FisyMat
Máster en Matemática Aplicada

UNIVERSIDAD DE GRANADA

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6. Nuclear processes in stars

Nuclear Fusion

For a star in hydrodynamic and thermal equilibrium, an internal energy source is required to balance the radiative energy loss from the surface.

Chemical composition of the star is modified because of this

Nuclear reactions do not determine the star L (energy transport; $\boldsymbol{\kappa}$), but they determine for how long it is able to sustain it.

During fusion nuclear reaction of two nuclei/particles (*i* and *j*), producing new nuclei/particles *k* and *l*, the reaction conserves charge (Z), baryonic mass (A), and lepton number:

$$i + j \to k + l$$ demands $$\begin{aligned} Z_i + Z_j &= Z_k + Z_l \\ A_i + A_j &= A_k + A_l \end{aligned}$$

Energy production

However mass is not conserved.

Masses of atomic nuclei are not the sum of individual nucleons masses, because they are bound together by strong nuclear force

$$\underbrace{E_{B,i}}_{\text{Binding energy}} = \overbrace{\left[(A_i - Z_i)m_n + Z_i m_p - \underbrace{m_i}_{\text{Mass of nucleus } i}\right]}^{\Delta m \text{ = mass defect}} c^2$$

Δm c² = E released in a reaction

And the energy released by a reaction i+j -> k +l

$$Q_{ij,k} = (m_i + m_j - m_k - m_l)c^2$$

where $Q_{ij,k} < 0$ end.

$Q_{ij,k} > 0$ exo.

The mass defect is usually written in terms of atomic (i.o. nuclear) mass

$$m_u c^2 = (1/12) m_{^{12}C} = 931.494 \text{ MeV}$$

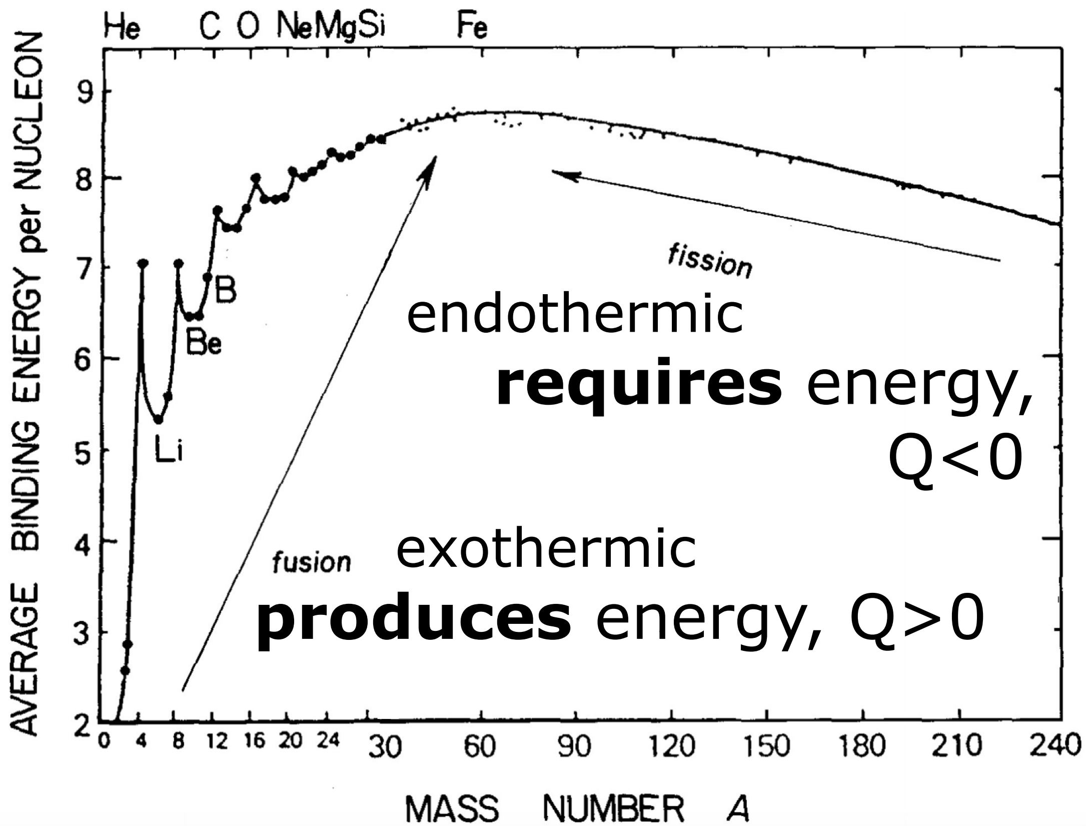
$$\Delta M_i = (m_i - A_i m_u)c^2$$ so $$Q_{ij,k} = \Delta M_i + \Delta M_j - \Delta M_k - \Delta M_l$$

Binding energy per nucleon E_B/A

Atomic masses

element	Z	A	M/m_u	element	Z	A	M/m_u	element	Z	A	M/m_u
n	0	1	1.008665	C	6	12	12.000000	Ne	10	20	19.992441
H	1	1	1.007825		6	13	13.003354	Mg	12	24	23.985043
	1	2	2.014101	N	7	13	13.005738	Si	14	28	27.976930
He	2	3	3.016029		7	14	14.003074	Fe	26	56	55.934940
	2	4	4.002603		7	15	15.000108	Ni	28	56	55.942139
Li	3	6	6.015124	O	8	15	15.003070				
	3	7	7.016003		8	16	15.994915				
Be	4	7	7.016928		8	17	16.999133				
	4	8	8.005308		8	18	17.999160				

$$\frac{E_B}{A}$$



^{56}Fe fusion is endothermic

It is the natural endpoint of the stellar nuclear reaction cycles

The decrease is due to the increase in the number of protons Z, which experience a repulsive Coulomb force

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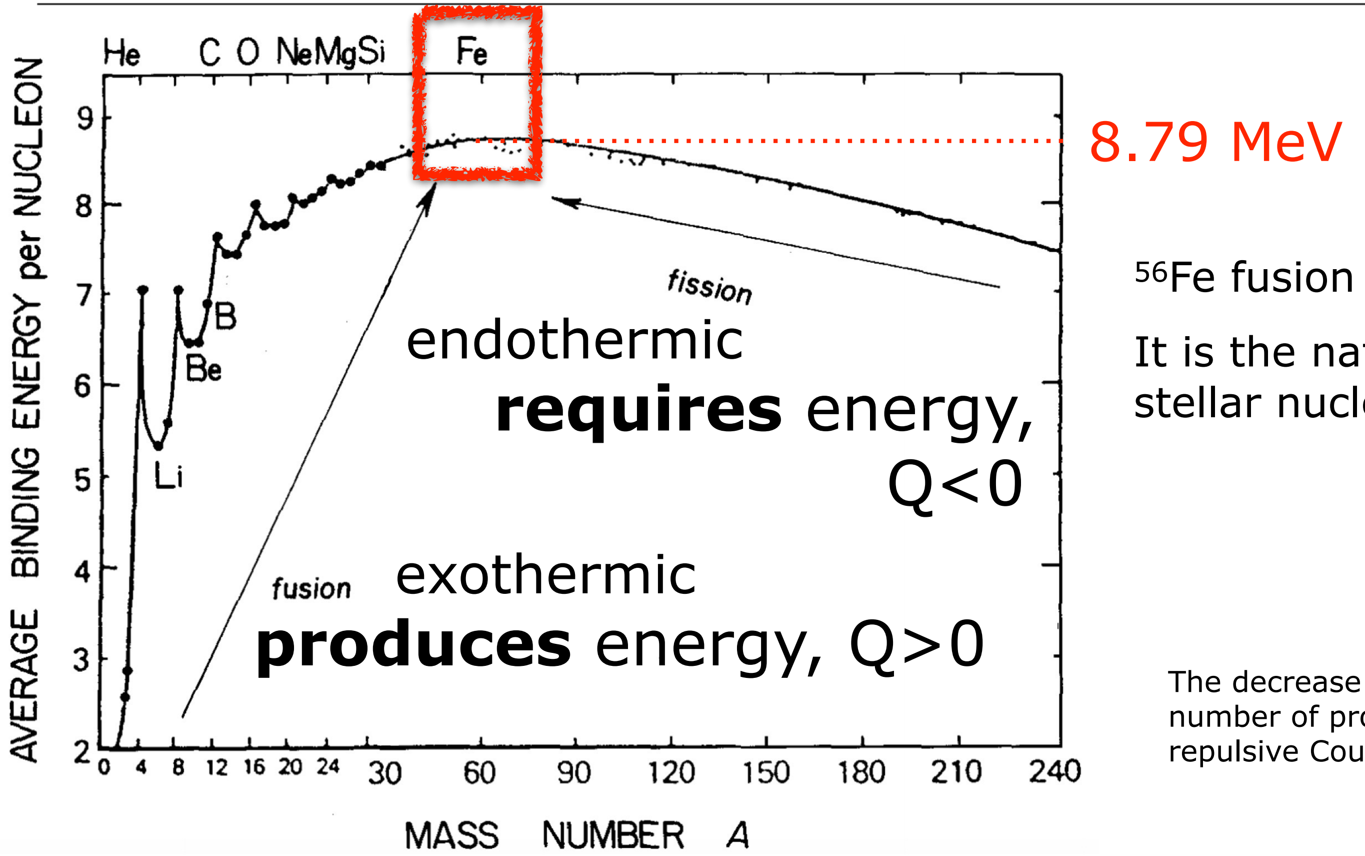

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Reaction rates

The rate of a nuclear reaction depends on the interaction cross section σ

Being the flux of incident particles $n_j v$, the number of reactions is $n_j v\sigma$
per sec.

The number of reactions per sec and cm³,

$$r_{ij} = \frac{n_i n_j v\sigma(v)}{1+\delta_{ij}} = \frac{n_i n_j}{1+\delta_{ij}} \int_0^\infty \underline{\phi(v)}\sigma(v) v dv = \frac{n_i n_j}{1+\delta_{ij}} \langle \sigma v \rangle$$

$\delta_{ij} = 1 \quad \text{if } i = j$

reaction between same nuclei consumes 2 nuclei

norm. distr. velocities

For an ideal gas, $\phi(v)$ is described by a Maxwell-Boltzmann distribution

$$\phi(v) = 4\pi v^2 \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left(-\frac{mv^2}{2kT}\right) \qquad \text{with} \qquad m = \frac{m_i m_j}{m_i + m_j}$$

and, replacing v with $E = \frac{1}{2} m v^2$

Only depends on T

$$\langle \sigma v \rangle = \left(\frac{8}{\pi m}\right)^{1/2} (kT)^{-3/2} \int_0^\infty \sigma(E) E \exp\left(-\frac{E}{kT}\right) dE$$

Cross-sections

Cross-sections are really difficult to calculate

classical approx. $\sigma = \pi (R_i + R_j)^2$ $\quad R_i \approx R_0 A_i^{1/3}$, $\quad R_0 = 1.44 \times 10^{-13}$ cm

QM approx. $\sigma = \pi \lambda^2$ $\quad \lambda = \frac{\hbar}{p} = \frac{\hbar}{(1mE)^{1/2}}$

de Broglie

However, the real situation is more complicated due to:

- *Coulomb force*. Although weaker than strong nuclear force, charged nuclei experience a repulsive force that would prevent any reaction. However, in QM we need to consider the tunnel effect

- *Nature of forces involved*. If the reaction emits a γ, only the strong force is involved and σ is similar to geometrical. But if it emits e^-+ν, the electromagnetic and weak nuclear force are involved: σ smaller

