The 13th International Workshop on the Algorithmic Foundations of Robotics

An Approximation Algorithm for Risk-averse Submodular Optimization

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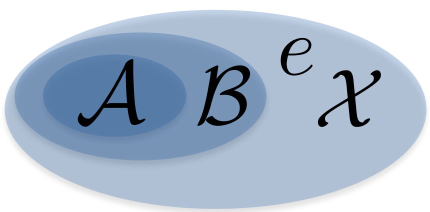
Submodular function

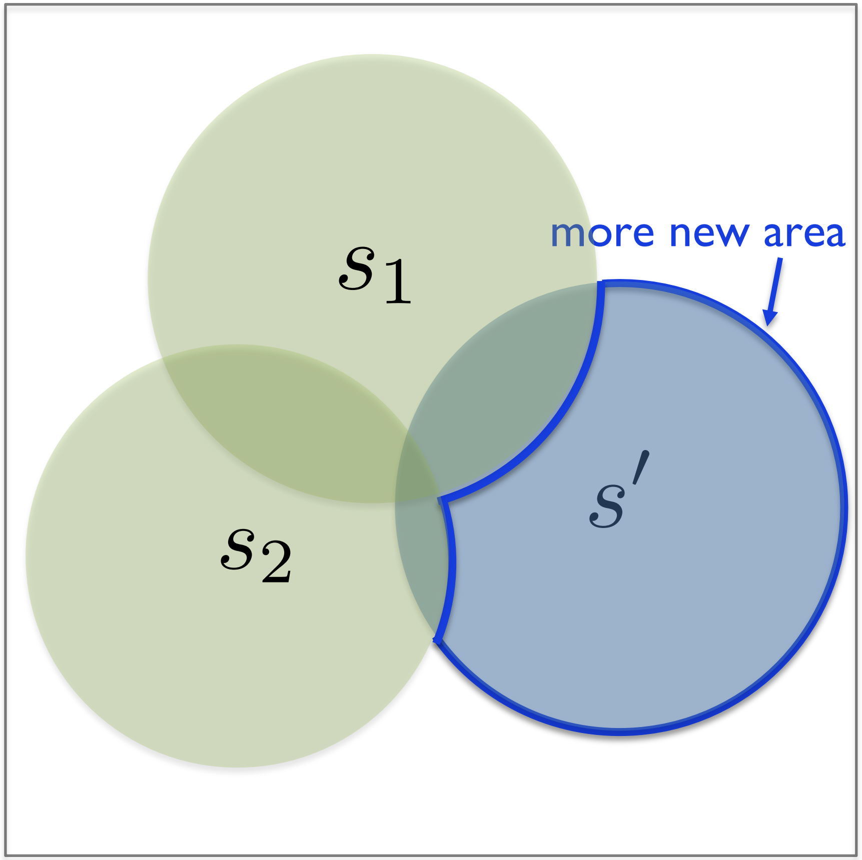
A set function $f : 2^{\mathcal{X}} \rightarrow \mathbb{R}$ is submodular if for every $\mathcal{A} \subseteq \mathcal{B} \subseteq \mathcal{X}$ and $e \in \mathcal{X} \backslash \mathcal{B}$ it holds



