

```
In[1]:= Clear["Global`*"]
```

# Parameters

```
In[2]:= ρmax = 1 (*maximum carbon uptake rate (d-1)*);
αmax = 1.5 * 10-9 (*attack rate of mixotroph on bacteria (cm2*d-1*cellM-1)*);
b = .15 (*conversion rate of bacteria to mixotroph (cellM*cellB-1)*);
KB = 1 * 108; (*carrying capacity of bacteria (cellB*cm-2)*);
r = .693 (*growth rate of bacteria (d-1)*);
h = 250 (*half saturation constant for photosynthesis (μmol quanta*m2*s-1)*);
Iin = 100 (*incident light (μmol quanta*m2*s-1)*);
k = 5 * 10^(-7) (*mixotroph light absorbance constant (cm2*cellM-1)*);
l = .05 (*mixotroph mortality rate (d-1)*);
mρ = .1;
(*photosynthetic temeprature sensitivity coefficient (°C-1)*);
mα = .25;
(*heterotrophic temperature sensitivity coefficient (°C-1)*);
T0 = 13; (*baseline temperature (°C)*);
T0α = T0 -  $\frac{1}{m_{\alpha}}$  (*minmimum temperature for heterotrophy (°C)*);
T0ρ = T0 -  $\frac{1}{m_{\rho}}$  (*minmimum temperature for photosynthesis (°C)*);
kb = 8.62 * 10-5 (*Boltzmann constant (eV*K-1)*);
Eaρ = .5 (*photosynthetic activation energy (eV)*);
Eaα = .85 (*heterotrophic activation energy (eV)*);
r0ρ = 6.4279909706*^8 (*photosynthetic normalization constant*);
r0α = 9.412997398*^14 (*heterotrophic normalization constant*);
```

# Equations/Functions for generating outputs

## Equations

```
In[21]:= (*temperature-dependent photosynthetic rate*)
ρ[θ-, z-, T-] := ρmax * (1 - θ2z) $\frac{1}{2^z}$  (mρ (T - T0ρ))
```

$$\rho \text{Exp}[\theta_-, z_-, T_-] := \rho_{\max} * \left(1 - \theta^{2^z}\right)^{\frac{1}{2^z}} r \theta \rho E^{\frac{-E_{\text{ap}}}{k_b (273+T)}}$$

(\*temperature-dependent grazing rate\*)

$$\alpha[\theta_-, T_-] := \alpha_{\max} * \theta \left(m_{\alpha} (T - T_0 \alpha)\right)$$

$$\alpha \text{Exp}[\theta_-, T_-] := \alpha_{\max} * \theta r \theta \alpha E^{\frac{-E_{\text{aa}}}{k_b (273+T)}}$$

(\*solves for mixotroph and bacteria population density at equilibrium\*)

$$\text{eqs}[\theta_-, z_-, T_-] := \text{FindRoot}[\{\text{dM}[\theta, z, T] == 0, \text{dB}[\theta, T] == 0\},$$

$$\{\{M, 10^7\}, \{B, 10^7\}\}, \text{AccuracyGoal} \rightarrow \text{Infinity}]$$

$$\text{eqsExp}[\theta_-, z_-, T_-] := \text{FindRoot}[\{\text{dMExp}[\theta, z, T] == 0, \text{dBEExp}[\theta, T] == 0\},$$

$$\{\{M, 10^7\}, \{B, 10^7\}\}, \text{AccuracyGoal} \rightarrow \text{Infinity}]$$

(\*mixotroph per capita growth rate\*)

$$\text{dM}[\theta_-, z_-, T_-] := \left( \frac{\rho[\theta, z, T]}{k M} \text{Log} \left[ \frac{(h + I_{in})}{(h + I_{in} * \text{Exp}[-k M])} \right] - l + \alpha[\theta, T] b B \right)$$

$$\text{dMExp}[\theta_-, z_-, T_-] := \left( \frac{\rho \text{Exp}[\theta, z, T]}{k M} \text{Log} \left[ \frac{(h + I_{in})}{(h + I_{in} * \text{Exp}[-k M])} \right] - l + \alpha \text{Exp}[\theta, T] b B \right)$$

(\*bacteria per capita growth rate\*)

$$\text{dB}[\theta_-, T_-] := \left( r \left( 1 - \left( \frac{B}{K_B} \right) \right) - \alpha[\theta, T] M \right)$$

$$\text{dBEExp}[\theta_-, T_-] := \left( r \left( 1 - \left( \frac{B}{K_B} \right) \right) - \alpha \text{Exp}[\theta, T] M \right)$$

(\*mutant fitness equation\*)

$$\text{Fitness}[\theta m_-, z_-, M_-, B_-, T_-] := -l + \alpha[\theta m, T] b B + \frac{\rho[\theta m, z, T] * \text{Log} \left[ \frac{h + I_{in}}{h + e^{-(k M)} I_{in}} \right]}{k M}$$

$$\text{FitnessEmpty}[\theta m_-, \theta_-, z_-, T_-] := -l + \alpha[\theta m, T] b K_B + \frac{I_{in} \rho[\theta m, z, T]}{I_{in} + h}$$

$$\text{FitnessExp}[\theta m_-, z_-, M_-, B_-, T_-] :=$$

$$-l + \alpha \text{Exp}[\theta m, T] b B + \frac{\rho \text{Exp}[\theta m, z, T] * \text{Log} \left[ \frac{h + I_{in}}{h + e^{-(k M)} I_{in}} \right]}{k M}$$

(\*selection gradient\*)

$$\text{SelectionGrad}[\theta m_-, z_-, M_-, B_-, T_-] :=$$

$$\alpha_{\max} \left( m_{\alpha} (T - T_0 \alpha) \right) b B - \frac{\rho[\theta m, z, T] \theta m^{-1+2^z} (1 - \theta m^{2^z})^{-1} \text{Log} \left[ \frac{h + I_{in}}{h + e^{-k M} I_{in}} \right]}{k M}$$

$$\text{SelectionGradExp}[\theta m_-, z_-, M_-, B_-, T_-] :=$$

$$b B e^{\frac{-E_{\text{aa}}}{(273+T) k_b}} r \theta \alpha \alpha_{\max} - \frac{e^{\frac{-E_{\text{ap}}}{(273+T) k_b}} r \theta \rho \theta m^{-1+2^z} (1 - \theta m^{2^z})^{-1+2^{-z}} \text{Log} \left[ \frac{h + I_{in}}{h + e^{-k M} I_{in}} \right] \rho_{\max}}{k M}$$

$\ln[\oplus] :=$

$\ln[\oplus] :=$

## Pairwise invasibility plots (PIP)

```

In[36]:= (*Uses a chosen shape parameter z, temperature T,
and color to generate a pairwise invasibility plot using the
fitness function for an invading mutant mixotroph. Regions in
which mutant fitness is positive are shaded. Function has been
modified to only shade regions in which resident is viable*)
MakePIP[z_, T_, color_] :=
  RegionPlot[If[(M /. eqs[θ, z, T]) > 0 && (B /. eqs[θ, z, T]) > 0, Fitness[θm, z,
    M /. eqs[θ, z, T], B /. eqs[θ, z, T], T], FitnessEmpty[θm, θ, z, T]] ≥ 0, {θ, 0, 1},
    {θm, 0, 1}, PlotStyle → {color}, BoundaryStyle → {color}, Frame → True,
    FrameLabel → {Style["Resident Heterotrophic investment (θres)", 12, Black],
      Style["Mutant Heterotrophic investment (θmut)", 12, Black]},
    FrameTicksStyle → Directive[Black, 12], ImageSize → Medium]
(*generates a RegionPlot without checking for viability. This
is overlaid with a RegionPlot made using the function
above to show regions with nonviable residents*)
MakePIPNV[z_, T_, color_] := RegionPlot[(M /. eqs[θ, z, T]) ≤ 0, {θ, 0, 1},
  {θm, 0, 1}, PlotStyle → {color, Opacity[.4]}, BoundaryStyle → None, Frame → True,
  FrameLabel → {Style["Resident Heterotrophic investment (θres)", 12, Black],
    Style["Mutant Heterotrophic investment (θmut)", 12, Black]},
  FrameTicksStyle → Directive[Black, 12], ImageSize → Medium]

MakePIPExp[z_, T_, color_] :=
  RegionPlot[Fitness[θm, z, If[(M /. eqsExp[θ, z, T]) ≥ 0, M /. eqsExp[θ, z, T], 0],
    If[(B /. eqsExp[θ, z, T]) ≥ 0, B /. eqsExp[θ, z, T], 0], T] ≥ 0, {θ, 0, 1},
    {θm, 0, 1}, PlotStyle → color, BoundaryStyle → {Bold, Dashed, Black}, Frame → True,
    FrameLabel → {Style["Resident Heterotrophic investment (θres)", 12, Black],
      Style["Mutant Heterotrophic investment (θmut)", 12, Black]},
    FrameTicksStyle → Directive[Black, 12], ImageSize → Medium]
MakePIPNVexp[z_, T_, color_, colorm_] :=
  RegionPlot[Fitness[θm, z, M /. eqsExp[θ, z, T], B /. eqsExp[θ, z, T], T] ≥ 0,
    {θ, 0, 1}, {θm, 0, 1}, PlotStyle → color, Mesh → 20, MeshStyle → colorm,
    BoundaryStyle → {Bold, Dashed, Black}, Frame → True,
    FrameLabel → {Style["Resident Heterotrophic investment (θres)", 12, Black],
      Style["Mutant Heterotrophic investment (θmut)", 12, Black]},
    FrameTicksStyle → Directive[Black, 12], ImageSize → Medium]

```

## Generating ESS vs Temp plots for generalist and specialist mixotrophs

(\*makeθListGen[], makeθListLin[], and makeθListSpec[] generate lists containing the evolutionarily stable investment strategy  $\theta_{ESS}$  as a function of temperature for generalist tradeoff, linear, and specialist tradeoff mixotrophs respectively\*)

```
In[111]:= makeθListGen[] :=
{
  θESSgen = {};
  Quiet[For[T = 1, T < 41, T++,
    currentθ = θm /. FindRoot[
      SelectionGrad[θm, 1, M /. eqs[θm, 1, T], B /. eqs[θm, 1, T], T] == 0, {θm, .99}];
    If[Re[currentθ] > 1, AppendTo[θESSgen, 1], (*if calculated  $\theta_{ESS}$  >
      1 (maximum heterotrophic investment), 1 is added to list*)
    If[Re[currentθ] < 0, AppendTo[θESSgen, 0.], AppendTo[θESSgen, currentθ]]]
    (*if calculate  $\theta_{ESS}$  < 0 (minimum heterotrophic investment),
      0 is added to list*)
  ]];
  θESSgen
}
makeθListLin[] :=
{
  θESSlin = {};
  Quiet[For[T = 1, T < 41, T++,
    currentθ = θm /. FindRoot[
      SelectionGrad[θm, 0, M /. eqs[θm, 0, T], B /. eqs[θm, 0, T], T] == 0, {θm, .5}];
    (*getting around issue where FindRoot identifies the incorrect,
      evolutionarily unstable root in some cases*)
    If[ $\frac{1}{.0001}$  (SelectionGrad[.0002, 0, M /. eqs[.0002, 0, T], B /. eqs[.0002, 0, T], T] -
      SelectionGrad[.0001, 0, M /. eqs[.0001, 0, T], B /. eqs[.0001, 0, T], T]) > 0,
      AppendTo[θESSlin, 1], If[Re[currentθ] > 1, AppendTo[θESSlin, 1], If[
        Re[currentθ] < 0, AppendTo[θESSlin, 0.000], AppendTo[θESSlin, currentθ]]]]
  ]];
  θESSlin
}

makeθListGenExp[] :=
{
  θESSgen = {};
```

```

Quiet[For[T = 1, T < 41, T++,
  current $\theta$  =  $\theta_m$  /. FindRoot[SelectionGradExp[ $\theta_m$ , 1,
    M /. eqsExp[ $\theta_m$ , 1, T], B /. eqsExp[ $\theta_m$ , 1, T], T] == 0, { $\theta_m$ , .99}]];
If[Re[current $\theta$ ] > 1, AppendTo[ $\theta$ ESSgen, 1], (*if calculated  $\theta_{ESS}$  >
  1 (maximum heterotrophic investment), 1 is added to list*)
  If[Re[current $\theta$ ] < 0, AppendTo[ $\theta$ ESSgen, 0.], AppendTo[ $\theta$ ESSgen, current $\theta$ ]]]
  (*if calculate  $\theta_{ESS}$  < 0 (minimum heterotrophic investment),
  0 is added to list*)
  0 is added to list*)
  ];
 $\theta$ ESSgen
}
make $\theta$ ListLinExp[] :=
{
 $\theta$ ESSlin = {};
Quiet[For[T = 1, T < 41, T++,
  current $\theta$  =  $\theta_m$  /. FindRoot[SelectionGradExp[ $\theta_m$ , 0,
    M /. eqsExp[ $\theta_m$ , 0, T], B /. eqsExp[ $\theta_m$ , 0, T], T] == 0, { $\theta_m$ , .5}]];
(*getting around issue where FindRoot identifies the incorrect,
  evolutionarily unstable root in some cases*)
  If[ $\frac{1}{.0001}$  (SelectionGradExp[.0002, 0, M /. eqsExp[.0002, 0, T],
    B /. eqsExp[.0002, 0, T], T] - SelectionGradExp[.0001, 0,
    M /. eqsExp[.0001, 0, T], B /. eqsExp[.0001, 0, T], T]) > 0,
    AppendTo[ $\theta$ ESSlin, 1], If[Re[current $\theta$ ] > 1, AppendTo[ $\theta$ ESSlin, 1], If[
      Re[current $\theta$ ] < 0, AppendTo[ $\theta$ ESSlin, 0.000], AppendTo[ $\theta$ ESSlin, current $\theta$ ]]]
    ];
 $\theta$ ESSlin
}
make $\theta$ ListGen[] :=
{
 $\theta$ ESSgen = {};
Quiet[For[T = 1, T < 41, T++,
  current $\theta$  =  $\theta_m$  /. FindRoot[
    SelectionGrad[ $\theta_m$ , 1, M /. eqs[ $\theta_m$ , 1, T], B /. eqs[ $\theta_m$ , 1, T], T] == 0, { $\theta_m$ , .99}]];
  If[Re[current $\theta$ ] > 1, AppendTo[ $\theta$ ESSgen, 1], (*if calculated  $\theta_{ESS}$  >
    1 (maximum heterotrophic investment), 1 is added to list*)
    If[Re[current $\theta$ ] < 0, AppendTo[ $\theta$ ESSgen, 0.], AppendTo[ $\theta$ ESSgen, current $\theta$ ]]]
    (*if calculate  $\theta_{ESS}$  < 0 (minimum heterotrophic investment),
    0 is added to list*)
    0 is added to list*)
    ];
 $\theta$ ESSgen
}
make $\theta$ ListLin[] :=

```

```

{
  θESSlin = {};
  Quiet[For[T = 1, T < 41, T++,
    currentθ = θm /. FindRoot[
      SelectionGrad[θm, 0, M /. eqs[θm, 0, T], B /. eqs[θm, 0, T], T] == 0, {θm, .5}];
    (*getting around issue where FindRoot identifies the incorrect,
    evolutionarily unstable root in some cases*)
    If[ $\frac{1}{.0001}$  (SelectionGrad[.0002, 0, M /. eqs[.0002, 0, T], B /. eqs[.0002, 0, T], T] -
      SelectionGrad[.0001, 0, M /. eqs[.0001, 0, T], B /. eqs[.0001, 0, T], T]) > 0,
      AppendTo[θESSlin, 1], If[Re[currentθ] > 1, AppendTo[θESSlin, 1], If[
        Re[currentθ] < 0, AppendTo[θESSlin, 0.000], AppendTo[θESSlin, currentθ]]]
    ]];
  θESSlin
}
makeθListGenExp[] :=
{
  θESSgen = {};
  Quiet[For[T = 1, T < 41, T++,
    currentθ = θm /. FindRoot[SelectionGradExp[θm, 1,
      M /. eqsExp[θm, 1, T], B /. eqsExp[θm, 1, T], T] == 0, {θm, .99}];
    If[Re[currentθ] > 1, AppendTo[θESSgen, 1], (*if calculated θESS >
      1 (maximum heterotrophic investment), 1 is added to list*)
    If[Re[currentθ] < 0, AppendTo[θESSgen, 0.], AppendTo[θESSgen, currentθ]]]
    (*if calculate θESS < 0 (minimum heterotrophic investment),
    0 is added to list*)
    ]];
  θESSgen
}
makeθListLinExp[] :=
{
  θESSlin = {};
  Quiet[For[T = 1, T < 41, T++,
    currentθ = θm /. FindRoot[SelectionGradExp[θm, 0,
      M /. eqsExp[θm, 0, T], B /. eqsExp[θm, 0, T], T] == 0, {θm, .5}];
    (*getting around issue where FindRoot identifies the incorrect,
    evolutionarily unstable root in some cases*)
    If[ $\frac{1}{.0001}$  (SelectionGradExp[.0002, 0, M /. eqsExp[.0002, 0, T],
      B /. eqsExp[.0002, 0, T], T] - SelectionGradExp[.0001, 0,
      M /. eqsExp[.0001, 0, T], B /. eqsExp[.0001, 0, T], T]) > 0,
      AppendTo[θESSlin, 1], If[Re[currentθ] > 1, AppendTo[θESSlin, 1], If[
        Re[currentθ] < 0, AppendTo[θESSlin, 0.000], AppendTo[θESSlin, currentθ]]]
    ]];
  θESSlin
}

```

```

    ]];
    θESSlin
  }

makeθListSpec[] :=
{
  θESSspec1viable = Table[Null, 40];
  θESSspec0 = {};
  Quiet[For[T = 1, T < 41, T++,
    If[eqs[1, -1, T][[1]][[2]] > 0, θESSspec1viable[[T ;; T]] = 1];
    AppendTo[θESSspec0, 0];
  ]];
  θESSspec = Table[θESSspec1viable, θESSspec0];
  θESSspec
}

makeθListSpecExp[] :=
{
  θESSspec1viable = Table[Null, 40];
  θESSspec0 = {};
  Quiet[For[T = 1, T < 41, T++,
    If[eqsExp[1, -1, T][[1]][[2]] > 0, θESSspec1viable[[T ;; T]] = 1];
    AppendTo[θESSspec0, 0];
  ]];
  θESSspec = Table[θESSspec1viable, θESSspec0];
  θESSspec
}

```

*In[ ]:=*

## Comparing evolved vs unevolved mixotrophs for carbon cycling

*In[90]:=* (\*generates plots comparing mixotroph and bacteria populations,  
and growth rate components derived from photosynthesis,  
 $P(\theta, z, I, T, M^*)$ , and heterotrophy,  $G(\theta, T, B^*)$ ,  
between evolving mixotrophs whos heterotrophic investment  $\theta$  varies  
as a function of temperature and genetically static mixotrophs with  
fixed  $\theta$ . This allows evolutionary and strictly thermal responses  
to be compared. Function inputs consist of the shape parameter,  $z$ ,  
the chosen lower bounds for each set of plots ( $l1$ ,  $l2$ , and  $l3$ ) and  
the chosen upper bounds for each set of plots ( $u1$ ,  $u2$ , and  $u3$ \*)  
Ccycling[z\_, l1\_, u1\_, l2\_, u2\_, l3\_, u3\_] :=



```

{makeΘListLin[];
 makeΘListGen[];
 ΘList = List[];
 If[z == 0, ΘList = ΘESSlin, ΘList = ΘESSgen];

Mpopsevo = List[];
Mpopsnoevo = List[];
Quiet[For[t = 1, t < 100, t++, AppendTo[Mpopsevo, M /. eqs[ΘList[[t]], z, t]]];
Quiet[For[t = 1, t < 100, t++, AppendTo[Mpopsnoevo, M /. eqs[ΘList[[T0]], z, t]]];

bpopsevo = List[];
bpopsnoevo = List[];
Quiet[For[t = 1, t < 100, t++, AppendTo[bpopsevo, B /. eqs[ΘList[[t]], z, t]]];
Quiet[For[t = 1, t < 100, t++, AppendTo[bpopsnoevo, B /. eqs[ΘList[[T0]], z, t]]];

photgrowthevo = List[];
photgrowthnoevo = List[];
Quiet[For[t = 1, t < 100, t++,
 AppendTo[photgrowthevo, (M /. eqs[ΘList[[t]], z, t]) * (

$$\left( \rho[\Theta\text{List}[[t]], z, t] \times \frac{\log\left[\frac{h + I_{in}}{h + e^{-(k(M /. eqs[\Theta\text{List}[[t]], z, t))} I_{in})}}{I_{in}} \right) / (k(M /. eqs[\Theta\text{List}[[t]], z, t))) \right) ]];
Quiet[For[t = 1, t < 100, t++, AppendTo[photgrowthnoevo,
 (M /. eqs[ΘList[[T0]], z, t]) *

$$\left( \left( \rho[\Theta\text{List}[[T0]], z, t] \times \log\left[\frac{h + I_{in}}{h + e^{-(k(M /. eqs[\Theta\text{List}[[T0]], z, t))} I_{in})}} \right) / \right. \right. \\ \left. \left. (k(M /. eqs[\Theta\text{List}[[T0]], z, t))) \right) ]];

hetgrowthevo = List[];
hetgrowthnoevo = List[];
Quiet[For[t = 1, t < 100, t++, AppendTo[hetcrowthevo,
 (B /. eqs[ΘList[[t]], z, t]) * (M /. eqs[ΘList[[t]], z, t]) α[ΘList[[t]], t] b]]];
Quiet[For[t = 1, t < 100, t++, AppendTo[hetcrowthnoevo,
 (B /. eqs[ΘList[[T0]], z, t]) *
 (M /. eqs[ΘList[[T0]], z, t]) α[ΘList[[T0]], t] b]]];

List[ListPlot[{Mpopsevo, Mpopsnoevo}, Joined → True, PlotRange →
 {{T0, 33}, {l1, u1}}, PlotStyle → {{If[z > 0, RGBColor["#B09771"], Black}},
 {If[z > 0, RGBColor["#B09771"], Black], Dashed}}, Frame → True,
 FrameLabel → {Style["Temperature (°C)", 15, Black], Style["M*", 15, Black]},$$$$

```

```

    FrameTicksStyle → Directive[Black, 12], ImageSize → Medium],
ListPlot[{bpopsevo, bpopsnoevo}, Joined → True, PlotRange → {{T0, 33}, {l2, u2}},
    PlotStyle → {{If[z > 0, RGBColor["#B09771"], Black]},
        {If[z > 0, RGBColor["#B09771"], Black], Dashed}}, Frame → True,
    FrameLabel → {Style["Temperature (°C)", 15, Black], Style["B*", 15, Black]},
    FrameTicksStyle → Directive[Black, 12], ImageSize → Medium],
ListPlot[{photgrowthevo, photgrowthnoevo, hetgrowthevo, hetgrowthnoevo},
    Joined → True, PlotRange → {{T0, 33}, {l3, u3}},
    PlotStyle → {{Darker[Green]}, {Darker[Green], Dashed}, {Black}, {Black, Dashed}},
    Frame → True, FrameLabel → {Style["Temperature (°C)", Black, 15],
        Style["P(θ,z,I,T,M*)*M, G(θ,T,B*)*M", Black, 12]},
    FrameTicksStyle → Directive[Black, 12], ImageSize → Medium]]
}
Ccyclingspec[z_, l1_, u1_, l2_, u2_, l3_, u3_] :=
{makeΘListSpec[];
    ΘList0 = Table[0, 40];
    ΘList1 = Table[1, 40];

    Mpopsevo0 = List[];
    Mpopsnoevo0 = List[];
    Quiet[For[t = 1, t < 100, t++, AppendTo[Mpopsevo0, M /. eqs[ΘList0[[t]], z, t]]];
    Quiet[
        For[t = 1, t < 100, t++, AppendTo[Mpopsnoevo0, M /. eqs[ΘList0[[T0]], z, t]]];

    Mpopsevo1 = List[];
    Mpopsnoevo1 = List[];
    Quiet[For[t = 1, t < 100, t++, AppendTo[Mpopsevo1, M /. eqs[ΘList1[[t]], z, t]]];
    Quiet[
        For[t = 1, t < 100, t++, AppendTo[Mpopsnoevo1, M /. eqs[ΘList1[[T0]], z, t]]];

    bpopsevo0 = List[];
    bpopsnoevo0 = List[];
    Quiet[For[t = 1, t < 100, t++, AppendTo[bpopsevo0, B /. eqs[ΘList0[[t]], z, t]]];
    Quiet[
        For[t = 1, t < 100, t++, AppendTo[bpopsnoevo0, B /. eqs[ΘList0[[T0]], z, t]]];

    bpopsevo1 = List[];
    bpopsnoevo1 = List[];
    Quiet[For[t = 1, t < 100, t++, AppendTo[bpopsevo1, B /. eqs[ΘList1[[t]], z, t]]];
    Quiet[
        For[t = 1, t < 100, t++, AppendTo[bpopsnoevo1, B /. eqs[ΘList1[[T0]], z, t]]];

    photgrowthevo0 = List[];