- *Nuclear structure effects.* Such as resonant interactions

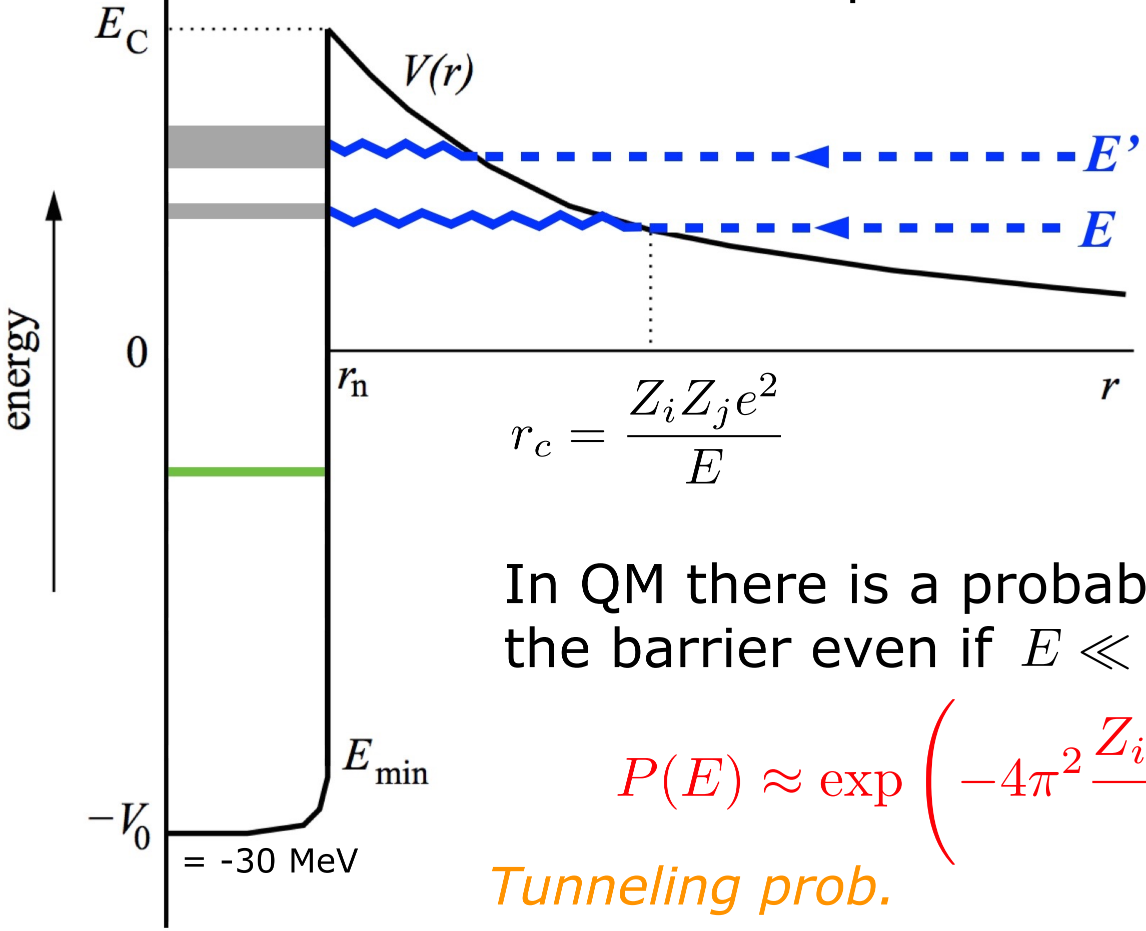
Coulomb barrier and tunnel effect

Two nuclei experience a repulsive Coulomb potential $V(r) = \dfrac{Z_i Z_j e^2}{r}$

To experience the attractive nuclear force they should be at $r_n \approx R_0 A^{1/3}$

So particles must overcome a Coulomb barrier $E_C = V(r_n) \approx Z_i Z_j \text{ MeV}$

Fusion requires that they get over it…



$$r_c = \frac{Z_i Z_j e^2}{E}$$

Classically, a particle with energy E would only reach r_C

$$\langle E \rangle = \frac{3}{2} kT \approx 1.3 \text{ keV at } 10^7 \text{ K}$$

So reactions have no chance of happening in CM

In QM there is a probability that the particle penetrates the barrier even if $E \ll E_C$

$$P(E) \approx \exp\left(-4\pi^2 \frac{Z_i Z_j e^2 \left(\frac{1}{2}m\right)^{1/2}}{h E^{1/2}}\right) \sim \exp\left(-\frac{b}{\sqrt{E}}\right)$$

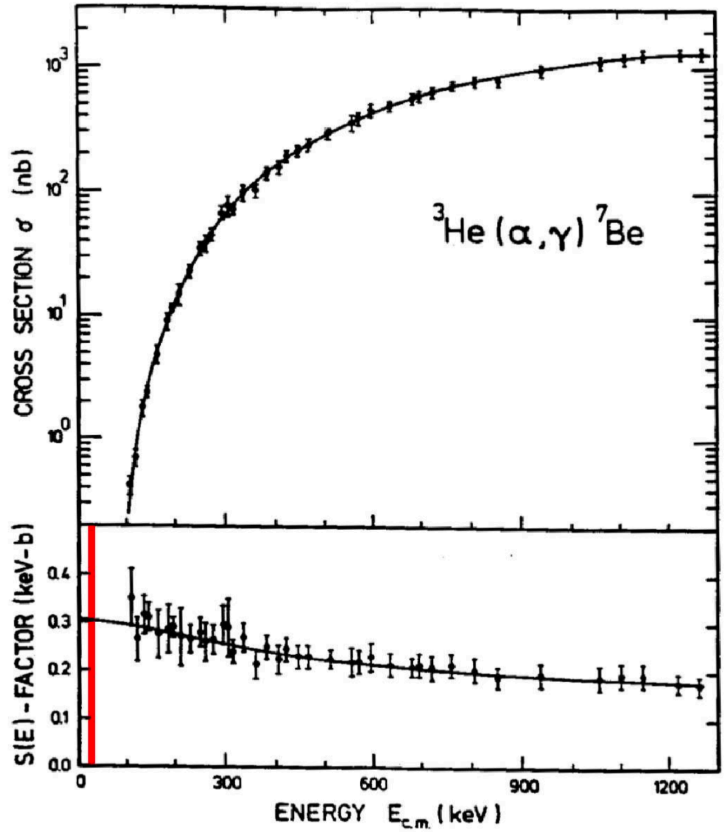
Tunneling prob.

(b = constant that depends on reaction)

Cross-section

Following the QM approx. $\sigma \approx \pi \lambda^2 P(E)$ $\lambda = \frac{\hbar}{p} = \frac{\hbar}{(1mE)^{1/2}} \approx \frac{1}{\sqrt{E}}$

$$\sigma(E) = \frac{S(E)}{E} \exp\left(-\frac{b}{\sqrt{E}}\right)$$



Where *S(E)* is the *astrophysical S-factor*, a smooth function of E, which includes all remaining effects (structure and resonances)

And $\langle \sigma v \rangle$ becomes,

$$\langle \sigma v \rangle = \left(\frac{8}{\pi m}\right)^{1/2} (kT)^{-3/2} \int_0^\infty S(E) \exp\left(-\frac{E}{kT} - \frac{b}{\sqrt{E}}\right) dE$$

M-B prob. Tuneling prob.

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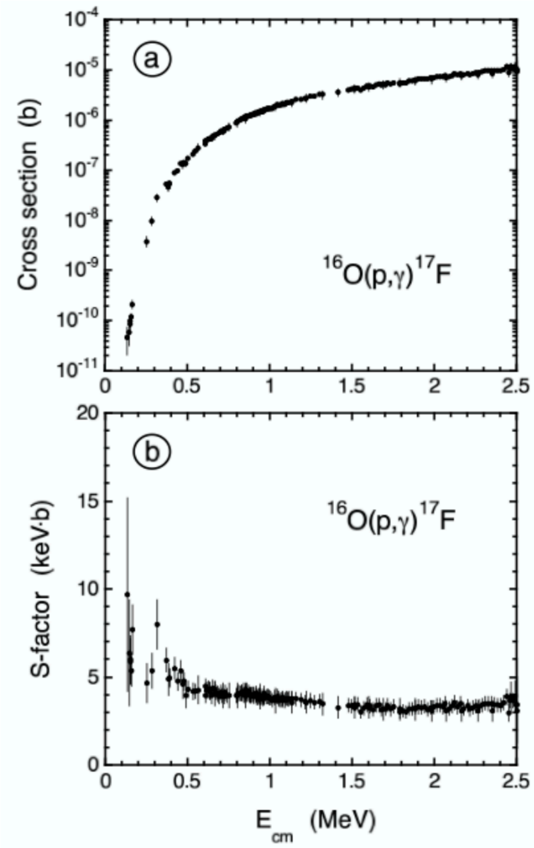

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M-B prob. Tuneling prob.