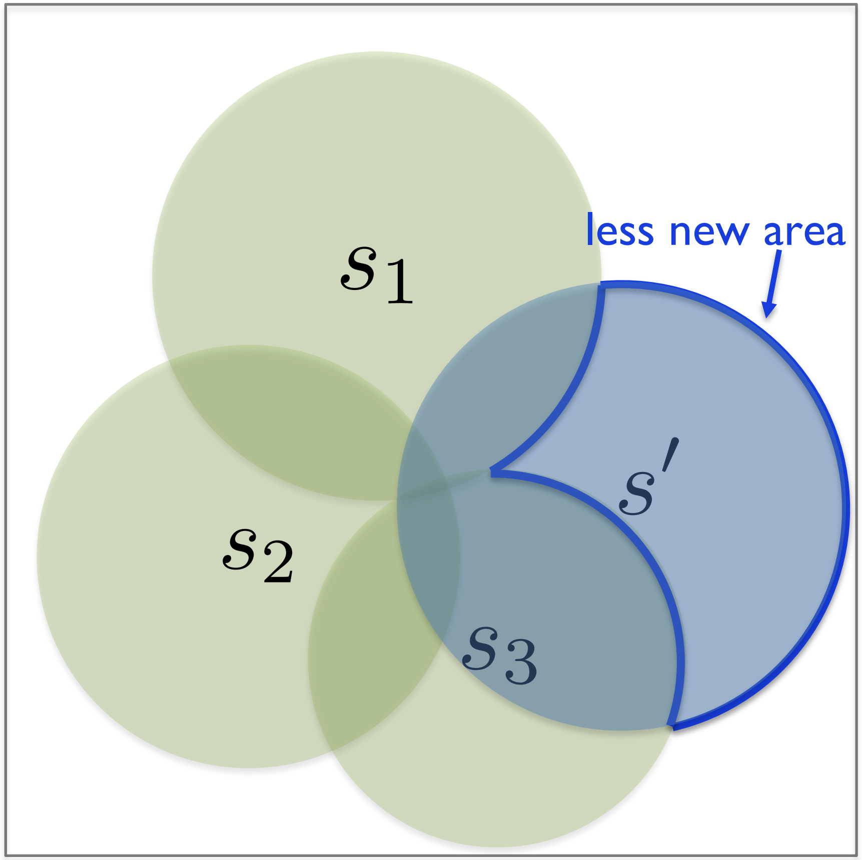
Marginal gain

$$f(\mathcal{A} \cup e) - f(\mathcal{A}) \geq f(\mathcal{B} \cup e) - f(\mathcal{B})$$

Diminishing return



Add s' to set $\{s_1, s_2\}$



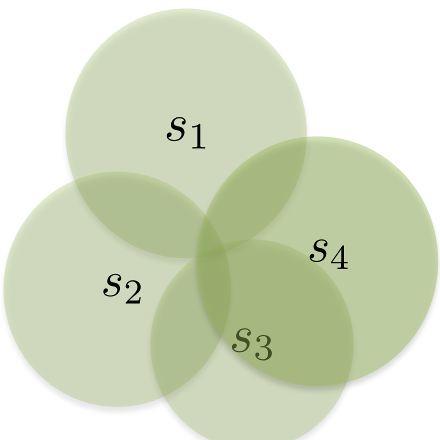
Add s' to superset $\{s_1, s_2, s_3\}$

Sensor placement

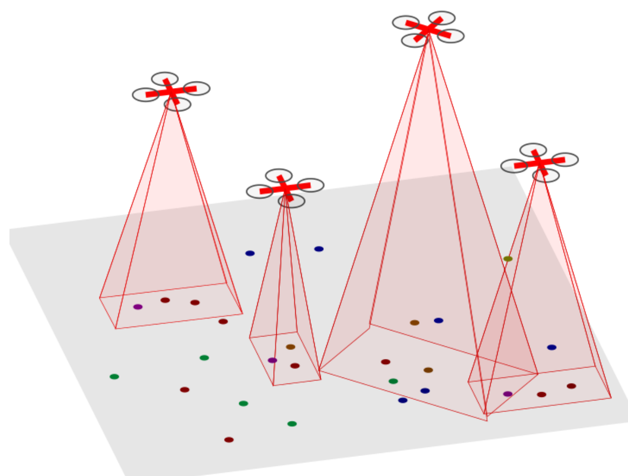
Submodular maximization

$$\max\ f(\mathcal{S}),\ \text{s.t. } \mathcal{S} \subseteq \mathcal{X}, \mathcal{S} \in \mathcal{I}$$

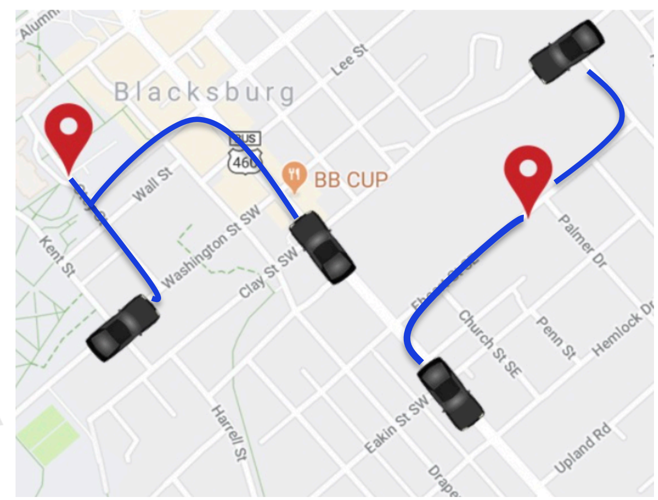
- Applications: sensor placement, information gathering, target tracking, task assignment …



Sensor placement



Multi-target tracking



Redundant task assignment

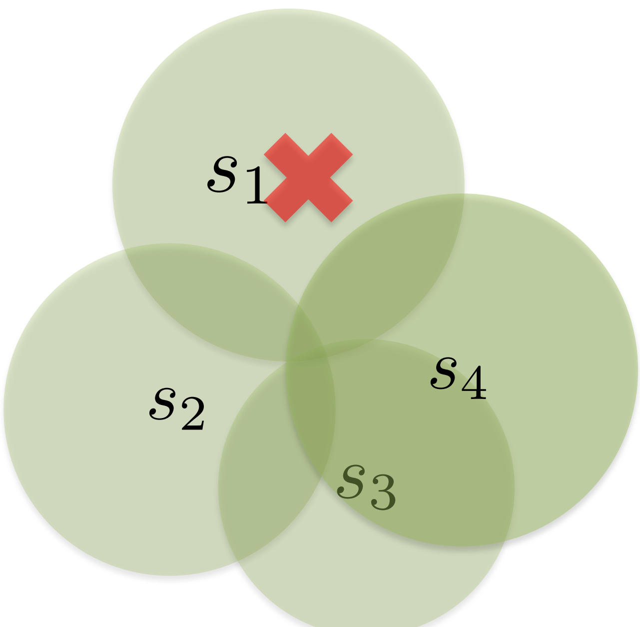
Greedy algorithm

$$\max\ f(\mathcal{S}),\ \text{s.t. } \mathcal{S} \subseteq \mathcal{X}, \mathcal{S} \in \mathcal{I}$$

