THE IMPACT OF NONRECIPROCAL HIGHER-ORDER INTERACTIONS ON SYNCHRONIZATION

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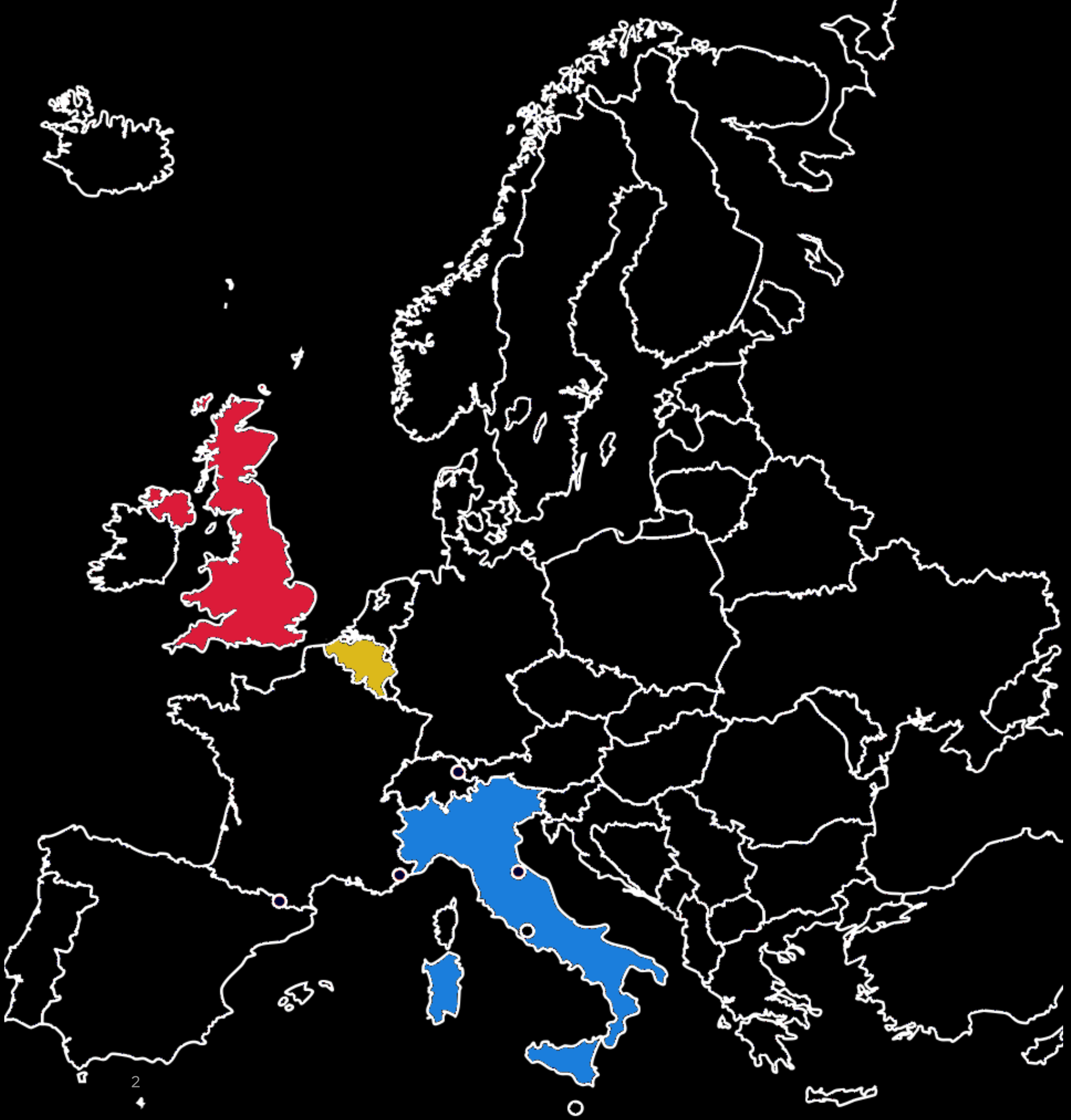
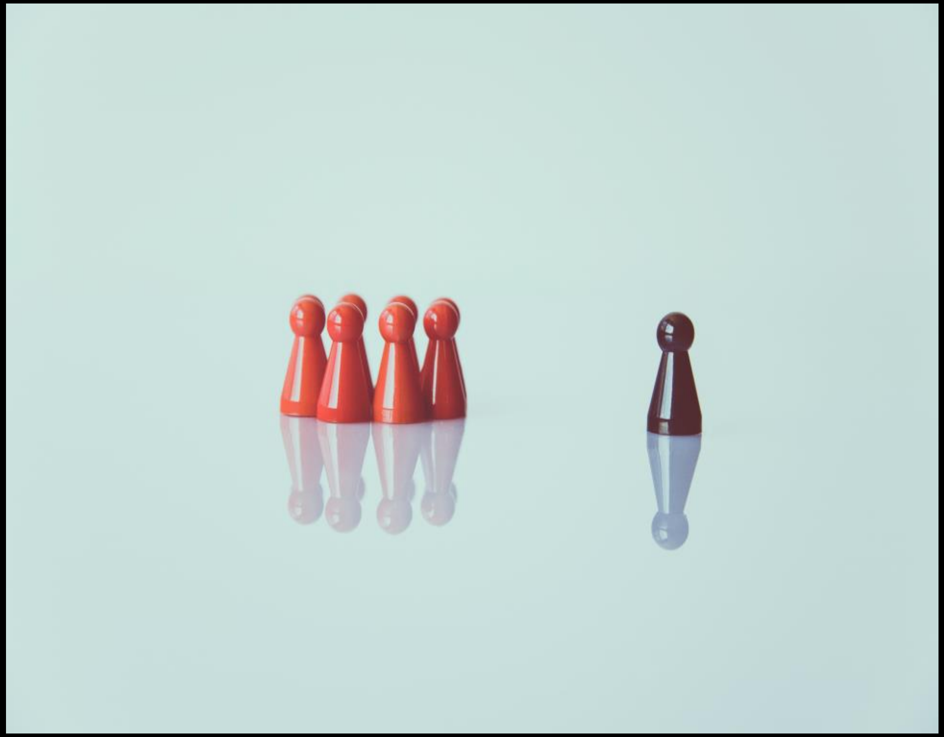