```

```

photgrowthnoevo0 = List[];
Quiet[
  For[t = 1, t < 100, t++, AppendTo[photgrowthevo0, (M /. eqs[θList0[[t]], z, t]) *
    (
      (
        (
          ρ[θList0[[t]], z, t] × Log[ $\frac{h + I_{in}}{h + e^{-(k (M /. eqs[θList0[[t]], z, t)])} I_{in}}$ )] /
          (k (M /. eqs[θList0[[t]], z, t]))
        )
      )
    )
  ];
Quiet[For[t = 1, t < 100, t++, AppendTo[photgrowthnoevo0,
  (M /. eqs[θList0[[T0]], z, t]) *
    (
      (
        (
          ρ[θList0[[T0]], z, t] × Log[ $\frac{h + I_{in}}{h + e^{-(k (M /. eqs[θList0[[T0]], z, t)])} I_{in}}$ )] /
          (k (M /. eqs[θList0[[T0]], z, t]))
        )
      )
    )
  ];

photgrowthevo1 = List[];
photgrowthnoevo1 = List[];
Quiet[
  For[t = 1, t < 100, t++, AppendTo[photgrowthevo1, (M /. eqs[θList1[[t]], z, t]) *
    (
      (
        (
          ρ[θList1[[t]], z, t] × Log[ $\frac{h + I_{in}}{h + e^{-(k (M /. eqs[θList1[[t]], z, t)])} I_{in}}$ )] /
          (k (M /. eqs[θList1[[t]], z, t]))
        )
      )
    )
  ];
Quiet[For[t = 1, t < 100, t++, AppendTo[photgrowthnoevo1,
  (M /. eqs[θList1[[T0]], z, t]) *
    (
      (
        (
          ρ[θList1[[T0]], z, t] × Log[ $\frac{h + I_{in}}{h + e^{-(k (M /. eqs[θList1[[T0]], z, t)])} I_{in}}$ )] /
          (k (M /. eqs[θList1[[T0]], z, t]))
        )
      )
    )
  ];

hetgrowthevo0 = List[];
hetgrowthnoevo0 = List[];
Quiet[
  For[t = 1, t < 100, t++, AppendTo[hetergrowthevo0, (B /. eqs[θList0[[t]], z, t]) *
    (M /. eqs[θList0[[t]], z, t]) α[θList0[[t]], t] b]]];
Quiet[For[t = 1, t < 100, t++, AppendTo[hetergrowthnoevo0,
  (B /. eqs[θList0[[T0]], z, t]) *
    (M /. eqs[θList0[[T0]], z, t]) α[θList0[[T0]], t] b]]];

hetgrowthevo1 = List[];
hetgrowthnoevo1 = List[];
Quiet[
  For[t = 1, t < 100, t++, AppendTo[hetergrowthevo1, (B /. eqs[θList1[[t]], z, t]) *

```

```

(M /. eqs[θList1[[t]], z, t]) α[θList1[[t]], t] b]]];
Quiet[For[t = 1, t < 100, t++, AppendTo[hetrogrowthnoevo1,
(B /. eqs[θList1[[T0]], z, t]) *
(M /. eqs[θList1[0[T0]], z, t]) α[θList1[[T0]], t] b]]];

List[ListPlot[{Mpopsevo1, Mpopsevo0, Mpopsnoevo1, Mpopsnoevo0}, Joined → True,
PlotRange → {{9, 33}, {l1, u1}}, PlotStyle → {{Lighter[RGBColor["#287DAB"]]},
{RGBColor["#287DAB"]}}, {Lighter[RGBColor["#287DAB"]], Dashed},
{RGBColor["#287DAB"], Dashed}}, Frame → True,
FrameLabel → {Style["Temperature (°C)", 15, Black], Style["M*", 15, Black]},
FrameTicksStyle → Directive[Black, 12], ImageSize → Medium],
ListPlot[{bpopsevo1, bpopsevo0, bpopsnoevo1, bpopsnoevo0}, Joined → True,
PlotRange → {{9, 33}, {l2, u2}}, PlotStyle → {{Lighter[RGBColor["#287DAB"]]},
{RGBColor["#287DAB"]}}, {Lighter[RGBColor["#287DAB"]], Dashed},
{RGBColor["#287DAB"], Dashed}}, Frame → True,
FrameLabel → {Style["Temperature (°C)", 15, Black], Style["B*", 15, Black]},
FrameTicksStyle → Directive[Black, 12], ImageSize → Medium],
ListPlot[{photgrowthevo0, photgrowthevo1, photgrowthnoevo0, photgrowthnoevo1,
hetgrowthevo0, hetgrowthevo1, hetgrowthnoevo0, hetgrowthnoevo1},
Joined → True, PlotRange → {{9, 33}, {l3, u3}},
PlotStyle → {{Darker[Green]}, {Darker[Green]}, {Darker[Green], Dashed},
{Darker[Green], Dashed}, {Black}, {Black}, {Black, Dashed}, {Black, Dashed}},
Frame → True, FrameLabel → {Style["Temperature (°C)", Black, 15],
Style["P(θ,z,I,T,M*)*M, G(θ,T,B*)*M", Black, 12]},
FrameTicksStyle → Directive[Black, 12], ImageSize → Medium]]
}

```

In[ ]:=

In[ ]:=

# $\theta$ vs. Temperature plots

$I_{in}=100$

## Linear temperature dependence

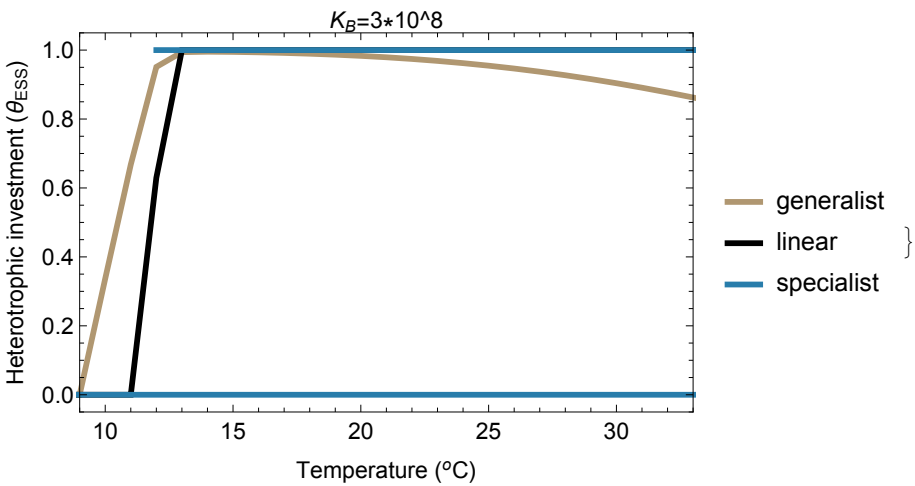
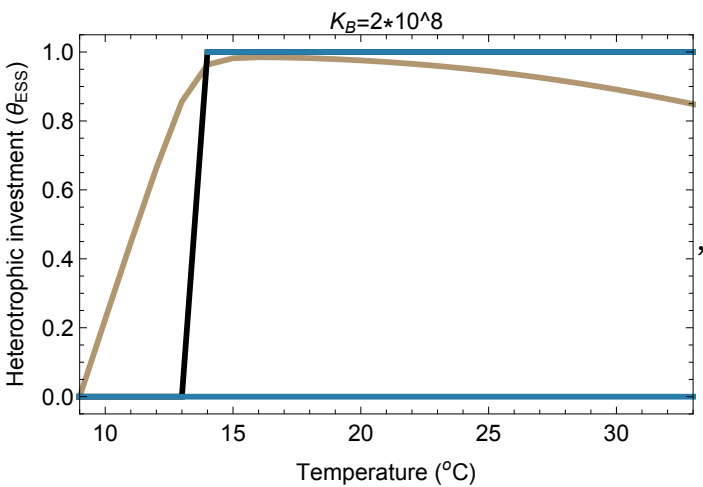
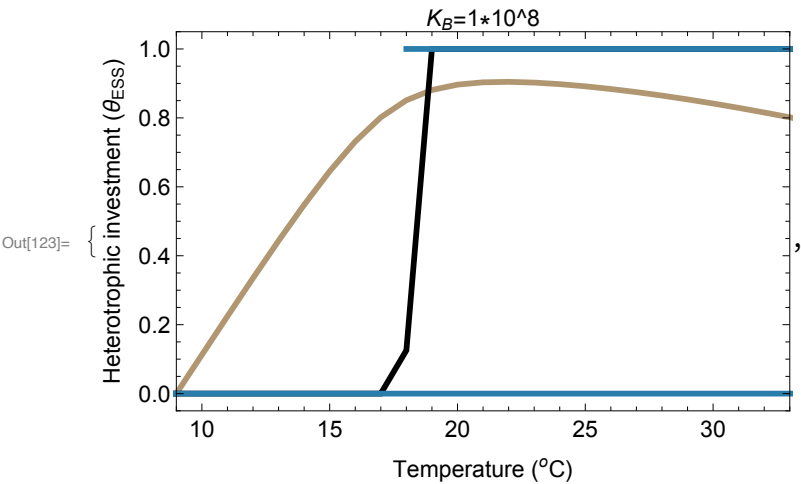
```
In[123]:= List[KB = 1 × 108; Iin = 100;
ListPlot[{makeΘListGen[] // Flatten, makeΘListLin[] // Flatten,
  makeΘListSpec[][[1]][[1]], makeΘListSpec[][[1]][[2]]},
  Joined → True, PlotRange → {{9, 33}, {- .05, 1.05}},
  PlotStyle → {{RGBColor["#B09771"], AbsoluteThickness[3]},
    {Black, AbsoluteThickness[3]}, {RGBColor["#287DAB"], AbsoluteThickness[3]},
    {RGBColor["#287DAB"], AbsoluteThickness[3]}}, Frame → True,
  ImageSize → Medium, FrameLabel → {Style["Temperature (°C)", 12, Black],
    Style["Heterotrophic investment (θESS)", 12, Black]},
  FrameTicksStyle → Directive[Black, 12],
  PlotLabel → Style["KB=1*108", 12, Black]}, KB = 2 × 108;
Iin = 100;
ListPlot[{makeΘListGen[] // Flatten, makeΘListLin[] // Flatten,
  makeΘListSpec[][[1]][[1]], makeΘListSpec[][[1]][[2]]},
  Joined → True, PlotRange → {{9, 33}, {- .05, 1.05}},
  PlotStyle → {{RGBColor["#B09771"], AbsoluteThickness[3]},
    {Black, AbsoluteThickness[3]}, {RGBColor["#287DAB"], AbsoluteThickness[3]},
    {RGBColor["#287DAB"], AbsoluteThickness[3]}}, Frame → True,
  ImageSize → Medium, FrameLabel → {Style["Temperature (°C)", 12, Black],
    Style["Heterotrophic investment (θESS)", 12, Black]},
  FrameTicksStyle → Directive[Black, 12],
  PlotLabel → Style["KB=2*108", 12, Black]}, KB = 3 × 108;
Iin = 100;
ListPlot[{makeΘListGen[] // Flatten, makeΘListLin[] // Flatten,
  makeΘListSpec[][[1]][[1]], makeΘListSpec[][[1]][[2]]},
  Joined → True, PlotRange → {{9, 33}, {- .05, 1.05}},
  PlotStyle → {{RGBColor["#B09771"], AbsoluteThickness[3]},
    {Black, AbsoluteThickness[3]}, {RGBColor["#287DAB"], AbsoluteThickness[3]},
    {RGBColor["#287DAB"], AbsoluteThickness[3]}}, Frame → True,
  ImageSize → Medium, FrameLabel → {Style["Temperature (°C)", 12, Black],
    Style["Heterotrophic investment (θESS)", 12, Black]},
  PlotLegends → {"generalist", "linear", "specialist"},
  FrameTicksStyle → Directive[Black, 12], PlotLabel → Style["KB=3*108", 12, Black]]]
```

... Table: Non-list iterator  $\theta_{ESS}spec0$  at position 2 does not evaluate to a real numeric value.

... **Table:** Non-list iterator  $\theta_{ESS}^{spec0}$  at position 2 does not evaluate to a real numeric value.

... **Table:** Non-list iterator  $\theta_{ESS}^{spec0}$  at position 2 does not evaluate to a real numeric value.

... **General:** Further output of Table::nliter will be suppressed during this calculation.



## Exponential temperature dependence

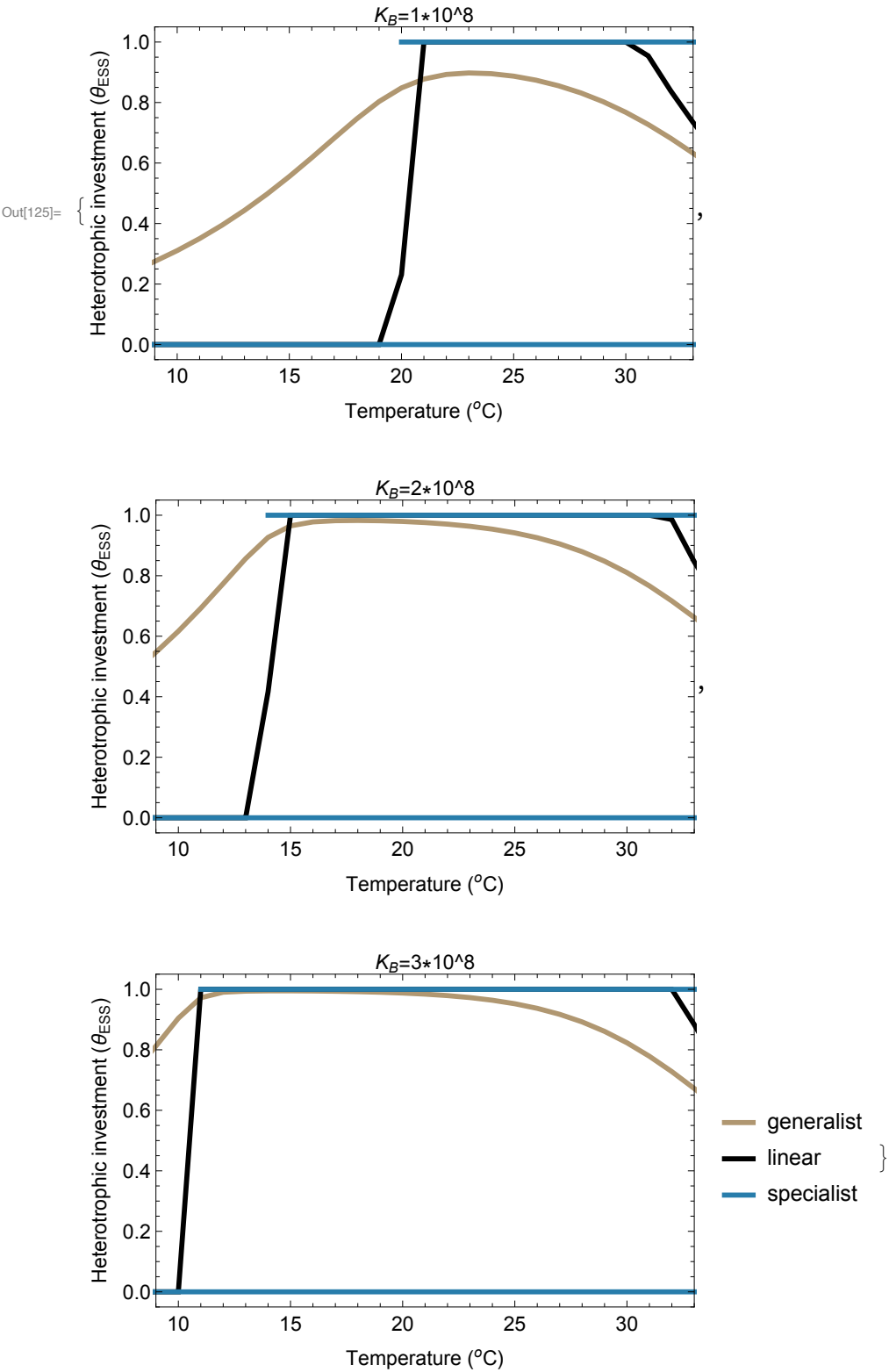
```
In[125]:= List[KB = 1 × 108; Iin = 100;
ListPlot[{makeθListGenExp[] // Flatten, makeθListLinExp[] // Flatten,
  makeθListSpecExp[][[1]][[1]], makeθListSpecExp[][[1]][[2]]},
  Joined → True, PlotRange → {{9, 33}, {- .05, 1.05}},
  PlotStyle → {{RGBColor["#B09771"], AbsoluteThickness[3]},
    {Black, AbsoluteThickness[3]}, {RGBColor["#287DAB"], AbsoluteThickness[3]},
    {RGBColor["#287DAB"], AbsoluteThickness[3]}}, Frame → True,
  ImageSize → Medium, FrameLabel → {Style["Temperature (°C)", 12, Black],
    Style["Heterotrophic investment (θESS)", 12, Black]},
  FrameTicksStyle → Directive[Black, 12],
  PlotLabel → Style["KB=1*108", 12, Black]}, KB = 2 × 108;
Iin = 100;
ListPlot[{makeθListGenExp[] // Flatten, makeθListLinExp[] // Flatten,
  makeθListSpecExp[][[1]][[1]], makeθListSpecExp[][[1]][[2]]},
  Joined → True, PlotRange → {{9, 33}, {- .05, 1.05}},
  PlotStyle → {{RGBColor["#B09771"], AbsoluteThickness[3]},
    {Black, AbsoluteThickness[3]}, {RGBColor["#287DAB"], AbsoluteThickness[3]},
    {RGBColor["#287DAB"], AbsoluteThickness[3]}}, Frame → True,
  ImageSize → Medium, FrameLabel → {Style["Temperature (°C)", 12, Black],
    Style["Heterotrophic investment (θESS)", 12, Black]},
  FrameTicksStyle → Directive[Black, 12],
  PlotLabel → Style["KB=2*108", 12, Black]}, KB = 3 × 108;
Iin = 100;
ListPlot[{makeθListGenExp[] // Flatten, makeθListLinExp[] // Flatten,
  makeθListSpecExp[][[1]][[1]], makeθListSpecExp[][[1]][[2]]},
  Joined → True, PlotRange → {{9, 33}, {- .05, 1.05}},
  PlotStyle → {{RGBColor["#B09771"], AbsoluteThickness[3]},
    {Black, AbsoluteThickness[3]}, {RGBColor["#287DAB"], AbsoluteThickness[3]},
    {RGBColor["#287DAB"], AbsoluteThickness[3]}}, Frame → True,
  ImageSize → Medium, FrameLabel → {Style["Temperature (°C)", 12, Black],
    Style["Heterotrophic investment (θESS)", 12, Black]},
  PlotLegends → {"generalist", "linear", "specialist"},
  FrameTicksStyle → Directive[Black, 12], PlotLabel → Style["KB=3*108", 12, Black]}}
```

... **Table:** Non-list iterator  $\theta_{ESS}spec0$  at position 2 does not evaluate to a real numeric value.

... **Table:** Non-list iterator  $\theta_{ESS}spec0$  at position 2 does not evaluate to a real numeric value.

... **Table:** Non-list iterator  $\theta_{ESS}spec0$  at position 2 does not evaluate to a real numeric value.

... **General:** Further output of Table::nliter will be suppressed during this calculation.





$I_{in}=150$

`In[ ]:=`

## Linear temperature dependence

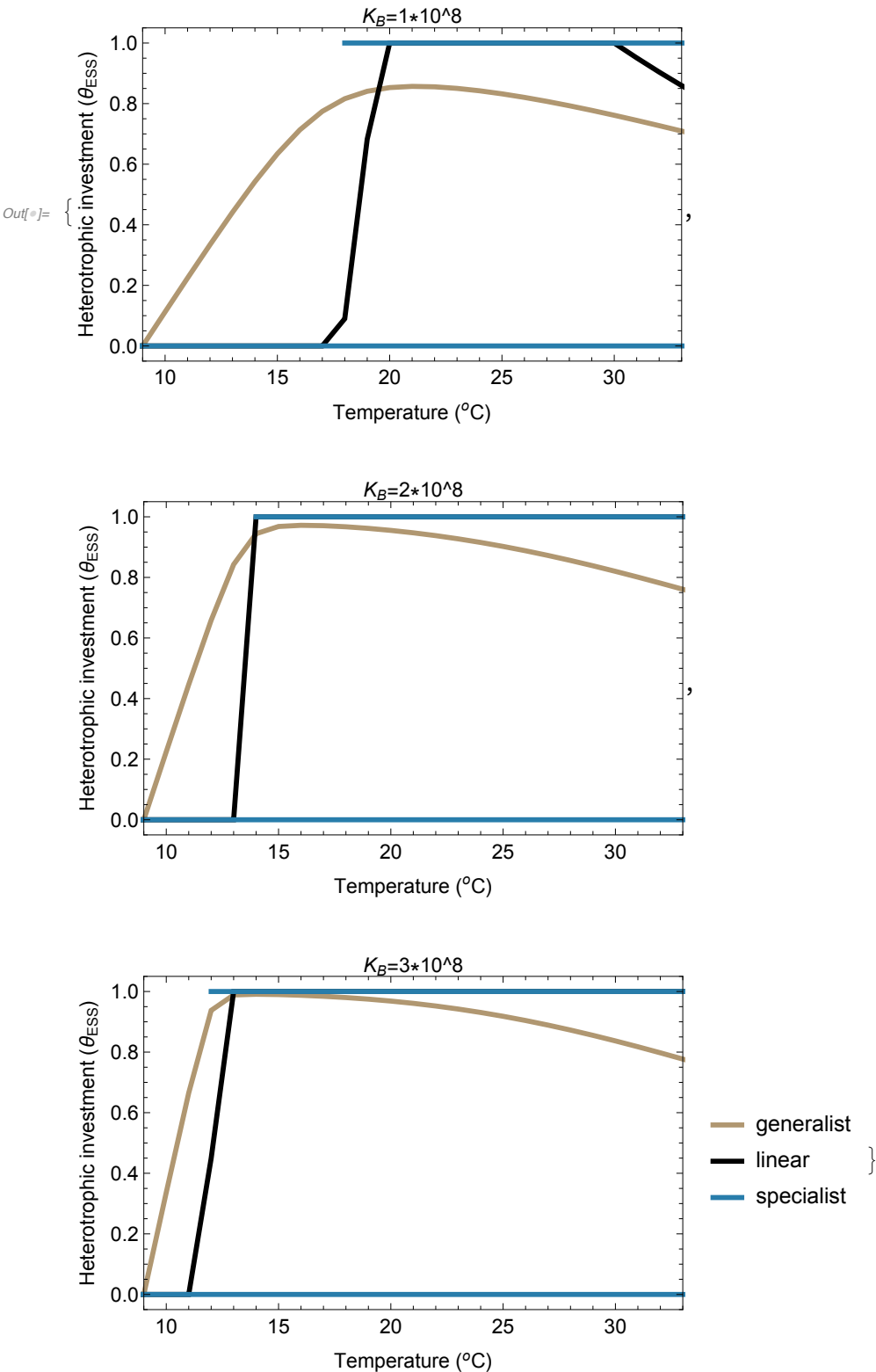
```
In[ ]:= ones = Table[1, 100];
zeros = Table[0, 100];
List[KB = 1 × 108; Iin = 150;
ListPlot[{makeΘListGen[] // Flatten, makeΘListLin[] // Flatten,
  makeΘListSpec[][[1]][[1]], makeΘListSpec[][[1]][[2]]},
  Joined → True, PlotRange → {{9, 33}, {- .05, 1.05}},
  PlotStyle → {{RGBColor["#B09771"], AbsoluteThickness[3]},
    {Black, AbsoluteThickness[3]}, {RGBColor["#287DAB"], AbsoluteThickness[3]},
    {RGBColor["#287DAB"], AbsoluteThickness[3]}}, Frame → True,
  ImageSize → Medium, FrameLabel → {Style["Temperature (°C)", 12, Black],
    Style["Heterotrophic investment (θESS)", 12, Black]},
  FrameTicksStyle → Directive[Black, 12],
  PlotLabel → Style["KB=1*108", 12, Black]}, KB = 2 × 108;
Iin = 150;
ListPlot[{makeΘListGen[] // Flatten, makeΘListLin[] // Flatten,
  makeΘListSpec[][[1]][[1]], makeΘListSpec[][[1]][[2]]},
  Joined → True, PlotRange → {{9, 33}, {- .05, 1.05}},
  PlotStyle → {{RGBColor["#B09771"], AbsoluteThickness[3]},
    {Black, AbsoluteThickness[3]}, {RGBColor["#287DAB"], AbsoluteThickness[3]},
    {RGBColor["#287DAB"], AbsoluteThickness[3]}}, Frame → True,
  ImageSize → Medium, FrameLabel → {Style["Temperature (°C)", 12, Black],
    Style["Heterotrophic investment (θESS)", 12, Black]},
  FrameTicksStyle → Directive[Black, 12],
  PlotLabel → Style["KB=2*108", 12, Black]}, KB = 3 × 108;
Iin = 150;
ListPlot[{makeΘListGen[] // Flatten, makeΘListLin[] // Flatten,
  makeΘListSpec[][[1]][[1]], makeΘListSpec[][[1]][[2]]},
  Joined → True, PlotRange → {{9, 33}, {- .05, 1.05}},
  PlotStyle → {{RGBColor["#B09771"], AbsoluteThickness[3]},
    {Black, AbsoluteThickness[3]}, {RGBColor["#287DAB"], AbsoluteThickness[3]},
    {RGBColor["#287DAB"], AbsoluteThickness[3]}}, Frame → True,
  ImageSize → Medium, FrameLabel → {Style["Temperature (°C)", 12, Black],
    Style["Heterotrophic investment (θESS)", 12, Black]},
  PlotLegends → {"generalist", "linear", "specialist"},
  FrameTicksStyle → Directive[Black, 12], PlotLabel → Style["KB=3*108", 12, Black]}}
```

... **Table:** Non-list iterator  $\theta_{ESS}spec0$  at position 2 does not evaluate to a real numeric value.

... **Table:** Non-list iterator  $\theta_{ESS}^{spec0}$  at position 2 does not evaluate to a real numeric value.

... **Table:** Non-list iterator  $\theta_{ESS}^{spec0}$  at position 2 does not evaluate to a real numeric value.

... **General:** Further output of Table::nliter will be suppressed during this calculation.