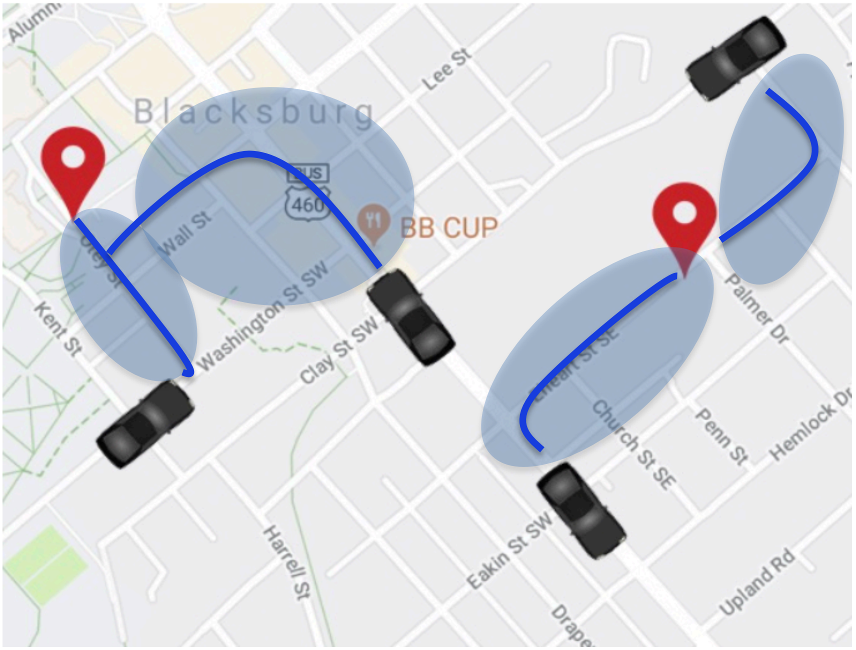
- Incrementally picks the element with the maximal marginal gain
- Gives $1 - 1/e$ approximation when the function value is known exactly