ASYMMETRICAL MANY-BODY PROCESSES



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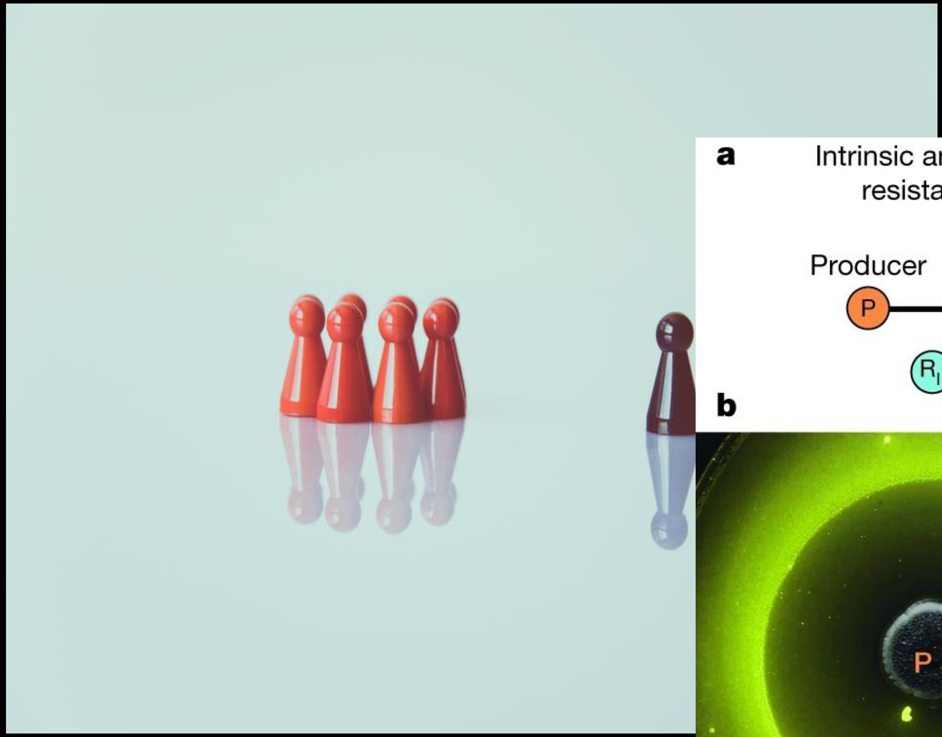

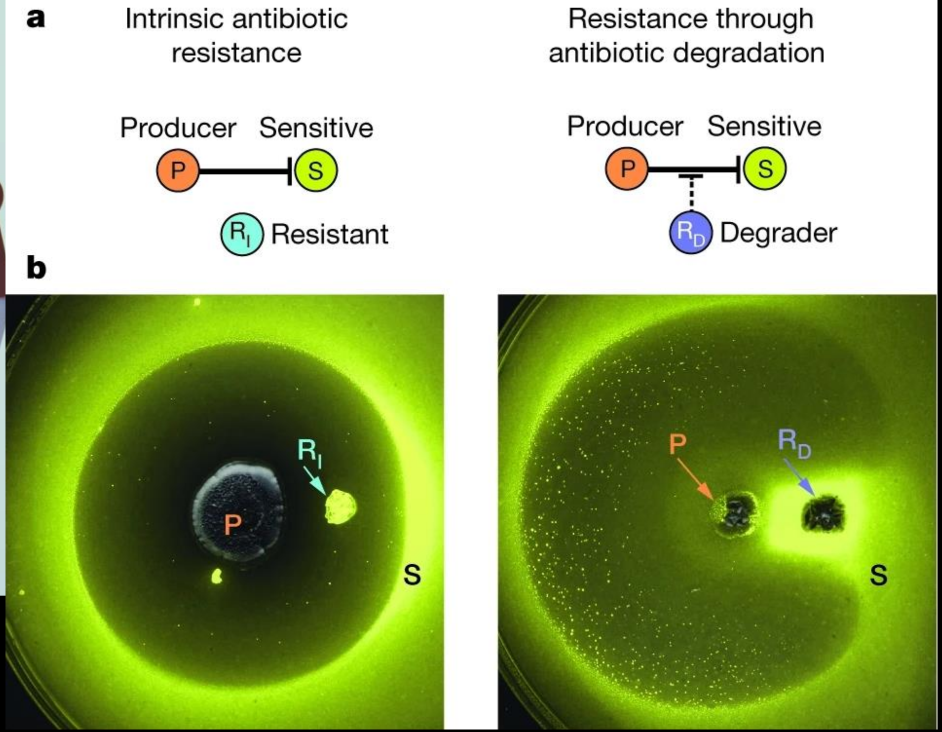

Kelsic, E.D., Zhao, J., Vetsigian, K., Kishony, R.,
Nature, 521, 7553 (2017)

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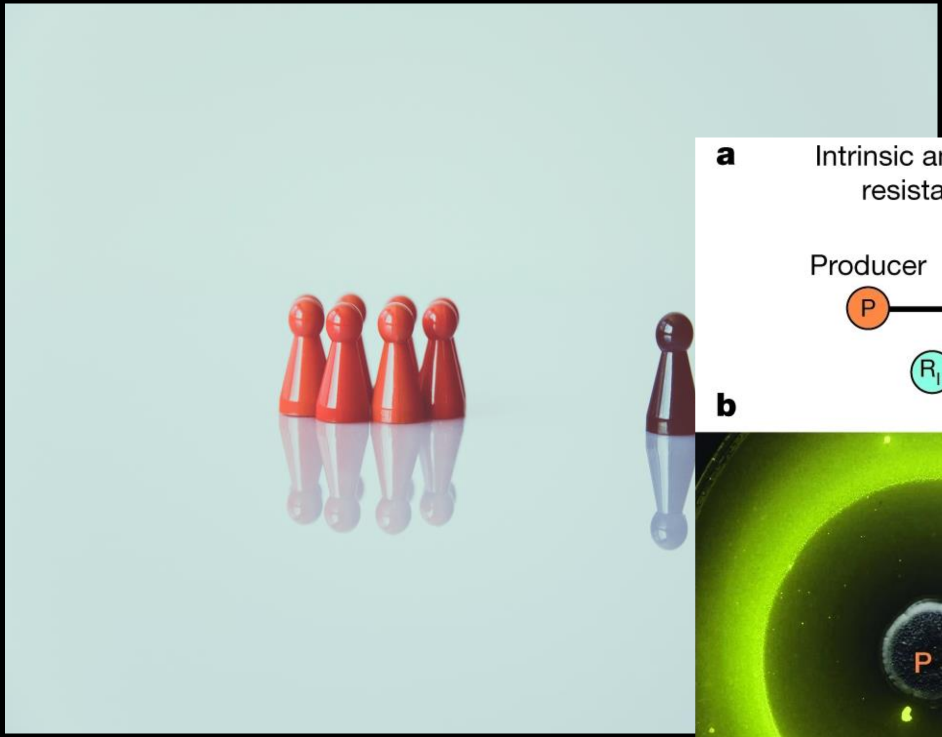

