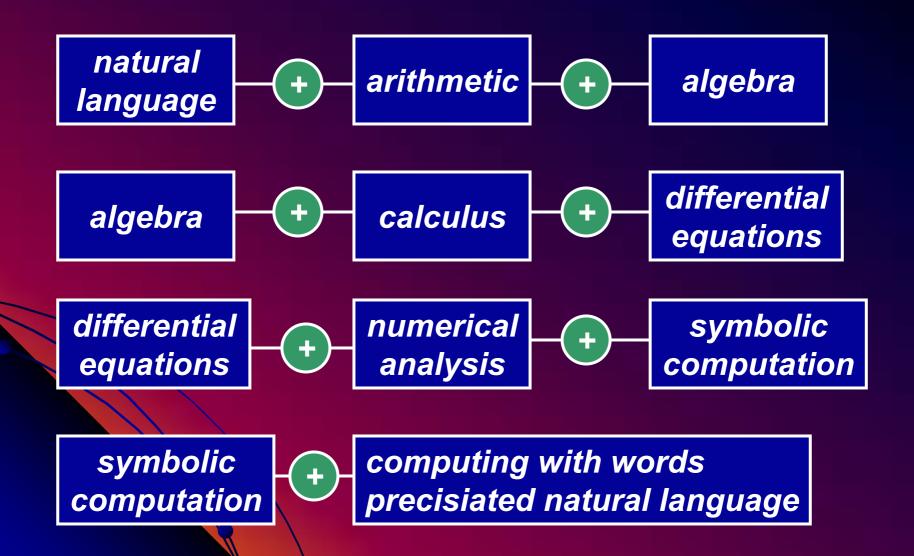
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Hong Kong

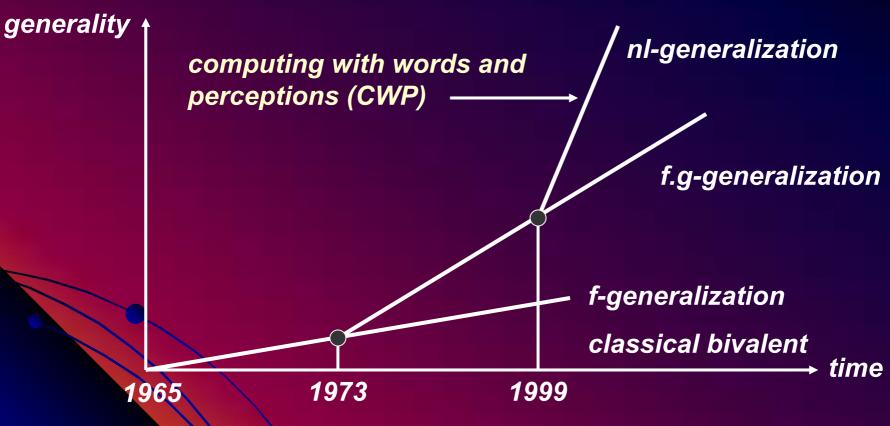
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EVOLUTION OF COMPUTATION



EVOLUTION OF FUZZY LOGIC—A PERSONAL PERSPECTIVE



1965: crisp sets —————— fuzzy sets

1999: measurements ---- perceptions

- In bivalent logic, BL, truth is bivalent, implying that every proposition, p, is either true or false, with no degrees of truth allowed
- In multivalent logic, ML, truth is a matter of degree
- In fuzzy logic, FL:
 - everything is, or is allowed to be, to be partial, i.e., a matter of degree
 - everything is, or is allowed to be, imprecise (approximate)
 - everything is, or is allowed to be, granular (linguistic)
 - everything is, or is allowed to be, perception based

 Fuzzy logic is much more general than traditional logical systems. The greater generality of fuzzy logic is needed to deal with complex problems in the realms of search, question-answering decision and control. Fuzzy logic provides a foundation for the development of new tools for dealing with natural languages and knowledge representation. Among these tools are: Computing with Words (CW); Precisiated Natural Language (PNL); Computational Theory of Perceptions (CTP); Protoform Theory (PT); Theory of Hierarchical Definability (THD); Perception-Based Probability Theory (PTp); Unified Theory of Uncertainty (UTU).

WHAT IS FUZZY LOGIC?

fuzzy logic (FL) is aimed at a formalization of modes of reasoning which are approximate rather than exact

examples:

exact all men are mortal

Socrates is a man

Socrates is mortal

approximate n

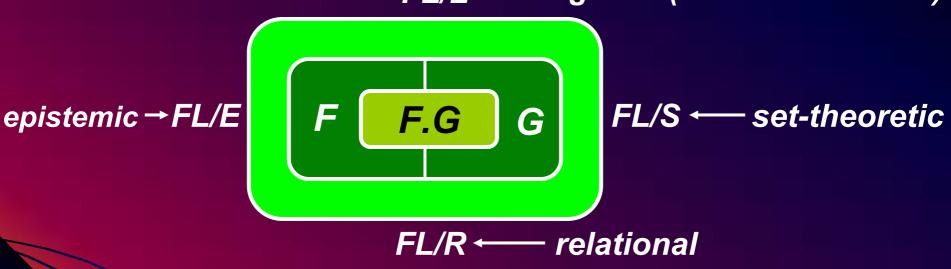
most Swedes are tall

Magnus is a Swede

it is likely that Magnus is tall

fuzzy logic (FL) has four principal facets

FL/L ← logical (narrow sense FL)



F: fuzziness/ fuzzification

G: granularity/ granulation

F.G: F and G

- The logical facet, FL/L, is focused on logical systems in which truth is a matter of degree
 a degree which is allowed to be a fuzzy set
- The set-theoretic facet, FL/S, is concerned, in the main, with the theory of fuzzy sets.
 Most of the mathematical literature on fuzzy logic relates to FL/S
- The relational facet, FL/R, is focused on fuzzy dependencies, granulation, linguistic variables and fuzzy rule sets. Most practical applications of fuzzy logic relate to FL/R

 The epistemic facet, FL/E, is concerned, in the main, with knowledge representation, natural languages, semantics and expert systems. Probabilistic and possibilistic modes of reasoning are a part of this facet as well as FL/L and FL/R

- fuzzy logic has been and still is, though to a lesser degree, an object of controversy
- for the most part, the controversies are rooted in misperceptions, especially a misperception of the relation between fuzzy logic and probability theory
- a source of confusion is that the label "fuzzy logic" is used in two different senses
 - (a) narrow sense: fuzzy logic is a logical system
 (b) wide sense: fuzzy logic is coextensive with fuzzy set theory
- today, the label "fuzzy logic" (FL) is used for the most part in its wide sense

SOME COMMENTS ON FUZZY LOGIC

R.E. Kalman (1972)

Let me say quite categorically that there is no such thing as a fuzzy concept, We do talk about fuzzy things but they are not scientific concepts. Some people in the past have discovered certain interesting things, formulated their findings in a non-fuzzy way, and therefore we have progressed in science.

Professor William Kahan (1975)

"Fuzzy theory is wrong, wrong, and pernicious." says William Kahan, a professor of computer sciences and mathematics at Cal whose Evans Hall office is a few doors from Zadeh's. "I can not think of any problem that could not be solved better by ordinary logic."

"What Zadeh is saying is the same sort of things 'Technology got us into this mess and now it can't get us out." Kahan says. "Well, technology did not get us into this mess. Greed and weakness and ambivalence got us into this mess. What we need is more logical thinking, not less. The danger of fuzzy theory is that it will encourage the sort of imprecise thinking that has brought us so much trouble."

STATISTICS

Count of papers containing the word "fuzzy" in title, as cited in INSPEC and MATH.SCI.NET databases. (data for 2003 are not complete)

Compiled by Camille Wanat, Head, Engineering Library, UC Berkeley, November 20, 2003

INSPEC/TUZZY		watn.5ci.net/f
1970-1979	569	443
1980-1989	2,404	2,465
1990-1999	23,207	5,479
2000-present	9,945	2,865
1970-present	36,125	11,252

NUMBERS ARE RESPECTED—WORDS ARE NOT

 in science and engineering there is a deepseated tradition of according much more respect to numbers than to words. The essence of this tradition was stated succinctly by Lord Kelvin in 1883.

 "In physical science the first essential step in the direction of learning any subject is to find principles of numerical reckoning and practicable methods for measuring some quality connected with it. I often say that when you can measure what you are speaking about and express it in numbers, you know something about it; but when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meager and unsatisfactory kind: it may be the beginning of knowledge but you have scarcely, in your thoughts, advanced to the state of science, whatever the matter may be."

- The risk of a 6.0 quake—which could be more damaging, with one-tenth the destructive power of the October 17 quake—is 11 percent during the next two months, the survey's scientists say.
- The seismologists in Menlo Park say the probability of an aftershock of a magnitude of 5 or more in the next two months is 45 percent.
- It is very unusual for a quake of this size not to come close to the surface. As a result, Dr. Holzer said, geologists have begun to doubt their ability to make reliable estimates for future major earthquakes and to recognize active faults.

Washington Analysis Corporation (The New York Times)

 Bruce Likness, a farm equipment dealer and long-time friend of Waletich, estimates that a beginner needs \$409,780 to \$526,487 worth of machinery to have a chance of success on a 1,500-acre farm.

- Reducing smog would save lives, Bay report says (San Francisco Examiner)
- Expected to attract national attention, the Santa Clara Criteria Air Pollutant Benefit Analysis is the first to quantify the effects on health of air pollution in California
- Removing lead from gasoline could save the lives of 26.7 Santa Clara County residents and spare them 18 strokes, 27 heart attacks, 722 nervous system problems and 1,668 cases where red blood cell production is affected

 Study projects S.F. 5-year AIDS toll (S.F. Chronicle July 15, 1992)

 The report projects that the number of new AIDS cases will reach a record 2,173 this year and decline thereafter to 2,007 new cases in 1997

Robert Shuster

(Ned Davis Research)

We classify a bear market as a 30 percent decline after 50 days, or a 13 percent decline after 145 days.

Warren Buffet

(Fortune 4-4-94)

It is better to be approximately right than precisely wrong.