Robot failures and uncertainty

$f(\mathcal{S})$ is a random variable



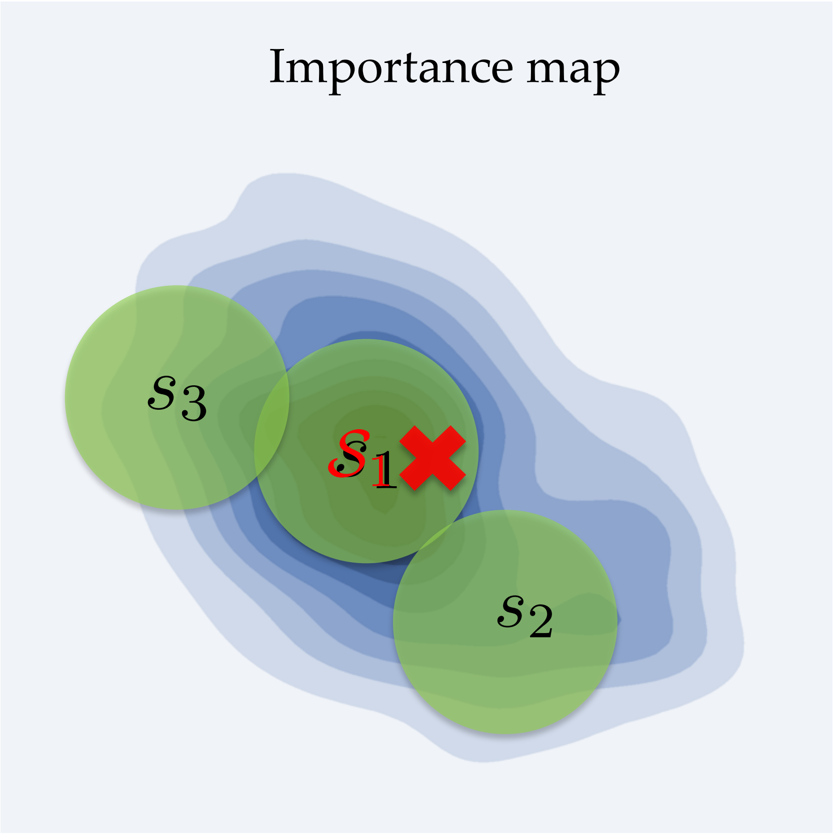
Sensor/Robot failures



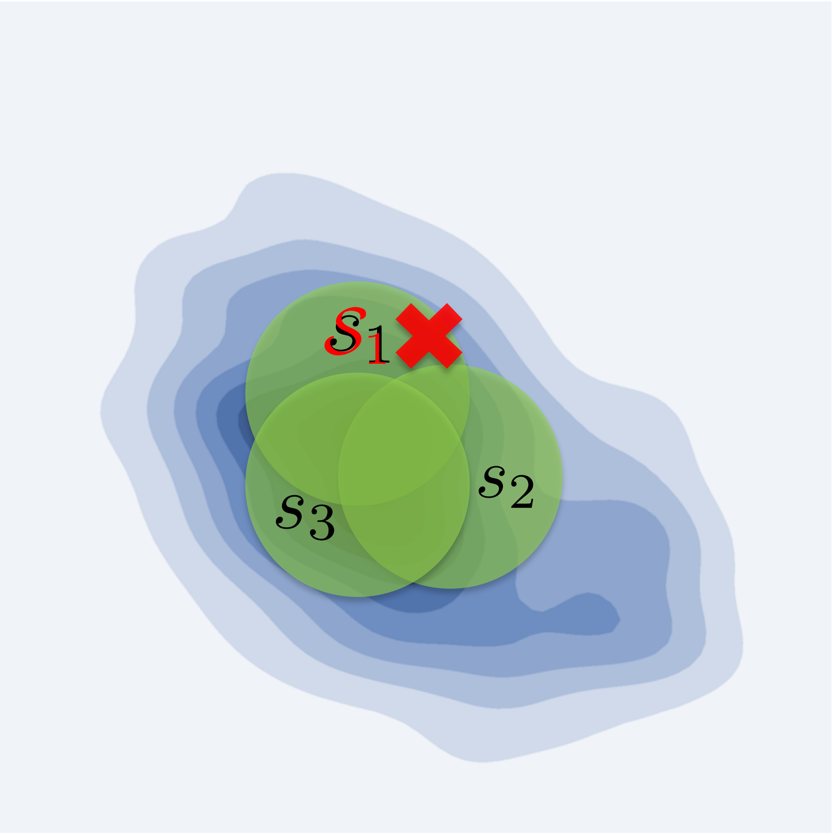
Uncertain travel time

Handling failure is challenging

Greedy placement
vulnerable to sensor failure



Robust placement
Large overlap

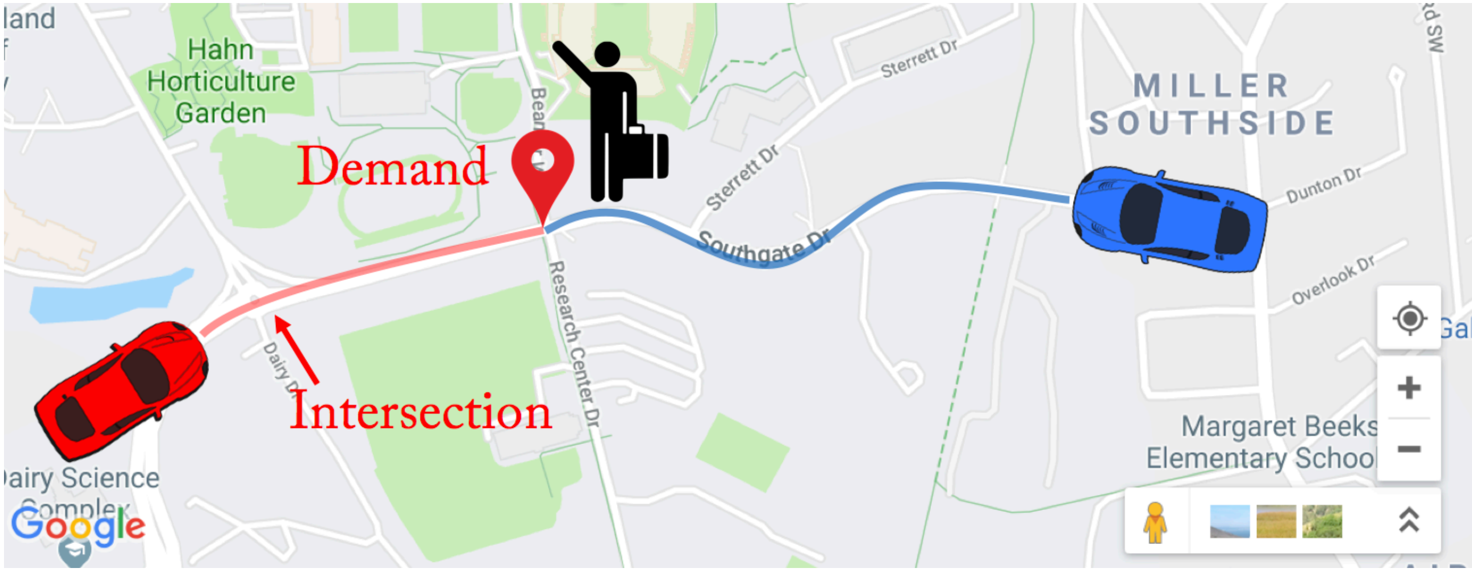


Stochastic submodular function

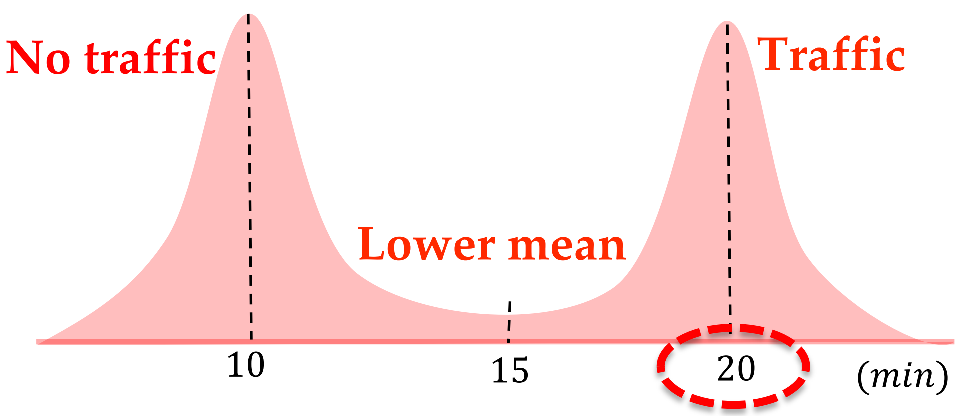
- $f(\mathcal{S}, y)$ is submodular in decision set $\mathcal{S}$