## Exponential temperature dependence

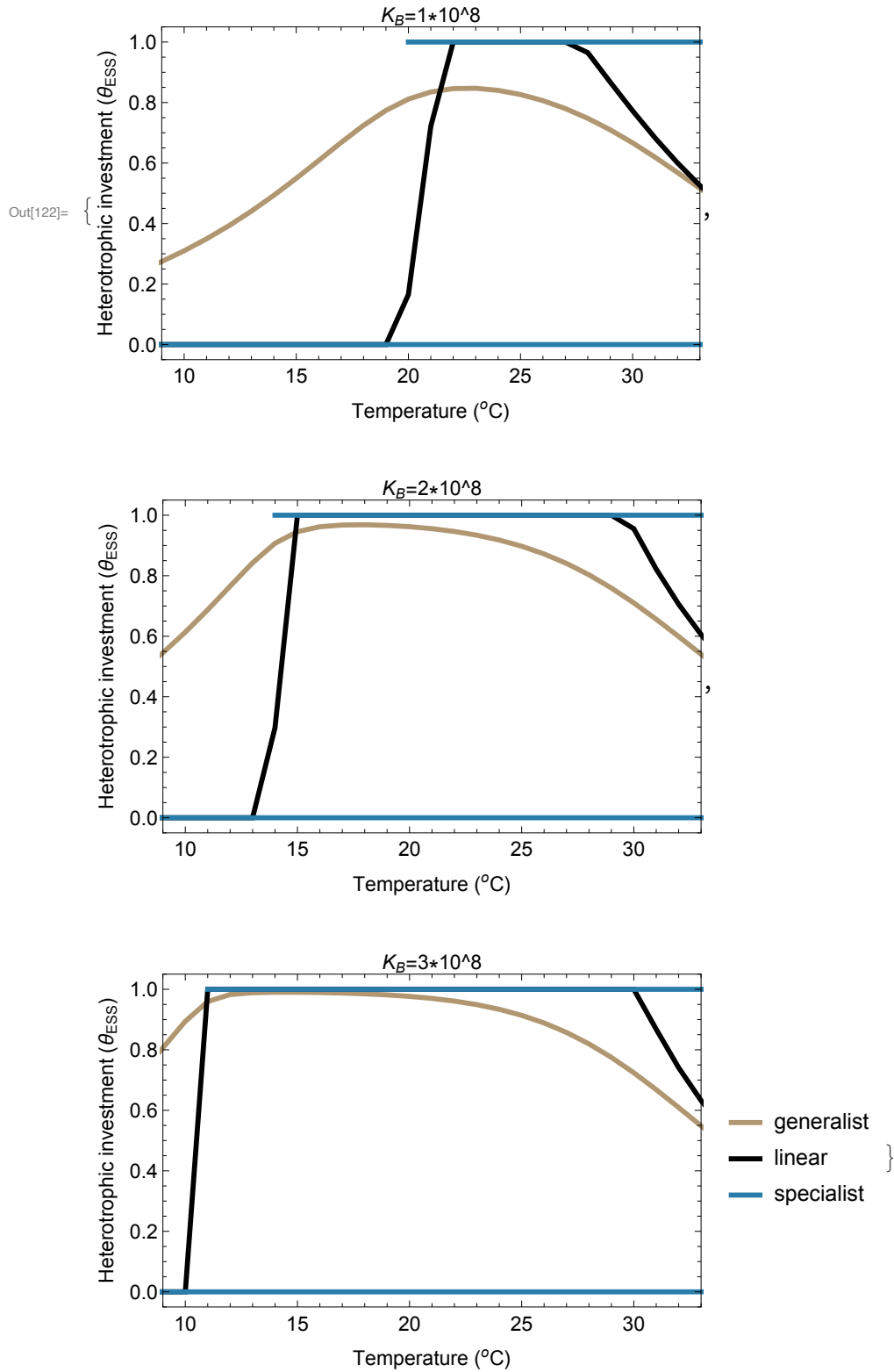
```
In[122]:= List[KB = 1 × 108; Iin = 150;
  ListPlot[{makeθListGenExp[] // Flatten, makeθListLinExp[] // Flatten,
    makeθListSpecExp[[1]][[1]], makeθListSpecExp[[1]][[2]]},
  Joined → True, PlotRange → {{9, 33}, {-.05, 1.05}},
  PlotStyle → {{RGBColor["#B09771"], AbsoluteThickness[3]},
    {Black, AbsoluteThickness[3]}, {RGBColor["#287DAB"], AbsoluteThickness[3]},
    {RGBColor["#287DAB"], AbsoluteThickness[3]}}, Frame → True,
  ImageSize → Medium, FrameLabel → {Style["Temperature (°C)", 12, Black],
    Style["Heterotrophic investment (θESS)", 12, Black]},
  FrameTicksStyle → Directive[Black, 12],
  PlotLabel → Style["KB=1*108", 12, Black]}, KB = 2 × 108;
  Iin = 150;
  ListPlot[{makeθListGenExp[] // Flatten, makeθListLinExp[] // Flatten,
    makeθListSpecExp[[1]][[1]], makeθListSpecExp[[1]][[2]]},
  Joined → True, PlotRange → {{9, 33}, {-.05, 1.05}},
  PlotStyle → {{RGBColor["#B09771"], AbsoluteThickness[3]},
    {Black, AbsoluteThickness[3]}, {RGBColor["#287DAB"], AbsoluteThickness[3]},
    {RGBColor["#287DAB"], AbsoluteThickness[3]}}, Frame → True,
  ImageSize → Medium, FrameLabel → {Style["Temperature (°C)", 12, Black],
    Style["Heterotrophic investment (θESS)", 12, Black]},
  FrameTicksStyle → Directive[Black, 12],
  PlotLabel → Style["KB=2*108", 12, Black]}, KB = 3 × 108;
  Iin = 150;
  ListPlot[{makeθListGenExp[] // Flatten, makeθListLinExp[] // Flatten,
    makeθListSpecExp[[1]][[1]], makeθListSpecExp[[1]][[2]]},
  Joined → True, PlotRange → {{9, 33}, {-.05, 1.05}},
  PlotStyle → {{RGBColor["#B09771"], AbsoluteThickness[3]},
    {Black, AbsoluteThickness[3]}, {RGBColor["#287DAB"], AbsoluteThickness[3]},
    {RGBColor["#287DAB"], AbsoluteThickness[3]}}, Frame → True,
  ImageSize → Medium, FrameLabel → {Style["Temperature (°C)", 12, Black],
    Style["Heterotrophic investment (θESS)", 12, Black]},
  PlotLegends → {"generalist", "linear", "specialist"},
  FrameTicksStyle → Directive[Black, 12], PlotLabel → Style["KB=3*108", 12, Black]}}
```

... **Table**: Non-list iterator  $\theta_{ESS}spec0$  at position 2 does not evaluate to a real numeric value.

... **Table**: Non-list iterator  $\theta_{ESS}spec0$  at position 2 does not evaluate to a real numeric value.

... **Table**: Non-list iterator  $\theta_{ESS}spec0$  at position 2 does not evaluate to a real numeric value.

... **General**: Further output of Table::nliter will be suppressed during this calculation.



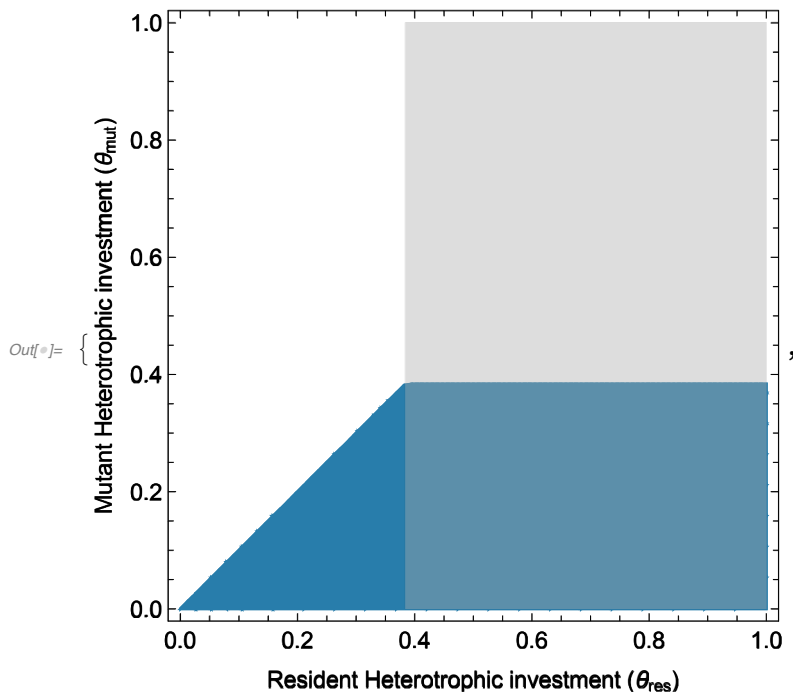
# Pairwise invasibility plots

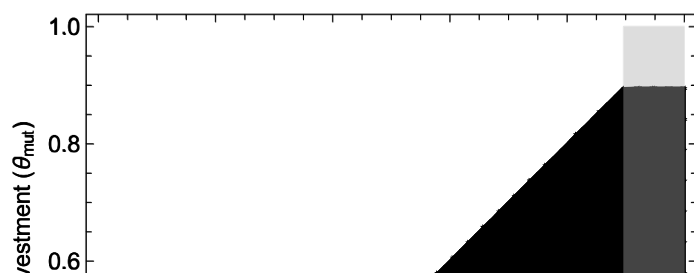
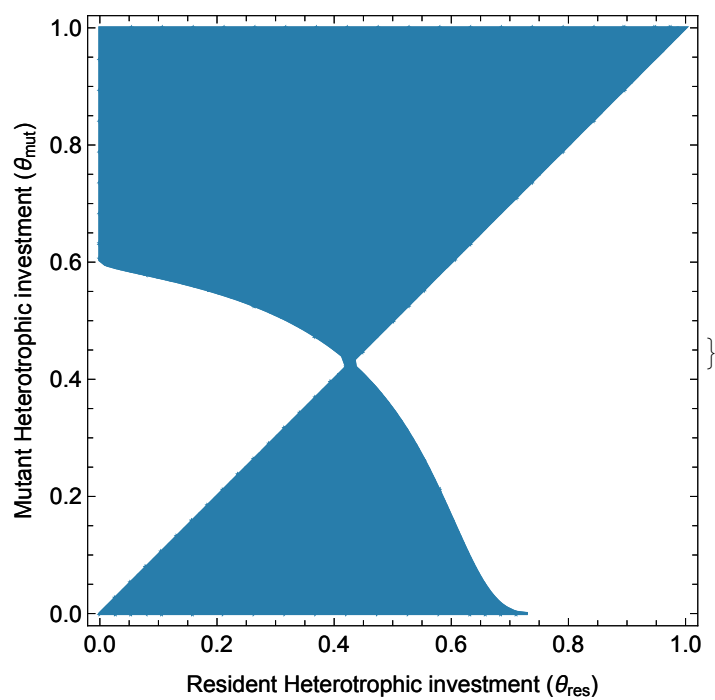
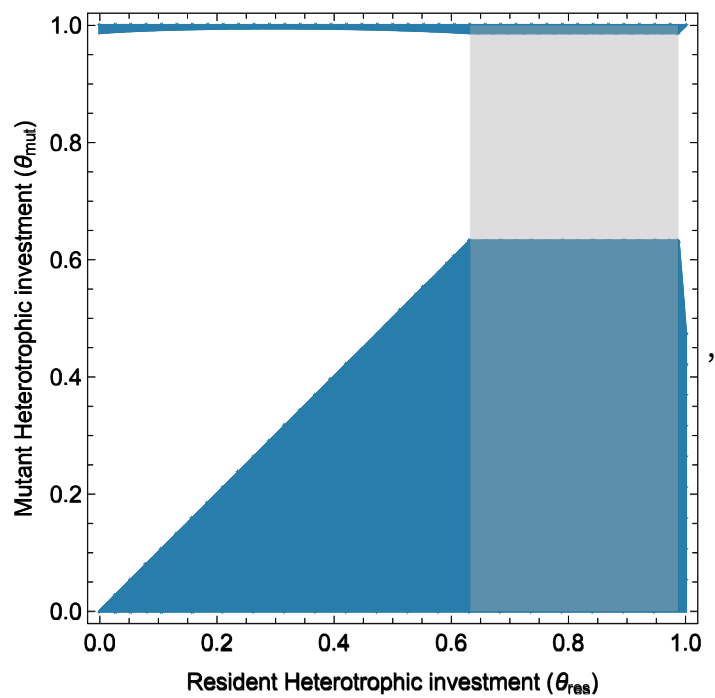
In[ ]:=

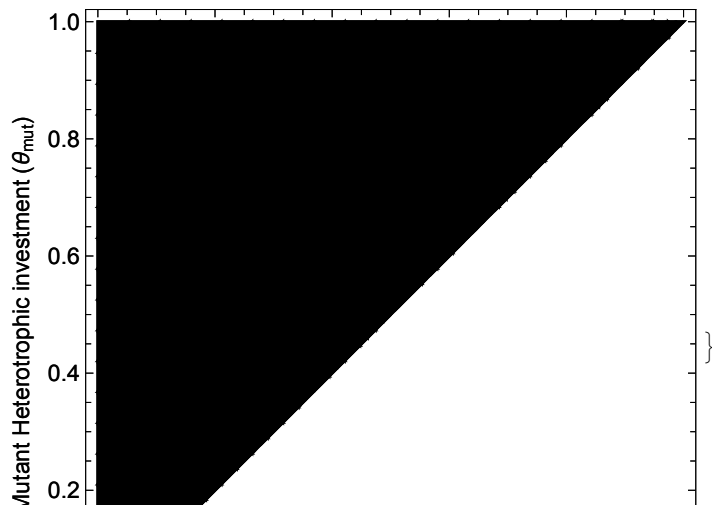
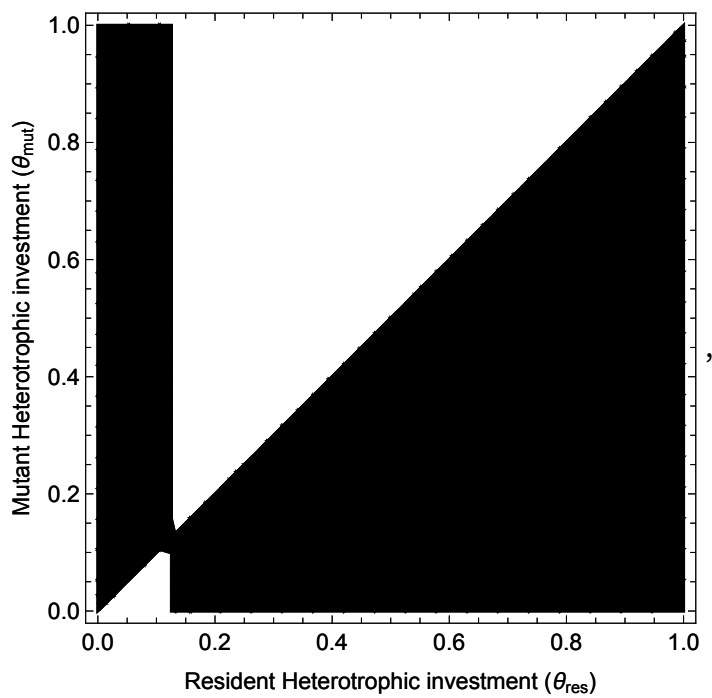
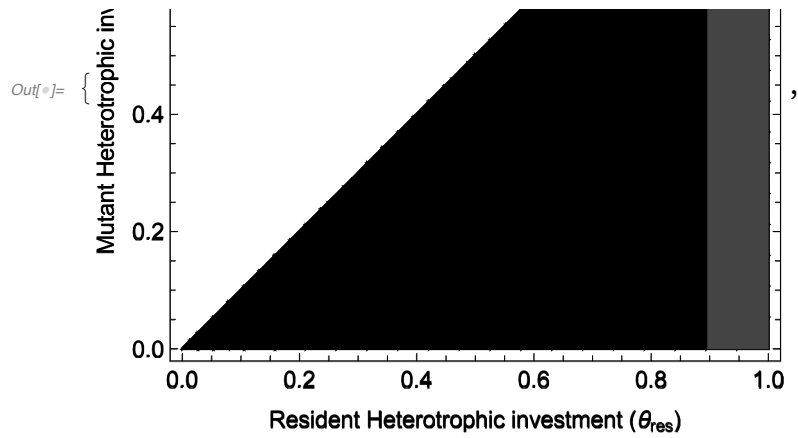
$K_B = 1 \times 10^8$ ,  $I_{in} = 100$

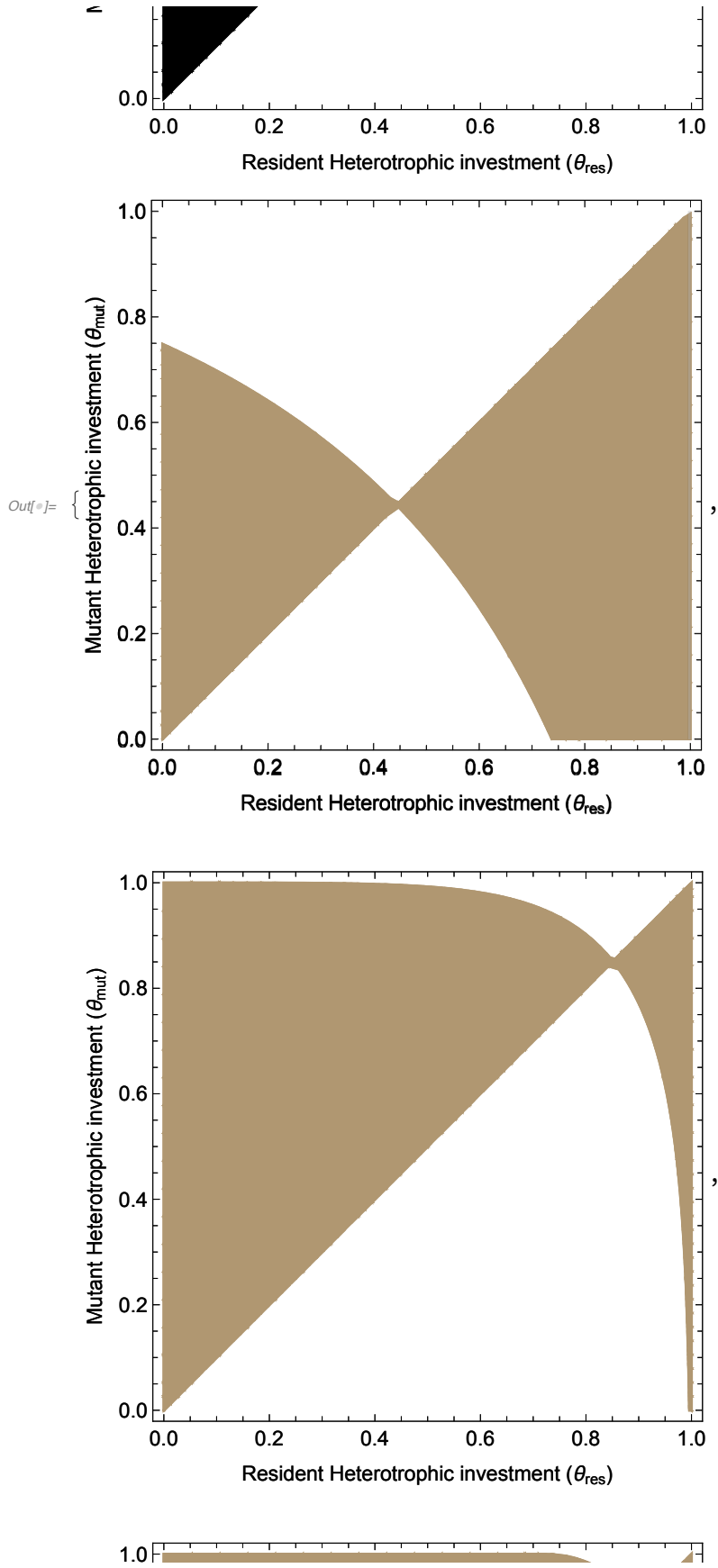
In[ ]:=  $K_B = 1 \times 10^8$ ;  $I_{in} = 100$ ;

```
Quiet[List[
  Overlay[{MakePIP[-1, T0, RGBColor["#287DAB"]], MakePIPNV[-1, T0, Lighter[Gray]]}],
  Overlay[{MakePIP[-1, T0 + 5, RGBColor["#287DAB"]],
    MakePIPNV[-1, T0 + 5, Lighter[Gray]]}],
  Overlay[{MakePIP[-1, T0 + 10, RGBColor["#287DAB"]],
    MakePIPNV[-1, T0 + 10, Lighter[Gray]]}]]],
Quiet[List[Overlay[{MakePIP[0, T0, Black], MakePIPNV[0, T0, Lighter[Gray]]}],
  Overlay[{MakePIP[0, T0 + 5, Black], MakePIPNV[0, T0 + 5, Lighter[Gray]]}], Overlay[
    {MakePIP[0, T0 + 10, Black], MakePIPNV[0, T0 + 10, Lighter[Gray]]}]]] (*linear*)
Quiet[List[Overlay[{MakePIP[1, T0, RGBColor["#B09771"]],
  MakePIPNV[1, T0, Lighter[Gray]]}], Overlay[
    {MakePIP[1, T0 + 5, RGBColor["#B09771"]], MakePIPNV[1, T0 + 5, Lighter[Gray]]}],
  Overlay[{MakePIP[1, T0 + 10, RGBColor["#B09771"]],
    MakePIPNV[1, T0 + 10, Lighter[Gray]]}]]]
(*generalist*)
```

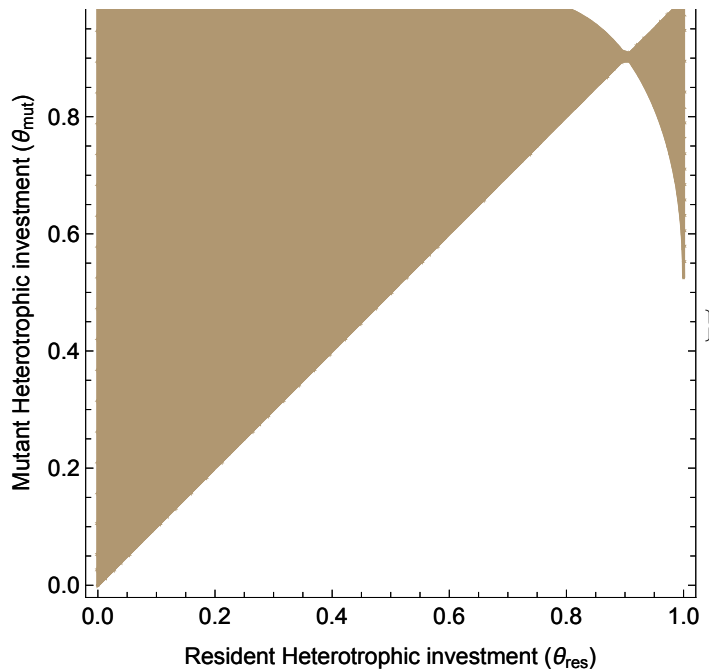












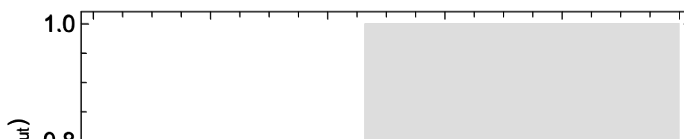
In[ ]:=

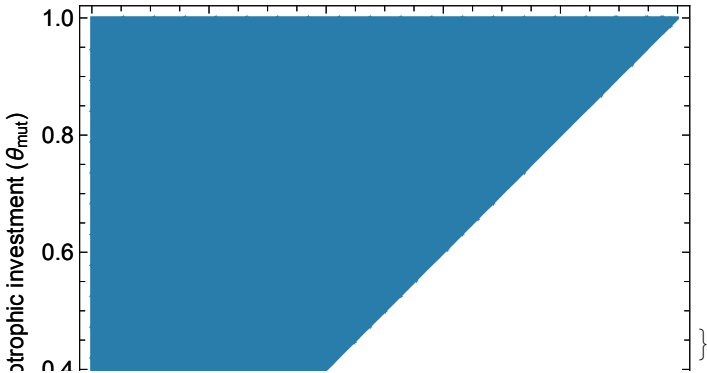
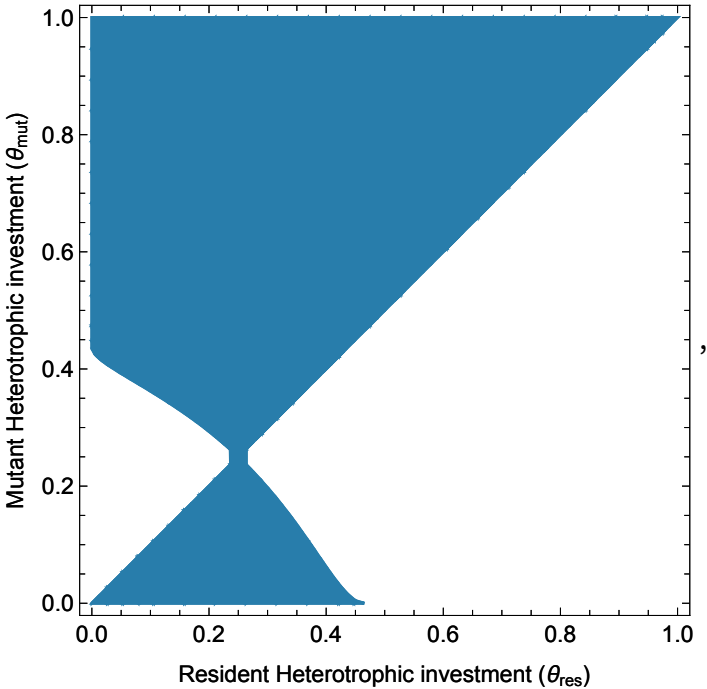
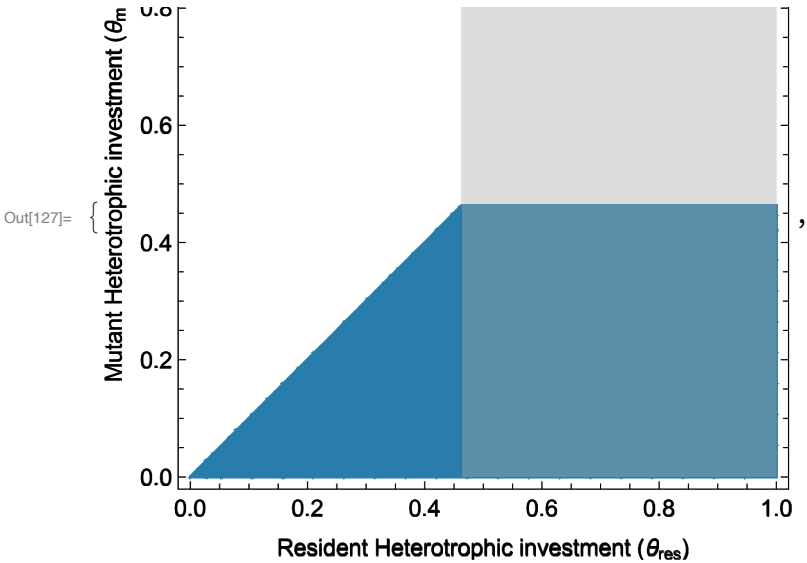
In[ ]:=