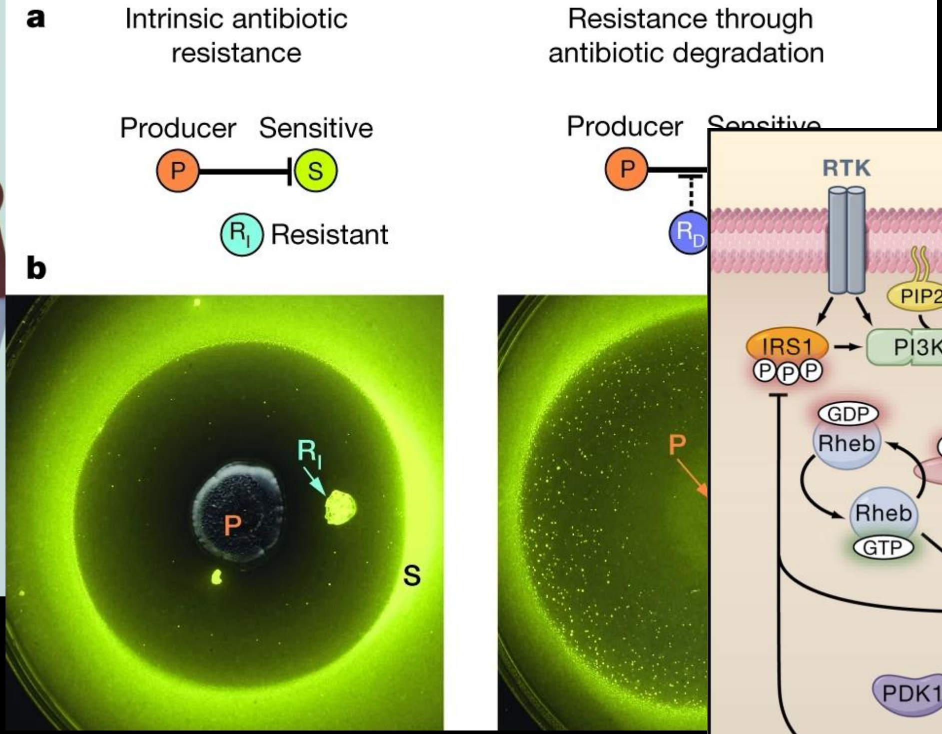

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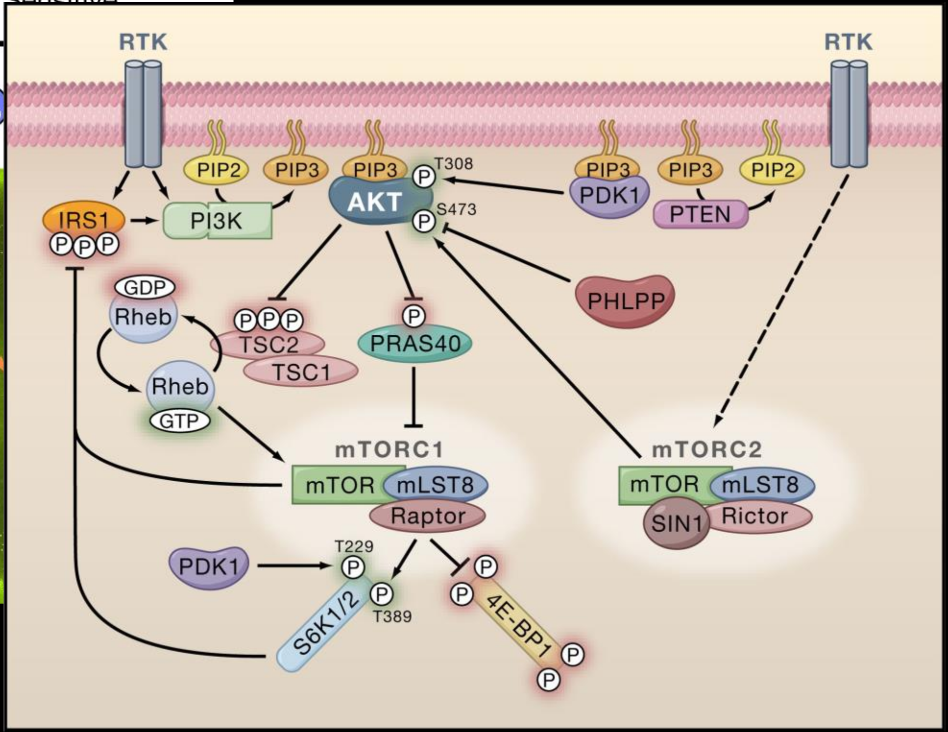
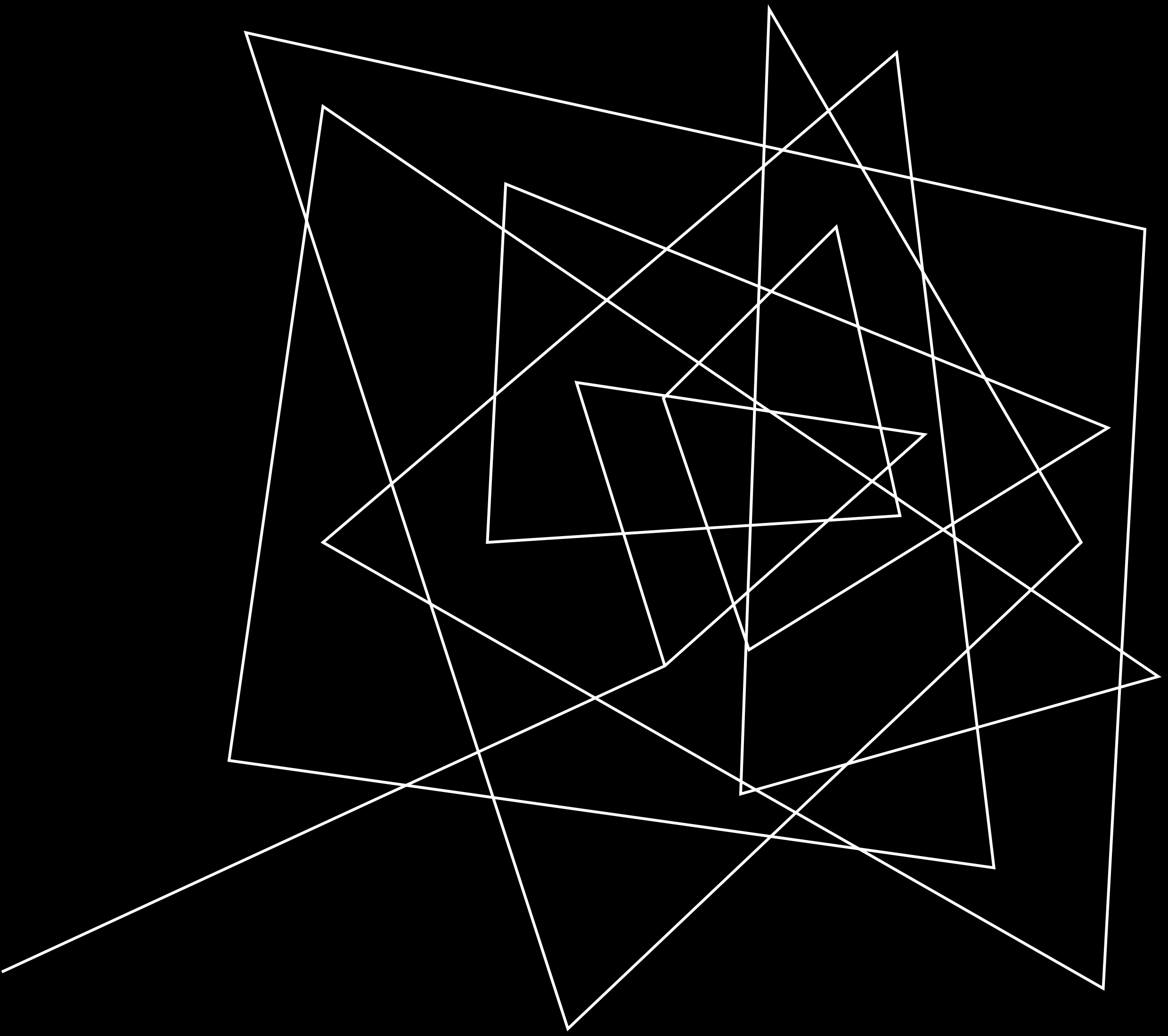
Manning, B.D., Cantley, L.C., Cell, 129, 7 (2007)





HOW CAN WE MODEL DYNAMICAL PROCESSES IN PRESENCE OF NONRECIPROCAL MANY-BODY INTERACTIONS?

