### **Parameters**

```
\ln[3935] = \rho_{\text{max}} = 1 \ (*\text{maximum carbon uptake rate } (d^{-1})*);
       \alpha_{\text{max}} = 1.5 * 10^{\text{A}} - 9 (*attack rate of mixotroph on bacteria (cm<sup>2</sup>*d<sup>-1</sup>*cell<sub>M</sub><sup>-1</sup>)*);
       b = .15(*conversion rate of bacteria to mixotroph (cell<sub>M</sub>*cell<sub>B</sub><sup>-1</sup>)*);
       K_B = 1 \times 10^8; (*carrying capacity of bacteria (cell<sub>B</sub>*cm<sup>-2</sup>)*);
       r = .693(*growth rate of bacteria (d^{-1})*);
       h = 250(*half saturation constant for photosynthesis (µmol quanta*m²*s⁻¹)*);
       I_{in} = 100 (*incident light (\mu mol quanta*m^2*s^{-1})*);
       k = 5 * 10^{(-7)} (*mixotroph light absorbance constant <math>(cm^2 * cell_{M}^{-1}) *);
       l = .05 (*mixotroph mortality rate (d^{-1})*);
       m_o = .1;
       (*photosynthetic temeprature sensitivity coefficient (°C<sup>-1</sup>)*);
       m_{\alpha} = .25;
       (*heterotrophic temperature sensitivity coefficient (°C<sup>-1</sup>)*);
       T0 = 13; (*baseline temperature (^{\circ}C)*);
       T\theta\alpha = T\theta - \frac{1}{m} (*minmimum temperature for heterotrophy (°C)*);
       T\theta \rho = T\theta - \frac{1}{m_0} (*minmimum temperature for photosynthesis (°C)*);
       k_b = 8.62 * 10^{-5} (*Boltzmann constant (eV*K^{-1})*);
       E_{a\rho} = .5 (*photosynthetic activation energy (eV)*);
       E_{a\alpha} = .85 (*heterotrophic activation energy (eV)*);
       r0p = 6.4279909706*^8 (*photosynthetic normalization constant*);
       r0\alpha = 9.412997398*^14 (*heterotrophic normalization constant*);
```

# Equations/Functions for generating outputs

#### **Equations**

```
 \begin{array}{ll} & \text{ln} \text{[3954]:=} & \textbf{(*temperature-dependent photosynthetic rate*)} \\ & \rho \left[\theta_{-}, z_{-}, T_{-}\right] := \rho_{\text{max}} * \left(1 - \theta^{2^{z}}\right)^{\frac{1}{2^{z}}} \left(\mathfrak{m}_{\rho} \left(T - \mathsf{T0}\rho\right)\right) \\ \end{array}
```

$$\begin{split} \rho & \text{Exp}\left[\theta_{-},\,z_{-},\,T_{-}\right] := \rho_{\text{max}} * \left(1-\theta^{2^{z}}\right)^{\frac{1}{2^{z}}} r\theta \rho \; E^{\frac{-E_{a_{\rho}}}{k_{b}}(273+T)} \\ & (*\text{temperature-dependent grazing rate*}) \\ & \alpha [\theta_{-},\,T_{-}] := \alpha_{\text{max}} * \theta \; \left(m_{\alpha} \left(T-T\theta\alpha\right)\right) \\ & \alpha \text{Exp}\left[\theta_{-},\,T_{-}\right] := \alpha_{\text{max}} * \theta \; r\theta\alpha \; E^{\frac{-E_{a_{\rho}}}{k_{b}}(273+T)} \end{split} \\ & (*\text{solves for mixotroph and bacteria population density at equilibrium*}) \\ & \text{eqs}\left[\theta_{-},\,z_{-},\,T_{-}\right] := \text{FindRoot}\left[\left\{dM\left[\theta,\,\,z,\,T\right] := \theta,\,dB\left[\theta,\,T\right] := \theta\right\}, \\ & \left\{\left\{M,\,10^{\wedge}7\right\},\,\left\{B,\,10^{\wedge}7\right\}\right\},\,\text{AccuracyGoal} \to \text{Infinity}\right] \\ & \text{eqsExp}\left[\theta_{-},\,z_{-},\,T_{-}\right] := \text{FindRoot}\left[\left\{dM\text{Exp}\left[\theta,\,z,\,T\right] := \theta,\,dB\text{Exp}\left[\theta,\,T\right] := \theta\right\}, \end{split}$$

{{M, 10^7}, {B, 10^7}}, AccuracyGoal → Infinity]

(\*mixotroph per capita growth rate\*)

$$\begin{split} & dM[\theta_-, \ z_-, T_-] := \left(\frac{\rho[\theta, z, T]}{k \, M} \, Log\left[\frac{\left(h + I_{in}\right)}{\left(h + I_{in} * \, Exp[-k \, M \,]\right)}\right] - l + \alpha[\theta, T] \, b \, B\right) \\ & dMExp[\theta_-, \ z_-, T_-] := \left(\frac{\rho Exp[\theta, z, T]}{k \, M} \, Log\left[\frac{\left(h + I_{in}\right)}{\left(h + I_{in} * \, Exp[-k \, M \,]\right)}\right] - l + \alpha Exp[\theta, T] \, b \, B\right) \end{split}$$

(\*bacteria per capita growth rate\*)

$$\begin{split} dB\left[\theta_{-},\,T_{-}\right] &:= \left(r\left(1 - \left(\frac{B}{K_{B}}\right)\right) - \alpha\left[\theta\,,\,T\right]\,\,M\right) \\ dBExp\left[\theta_{-},\,T_{-}\right] &:= \left(r\left(1 - \left(\frac{B}{K_{B}}\right)\right) - \alphaExp\left[\theta\,,\,T\right]\,\,M\right) \end{split}$$

(\*mutant fitness equation\*)

Fitness[
$$\theta$$
m\_, z\_, M\_, B\_, T\_] := -l +  $\alpha$ [ $\theta$ m, T] bB + 
$$\frac{\rho[\theta m, z, T] Log\left[\frac{h + I_{in}}{h + e^{-(k M)} I_{in}}\right]}{k M}$$
FitnessExp[ $\theta$ m\_, z\_, M\_, B\_, T\_] := -l +  $\alpha$ Exp[ $\theta$ m, T] bB + 
$$\frac{\rho Exp[\theta m, z, T] Log\left[\frac{h + I_{in}}{h + e^{-(k M)} I_{in}}\right]}{k M}$$

(\*selection gradient\*)

SelectionGrad[
$$\theta$$
m\_, z\_, M\_, B\_, T\_] :=

$$\alpha_{\text{max}} \left( \text{M}_{\alpha} \left( \text{T} - \text{T0}\alpha \right) \right) \text{ b B } - \frac{\rho \left[ \theta \text{m, z, T} \right] \theta \text{m}^{-1+2^z} \left( 1 - \theta \text{m}^{2^z} \right)^{-1} \text{ Log} \left[ \frac{\text{h+I}_{\text{in}}}{\text{h+e}^{\text{-k} \, \text{M}} \, \text{I}_{\text{in}}} \right]}{\text{k M}}$$

SelectionGradExp[⊕m\_, z\_, M\_, B\_, T\_] :=

$$b \; B \; e^{-\frac{e_{a\alpha}}{(273+T)\; k_b}} \; r \theta \alpha \; \alpha_{\text{max}} \; - \; \frac{e^{-\frac{e_{a\alpha}}{(273+T)\; k_b}} \; r \theta \rho \; \theta \text{m}^{-1+2^z} \; \left(1 - \theta \text{m}^{2^z}\right)^{-1+2^{-z}} \; \text{Log}\left[\frac{h + \hat{\textbf{i}}_{:n}}{h + e^{-kM}\; \hat{\textbf{i}}_{:n}}\right] \; \rho_{\text{max}}}{k \; M}$$

```
In[•]:=
In[ • ]:=
```

#### Pairwise invasibility plots (PIP)

```
In[3968]:= (*Uses a chosen shape parameter z, temperature T,
       and color to generate a pairwise invasibility plot using
        the fitness function for an invading mutant mixotroph. Regions
        in which mutant fitness is positive are shaded*)
       MakePIP[z , T , color ] :=
        RegionPlot[Fitness[\theta m, z, If[(M /. eqs[\theta, z, T]) \geq 0, M /. eqs[\theta, z, T], 0],
            If [(B/.eqs[\theta, z, T]) \ge 0, B/.eqs[\theta, z, T], 0], T] \ge 0, \{\theta, 0, 1\}, \{\theta m, 0, 1\},
         PlotStyle → color, BoundaryStyle → {Bold, Dashed, Black}, Frame → True,
          FrameLabel \rightarrow {Style["Resident Heterotrophic investment (\theta_{res})", 12, Black],
            Style["Mutant Heterotrophic investment (\theta_{mut})", 12, Black]},
         FrameTicksStyle → Directive[Black, 12], ImageSize → Medium]
      MakePIPNV[z_, T_, color_, colorm_] :=
        RegionPlot[Fitness[\thetam, z, M /. eqs[\theta, z, T], B /. eqs[\theta, z, T], T] \geq 0,
          \{\theta, 0, 1\}, \{\theta m, 0, 1\}, PlotStyle \rightarrow color, Mesh \rightarrow 20, MeshStyle \rightarrow colorm,
         BoundaryStyle → {Bold, Dashed, Black}, Frame → True,
         FrameLabel \rightarrow {Style["Resident Heterotrophic investment (\theta_{res})", 12, Black],
            Style["Mutant Heterotrophic investment (\theta_{mut})", 12, Black]},
         FrameTicksStyle → Directive[Black, 12], ImageSize → Medium]
      MakePIPexp[z_, T_, color_] :=
        RegionPlot[Fitness[\thetam, z, If[(M /. eqsExp[\theta, z, T]) \geq 0, M /. eqsExp[\theta, z, T], 0],
            If [(B/. eqsExp[\theta, z, T]) \ge 0, B/. eqsExp[\theta, z, T], 0], T] \ge 0, \{\theta, 0, 1\},
          \{\theta m, 0, 1\}, PlotStyle \rightarrow color, BoundaryStyle \rightarrow {Bold, Dashed, Black}, Frame \rightarrow True,
         FrameLabel \rightarrow {Style["Resident Heterotrophic investment (\theta_{res})", 12, Black],
            Style["Mutant Heterotrophic investment (\theta_{mut})", 12, Black]},
         FrameTicksStyle → Directive[Black, 12], ImageSize → Medium]
       MakePIPNVexp[z_, T_, color_, colorm_] :=
        RegionPlot[Fitness[\thetam, z, M /. eqsExp[\theta, z, T], B /. eqsExp[\theta, z, T], T] \geq 0,
          \{\theta, 0, 1\}, \{\theta m, 0, 1\}, PlotStyle \rightarrow color, Mesh \rightarrow 20, MeshStyle \rightarrow colorm,
         BoundaryStyle → {Bold, Dashed, Black}, Frame → True,
         FrameLabel \rightarrow {Style["Resident Heterotrophic investment (\theta_{res})", 12, Black],
            Style["Mutant Heterotrophic investment (\theta_{mut})", 12, Black]},
          FrameTicksStyle → Directive[Black, 12], ImageSize → Medium]
```