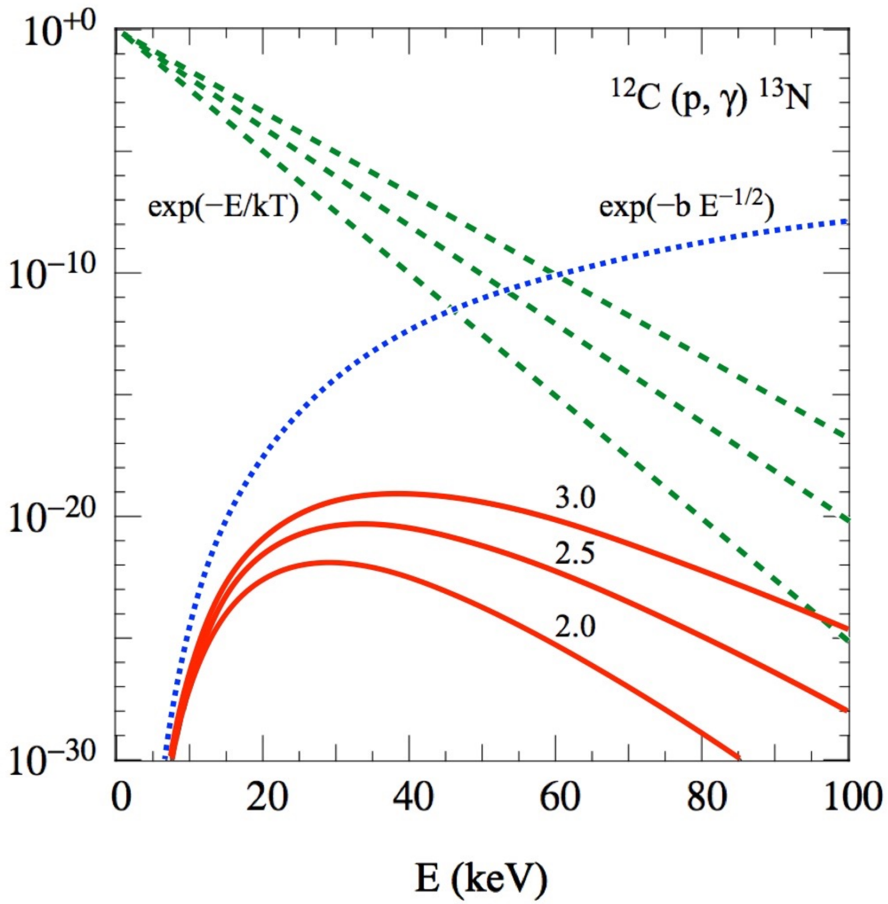
Gamow peak

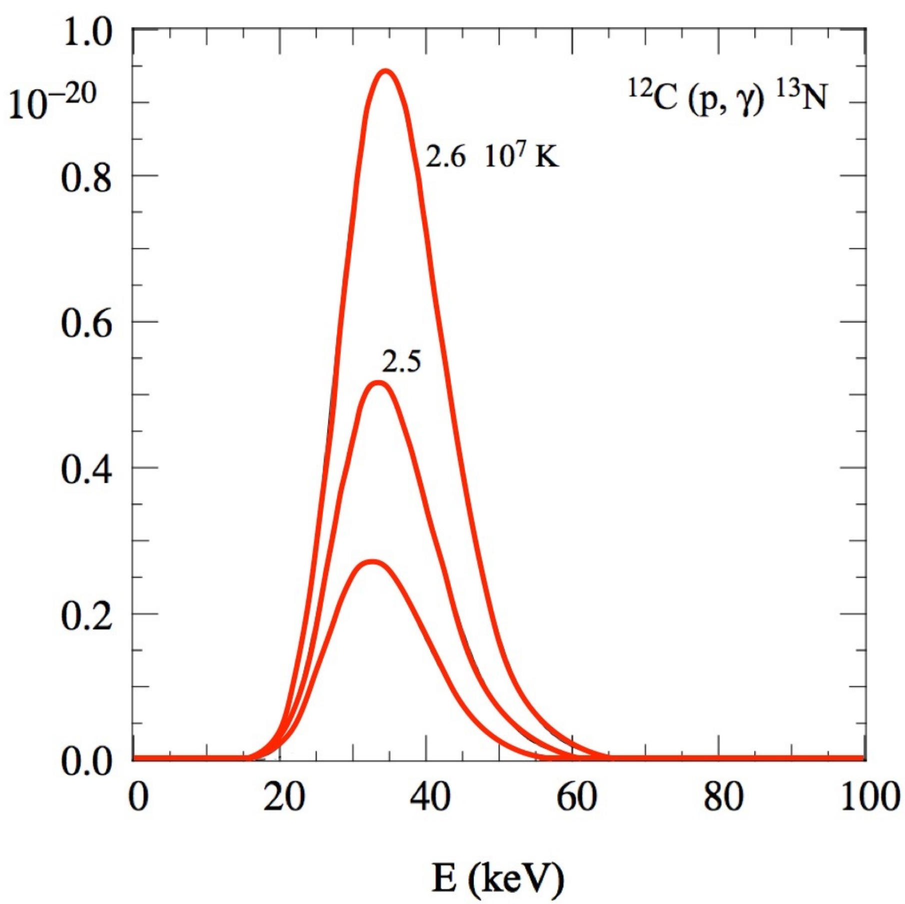
The product of the two exponentials is called the Gamow peak

$$f(E) = \exp\left(-\frac{E}{kT} - \frac{b}{\sqrt{E}}\right)$$

Which is a function of E with a peak at $df/dE = 0$

$$E_0 = \left(\frac{1}{2}bkT\right)^{2/3} \sim 5.665\left(Z_i Z_j A^{1/2}\frac{T}{10^6}\right)^{2/3}\ \text{keV}$$





Cross-section factor

Therefore the factor f(E) can be approx. to $f(E) \approx f(E_0)\exp\left[-\left(\frac{E-E_0}{\Delta E}\right)^2\right]$

Where $f(E_0) = \exp\left(-\frac{3E_0}{kT}\right) \equiv \exp(-\tau)$

and expanding f(E) in a Taylor series, we can then obtain,

$$\int_0^\infty f(E)dE \approx e^{-\tau}\int_0^\infty \exp\left[-\left(\frac{E-E_0}{\Delta E}\right)^2\right]dE \approx e^{-\tau}\sqrt{\pi}\Delta E$$

Finally, the $\langle\sigma v\rangle$ factor ends up being

$$\langle\sigma v\rangle \approx \frac{7.21\times 10^5}{Z_i Z_j A}\left(\frac{S(E_0)}{\mathrm{keV\ cm^2}}\right)\tau^2 e^{-\tau} \quad \propto \frac{1}{T^{2/3}}\exp\left(-\frac{C}{T^{1/3}}\right)$$

properties of $\langle\sigma v\rangle$: - increases very strongly with T

- decreases strongly with the Coulomb barrier

Energy generation and composition rates

The E generation rate per unit mass and time

$$n_i = \frac{X_i \rho}{A_i m_u}$$

$$\epsilon_{ij} = \frac{Q_{ij} r_{ij}}{\rho} = \frac{Q_{ij}}{\rho} \frac{n_i n_j}{1+\delta_{ij}} \langle \sigma v \rangle = \frac{Q_{ij}}{(1+\delta_{ij}) A_i A_j m_u^2} \rho X_i X_j \langle \sigma v \rangle$$

So then, $\epsilon_{\rm nuc} = \sum_{i,j} \epsilon_{ij}$

Reaction rates also determine the composition change rate

The rate of change of number density of I

$$\frac{dn_i}{dt} = -\underbrace{\sum_{j,k} r_{ij,k}}_{\text{Consume}} + \underbrace{\sum_{k,l} r_{kl,i}}_{\text{Produce}}$$

Where, using $n_i = \frac{X_i \rho}{A_i m_u}$ we can obtain the change in mass fraction

$$\frac{dX_i}{dt} = A_i \frac{m_u}{\rho} \left(\sum_{k,l} r_{kl,i} - \sum_{j,k} r_{ij,k} \right)$$

Main nuclear burning cycles

In principle, many different nuclear reactions can occur simultaneously in a stellar interior.

But for the calculation of the structure and evolution of a star usually a much simpler procedure is sufficient

- nuclear fusions of different possible fuels are well separated by substantial *T* differences. The evolution of a star therefore proceeds through several distinct *nuclear burning cycles*
- For each nuclear burning cycle, only a handful of reactions contribute significantly to the energy production
- In a chain of subsequent reactions, often one reaction is by far the slowest and determines the rate of the whole chain

Hydrogen burning

Net reaction: $4\,{}^1\mathrm{H} \rightarrow {}^4\mathrm{He} + 2\mathrm{e}^+ + 2\nu + 2\gamma$

The total energy release is $Q = 26.734\ \mathrm{MeV}$

in order to create 1 ***He*** nucleus,
2 ***p*** have to be converted into ***n***
-> 2 neutrinos are released

$$p \rightarrow n + e^+ + \nu$$

Since a simultaneous reaction between four ***p*** is extremely unlikely, a chain of reactions is always necessary for ***H*** burning.

p-p chain & ***CNO cycle***

$(5 \times 10^6 < T < 15 \times 10^6\ \mathrm{K})$ $(T > 15 \times 10^6\ \mathrm{K})$

The p-p chain

The 1st reaction is the p-p decay:

$$^{1}\mathrm{H} + {}^{1}\mathrm{H} \rightarrow {}^{2}\mathrm{H} + \mathrm{e}^{+} + \nu$$

or

$$\mathrm{p} + \mathrm{p} \rightarrow \mathrm{D} + \mathrm{e}^{+} + \overset{0.265\ \mathrm{MeV}}{\nu}$$

D then reacts with other p: $^{2}\mathrm{H} + {}^{1}\mathrm{H} \rightarrow {}^{3}\mathrm{He} + \gamma$

And 3 different chains are possible

69%

$$^{3}\mathrm{He} + {}^{3}\mathrm{He} \rightarrow {}^{4}\mathrm{He} + 2\,{}^{1}\mathrm{H}$$

PPI 26.20 MeV $T < 8 \times 10^{6}$ K

31%

$$^{3}\mathrm{He} + {}^{4}\mathrm{He} \rightarrow {}^{7}\mathrm{Be} + \gamma$$

$$8 \times 10^{6} < T < 15 \times 10^{6}$$

99.7%

$$^{7}\mathrm{Be} + \mathrm{e}^{-} \rightarrow {}^{7}\mathrm{Li} + \overset{0.814\ \mathrm{MeV}}{\nu}$$

$$^{7}\mathrm{Li} + {}^{1}\mathrm{H} \rightarrow {}^{4}\mathrm{He} + {}^{4}\mathrm{He}$$

PPII 25.66 MeV

0.3%

$$^{7}\mathrm{Be} + {}^{1}\mathrm{H} \rightarrow {}^{8}\mathrm{B} + \gamma$$

$$^{8}\mathrm{B} \rightarrow {}^{8}\mathrm{Be} + \mathrm{e}^{+} + \overset{6.71\ \mathrm{MeV}}{\nu}$$

$$^{8}\mathrm{Be} \rightarrow {}^{4}\mathrm{He} + {}^{4}\mathrm{He}$$

PPIII 19.76 MeV

$$\epsilon_{\mathrm{pp}} \sim X^{2} \rho T^{4}$$

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The CNO chain

CN-cycle

$$
\begin{aligned}
{}^{12}\mathrm{C} + {}^{1}\mathrm{H} &\rightarrow {}^{13}\mathrm{N} + \gamma \\
{}^{13}\mathrm{N} &\rightarrow {}^{13}\mathrm{C} + e^{+} + \nu \\
{}^{13}\mathrm{C} + {}^{1}\mathrm{H} &\rightarrow {}^{14}\mathrm{N} + \gamma \\
{}^{14}\mathrm{N} + {}^{1}\mathrm{H} &\rightarrow {}^{15}\mathrm{O} + \gamma \\
{}^{15}\mathrm{O} &\rightarrow {}^{15}\mathrm{N} + e^{+} + \nu \\
{}^{15}\mathrm{N} + {}^{1}\mathrm{H} &\rightarrow {}^{12}\mathrm{C} + {}^{4}\mathrm{He}
\end{aligned}
$$

${}^{12}C$ nucleus is reproduced after the cycle, so it only acts as a catalyst

$$(T > 15 \times 10^6 \ \mathrm{K})$$

$$\epsilon \sim X X_{14} \rho T^{18}$$

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$$

ON-cycle I

10^{-3} probability

$$
\begin{aligned}
{}^{15}\mathrm{N} + {}^{1}\mathrm{H} &\rightarrow {}^{16}\mathrm{O} + \gamma \\
{}^{16}\mathrm{O} + {}^{1}\mathrm{H} &\rightarrow {}^{17}\mathrm{F} + \gamma \\
{}^{17}\mathrm{F} \quad &\rightarrow {}^{17}\mathrm{O} + e^{+} + \nu \\
{}^{17}\mathrm{O} + {}^{1}\mathrm{H} &\rightarrow {}^{14}\mathrm{N} + {}^{4}\mathrm{He}
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The CNO chain

CN-cycle ON-cycle I ON-cycle II

$$^{12}C + {}^{1}H \rightarrow {}^{13}N + \gamma$$
$$^{13}N \rightarrow {}^{13}C + e^{+} + \nu$$
$$^{13}C + {}^{1}H \rightarrow {}^{14}N + \gamma$$
$$^{14}N + {}^{1}H \rightarrow {}^{15}O + \gamma$$
$$^{15}O \rightarrow {}^{15}N + e^{+} + \nu$$
$$^{15}N + {}^{1}H \rightarrow {}^{12}C + {}^{4}He$$

