IEEE International Conference on Robotics and Automation Workshop on Multi-robot Perception-Driven Control and Planning 2017

# A Lower Bound on Observability for Target Tracking with Range Sensors and its Application to Sensor Assignment

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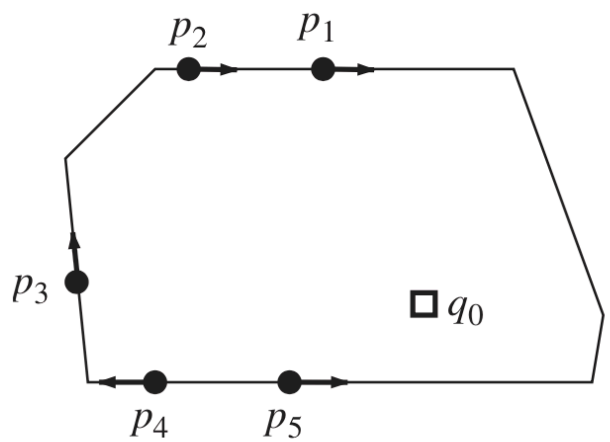
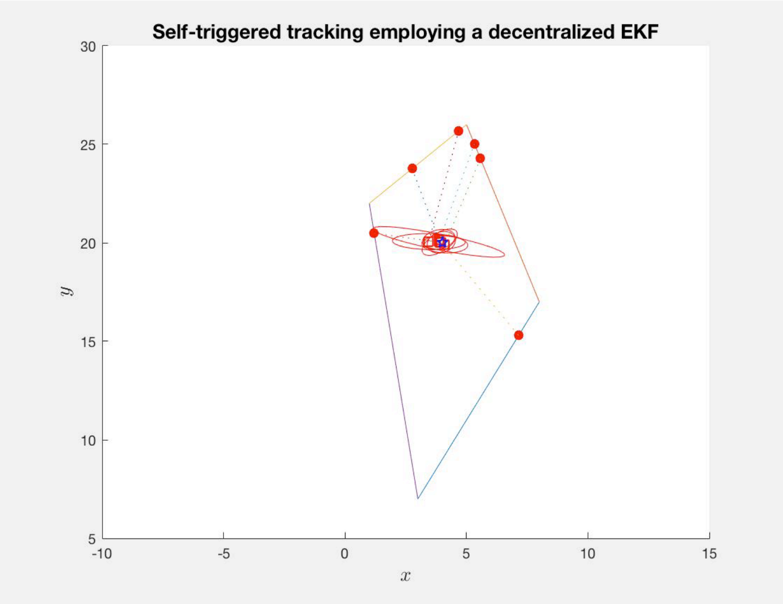
# Motivation

*Reduce the communication in decentralized target tracking*




$p_1$~$p_5$ are five range sensors
$q_o$ is the target

[Martinez & Bullo. *Automatica '06*]