Generating ESS vs Temp plots for generalist and specialist mixotrophs

```
_{\mathit{In[v]}:=} (*makeListGen[] and makeListLin[] generate lists containing the
        evolutionarily stable investment strategy oldsymbol{arTheta}_{	extsf{ESS}} as a function of temperature
        for generalist tradeoff and linear tradeoff mixotrophs respectively*)
In[3972]:= makeθListGen[] :=
         \thetaESSgen = {};
         Quiet[For[T = 1, T < 41, T++,
           current0 = 0m /. FindRoot[
                SelectionGrad[\thetam, 1, M /. eqs[\thetam, 1, T], B /. eqs[\thetam, 1, T], T] == 0, {\thetam, .99}];
           If [Re[current\theta] > 1, AppendTo[\thetaESSgen, 1], (*if calculated \theta<sub>ESS</sub> >
              1 (maximum heterotrophic investment), 1 is added to list*)
             If[Re[currentθ] < 0, AppendTo[θESSgen, 0.], AppendTo[θESSgen, currentθ]]]</pre>
             (*if calculate \theta_{ESS} < 0 (minimum heterotrophic investment),
           0 is added to list*)
          ]];
         <del>0</del>ESSgen
      makeθListLin[] :=
         θESSlin = {};
         Quiet[For[T = 1, T < 41, T++,
            currentθ = θm /. FindRoot[
                SelectionGrad[\thetam, 0, M /. eqs[\thetam, 0, T], B /. eqs[\thetam, 0, T], T] == 0, {\thetam, .5}];
            (*getting around issue where FindRoot identifies the incorrect,
            evolutionarily unstable root in some cases*)
           If[ 1/.0001 (SelectionGrad[.0002, 0, M /. eqs[.0002, 0, T], B /. eqs[.0002, 0, T], T] -
                  SelectionGrad[.0001, 0, M /. eqs[.0001, 0, T], B /. eqs[.0001, 0, T], T]) > 0,
             AppendTo[\thetaESSlin, 1], If[Re[current\theta] > 1, AppendTo[\thetaESSlin, 1], If[
                Re[currentθ] < 0, AppendTo[θESSlin, 0.000], AppendTo[θESSlin, currentθ]]]]
          ]];
         ⊕ESSlin
      makeθListGenExp[] :=
         \thetaESSgen = {};
         Quiet[For[T = 1, T < 41, T++,
            current\theta = \theta m / . FindRoot[SelectionGradExp[\theta m, 1,
                  M /. eqsExp[\Theta m, 1, T], B /. eqsExp[\Theta m, 1, T], T] == 0, {\Theta m, .99}];
           If[Re[current\theta] > 1, AppendTo[\thetaESSgen, 1], (*if calculated \theta_{ESS} >
```

```
1 (maximum heterotrophic investment), 1 is added to list*)
      If[Re[current0] < 0, AppendTo[0ESSgen, 0.], AppendTo[0ESSgen, current0]]]</pre>
      (*if calculate \theta_{ESS} < 0 (minimum heterotrophic investment),
     0 is added to list*)
    ]];
  <del>0</del>ESSgen
make⊖ListLinExp[] :=
  \thetaESSlin = {};
  Quiet[For[T = 1, T < 41, T++,
     current0 = 0m /. FindRoot[SelectionGradExp[0m, 0,
           M / . eqsExp[\Theta m, 0, T], B / . eqsExp[\Theta m, 0, T], T] == 0, {\Theta m, .5}];
     (*getting around issue where FindRoot identifies the incorrect,
     evolutionarily unstable root in some cases*)
     If \left[\frac{1}{.0001}\right] (SelectionGradExp[.0002, 0, M /. eqsExp[.0002, 0, T],
             B /. eqsExp[.0002, 0, T], T] - SelectionGradExp[.0001, 0,
            M /. eqsExp[.0001, 0, T], B /. eqsExp[.0001, 0, T], T]) > 0,
      AppendTo[\thetaESSlin, 1], If[Re[current\theta] > 1, AppendTo[\thetaESSlin, 1], If[
         Re[current0] < 0, AppendTo[0ESSlin, 0.000], AppendTo[0ESSlin, current0]]]
    ]];
  \thetaESSlin
make⊖ListGen[] :=
  \thetaESSgen = {};
  Quiet[For[T = 1, T < 41, T++,
     current0 = 0m /. FindRoot[
         SelectionGrad[0m, 1, M /. eqs[0m, 1, T], B /. eqs[0m, 1, T], T] == 0, {0m, .99}];
     If [Re[current\theta] > 1, AppendTo[\thetaESSgen, 1], (*if calculated \theta<sub>ESS</sub> >
       1 (maximum heterotrophic investment), 1 is added to list*)
      If[Re[current0] < 0, AppendTo[0ESSgen, 0.], AppendTo[0ESSgen, current0]]]</pre>
      (*if calculate \theta_{ESS} < 0 (minimum heterotrophic investment),
     0 is added to list*)
    ]];
  <del>0</del>ESSgen
makeθListLin[] :=
  \thetaESSlin = {};
  Quiet[For[T = 1, T < 41, T++,
     current0 = 0m /. FindRoot[
```

```
SelectionGrad[\thetam, 0, M /. eqs[\thetam, 0, T], B /. eqs[\thetam, 0, T], T] == 0, {\thetam, .5}];
     (*getting around issue where FindRoot identifies the incorrect,
     evolutionarily unstable root in some cases*)
     If [ 1 (SelectionGrad[.0002, 0, M /. eqs[.0002, 0, T], B /. eqs[.0002, 0, T], T] -
           SelectionGrad[.0001, 0, M /. eqs[.0001, 0, T], B /. eqs[.0001, 0, T], T]) > 0,
      AppendTo[θESSlin, 1], If[Re[currentθ] > 1, AppendTo[θESSlin, 1], If[
         Re[currentθ] < 0, AppendTo[θESSlin, 0.000], AppendTo[θESSlin, currentθ]]]]
    ]];
  ⊕ESSlin
make⊖ListGenExp[] :=
  \thetaESSgen = {};
  Quiet[For[T = 1, T < 41, T++,
     currentθ = θm /. FindRoot[SelectionGradExp[θm, 1,
           M /. eqsExp[\thetam, 1, T], B /. eqsExp[\thetam, 1, T], T] == 0, {\thetam, .99}];
     If[Re[current\theta] > 1, AppendTo[\thetaESSgen, 1], (*if calculated \theta<sub>ESS</sub> >
       1 (maximum heterotrophic investment), 1 is added to list*)
      If[Re[currentθ] < 0, AppendTo[θESSgen, 0.], AppendTo[θESSgen, currentθ]]]</pre>
      (*if calculate \theta_{ESS} < 0 (minimum heterotrophic investment),
     0 is added to list*)
    ]];
  OESSgen
makeθListLinExp[] :=
  \thetaESSlin = {};
  Quiet \lceil For \rceil T = 1, T < 41, T++,
     currentθ = θm /. FindRoot[SelectionGradExp[θm, 0,
           M /. eqsExp[\Theta m, 0, T], B /. eqsExp[\Theta m, 0, T], T] == 0, {\Theta m, .5}];
     (*getting around issue where FindRoot identifies the incorrect,
     evolutionarily unstable root in some cases*)
    If \left[\frac{1}{.0001}\right] (SelectionGradExp[.0002, 0, M /. eqsExp[.0002, 0, T],
            B /. eqsExp[.0002, 0, T], T] - SelectionGradExp[.0001, 0,
            M /. eqsExp[.0001, 0, T], B /. eqsExp[.0001, 0, T], T]) > 0,
      AppendTo[\thetaESSlin, 1], If[Re[current\theta] > 1, AppendTo[\thetaESSlin, 1], If[
         Re[current0] < 0, AppendTo[0ESSlin, 0.000], AppendTo[0ESSlin, current0]]]
    ]];
  \thetaESSlin
```

```
makeθListSpec[] :=
           OESSspec1viable = Table[Null, 40];
           \thetaESSspec0 = {};
           Quiet[For[T = 1, T < 41, T++,
                      If[eqs[1, -1, T][[1]][[2]] > 0,
                          If [10^{15} > eqs[.95, -1, T][[1]][[2]] > 0, \thetaESSspec1viable[[T;; T]] = 1];
                    AppendTo[θESSspec0, 0];
                ]];
           θESSspec = Table[θESSspec1viable, θESSspec0];
           ⊕ESSspec
makeθListSpecExp[] :=
           OESSspec1viable = Table[Null, 40];
           \thetaESSspec0 = {};
           Quiet[For[T = 1, T < 41, T++,
                      If[eqsExp[1, -1, T][[1]][[2]] > 0 ,
                          If [10^{15} > eqsExp[.95, -1, T][[1]][[2]] > 0, \theta ESS spec1viable[[T;; T]] = 1];
                     AppendTo[\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\textit{\text
           θESSspec = Table[θESSspec1viable, θESSspec0];
           <del>0ESSspec</u></del>
```

In[•]:=

#### Comparing evolved vs unevolved mixotrophs for carbon cycling

```
In[3991]:= (*generates plots comparing mixotroph and bacteria populations,
      and growth rate components derived from photosynthesis,
     P(\theta,z,I,T,M^*), and heterotrophy, G(\theta,T,B^*),
     between evolving mixotrophs whos heterotrophic investment \theta varies
       as a function of temperature and genetically static mixotrophs with
       fixed \theta. This allows evolutionary and strictly thermal responses
       to be compared. Function inputs consist of the shape parameter, z,
     the chosen lower bounds for each set of plots (l1, l2, and l3) and
       the chosen upper bounds for each set of plots (u1, u2, and u3)*)
     Ccycling[z_, l1_, u1_, l2_, u2_, l3_, u3_] :=
       {make⊖ListLin[];
        make⊖ListGen[];
```

```
θList = List[];
If [z = 0, \theta List = \theta ESSlin, \theta List = \theta ESSgen];
Mpopsevo = List[];
Mpopsnoevo = List[];
Quiet[For[t = 1, t < 100, t++, AppendTo[Mpopsevo, M /. eqs[θList[[t]], z, t]]]];
Quiet[For[t = 1, t < 100, t++, AppendTo[Mpopsnoevo, M /. eqs[θList[[T0]], z, t]]]];
bpopsevo = List[];
bpopsnoevo = List[];
Quiet[For[t = 1, t < 100, t++, AppendTo[bpopsevo, B /. eqs[∂List[[t]], z, t]]]];
Quiet[For[t = 1, t < 100, t++, AppendTo[bpopsnoevo, B /. eqs[\textit{\textit{eqs}}], z, t]]]];</pre>
photgrowthevo = List[];
photgrowthnoevo = List[];
Quiet[For[t = 1, t < 100, t++,
  AppendTo[photgrowthevo, (M /. eqs[\thetaList[[t]], z, t]) * (\rho[\thetaList[[t]], z, t]
           Log\left[\frac{h+I_{in}}{h+e^{-(k(M/.eqs[\theta List[[t]],z,t]))}I_{in}}\right] \bigg/ \left(k\left(M/.eqs[\theta List[[t]],z,t]\right)\right)\right]\right];
Quiet[For[t = 1, t < 100, t++, AppendTo[photgrowthnoevo,
    (M /. eqs[θList[[T0]], z, t]) *
     \left(\left[\rho[\theta List[[T0]], z, t] Log\left[\frac{h + I_{in}}{h + e^{-(k (M/.eqs[\theta List[[T0]], z, t]))} I_{in}}\right]\right) / e^{-(k (M/.eqs[\theta List[[T0]], z, t]))} I_{in}\right]\right)
        (k (M /. eqs[θList[[T0]], z, t])))]]];
hetgrowthevo = List[];
hetgrowthnoevo = List[];
Quiet[For[t = 1, t < 100, t++, AppendTo[hetgrowthevo,
    (B /. eqs[\thetaList[[t]], z, t]) * (M /. eqs[\thetaList[[t]], z, t]) \alpha[\thetaList[[t]], t] b]]];
Quiet[For[t = 1, t < 100, t++, AppendTo[hetgrowthnoevo,
    (B /. eqs[\thetaList[[T0]], z, t]) *
     (M /. eqs[\thetaList[[T0]], z, t]) \alpha[\thetaList[[T0]], t] b]]];
List[ListPlot[{Mpopsevo, Mpopsnoevo}, Joined → True, PlotRange →
    \{T0, 33\}, \{l1, u1\}\}, PlotStyle \rightarrow \{\{If[z > 0, RGBColor["#B09771"], Black]\}, \}
     {If[z > 0, RGBColor["#B09771"], Black], Dashed}}, Frame → True,
  FrameLabel → {Style["Temperature (°C)", 15, Black], Style["M*", 15, Black]},
  FrameTicksStyle → Directive[Black, 12], ImageSize → Medium],
 ListPlot[{bpopsevo, bpopsnoevo}, Joined → True, PlotRange → {{T0, 33}, {l2, u2}},
```

```
PlotStyle → {{If[z > 0, RGBColor["#B09771"], Black]},
       \{If[z > 0, RGBColor["#B09771"], Black], Dashed\}\}, Frame \rightarrow True,
    FrameLabel → {Style["Temperature (°C)", 15, Black], Style["B*", 15, Black]},
    FrameTicksStyle → Directive[Black, 12], ImageSize → Medium],
   ListPlot [{photgrowthevo, photgrowthnoevo, hetgrowthevo, hetgrowthnoevo},
    Joined \rightarrow True, PlotRange \rightarrow {{T0, 33}, {13, u3}},
    PlotStyle → {{Darker[Green]}, {Darker[Green], Dashed}, {Black, Dashed}},
    Frame → True, FrameLabel → {Style["Temperature (°C)", Black, 15],
       Style["P(\theta,z,I,T,M*)*M, G(\theta,T,B*)*M", Black, 12]\},
    FrameTicksStyle → Directive[Black, 12], ImageSize → Medium]]
Ccyclingspec[z_, l1_, u1_, l2_, u2_, l3_, u3_] :=
 {make⊖ListSpec[];
  \thetaList0 = Table[0, 40];
  θList1 = Table[1, 40];
  Mpopsevo0 = List[];
  Mpopsnoevo0 = List[];
  Quiet[For[t = 1, t < 100, t++, AppendTo[Mpopsevo0, M /. eqs[\thetaList0[[t]], z, t]]]];
  Quiet[
   For [t = 1, t < 100, t++, AppendTo [Mpopsnoevo0, M /. eqs[\thetaList0[[T0]], z, t]]]];
  Mpopsevo1 = List[];
  Mpopsnoevo1 = List[];
  Quiet[For[t = 1, t < 100, t++, AppendTo[Mpopsevo1, M /. eqs[θList1[[t]], z, t]]]];
  Quiet[
   For [t = 1, t < 100, t++, AppendTo [Mpopsnoevo1, M /. eqs[\thetaList1[[T0]], z, t]]]];
  bpopsevo0 = List[];
  bpopsnoevo0 = List[];
  Quiet[For[t = 1, t < 100, t++, AppendTo[bpopsevo0, B /. eqs[\thetaList0[[t]], z, t]]]];
   For [t = 1, t < 100, t++, AppendTo[bpopsnoevo0, B /. eqs[\thetaList0[[T0]], z, t]]]];
  bpopsevo1 = List[];
  bpopsnoevo1 = List[];
  Quiet[For[t = 1, t < 100, t++, AppendTo[bpopsevo1, B /. eqs[\thetaList1[[t]], z, t]]]];
  Quiet[
   For [t = 1, t < 100, t++, AppendTo[bpopsnoevo1, B /. eqs[\thetaList1[[T0]], z, t]]]];
  photgrowthevo0 = List[];
  photgrowthnoevo0 = List[];
  Quiet[
```

```
For [t = 1, t < 100, t++, AppendTo[photgrowthevo0, (M /. eqs[<math>\thetaList0[[t]], z, t]) *
       \left(\left[\rho[\theta \text{List0}[[t]], z, t] \text{Log}\left[\frac{h + I_{\text{in}}}{h + e^{-(k (M/.eqs[\theta \text{List0}[[t]], z, t]))} I_{\text{in}}}\right]\right) / e^{-(k (M/.eqs[\theta \text{List0}[[t]], z, t]))} I_{\text{in}}\right]\right)
           (k (M /. eqs[θList0[[t]], z, t])))]]];
Quiet[For[t = 1, t < 100, t++, AppendTo[photgrowthnoevo0,
      (M /. eqs[θList0[[T0]], z, t]) *
       \left( \left[ \rho[\theta \text{List0[[T0]]}, z, t] \text{ Log} \left[ \frac{h + I_{in}}{h + e^{-(k \, (\text{M/.eqs[}\theta \text{List0[[T0]]}, z, t]))} \, T_{in}} \right] \right) \middle/ e^{-(k \, (\text{M/.eqs[}\theta \text{List0[[T0]]}, z, t]))} \right) \right)
           (k (M /. eqs[θList0[[T0]], z, t])))]]];
photgrowthevo1 = List[];
photgrowthnoevo1 = List[];
Quiet[
  For [t = 1, t < 100, t++, AppendTo [photgrowthevo1, (M /. eqs[\thetaList1[[t]], z, t]) *
       \left(\left[\rho[\theta List1[[t]], z, t] Log\left[\frac{h + I_{in}}{h + e^{-(k (M/.eqs[\theta List1[[t]], z, t]))} I_{in}}\right]\right) / e^{-(k (M/.eqs[\theta List1[[t]], z, t]))} I_{in}\right]\right)
           (k (M /. eqs[θList1[[t]], z, t])))]]];
Quiet[For[t = 1, t < 100, t++, AppendTo[photgrowthnoevo1,</pre>
      (M /. eqs[\theta List1[[T0]], z, t]) *
       \left(\left[\rho[\theta List1[[T0]], z, t] Log\left[\frac{h + I_{in}}{h + e^{-(k (M/.eqs[\theta List1[[T0]], z, t]))} I_{in}}\right]\right) / e^{-(k (M/.eqs[\theta List1[[T0]], z, t]))} I_{in}\right]\right)
           (k (M /. eqs[θList1[[T0]], z, t])))]]];
hetgrowthevo0 = List[];
hetgrowthnoevo0 = List[];
Quiet[
  For [t = 1, t < 100, t++, AppendTo[hetgrowthevo0, (B /. eqs[\thetaList0[[t]], z, t]) *
       (M /. eqs[\thetaList0[[t]], z, t]) \alpha[\thetaList0[[t]], t] b]]];
Quiet[For[t = 1, t < 100, t++, AppendTo[hetgrowthnoevo0,
      (B /. eqs[θList0[[T0]], z, t]) *
       (M /. eqs[\thetaList0[0[T0]], z, t]) \alpha[\thetaList0[[T0]], t] b]]];
hetgrowthevo1 = List[];
hetgrowthnoevo1 = List[];
Quiet[
  For [t = 1, t < 100, t++, AppendTo [hetgrowthevo1, (B /. eqs[\thetaList1[[t]], z, t]) *
        (M /. eqs[\thetaList1[[t]], z, t]) \alpha[\thetaList1[[t]], t] b]]];
Quiet[For[t = 1, t < 100, t++, AppendTo[hetgrowthnoevol,
```

```
(B /. eqs[θList1[[T0]], z, t]) *
           (M /. eqs[\thetaList1[0[T0]], z, t]) \alpha[\thetaList1[[T0]], t] b]]];
List[ListPlot[{Mpopsevo1, Mpopsevo0, Mpopsnoevo1, Mpopsnoevo0}, Joined → True,
     PlotRange \rightarrow {{T0, 33}, {l1, u1}}, PlotStyle \rightarrow {{Lighter[RGBColor["#287DAB"]]}},
           {RGBColor["#287DAB"]}, {Lighter[RGBColor["#287DAB"]], Dashed},
           {RGBColor["#287DAB"], Dashed}}, Frame → True,
     FrameLabel → {Style["Temperature (°C)", 15, Black], Style["M*", 15, Black]},
     FrameTicksStyle → Directive[Black, 12], ImageSize → Medium],
   ListPlot[{bpopsevo1, bpopsevo0, bpopsnoevo1, bpopsnoevo0}, Joined → True,
     PlotRange \rightarrow \{\{T0, 33\}, \{l2, u2\}\}, PlotStyle \rightarrow \{\{Lighter[RGBColor["#287DAB"]]\}, \{lage of the context of the co
           {RGBColor["#287DAB"]}, {Lighter[RGBColor["#287DAB"]], Dashed},
           {RGBColor["#287DAB"], Dashed}}, Frame → True,
     FrameLabel → {Style["Temperature (°C)", 15, Black], Style["B*", 15, Black]},
     FrameTicksStyle → Directive[Black, 12], ImageSize → Medium],
   ListPlot[{photgrowthevo0, photgrowthevo1, photgrowthnoevo0, photgrowthnoevo1,
       hetgrowthevo0, hetgrowthevo1, hetgrowthnoevo0, hetgrowthnoevo1},
     Joined \rightarrow True, PlotRange \rightarrow {{T0, 33}, {13, u3}},
     PlotStyle → {{Darker[Green]}, {Darker[Green]}, {Darker[Green], Dashed},
           {Darker[Green], Dashed}, {Black}, {Black, Dashed}, {Black, Dashed}},
     Frame → True, FrameLabel → {Style["Temperature (°C)", Black, 15],
           Style["P(\theta,z,I,T,M*)*M, G(\theta,T,B*)*M", Black, 12]},
     FrameTicksStyle → Directive[Black, 12], ImageSize → Medium]]
```