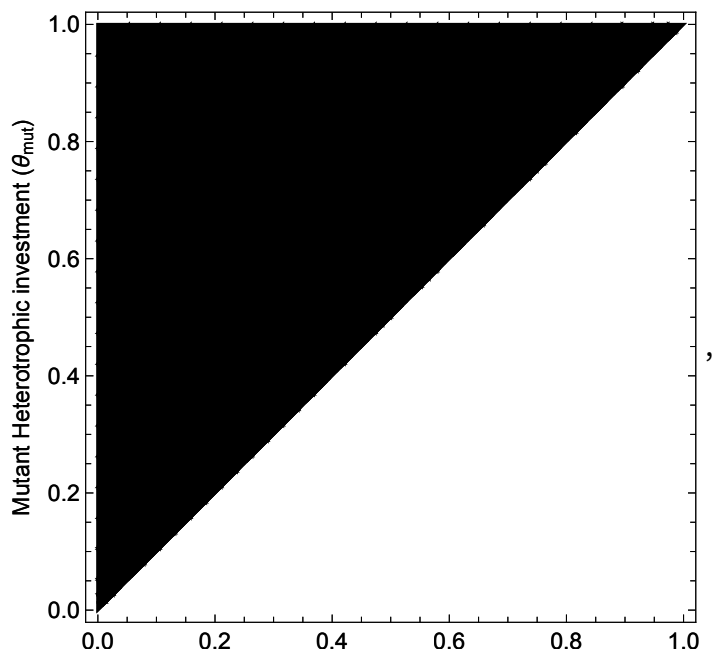
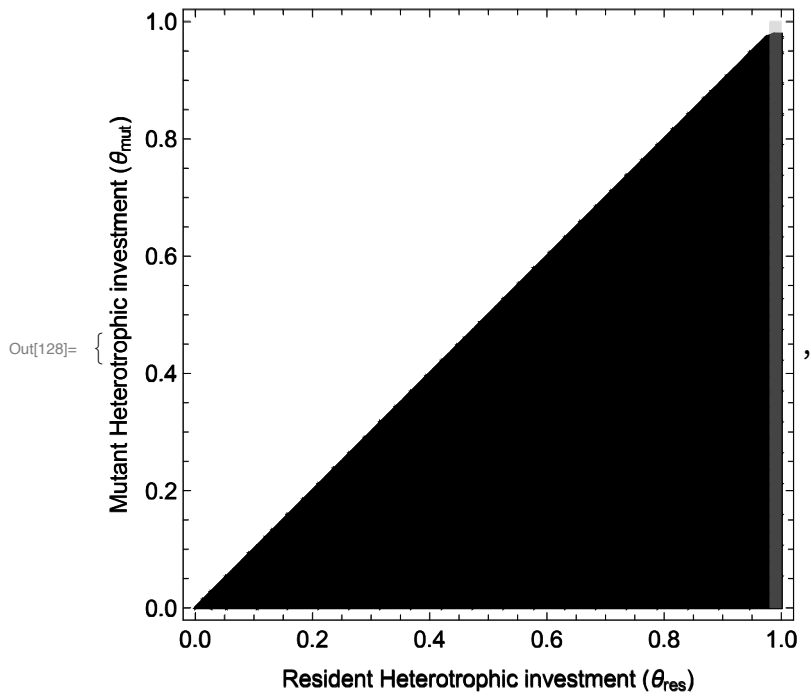
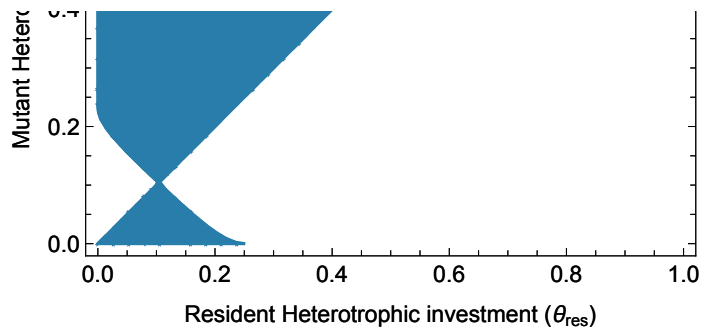
$$K_B = 2 \times 10^8, I_{in} = 100$$

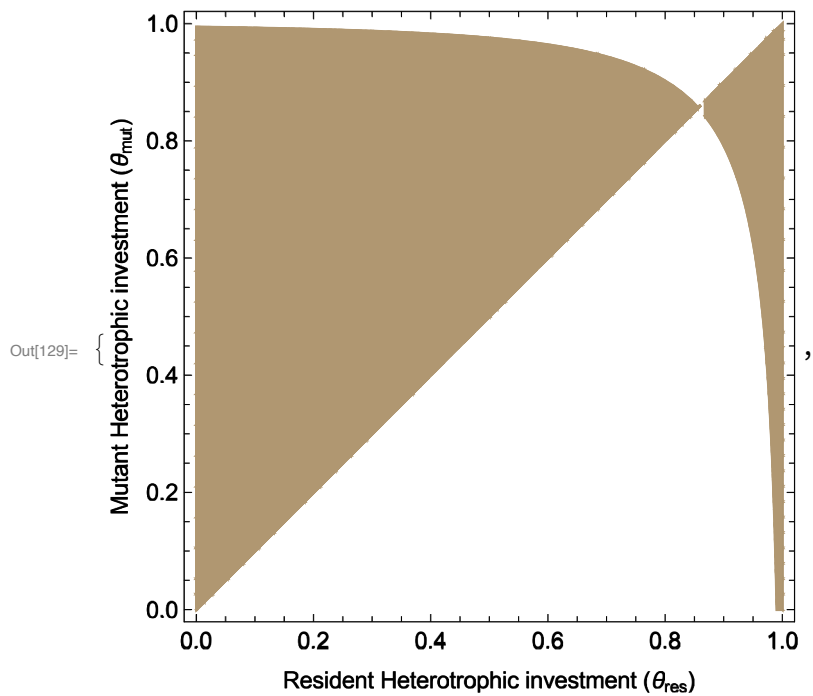
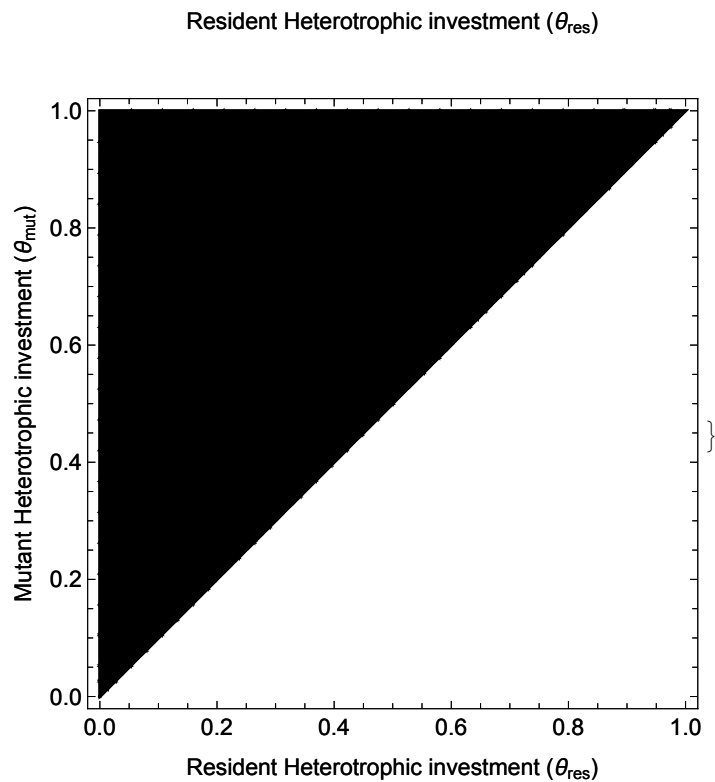
In[126]:=  $K_B = 2 \times 10^8$ ;  $I_{in} = 100$ ;

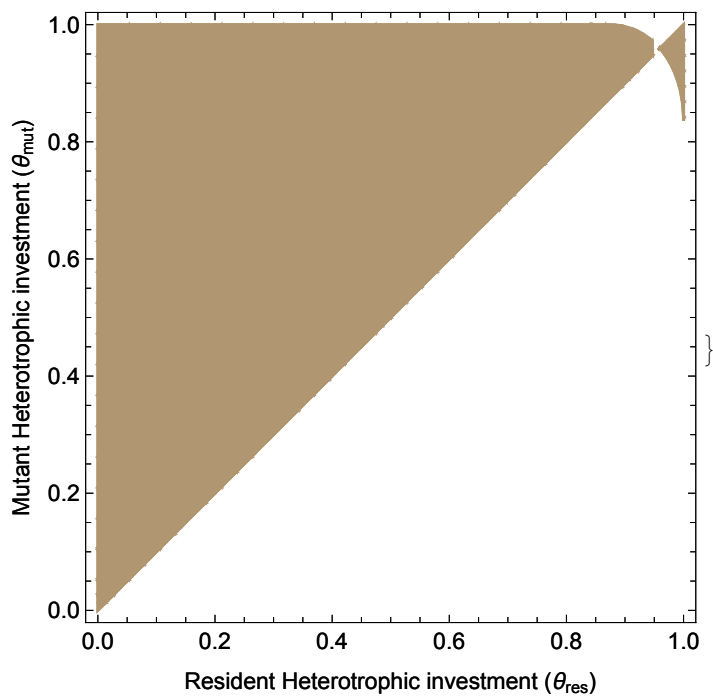
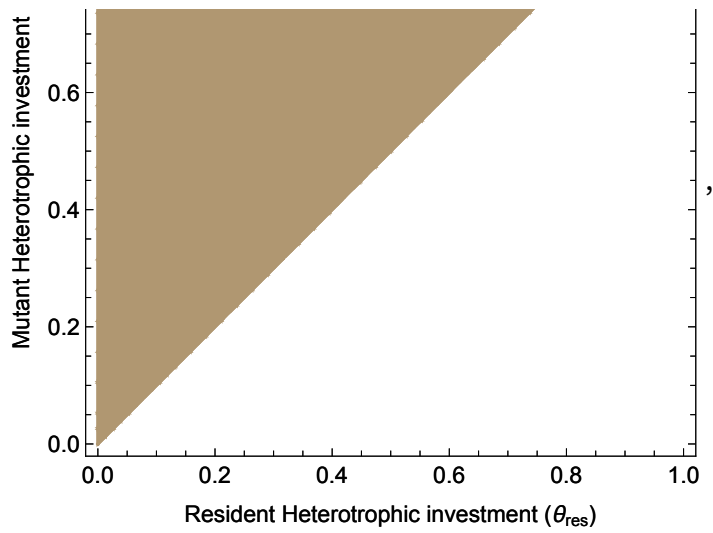
```
Quiet[List[
  Overlay[{MakePIP[-1, T0, RGBColor["#287DAB"]], MakePIPNV[-1, T0, Lighter[Gray]]}],
  Overlay[{MakePIP[-1, T0 + 5, RGBColor["#287DAB"]],
    MakePIPNV[-1, T0 + 5, Lighter[Gray]]}],
  Overlay[{MakePIP[-1, T0 + 10, RGBColor["#287DAB"]],
    MakePIPNV[-1, T0 + 10, Lighter[Gray]]}]]]
Quiet[List[Overlay[{MakePIP[0, T0, Black], MakePIPNV[0, T0, Lighter[Gray]]}],
  Overlay[{MakePIP[0, T0 + 5, Black], MakePIPNV[0, T0 + 5, Lighter[Gray]]}], Overlay[
    {MakePIP[0, T0 + 10, Black], MakePIPNV[0, T0 + 10, Lighter[Gray]]}]]] (*linear*)
Quiet[List[Overlay[{MakePIP[1, T0, RGBColor["#B09771"]],
  MakePIPNV[1, T0, Lighter[Gray]]}], Overlay[
    {MakePIP[1, T0 + 5, RGBColor["#B09771"]], MakePIPNV[1, T0 + 5, Lighter[Gray]]}],
  Overlay[{MakePIP[1, T0 + 10, RGBColor["#B09771"]],
    MakePIPNV[1, T0 + 10, Lighter[Gray]]}]]]
(*generalist*)
```







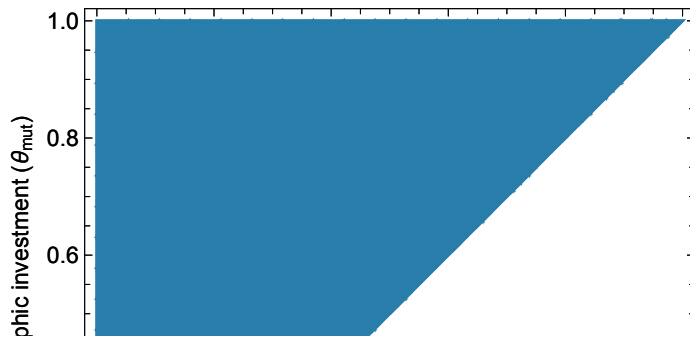
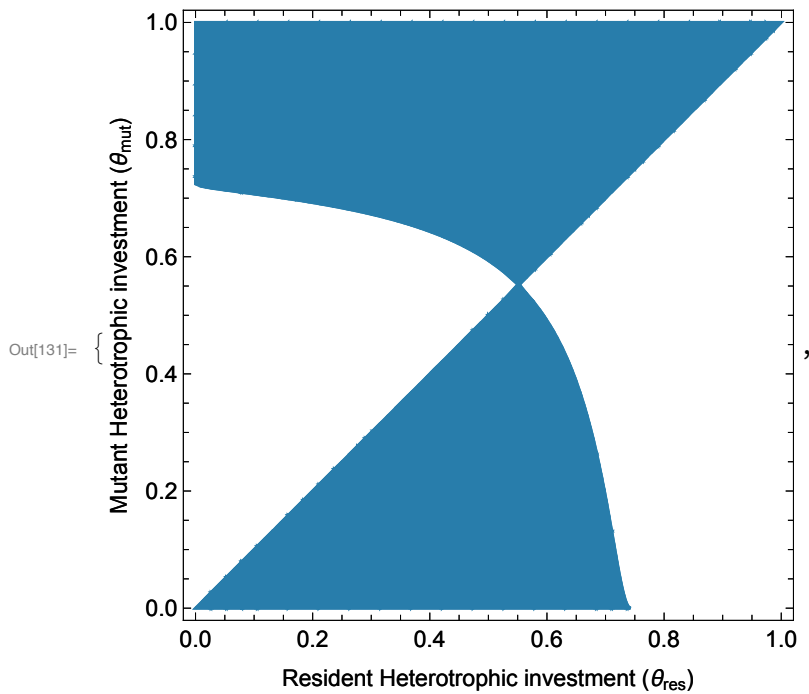


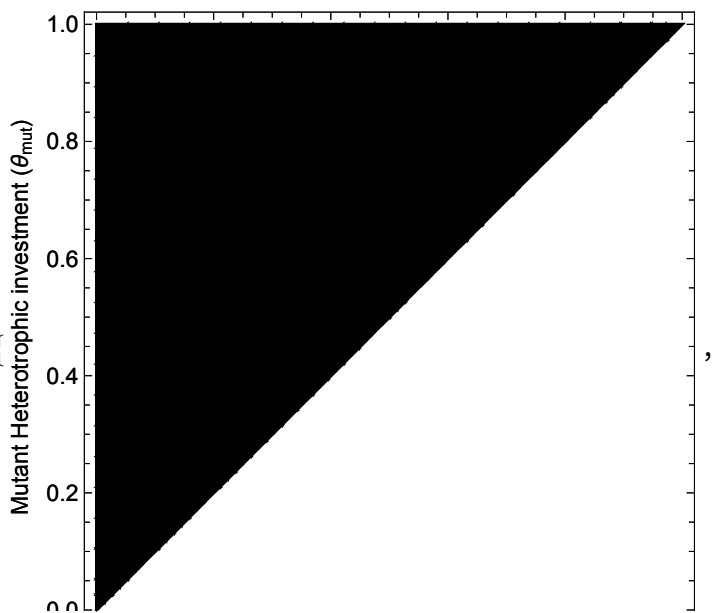
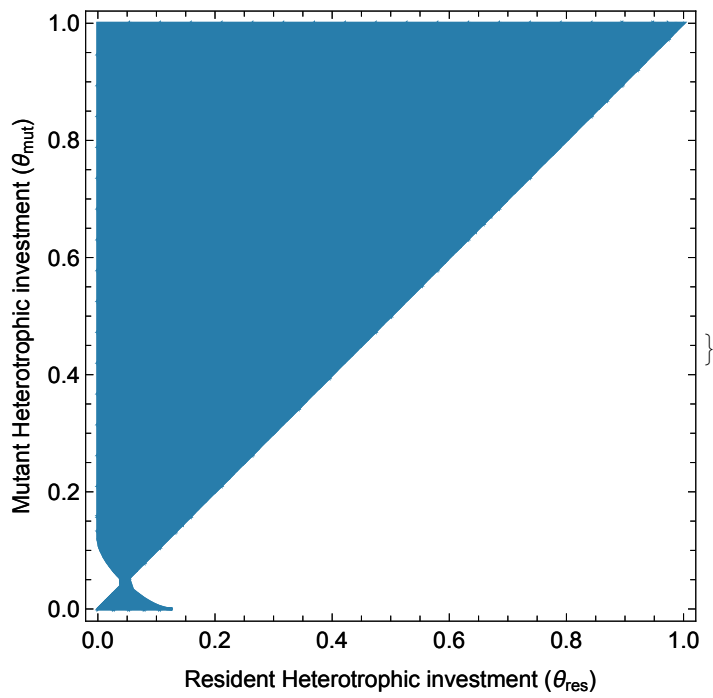
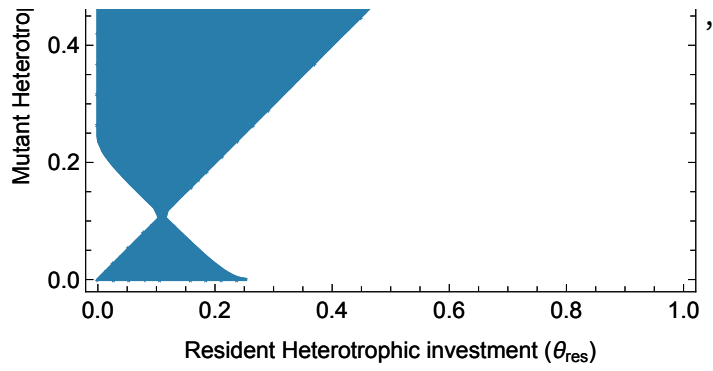


$$K_B = 3 \cdot 10^8, I_{in} = 100$$

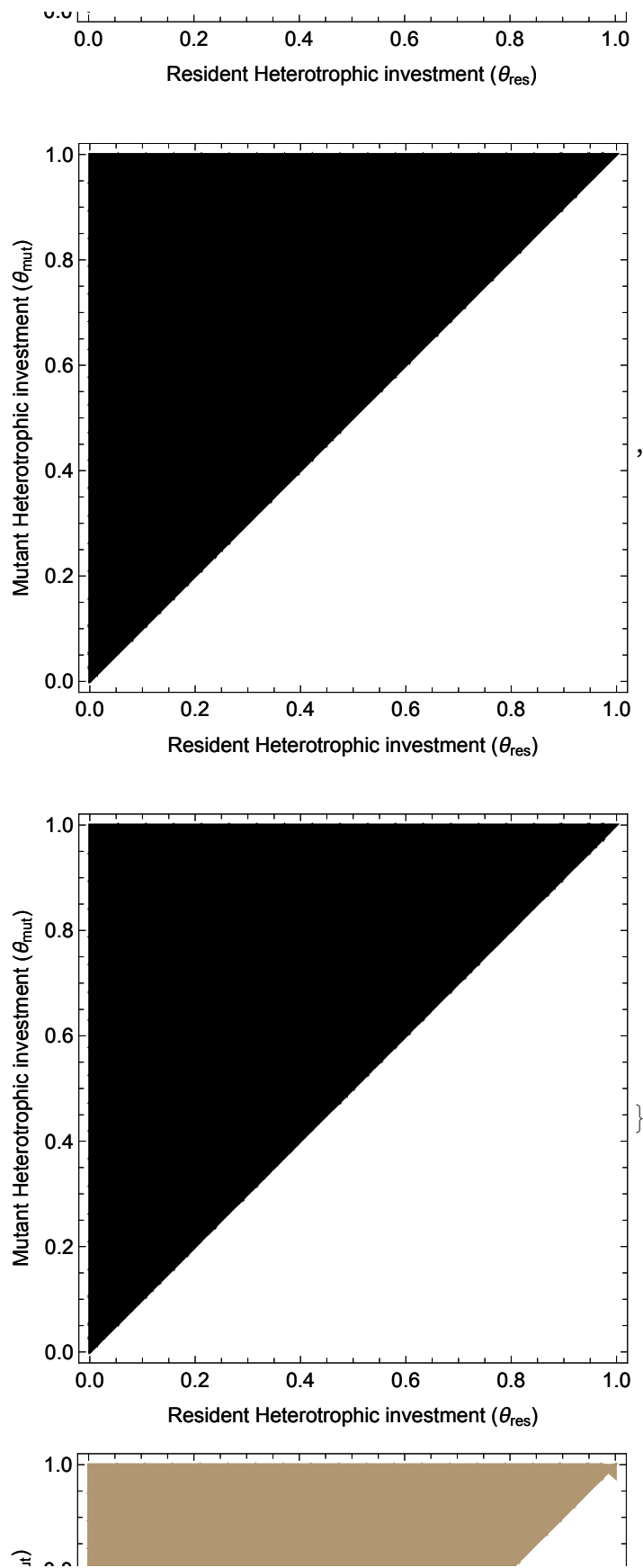
```
In[130]:= KB = 3 × 108; Iin = 100;
```

```
Quiet[List[
  Overlay[{MakePIP[-1, T0, RGBColor["#287DAB"]], MakePIPNV[-1, T0, Lighter[Gray]]}],
  Overlay[{MakePIP[-1, T0 + 5, RGBColor["#287DAB"]],
    MakePIPNV[-1, T0 + 5, Lighter[Gray]]}],
  Overlay[{MakePIP[-1, T0 + 10, RGBColor["#287DAB"]],
    MakePIPNV[-1, T0 + 10, Lighter[Gray]]}]]],
Quiet[List[Overlay[{MakePIP[0, T0, Black], MakePIPNV[0, T0, Lighter[Gray]]}],
  Overlay[{MakePIP[0, T0 + 5, Black], MakePIPNV[0, T0 + 5, Lighter[Gray]]}], Overlay[
  {MakePIP[0, T0 + 10, Black], MakePIPNV[0, T0 + 10, Lighter[Gray]]}]]] (*linear*)
Quiet[List[Overlay[{MakePIP[1, T0, RGBColor["#B09771"]],
  MakePIPNV[1, T0, Lighter[Gray]]}], Overlay[
  {MakePIP[1, T0 + 5, RGBColor["#B09771"]], MakePIPNV[1, T0 + 5, Lighter[Gray]]}],
  Overlay[{MakePIP[1, T0 + 10, RGBColor["#B09771"]],
    MakePIPNV[1, T0 + 10, Lighter[Gray]]}]]]
(*generalist*)
```

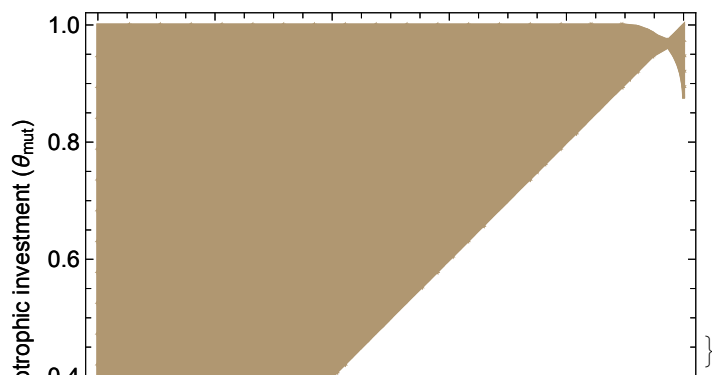
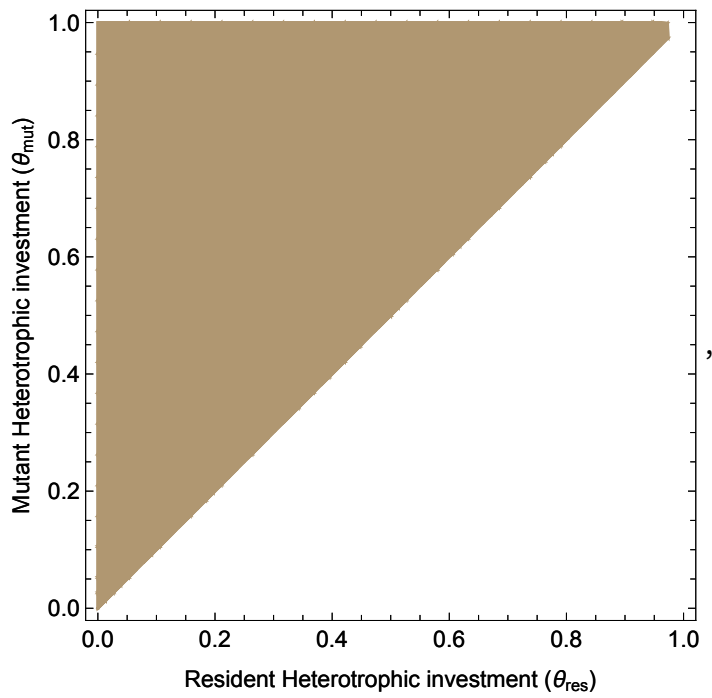
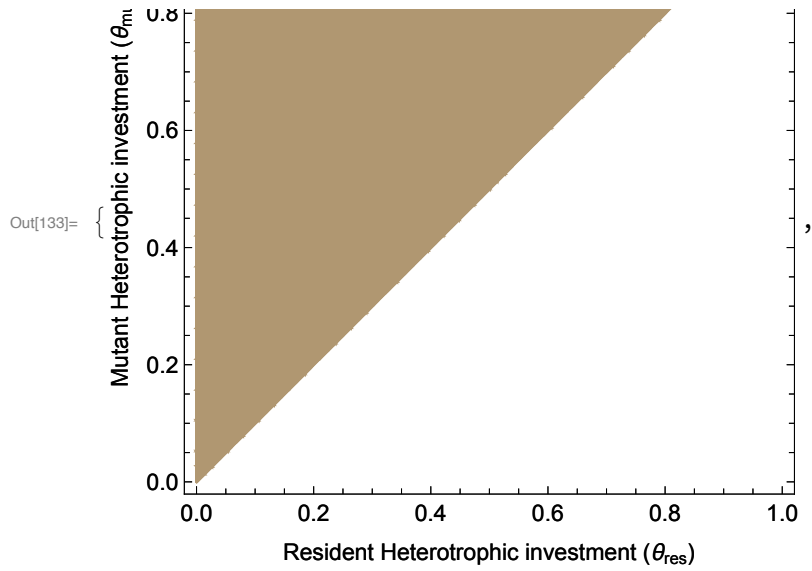