RATIONALE FOR FUZZY LOGIC

 In the evolution of science a time comes when alongside the brilliant successes of a theory, T, what become visible are classes of problems which fall beyond the reach of T. At that point, the stage is set for a progression from T to T*--a generalization of T

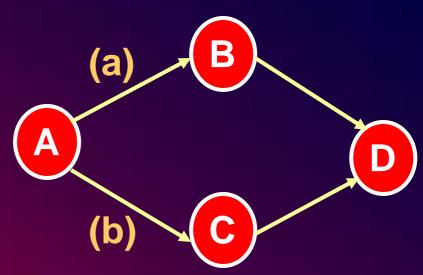
Among the many historical examples are the transitions from Newtonian mechanics to quantum mechanics; from linear system theory to nonlinear system theory; and from deterministic models to probabilistic models in economics and decision analysis

Fuzzy logic is a better approximation to reality

- In this perspective, a fundamental point-- a point which is not as yet widely recognized-- is that there are many classes of problems which cannot be addressed by any theory, T, which is based on bivalent logic. The problem with bivalent logic is that it is in fundamental conflict with reality- a reality in which almost everything is a matter of degree
- To address such problems what is needed is a logic for modes of reasoning which are approximate rather than exact. This is what fuzzy logic is aimed at.

THE TRIP-PLANNING PROBLEM

- I have to fly from A to D, and would like to get there as soon as possible
- I have two choices: (a) fly to D with a connection in B; or
 (b) fly to D with a connection in C



- if I choose (a), I will arrive in D at time t₁
- if I choose (b), I will arrive in D at time t₂
- t_1 is earlier than t_2
- therefore, I should choose (a) ?

- now, let us take a closer look at the problem
- the connection time, c_B, in B is short
- should I miss the connecting flight from B to D, the next flight will bring me to D at t₃
- t₃ is later than t₂
- what should I do?

 $decision = f(t_1, t_2, t_3, c_B, c_C)$

existing methods of decision analysis do not have the capability to compute f

reason: nominal values of decision variables + observed values of decision variables

- the problem is that we need information about the probabilities of missing connections in B and C.
- I do not have, and nobody has, measurement-based information about these probabilities
- whatever information I have is perception-based
- with this information, I can compute perception-based granular probability distributions of arrival times in D for (a) and (b)
- the problem is reduced to ranking of granular probability distributions

Note: subjective probability = perception of likelihood

DEEP STRUCTURE (PROTOFORM)

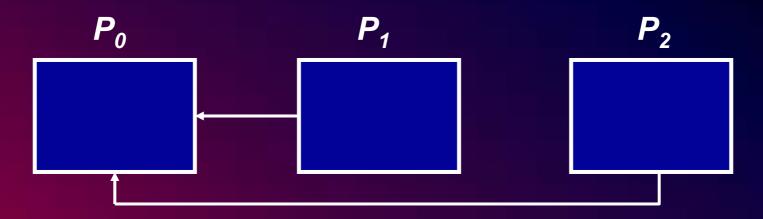
time of arrival $t_3
ightharpoonup - missed connection$ $t_1
ightharpoonup t_2$ $t_1
ightharpoonup - alternatives$

- decision is a function of t₁, t₂, t₃ and the perceived probability of missing connection
- strength of decision

THE PARKING PROBLEM

 I have to drive to the post office to mail a package. The post office closes at 5 pm. As I approach the post office, I come across two parking spots, P_1 and P_2 , P_1 is closer to the post office but it is in a yellow zone. If I park my car in P₁ and walk to the post office, I may get a ticket, but it is likely that I will get to the post office before it closes. If I park my car in P2 and walk to the post office, it is likely that I will not get there before the post office closes. Where should I park my car?

THE PARKING PROBLEM



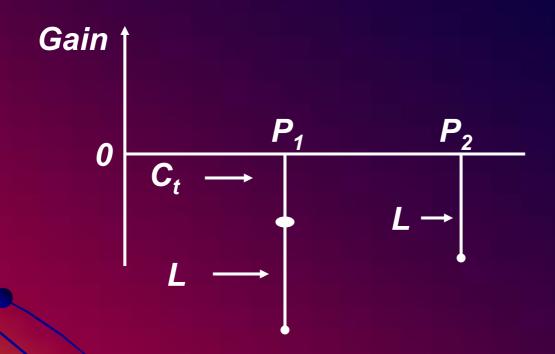
- P₁: probability of arriving at the post office after it closes, starting in P₁
- Pt: probability of getting a ticket
- C_t: cost of ticket
- P₂: probability of arriving at the post office after it closes, starting in P₂
- L: loss if package is not mailed

 C_t : expected cost of parking in P_1 $C_1 = C_t + p_1 L$

 C_2 : expected cost of parking in C_2 $C_2 = p_2L$

- standard approach: minimize expected cost
- standard approach is not applicable when the values of variables and parameters are perception-based (linguistic)

DEEP STRUCTURE (PROTOFORM)



MEASUREMENTS VS. PERCEPTIONS

- what we are beginning to appreciate—and what Lord Kelvin did not—is the fundamental importance of the remarkable human capability to perform a wide variety of physical and mental tasks without any measurements and any computations.
- in performing such tasks, exemplified by driving a car in city traffic, we employ perceptions of distance, speed, time, position, shape, likelihood, intent, similarity and other attributes of physical and mental objects.

MEASUREMENT-BASED VS. PERCEPTION-BASED INFORMATION



measurement-based numerical perception-based linguistic

- •it is 35 C°
- •Eva is 28

- It is very warm
- Eva is young
- •it is cloudy
- traffic is heavy
- it is hard to find parking near the campus

 measurement-based information may be viewed as special case of perception-based information

COMPUTATION WITH PERCEPTIONS

Dana is young

Tandy is a few years older than Dana

Tandy is ?A

Y is several times larger than X Y is large

X is ?A

small × X + small × Y = medium medium × X + large × Y = large X is ?A, Y is ?B

REASONING WITH PERCEPTIONS

simple examples

Dana is young
Tandy is a few years older than Dana
Tandy is (young + few)

most Swedes are tall most Swedes are blond (2most-1) Swedes are tall and blond

most Swedes are tall most² Swedes are very tall

FROM NUMBERS TO WORDS

- There is a deep-seated tradition in science of striving for the ultimate in rigor and precision
- Words are less precise than numbers
- Why and where, then, should words be used?
- 1. When the available information is perception-based or not precise enough to justify the use of numbers
- 2. When there is a tolerance for imprecision which can be exploited to achieve tractability, simplicity, robustness and low solution cost
- 3. When the expressive power of words is greater than the expressive power of numbers

VARIABLES AND LINGUISTIC VARIABLES

- one of the most basic concepts in science is that of a variable
- variable -numerical (X=5; X=(3, 2); ...)
 -linguistic (X is small; (X, Y) is much larger)
- a linguistic variable is a variable whose values are words or sentences in a natural or synthetic language (Zadeh 1973)
- the concept of a linguistic variable plays a central role in fuzzy logic and underlies most of its applications

LINGUISTIC VARIABLES AND F-GRANULATION (1973)

example: Age

primary terms: young, middle-aged, old

modifiers: not, very, quite, rather, ...

linguistic values: young, very young, not very young

and not very old, ...



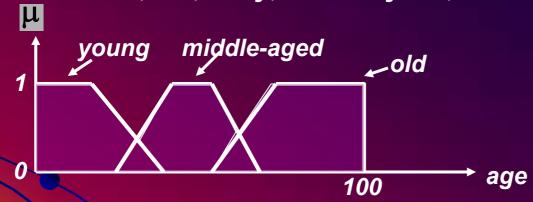
EXAMPLES OF F-GRANULATION (LINGUISTIC VARIABLES)

color: red, blue, green, yellow, ...

age: young, middle-aged, old, very old

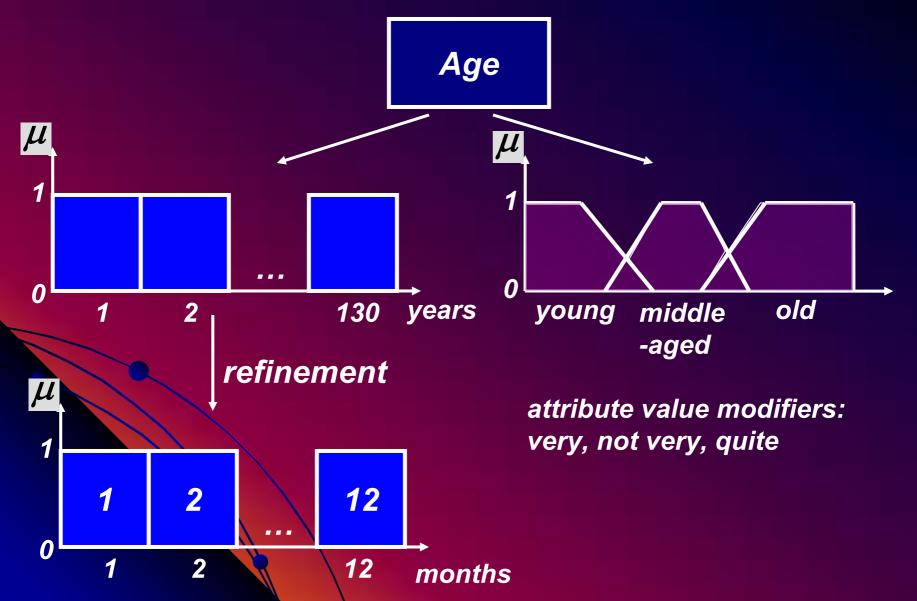
size: small, big, very big, ...

distance: near, far, very, not very far, ...