- $f(\mathcal{S}, y)$ is a random variable, the randomness is induced by y. Assume y is independent of $\mathcal{S}$

- Standard measure: Expectation, $\mathbb{E}_y(f(\mathcal{S}, y))$
- Expectation is not risk-aware

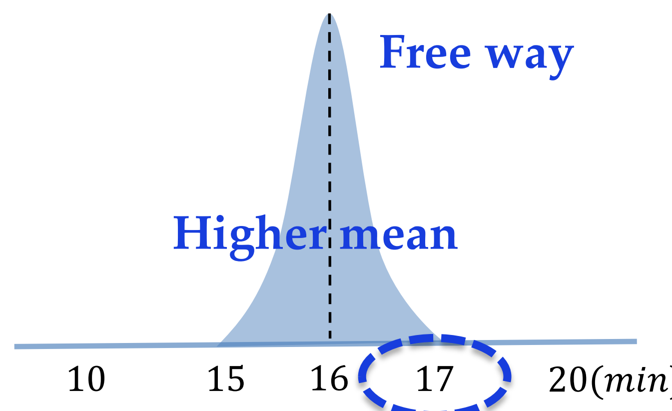
Risk-aware measure instead of expectation



Which vehicle arrives first?



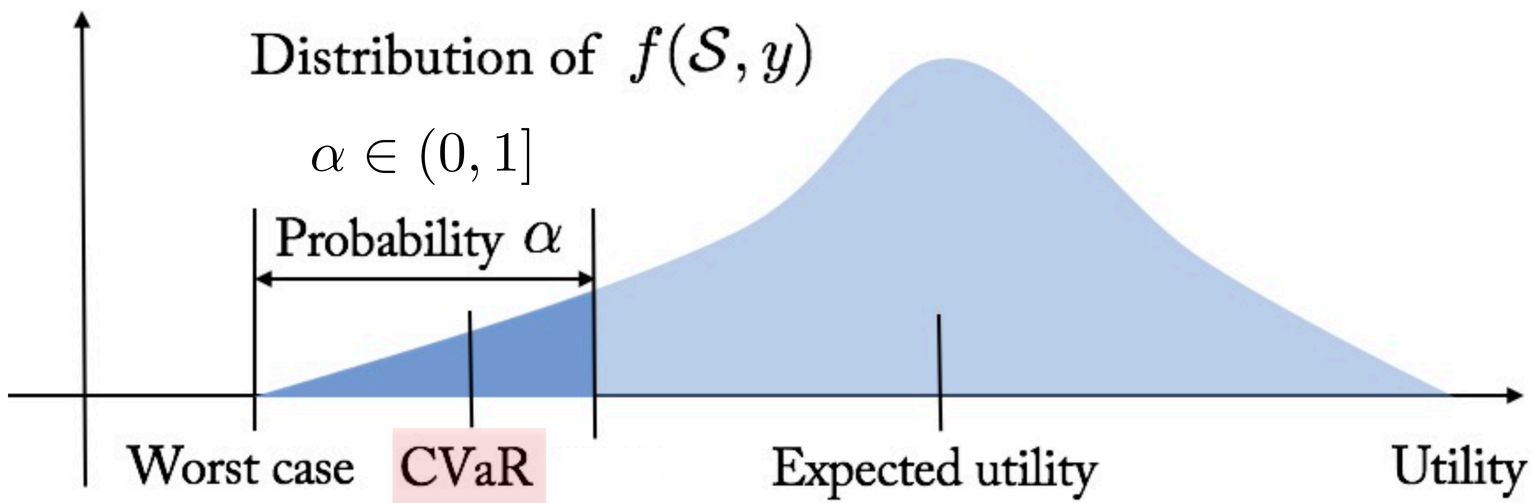
Travel time: Red vehicle



Travel Time: Blue vehicle

Conditional-value-at-risk

- CVaR is the expected performance in the worst $\alpha\%$ of cases



Maximizing CVaR

- Optimization Problem:

