An Approximation Algorithm for Distributed Resilient Submodular Maximization

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MOTIVATION

- **Objective:** Choose a strategy set for a group of robots to maximize a submodular function on the strategy set.
- **Challenges:**
  - Adversaries can attack robots and make them lose contribution to the function (Fig. 1).
  - The robot has a limited communication range and communicates with other robots within a clique (Fig. 1).

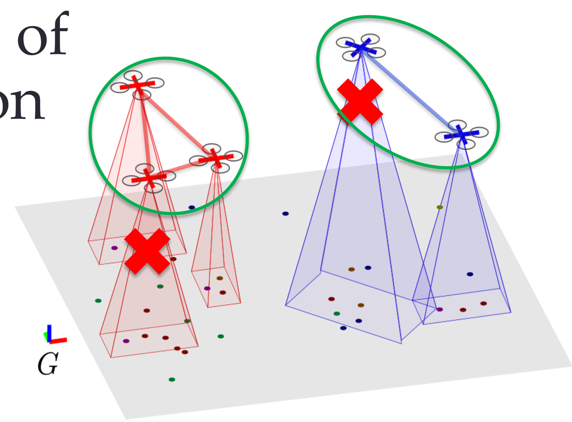


Fig. 1. A multi-robot multi-target tracking scenario where adversaries attack a number of robots. Each robot can only communicate with other robots in a clique.

PROBLEM FORMULATION

- **Problem (Distributed Resilient Submodular Maximization):**

$$\max_{\mathcal{S}\subseteq\mathcal{X},|\mathcal{S}|\leq N} \quad \min_{\mathcal{A}\subseteq\mathcal{S},|\mathcal{A}|\leq\alpha} \quad f(\mathcal{S}\setminus\mathcal{A}):$$

$$\mathcal{S}=\mathcal{S}_1\cup\cdots\cup\mathcal{S}_{\mathcal{K}(\mathcal{G})},\ |\mathcal{S}_k|\leq n_k;$$

$$n_1+\cdots+n_{\mathcal{K}(\mathcal{G})}=N;$$

$$|\mathcal{A}|\leq\alpha,\ \alpha<N.$$

- $\mathcal{S}$, the set of selected strategies by N robots.
- $\mathcal{A}$, a set of strategies removed / attacked from the selected set.
- $\mathcal{K}(\mathcal{G})$, the number of cliques on communication graph $\mathcal{G}$.
- $\mathcal{S}_k,\ k\in\{1,\cdots,\mathcal{K}(\mathcal{G})\}$, the set of selected strategies by clique $\mathcal{C}_k(\mathcal{G})$.
- $n_k,\ k\in\{1,\cdots,\mathcal{K}(\mathcal{G})\}$, the number of robots in clique $\mathcal{C}_k(\mathcal{G})$.

EVALUATION

- **Multi-robot multi-target tracking:** 10 aerial robots are tasked to track 50 ground targets (Fig. 4 (a) & (b)). Each robot has 4 candidate trajectories (Fig. 4 (c)). The robot will follow one at each time step.

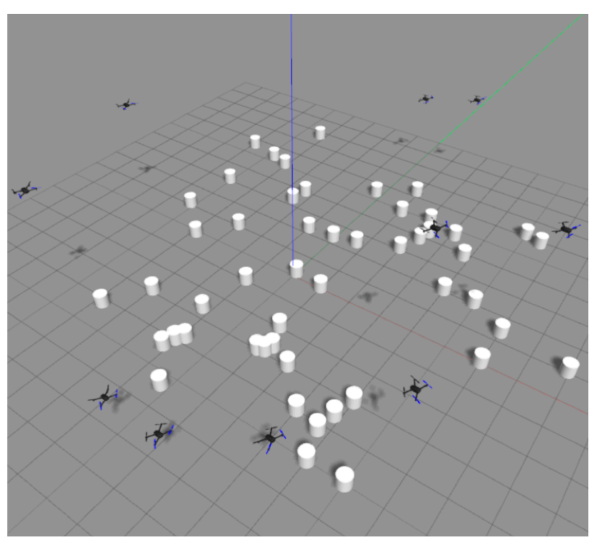


Fig. 4. (a) Gazebo environment.