ELEMENTARY DECOMPOSITION: THREE-BODY INTERACTIONS

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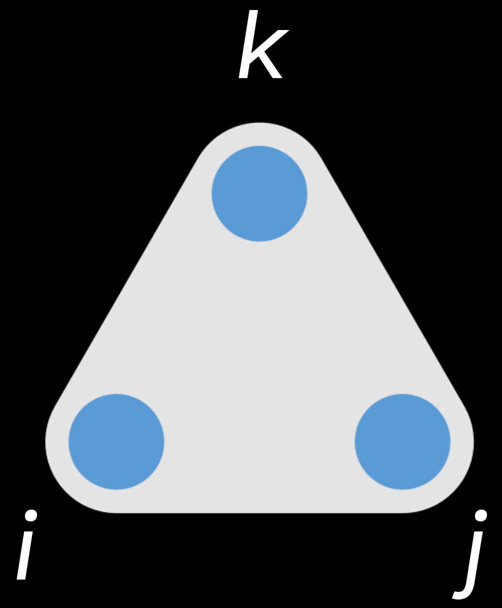
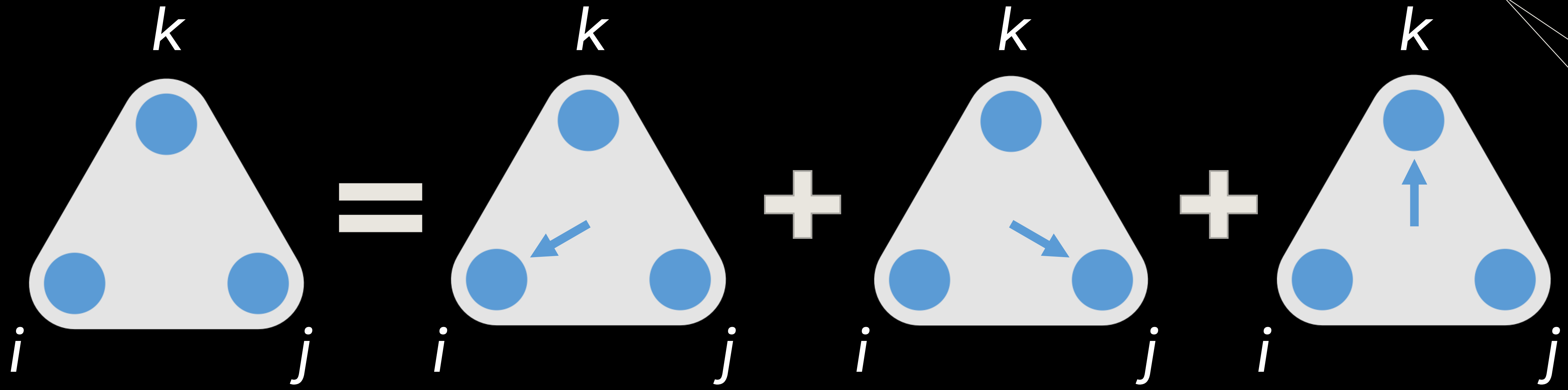


ELEMENTARY DECOMPOSITION: THREE-BODY INTERACTIONS



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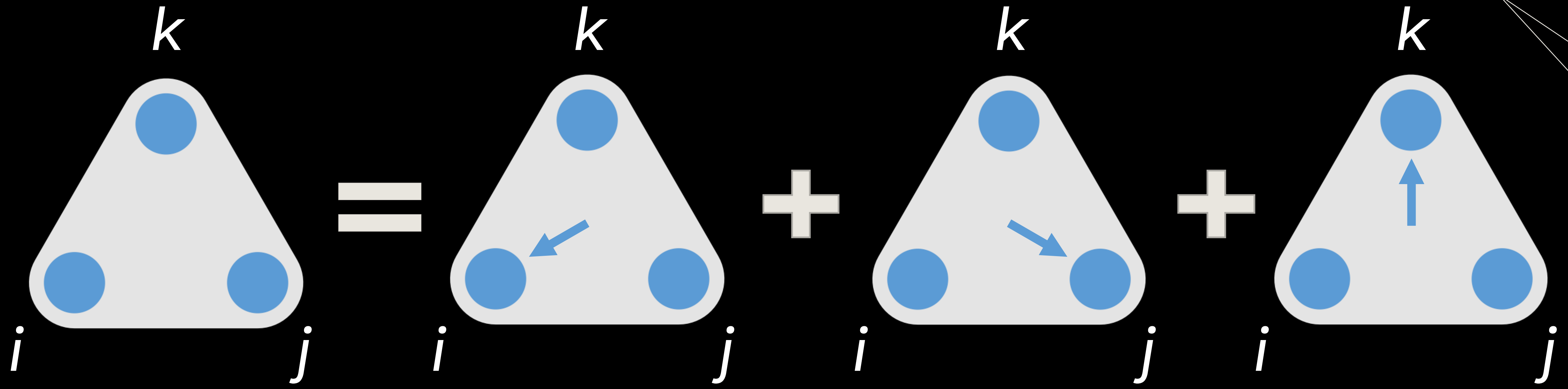

$$A^{(2)}_{\pi(ijk)} = 1 \qquad A^{(2)}_{i\pi(jk)} = 1 \qquad A^{(2)}_{j\pi(ik)} = 1 \qquad A^{(2)}_{k\pi(ij)} = 1$$

SYNCHRONIZATION: AN EMERGENT AND UBIQUITOUS PHENOMENON

SYNCHRONIZATION OF RÖSSLER OSCILLATORS WITH NONRECIPROCAL HIGHER-ORDER INTERACTIONS

$$\begin{cases} \dot{x}_i = -y_i - z_i + \sigma_1 \sum_{j=1}^{N} A_{ij}^{(1)}(x_j^3 - x_i^3) + \sigma_2 \sum_{j,k=1}^{N} A_{ijk}^{(2)}(x_j^2 x_k - x_i^3) \\ \dot{y}_i = x_i + a y_i \\ \dot{z}_i = b + z_i(x_i - c), \end{cases}$$

