A Lower Bound on Observability for Target Tracking with Range Sensors and its Application to Sensor Assignment

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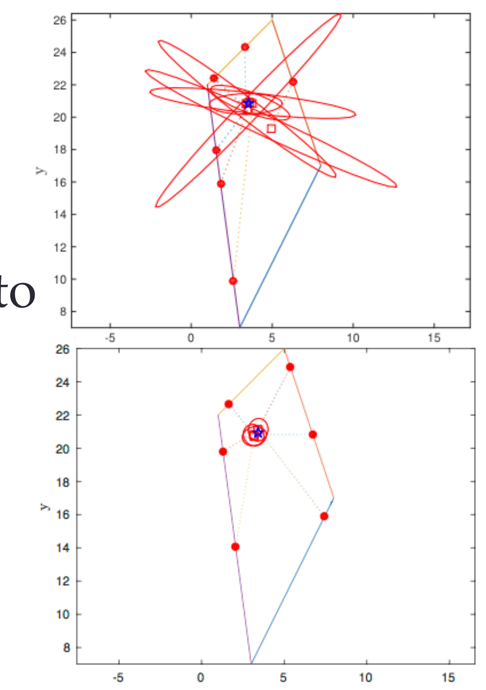
MOTIVATION

The motivation originates from our ICRA work, "Active Target Tracking with Self-Triggered Communications". We focus on a target tracking scenario where a potentially target is measured by range-only sensors with EKF estimators. Sensor can communicate and share the measurement with its neighbors at triggering time instants. However, during no communication slot, each sensor can only have its own estimation of the target, which leads to a bad tracking performance. Since sensor estimation can be improved by better positioning of sensors relative to the target. Observability matrix is often used to find this better position. So we want to improve the state estimation by looking into the observability of the sensor-target system.



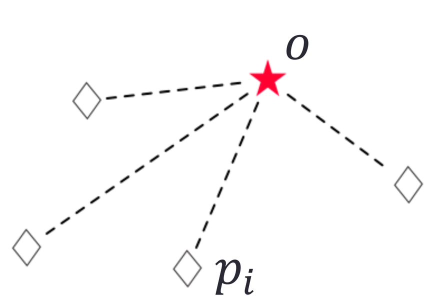
- **Objective:** Select sensors to improve the observability in tracking a potentially mobile target
- **Challenge:** The control input of the target is unknown to sensors, so the condition number of the local nonlinear observability matrix cannot be calculated exactly.

PARTIALLY KNOWN OBSERVABILITY MATRIX

- **Target's Motion and Measurement Model**

$$\dot{o} = u_o,$$
$$z_i = h_i(o) = \tfrac{1}{2}\|p_i - o\|_2^2,\ i = 1, \dots, N$$

Sensors measure the square of the range to the target



- **Partitioning the local nonlinear observability matrix**

$$O(o, u_o) = \begin{bmatrix} o_x - p_{1x}, o_y - p_{1y} \\ \vdots \\ o_x - p_{Nx}, o_y - p_{Ny} \\ u_{ox}, u_{oy} \end{bmatrix}$$

$$O(o) := \begin{bmatrix} o_x - p_{1x}, o_y - p_{1y} \\ o_x - p_{2x}, o_y - p_{2y} \\ \vdots \\ o_x - p_{Nx}, o_y - p_{Ny} \end{bmatrix},$$

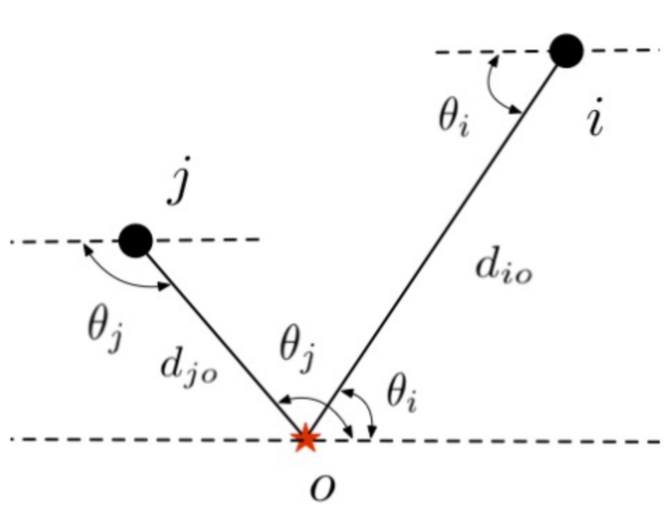
Known

$$O(u_o) := \begin{bmatrix} u_{ox}, u_{oy} \end{bmatrix},$$

Unknown

SENSOR PAIR SELECTION

- **Pair-sensor-target system**



$$\theta_{ji} := \theta_j - \theta_i$$
$$\alpha := d_{jo}/d_{io}$$

LOWER BOUND FOR THE UNKNOWN OBSERVABILITY METRIC

- **Inverse of condition number**

$$C^{-1}(O(o, u_o)) = \frac{\sigma_{\min}(O(o, u_o))}{\sigma_{\max}(O(o, u_o))}.$$