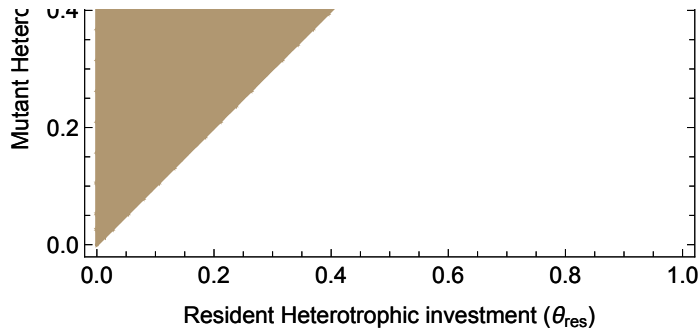


Out[132]=





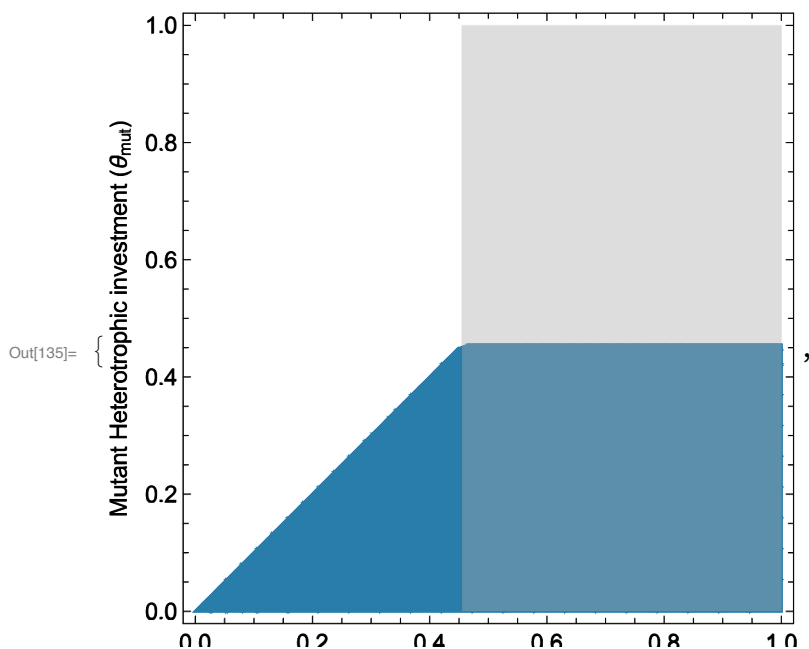


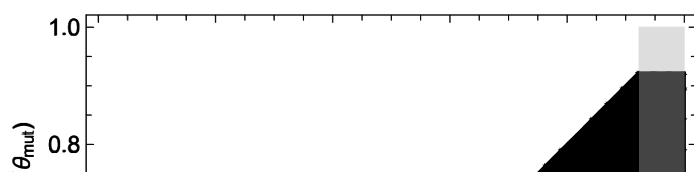
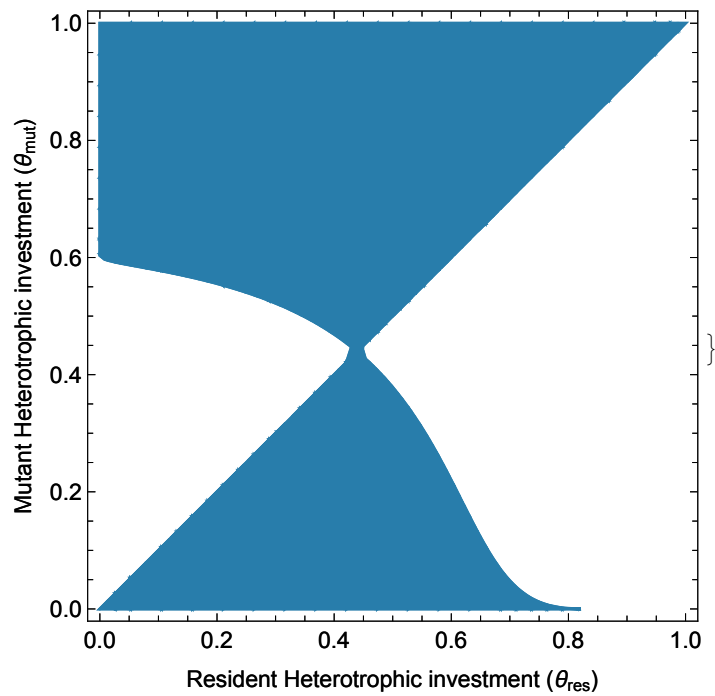
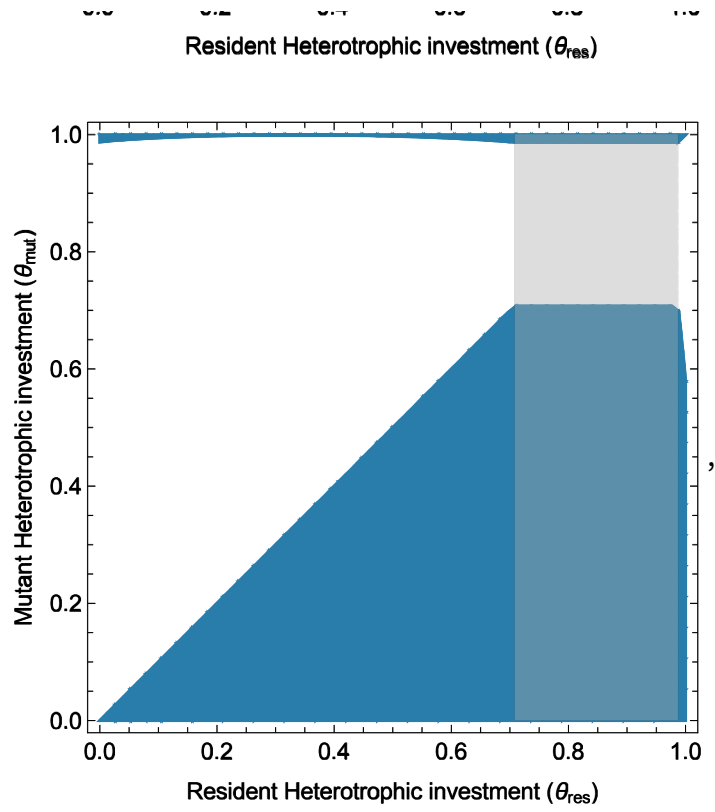


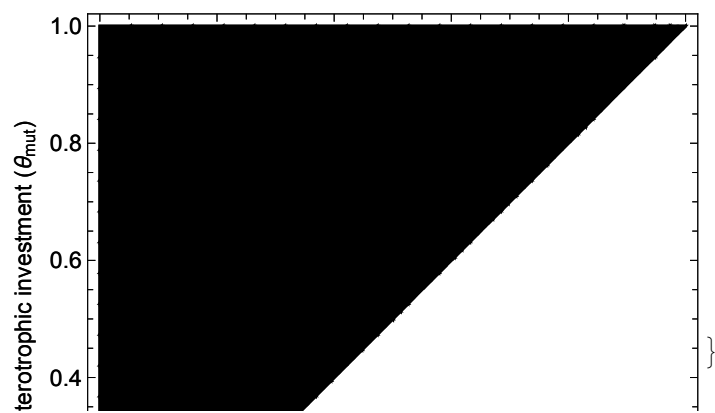
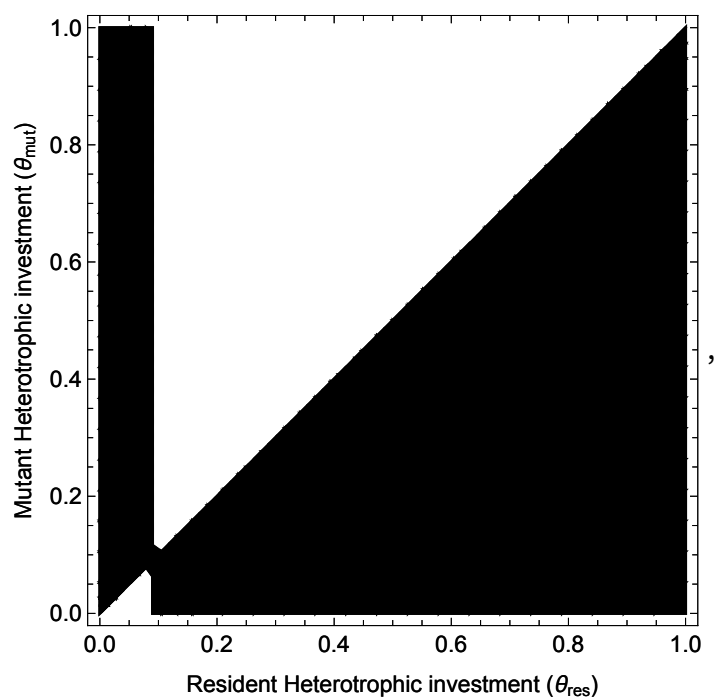
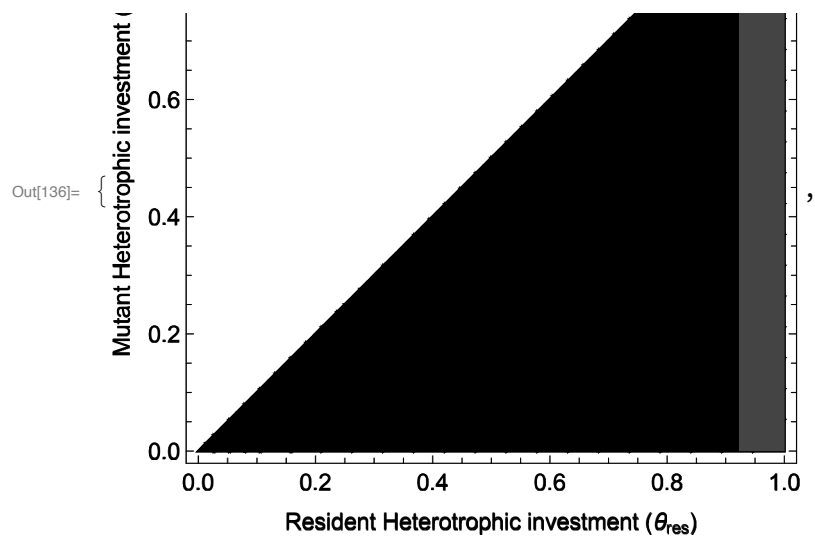
$$K_B = 1 \times 10^8, I_{in} = 150$$

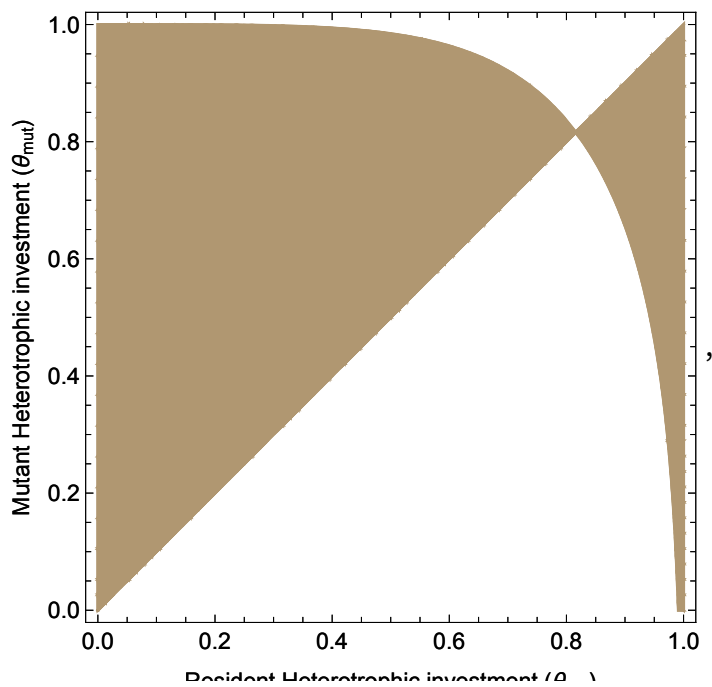
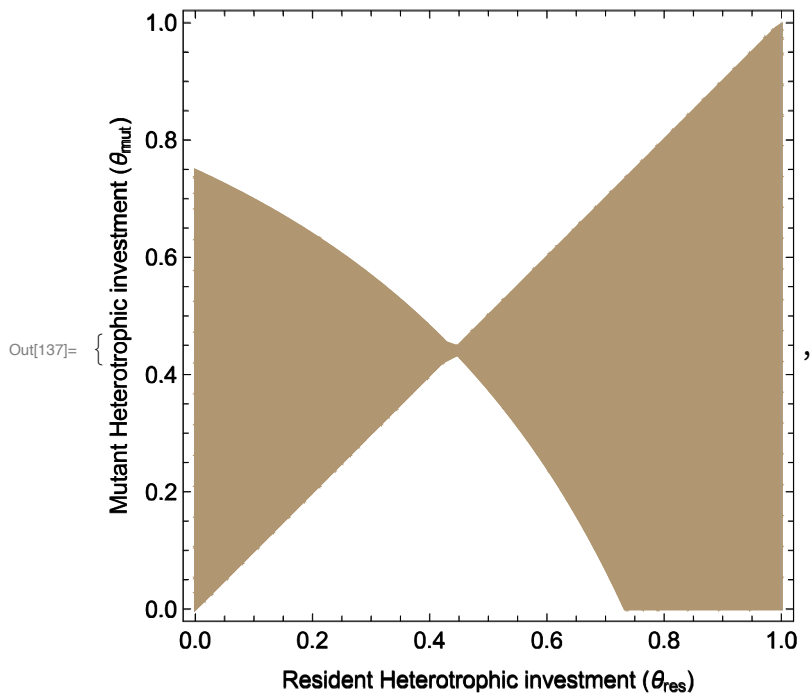
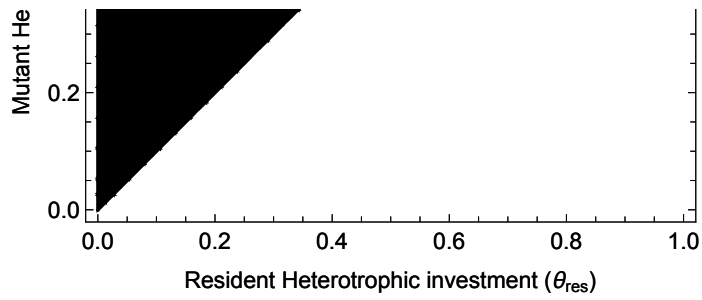
In[134]:=  $K_B = 1 \times 10^8$ ;  $I_{in} = 150$ ;

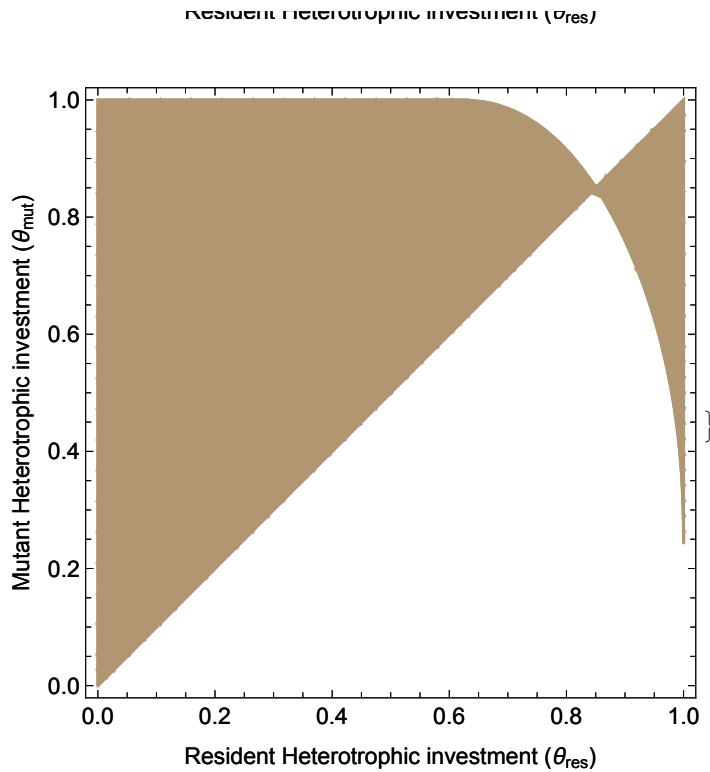
```
Quiet[List[
  Overlay[{MakePIP[-1, T0, RGBColor["#287DAB"]], MakePIPNV[-1, T0, Lighter[Gray]]}],
  Overlay[{MakePIP[-1, T0 + 5, RGBColor["#287DAB"]],
    MakePIPNV[-1, T0 + 5, Lighter[Gray]]}],
  Overlay[{MakePIP[-1, T0 + 10, RGBColor["#287DAB"]],
    MakePIPNV[-1, T0 + 10, Lighter[Gray]]}]]],
Quiet[List[Overlay[{MakePIP[0, T0, Black], MakePIPNV[0, T0, Lighter[Gray]]}],
  Overlay[{MakePIP[0, T0 + 5, Black], MakePIPNV[0, T0 + 5, Lighter[Gray]]}], Overlay[
    {MakePIP[0, T0 + 10, Black], MakePIPNV[0, T0 + 10, Lighter[Gray]]}]]] (*linear*)
Quiet[List[Overlay[{MakePIP[1, T0, RGBColor["#B09771"]],
  MakePIPNV[1, T0, Lighter[Gray]]}], Overlay[
    {MakePIP[1, T0 + 5, RGBColor["#B09771"]], MakePIPNV[1, T0 + 5, Lighter[Gray]]}],
  Overlay[{MakePIP[1, T0 + 10, RGBColor["#B09771"]],
    MakePIPNV[1, T0 + 10, Lighter[Gray]]}]]]
(*generalist*)
```







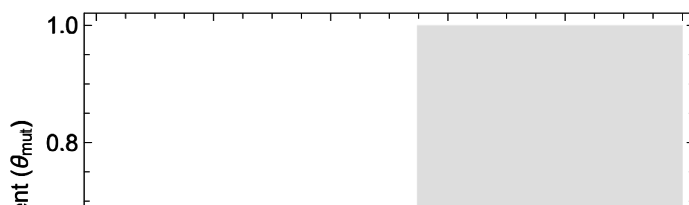


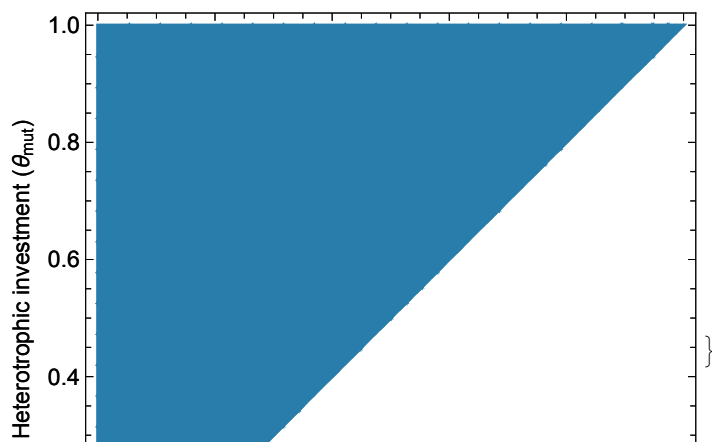
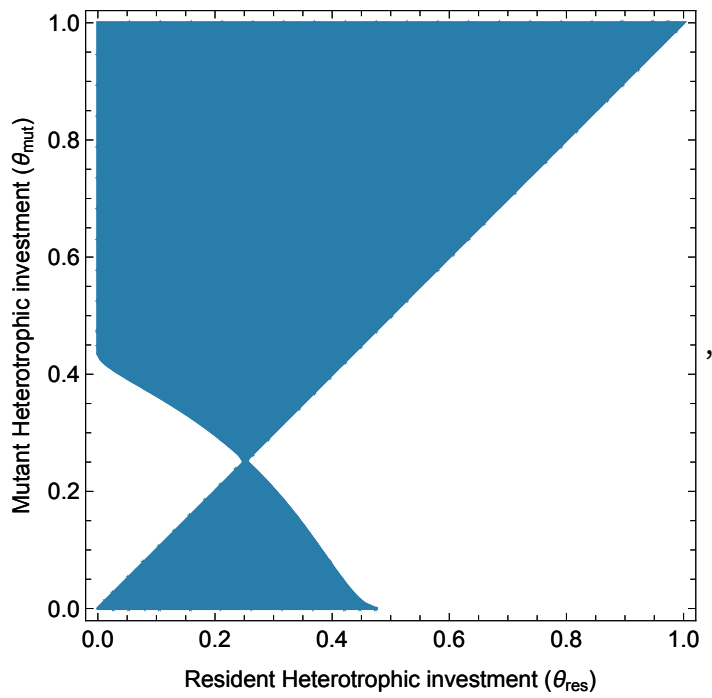
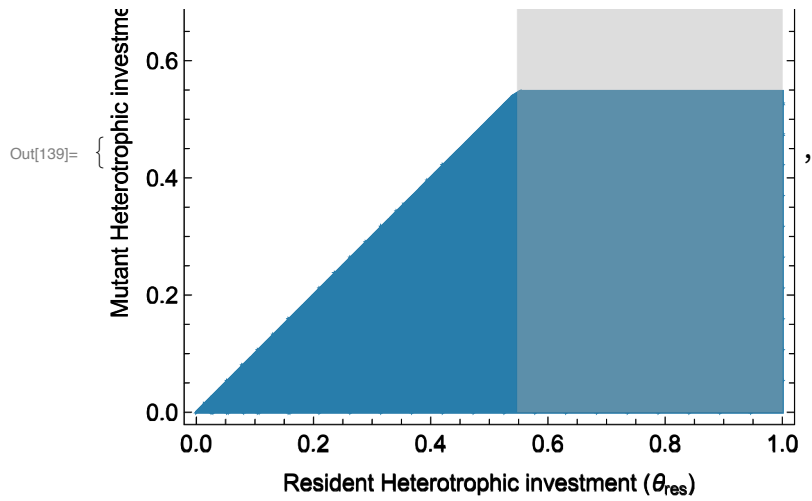


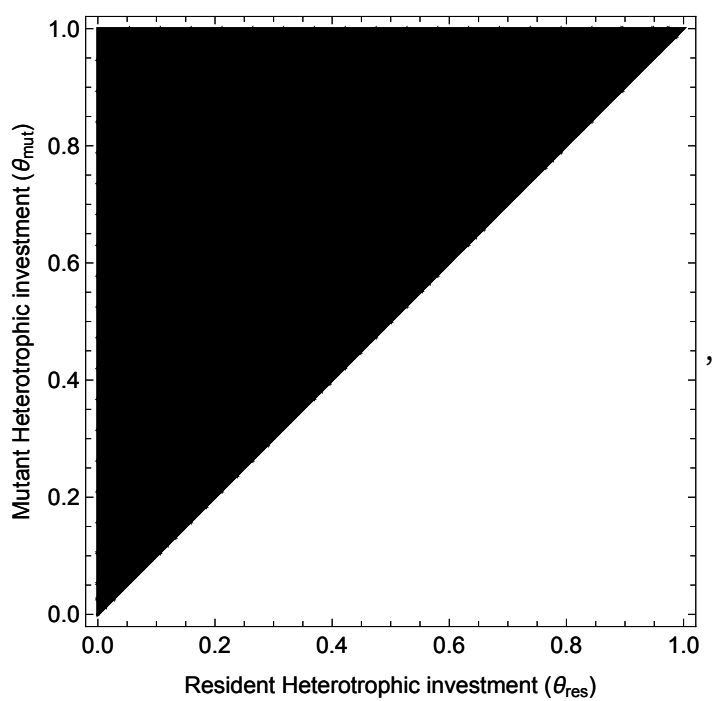
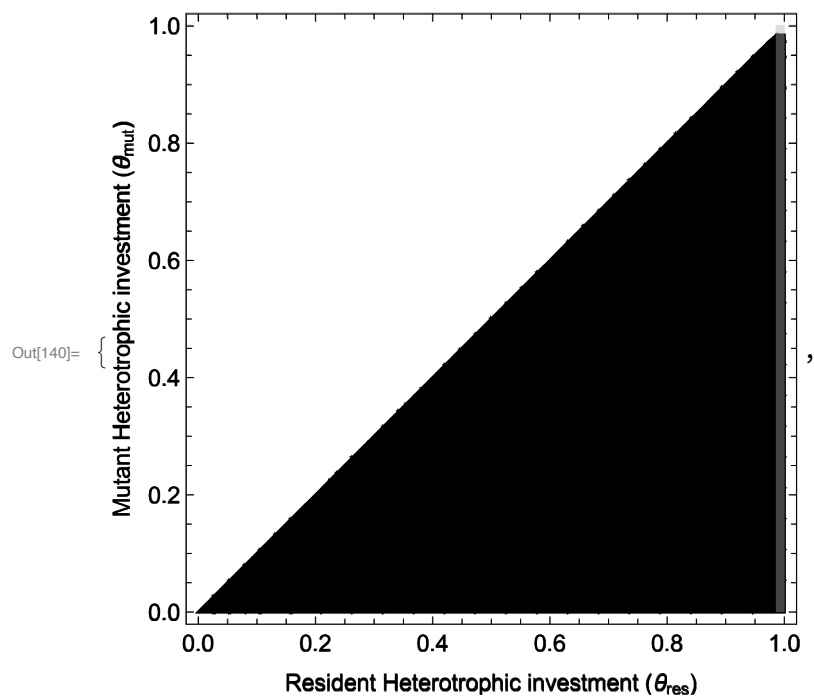
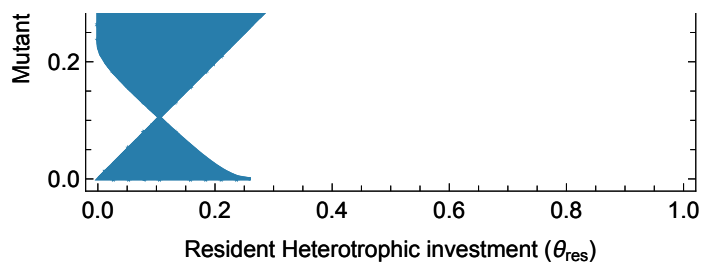
$$K_B = 2 \times 10^8, I_{in} = 150$$

```
In[138]:= KB = 2 × 108; Iin = 150;
```