^{12}C nucleus is reproduced after the cycle, so it only acts as a catalyst

10^{-3} probability

$$^{15}N + {}^{1}H \rightarrow {}^{16}O + \gamma$$
$$^{16}O + {}^{1}H \rightarrow {}^{17}F + \gamma$$
$$^{17}F \rightarrow {}^{17}O + e^{+} + \nu$$
$$^{17}O + {}^{1}H \rightarrow {}^{14}N + {}^{4}He$$

$$^{17}O + {}^{1}H \rightarrow {}^{18}F + \gamma$$
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$$^{18}O + {}^{1}H \rightarrow {}^{15}N + {}^{4}He$$

$$(T > 15 \times 10^{6}\ \mathrm{K})$$

$$\epsilon \sim X X_{14} \rho T^{18}$$

The CNO chain

CN-cycle | **ON-cycle I** | **ON-cycle II** | **ON-cycle III**

$^{12}C + {}^{1}H \rightarrow {}^{13}N + \gamma$
$^{13}N \rightarrow {}^{13}C + e^{+} + \nu$
$^{13}C + {}^{1}H \rightarrow {}^{14}N + \gamma$
$^{14}N + {}^{1}H \rightarrow {}^{15}O + \gamma$
$^{15}O \rightarrow {}^{15}N + e^{+} + \nu$
$^{15}N + {}^{1}H \rightarrow {}^{12}C + {}^{4}He$

^{12}C nucleus is reproduced after the cycle, so it only acts as a catalyst

10^{-3} probability

$^{15}N + {}^{1}H \rightarrow {}^{16}O + \gamma$
$^{16}O + {}^{1}H \rightarrow {}^{17}F + \gamma$
$^{17}F \rightarrow {}^{17}O + e^{+} + \nu$
$^{17}O + {}^{1}H \rightarrow {}^{14}N + {}^{4}He$

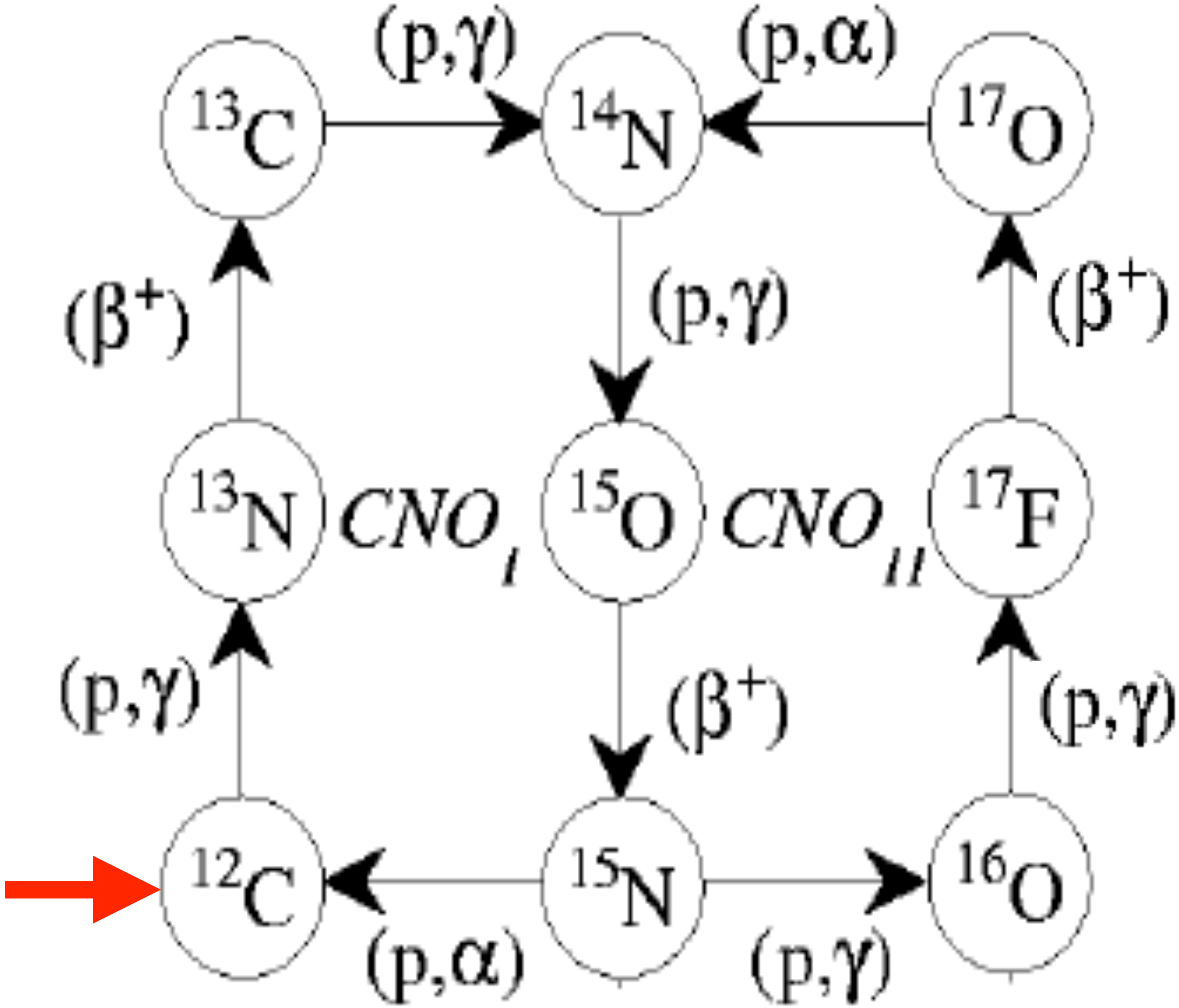
$^{17}O + {}^{1}H \rightarrow {}^{18}F + \gamma$
$^{18}F \rightarrow {}^{18}O + e^{+} + \nu$
$^{18}O + {}^{1}H \rightarrow {}^{15}N + {}^{4}He$

$^{18}O + {}^{1}H \rightarrow {}^{19}F + \gamma$
$^{19}F + {}^{1}H \rightarrow {}^{16}O + {}^{4}He$