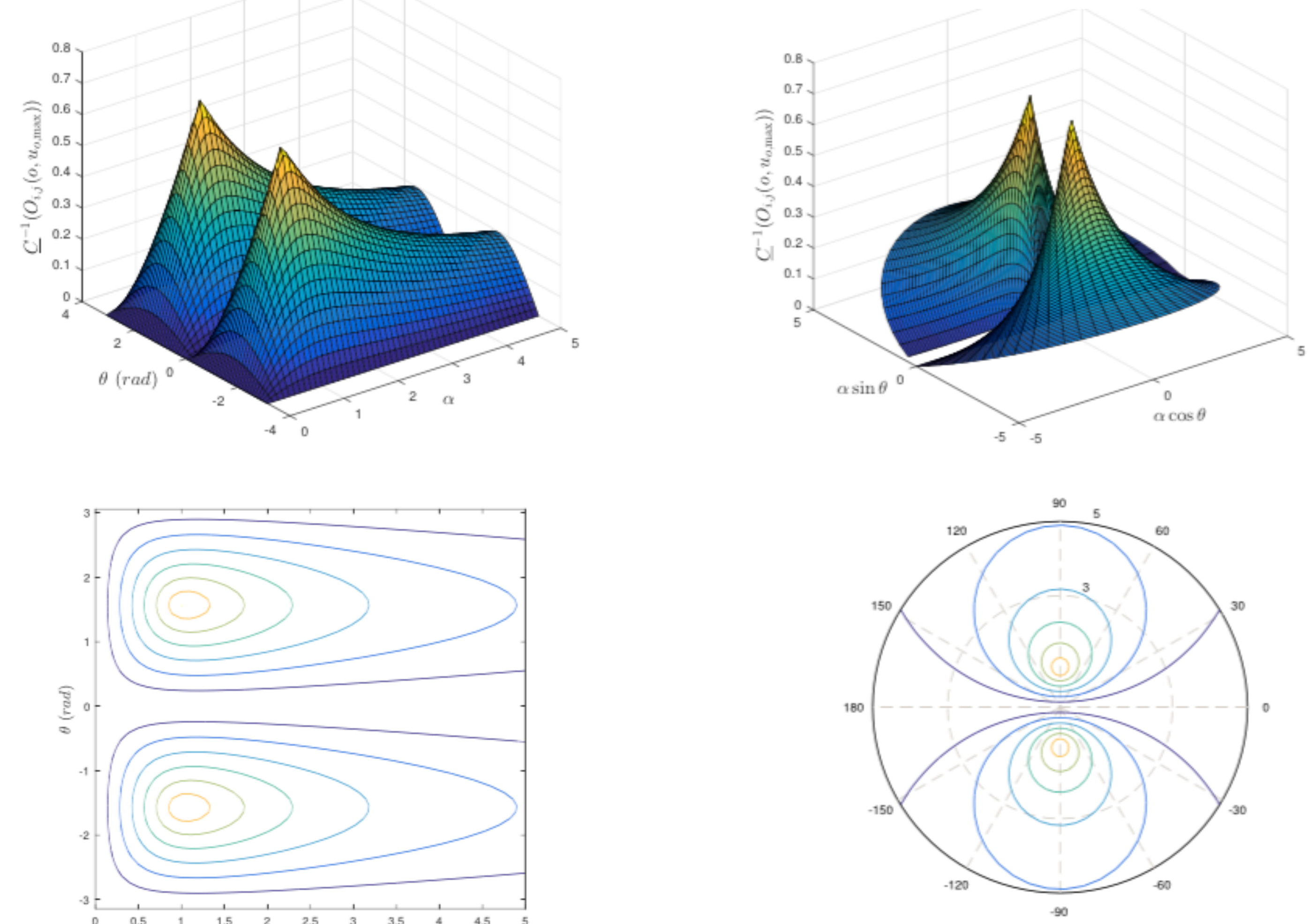
The inverse of the condition number measures the degree of observability. A larger inverse of condition number suggests better observability. It has the range [0, 1].