- humans have a remarkable capability to perform a wide variety of physical and mental tasks, e.g., driving a car in city traffic, without any measurements and any computations
- one of the principal aims of CTP is to develop a better understanding of how this capability can be added to machines

GRANULATION OF AGE

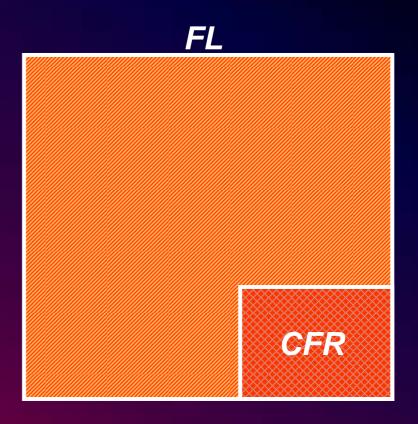


F-GRANULARITY AND F-GRANULATION

- perceptions are f-granular (fuzzy and granular)
 fuzzy: unsharp class boundaries
 gradual transition from membership to non-membership
 - granular: class elements are grouped into granules, with a granule being a clump of elements drawn together by indistinguishability, similarity, proximity or functionality
- f-granular is a manifestation of a fundamental limitation on the cognitive ability of humans to resolve detail and store information
- f-granulation serves two major purposes:
 - (a) Data compression
 - (a') Suppression of decision-irrelevant detail
 - (b) Divide and conquer

PRINCIPAL APPLICATIONS OF FUZZY LOGIC

- control
- consumer products
- industrial systems
- automotive
- decision analysis
- medicine
- geology
- pattern recognition
- robotics



CFR: calculus of fuzzy rules

EMERGING APPLICATIONS OF FUZZY LOGIC

- computational theory of perceptions
- natural language processing
- financial engineering
- biomedicine
- legal reasoning
- forecasting

CALGULUS OF FUZZY RULES

CALCULUS OF FUZZY RULES (CFR)

- <u>syntax</u>: legal forms of rules
 if X is A then Y is B
 if X is A then Y is B unless Z is C
- taxonomy: classification of rules
 - categoricalif X is then Y is B
 - qualified
 if X is A then usually (Y is B)
- semantics: meaning of rules
 - single rule
 - collection of rules

FUZZY IF-THEN RULES

examples (free form) simple: If pressure is high then volume is low compound: if inflation is very low and unemployment is very high then a substantial reduction in the interest rate is called for dynamic: if goal is right turn and light is red then stop; then if intersection is clear make right turn fact: pressure is low command: reduce speed if road is slippery dispositional: usually it is foggy in San Francisco in July and August gradual: the more a tomato is ripe the more it is red exceptional: a tomato is red unless it is unripe

DEPENDENCY AND COMMAND

Dependency

Y is large if X is small

Y is medium if X is medium

Y is small if X is large

Command
 reduce Y slightly if X is small
 reduce Y substantially if X is not small

TAXONOMY OF RULES IN FDCL

```
categorical (examples)
                                               (fact)
X is A
if X is A then Y is B or equivalently Y is B if X is A
if X is A and Y is B then U is C and W is D
if X is A then Y is f(A)
if X is A then Action is B
                                               (command)
if X is A and Context is B then replace X is A with X is C
                                            (replacement)
if X is A then delete (if X is B then Y is C)
                                               (metarule)
if X is A then add (if X is B then Y is C)
                                               (metarule)
the more X is A the more Y is B
                                               (gradual)
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TAXONOMY OF RULES IN FDCL

• qualified (examples) if X is A then Y is B unless Z is E (exception) if X is A then usually (Y is B) (usuality qualified) usually (if X is A then Y is B) if X is A and Prob $\{Y \text{ is } B | X \text{ is } A\}$ is C then Action is D if X is A then possibly (Y is B) (possibility qualified) (if X is A then Y is B) is possible α (possibilistic) (if X is A then Y is B) is true α (truth qualified)

hybrid (examples)
usually (the more X is A the more Y is B)
If X is A then very likely (Y is B) unless Z is E

...

SEMANTICS OF SINGLE RULES

categorical

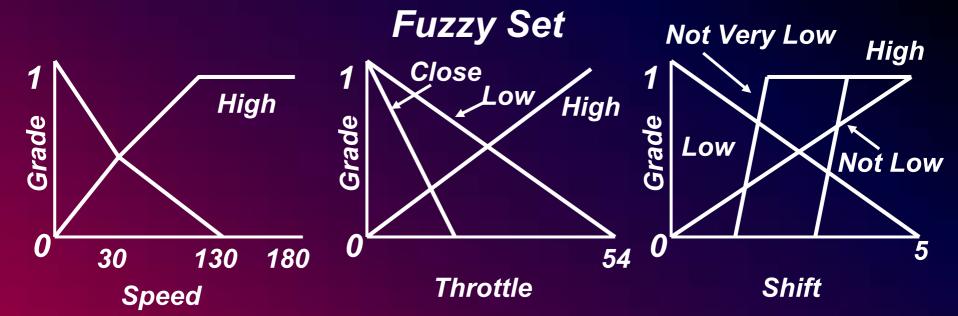
- If X1 is A1 and ... Xn is An then Y is B1 and Yn is Bn
- (sugeno) If X1 is A1 and ... Xn is An then Y is (b0 + D) bi

exception if X is A then Y is B unless Z is E truth qualified if X is A then Y is B is very true probability-qualified if X is A then Y is B is likely possibility-qualified if X is A then Y is B is quite possible

FUZZY IF-THEN RULES

- increase interest rates slightly if unemployment is low and inflation is moderate
- increase interest rates sharply if unemployment is low and inflation is moderate but rising sharply
- decrease interest rates slightly if unemployment is low but increasing and inflation rate is low and stable

HONDA FUZZY LOGIC TRANSMISSION



Control Rules:

- 1. If (speed is low) and (shift is high) then (-3)
- 2. If (speed is high) and (shift is low) then (+3)
- 3. If (throt is low) and (speed is high) then (+3)
- 4. If (throt is low) and (speed is low) then (+1)
- 5. If (throt is high) and (speed is high) then (-1)
- 6. If (throt is high) and (speed is low) then (-3)

INTERPOLATION

Y is B_1 if X is A_1 Y is B_2 if X is A_2

Y is B_n if X is A_n Y is ?B if X is A

 $A \neq A_1, \ldots, A_n$

Conjuctive approach (Zadeh 1973)

Disjunctive approach (Zadeh 1971, Zadeh 1973, Mamdani 1974)

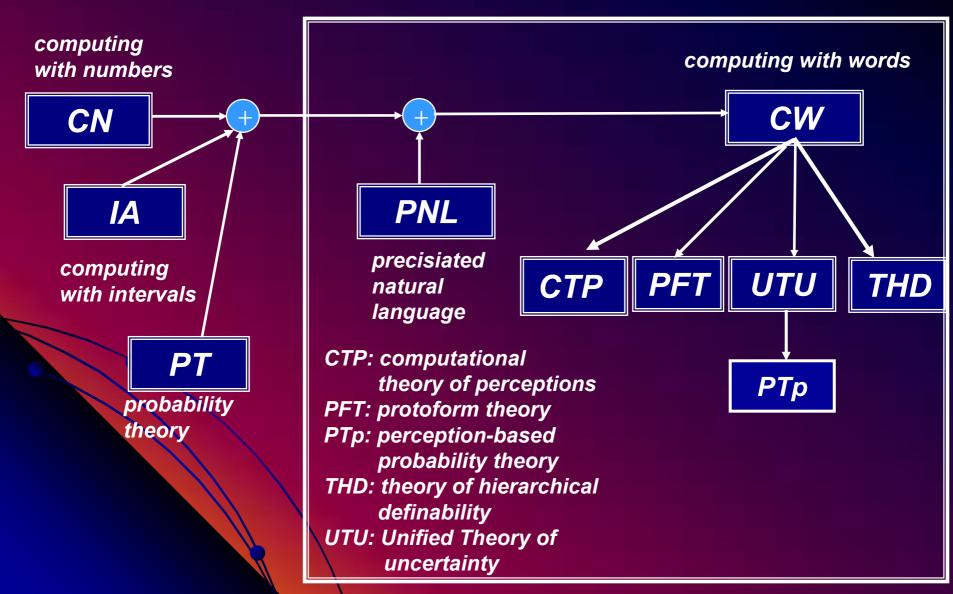
THE "IT IS POSSIBLE BUT NOT PROBABLE" DILEMMA—THE ROCK ON WHICH MANY CRISP THEORIES FOUNDER

- decision is based on information
- in most real-world settings, decision-relevant information is incomplete, uncertain and imprecise
- to assess the consequences of a decision when decision-relevant information is not complete, requires consideration of all possible scenarios
- among such scenarios, a scenario that plays a pivotal role is the worst-case scenario

THE DILEMMA

- worst-case scenario is possible
- what is the probability of the worst-case scenario?
- the problem is that, in general, the probability of worst-case scenario does not lend itself to crisp assessment
- this problem is a rock on which many crisp theories founder

NEW TOOLS



GRANULAR COMPUTING GENERALIZED VALUATION

valuation = assignment of a value to a variable

X = 5 point $0 \le X \le 5$ X is small X isr R interval fuzzy interval generalized

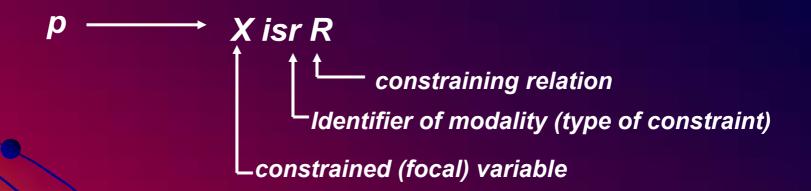
singular value measurement-based

granular values
perception-based



THE BASICS OF PNL

- The point of departure in PNL is the key idea:
 - A proposition, p, drawn from a natural language, NL, is precisiated by expressing its meaning as a generalized constraint



- In general, X, R, r are implicit in p
- precisiation of p ← explicitation of X, R, r

SIMPLE EXAMPLE

Eva is young Age(Eva) is young

- Annotated representation
 - X/Age(Eva) is R/young

KEY POINTS

A proposition is an answer to a question

example:

p: Eva is young

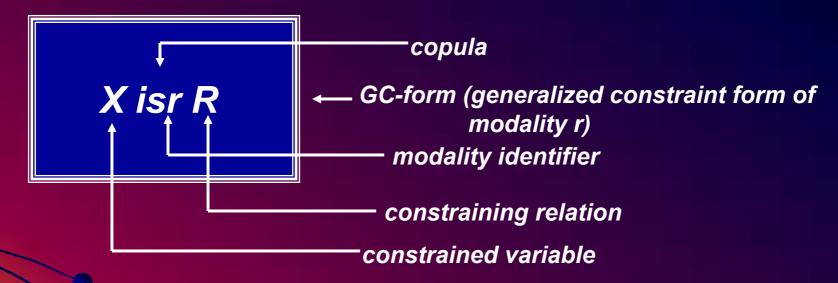
is an answer to the question q: How old is Eva?

 The concept of a generalized constraint serves as a basis for generalized-constraintbased semantics of natural languages

THE CENTERPIECE OF PNL IS THE CONCEPT OF A GENERALIZED CONSTRAINT (ZADEH 1986)

GENERALIZED CONSTRAINT

- •standard constraint: X ∈ C
- •generalized constraint: X isr R



- $\bullet X = (X_1, ..., X_n)$
- •X may have a structure: X=Location (Residence(Carol))
- •X may be a function of another variable: X=f(Y)
- X may be conditioned: (X/Y)
- r := *l* ≤ *l* ... *l* ⊂ *l* ⊃ *l* blank *l* v *l* p *l* u *l* rs *l* fg *l* ps *l* ...

GENERALIZED CONSTRAINT—MODALITY r

X isr R

r:=

equality constraint: X=R is abbreviation of X is=R

r: ≤ inequality constraint: X ≤ R subsethood constraint: X ⊂ R r:c possibilistic constraint; X is R; R is the possibility r: blank distribution of X veristic constraint; X isv R; R is the verity distribution of X probabilistic constraint; X isp R; R is the r: p probability distribution of X

CONTINUED

- r: rs random set constraint; X isrs R; R is the setvalued probability distribution of X
- r: fg fuzzy graph constraint; X isfg R; X is a function and R is its fuzzy graph
- r: u usuality constraint; X isu R means usually (X is R)
- r: ps Pawlak set constraint: X isps (X, X) means that X is a set and X and X are the lower and upper approximations to X

CONSTRAINT QUALIFICATION

- verity (truth) qualification
 (X isr R) is τ
- probability qualification (X isr R) is p
- possibility qualification (X isr R) is π
- truth, probability and possibility are attributes of propositions

GENERALIZED CONSTRAINT LANGUAGE (GCL)

- GCL is an abstract language
- GCL is generated by combination, qualification and propagation of generalized constraints
- examples of elements of GCL
 - (X isp R) and (X,Y) is S)
 - (X isr R) is unlikely) and (X iss S) is likely
 - If X is A then Y is B
- the language of fuzzy if-then rules is a sublanguage of GCL

deduction= generalized constraint propagation

EXAMPLE OF DEDUCTION

compositional rule of inference in FL

$$\mu_{A\circ B}(v)=v_u(\mu_A(u)\wedge\mu_B(v,u))$$