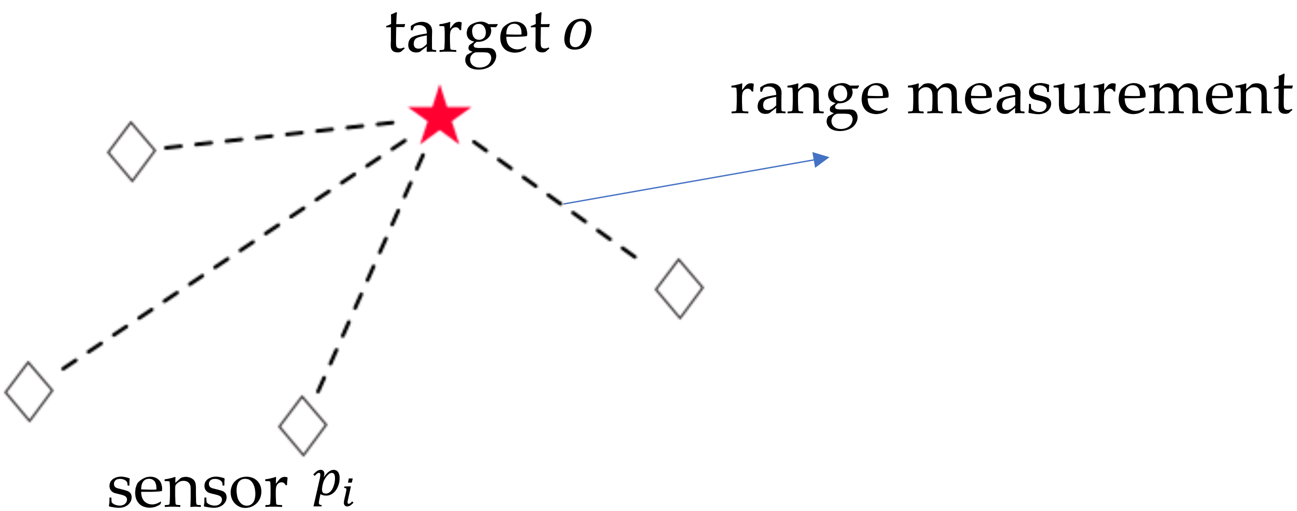

[Zhou & Tokekar. *ICRA '17]*

*Select sensors to improve the observability in tracking a potentially mobile target*

# Target's Motion and Measurement Model

$$\begin{cases} \dot{o} = u_o, \\ z_i = h_i(o) = \frac{1}{2}\|p_i - o\|_2^2, \;\; i = 1, \ldots, N \end{cases}$$

$$||u_o||_2 \le u_{o,\max}.$$



# Partially Known Observability Matrix

$$O(o, u_o) \quad = \begin{bmatrix} o_x - p_{1x}, o_y - p_{1y} \\ o_x - p_{2x}, o_y - p_{2y} \\ \vdots \\ o_x - p_{Nx}, o_y - p_{Ny} \\ u_{ox}, u_{oy} \end{bmatrix}$$

*Known (relative position)*

$$O(o) := \begin{bmatrix} o_x - p_{1x}, o_y - p_{1y} \\ o_x - p_{2x}, o_y - p_{2y} \\ \vdots \\ o_x - p_{Nx}, o_y - p_{Ny} \end{bmatrix}$$