- **Lower bound**

$$\underline{C}^{-1}(O(o, u_o)) = \frac{\sigma_{\min}(O(o))}{\sqrt{\sigma^2_{\max}(O(o)) + u_o^2}} \quad \|u_o\|_2 \leq u_{o,\max}.$$

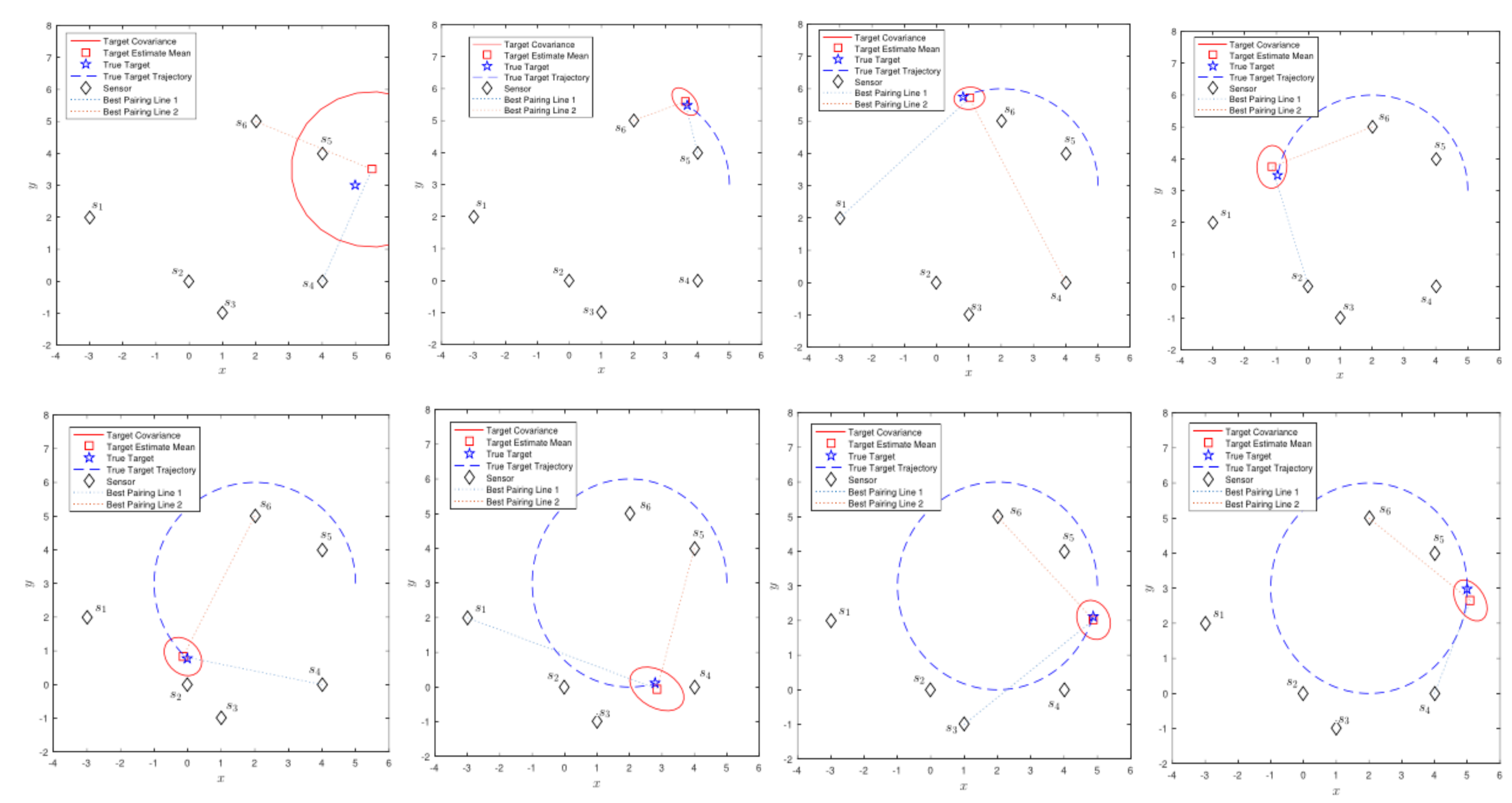
Lower bound of observability metric for pair-sensor-target system

$$\underline{C}^{-1}(O_{i,j}(o, u_o)) = \sqrt{\frac{1 + \alpha^2 - \sqrt{1 + \alpha^4 + 2\alpha^2 \cos(2\theta_{ji})}}{1 + \alpha^2 + \sqrt{1 + \alpha^4 + 2\alpha^2 \cos(2\theta_{ji})} + 2u_o^2/d_{io}^2}}.$$