```
\wedge = min (t-norm)

\wedge = max (t-conorm)
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INFORMATION AND GENERALIZED CONSTRAINTS—KEY POINTS

In CW, the carriers of information are propositions

p: proposition

GC(p): X isr R

p is a carrier of information about X

GC(p) is the information about X carried by p

MODALITIES OF INFORMATION

- Probability-based: X isp R
- Verity-based: X isv R
- Possibility-based: X is R
- Generalized: X isr R
- Hybrid: (X isr R) ∧ (X iss S)
- unimodal, bimodal, trimodal

STATISTICAL INFORMATION THEORY (SHANNON)

Modality
p: X isp R
R is probability distribution of X

 statistical information theory is concerned with measure of information rather than with its meaning

PRECISIATION—KEY POINTS

- precisiation of p = translation of p into GCL
- GCL plays the role of a precisiation language
- precisiation of p ≠ representation of meaning of p
- precisiation of p = precisiation of meaning of p example
- Brian is much taller than most of his close friends I understand what you say but could you be more precise?
- not every proposition is precisiable
- GCL is maximally expressive

PRECISIATION / MARIA'S AGE PROBLEM

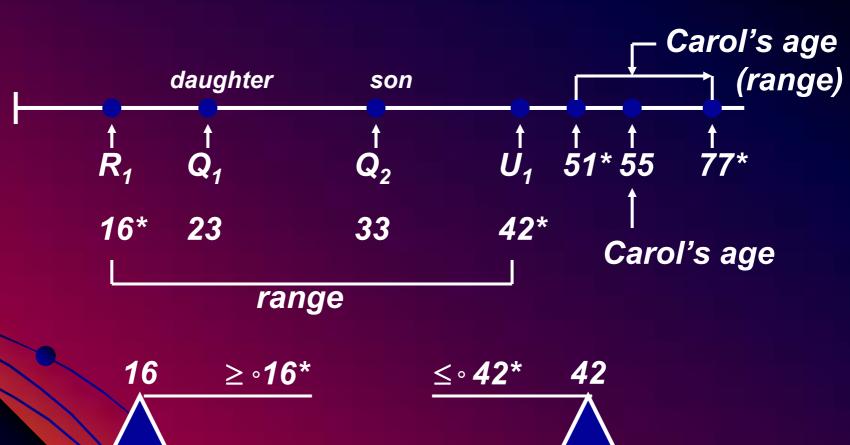
- p₁: Maria is about ten years older than Carol
- p₂: Carol has two children: a son, in mid-twenties; and a daughter, in mid-thirties
- q: How old is Maria?

PNL-based analysis

p₁: X/Age(Maria) is (Y/Age(Carol) + 10*)

Go to World Knowledge database w: child-bearing age ranges from about 16 to about 42

PRECISIATION





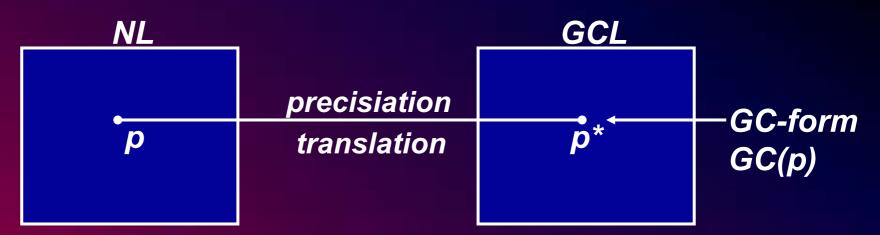
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PRECISIATION