In[•]:=

In[ • ]:=

## $\theta$ vs. Temperature plots

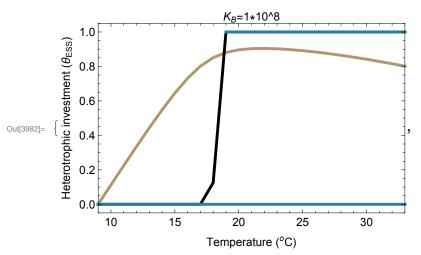
#### I<sub>in</sub>=100

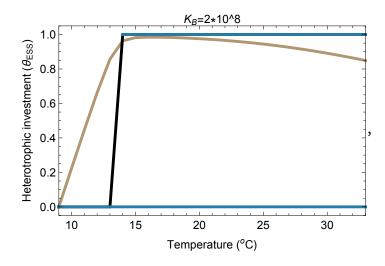
#### Linear temperature dependence

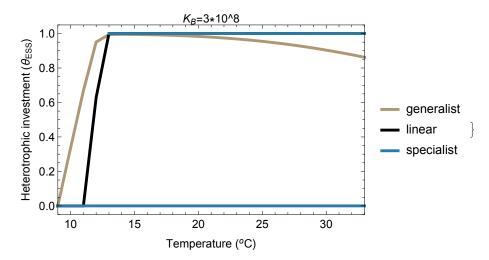
```
In[3982]:= List[K_B = 1 \times 10^8; I_{in} = 100;
       ListPlot[{make⊖ListGen[] // Flatten, make⊖ListLin[] // Flatten,
          makeθListSpec[][[1]][[1]], makeθListSpec[][[1]][[2]]},
         Joined \rightarrow True, PlotRange \rightarrow {{9, 33}, {-.05, 1.05}},
         PlotStyle → {{RGBColor["#B09771"], AbsoluteThickness[3]},
           {Black, AbsoluteThickness[3]}, {RGBColor["#287DAB"], AbsoluteThickness[3]},
           {RGBColor["#287DAB"], AbsoluteThickness[3]}}, Frame → True,
         ImageSize → Medium, FrameLabel → {Style["Temperature (°C)", 12, Black],
           Style["Heterotrophic investment (\theta_{ESS})", 12, Black]},
         FrameTicksStyle → Directive[Black, 12],
         PlotLabel \rightarrow Style["K<sub>B</sub>=1*10<sup>8</sup>", 12, Black]], K<sub>B</sub> = 2 × 10<sup>8</sup>;
       ListPlot[{makeθListGen[] // Flatten, makeθListLin[] // Flatten,
          makeθListSpec[][[1]][[1]], makeθListSpec[][[1]][[2]]},
         Joined → True, PlotRange → \{\{9, 33\}, \{-.05, 1.05\}\},
         PlotStyle → {{RGBColor["#B09771"], AbsoluteThickness[3]},
           {Black, AbsoluteThickness[3]}, {RGBColor["#287DAB"], AbsoluteThickness[3]},
           {RGBColor["#287DAB"], AbsoluteThickness[3]}}, Frame → True,
         ImageSize → Medium, FrameLabel → {Style["Temperature (°C)", 12, Black],
           Style["Heterotrophic investment (\theta_{ESS})", 12, Black]},
         FrameTicksStyle → Directive[Black, 12],
         PlotLabel \rightarrow Style["K<sub>B</sub>=2*10^8", 12, Black]], K<sub>B</sub> = 3 × 10^8;
       I_{in} = 100;
       ListPlot[{make⊖ListGen[] // Flatten, make⊖ListLin[] // Flatten,
          makeθListSpec[][[1]][[1]], makeθListSpec[][[1]][[2]]},
         Joined \rightarrow True, PlotRange \rightarrow \{\{9, 33\}, \{-.05, 1.05\}\},
         PlotStyle → {{RGBColor["#B09771"], AbsoluteThickness[3]},
           {Black, AbsoluteThickness[3]}, {RGBColor["#287DAB"], AbsoluteThickness[3]},
           {RGBColor["#287DAB"], AbsoluteThickness[3]}}, Frame \rightarrow True,
         ImageSize → Medium, FrameLabel → {Style["Temperature (°C)", 12, Black],
           Style["Heterotrophic investment (\theta_{ESS})", 12, Black]},
         PlotLegends → {"generalist", "linear", "specialist"},
         FrameTicksStyle \rightarrow Directive[Black, 12], PlotLabel \rightarrow Style["K_B=3*10^8", 12, Black]]
```

 $\overline{}$  Table: Non-list iterator θESSspec0 at position 2 does not evaluate to a real numeric value.

- Table: Non-list iterator  $\theta$ ESSspec0 at position 2 does not evaluate to a real numeric value.
- Table: Non-list iterator  $\theta$ ESSspec0 at position 2 does not evaluate to a real numeric value.
- General: Further output of Table::nliter will be suppressed during this calculation.