*Unknown (control input)*

$$O(u_o) := [u_{ox}, u_{oy}]$$

# Lower Bound for the Unknown Observability Metric

- Inverse of condition number

$$C^{-1}(O(o,u_o)) = \frac{\sigma_{\min}(O(o,u_o))}{\sigma_{\max}(O(o,u_o))}.$$

- Lower bound

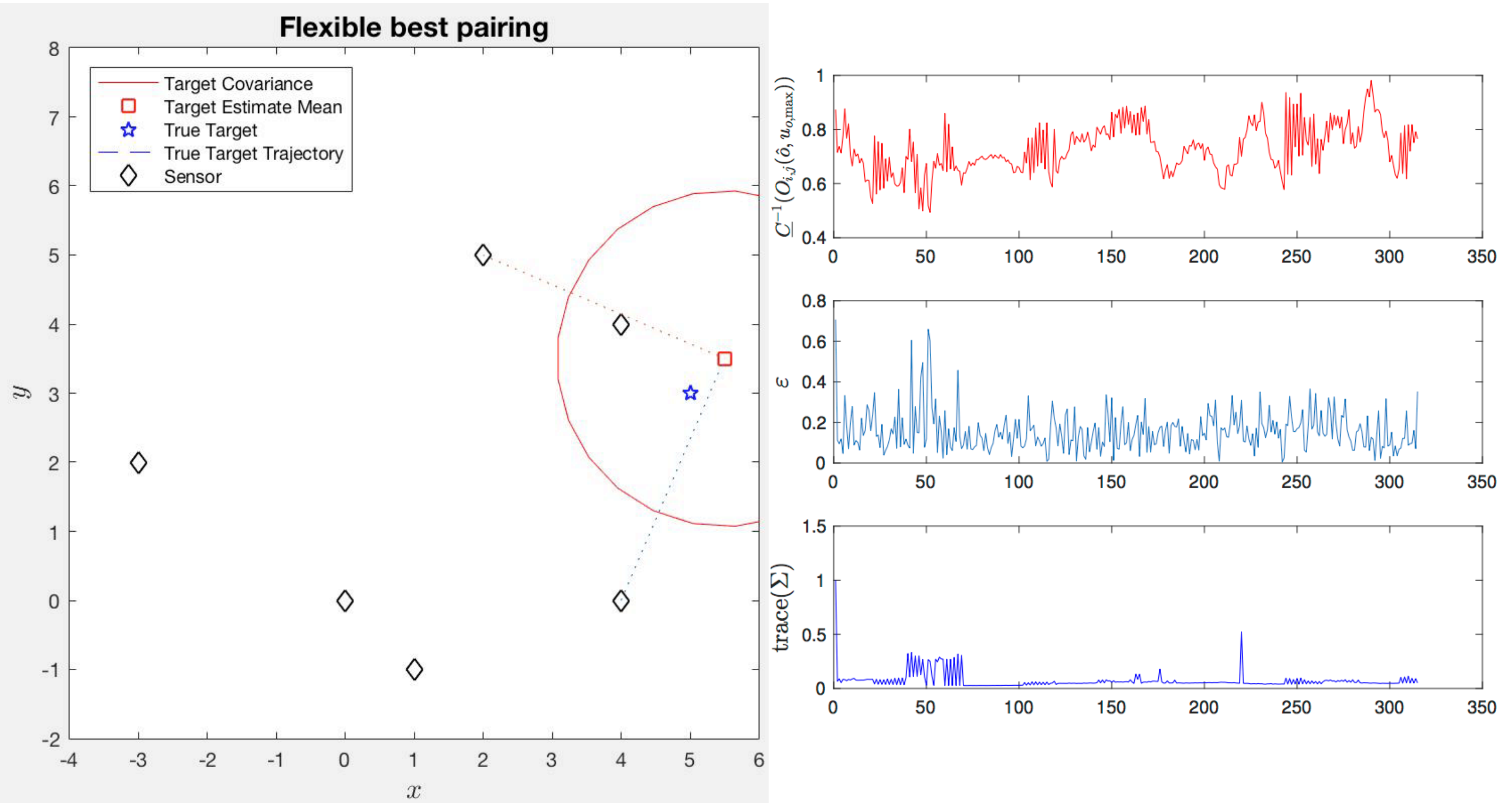
$$\underline{C}^{-1}(O(o,u_o)) = \frac{\sigma_{\min}(O(o))}{\sqrt{\sigma^2_{\max}(O(o)) + u_o^2}}$$