Function $H(\mathcal{S},\tau)$

$$\max \ \mathrm{CVaR}_\alpha(\mathcal{S}) \qquad s.t.\ \mathcal{S}\in\mathcal{I},\ \mathcal{S}\subseteq\mathcal{X}$$

$$\max \ \tau - \frac{1}{\alpha}\mathbb{E}[(\tau - f(\mathcal{S},y))_+] \qquad s.t.\ \mathcal{S}\in\mathcal{I},\ \mathcal{S}\subseteq\mathcal{X},\ \tau\in[0,\Gamma]$$

Upper bound on τ

- Maximize $\mathrm{CVaR}_\alpha(\mathcal{S})$ on the decision set $\mathcal{S}$ is equivalent to maximizing $H(\mathcal{S},\tau)$ over $\mathcal{S}$ and τ

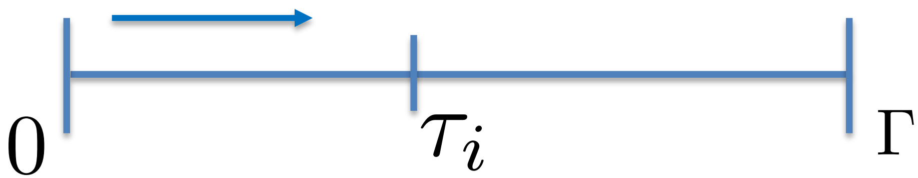
Contributions

- We present a sequential greedy algorithm (SGA)
- SGA gives a bounded approximation of the optimal
- SGA runs in polynomial time

Sequential greedy algorithm

- Sequentially search for τ

$$H(\mathcal{S},\tau) = \tau - \frac{1}{\alpha}\mathbb{E}[(\tau - f(\mathcal{S},y))_+]$$



- For each τ_i, solve a subproblem by the greedy algorithm to select set: $\mathcal{S}_i^G$

$$\max \quad \tau_i - \frac{1}{\alpha}\mathbb{E}[(\tau_i - f(\mathcal{S},y))_+]$$

$$s.t. \quad \mathcal{S} \in \mathcal{I}, \mathcal{S} \subseteq \mathcal{X}$$

$H(\mathcal{S},\tau_i)$
Monotone and Submodular in $\mathcal{S}$

- Find the optimal $(\mathcal{S}_i^G, \tau_i)$ that maximizes $H(\mathcal{S}_i^G, \tau_i)$

$$(\mathcal{S}^G, \tau^G) = \underset{\{(\mathcal{S}_i^G,\tau_i)\}}{\operatorname{argmax}} \; H(\mathcal{S}_i^G, \tau_i)$$

Output $\mathcal{S}^G$

Sequential greedy algorithm: bounded approximation

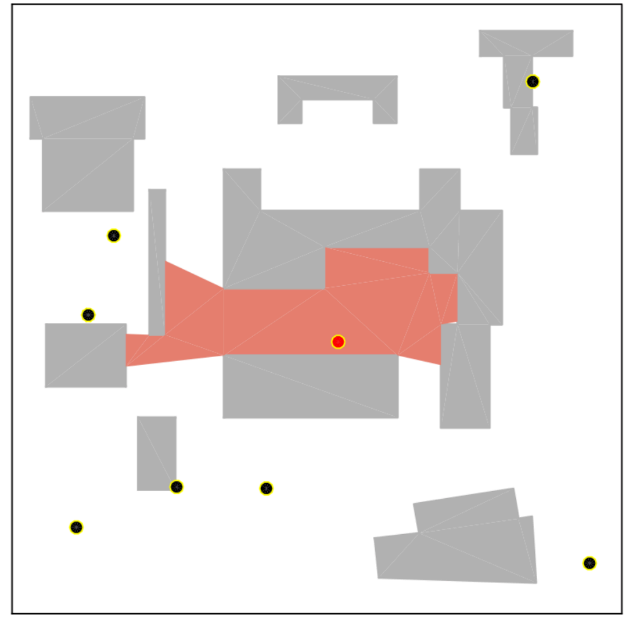
$$H(\mathcal{S}^G, \tau^G) \geq \boxed{\frac{1}{1+k_f}}(H(\mathcal{S}^\star, \tau^\star) - \Delta) \boxed{- \frac{k_f}{1+k_f}\Gamma(\frac{1}{\alpha} - 1)}$$

Constant-factor Approximation Error

k_f measures how far the submodular function is from modularity or linearity
Δ is the user-defined searching separation for τ
Γ is the upper bound on τ

Robust environmental monitoring

(a) Part of Virgina Tech campus from Google Earth.