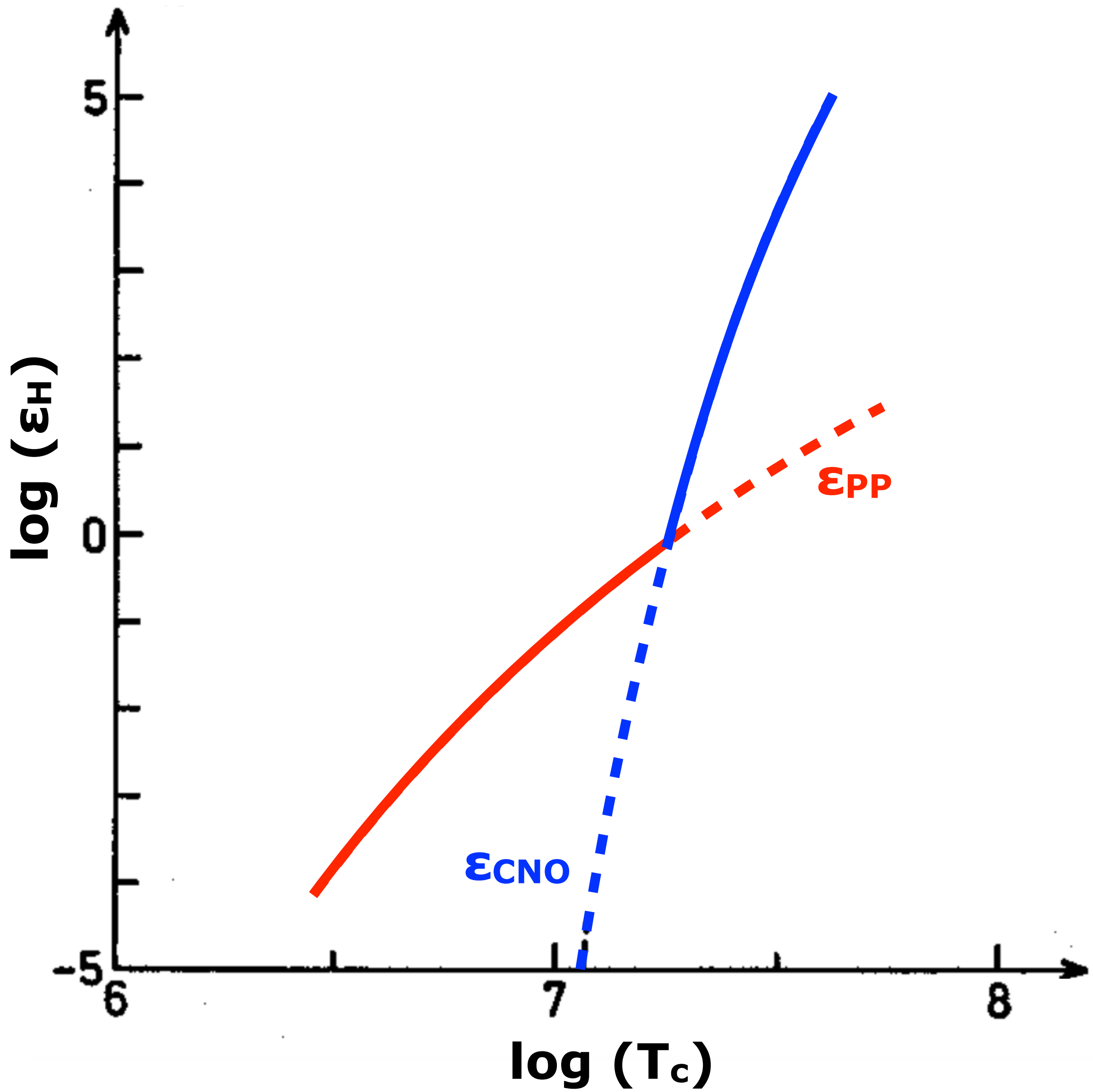
$$(T > 15 \times 10^{6}\ \mathrm{K})$$

$$\epsilon \sim X X_{14} \rho T^{18}$$

The CNO chain



Total energy generation H->He



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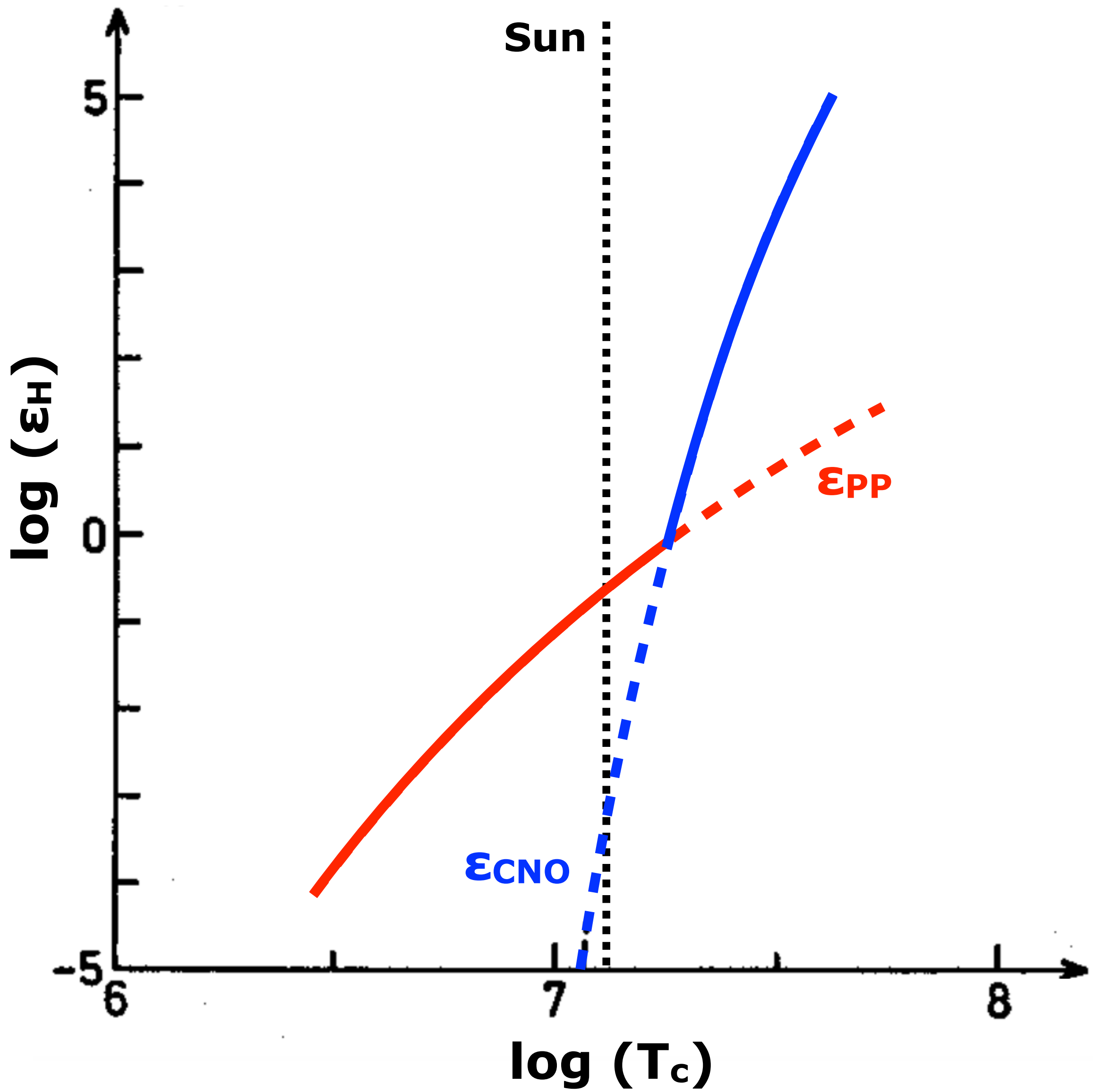

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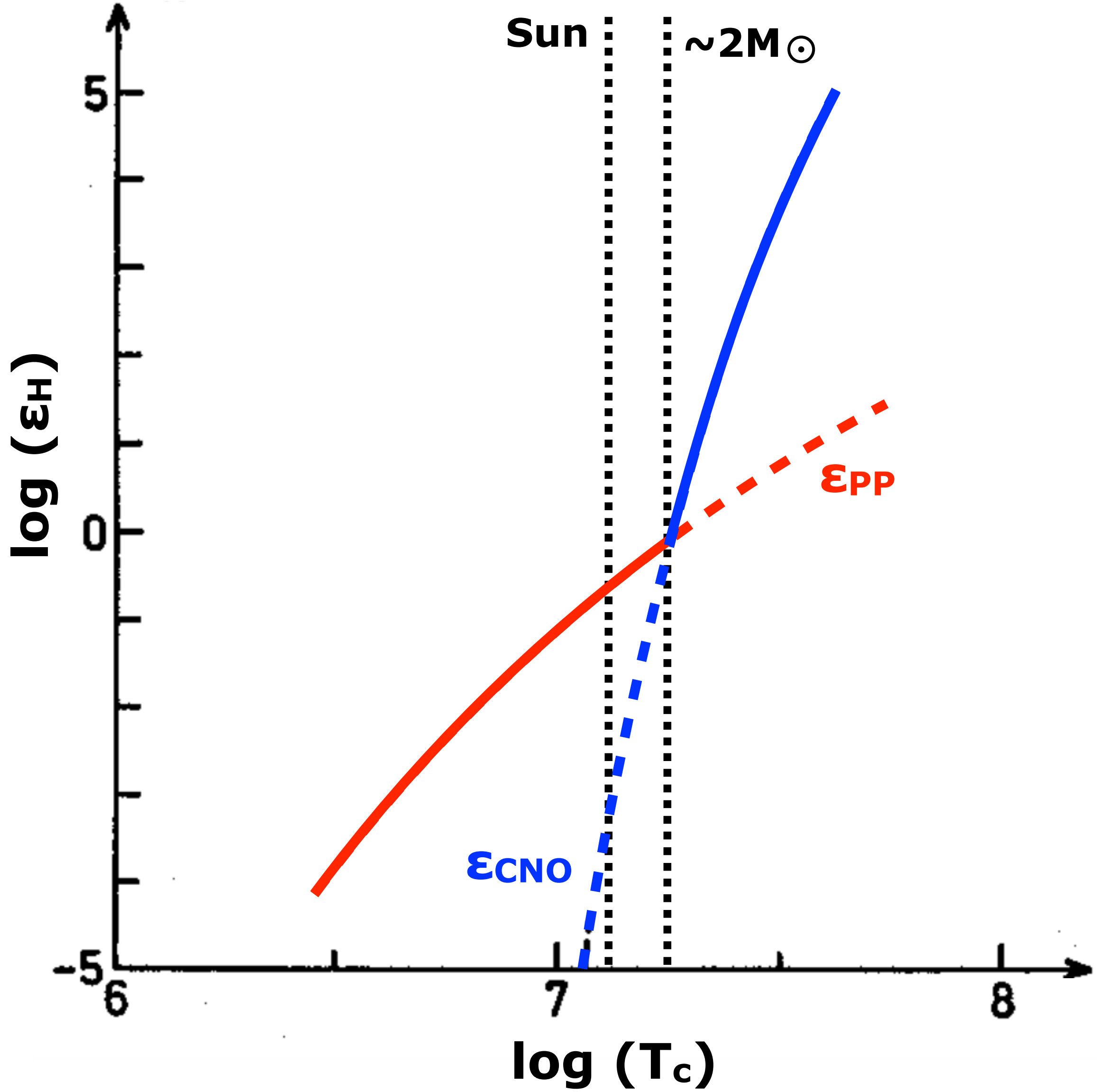

Helium burning

Helium burning into ^{12}C and ^{16}O occurs at $T>10^8$ K

- Coulomb barrier for He is higher than that of H
- No stable ion with A=8, so 2 steps are needed:

$$^{4}\mathrm{He} + {}^{4}\mathrm{He} \rightarrow {}^{8}\mathrm{Be}$$

^{8}Be lifetime ~3×10^{-16} s!

$$^{8}\mathrm{Be} + {}^{4}\mathrm{He} \rightarrow {}^{12}\mathrm{C}^{*} \rightarrow {}^{12}\mathrm{C} + \gamma$$

The second reaction takes place because ^{12}C has a resonant energy level precisely at the Gamow peak of the ^{8}Be + ^{4}He reaction

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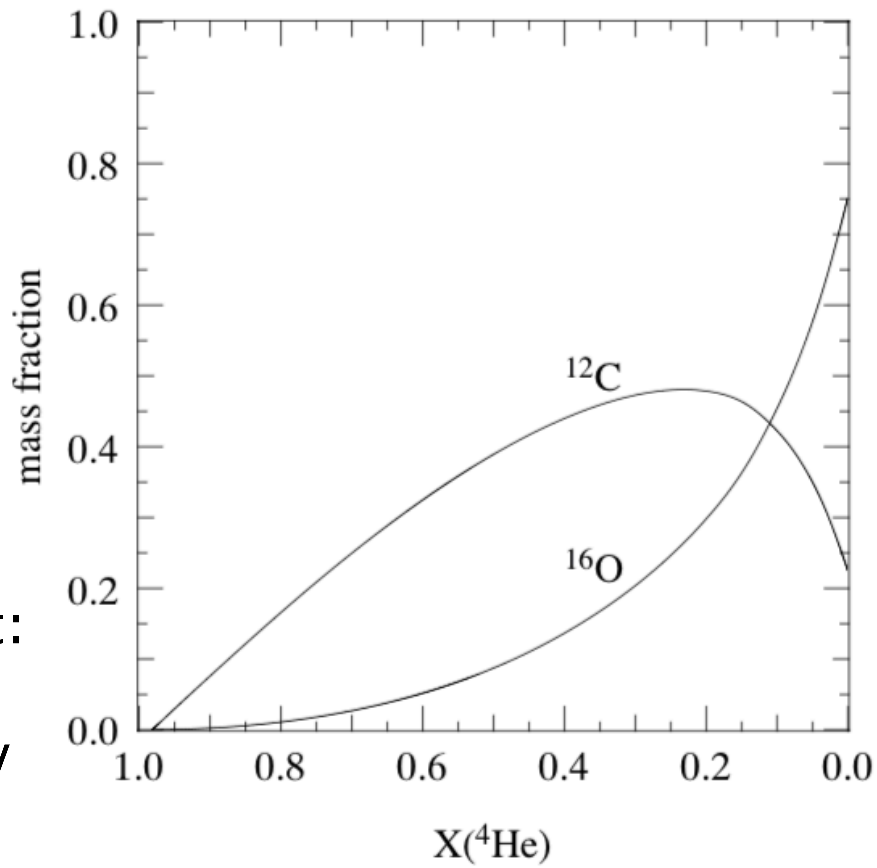
The effect of the 2 reactions is called the triple-α process

$$3\,{}^4\mathrm{He} \rightarrow {}^{12}\mathrm{C} + \gamma$$ Q=7.275 MeV

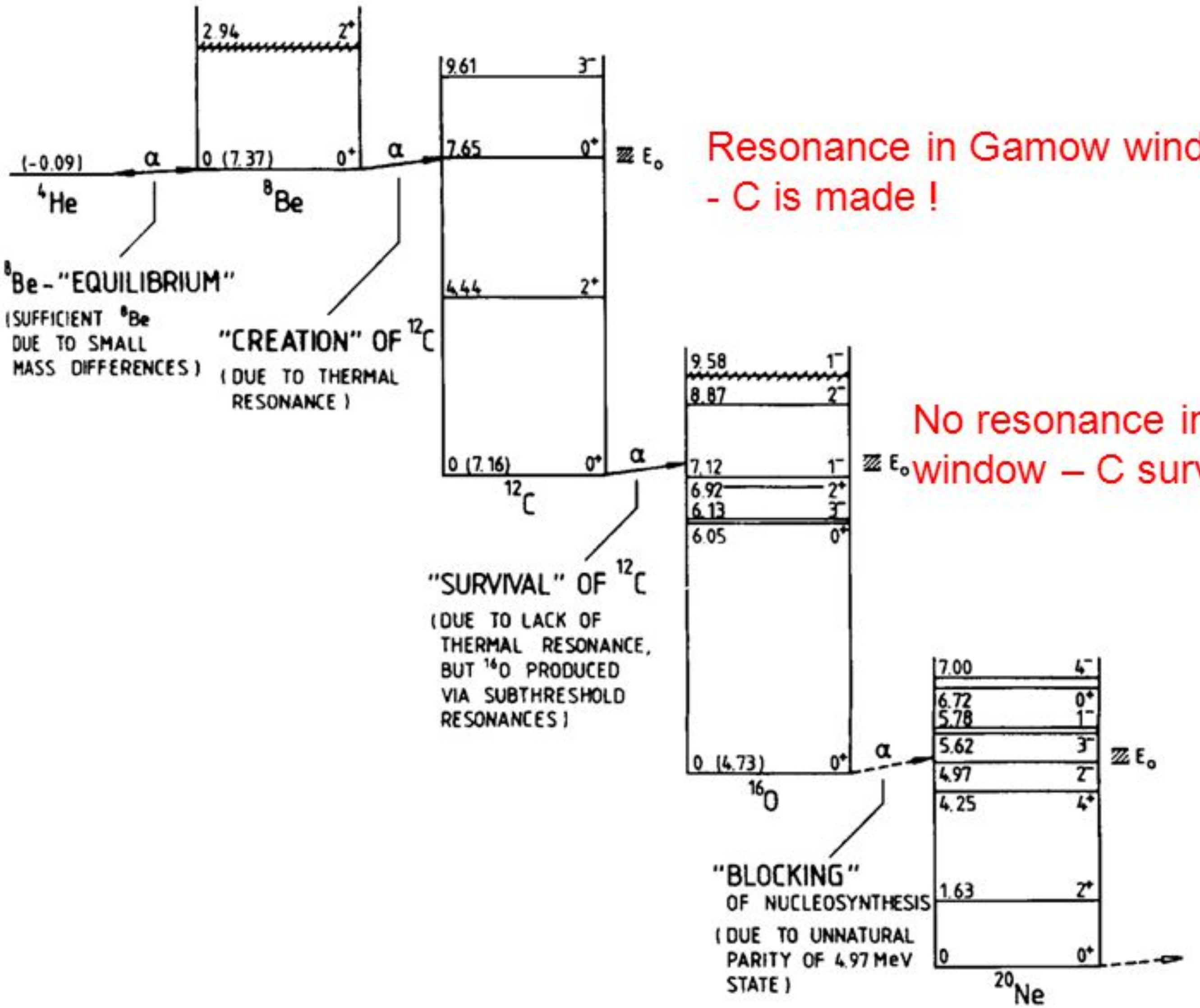
Once a sufficient amount of ^{12}C is created (near the end of the He-fusion phase) we get:

