k-Rank-2-Nash

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Problem

Bimatrix Game: played by two players ROW and COLUMN

Two payoff matrices A, B $\in Q^{m \times n}$

0	1	-2
0	2	2
1	2	-1

ROW chooses i

0	2	0
0	-2	2
1	1	1

COLUMN chooses j

Pure Nash Equilibrium

Example: Rock, Paper, Scissors

$$\begin{array}{|c|}\hline 1/3 \\ \hline 1/3 \\ \hline 1/3 \\ \hline \end{array} \left\{ \begin{array}{|c|c|c|}\hline 0 & -1 & 1 \\ \hline 1 & 0 & -1 \\ \hline -1 & 1 & 0 \\ \hline \end{array} \right. \quad \begin{array}{|c|c|c|}\hline 0 & 1 & -1 \\ \hline -1 & 0 & 1 \\ \hline 1 & -1 & 0 \\ \hline \end{array}$$

Mixed Nash Equilibrium

Nash Equilibrium : no player can gain by a unilateral change of strategy

Bounded Rank

Rank of matrix: # of linearly independent rows

Rank of Game: rank(A+B)

$$\begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline -2 & -2 & 1 \\ \hline 3 & 5 & 0 \\ \hline \end{array} \; : \begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline -2 & -2 & 1 \\ \hline 3 & 5 & 0 \\ \hline \end{array} \; R_2 \longrightarrow 2r_1+r_2 \quad \begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 0 & 1 & 3 \\ \hline 3 & 5 & 0 \\ \hline \end{array} \; R_3 \longrightarrow -3r_1+r_3 \quad \begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 0 & 1 & 3 \\ \hline 0 & -1 & -3 \\ \hline \end{array} \; R_3$$

$$\longrightarrow r_2+r_3 \quad \begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 0 & 1 & 3 \\ \hline 0 & 0 & 0 \\ \hline \end{array} \; R_1 \longrightarrow -2r_2+r_1 \quad \begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 0 & 1 & 3 \\ \hline 0 & 0 & 0 \\ \hline \end{array} \Big\}$$

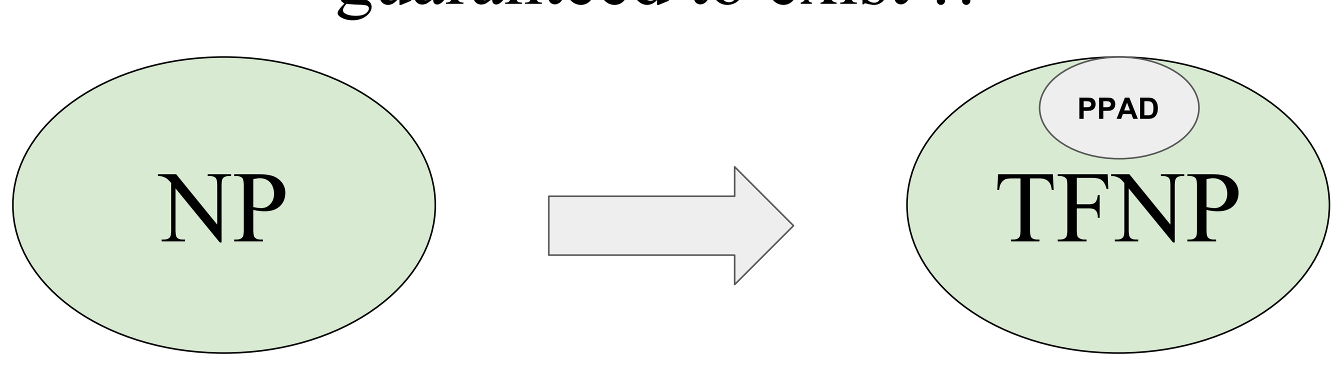
Rank = 2, # non zero rows in echelon form

Bound on rank ⇒ Constraints on # of independent payoff values

> *k-Rank-2-Nash*
> **Input**: A bimatrix game *G(A,B)* and an integer *k* s.t. *rank(A)*, *rank(B)* are both at most *k*.
> **Parameter**: *k*
> **Question**: Compute the Nash equilibrium of *G(A,B)*.

Significance

Different notion of hardness because solution is guaranteed to exist !!



Applications of Finding Nash Equilibrium:

- Political Elections.
- Stock Market Analysis.
- Cryptography, Secure Multiparty Computation.
- Social Interactions.

Related Work

2-Nash is proven to be PPAD-complete.

(polynomial parity argument on directed graphs).

k-rank 2-Nash is proven to be PPAD-hard.

Bounding the # of values is promising because:

- Sparse Matrix games
- Bounded treewidth games
- K-unbalanced games are already solved in FPT parameterized on # payoff values.