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Local dynamics
$\mathbf{f}(\mathbf{x}_i)$

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Two-body interactions
$\mathbf{h}^{(1)}(\mathbf{x}) = x^3$

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\end{cases}
$$

Local dynamics
$\mathbf{f}(\mathbf{x}_i)$

Two-body interactions
$\mathbf{h}^{(1)}(\mathbf{x}) = x^3$

Three-body interactions
$\mathbf{h}^{(2)}(\mathbf{x}_1, \mathbf{x}_2) = x_1^2 x_2$

THE MASTER STABILITY FUNCTION APPROACH:

Nonlinear dynamics

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Nonlinear dynamics

Linearized dynamics

Linearization around the synchronous state

THE MASTER STABILITY FUNCTION APPROACH:

Nonlinear dynamics

Linearization around the synchronous state

Linearized dynamics

Projection onto the eigenvectors of a Laplacian matrix

Decoupled dynamics

THE MASTER STABILITY FUNCTION APPROACH IN DIRECTED HYPERGRAPHS

$$\boxed{\dot{\eta}_i = [\mathbf{JF}(\mathbf{x}^s) - \sigma_1 \mu_i \mathbf{JH}^{(1)}(\mathbf{x}^s)]\eta_i}$$

THE MASTER STABILITY FUNCTION APPROACH IN DIRECTED HYPERGRAPHS

$$\boxed{\dot{\eta}_i = [\mathbf{JF}(\mathbf{x}^s) - \sigma_1 \mu_i \mathbf{JH}^{(1)}(\mathbf{x}^s)]\eta_i}$$

Nontrivial eigenvalues of $\mathcal{M} = L^{(1)} + \frac{\sigma_2}{\sigma_1} L^{(2)}$

THE MASTER STABILITY FUNCTION APPROACH IN DIRECTED HYPERGRAPHS

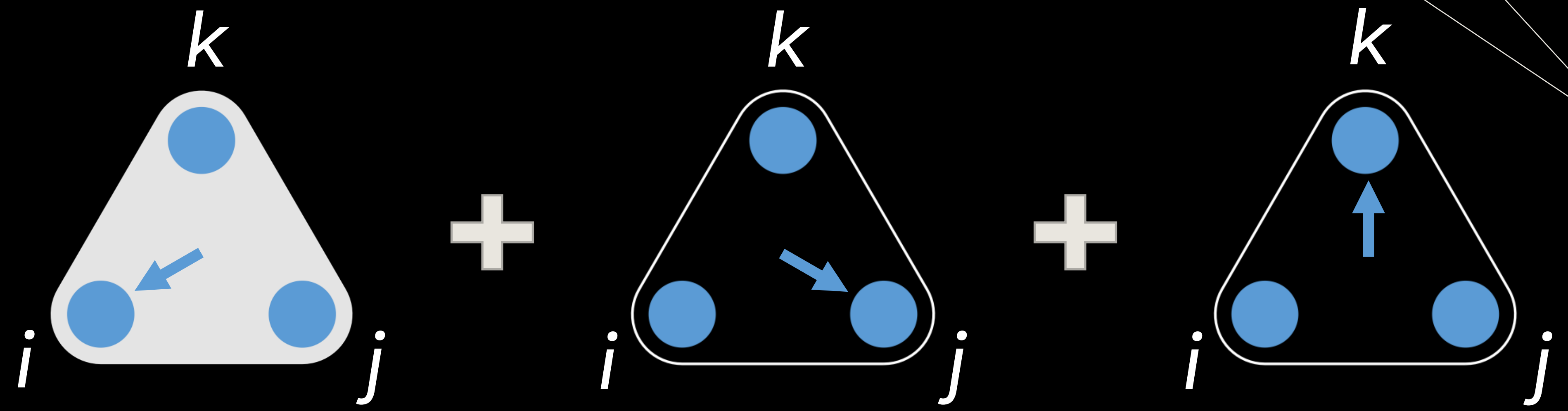
$$\boxed{\dot{\eta}_i = [\mathbf{JF}(\mathbf{x}^s) - \sigma_1 \mu_i \mathbf{JH}^{(1)}(\mathbf{x}^s)]\eta_i}$$

$$\boxed{\dot{\zeta} = [\mathbf{JF}(\mathbf{x}^s) - (\alpha + \mathrm{i}\beta)\mathbf{JH}^{(1)}]\zeta}$$