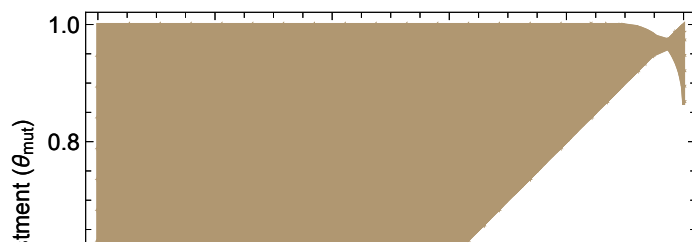
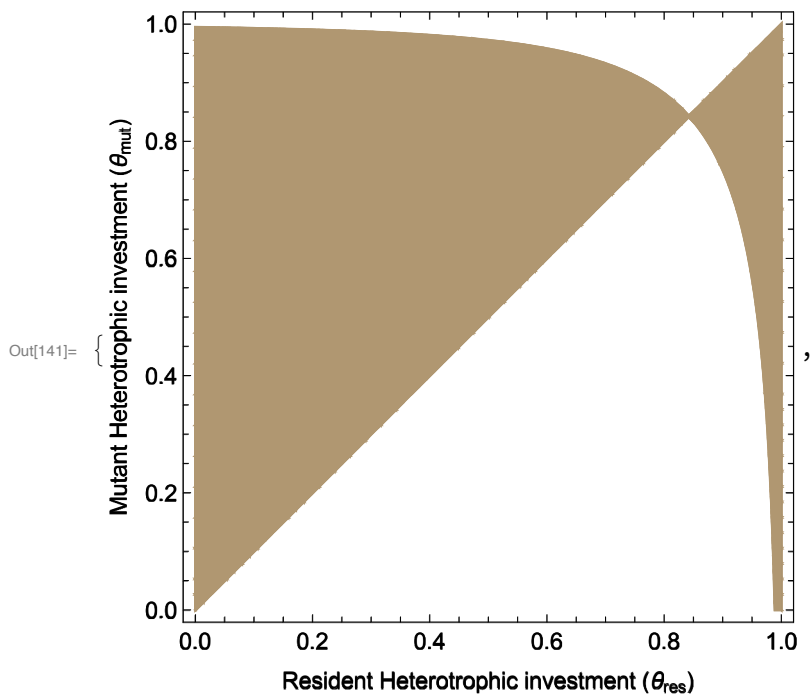
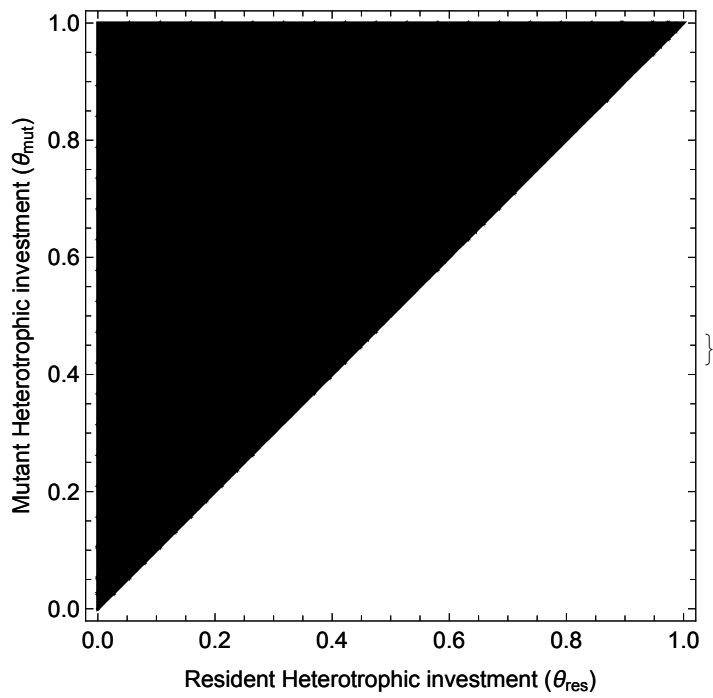
```
Quiet[List[
  Overlay[{MakePIP[-1, T0, RGBColor["#287DAB"]], MakePIP[NV[-1, T0, Lighter[Gray]]}],
  Overlay[{MakePIP[-1, T0 + 5, RGBColor["#287DAB"]],
    MakePIP[NV[-1, T0 + 5, Lighter[Gray]]}],
  Overlay[{MakePIP[-1, T0 + 10, RGBColor["#287DAB"]],
    MakePIP[NV[-1, T0 + 10, Lighter[Gray]]}]]]
Quiet[List[Overlay[{MakePIP[0, T0, Black], MakePIP[NV[0, T0, Lighter[Gray]]}],
  Overlay[{MakePIP[0, T0 + 5, Black], MakePIP[NV[0, T0 + 5, Lighter[Gray]]}], Overlay[
    {MakePIP[0, T0 + 10, Black], MakePIP[NV[0, T0 + 10, Lighter[Gray]]}]]] (*linear*)
Quiet[List[Overlay[{MakePIP[1, T0, RGBColor["#B09771"]],
  MakePIP[NV[1, T0, Lighter[Gray]]}], Overlay[
    {MakePIP[1, T0 + 5, RGBColor["#B09771"]], MakePIP[NV[1, T0 + 5, Lighter[Gray]]}],
  Overlay[{MakePIP[1, T0 + 10, RGBColor["#B09771"]],
    MakePIP[NV[1, T0 + 10, Lighter[Gray]]}]]]
(*generalist*)
```

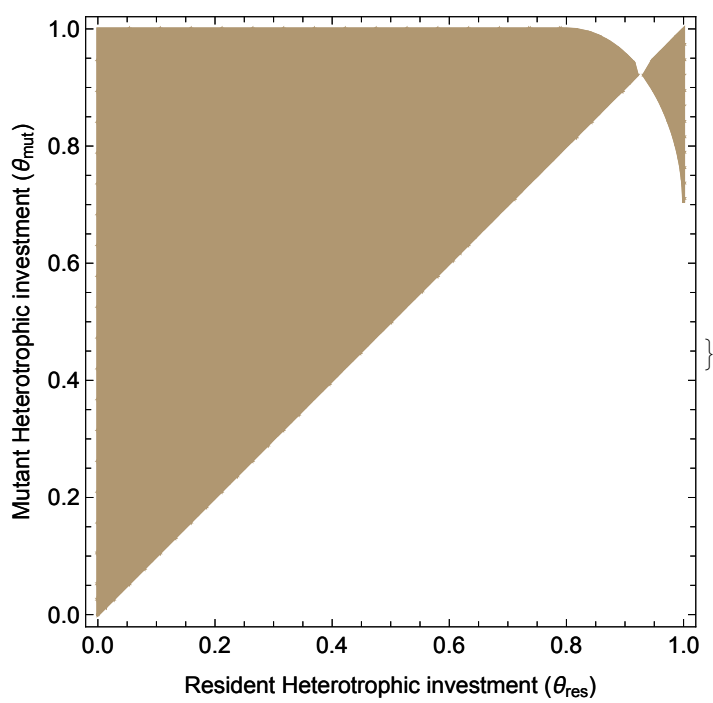
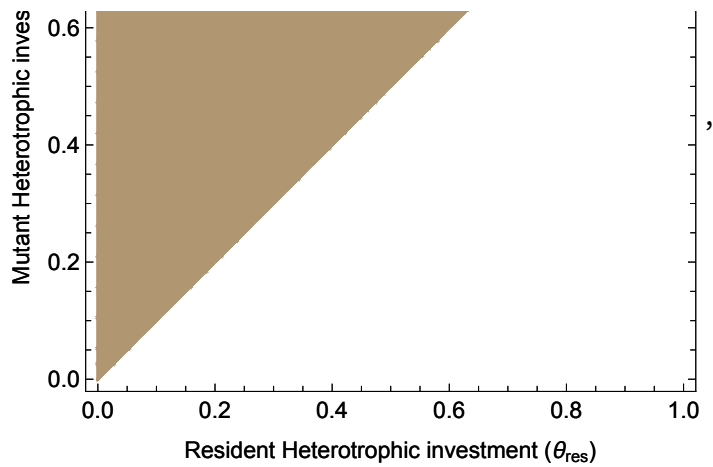








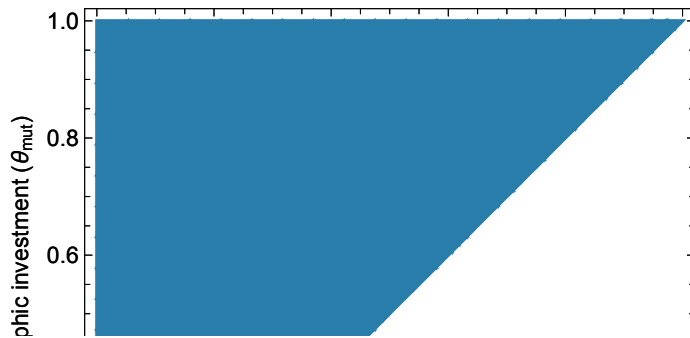
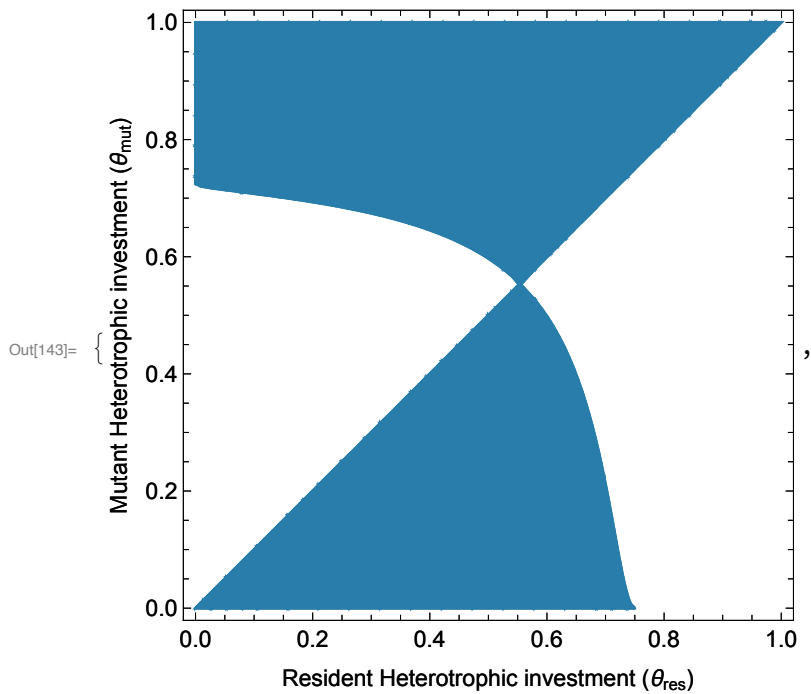


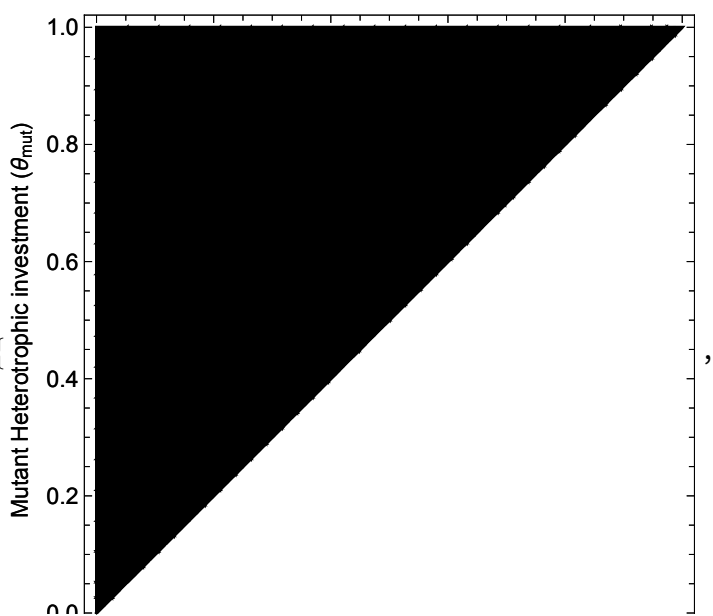
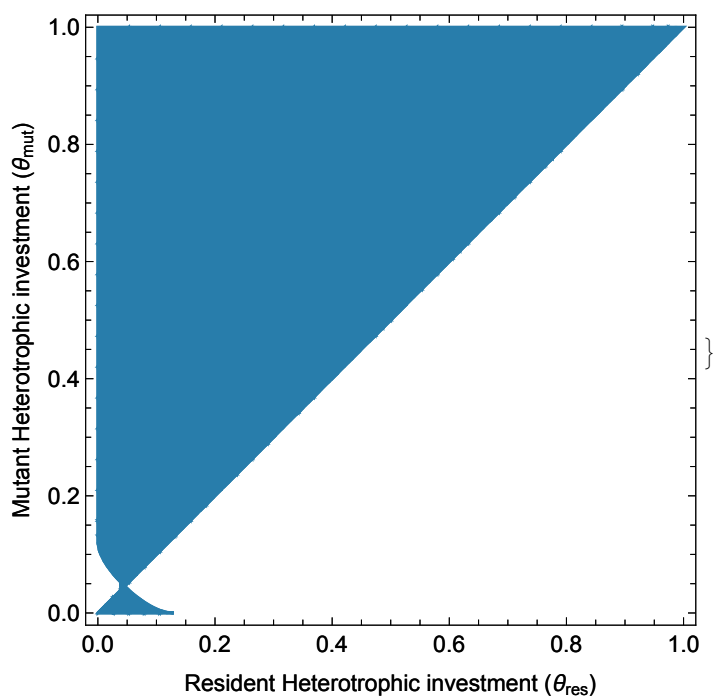
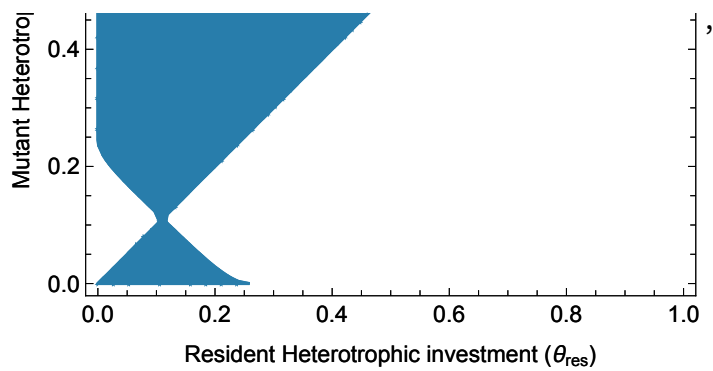


$$K_B = 3 \cdot 10^8, I_{\text{in}} = 150$$

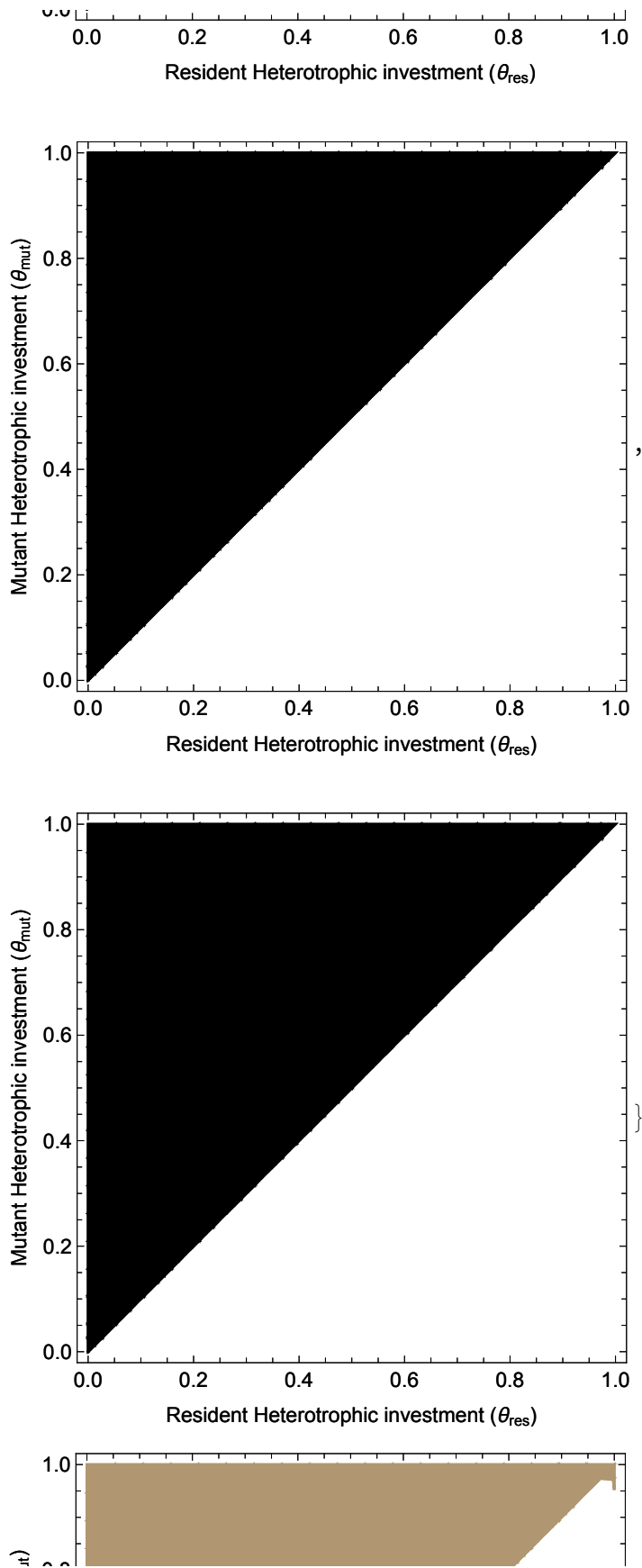
```
In[142]:= KB = 3 × 108; Iin = 150;
```

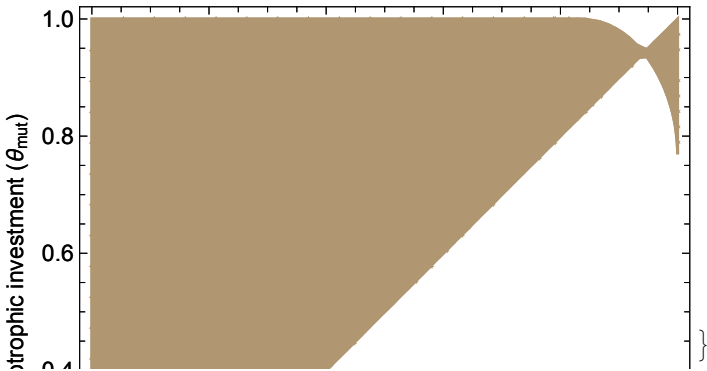
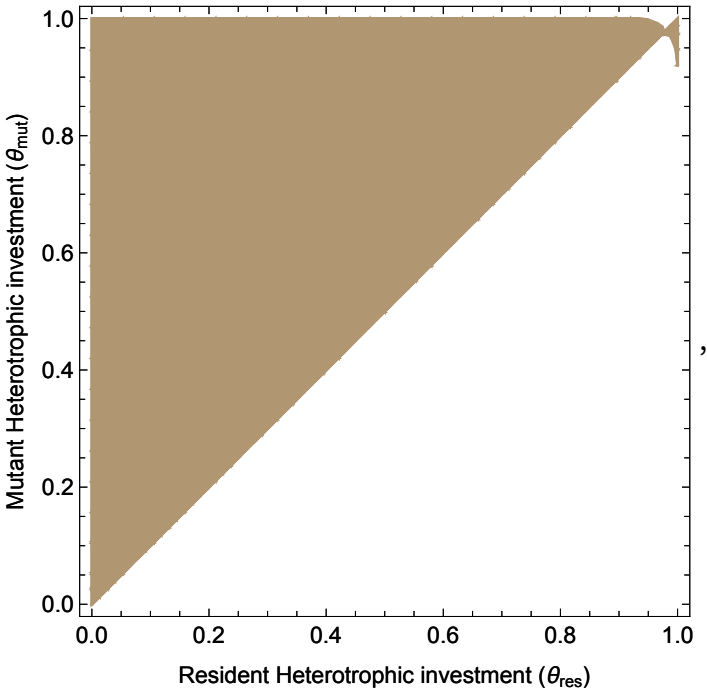
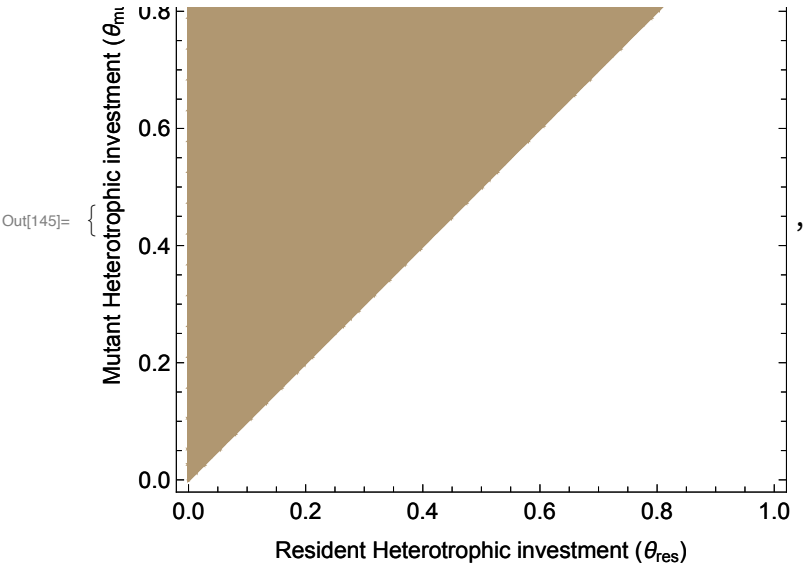
```
Quiet[List[
  Overlay[{MakePIP[-1, T0, RGBColor["#287DAB"]], MakePIPNV[-1, T0, Lighter[Gray]]}],
  Overlay[{MakePIP[-1, T0 + 5, RGBColor["#287DAB"]],
    MakePIPNV[-1, T0 + 5, Lighter[Gray]]}],
  Overlay[{MakePIP[-1, T0 + 10, RGBColor["#287DAB"]],
    MakePIPNV[-1, T0 + 10, Lighter[Gray]]}]]],
Quiet[List[Overlay[{MakePIP[0, T0, Black], MakePIPNV[0, T0, Lighter[Gray]]}],
  Overlay[{MakePIP[0, T0 + 5, Black], MakePIPNV[0, T0 + 5, Lighter[Gray]]}], Overlay[
  {MakePIP[0, T0 + 10, Black], MakePIPNV[0, T0 + 10, Lighter[Gray]]}]]] (*linear*)
Quiet[List[Overlay[{MakePIP[1, T0, RGBColor["#B09771"]],
  MakePIPNV[1, T0, Lighter[Gray]]}], Overlay[
  {MakePIP[1, T0 + 5, RGBColor["#B09771"]], MakePIPNV[1, T0 + 5, Lighter[Gray]]}],
  Overlay[{MakePIP[1, T0 + 10, RGBColor["#B09771"]],
    MakePIPNV[1, T0 + 10, Lighter[Gray]]}]]]
(*generalist*)
```