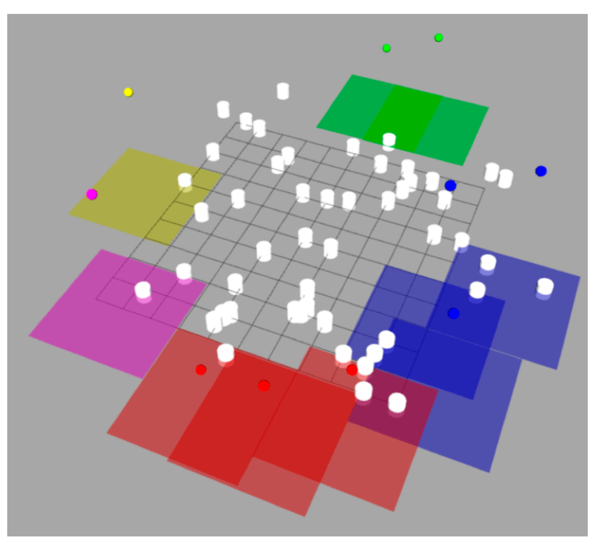


Fig. 4. (b) Rviz environment.

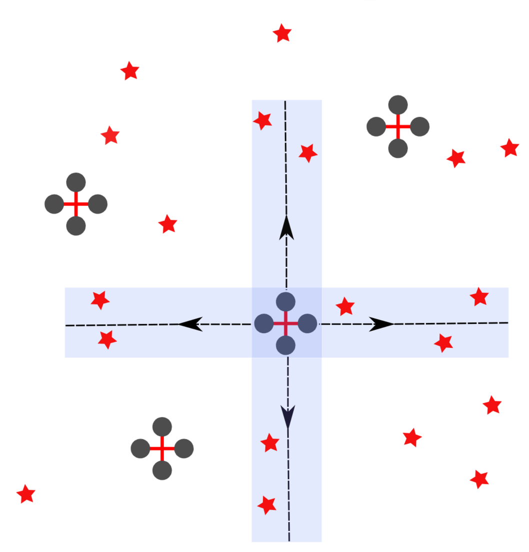
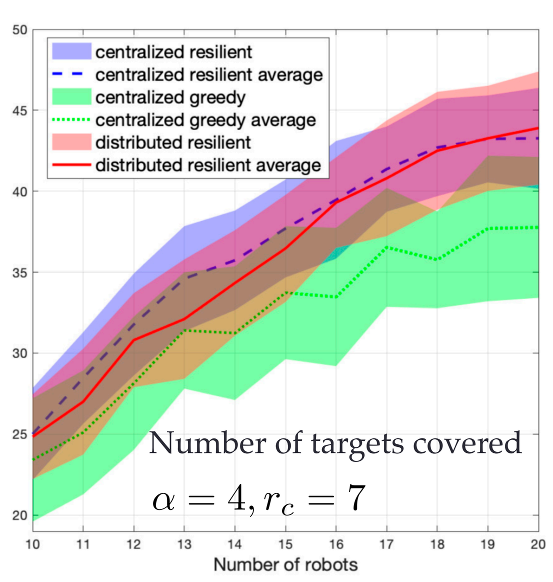
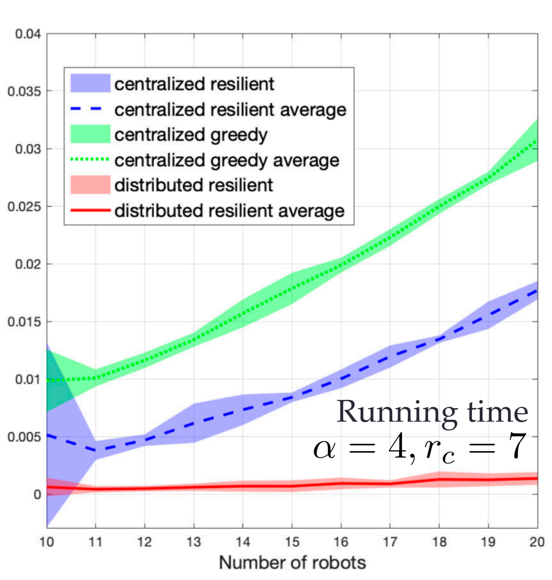
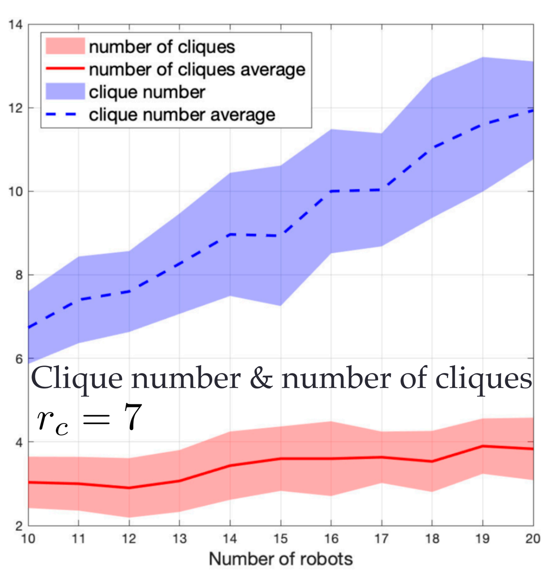