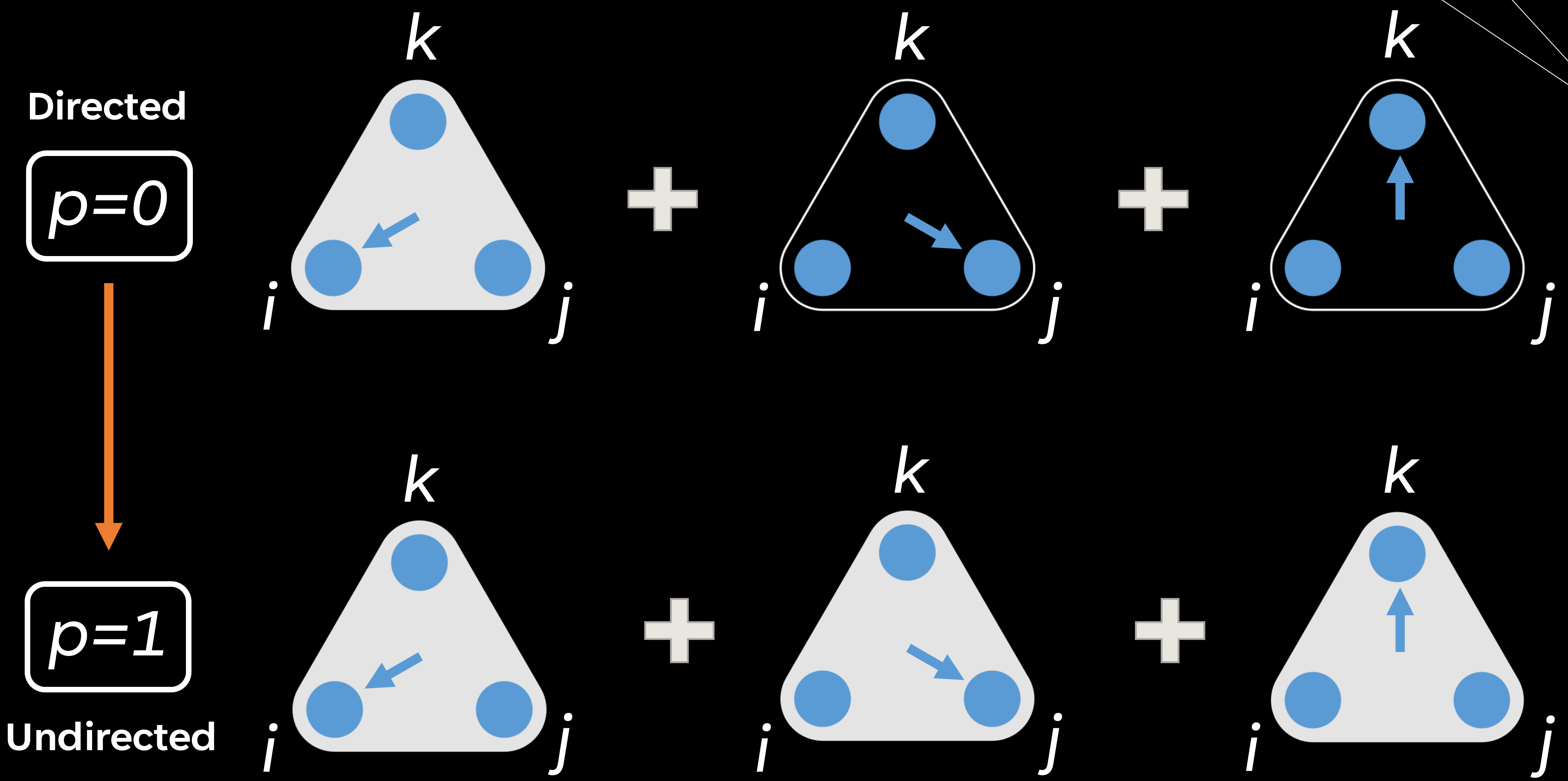
SYMMETRIZATION

Directed

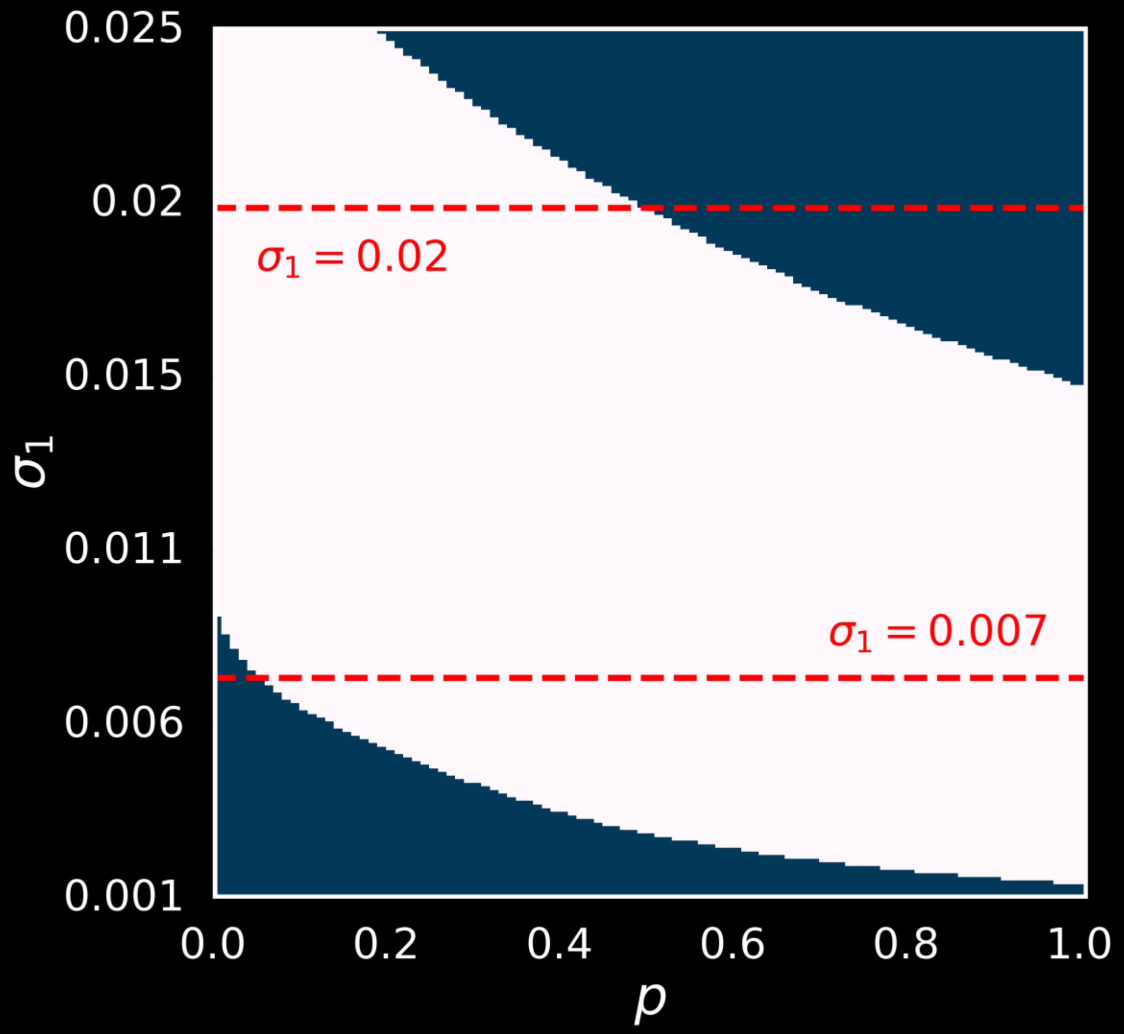
$\boxed{p=0}$



SYMMETRIZATION



NONRECIPROCAL INTERACTIONS