

2801.001 Spring 2018 Homework 1

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Submission instructions: Same groups as HW1. Refer to instructions from TA.

Note: Problems 1 and 2 require the datafile posted online which contains prices and implied volatilities of one-year (349 calendar days) listed options on the Nasdaq 100 index (NDX), as well as other market data.

Problem 1

- (a) Estimate the market price of the 5% call spread (i.e. with strikes ATM and 5% OTM). What about the 5% put spread?

Solution.

TODO

- (b) If you were to price the spreads in the Black-Scholes model using a single volatility parameter σ , what value of σ would match the theoretical price with the market price? Comment on your results.

Solution.

TODO

Problem 2

- (a) Using the numerical package of your choice, calibrate the parameters of the SVI model against the market-implied volatility data. Show a comparative graph of the SVI curve and the actual implied volatility data points.

Solution.

TODO

- (b) Compute or estimate the price of an at-the-money digital call option paying off \$1 if in one year NDX is greater than its current spot level, and zero otherwise: (i) in the Black-Scholes model, (ii) using $\pm 1\%$ call spreads, (iii) using the smile-adjusted formula on page 19.

Solution.

TODO

- (c) Graph the implied distribution corresponding to the SVI model calibration.

Solution.

TODO

- (d) Use the implied distribution to compute the price of the following European exotic options, where X_0 is the current index level and X_T is the final index level:

Solution.

- (i) Digital call defined in question (b);

TODO

- (ii) "Reverse convertible" paying off $\max\left(100\%, 100\% + p \times \frac{X_T - X_0}{X_0}\right)$ if $\frac{X_T}{X_0} > 75\%$ and $\frac{X_T}{X_0}$ otherwise, where $p = 50\%$. Then solve for p to get a price of 100%;

TODO

- (iii) Option paying off $\max\left(0, \frac{X_T - X_0}{X_T}\right)$;

TODO

- (iv) Log-contract paying off $-2 \log\left(\frac{X_T}{X_0}\right)$. Price interpretation;

TODO

Problem 3

Find conditions on the SVI model parameters to satisfy Lees asymptotic bounds on p. 22:

$$\sigma^{\star^2}(k_F, T) \leq \beta T |\log k_F|, \beta \in [0, 2]$$

Solution.

TODO

Problem 4

(Problem 4.3 p. 56 in textbook, with corrections): Consider an underlying stock S currently trading at $S_0 = 100$ which does not pay any dividend. Assume the local volatility function is $\sigma_{loc}(t, S) = \frac{0.1 - 0.15 \times \log\left(\frac{S}{S_0}\right)}{\sqrt{t}}$, and that interest rates are zero.

- (a) Produce the graph of the local volatility surface for spots 0 to 200 and maturities 0 to 5 years.

Solution.

TODO

- (b) Write a Monte-Carlo algorithm to price the following 1-year payoffs using 252 time steps and e.g. 10,000 paths:

Solution.

- (i) "Capped quadratic" option: $\min\left(1, \frac{S_1^2}{S_0^2}\right)$;

TODO

- (ii) Asian at-the-money-call: $\max\left(0, \frac{S_{0.25} + S_{0.5} + S_{0.75} + S_1}{4 \times S_0} - 1\right)$;

TODO

- (iii) Barrier call: $\max(0, S_1 - S_0)$ if S always traded above 80 using 252 daily observations, 0 otherwise;

TODO

Problem 5

The payoff of a 1-year at-the-money call on the geometric average return of two non-dividend paying stocks X, Y is given as:

$$f(X_T, Y_T) = \max \left(0, \sqrt{\frac{X_T Y_T}{X_0 Y_0}} - 1 \right)$$

where $T = 1$ year and X_t, Y_t are the respective underlying spot prices of X, Y at any time t .

- (a) Derive analytical formulas for the call value at any time $0 \leq t \leq T$ in the Black-Scholes model with constant correlation ρ (cf. Section 6 – 4 in the textbook, to be covered during Session 5.)

Solution.

TODO

- (b) Compute the value of the call using a 5% interest rate, 20% volatility for X , 30% volatility for Y , and $\rho = 0.4$. Use finite differences to estimate the deltas, gammas and cross-gamma of the call.

Solution.

TODO

- (c) You purchased the call on a \$10,000,000 notional. What actions would you take to delta-hedge your position? What would then be your instant $P\&L$ in the following matrix of scenarios. Generally, graph your instant $P\&L$ against percent changes x, y in underlying stock prices.

Solution.

TODO

$X \backslash Y$	−5%	+1%	+5%
−5%			
+1%			
+5%			