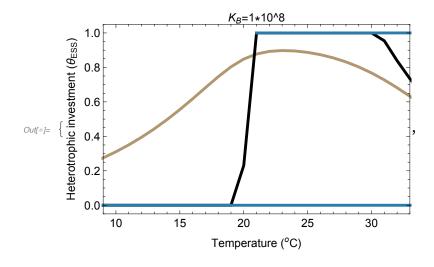


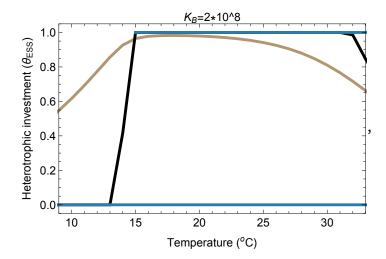


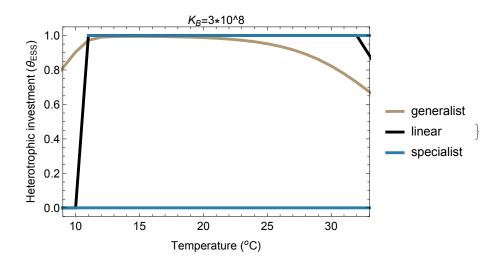


#### Exponential temperature dependence

```
In[\bullet]:= List[K_B = 1 \times 10^8; I_{in} = 100;
      ListPlot[{make⊖ListGenExp[] // Flatten, make⊖ListLinExp[] // Flatten,
         makeθListSpecExp[][[1]][[1]], makeθListSpecExp[][[1]][[2]]},
       Joined → True, PlotRange → \{\{9, 33\}, \{-.05, 1.05\}\},
       PlotStyle → {{RGBColor["#B09771"], AbsoluteThickness[3]},
          {Black, AbsoluteThickness[3]}, {RGBColor["#287DAB"], AbsoluteThickness[3]},
          {RGBColor["#287DAB"], AbsoluteThickness[3]}}, Frame → True,
       ImageSize → Medium, FrameLabel → {Style["Temperature (°C)", 12, Black],
          Style["Heterotrophic investment (\theta_{ESS})", 12, Black]},
       FrameTicksStyle → Directive[Black, 12],
       PlotLabel \rightarrow Style["K<sub>B</sub>=1*10<sup>8</sup>", 12, Black]], K<sub>B</sub> = 2 × 10<sup>8</sup>;
      I_{in} = 100;
      ListPlot[{make⊖ListGenExp[] // Flatten, make⊖ListLinExp[] // Flatten,
        makeθListSpecExp[][[1]][[1]], makeθListSpecExp[][[1]][[2]]},
       Joined \rightarrow True, PlotRange \rightarrow \{\{9, 33\}, \{-.05, 1.05\}\},
       PlotStyle → {{RGBColor["#B09771"], AbsoluteThickness[3]},
          {Black, AbsoluteThickness[3]}, {RGBColor["#287DAB"], AbsoluteThickness[3]},
          {RGBColor["#287DAB"], AbsoluteThickness[3]}}, Frame → True,
       ImageSize → Medium, FrameLabel → {Style["Temperature (°C)", 12, Black],
          Style["Heterotrophic investment (\theta_{ESS})", 12, Black]},
       FrameTicksStyle → Directive[Black, 12],
       PlotLabel \rightarrow Style["K<sub>B</sub>=2*10<sup>8</sup>", 12, Black]], K<sub>B</sub> = 3 × 10<sup>8</sup>;
      I_{in} = 100;
      ListPlot[{make⊖ListGenExp[] // Flatten, make⊖ListLinExp[] // Flatten,
        make\(\theta\)ListSpecExp[][[1]][[1]], make\(\theta\)ListSpecExp[][[1]][[2]]},
       Joined → True, PlotRange → \{\{9, 33\}, \{-.05, 1.05\}\},
       PlotStyle → {{RGBColor["#B09771"], AbsoluteThickness[3]},
          {Black, AbsoluteThickness[3]}, {RGBColor["#287DAB"], AbsoluteThickness[3]},
          {RGBColor["#287DAB"], AbsoluteThickness[3]}}, Frame → True,
       ImageSize → Medium, FrameLabel → {Style["Temperature (°C)", 12, Black],
          Style["Heterotrophic investment (\theta_{ESS})", 12, Black]},
       PlotLegends → {"generalist", "linear", "specialist"},
       FrameTicksStyle → Directive[Black, 12], PlotLabel → Style["K<sub>B</sub>=3*10^8", 12, Black]]
     \overline{} Table: Non-list iterator \thetaESSspec0 at position 2 does not evaluate to a real numeric value.
     Table: Non-list iterator \thetaESSspec0 at position 2 does not evaluate to a real numeric value.
     \overline{} Table: Non-list iterator \thetaESSspec0 at position 2 does not evaluate to a real numeric value.
     ... General: Further output of Table::nliter will be suppressed during this calculation.
```







#### I<sub>in</sub>=150

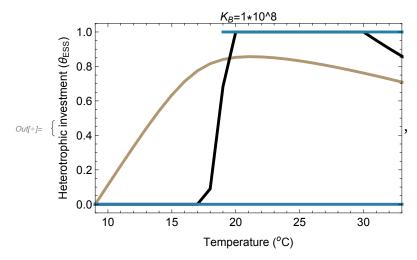
In[ • ]:=

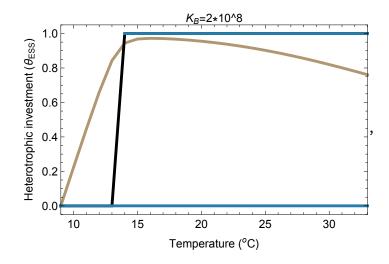
#### Linear temperature dependence

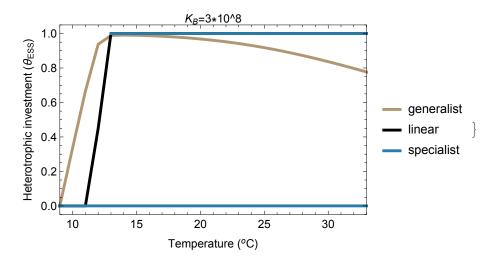
```
In[*]:= ones = Table[1, 100];
    zeros = Table[0, 100];
    List[K_B = 1 \times 10^8; I_{in} = 150;
      ListPlot[{makeθListGen[] // Flatten, makeθListLin[] // Flatten,
        makeθListSpec[][[1]][[1]], makeθListSpec[][[1]][[2]]},
       Joined → True, PlotRange → \{\{9, 33\}, \{-.05, 1.05\}\},
       PlotStyle → {{RGBColor["#B09771"], AbsoluteThickness[3]},
          {Black, AbsoluteThickness[3]}, {RGBColor["#287DAB"], AbsoluteThickness[3]},
          {RGBColor["#287DAB"], AbsoluteThickness[3]}}, Frame → True,
       ImageSize → Medium, FrameLabel → {Style["Temperature (°C)", 12, Black],
         Style["Heterotrophic investment (\theta_{ESS})", 12, Black]},
       FrameTicksStyle → Directive[Black, 12],
       PlotLabel \rightarrow Style["K<sub>B</sub>=1*10<sup>8</sup>", 12, Black]], K<sub>B</sub> = 2 × 10<sup>8</sup>;
      I_{in} = 150;
      ListPlot[{make⊖ListGen[] // Flatten, make⊖ListLin[] // Flatten,
        makeθListSpec[][[1]][[1]], makeθListSpec[][[1]][[2]]},
       Joined \rightarrow True, PlotRange \rightarrow \{\{9, 33\}, \{-.05, 1.05\}\},
       PlotStyle → {{RGBColor["#B09771"], AbsoluteThickness[3]},
          {Black, AbsoluteThickness[3]}, {RGBColor["#287DAB"], AbsoluteThickness[3]},
          {RGBColor["#287DAB"], AbsoluteThickness[3]}}, Frame → True,
       ImageSize → Medium, FrameLabel → {Style["Temperature (°C)", 12, Black],
         Style["Heterotrophic investment (\theta_{ESS})", 12, Black]},
       FrameTicksStyle → Directive[Black, 12],
       PlotLabel \rightarrow Style["K<sub>B</sub>=2*10<sup>8</sup>", 12, Black]], K<sub>B</sub> = 3 × 10<sup>8</sup>;
      I_{in} = 150;
      ListPlot[{make\thetaListGen[] // Flatten, make\thetaListLin[] // Flatten,
        makeθListSpec[][[1]][[1]], makeθListSpec[][[1]][[2]]},
       Joined → True, PlotRange → \{\{9, 33\}, \{-.05, 1.05\}\},
       PlotStyle → {{RGBColor["#B09771"], AbsoluteThickness[3]},
          {Black, AbsoluteThickness[3]}, {RGBColor["#287DAB"], AbsoluteThickness[3]},
          {RGBColor["#287DAB"], AbsoluteThickness[3]}}, Frame → True,
       ImageSize \rightarrow Medium, FrameLabel \rightarrow {Style["Temperature (°C)", 12, Black],
         Style["Heterotrophic investment (\theta_{ESS})", 12, Black]},
       PlotLegends → {"generalist", "linear", "specialist"},
       FrameTicksStyle \rightarrow Directive[Black, 12], PlotLabel \rightarrow Style["K_B=3*10^8", 12, Black]]
```

 $\overline{}$  Table: Non-list iterator  $\theta$ ESSspec0 at position 2 does not evaluate to a real numeric value.

- Table: Non-list iterator  $\theta$ ESSspec0 at position 2 does not evaluate to a real numeric value.
- Table: Non-list iterator  $\theta$ ESSspec0 at position 2 does not evaluate to a real numeric value.
- General: Further output of Table::nliter will be suppressed during this calculation.

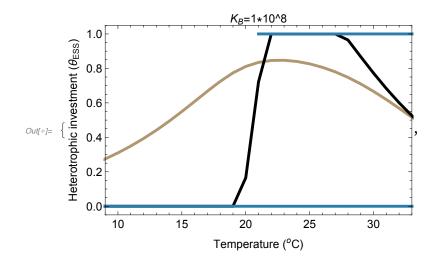


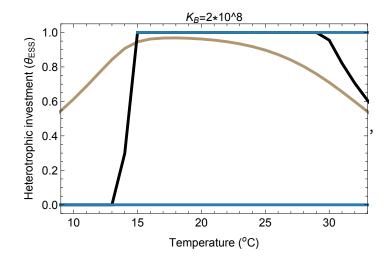


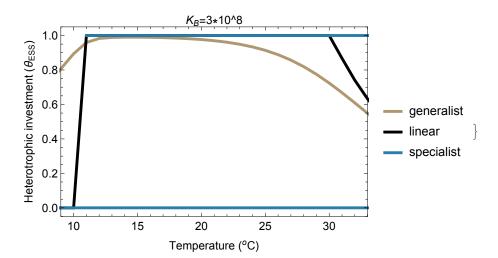


#### Exponential temperature dependence

```
ln[\bullet]:= List[K_B = 1 \times 10^8; I_{in} = 150;
      ListPlot[{make⊖ListGenExp[] // Flatten, make⊖ListLinExp[] // Flatten,
        makeθListSpecExp[][[1]][[1]], makeθListSpecExp[][[1]][[2]]},
       Joined → True, PlotRange → \{\{9, 33\}, \{-.05, 1.05\}\},
       PlotStyle → {{RGBColor["#B09771"], AbsoluteThickness[3]},
          {Black, AbsoluteThickness[3]}, {RGBColor["#287DAB"], AbsoluteThickness[3]},
          {RGBColor["#287DAB"], AbsoluteThickness[3]}}, Frame → True,
       ImageSize → Medium, FrameLabel → {Style["Temperature (°C)", 12, Black],
          Style["Heterotrophic investment (\theta_{ESS})", 12, Black]},
       FrameTicksStyle → Directive[Black, 12],
       PlotLabel \rightarrow Style["K<sub>B</sub>=1*10<sup>8</sup>", 12, Black]], K<sub>B</sub> = 2 × 10<sup>8</sup>;
      I_{in} = 150;
      ListPlot[{make⊖ListGenExp[] // Flatten, make⊖ListLinExp[] // Flatten,
        makeθListSpecExp[][[1]][[1]], makeθListSpecExp[][[1]][[2]]},
       Joined \rightarrow True, PlotRange \rightarrow \{\{9, 33\}, \{-.05, 1.05\}\},
       PlotStyle → {{RGBColor["#B09771"], AbsoluteThickness[3]},
          {Black, AbsoluteThickness[3]}, {RGBColor["#287DAB"], AbsoluteThickness[3]},
          {RGBColor["#287DAB"], AbsoluteThickness[3]}}, Frame → True,
       ImageSize → Medium, FrameLabel → {Style["Temperature (°C)", 12, Black],
          Style["Heterotrophic investment (\theta_{ESS})", 12, Black]},
       FrameTicksStyle → Directive[Black, 12],
       PlotLabel \rightarrow Style["K<sub>B</sub>=2*10<sup>8</sup>", 12, Black]], K<sub>B</sub> = 3 × 10<sup>8</sup>;
      I_{in} = 150;
      ListPlot[{make⊖ListGenExp[] // Flatten, make⊖ListLinExp[] // Flatten,
        make\(\theta\)ListSpecExp[][[1]][[1]], make\(\theta\)ListSpecExp[][[1]][[2]]},
       Joined → True, PlotRange → \{\{9, 33\}, \{-.05, 1.05\}\},
       PlotStyle → {{RGBColor["#B09771"], AbsoluteThickness[3]},
          {Black, AbsoluteThickness[3]}, {RGBColor["#287DAB"], AbsoluteThickness[3]},
          {RGBColor["#287DAB"], AbsoluteThickness[3]}}, Frame → True,
       ImageSize → Medium, FrameLabel → {Style["Temperature (°C)", 12, Black],
          Style["Heterotrophic investment (\theta_{ESS})", 12, Black]},
       PlotLegends → {"generalist", "linear", "specialist"},
       FrameTicksStyle → Directive[Black, 12], PlotLabel → Style["K<sub>B</sub>=3*10^8", 12, Black]]
     \overline{} Table: Non-list iterator \thetaESSspec0 at position 2 does not evaluate to a real numeric value.
     Table: Non-list iterator \thetaESSspec0 at position 2 does not evaluate to a real numeric value.
     \overline{} Table: Non-list iterator \thetaESSspec0 at position 2 does not evaluate to a real numeric value.
     ... General: Further output of Table::nliter will be suppressed during this calculation.
```