$$^{12}\mathrm{C} + {}^4\mathrm{He} \rightarrow {}^{16}\mathrm{O} + \gamma$$ Q=7.162 MeV



Helium burning

Helium burning 2 – the $^{12}C(\alpha,\gamma)$ rate



Resonance in Gamow window - C is made !

No resonance in Gamow window – C survives !

But some C is converted into O …

Carbon burning and beyond

At T > 5x10⁸ K the $^{12}C+^{12}C$ Coulomb barrier can be overcome

$$^{12}\mathrm{C} + {}^{12}\mathrm{C} \rightarrow {}^{24}\mathrm{Mg} + \gamma \begin{cases} \rightarrow {}^{20}\mathrm{Ne} + {}^{4}\mathrm{He} & Q = 4.616\ \mathrm{MeV}\ (\sim 50\%) \\ \rightarrow {}^{23}\mathrm{Na} + p & Q = 2.238\ \mathrm{MeV}\ (\sim 50\%) \end{cases}$$

At T > 1.5 x 10⁹ , *Neon burning* (Net reaction: $2^{20}\mathrm{Ne} \rightarrow {}^{16}\mathrm{O} + {}^{24}\mathrm{Mg}$)

$$^{20}\mathrm{Ne} + \gamma \rightarrow {}^{16}\mathrm{O} + {}^{4}\mathrm{He} \qquad Q = -4.73\ \mathrm{MeV}$$

$$^{20}\mathrm{Ne} + {}^{4}\mathrm{He} \rightarrow {}^{24}\mathrm{Mg} + \gamma \qquad Q = 9.31\ \mathrm{MeV}$$

At T > 2 x 10⁹ , *Oxygen burning*

$$^{16}\mathrm{O} + {}^{16}\mathrm{O} \begin{cases} \rightarrow {}^{28}\mathrm{Si} + {}^{4}\mathrm{He} & Q = 9.59\ \mathrm{MeV}\ (\sim 60\%) \\ \rightarrow {}^{31}\mathrm{P} + p & Q = 7.68\ \mathrm{MeV}\ (\sim 40\%) \end{cases}$$

and many other secondary reactions ending up with lots of ^{28}Si and ^{32}S

At T > 3 x 10⁹ , *Silicon burning*, but Coulomb barrier is very high. Instead,

Photo-desintegration $^{28}\mathrm{Si}(\gamma,\alpha)^{24}\mathrm{Mg}(\gamma,\alpha)^{20}\mathrm{Ne}(\gamma,\alpha)^{16}\mathrm{O}(\gamma,\alpha)^{12}\mathrm{C}(\gamma,\alpha)2\alpha$

α-capture $^{28}\mathrm{Si}(\alpha,\gamma)^{32}\mathrm{S}(\alpha,\gamma)^{36}\mathrm{Ar}(\alpha,\gamma)^{40}\mathrm{Ca}(\alpha,\gamma)^{44}\mathrm{Ti}(\alpha,\gamma)...^{56}\mathrm{Ni}$ which is unstable and decay:

Carbon burning and beyond

At T > 5x10^8 K the $^{12}C+^{12}C$ Coulomb barrier can be overcome

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At T > 1.5 x 10^9 , *Neon burning* (Net reaction: $2^{20}\mathrm{Ne} \rightarrow {}^{16}\mathrm{O} + {}^{24}\mathrm{Mg}$)

$$^{20}\mathrm{Ne} + \gamma \rightarrow {}^{16}\mathrm{O} + {}^{4}\mathrm{He} \qquad Q = -4.73 \text{ MeV}$$

$$^{20}\mathrm{Ne} + {}^{4}\mathrm{He} \rightarrow {}^{24}\mathrm{Mg} + \gamma \qquad Q = 9.31 \text{ MeV}$$

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$$\boxed{^{56}\mathrm{Ni} \rightarrow {}^{56}\mathrm{Co} \rightarrow {}^{56}\mathrm{Fe}}$$

Nuclear Fusion

Summary of the most important reaction rates in stars

Fuel (1)	Process (2)	T_{thresh} 10^6 K (3)	Product (4)	E_{net} MeV/nucl (5)	T_c 10^6 K * (6)	L_{net}/L (7) *	Duration yr (8) *
H	p-p chain	4	He	6.55	–	–	–
H	CNO cycle	15	He	6.25	35	0.94	1.1×10^7
He	3-α fusion	100	C,O	0.61	180	0.96	2.0×10^6
C	C-fusion	600	Ne,Mg,Na,O	0.54	810	0.16	2.0×10^3
Ne	Ne photdis	900	O,Mg,Si		1600	5.3×10^{-4}	0.7
O	O-fusion	1000	S,Si,P,Mg	0.30	1900	8.2×10^{-5}	2.6
Si	Si nucl equil.	3000	Fe,Ni,Cr,Ti	<0.18	3300	5.8×10^{-7}	0.05

Results:
- neutronization of the core
- E loss by neutrino emission
- no energy production -> break of HE and TE
- collapse of the core down to nuclear densities, $\rho \sim 10^{14}$ g/cm^3

*for $15 M_\odot$ star

Neutron capture

s-process and r-process

Free neutrons in a star can be captured by ions, creating neutron rich isotopes. The net chemical profile and stability of neutron capture products depends on the rate of capture.

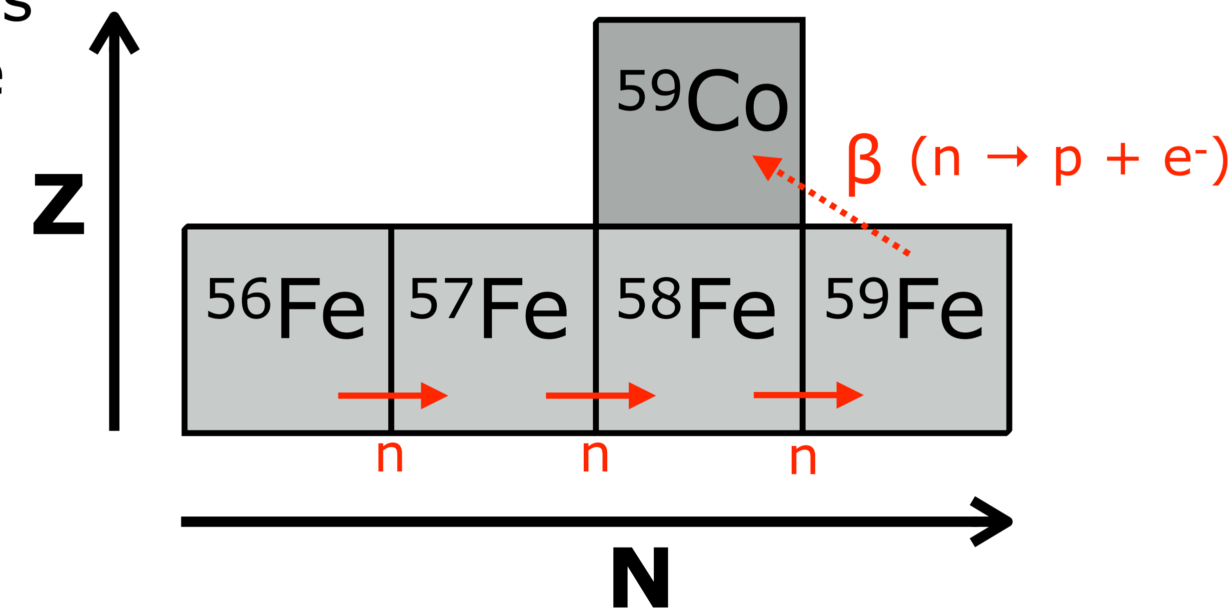
We can define $t_{capture}$, time between successive n capture, and t_{decay}, typical β-decay time for an unstable isotope.

$t_{capture} > t_{decay}$ = slow**-process**

Particle captures neutrons until it forms an unstable isotope, which then β-decays to a stable point.

Typical products: Zr, Sr, Ba, Pb

Common in: AGB stars



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$t_{capture} > t_{decay}$ = **s**low**-process**

$t_{capture} < t_{decay}$ = **r**apid**-process**

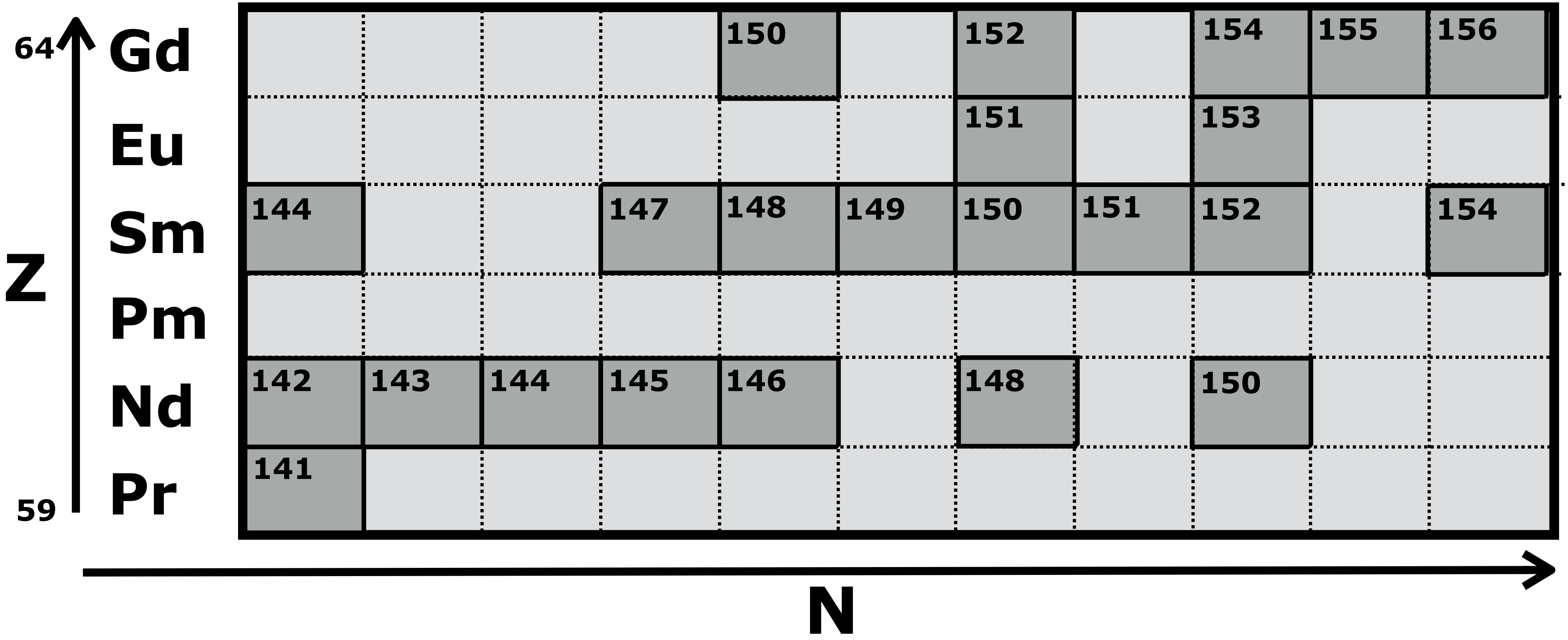
Unstable isotopes continue capturing neutrons, creating super-neutron-rich species. When neutron capture stops, isotopes undergo a series of β-decays until stable.