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w: Prob\{Q_1/Age(Carol).at.birth.daughter) is \geq \circ R_1/16^*\} is S_1/very.likely \land Prob\{Q_1 \ is < \circ R_1\} is T_1/unlikely \land Prob\{Q_2 \ is \le \circ U_1/42^*\} is S_1 \land Prob\{Q_2 \ is > \circ U_1\} is W_1/very.unlikely \land (same for son)
```

PRECISIATION = TRANSLATION INTO GCL



annotation

 $p \longrightarrow X/A$ isr R/B \longleftarrow GC-form of p

example

p: Carol lives in a small city near San Francisco
X/Location(Residence(Carol)) is R/NEAR[City] \(\times \) SMALL[City]

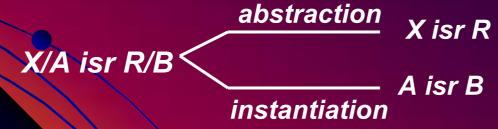
PRECISIATION

- Usually it does not rain in San Francisco in midsummer
- Brian is much taller than most of his close friends
- It is very unlikely that there will be a significant increase in the price of oil in the near future
- Mary loves books
- It is not quite true that Mary is very rich

GENERALIZED-CONSTRAINT-FORM(GC(p))

annotation

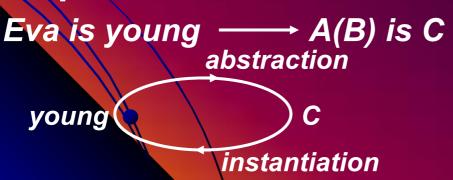
suppression



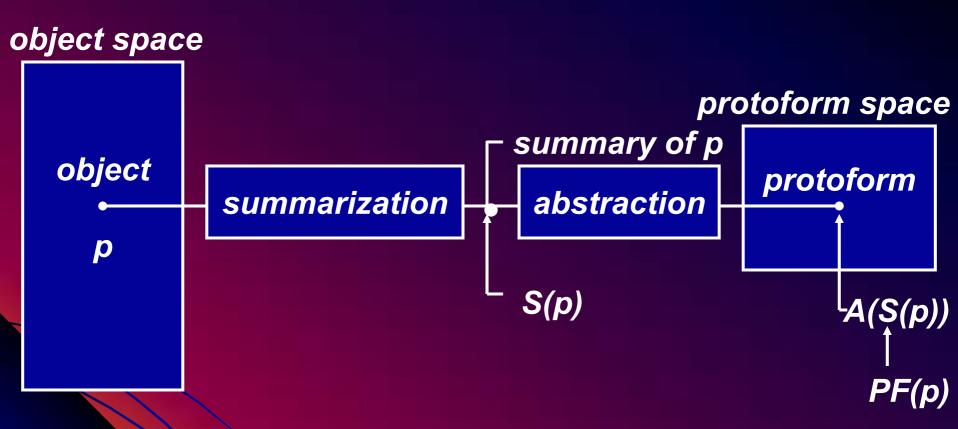
X isr R is a deep structure (protoform) of p

THE CONCEPT OF A PROTOFORM AND ITS BASIC ROLE IN KNOWLEDGE REPRESENTATION, DEDUCTION AND SEARCH

- Informally, a protoform—abbreviation of prototypical form—is an abstracted summary. More specifically, a protoform is a symbolic expression which defines the deep semantic structure of a construct such as a proposition, command, question, scenario, or a system of such constructs
- Example:



CONTINUED



PF(p): abstracted summary of p
deep structure of p

- protoform equivalence
- protoform similarity

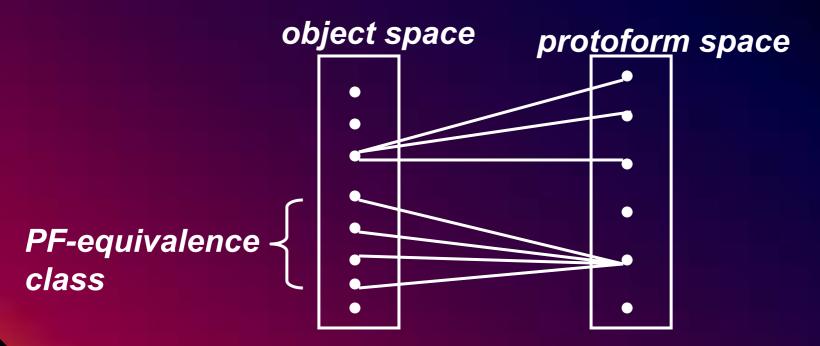
EXAMPLES



- Usually Robert returns from work at about 6:15pm ——
 Prob{Time(Return(Robert)} is 6:15*} is usually ——
 Prob{A(B) is C} is D

to usually
6:15*
Return(Robert)
Time

PROTOFORMS



 at a given level of abstraction and summarization, objects p and q are PF-equivalent if PF(p)=PF(q)

example

p: Most Swedes are tall

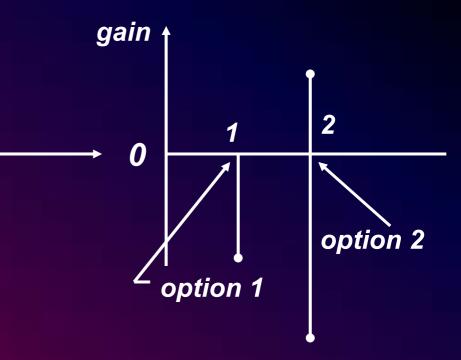
q: Few professors are rich

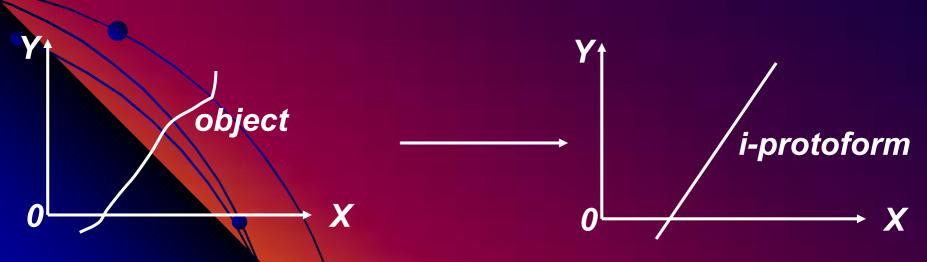
Count (A/B) is Q

Count (A/B) is Q

EXAMPLES

Alan has severe back pain. He goes to see a doctor. The doctor tells him that there are two options: (1) do nothing; and (2) do surgery. In the case of surgery, there are two possibilities: (a) surgery is successful, in which case Alan will be pain free; and (b) surgery is not successful, in which case Alan will be paralyzed from the neck down. Question: Should Alan elect surgery?





PF-EQUIVALENCE

Scenario A:

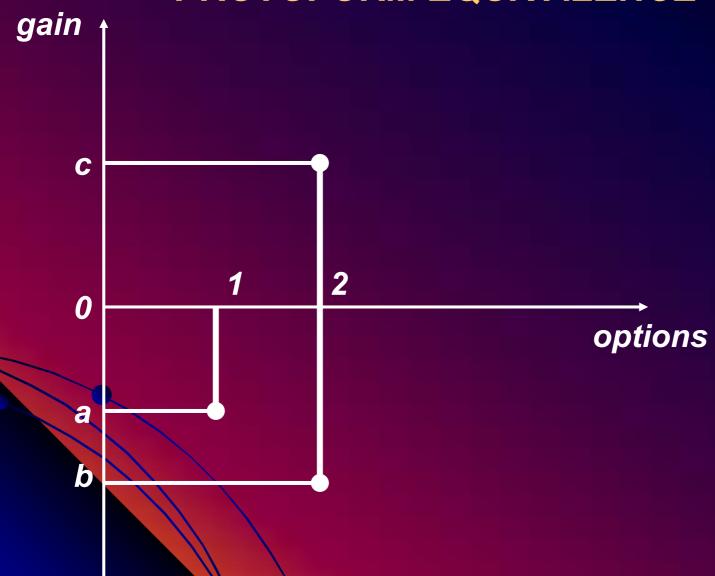
Alan has severe back pain. He goes to see a doctor. The doctor tells him that there are two options: (1) do nothing; and (2) do surgery. In the case of surgery, there are two possibilities: (a) surgery is successful, in which case Alan will be pain free; and (b) surgery is not successful, in which case Alan will be paralyzed from the neck down. Question: Should Alan elect surgery?

PF-EQUIVALENCE

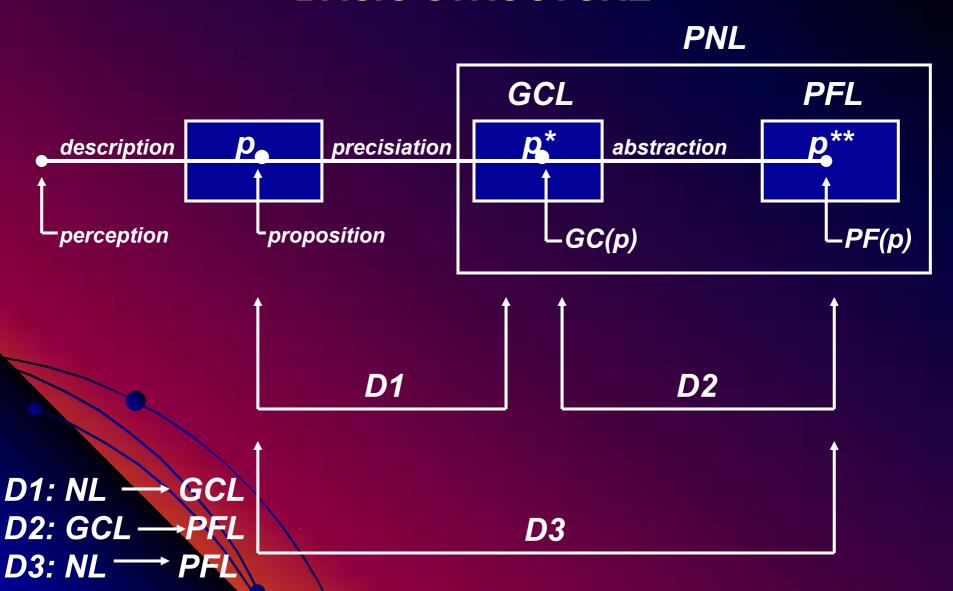
Scenario B:

Alan needs to fly from San Francisco to St. Louis and has to get there as soon as possible. One option is fly to St. Louis via Chicago and the other through Denver. The flight via Denver is scheduled to arrive in St. Louis at time a. The flight via Chicago is scheduled to arrive in St. Louis at time b, with a < b. However, the connection time in Denver is short. If the flight is missed, then the time of arrival in St. Louis will be c, with c>b. Question: Which option is best?





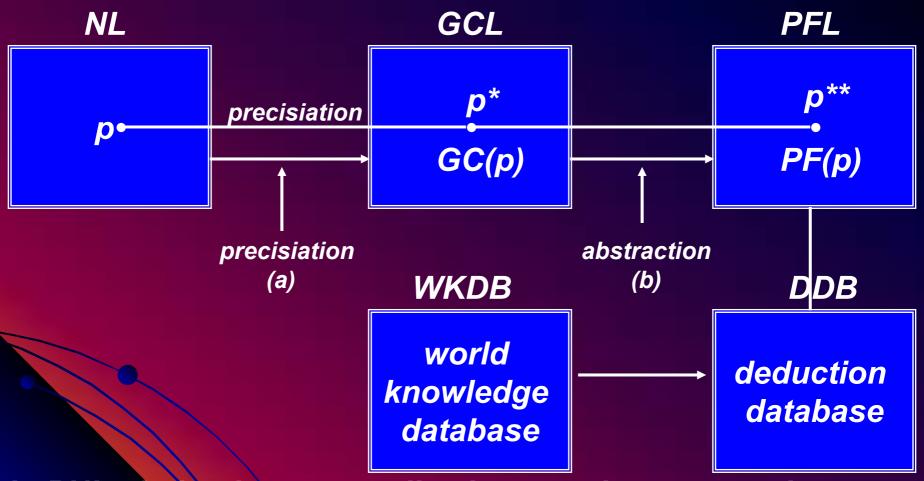
BASIC STRUCTURE



BASIC POINTS

- annotation: specification of class or type
 Eva is young A(B) is C
 A/attribute of B, B/name, C/value of A
- abstraction has levels, just as summarization does
 - most Swedes are tall —→most A's are tall —→
 most A's are B —→QA's are B's
- P and q are PF-equivalent (at level α) iff they have identical protoforms (at level α) most Swedes are tall=few professors are rich

BASIC STRUCTURE OF PNL



•In PNL, deduction=generalized constraint propagation DDB: deduction database=collection of protoformal rules governing generalized constraint propagation

WKDB: PNL-based

WORLD KNOWLEDGE

- There is an extensive literature on world knowledge. But there are two key aspects of world knowledge which are not addressed in the literature
- 1. Much of world knowledge is perception-based
 - Icy roads are slippery
 - Usually it does not rain in San Francisco in midsummer
- 2. Most concepts are fuzzy rather than bivalent, i.e., most concepts are a matter of degree rather than categorical

FUZZY CONCEPTS

- Relevance
- Causality
- Summary
- Cluster
- Mountain
- Valley
- In the existing literature, there are no operational definitions of these concepts

WORLD KNOWLEDGE

KEY POINT

 world knowledge—and especially knowledge about the underlying probabilities—plays an essential role in disambiguation, planning, search and decision processes

WORLD KNOWLEDGE

examples

- icy roads are slippery
- big cars are safer than small cars
- usually it is hard to find parking near the campus on weekdays between 9 and 5
- most Swedes are tall
- overeating causes obesity
- usually it does not rain in San Francisco in midsummer
- an academic degree is associated with a field of study
- Princeton employees are well paid

WORLD KNOWLEDGE: EXAMPLE

specific:

- if Robert works in Berkeley then it is likely that Robert lives in or near Berkeley
- if Robert lives in Berkeley then it is likely that Robert works in or near Berkeley

generalized:

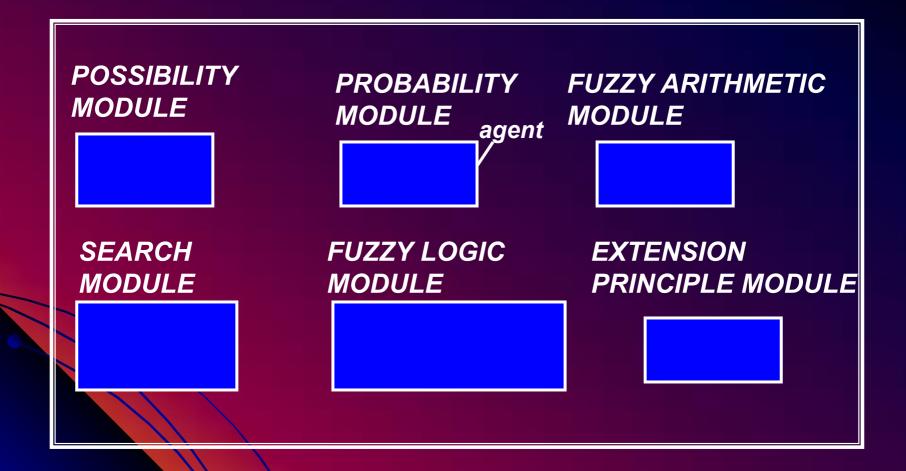
if A/Person works in B/City then it is likely that A lives in or near B

precisiated:

Distance (Location (Residence (A/Person), Location (Work (A/Person) isu near

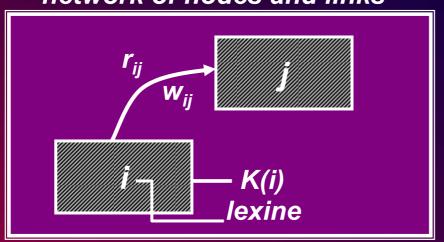
protoform: F (A (B (C)), A (D (C))) isu R

MODULAR DEDUCTION DATABASE



ORGANIZATION OF WORLD KNOWLEDGE EPISTEMIC (KNOWLEDGE-DIRECTED) LEXICON (EL)

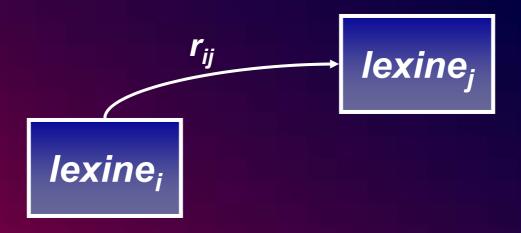
network of nodes and links



w_{ij}= granular strength of association between i and j

- i (lexine): object, construct, concept (e.g., car, Ph.D. degree)
- K(i): world knowledge about i (mostly perception-based)
- K(i) is organized into n(i) relations R_{ii}, ..., R_{in}
- entries in R_{ii} are bimodal-distribution-valued attributes of i
- values of attributes are, in general, granular and contextdependent

EPISTEMIC LEXICON



r_{ij}: i is an instance of j i is a subset of j i is a superset of j j is an attribute of i i causes j i and j are related (is or isu) (is or isu) (is or isu)

(or usually)

EPISTEMIC LEXICON

FORMAT OF RELATIONS

perception-based relation

lexine	A ₁	 A_m	←
	G ₁	G _m	←

---- attributes

← granular values

example

car	Make	Price	
	ford	G	
	chevy		

G: $20*\% \setminus 2/15k* + 40*\% \setminus [15k*, 25k*] + \bullet \bullet \bullet$

PROTOFORMAL SEARCH RULES