*unknown*

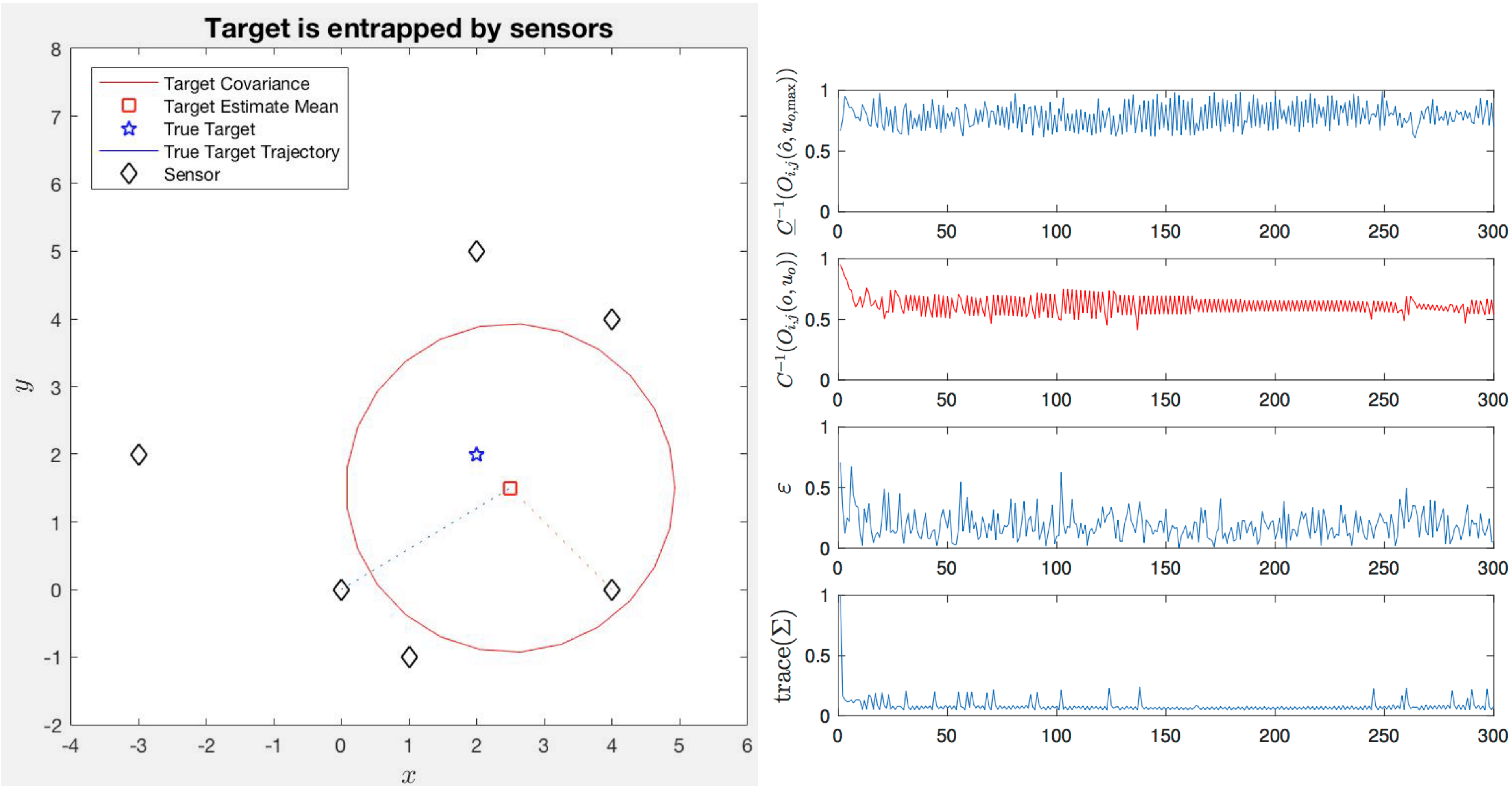
$$\geq \frac{\sigma_{\min}(O(o))}{\sqrt{\sigma^2_{\max}(O(o)) + u^2_{o,\max}}}$$

*known*

$$||u_o||_2 \leq u_{o,\max}.$$



Flexible best pairing
Target Covariance
Target Estimate Mean
True Target
True Target Trajectory
Sensor
$x$
$y$
$\underline{C}^{-1}(O_{i,j}(\hat{o}, u_{o,\max}))$
$\varepsilon$
$\mathrm{trace}(\Sigma)$



Target is entrapped by sensors
Target Covariance
Target Estimate Mean
True Target
True Target Trajectory
Sensor
$x$
$y$
$\underline{C}^{-1}(O_{i,j}(\hat{o}, u_{o,\max}))$
$C^{-1}(O_{i,j}(o, u_o))$
$\varepsilon$
$\mathrm{trace}(\Sigma)$

# Citations

MartíNez, S., & Bullo, F. (2006). Optimal sensor placement and motion coordination for target tracking. *Automatica*, *42*(4), 661-668.

Zhou, L., & Tokekar, P. (2017). Active Target Tracking with Self-Triggered Communications in Multi-Robot Teams. *arXiv preprint arXiv:1704.07475*.

# Thanks for listening!

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The material is based upon work supported by the National Science Foundation under Grant  Nos. 1566247 and 1637915

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