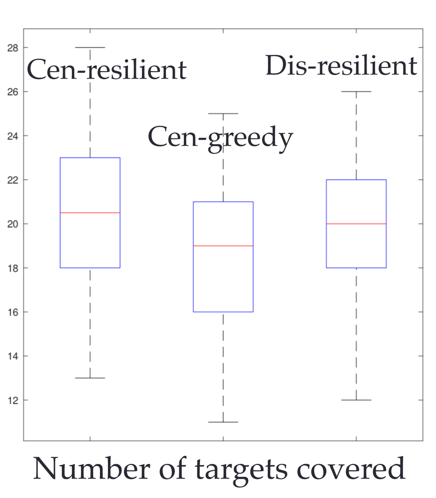
Fig. 4. (c) A top view of some robots and targets.

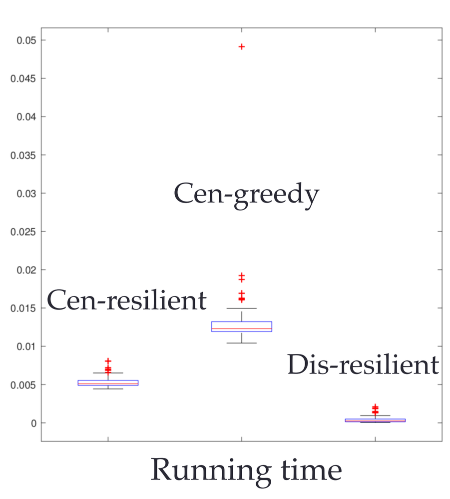
- **Comparison of the number of targets covered and running time: (1) centralized resilient, (2) centralized greedy, (3) distributed resilient**

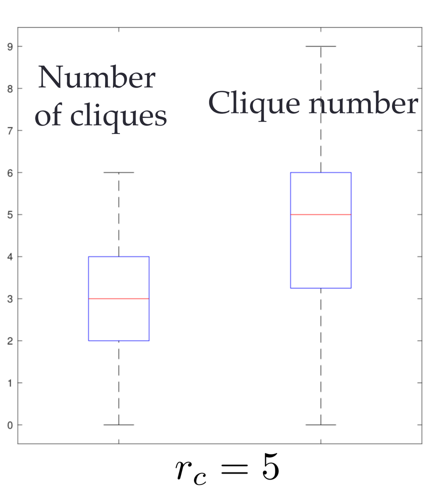












- Distributed resilient covers similar number of targets as centralized resilient, and covers more than centralized greedy.
- Distributed resilient runs faster than both centralized resilient and centralized greedy.

FRAMEWORK

- **Communication:** Robots $\mathcal{R}=\{1,\cdots,N\}$ have the same communication range r_c. They communicate on an undirected communication graph $\mathcal{G}$. The graph has a number of cliques $\mathcal{C}(\mathcal{G})$ (see an example in Fig. 2).
- **Strategy:** Each robot i has a candidate strategy set $\mathcal{X}_i$ from which it must choose one, $s_i\in\mathcal{X}_i$.
- **Objective:** A normalized, monotone (increasing) and submodular function $f:2^{\mathcal{X}}\rightarrow\mathbb{R}_{\geq 0}$ defined on robots' strategy set.
- **Attacks:** At any time, at most α robots will fail and stop contributing to f due to attacks.