```
example
```

query: What is the distance between the largest city in Spain and the largest city in Portugal?

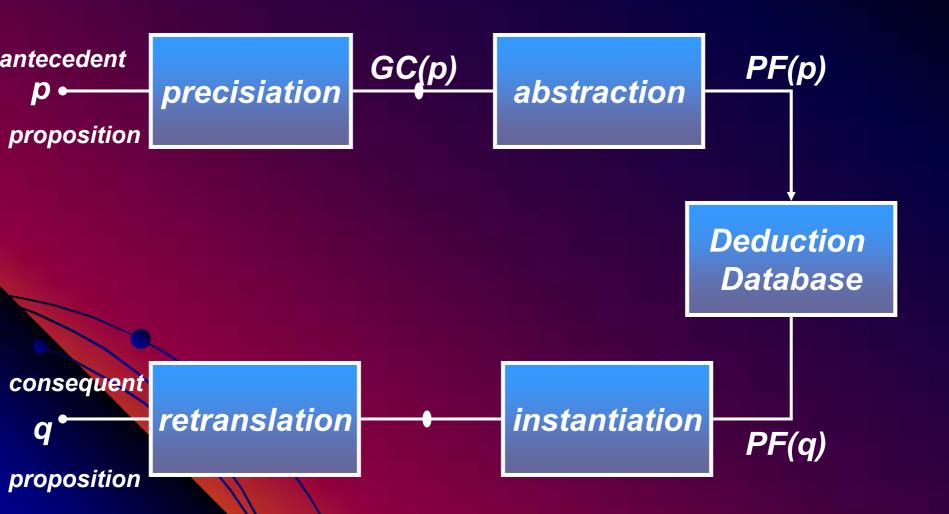
protoform of query: ?Attr (Desc(A), Desc(B))
procedure

query: ?Name (A)|Desc (A)

query: Name (B)|Desc (B)

query: ?Attr (Name (A), Name (B))

PROTOFORMAL (PROTOFORM-BASED) DEDUCTION



FORMAT OF PROTOFORMAL DEDUCTION RULES

protoformal rule

symbolic part

computational part

PROTOFORM DEDUCTION RULE: GENERALIZED MODUS PONENS

fuzzy logic

classical

$$\begin{array}{c}
A \\
A \longrightarrow B \\
\hline
B
\end{array}$$

X is A

If X is B then Y is C

Y is D

symbolic

computational 1—

$$D = A \circ (B \times C)$$

(fuzzy graph; Mamdani)

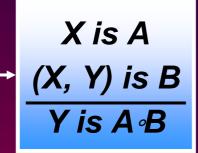
computational 2

$$D = A \circ (B \Rightarrow C)$$

(implication; conditional relation)

PROTOFORMAL RULES OF DEDUCTION

examples



$$\mu_{A\circ B}(v) = max_u(\mu_A(u) \wedge \mu_B(u,v))$$

symbolic part

Prob (X is A) is B
Prob (X is C) is D

$$\mu_D(u) = \max_q (\mu_B(\int_U \mu_A(u)g(u)du))$$

$$subject\ to:\ v = \int_U \mu_C(u)g(u)du$$

$$U$$

$$\int_U g(u)du = 1$$

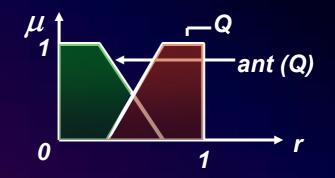
$$U$$

computational

part

COUNT-AND MEASURE-RELATED RULES

-crisp Q A's are B's ant (Q) A's are not B's



Q A's are B's Q^{1/2} A's are ²B's



most Swedes are tall ave (height) Swedes is ?h Q A's are B's ave (B|A) is ?C

$$\mu_{ave}(v) = \sup_{a} \mu_{Q}(\frac{1}{N} \sum_{i} \mu_{B}(a_{i}))$$
 , $a = (a_{1},...,a_{N})$

$$a = (a_1, ..., a_N)$$

$$v = \frac{1}{N} (\sum_{i} a_{i})$$

CONTINUED

 $not(QA's are B's) \longleftrightarrow (not Q) A's are B's$

 Q_1 A's are B's Q_2 (A&B)'s are C's Q_1 Q_2 A's are (B&C)'s

 Q_1 A's are B's Q_2 A's are C's $(Q_1 + Q_2 -1)$ A's are (B&C)'s

PROTOFORMAL CONSTRAINT PROPAGATION

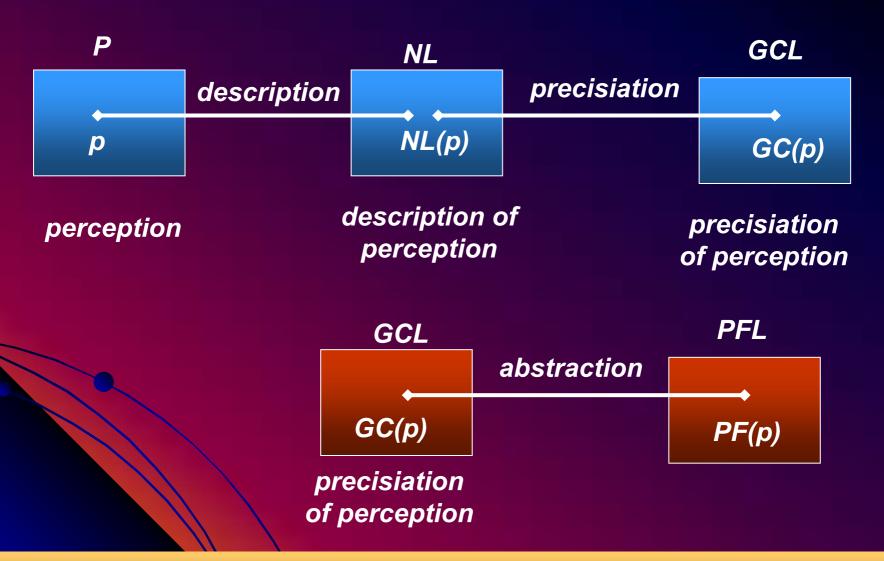
p	GC(p)	PF(p)	
Dana is young	Age (Dana) is young	X is A	
Tandy is a few years older than Dana	Age (Tandy) is (Age (Dana)) +few	Y is (X+B)	

X is A Y is (X+B) Y is A+B

Age (Tandy) is (young+few)

$$\mu_{A+B}(V) = \sup_{u}(\mu_{A}(u) + \mu_{B}(V - u)$$

CONCEPTUAL STRUCTURE OF PNL



GCL (Generalized Constraint Language) is maximally expressive

PRINCIPAL FUNCTIONS OF PNL

- perception description language
- knowledge representation language
- definition language
- specification language
- deduction language

PNL AS A DEFINITION / DESCRIPTION / SPECIFICATION LANGUAGE

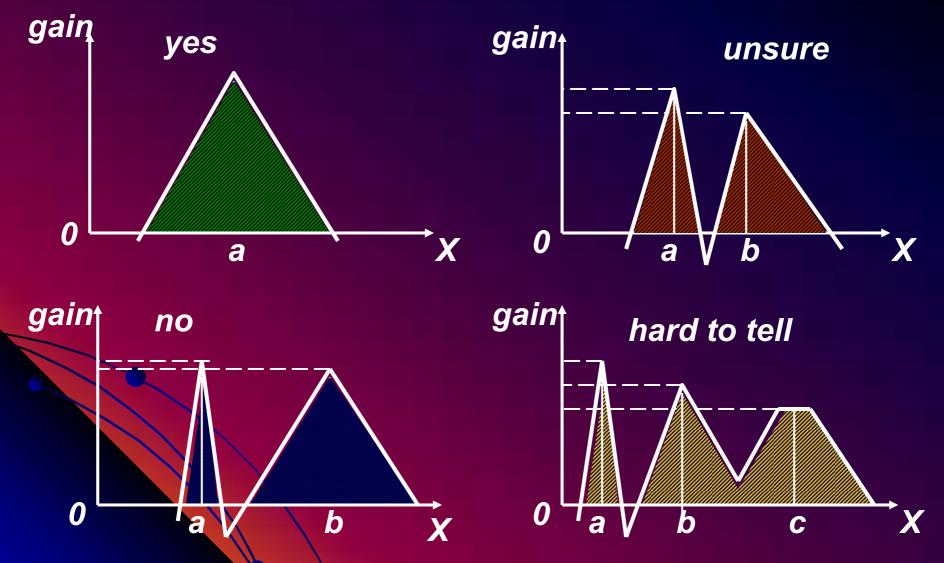
X: concept, description, specification

KEY IDEA

- Describe X in a natural language
- Precisiate description of X

Test: What is the definition of a mountain?

DEFINITION OF OPTIMALITY OPTIMIZATION=MAXIMIZATION?



definition of optimal X requires use of PNL

PNL AS A

DEFINITION LANGUAGE

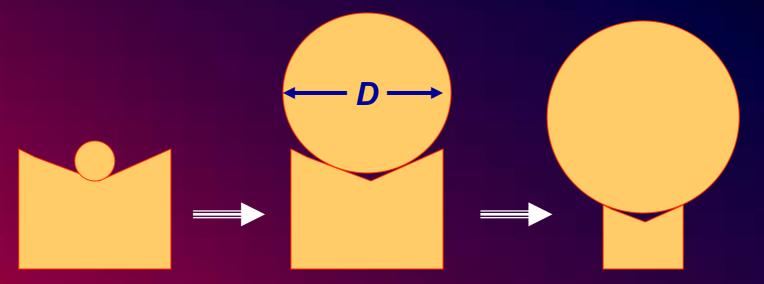
BRITTLENESS OF DEFINITIONS (THE SORITES PARADOX)

statistical independence

- A and B are independent $\leftarrow P_A(B) = P(B)$
- suppose that (a) P_A(B) and P(B) differ by an epsilon; (b) epsilon increases
- at which point will A and B cease to be independent?
 - statistical independence is a matter of degree
- degree of independence is contextdependent
- brittleness is a consequence of bivalence

STABILITY IS A FUZZY CONCEPT

•graduality of progression from stability to instability



- •Lyapounov's definition of stability leads to the counterintuitive conclusion that the system is stable no matter how large the ball is
- •In reality, stability is a matter of degree

SIMPLE QUESTIONS THAT ARE HARD TO ANSWER

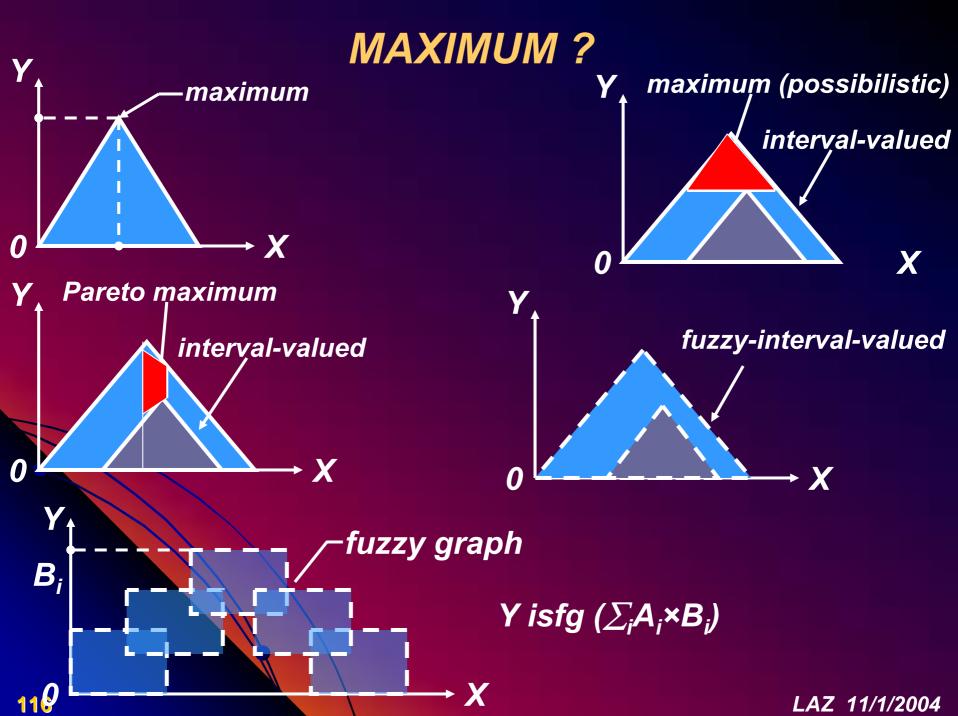
WHAT ARE THE DEFINITIONS OF:

- length
- volume
- edge
- cluster
- summary
- relevance
- density

EVERYDAY CONCEPTS WHICH CANNOT BE DEFINED REALISTICALY THROUGH THE USE OF BIVALENT-LOGIC-BASED CONCEPTS

- check-out time is 12:30 pm
- speed limit is 65 mph
- it is cloudy
- Eva has long hair
- economy is in recession
- Lam risk averse

•



HIERARCHY OF DEFINITION LANGUAGES



fuzzy-logic-based

bivalent-logic-based

NL: natural language

B language: standard mathematical bivalent-logic-based language

Flanguage: fuzzy logic language without granulation

F.G language: fuzzy logic language with granulation

PNL: Precisiated Natural Language

Note: the language of fuzzy if-then rules is a sublanguage of PNL

Note: a language in the hierarchy subsumes all lower languages

SIMPLIFIED HIERARCHY

PNL

fuzzy-logic-based

B language

bivalent-logic-based

NL

The expressive power of the B language – the standard bivalence-logic-based definition language – is insufficient

Insufficiency of the expressive power of the B language is rooted in the fundamental conflict between bivalence and reality

INSUFFICIENCY OF THE B LANGUAGE