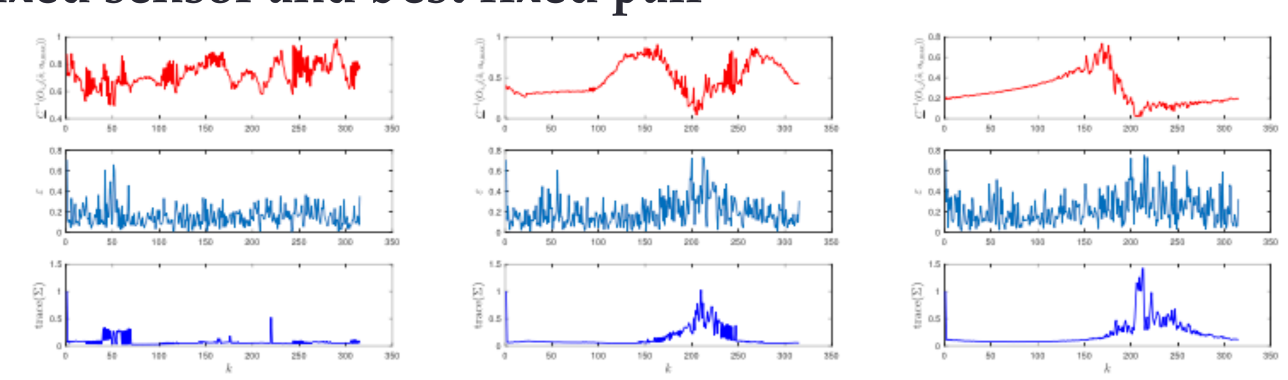


APPLICATIONS & SIMULATIONS

- **Best sensor selection**



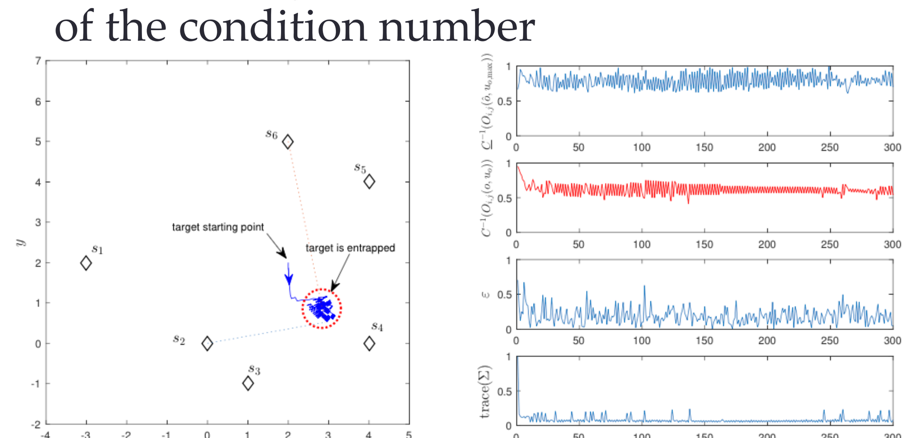
Comparison of flexible best pairing, flexible pair for fixed sensor and best fixed pair



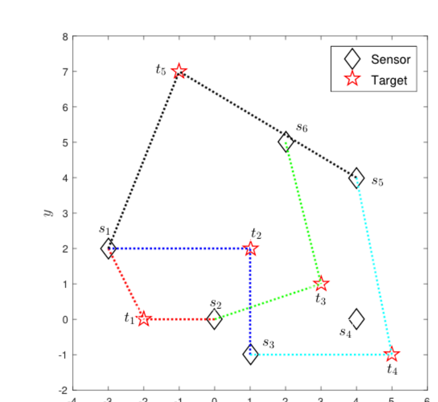
(a) flexible best pairing (b) best pairing for a fixed sensor (c) best fixed partner

- **Adversarial target**

The target uses one step planning to maximally decrease the inverse of the condition number



- **Multi-Sensor Multi-Target Assignment**



CONCLUSIONS

- Deduce the lower bound on the inverse of the condition number of the observability matrix with partially known information
- Improve tracking performance by by selecting the optimal set of sensors to improve the lower bound
- Apply to adversarial target and multi-sensor multi-target assignment
- Focus on sensor covering problem and sensor-target game in future work.