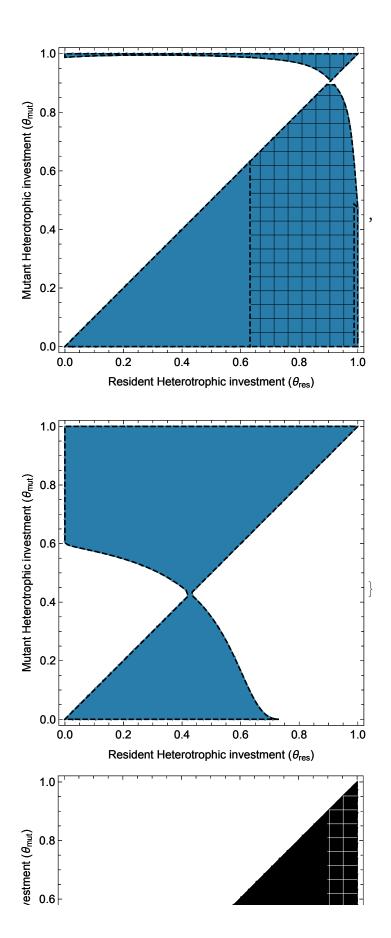


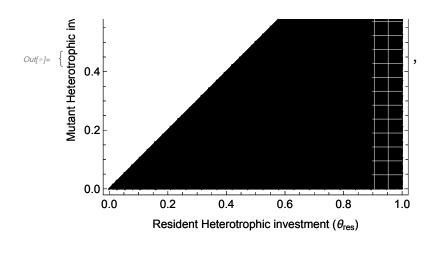


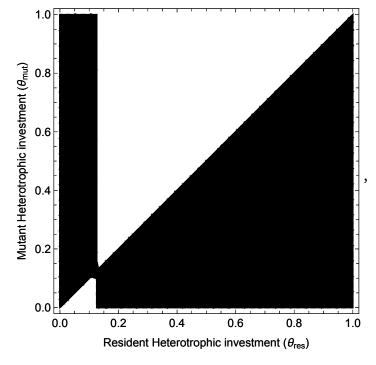
## Pairwise invasibility plots

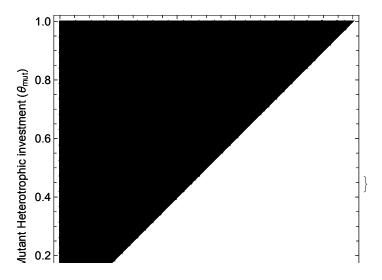
```
In[•]:=
K_B=1*10^8, I_{in}=100
 In[\bullet]:= K_B = 1 \times 10^8; I_{in} = 100;
     Quiet[List[Overlay[{MakePIPNV[-1, T0, RGBColor["#287DAB"], Black],
          MakePIP[-1, T0, RGBColor["#287DAB"]]}],
        Overlay[{MakePIPNV[-1, T0+5, RGBColor["#287DAB"], Black],
          MakePIP[-1, T0 + 5, RGBColor["#287DAB"]]}],
        Overlay[{MakePIPNV[-1, T0 + 10, RGBColor["#287DAB"], Black],
          MakePIP[-1, T0 + 10, RGBColor["#287DAB"]]}]]
     Quiet[List[Overlay[{MakePIPNV[0, T0, Black, White], MakePIP[0, T0, Black]}],
        Overlay[{MakePIPNV[0, T0+5, Black, White], MakePIP[0, T0+5, Black]}], Overlay[
         {MakePIPNV[0, T0 + 10, Black, White], MakePIP[0, T0 + 10, Black]}]]](*linear*)
     Quiet[List[Overlay[{MakePIPNV[1, T0, RGBColor["#B09771"], Black],
          MakePIP[1, T0, RGBColor["#B09771"]]}],
        Overlay[{MakePIPNV[1, T0 + 5, RGBColor["#B09771"], Black],
          MakePIP[1, T0 + 5, RGBColor["#B09771"]]}],
        Overlay[{MakePIPNV[1, T0 + 10, RGBColor["#B09771"], Black],
          MakePIP[1, T0 + 10, RGBColor["#B09771"]]}]]](*generalist*)
      Mutant Heterotrophic investment (	heta_{	ext{mut}})
         8.0
         0.6
         0.2
         0.0
```

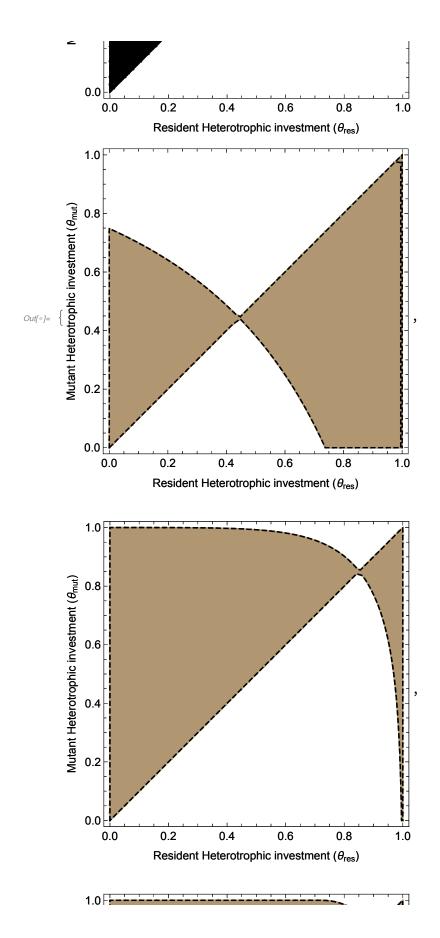
Resident Heterotrophic investment ( $\theta_{res}$ )

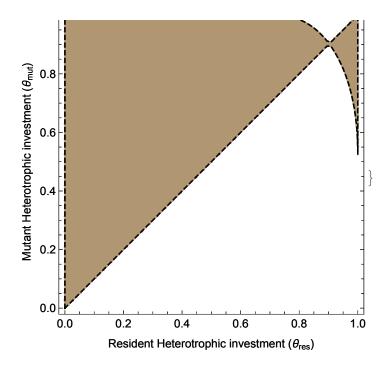








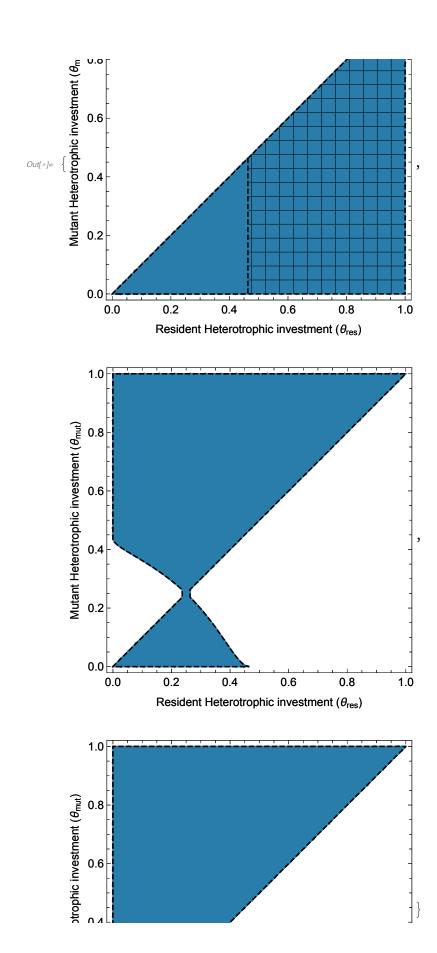


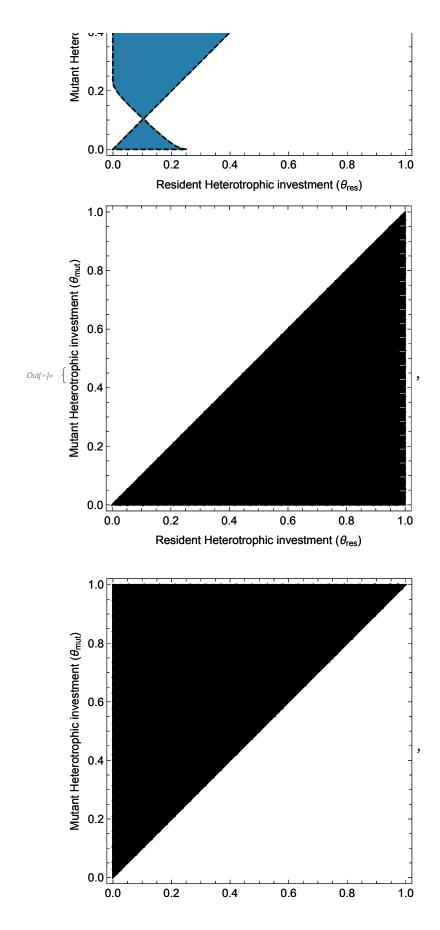


In[ • ]:= In[ • ]:=

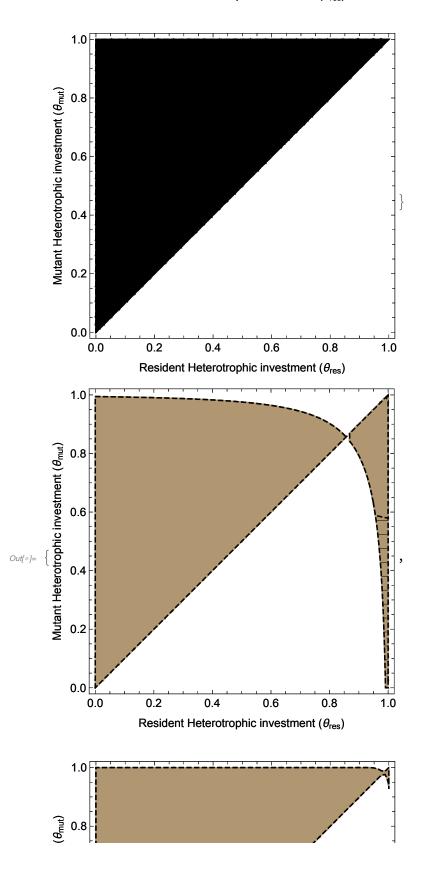
#### $K_B=2*10^8, I_{in}=100$

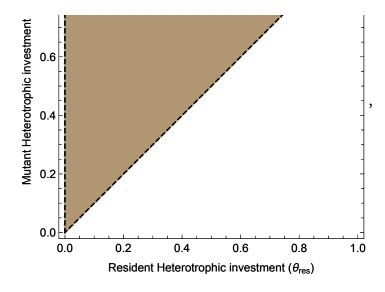
```
In[*]:= K_B = 2 \times 10^8; I_{in} = 100;
    Quiet[List[Overlay[{MakePIPNV[-1, T0, RGBColor["#287DAB"], Black],
        MakePIP[-1, T0, RGBColor["#287DAB"]]}],
      Overlay[{MakePIPNV[-1, T0+5, RGBColor["#287DAB"], Black],
        MakePIP[-1, T0 + 5, RGBColor["#287DAB"]]}],
      Overlay[{MakePIPNV[-1, T0 + 10, RGBColor["#287DAB"], Black],
        MakePIP[-1, T0 + 10, RGBColor["#287DAB"]]}]]]
    Quiet[List[Overlay[{MakePIPNV[0, T0, Black, White], MakePIP[0, T0, Black]}],
      Overlay[{MakePIPNV[0, T0 + 5, Black, White], MakePIP[0, T0 + 5, Black]}], Overlay[
        {MakePIPNV[0, T0 + 10, Black, White], MakePIP[0, T0 + 10, Black]}]]](*linear*)
    Quiet[List[Overlay[{MakePIPNV[1, T0, RGBColor["#B09771"], Black],
        MakePIP[1, T0, RGBColor["#B09771"]]}],
      Overlay[{MakePIPNV[1, T0 + 5, RGBColor["#B09771"], Black],
         MakePIP[1, T0 + 5, RGBColor["#B09771"]]}],
      Overlay[{MakePIPNV[1, T0 + 10, RGBColor["#B09771"], Black],
        MakePIP[1, T0 + 10, RGBColor["#B09771"]]}]]](*generalist*)
     on É
```

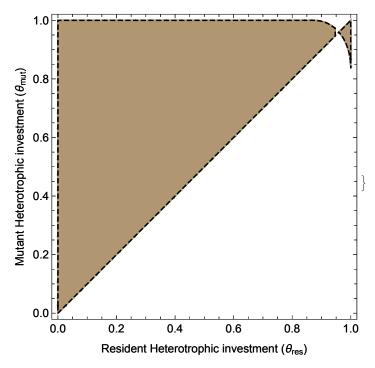




#### Resident Heterotrophic investment ( $\theta_{res}$ )

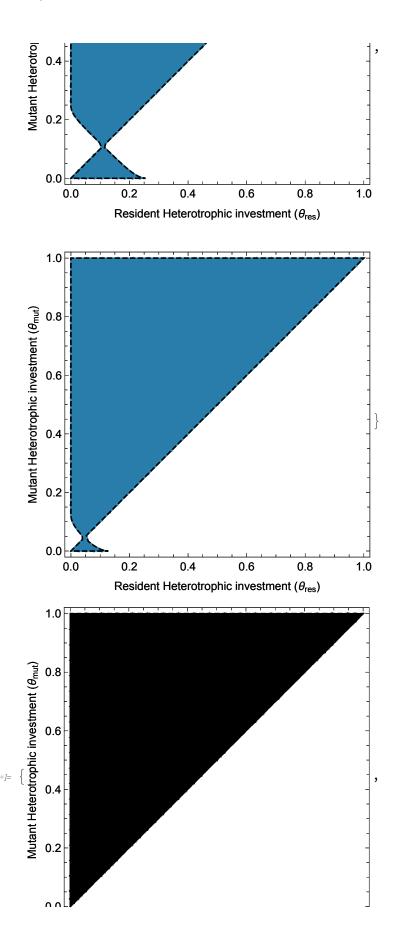


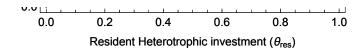


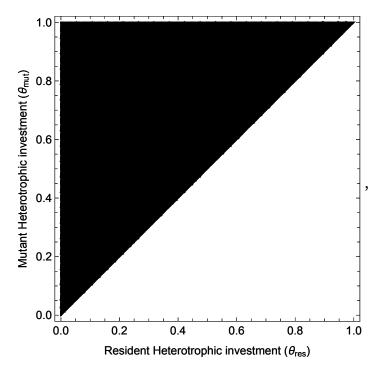


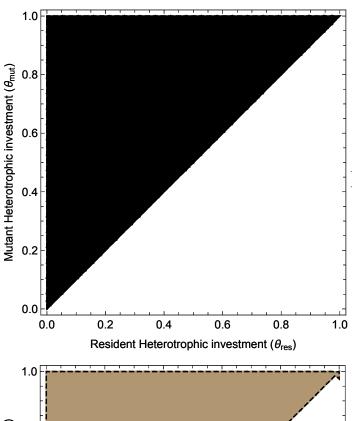
K<sub>B</sub>=3\*10^8, I<sub>in</sub>=100