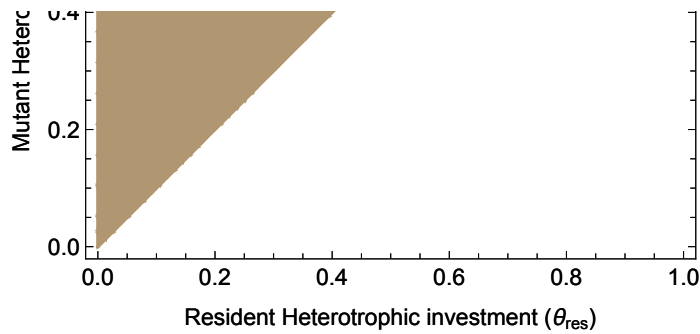




Out[144]=



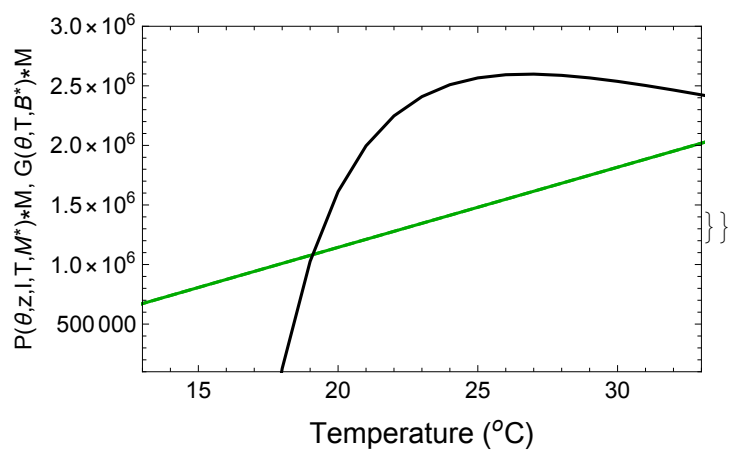
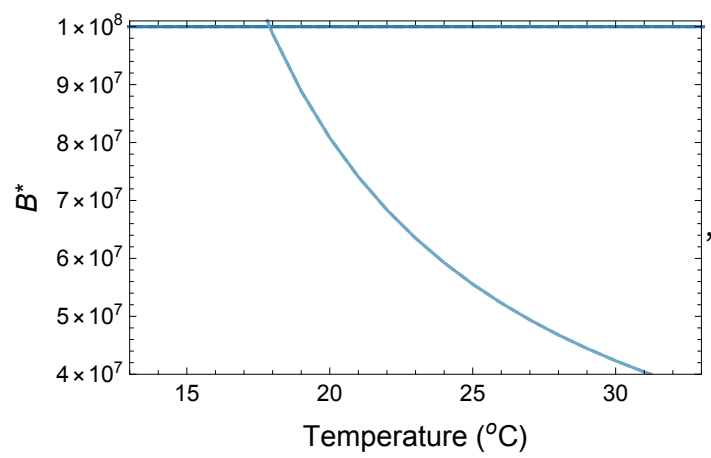
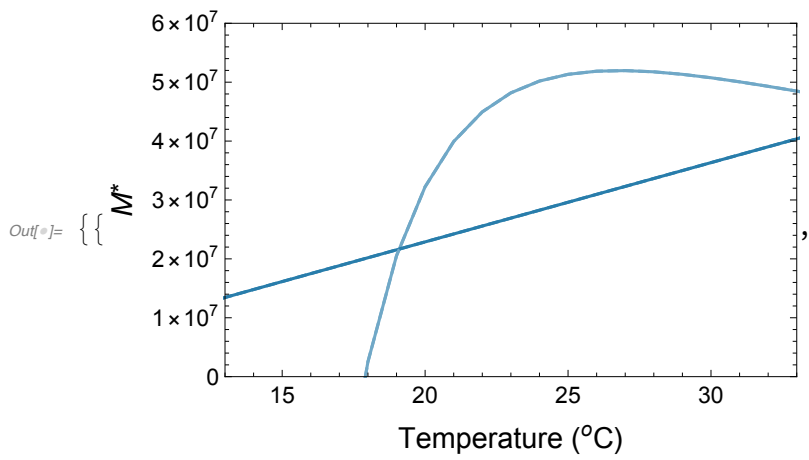




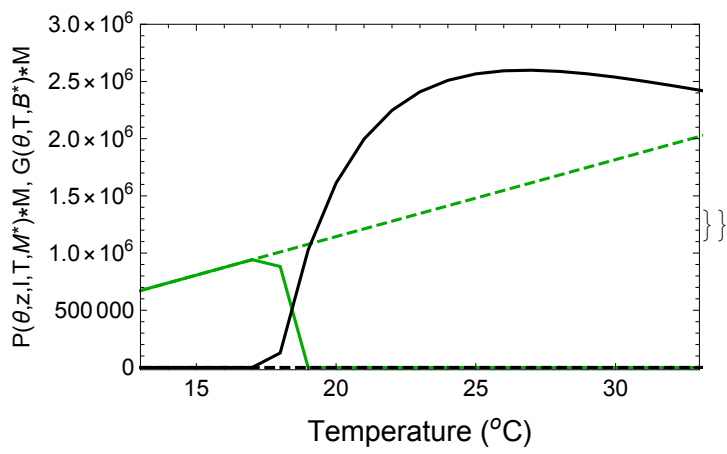
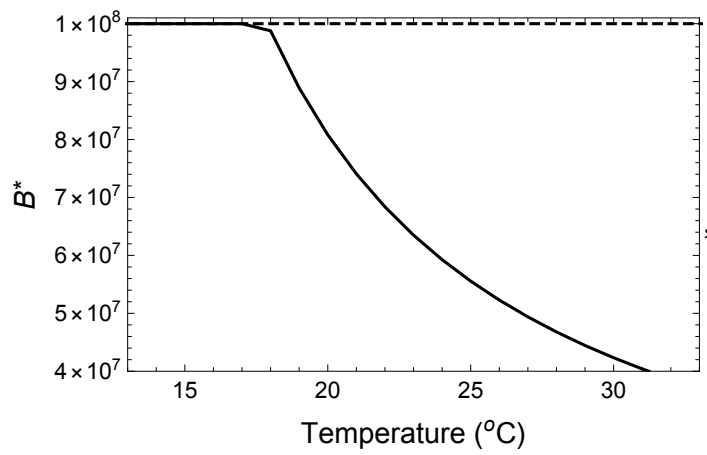
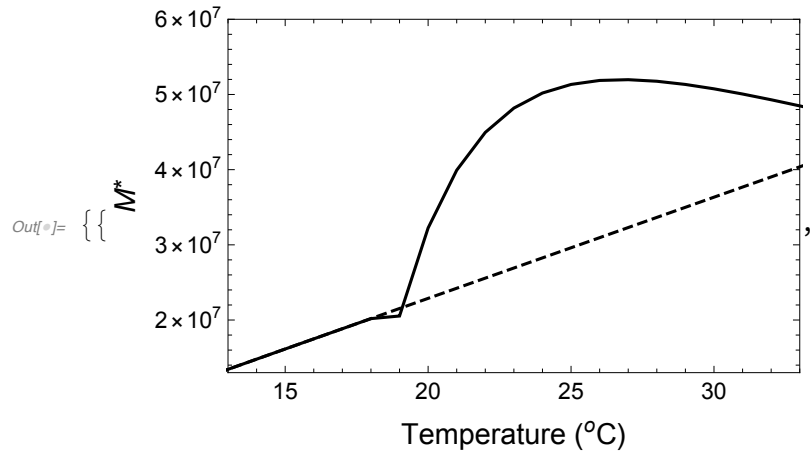
## C-cycling related figures (Dashed - genetically static, Solid - evolving)

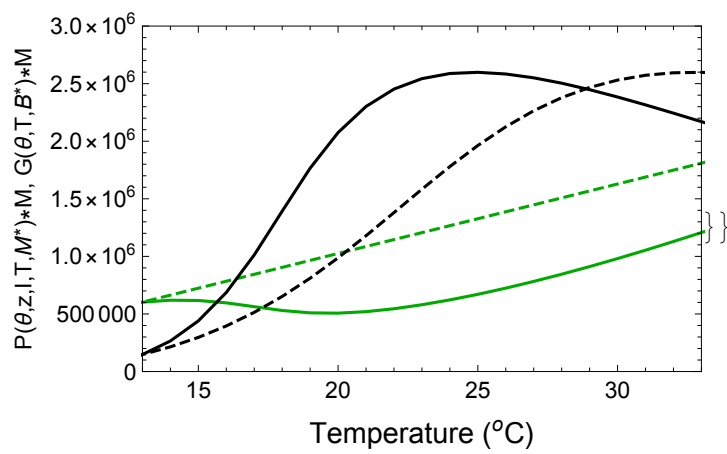
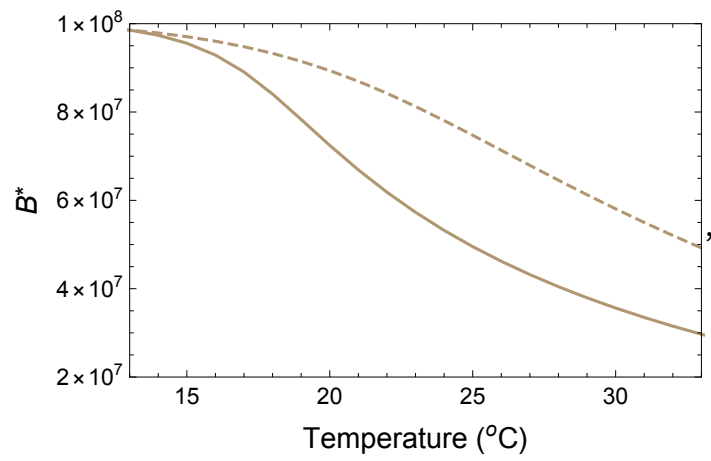
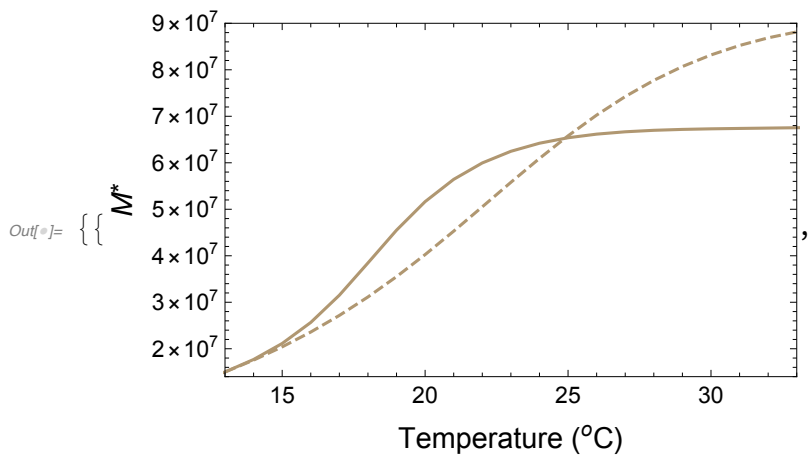
$K_B = 1 \times 10^8$ ,  $I_{in} = 100$

```
In[ ]:= KB = 1 × 108; Iin = 100;
Quiet[Ccyclingspec[-1, 0, 6 × 107, 4 × 107, 1.01 × 108, 105, 3 × 106]]
Quiet[Ccycling[0, 1.3 × 107, 6 × 107, 4 × 107, 1.01 × 108, -20 000, 3 × 106]]
Quiet[Ccycling[1, 1.4 × 107, 9 × 107, 2 × 107, 10 × 107, 0, 3 × 106]]
```







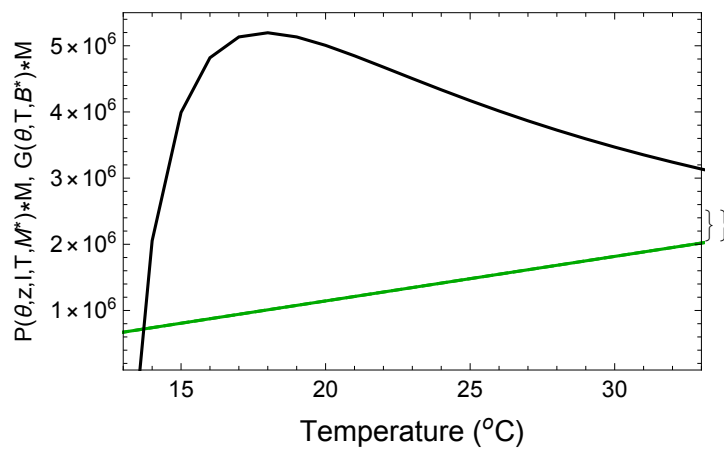
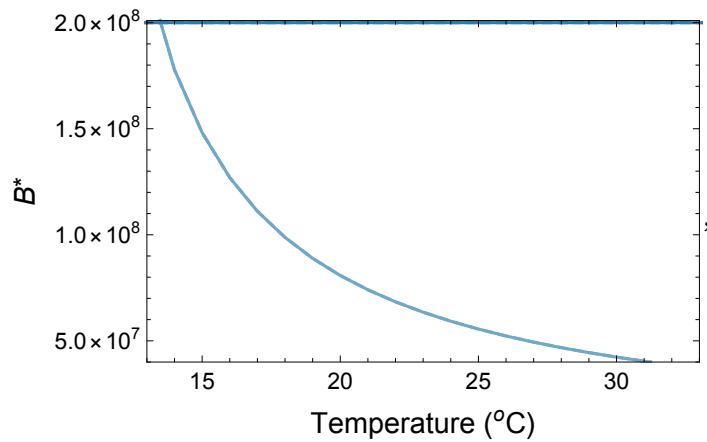
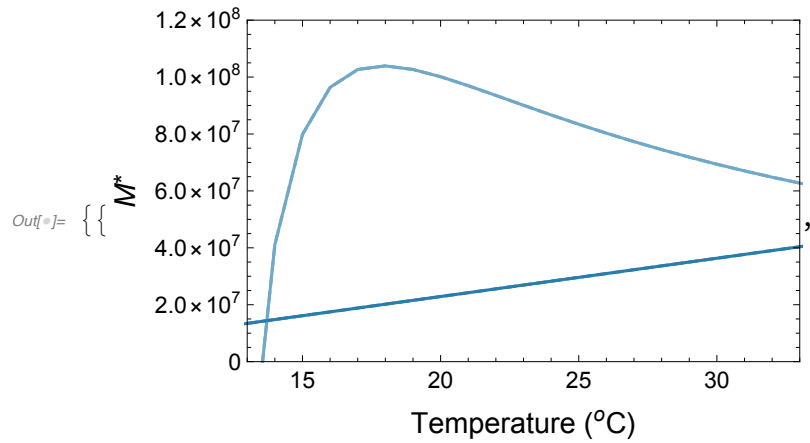


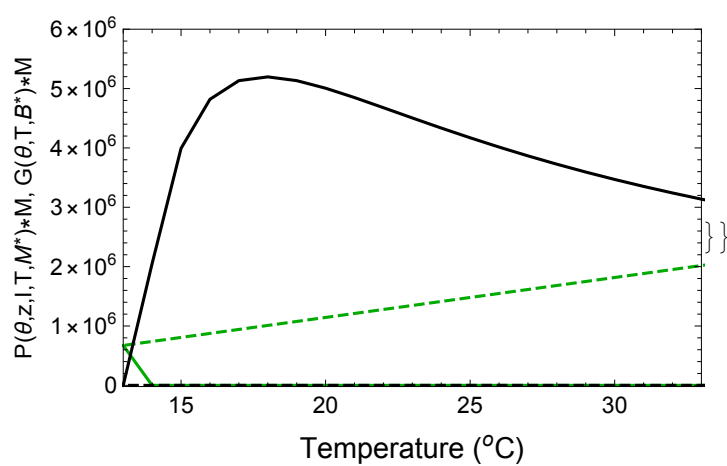
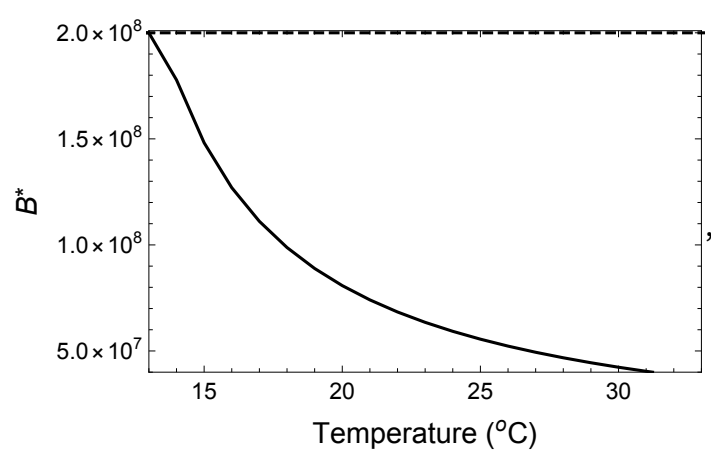
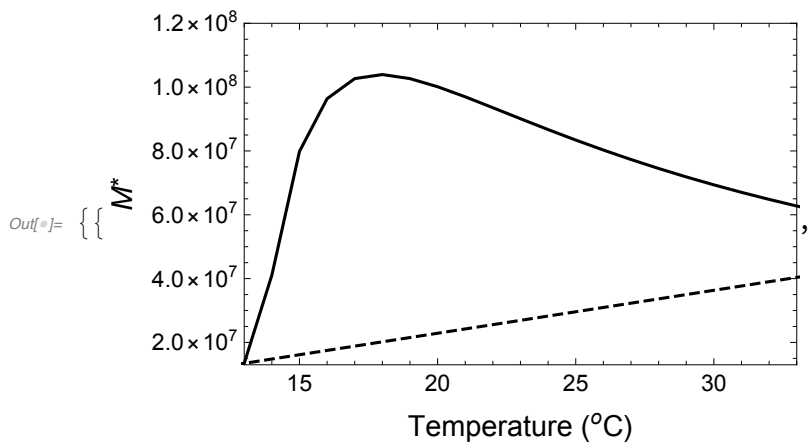
$$K_B = 2 \times 10^8, I_{in} = 100$$

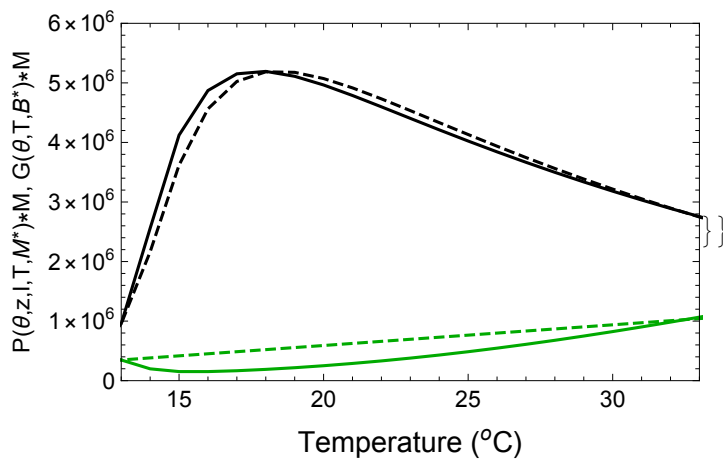
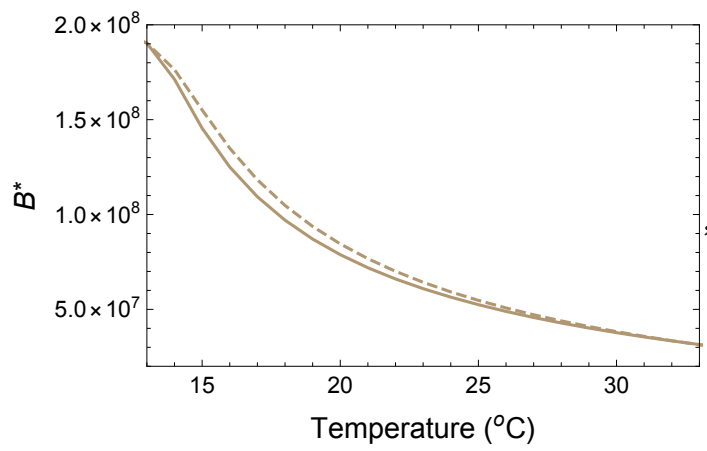
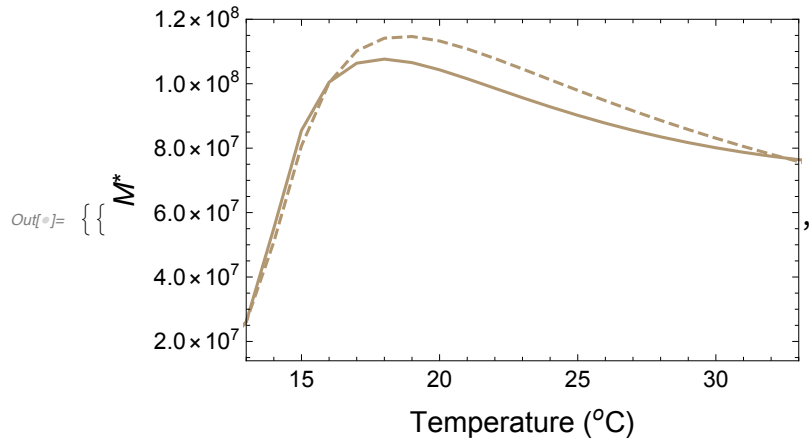
```

In[ ]:= KB = 2 × 108; Iin = 100;
Quiet[Ccyclingspec[-1, 0, 1.2 × 108, 4 × 107, 2.01 × 108, 105, 5.5 × 106]]
Quiet[Ccycling[0, 1.3 × 107, 1.2 × 108, 4 × 107, 2.01 × 108, -20 000, 6 × 106]]
Quiet[Ccycling[1, 1.4 × 107, 1.2 × 108, 2 × 107, 2 × 108, 0, 6 × 106]]

```







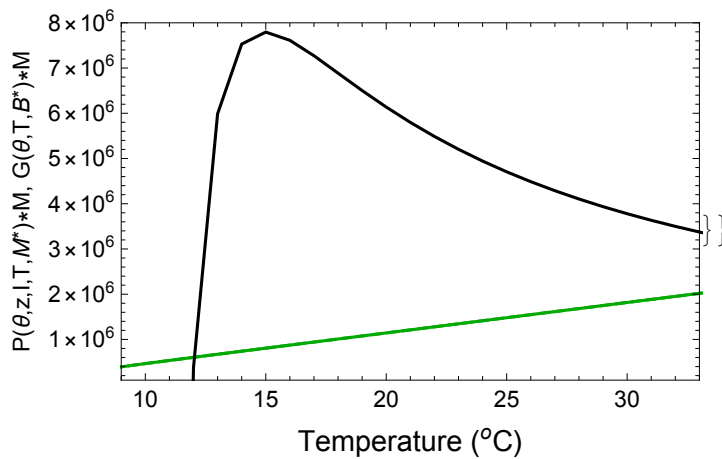
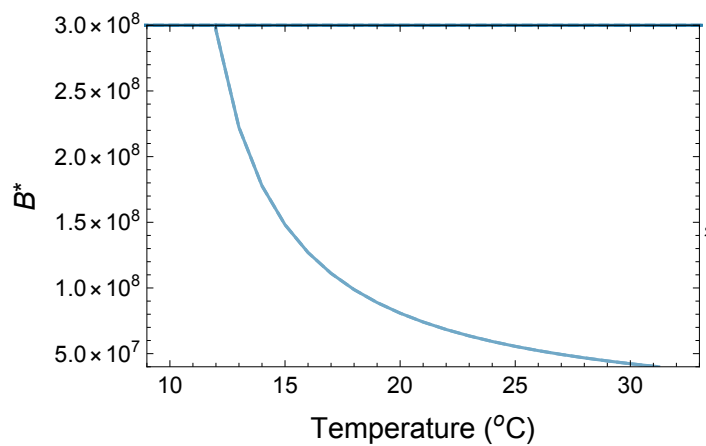
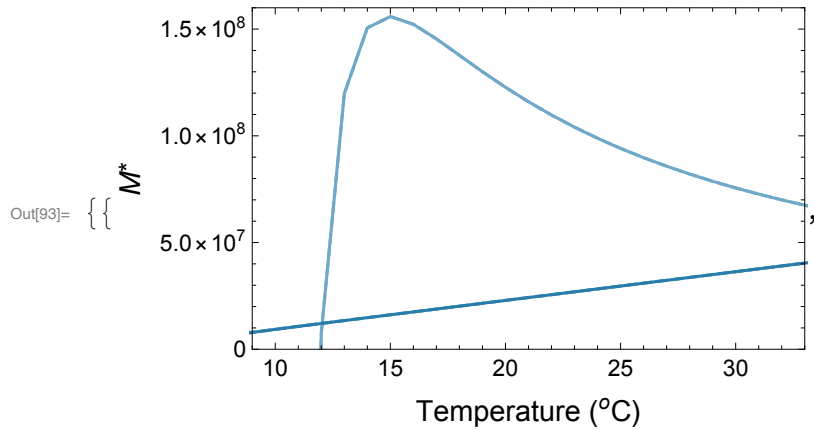
$$K_B = 3 \times 10^8, I_{in} = 100$$

In[92]:=  $K_B = 3 \times 10^8$ ;  $I_{in} = 100$ ;

Quiet[Ccylingspec[-1, 0,  $1.6 \times 10^8$ ,  $4 \times 10^7$ ,  $3 \times 10^8$ ,  $10^5$ ,  $8 \times 10^6$ ]]

Quiet[Ccycling[0,  $1.3 \times 10^7$ ,  $2 \times 10^8$ ,  $4 \times 10^7$ ,  $2.5 \times 10^8$ , -20000,  $8 \times 10^6$ ]]

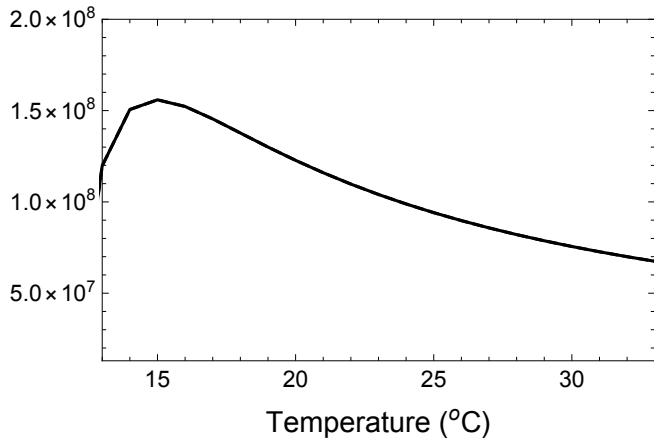
Quiet[Ccycling[1,  $1.4 \times 10^7$ ,  $2 \times 10^8$ , 0,  $3 \times 10^8$ , 0,  $8 \times 10^6$ ]]