CAN CHANGE STABILITY BEHAVIOR



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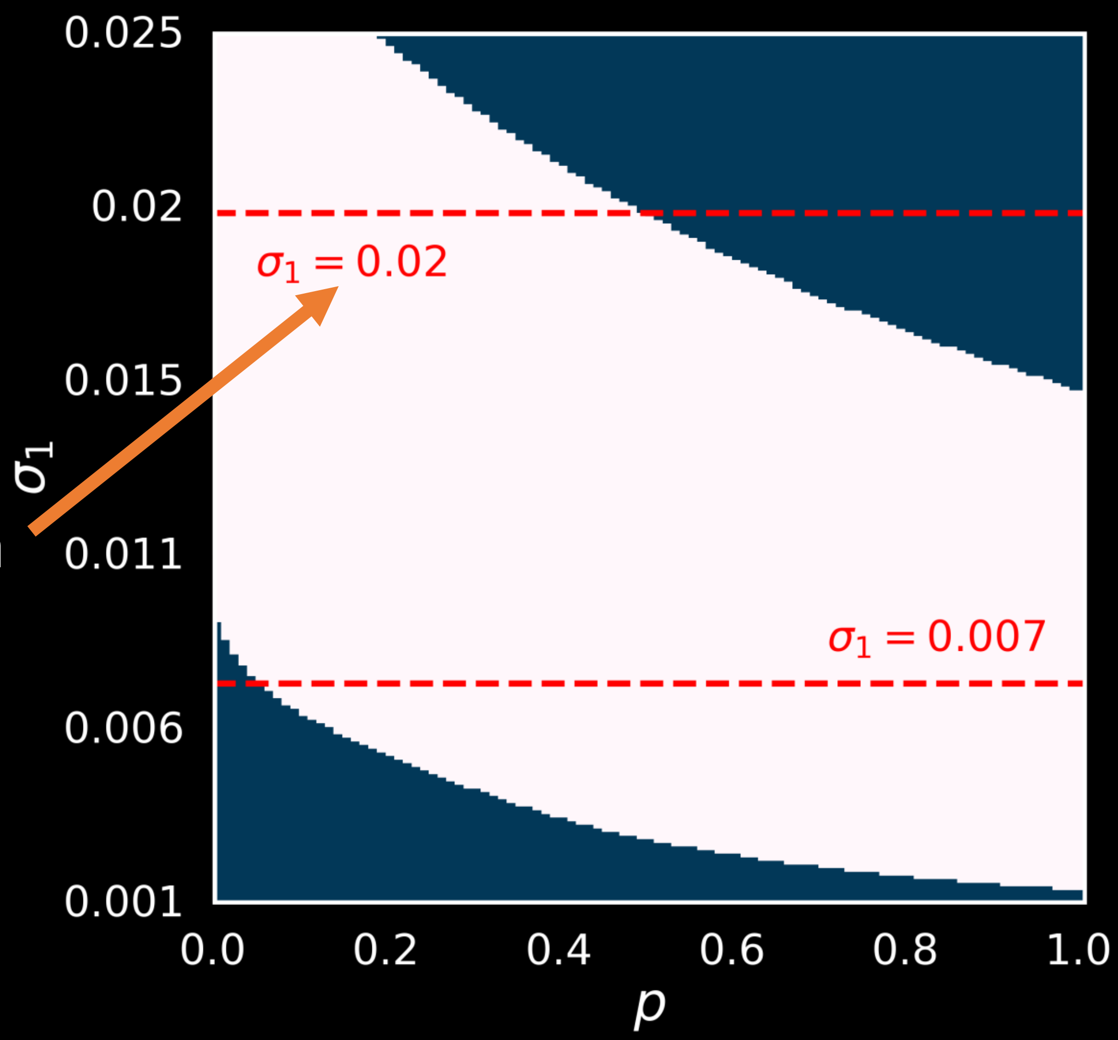
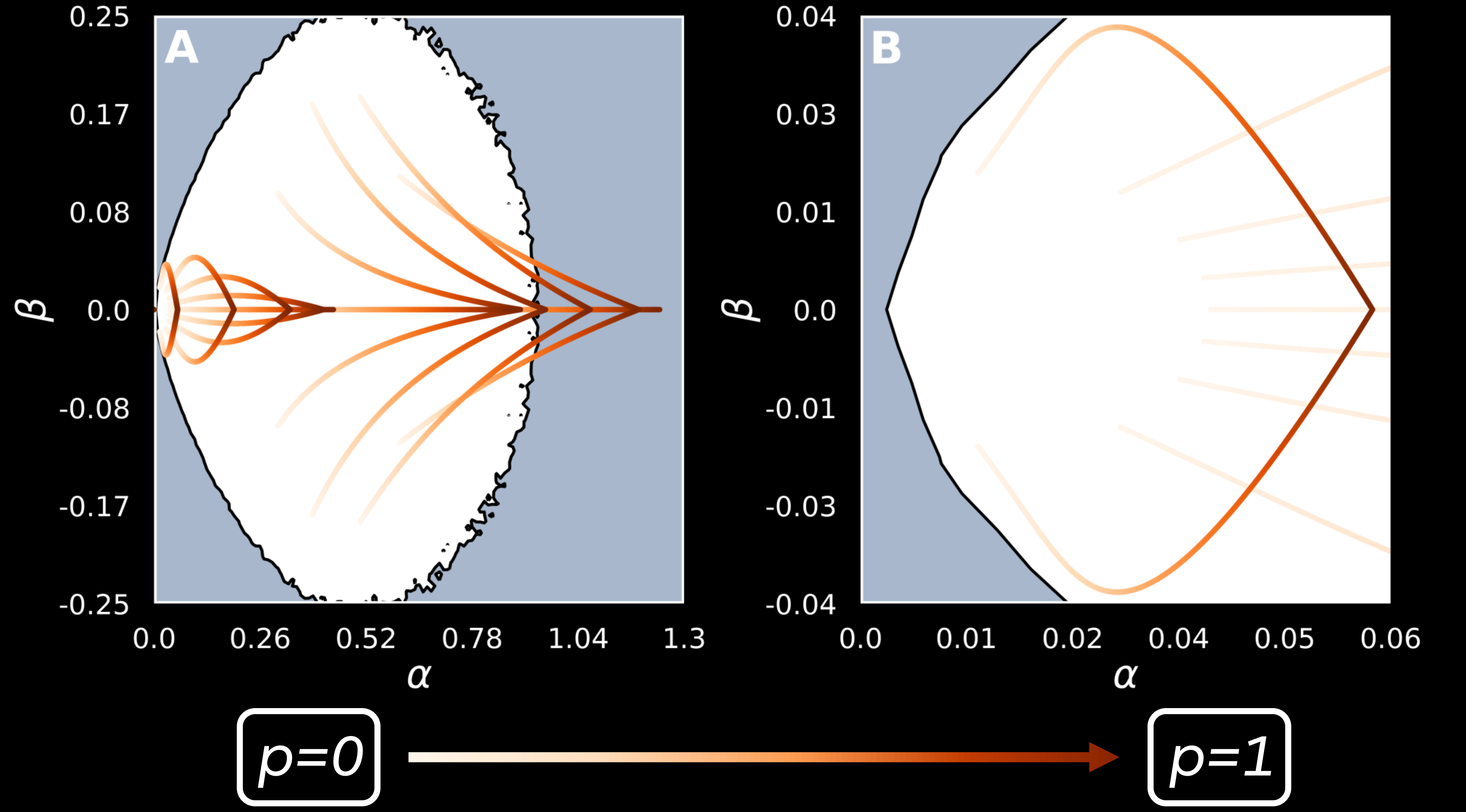