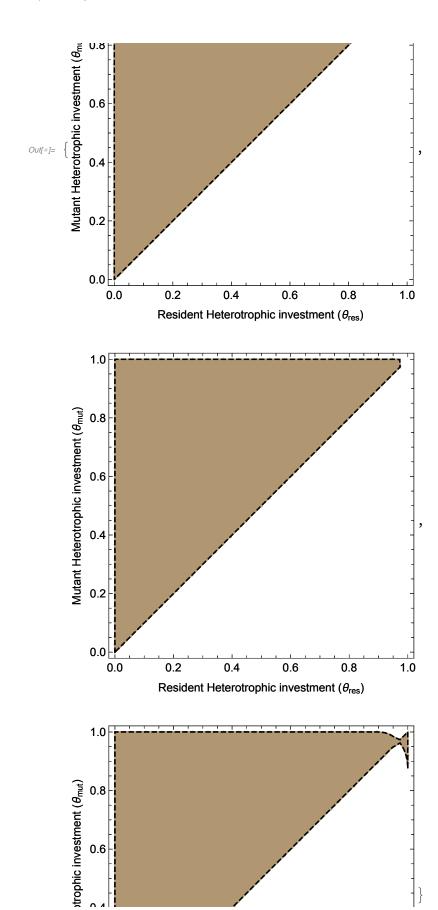
```
In[\bullet]:= K_B = 3 \times 10^8; I_{in} = 100;
    Quiet[List[Overlay[{MakePIPNV[-1, T0, RGBColor["#287DAB"], Black],
         MakePIP[-1, T0, RGBColor["#287DAB"]]}],
       Overlay[{MakePIPNV[-1, T0 + 5, RGBColor["#287DAB"], Black],
          MakePIP[-1, T0 + 5, RGBColor["#287DAB"]]}],
       Overlay[{MakePIPNV[-1, T0 + 10, RGBColor["#287DAB"], Black],
          MakePIP[-1, T0 + 10, RGBColor["#287DAB"]]}]]]
    Quiet[List[Overlay[{MakePIPNV[0, T0, Black, White], MakePIP[0, T0, Black]}],
       Overlay[{MakePIPNV[0, T0+5, Black, White], MakePIP[0, T0+5, Black]}], Overlay[
        {MakePIPNV[0, T0 + 10, Black, White], MakePIP[0, T0 + 10, Black]}]]](*linear*)
    Quiet[List[Overlay[{MakePIPNV[1, T0, RGBColor["#B09771"], Black],
          MakePIP[1, T0, RGBColor["#B09771"]]}],
       Overlay[{MakePIPNV[1, T0 + 5, RGBColor["#B09771"], Black],
          MakePIP[1, T0 + 5, RGBColor["#B09771"]]}],
       Overlay[{MakePIPNV[1, T0 + 10, RGBColor["#B09771"], Black],
         MakePIP[1, T0 + 10, RGBColor["#B09771"]]}]]](*generalist*)
     Mutant Heterotrophic investment (\theta_{
m mut})
         8.0
        0.6
         0.4
         0.2
         0.0
           0.0
                     0.2
                              0.4
                                       0.6
                                                 8.0
                                                          1.0
                    Resident Heterotrophic investment (\theta_{res})
         1.0
      phic investment (\theta_{mut})
         8.0
         0.6
```

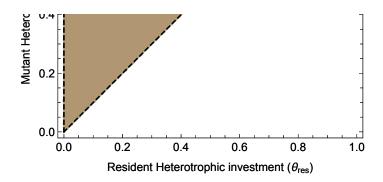






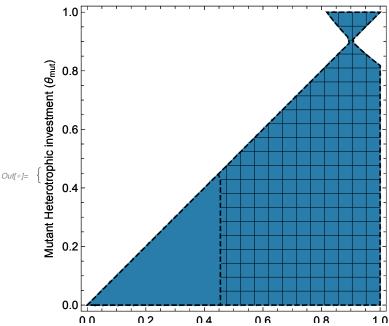




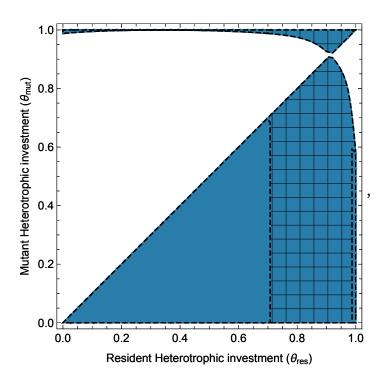


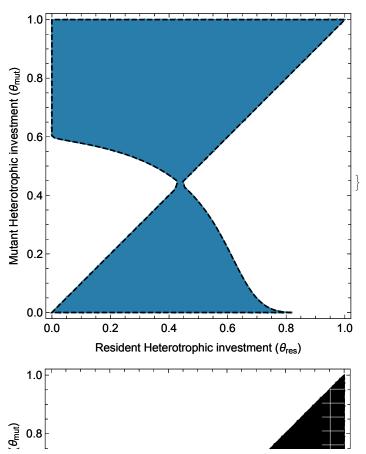
#### $K_B=1*10^8, I_{in}=150$

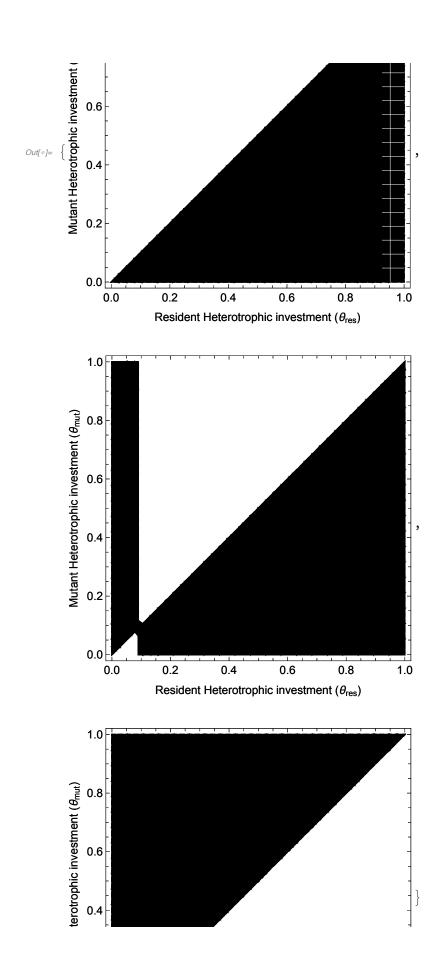
```
In[\bullet]:= K_B = 1 \times 10^8; I_{in} = 150;
    Quiet[List[Overlay[{MakePIPNV[-1, T0, RGBColor["#287DAB"], Black],
         MakePIP[-1, T0, RGBColor["#287DAB"]]}],
      Overlay[{MakePIPNV[-1, T0+5, RGBColor["#287DAB"], Black],
         MakePIP[-1, T0 + 5, RGBColor["#287DAB"]]}],
      Overlay[{MakePIPNV[-1, T0 + 10, RGBColor["#287DAB"], Black],
        MakePIP[-1, T0 + 10, RGBColor["#287DAB"]]}]]]
    Quiet[List[Overlay[{MakePIPNV[0, T0, Black, White], MakePIP[0, T0, Black]}],
      Overlay[{MakePIPNV[0, T0+5, Black, White], MakePIP[0, T0+5, Black]}], Overlay[
        {MakePIPNV[0, T0 + 10, Black, White], MakePIP[0, T0 + 10, Black]}]]](*linear*)
    Quiet[List[Overlay[{MakePIPNV[1, T0, RGBColor["#B09771"], Black],
        MakePIP[1, T0, RGBColor["#B09771"]]}],
      Overlay[{MakePIPNV[1, T0 + 5, RGBColor["#B09771"], Black],
         MakePIP[1, T0 + 5, RGBColor["#B09771"]]}],
      Overlay[{MakePIPNV[1, T0 + 10, RGBColor["#B09771"], Black],
         MakePIP[1, T0 + 10, RGBColor["#B09771"]]}]]](*generalist*)
```

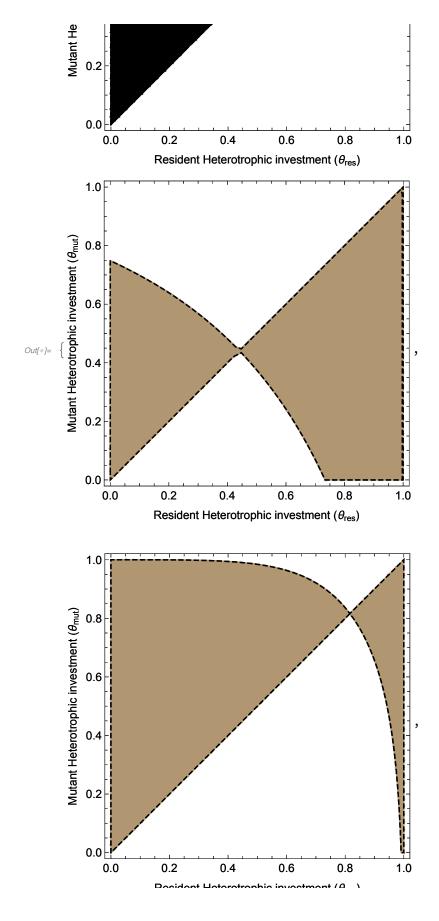




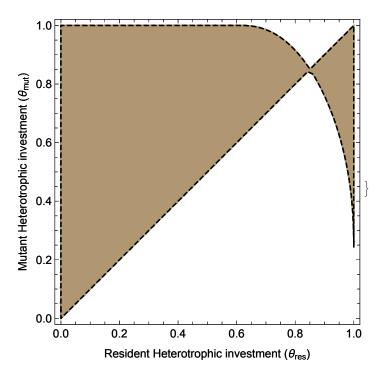






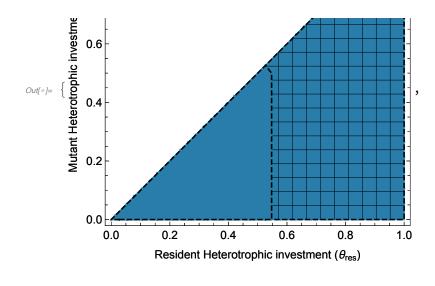


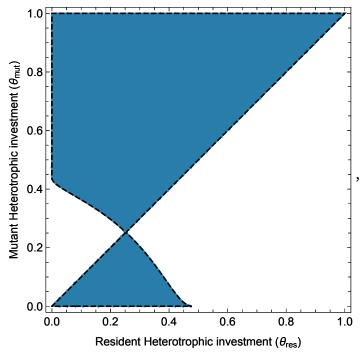
#### resident tierenomobilio invesiment (pres)