Concepts which cannot be defined

- causality
- relevance
- intelligence

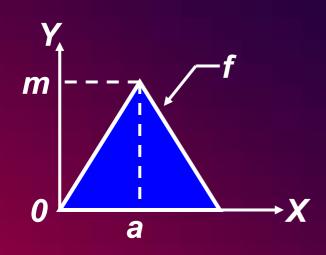
Concepts whose definitions are problematic

- stability
- optimality
- statistical independence
- stationarity

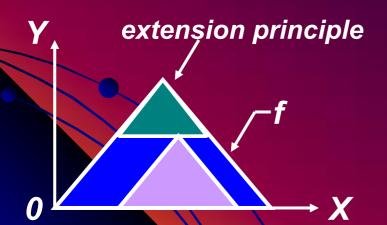
WHY IS EXPRESSIVE POWER AN IMPORTANT FACTOR?

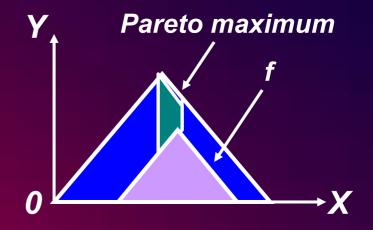
- Definition of a concept, construct or metric may be viewed as a precisiation of perception of the definiendum
- The language in which a definition is expressed is a definition language
- The expressive power of a definition language places a limit on the complexity of the definiendum and on the degree to which the definition of the definiendum is coextensive, that is, a good approximation to its perception

MAXIMUM?



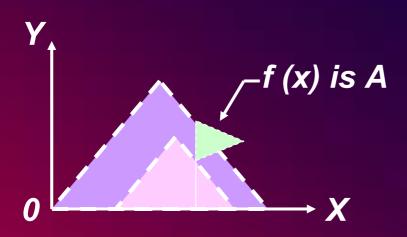
- a) $\forall x (f(x) \leq f(a))$
- b) $\sim (\exists x (f(x) > f(a))$

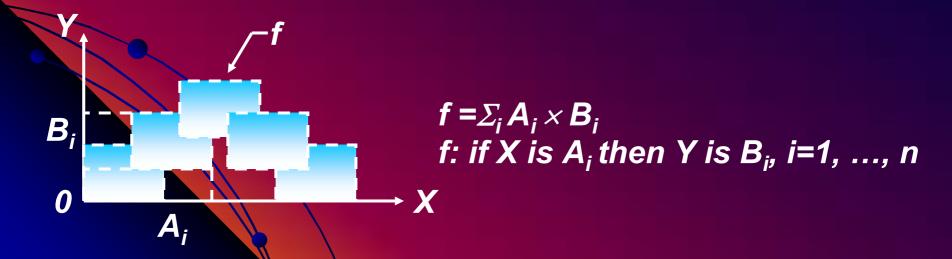




b) \sim ($\exists x$ (f (x) dominates f(a))

MAXIMUM?





EXAMPLE

- I am driving to the airport. How long will it take me to get there?
- Hotel clerk's perception-based answer: about 20-25 minutes
- "about 20-25 minutes" cannot be defined in the language of bivalent logic and probability theory
- To define "about 20-25 minutes" what is needed is PNL

PRECISIATION

s-precisiation

g-precisiation

conventional (degranulation)

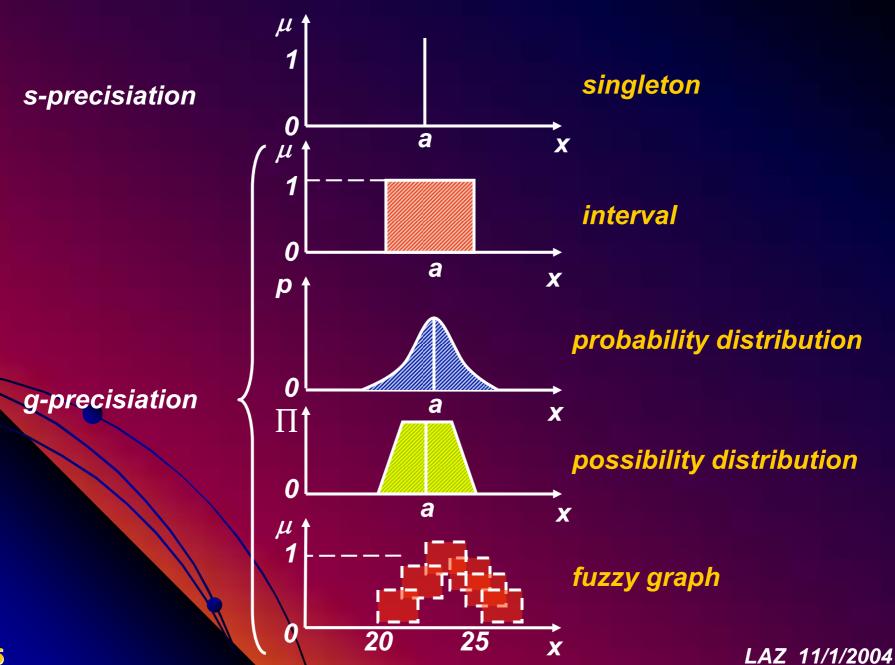
* a precisiation

approximately a

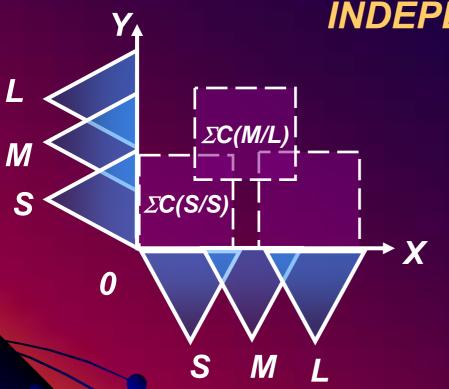
GCL-based (granulation)

common practice in probability theory

PRECISIATION OF "approximately a," *a



PNL-BASED DEFINITION OF STATISTICAL INDEPENDENCE



contingency table

3	L/S	L/M	L/L
2	M/S	M/M	M/L
1	S/S	S/M	S/L
<u></u>	1	2	3

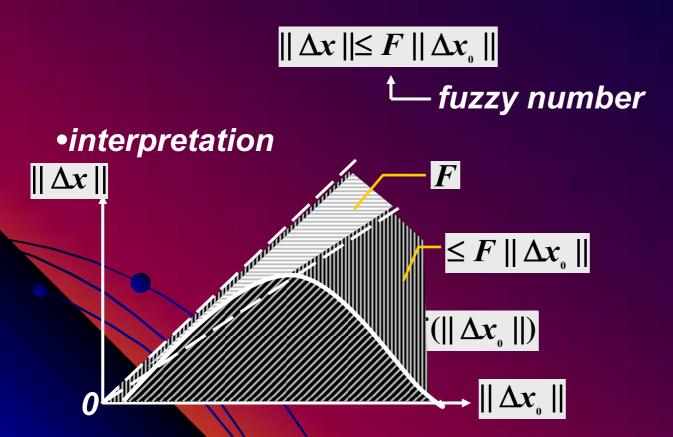
$$\Sigma (M/L) = \frac{\Sigma C (M \times L)}{\Sigma C (L)}$$

 degree of independence of Y from X= degree to which columns 1, 2, 3 are identical

PNL-based definition

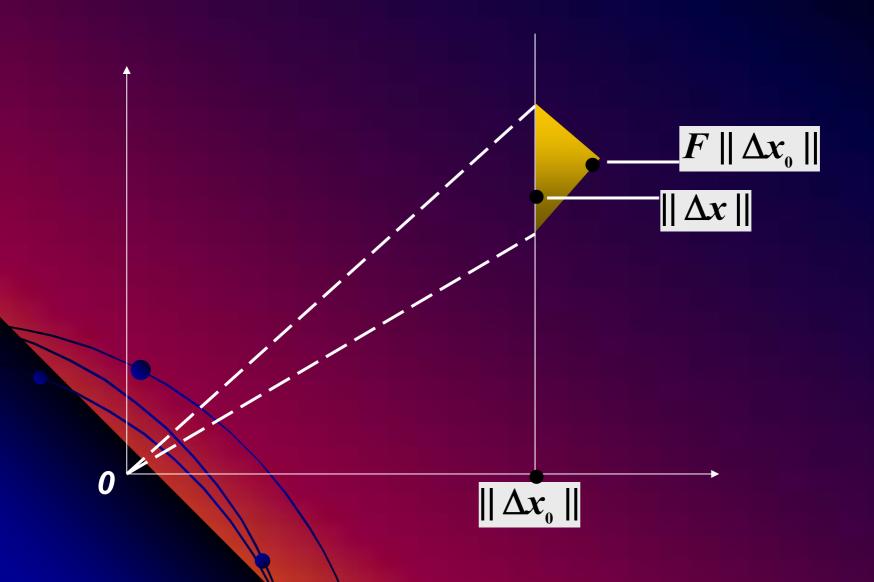
PNL-BASED DEFINITION OF STABILITY

 a system is F-stable if it satisfies the fuzzy Lipshitz condition



degree of stability=degree to which f is in $\leq F \|\Delta x_{0}\|$

F-STABILITY



CONCLUSION

- Existing scientific theories are based on bivalent logic—a logic in which everything is black or white, with no shades of gray allowed
- What is not recognized, to the extent that it should, is that bivalent logic is in fundamental conflict with reality
- Fuzzy logic is not in conflict with bivalent logic—it is a generalization of bivalent logic in which everything is, or is allowed to be, a matter of degree
- Fuzzy logic provides a foundation for the methodology of computing with words and perceptions

Factual Information About the Impact of Fuzzy Logic

PATENTS

- Number of fuzzy-logic-related patents applied for in Japan: 17,740
- Number of fuzzy-logic-related patents issued in Japan: 4,801
- Number of fuzzy-logic-related patents issued in the US: around 1,700

PUBLICATIONS

Count of papers containing the word "fuzzy" in title, as cited in INSPEC and MATH.SCI.NET databases. (Data for 2002 are not complete)
Compiled by Camille Wanat, Head, Engineering Library, UC Berkeley,
November 20, 2003

Number of papers in INSPEC and MathSciNet which have "fuzzy" in their titles:

INSPEC - "fuzzy" in the title

1970-1979: 569

1980-1989: 2,404

1990-1999: 23,207

2000-present: 9,945

Total: 36,125

MathSciNet - "fuzzy" in the title

1970-1979: 443

1980-1989: 2,465

1990-1999: 5,479

2000-present: 2,865

Total: 11,252

JOURNALS ("fuzzy" or "soft computing" in title)

- 1. Fuzzy Sets and Systems
- 2. IEEE Transactions on Fuzzy Systems
- 3. Fuzzy Optimization and Decision Making
- 4. Journal of Intelligent & Fuzzy Systems
- 5. Fuzzy Economic Review
- 6. International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems
- 7. Journal of Japan Society for Fuzzy Theory and Systems
- 8. International Journal of Fuzzy Systems
- 9. Soft Computing
- 10. International Journal of Approximate Reasoning--Soft Computing in Recognition and Search
- 11. Intelligent Automation and Soft Computing
- 12. Journal of Multiple-Valued Logic and Soft Computing
- 13. Mathware and Soft Computing
- 14. Biomedical Soft Computing and Human Sciences
- 15. Applied Soft Computing

APPLICATIONS

The range of application-areas of fuzzy logic is too wide for exhaustive listing. Following is a partial list of existing application-areas in which there is a record of substantial activity.

1.	Industrial control	19.	Financial engineering
2.	Quality control	20.	Assessment of credit-worthiness
<i>3.</i>	Elevator control and scheduling	21.	Fraud detection
4.	Train control	22.	Mine detection