Typical products: Eu, Au, Xe, Pt

Common in: supernovae

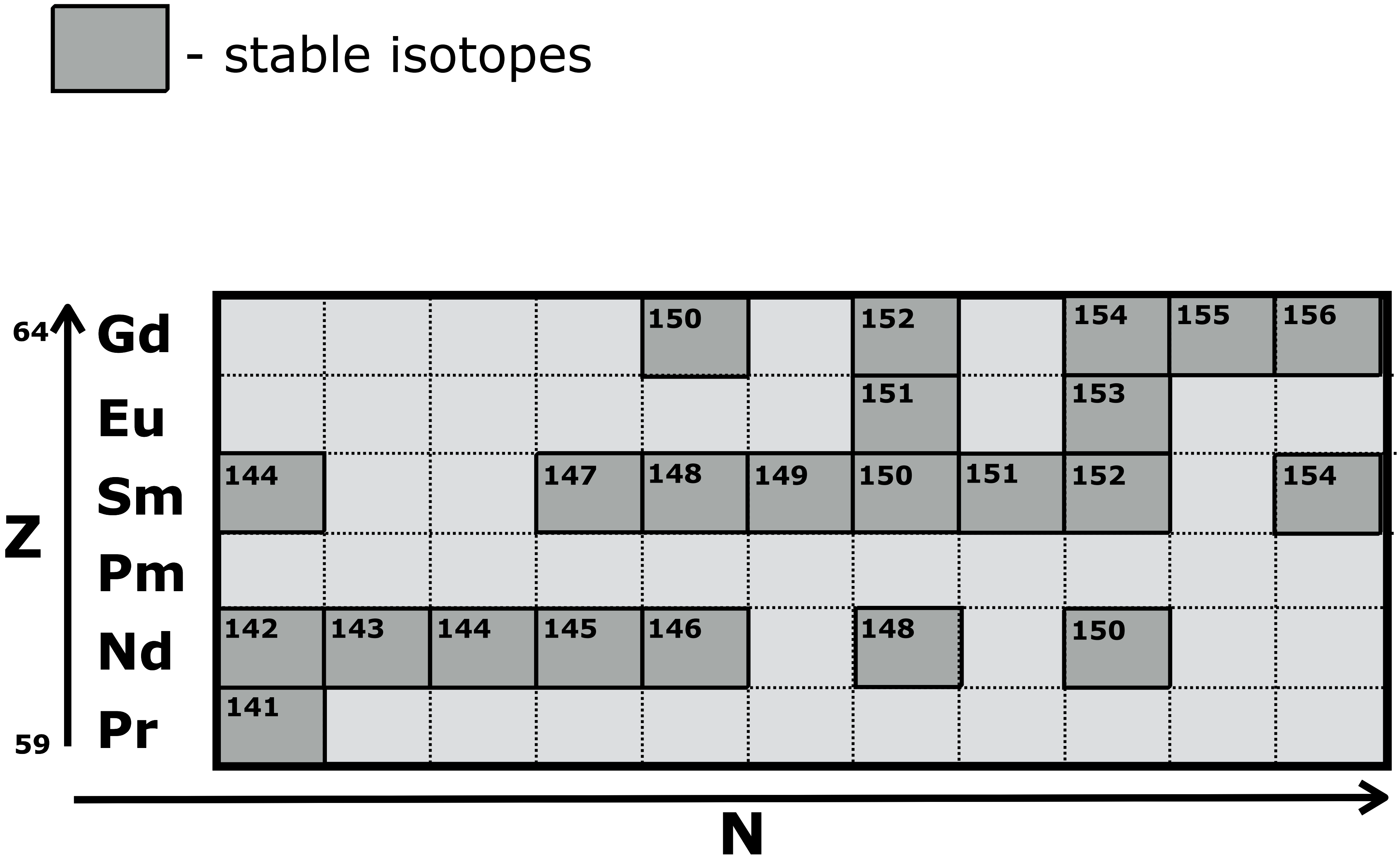
Formation of heavy elements

s-process and r-process



Formation of heavy elements

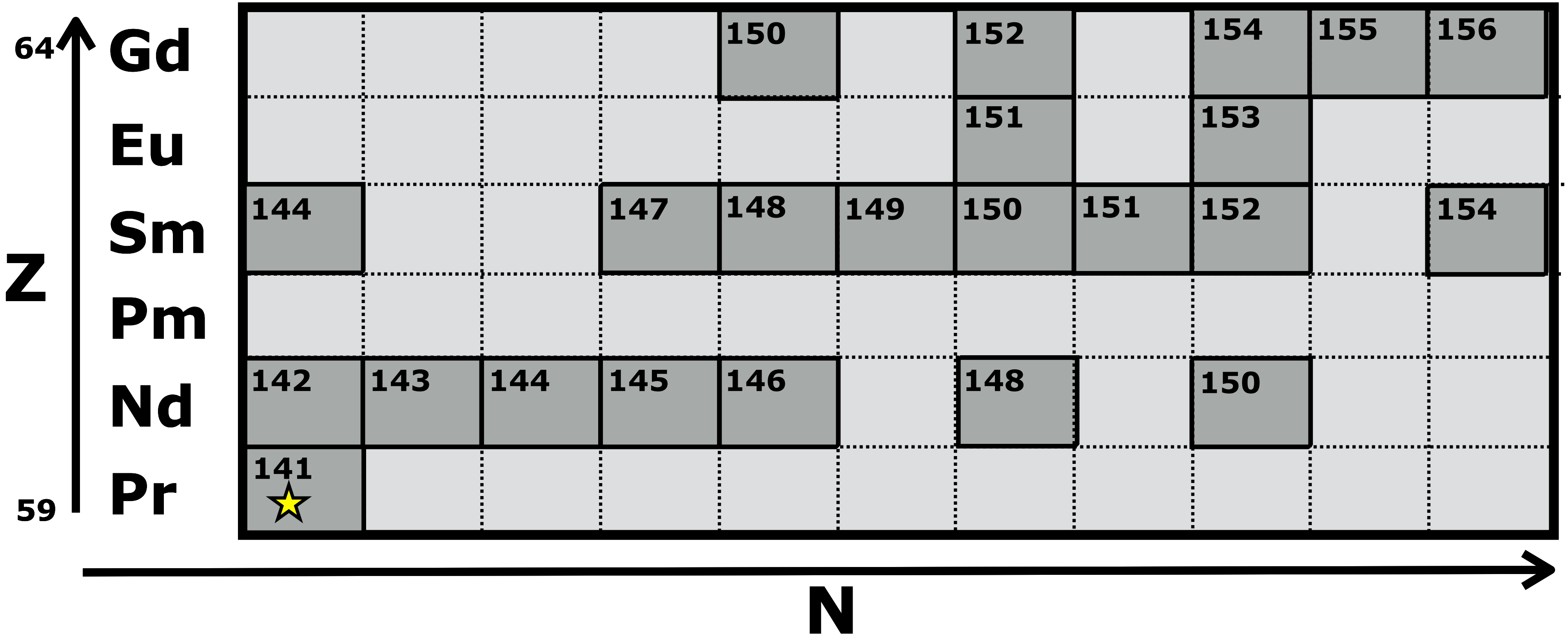
s-process and r-process



Formation of heavy elements

s-process and r-process

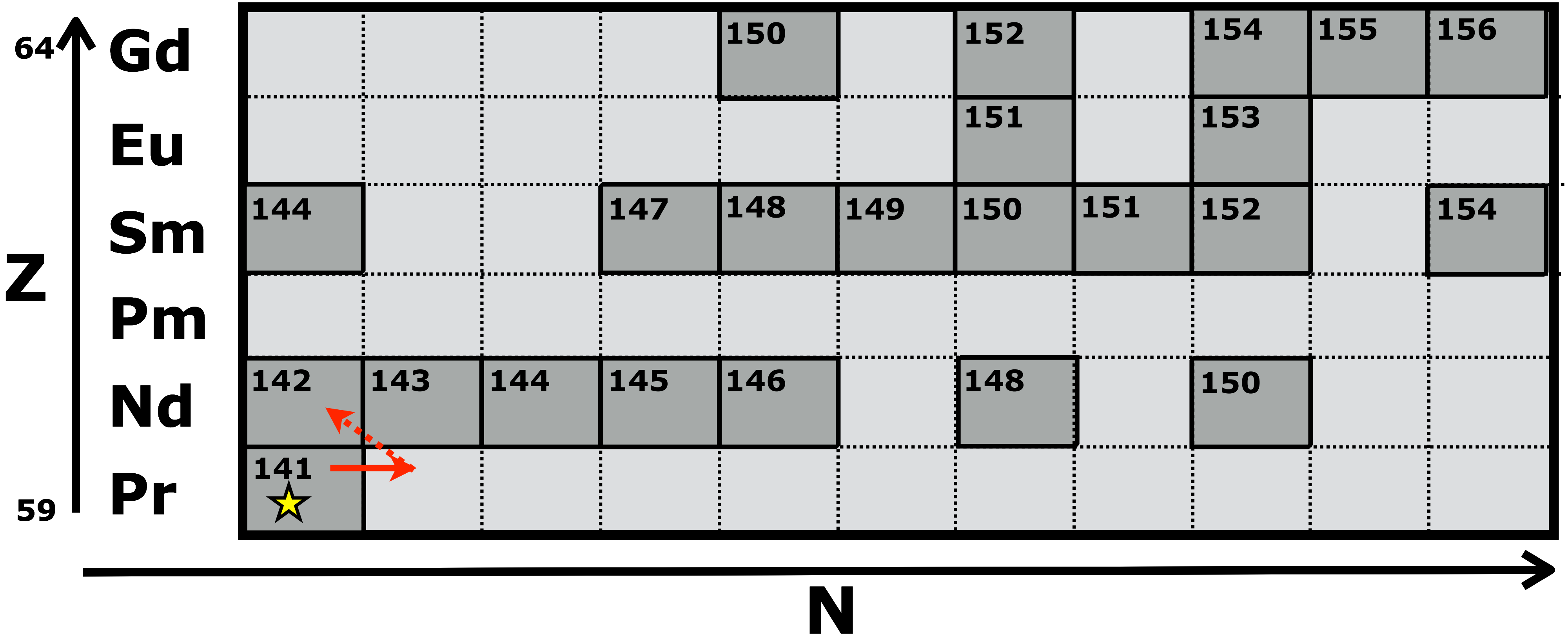
- stable isotopes



Formation of heavy elements