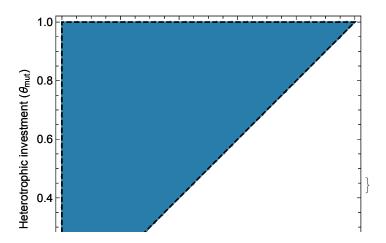


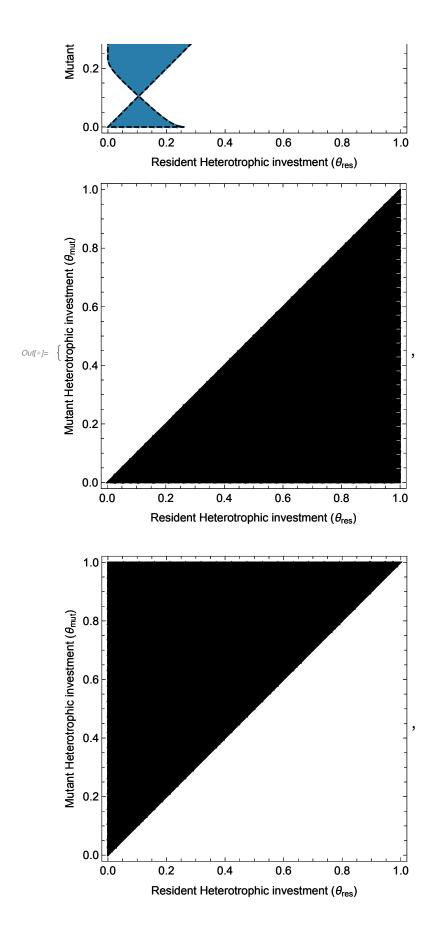
#### $K_B=2*10^8, I_{in}=150$

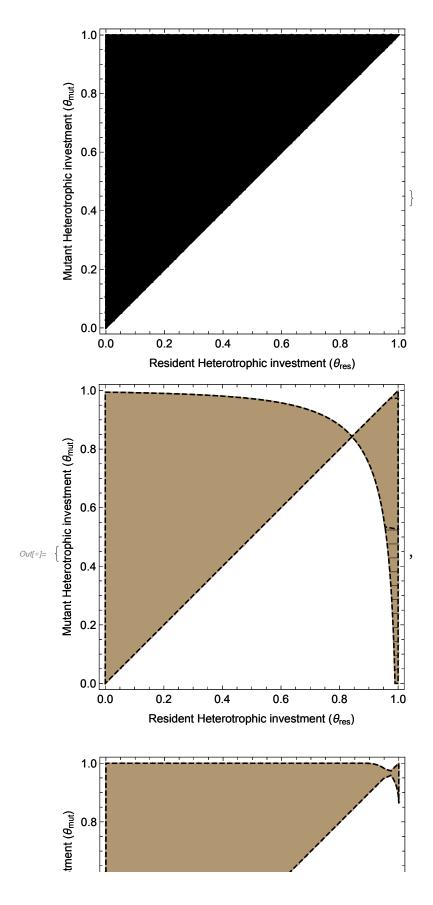
```
In[\bullet]:= K_B = 2 \times 10^8; I_{in} = 150;
    Quiet[List[Overlay[{MakePIPNV[-1, T0, RGBColor["#287DAB"], Black],
         MakePIP[-1, T0, RGBColor["#287DAB"]]}],
      Overlay[{MakePIPNV[-1, T0 + 5, RGBColor["#287DAB"], Black],
         MakePIP[-1, T0 + 5, RGBColor["#287DAB"]]}],
      Overlay[{MakePIPNV[-1, T0 + 10, RGBColor["#287DAB"], Black],
         MakePIP[-1, T0 + 10, RGBColor["#287DAB"]]}]]]
    Quiet[List[Overlay[{MakePIPNV[0, T0, Black, White], MakePIP[0, T0, Black]}],
      Overlay[{MakePIPNV[0, T0+5, Black, White], MakePIP[0, T0+5, Black]}], Overlay[
        {MakePIPNV[0, T0 + 10, Black, White], MakePIP[0, T0 + 10, Black]}]]](*linear*)
    Quiet[List[Overlay[{MakePIPNV[1, T0, RGBColor["#B09771"], Black],
         MakePIP[1, T0, RGBColor["#B09771"]]}],
      Overlay[{MakePIPNV[1, T0 + 5, RGBColor["#B09771"], Black],
         MakePIP[1, T0 + 5, RGBColor["#B09771"]]}],
      Overlay[{MakePIPNV[1, T0 + 10, RGBColor["#B09771"], Black],
         MakePIP[1, T0 + 10, RGBColor["#B09771"]]}]]](*generalist*)
     int (\theta_{mut}) 80
```

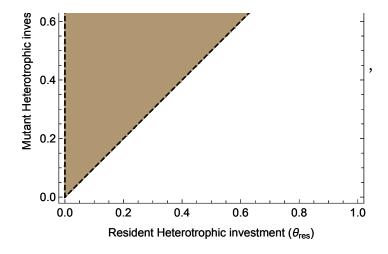


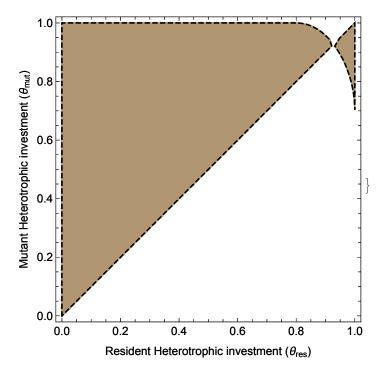






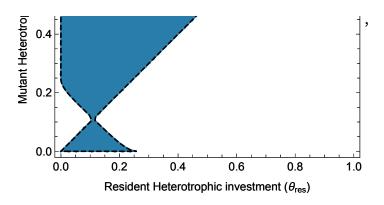


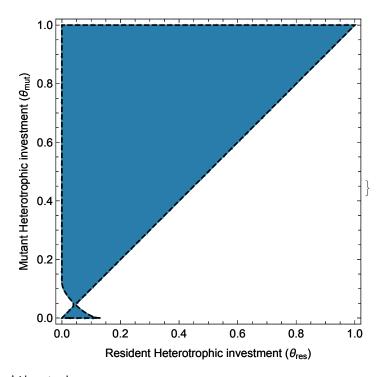




$$K_B=3*10^8, I_{in}=150$$

```
In[\bullet]:= K_B = 3 \times 10^8; I_{in} = 150;
    Quiet[List[Overlay[{MakePIPNV[-1, T0, RGBColor["#287DAB"], Black],
         MakePIP[-1, T0, RGBColor["#287DAB"]]}],
       Overlay[{MakePIPNV[-1, T0 + 5, RGBColor["#287DAB"], Black],
          MakePIP[-1, T0 + 5, RGBColor["#287DAB"]]}],
       Overlay[{MakePIPNV[-1, T0 + 10, RGBColor["#287DAB"], Black],
          MakePIP[-1, T0 + 10, RGBColor["#287DAB"]]}]]]
    Quiet[List[Overlay[{MakePIPNV[0, T0, Black, White], MakePIP[0, T0, Black]}],
       Overlay[{MakePIPNV[0, T0+5, Black, White], MakePIP[0, T0+5, Black]}], Overlay[
        {MakePIPNV[0, T0 + 10, Black, White], MakePIP[0, T0 + 10, Black]}]]](*linear*)
    Quiet[List[Overlay[{MakePIPNV[1, T0, RGBColor["#B09771"], Black],
          MakePIP[1, T0, RGBColor["#B09771"]]}],
       Overlay[{MakePIPNV[1, T0 + 5, RGBColor["#B09771"], Black],
          MakePIP[1, T0 + 5, RGBColor["#B09771"]]}],
       Overlay[{MakePIPNV[1, T0 + 10, RGBColor["#B09771"], Black],
         MakePIP[1, T0 + 10, RGBColor["#B09771"]]}]]](*generalist*)
     Mutant Heterotrophic investment (\theta_{
m mut})
         8.0
         0.6
         0.4
         0.2
         0.0
           0.0
                     0.2
                              0.4
                                       0.6
                                                 8.0
                                                          1.0
                    Resident Heterotrophic investment (\theta_{res})
         1.0
      phic investment (\theta_{mut})
         8.0
         0.6
```



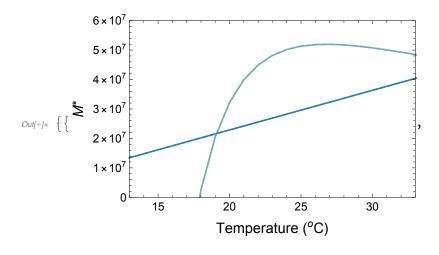


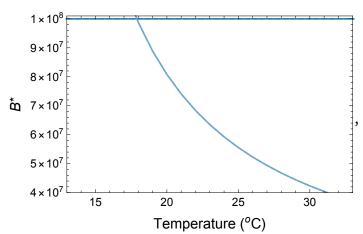
Out[\*]= \$Aborted  $Out[\bullet] = Aborted$ 

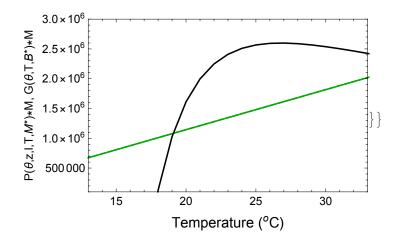
# C-cycling related figures (Dashed genetically static, Solid - evolving)

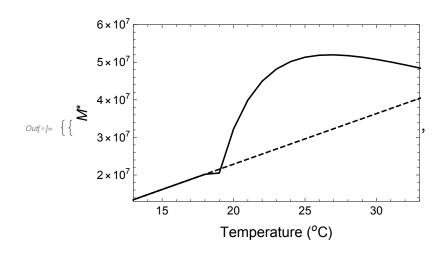
 $K_B=1*10^8, I_{in}=100$ 

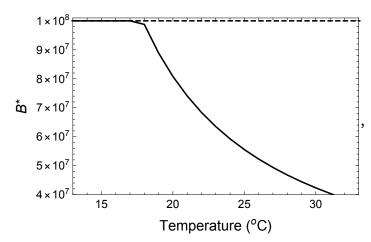
```
In[\bullet]:= K_B = 1 \times 10^8; I_{in} = 100;
      Quiet[Ccyclingspec[-1, 0, 6 \times 10^7, 4 \times 10^7, 1.01 \times 10^8, 10^5, 3 \times 10^6]
     Quiet[Ccycling[0, 1.3 \times 10^7, 6 \times 10^7, 4 \times 10^7, 1.01 \times 10^8, -20000, 3 \times 10^6]
     Quiet[Ccycling[1, 1.4×10<sup>7</sup>, 9×10<sup>7</sup>, 2×10<sup>7</sup>, 10×10<sup>7</sup>, 0, 3×10<sup>6</sup>]]
```

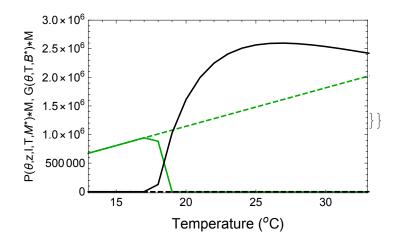


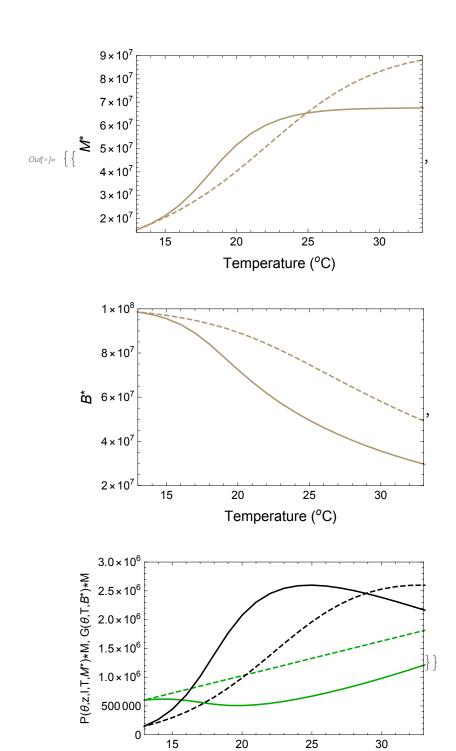








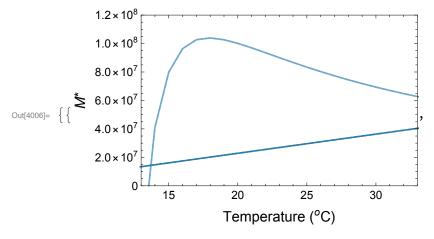


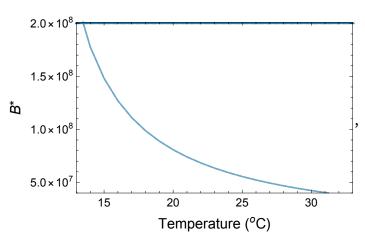


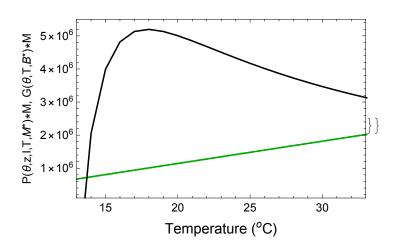
Temperature (°C)

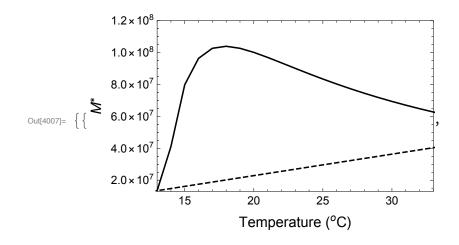
### $K_B=2*10^8, I_{in}=100$

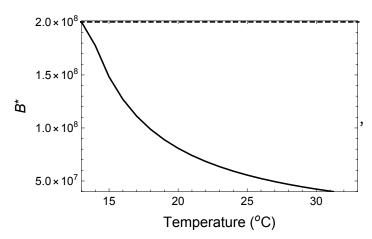
 $In[4005]:= K_B = 2 \times 10^8; I_{in} = 100;$ Quiet[Ccyclingspec[-1, 0, 1.2 × 10 ^ 8 , 4 × 10 ^ 7, 2.01 × 10 ^ 8, 10 ^ 5, 5.5 × 10 ^ 6]] Quiet[Ccycling[0,  $1.3 \times 10^7$ ,  $1.2 \times 10^8$ ,  $4 \times 10^7$ ,  $2.01 \times 10^8$ , -20000,  $6 \times 10^6$ ] Quiet[Ccycling[1, 1.4 × 10 ^ 7, 1.2 × 10 ^ 8, 2 × 10 ^ 7, 2 × 10 ^ 8, 0, 6 × 10 ^ 6]]