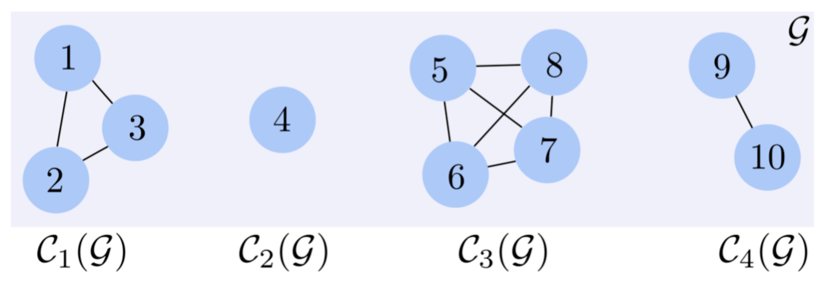


Fig. 2. A graph $\mathcal{G}$ that contains 10 robots and 4 cliques.

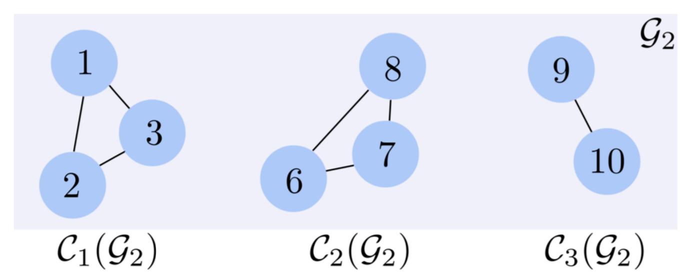


Fig. 3. Subgraph $\mathcal{G}_2$.

CONTRIBUTIONS

- **Distributed resilient algorithm:** all cliques of robots perform *a resilient algorithm in parallel*.
  - If $n_k>\alpha$, the robots in clique $\mathcal{C}_k(\mathcal{G})$: (1) obliviously choose the α most profitable strategies; (2) the remaining $n_k-\alpha$ robots greedily choose the following strategies.
  - Else if $n_k\leq\alpha$, the robots in clique $\mathcal{C}_k(\mathcal{G})$: obliviously choose the n_k most profitable strategies.
- **Performance of distributed resilient algorithm:**
  - **Approximation performance :** when $\mathcal{K}(\mathcal{G})\geq 2$

$$\frac{f(\mathcal{S}\setminus\mathcal{A}^\star(\mathcal{S}))}{f^\star}\geq$$

$$\max\left[\frac{1-k_f}{1+k_f},\frac{1}{(\alpha+1)\mathcal{K}(\mathcal{G}_2)\omega(\mathcal{G}_2)},\frac{1}{(N-\alpha)\mathcal{K}(\mathcal{G}_2)\omega(\mathcal{G}_2)}\right]$$

  where $k_f\in[0,1]$ is the curvature of function f. $\mathcal{G}_2$ is a subgraph of $\mathcal{G}$ (see an example in Fig. 3). $\mathcal{K}(\mathcal{G}_2)$ and $\omega(\mathcal{G}_2)$ are the number of cliques and clique number in $\mathcal{G}_2$.
  - **Running time:** It runs in $O(\omega^2(\mathcal{G})D^2)$ time. $\omega(\mathcal{G})$ and D are the clique number of $\mathcal{G}$ and the number of candidate strategies for each robot.
- **Analysis of the performance:**
  - When $k_f=0$, distributed resilient algorithm gives the optimal solution.
  - Distributed resilient algorithm runs faster than centralized resilient algorithm and the centralized greedy algorithm.

CENTRALIZED RESILIENT

- **All robots communicate with each other to counter α worst-case attacks while tracking targets.**

L. Zhou, V. Tzoumas, G. J. Pappas, and P. Tokekar. ICRA+ RA-L '19
Resilient Active Target Tracking With Multiple Robots
MoC1-23 Interactive Session, 220
Path Planning for Multi-Robot Systems II - 1.3.23