(b) Top view of part of a campus and ground sensor's visibility region.

$$f(\mathcal{S}, y) = \text{area}(\bigcup_{i=1:M} A_i),$$

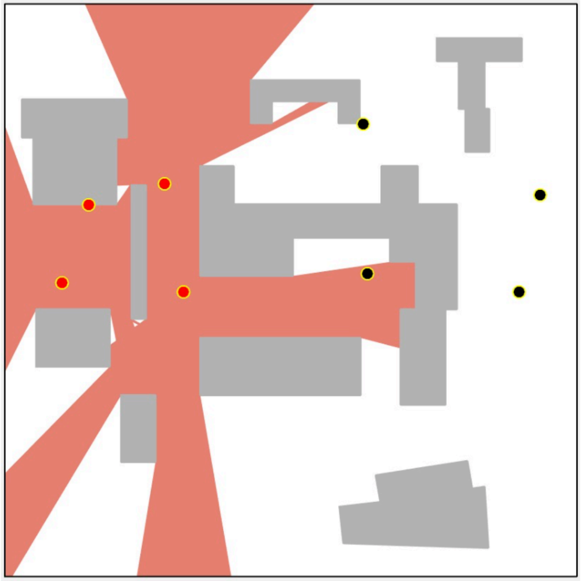
$$i \in \mathcal{S}, \mathcal{S} \subseteq \mathcal{I}.$$

Sensor $i \in \{1, \ldots, N\}$

A_i : random polygon

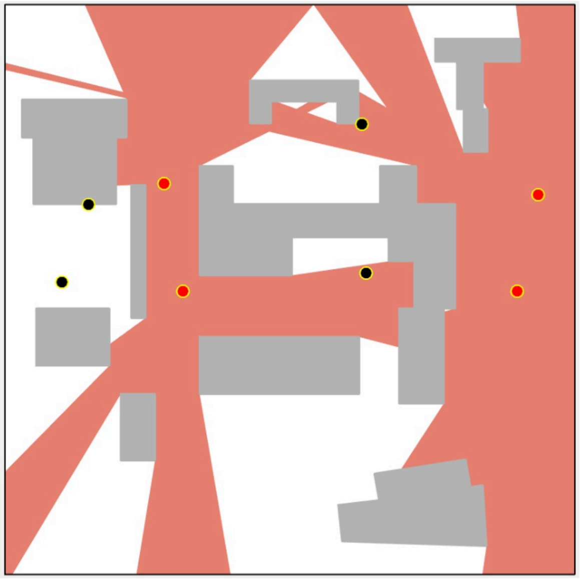
Low risk level VS expectation

Conservative



(a) Selection when $\alpha = 0.1$.

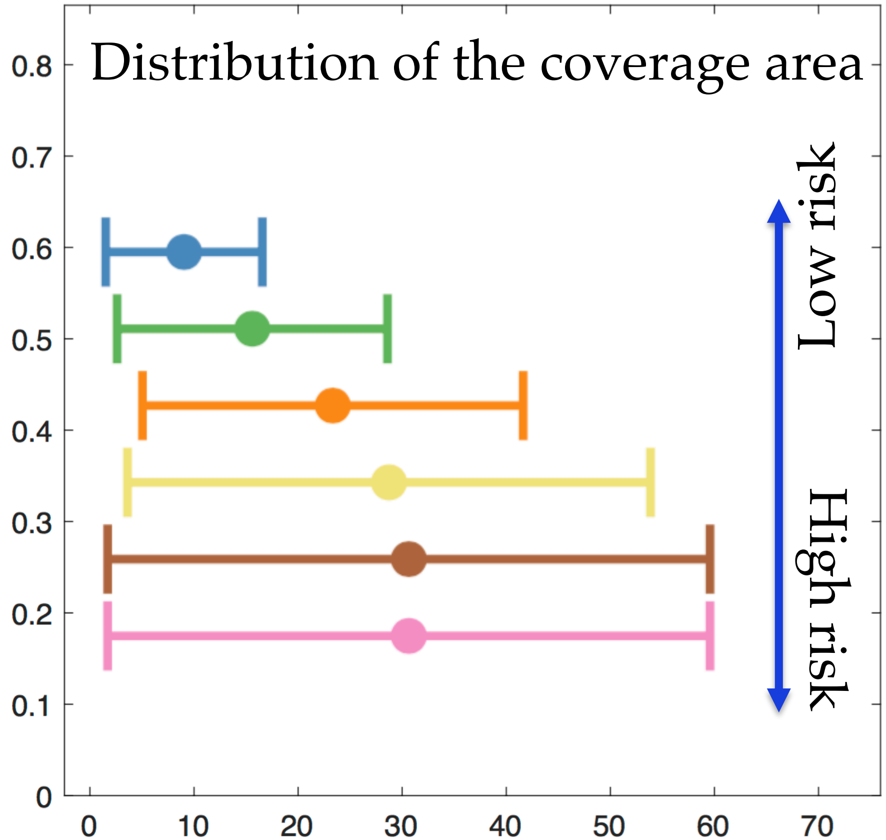
Riskier



(b) Selection when $\alpha = 1$ (Risk-neutral).