5.	Traffic control	23.	Pattern classification
6.	Loading crane control	24.	Oil exploration
7.	Reactor control	25.	Geology
8.	Automobile transmissions	26.	Civil Engineering
9.	Automobile climate control	27.	Chemistry
10.	Automobile body painting control	28.	Mathematics
11.	Automobile engine control	29.	Medicine
<i>12.</i>	Paper manufacturing	<i>30.</i>	Biomedical instrumentation
<i>13.</i>	Steel manufacturing	31.	Health-care products
14.	Power distribution control	32.	Economics
<i>15.</i>	Software engineerinf	33.	Social Sciences

34. Internet

35. Library and Information Science

18. Decision analysis

16. Expert systems

17. Operation research

Product Information Addendum 1

This addendum relates to information about products which employ fuzzy logic singly or in combination. The information which is presented came from SIEMENS and OMRON. It is fragmentary and far from complete. Such addenda will be sent to the Group from time to time.

SIEMENS:

- * washing machines, 2 million units sold
- * fuzzy guidance for navigation systems (Opel, Porsche)
- * OCS: Occupant Classification System (to determine, if a place in a car is occupied by

a person or something else; to control the airbag as well as the intensity of the airbag). Here FL is used in the product as well as in the design process (optimization of parameters).

* fuzzy automobile transmission (Porsche, Peugeot, Hyundai)

OMRON:

* fuzzy logic blood pressure meter, 7.4 million units sold, approximate retail value \$740 million dollars

Note: If you have any information about products and or manufacturing which may be of relevance please communicate it to Dr. Vesa Niskanen vesa.a.niskanen@helsinki.fi and Masoud Nikravesh Nikravesh@cs.berkeley.edu.

Product Information Addendum 2

This addendum relates to information about products which employ fuzzy logic singly or in combination. The information which is presented came from Professor Hideyuki Takagi, Kyushu University, Fukuoka, Japan. Professor Takagi is the co-inventor of neurofuzzy systems. Such addenda will be sent to the Group from time to time.

Facts on FL-based systems in Japan (as of 2/06/2004)

1. Sony's FL camcorders

Total amount of camcorder production of all companies in 1995-1998 times Sony's market share is the following. Fuzzy logic is used in all Sony's camcorders at least in these four years, i.e. total production of Sony's FL-based camcorders is 2.4 millions products in these four years.

1,228K units X 49% in 1995 1,315K units X 52% in 1996 1,381K units X 50% in 1997 1,416K units X 51% in 1998

2. FL control at Idemitsu oil factories

Fuzzy logic control is running at more than 10 places at 4 oil factories of Idemitsu Kosan Co. Ltd including not only pure FL control but also the combination of FL and conventional control.

They estimate that the effect of their FL control is more than 200 million YEN per year and it saves more than 4,000 hours per year.

3. Canon

Canon used (uses) FL in their cameras, camcorders, copy machine, and stepper alignment equipment for semiconductor production. But, they have a rule not to announce their production and sales data to public.

Canon holds 31 and 31 established FL patents in Japan and US, respectively.

4. Minolta cameras

Minolta has a rule not to announce their production and sales data to public, too.

whose name in US market was Maxxum 7xi. It used six FL systems in a camera and was put on the market in 1991 with 98,000 YEN (body price without lenses). It was produced 30,000 per month in 1991. Its sister cameras, alpha-9xi, alpha-5xi, and their successors used FL systems, too. But, total number of production is confidential.

5. FL plant controllers of Yamatake Corporation

Yamatake-Honeywell (Yamatake's former name) put FUZZICS, fuzzy software package for plant operation, on the market in 1992. It has been used at the plants of oil, oil chemical, chemical, pulp, and other industries where it is hard for conventional PID controllers to describe the plan process for these more than 10 years.

They planed to sell the FUZZICS 20 - 30 per year and total 200 million YEN.

As this software runs on Yamatake's own control systems, the software package itself is not expensive comparative to the hardware control systems.

6. Others

Names of 225 FL systems and products picked up from news articles in 1987 - 1996 are listed at http://www.adwin.com/elec/fuzzy/note 10.html in Japanese.)

Note: If you have any information about products and or manufacturing which may be of relevance please communicate it to Dr. Vesa Niskanen vesa.a.niskanen@helsinki.fi and Masoud Nikravesh Nikravesh@cs.berkeley.edu, with cc to me.