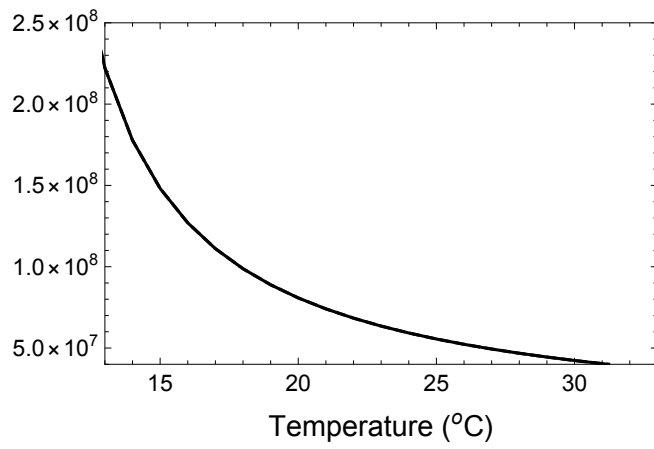
Out[94]=

{

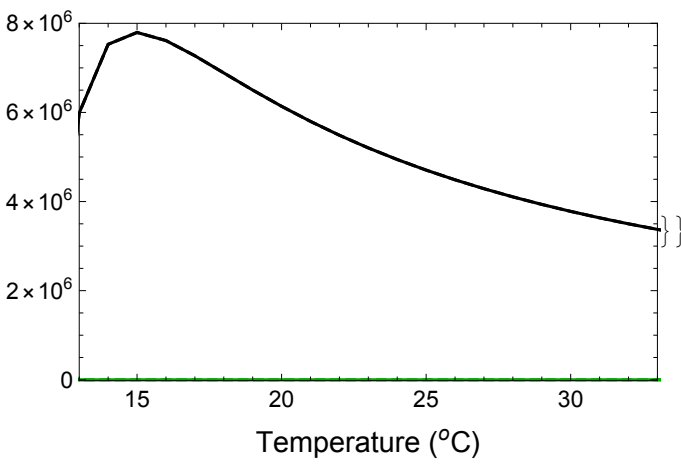
$M^*$

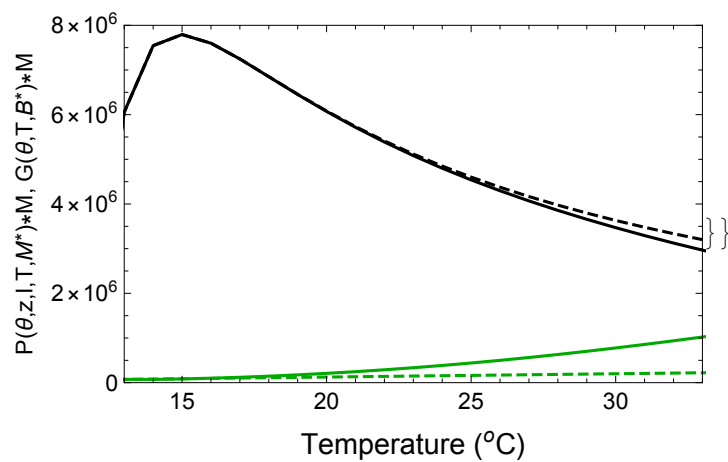
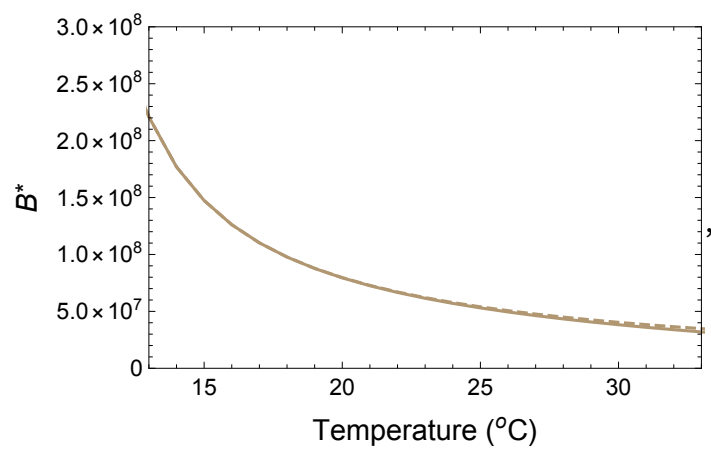
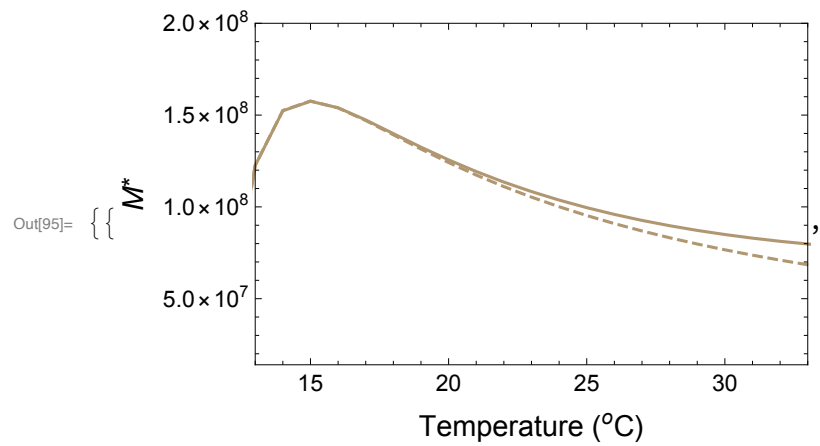


$B^*$



$P(\theta_z, I, T, M^*) * M, G(\theta, T, B^*) * M$

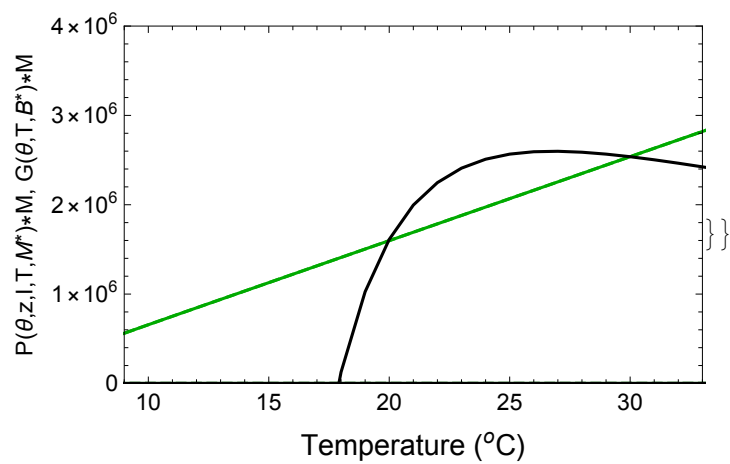
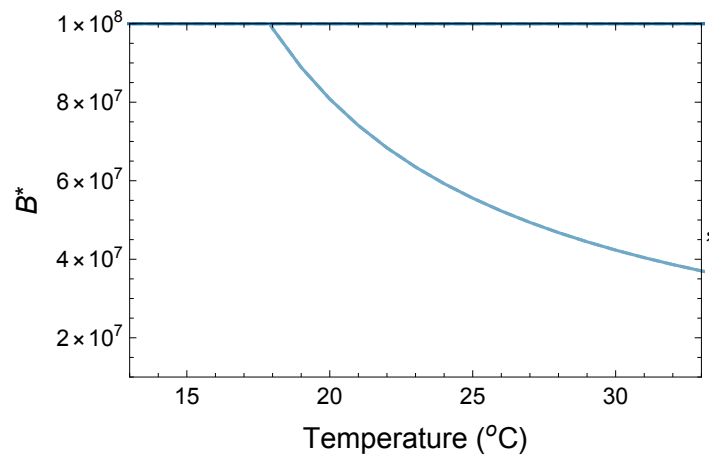
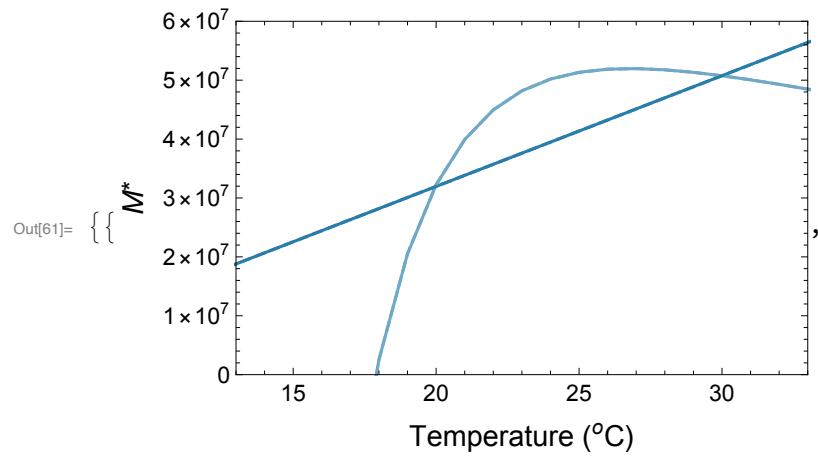


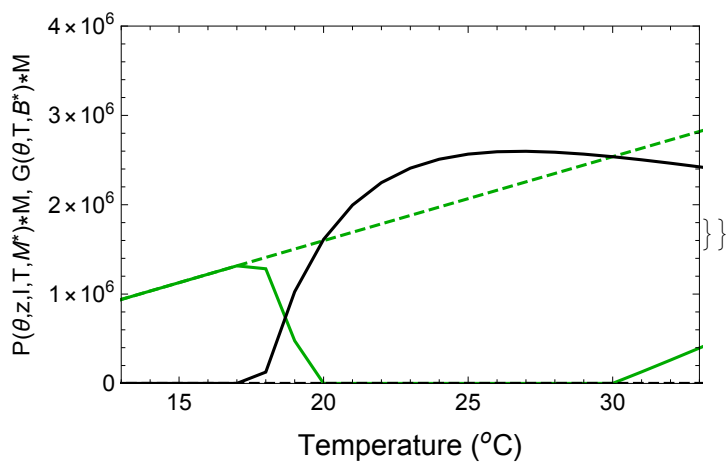
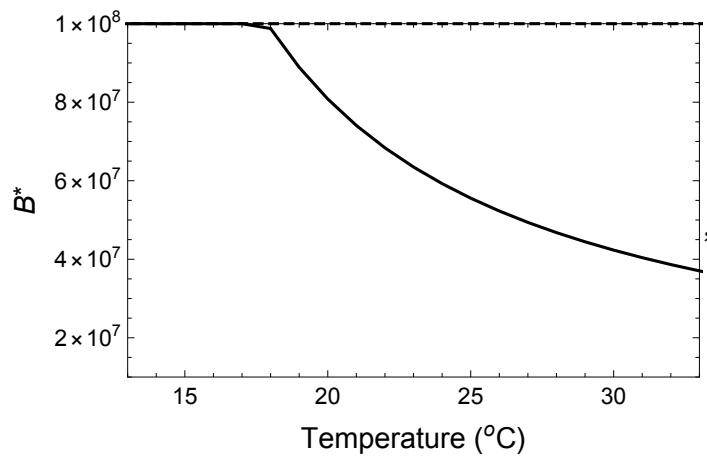
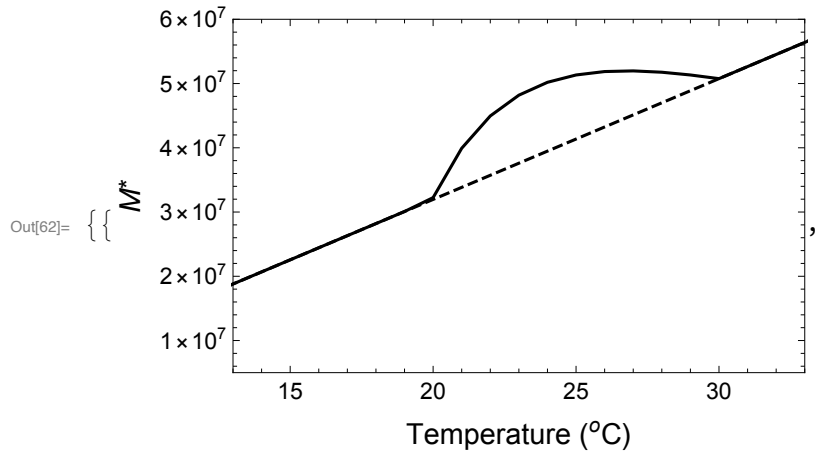


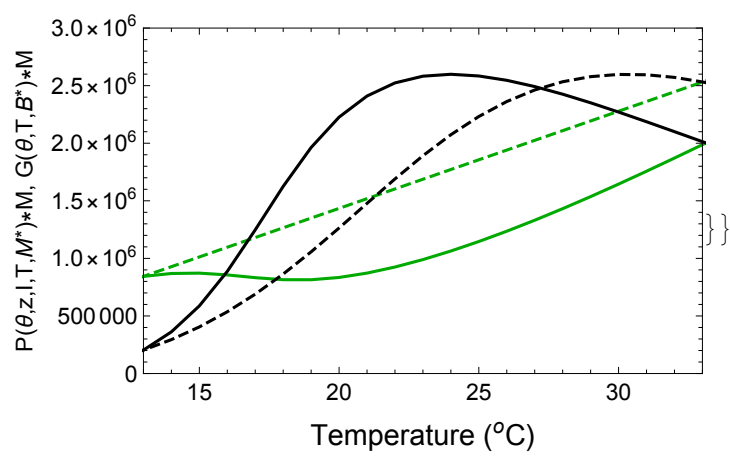
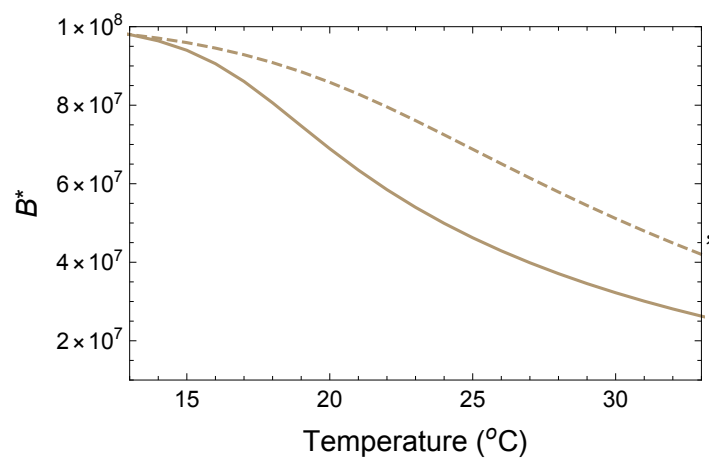
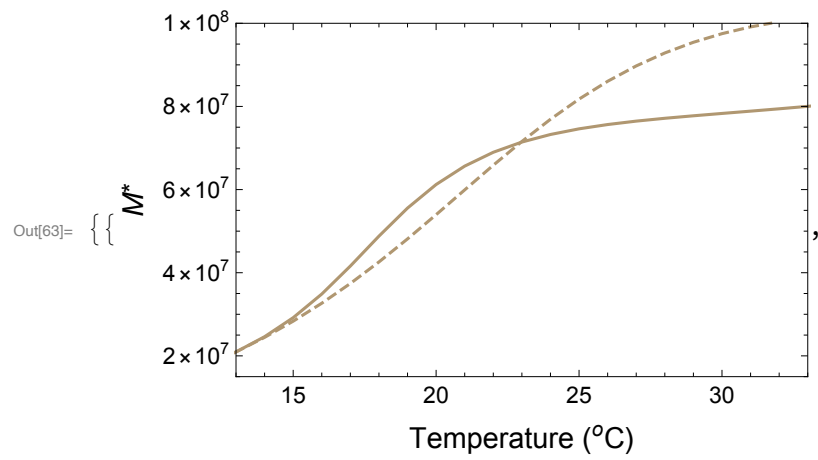


$$K_B = 1 \times 10^8, I_{in} = 150$$

```
In[60]:= KB = 1 × 108; Iin = 150;
Quiet[Ccyclingspec[-1, 0, 6.0 × 107, 1 × 107, 1 × 108, 0, 4 × 106]]
Quiet[Ccycling[0, .5 × 107, 6.0 × 107, 1 × 107, 1 × 108, 0, 4 × 106]]
Quiet[Ccycling[1, 1.5 × 107, 1 × 108, 1 × 107, 1 × 108, 0, 3 × 106]]
```

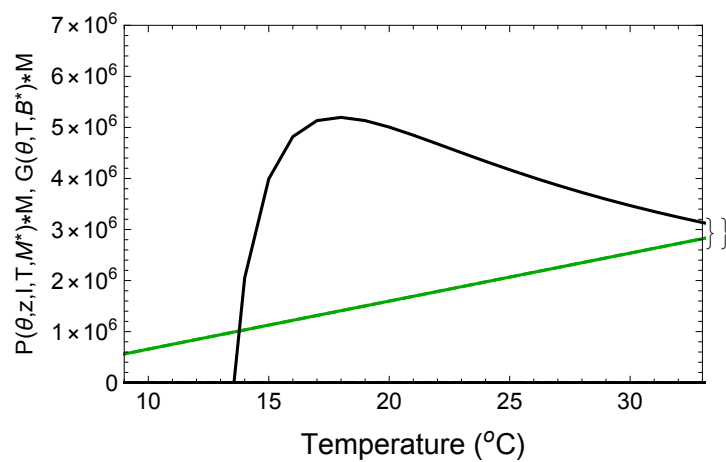
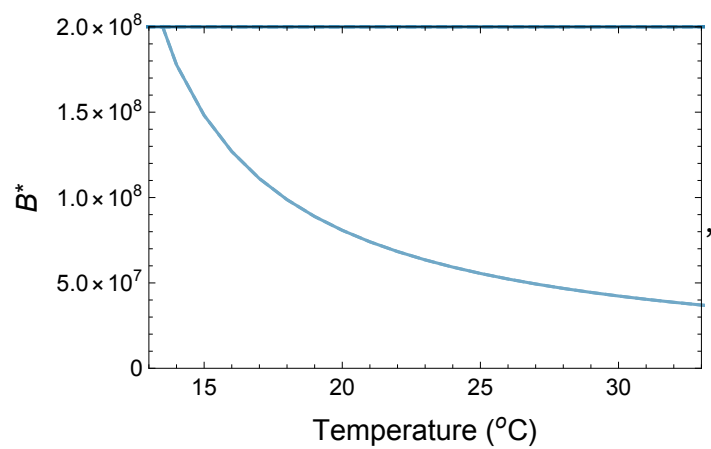
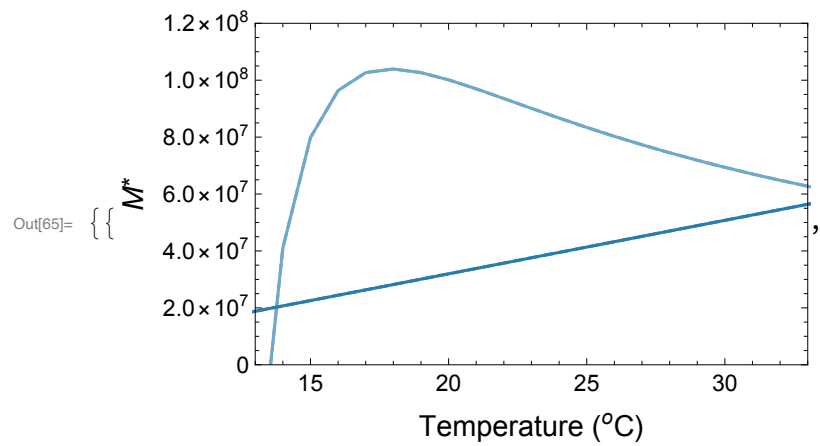


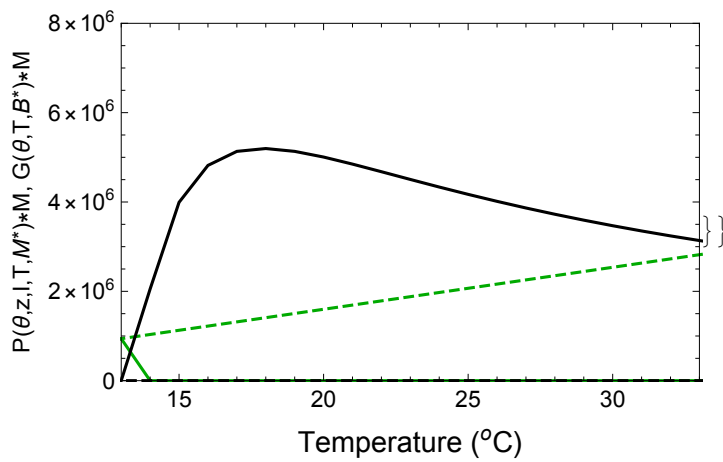
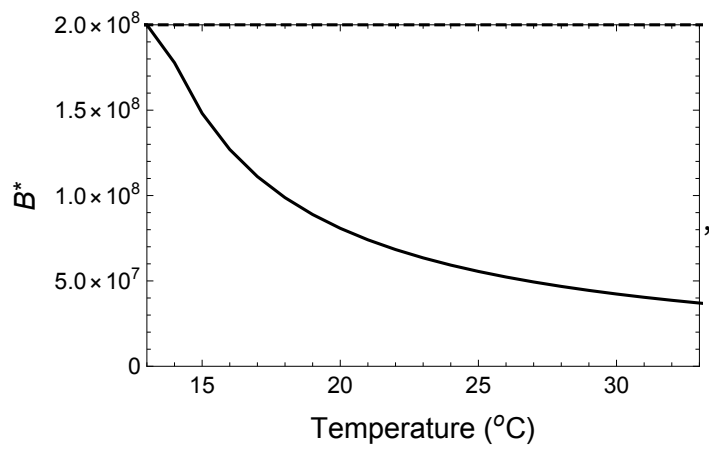
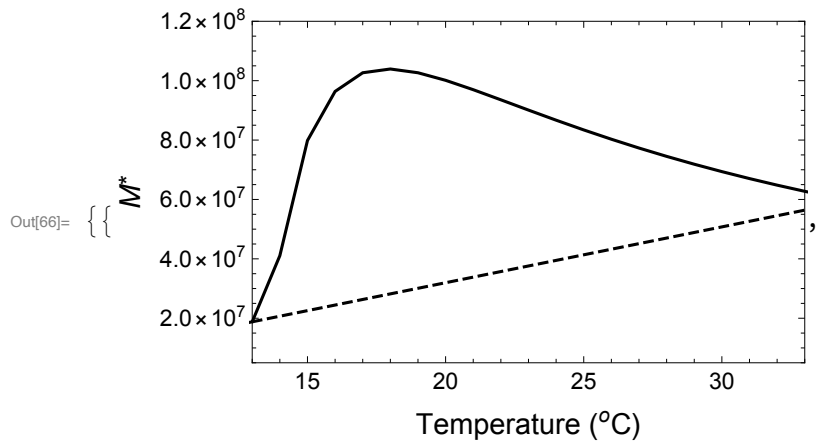


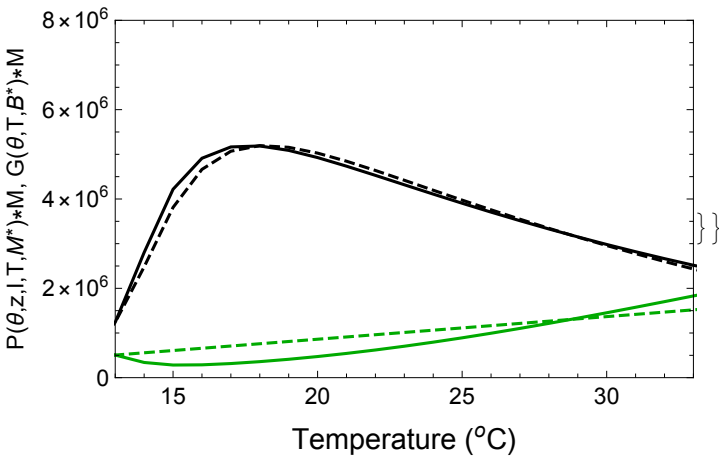
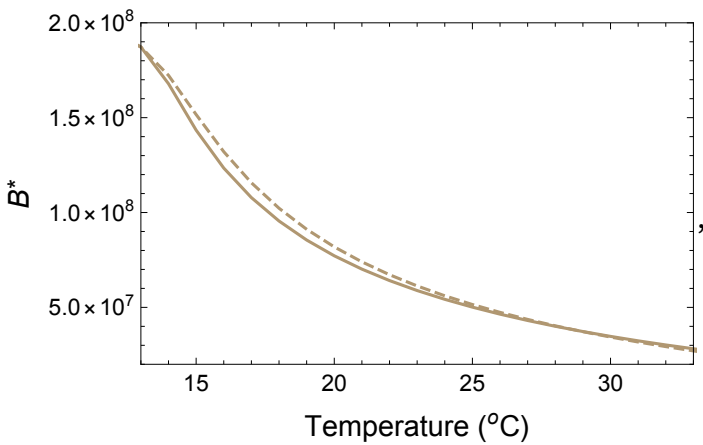
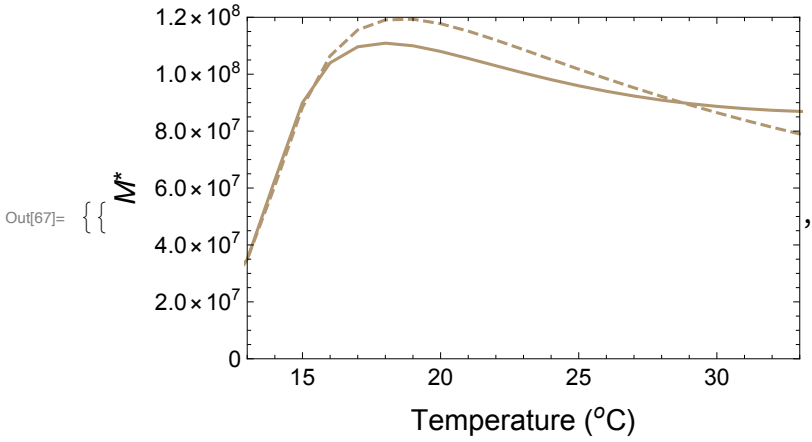


$$K_B = 2 \times 10^8, I_{in} = 150$$

```
In[64]:= KB = 2 × 108; Iin = 150;
Quiet[Ccyclingspec[-1, 0, 1.2 × 108, 0, 2 × 108, 0, 7 × 106]]
Quiet[Ccycling[0, .5 × 107, 1.2 × 108, 0, 2 × 108, 0, 8 × 106]]
Quiet[Ccycling[1, 0 × 107, 1.2 × 108, 2 × 107, 2 × 108, 0, 8 × 106]]
```









$$K_B = 3 \times 10^8, I_{in} = 150$$

```
In[96]:= KB = 3 × 108; Iin = 150;
Quiet[Ccyclingspec[-1, 0, 2 × 108, 0 × 107, 3 × 108, 0, 7 × 106]]
Quiet[Ccycling[0, 0 × 107, 2 × 108, 0 × 107, 2.5 × 108, 0, 8 × 106]]
Quiet[Ccycling[1, 0 × 107, 2 × 108, 2 × 107, 2.5 × 108, 0, 8 × 106]]
```