- Compare risk-averse with risk-neutral → Lower visibility region VS Higher visibility region

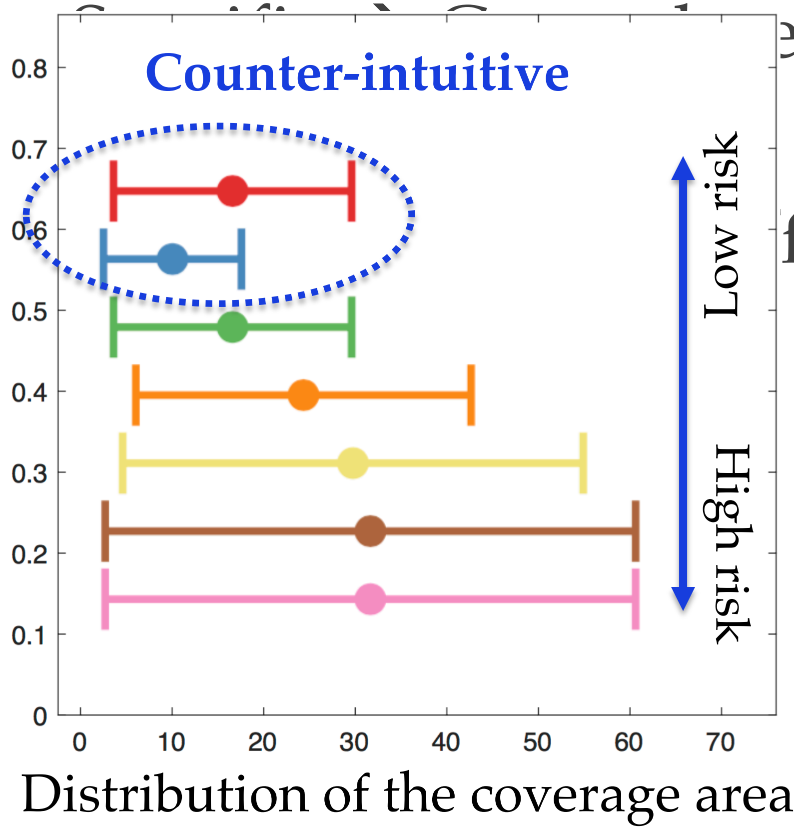
Coverage area w.r.t. risk levels

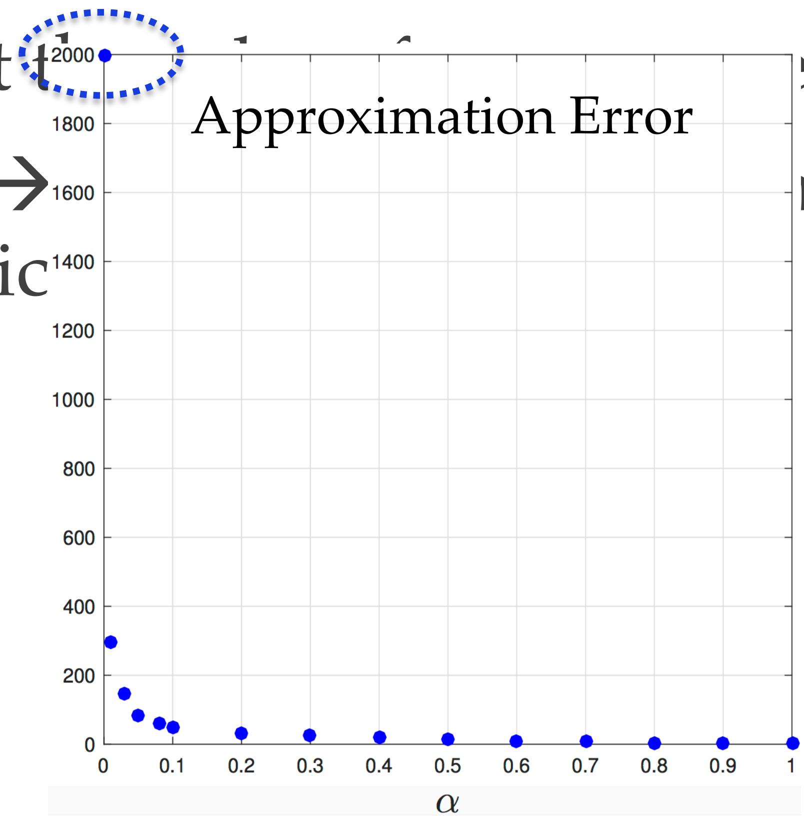


- Small risk level → Small coverage area (small visibility region & large overlap) → Conservative
- Large risk level → Large coverage area (large visibility region & small overlap) → Riskier

Dirty laundry and ongoing work

- Bounded approximation → Large approximation error → new algorithm for small risk level





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Thank you!

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