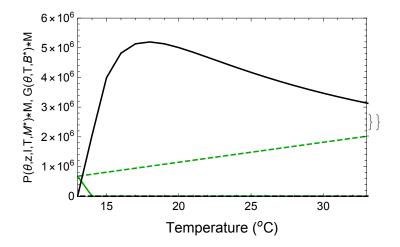


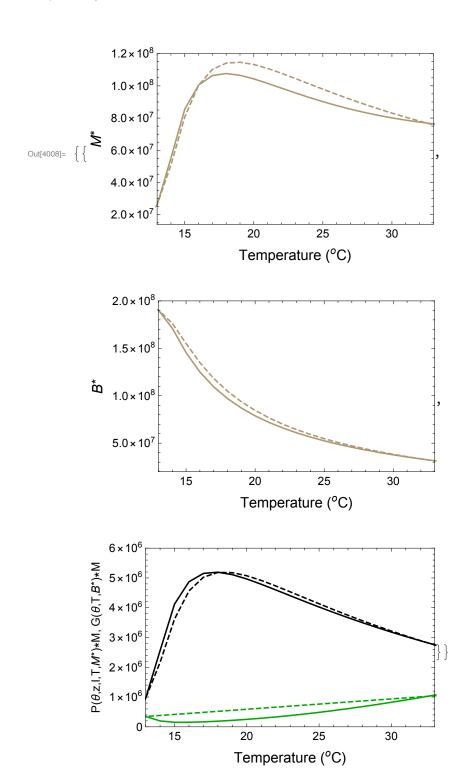






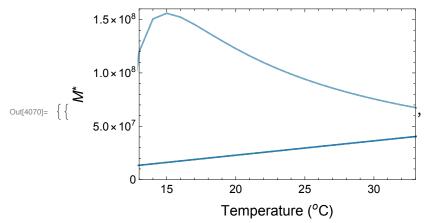


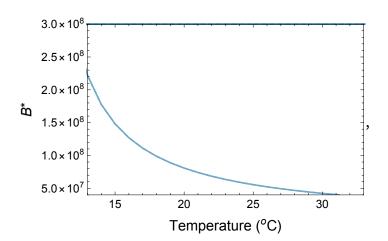


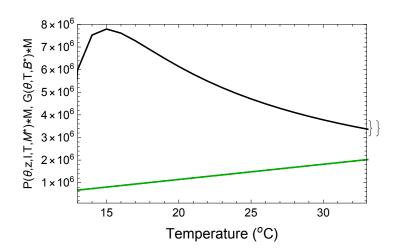


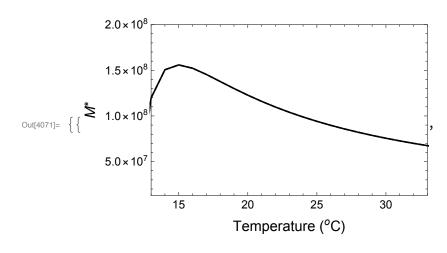
### $K_B=3*10^8, I_{in}=100$

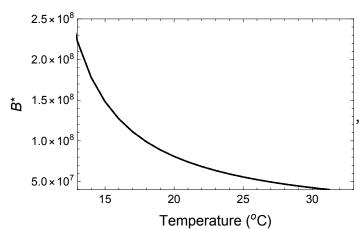
 $In[4069] = K_B = 3 \times 10^8; I_{in} = 100;$ Quiet[Ccyclingspec[-1, 0, 1.6 × 10 ^ 8 , 4 × 10 ^ 7, 3 × 10 ^ 8, 10 ^ 5, 8 × 10 ^ 6]] Quiet[Ccycling[0,  $1.3 \times 10^7$ ,  $2 \times 10^8$ ,  $4 \times 10^7$ ,  $2.5 \times 10^8$ , -20000,  $8 \times 10^6$ ] Quiet[Ccycling[1, 1.4 × 10 ^ 7, 2 × 10 ^ 8, 0, 3 × 10 ^ 8, 0, 8 × 10 ^ 6]]

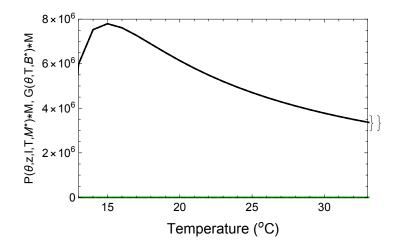


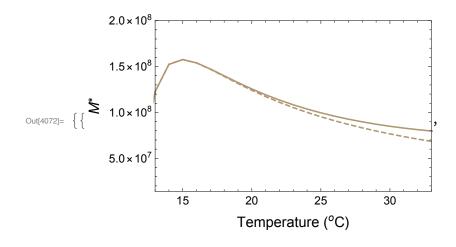


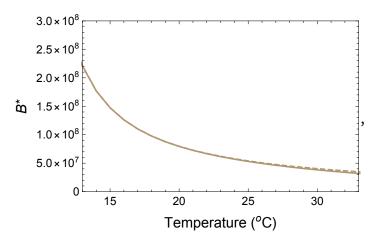


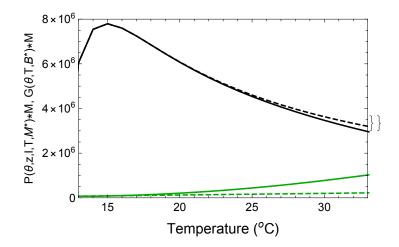






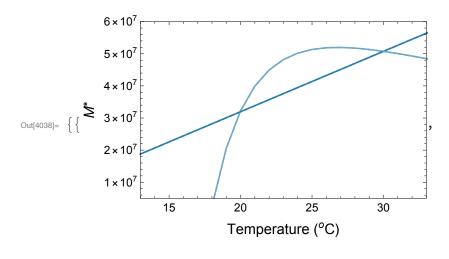


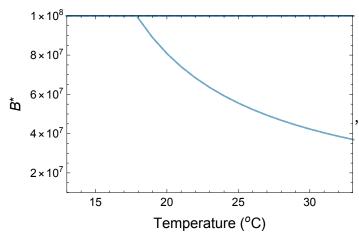


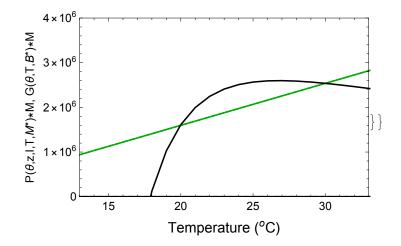


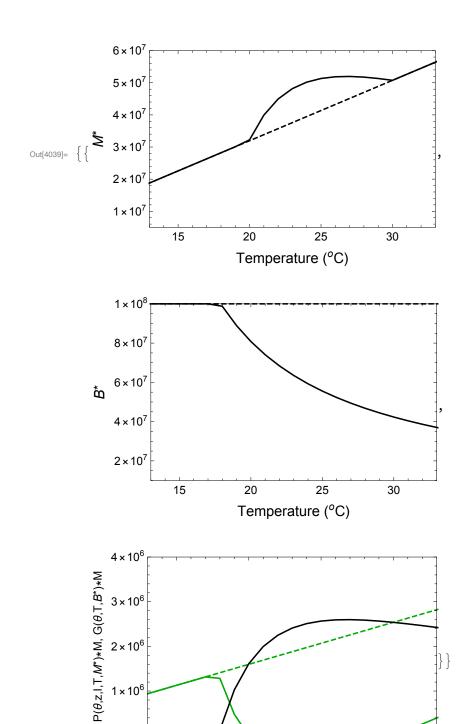
# $K_B=1*10^8, I_{in}=150$

 $In[4037]:= K_B = 1 \times 10^8; I_{in} = 150;$ Quiet[Ccyclingspec[-1,  $.5 \times 10^{7}$ ,  $6.0 \times 10^{7}$ ,  $1 \times 10^{7}$ ,  $1 \times 10^{8}$ , 0,  $4 \times 10^{6}$ ] Quiet[Ccycling[0, .5 × 10 ^ 7, 6.0 × 10 ^ 7, 1 × 10 ^ 7, 1 × 10 ^ 8, 0, 4 × 10 ^ 6]] Quiet[Ccycling[1, 1.5 × 10^7, 1 × 10^8, 1 × 10^7, 1 × 10^8, 0, 3 × 10^6]]

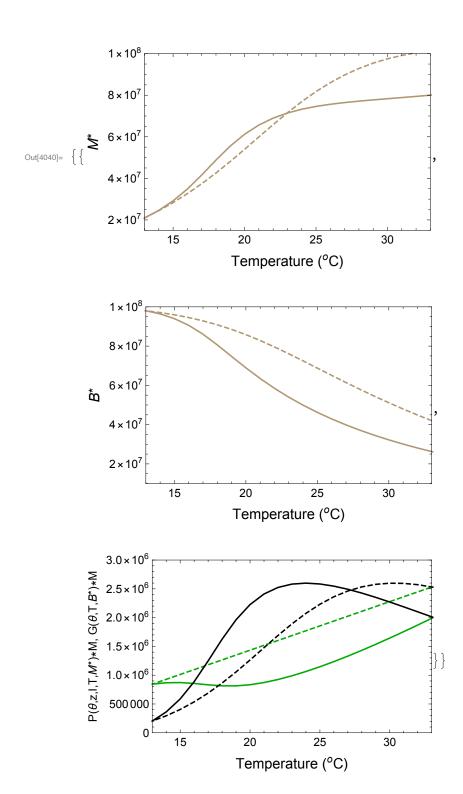






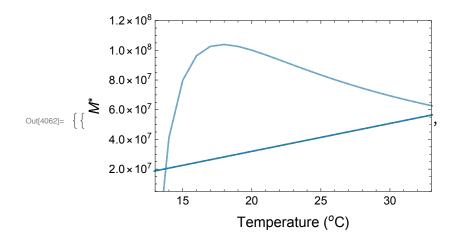


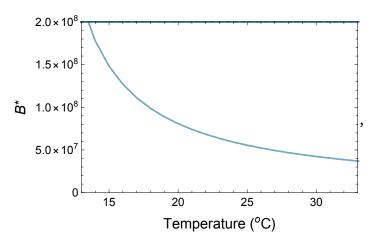
Temperature (°C)

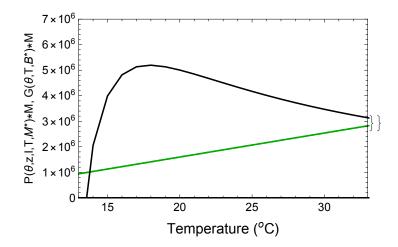


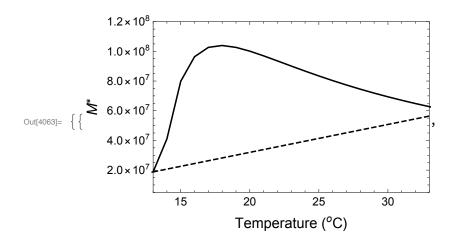
# $K_B=2*10^8, I_{in}=150$

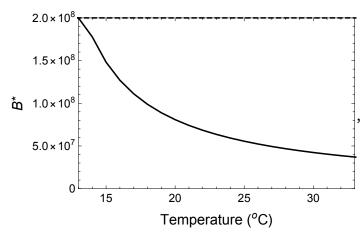
 $In[4061]:= K_B = 2 \times 10^8; I_{in} = 150;$ Quiet[Ccyclingspec[-1,  $.5 \times 10^{7}$ ,  $1.2 \times 10^{8}$ , 0,  $2 \times 10^{8}$ , 0,  $7 \times 10^{6}$ ] Quiet[Ccycling[0, .5 × 10 ^ 7, 1.2 × 10 ^ 8, 0, 2 × 10 ^ 8, 0, 8 × 10 ^ 6]] Quiet[Ccycling[1, 0 × 10 ^ 7, 1.2 × 10 ^ 8, 2 × 10 ^ 7, 2 × 10 ^ 8, 0, 8 × 10 ^ 6]]

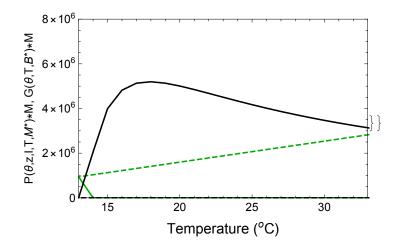


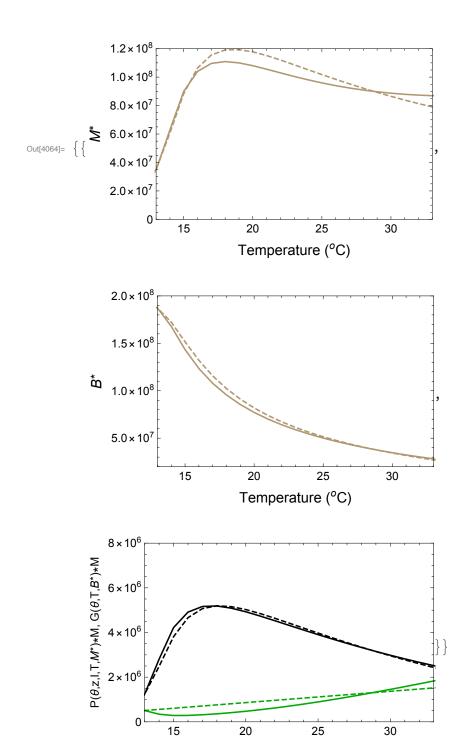












Temperature (°C)

# $K_B = 3*10^8, I_{in} = 150$

```
K_B = 3 \times 10^8; I_{in} = 150;
Quiet[Ccyclingspec[-1, 0 \times 10^{7}, 2 \times 10^{8}, 0 \times 10^{7}, 3 \times 10^{8}, 0, 7 \times 10^{6}]
Quiet[Ccycling[0, 0×10^7, 2×10^8, 0×10^7, 2.5×10^8, 0, 8×10^6]]
Quiet[Ccycling[1, 0 \times 10^{7}, 2 \times 10^{8}, 2 \times 10^{7}, 2.5 \times 10^{8}, 0, 8 \times 10^{6}]
```