Desynchronization

NONRECIPROCAL INTERACTIONS CAN FAVOR SYNCHRONIZATION



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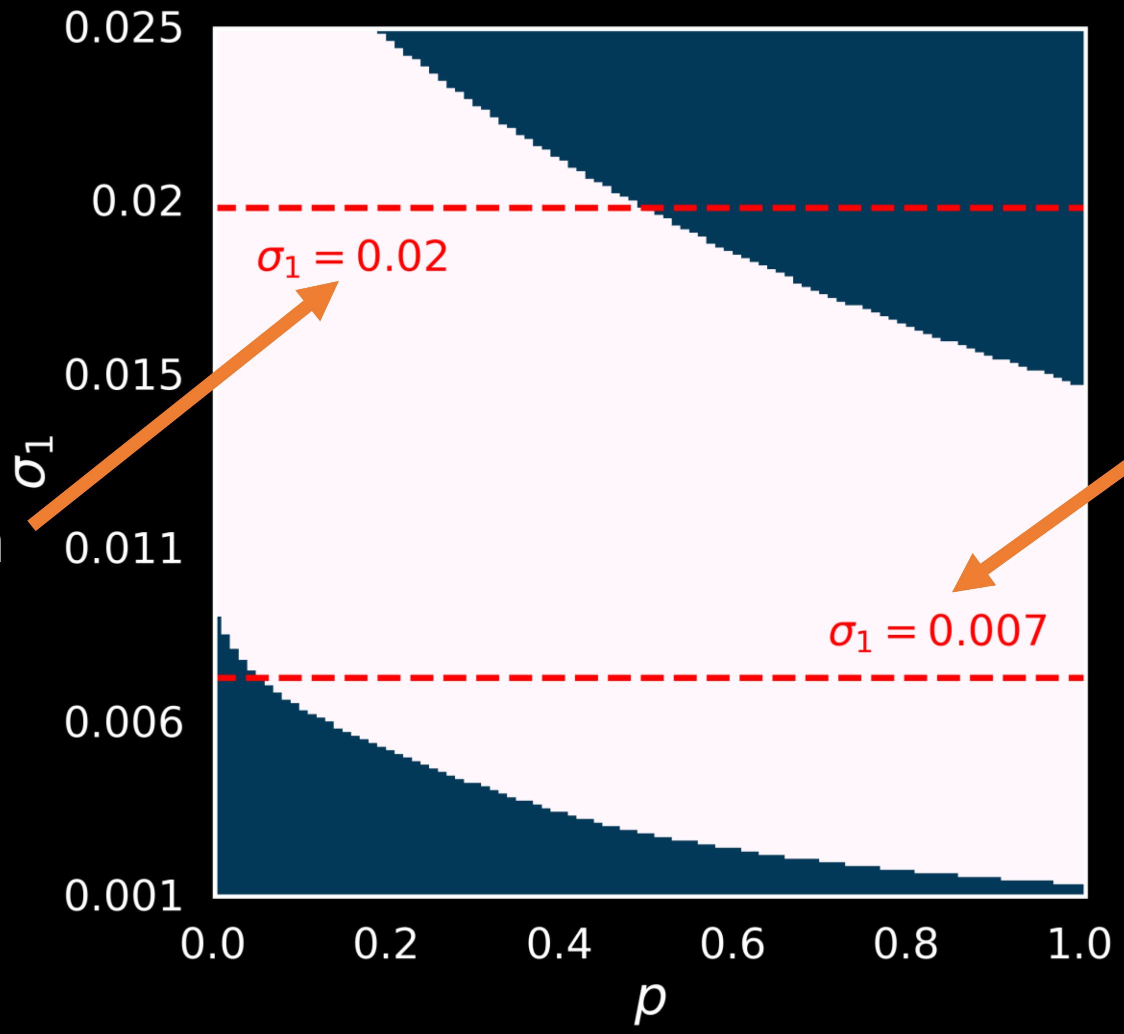
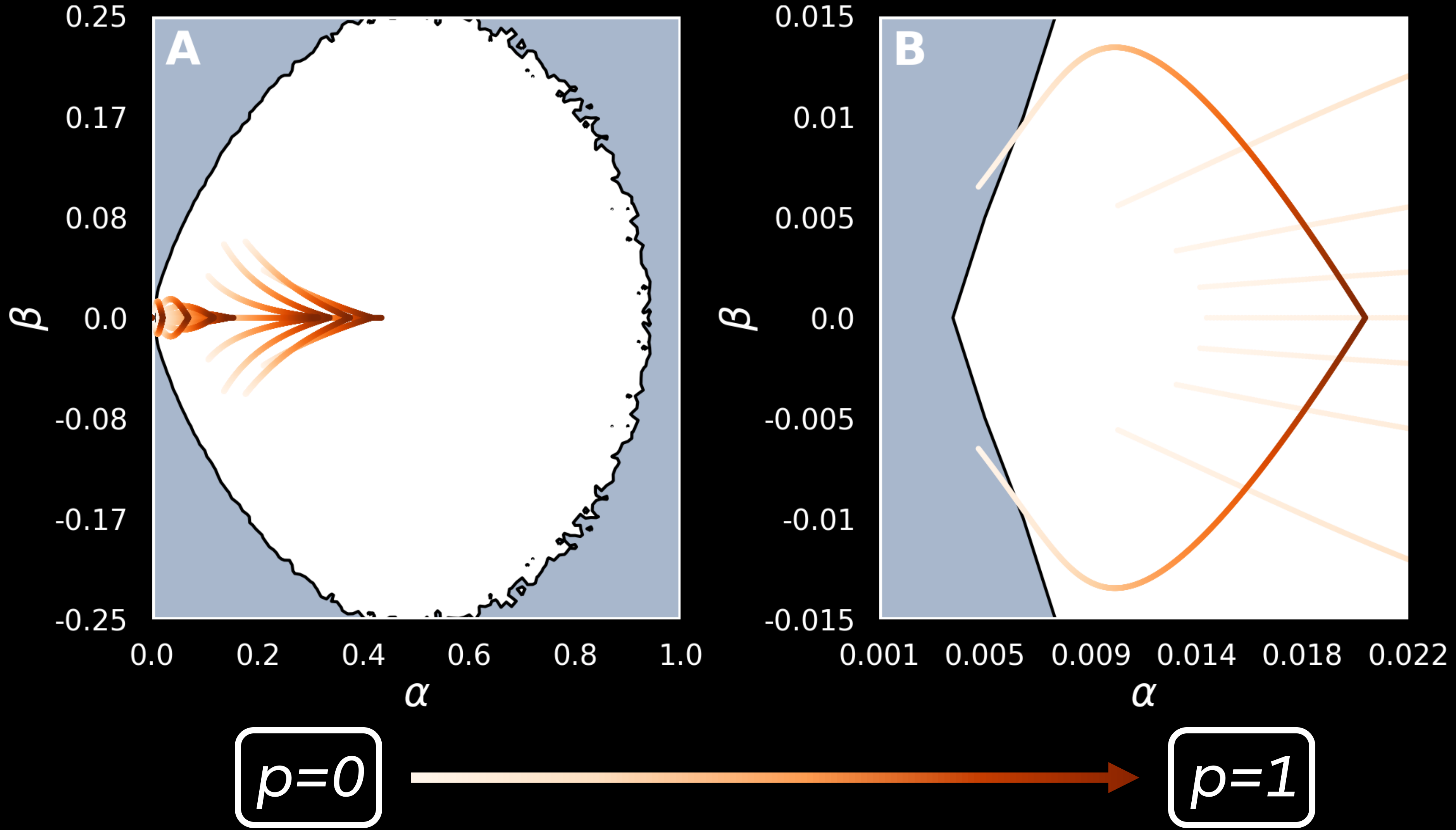


Desynchronization

Synchronization

NONRECIPROCAL INTERACTIONS CAN HAMPER SYNCHRONIZATION



CONCLUSION

- We have introduced a hypergraph representation to encode nonreciprocal group interactions
- Such formalism allowed us to extend the Master Stability Function approach to study synchronization of chaotic oscillators in presence of directed higher-order interactions
- We found that the stability of the synchronized state can be lost or gained as directionality in the higher-order structure is varied

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