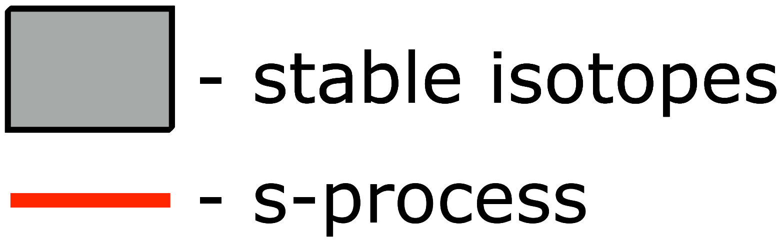
s-process and r-process

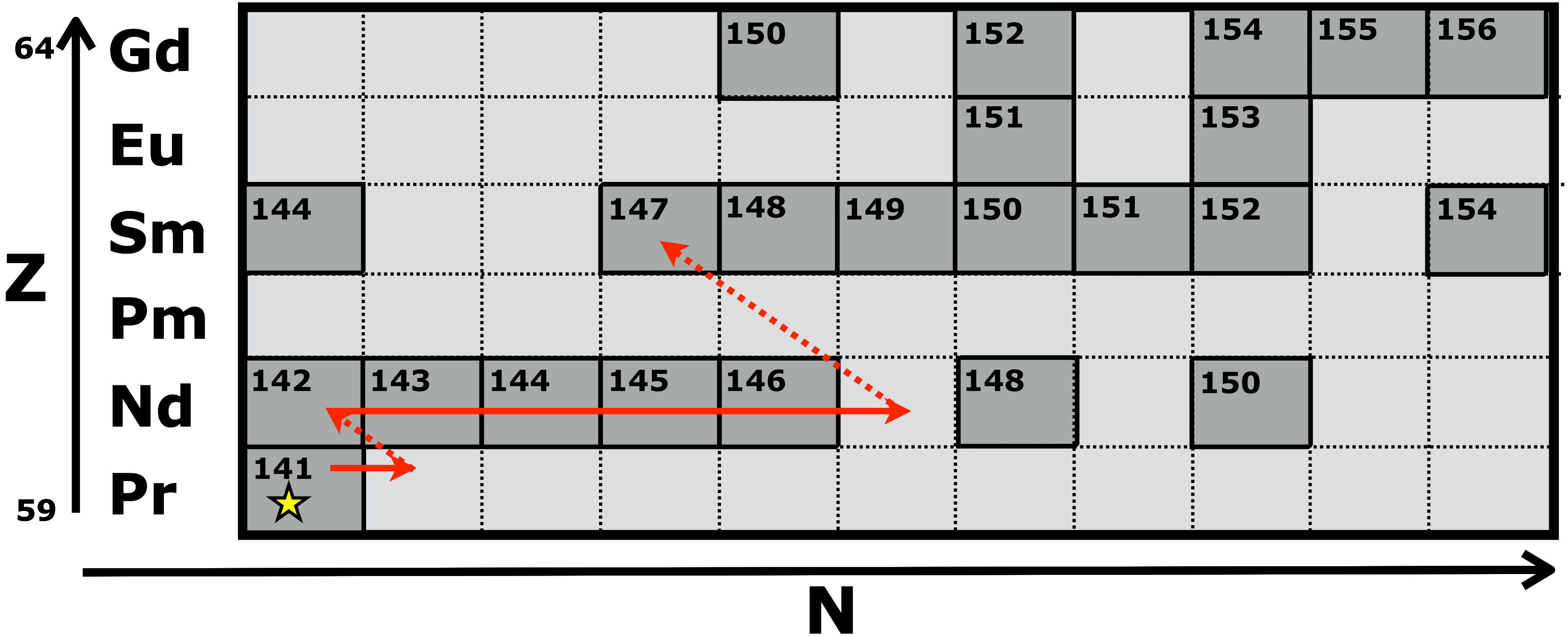
- stable isotopes
- s-process



Formation of heavy elements

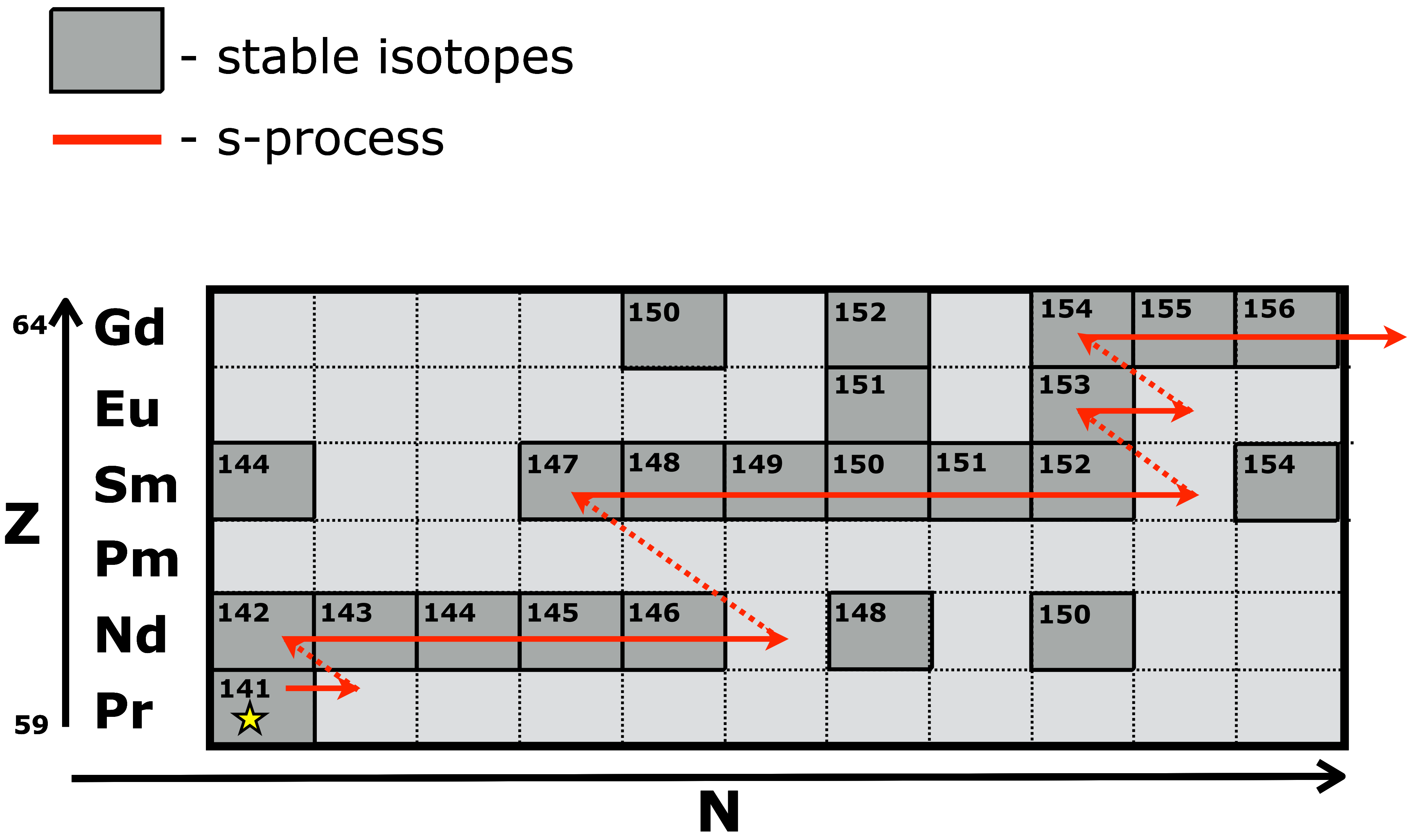
s-process and r-process





Formation of heavy elements

s-process and r-process

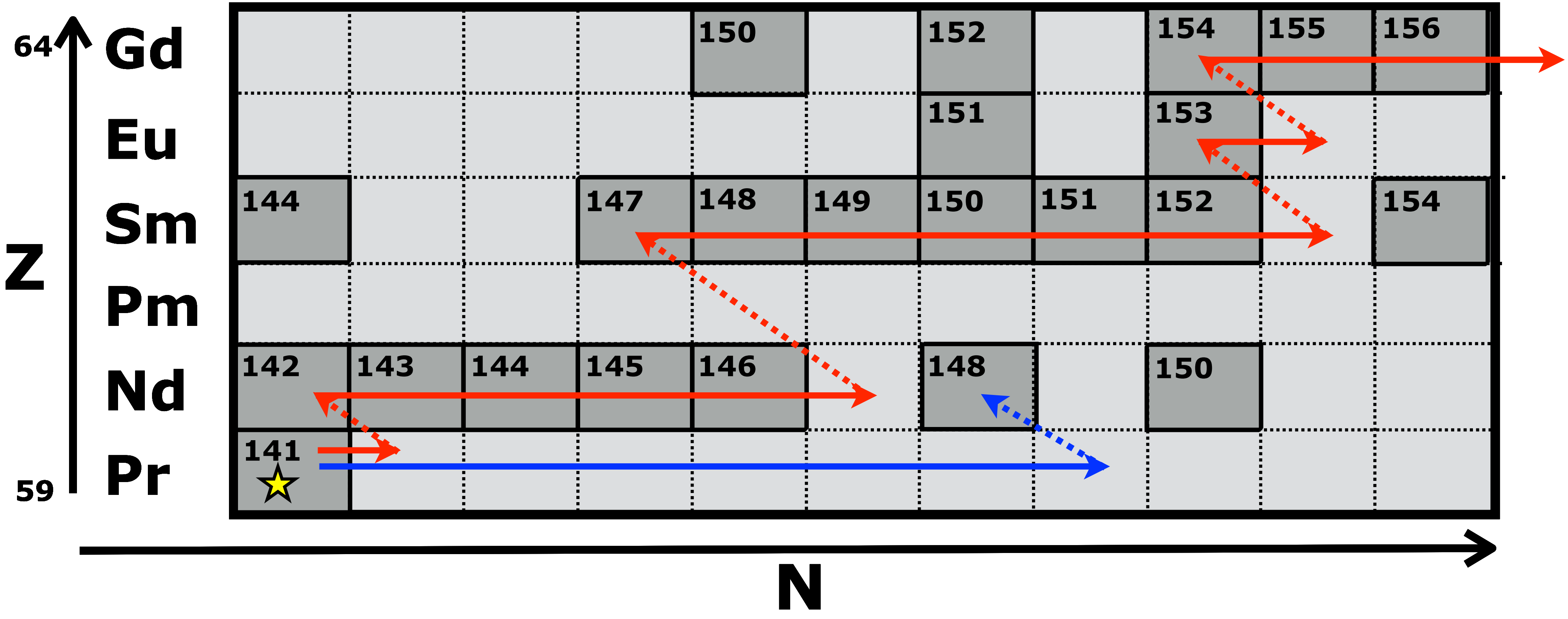


Formation of heavy elements

s-process and r-process

- stable isotopes

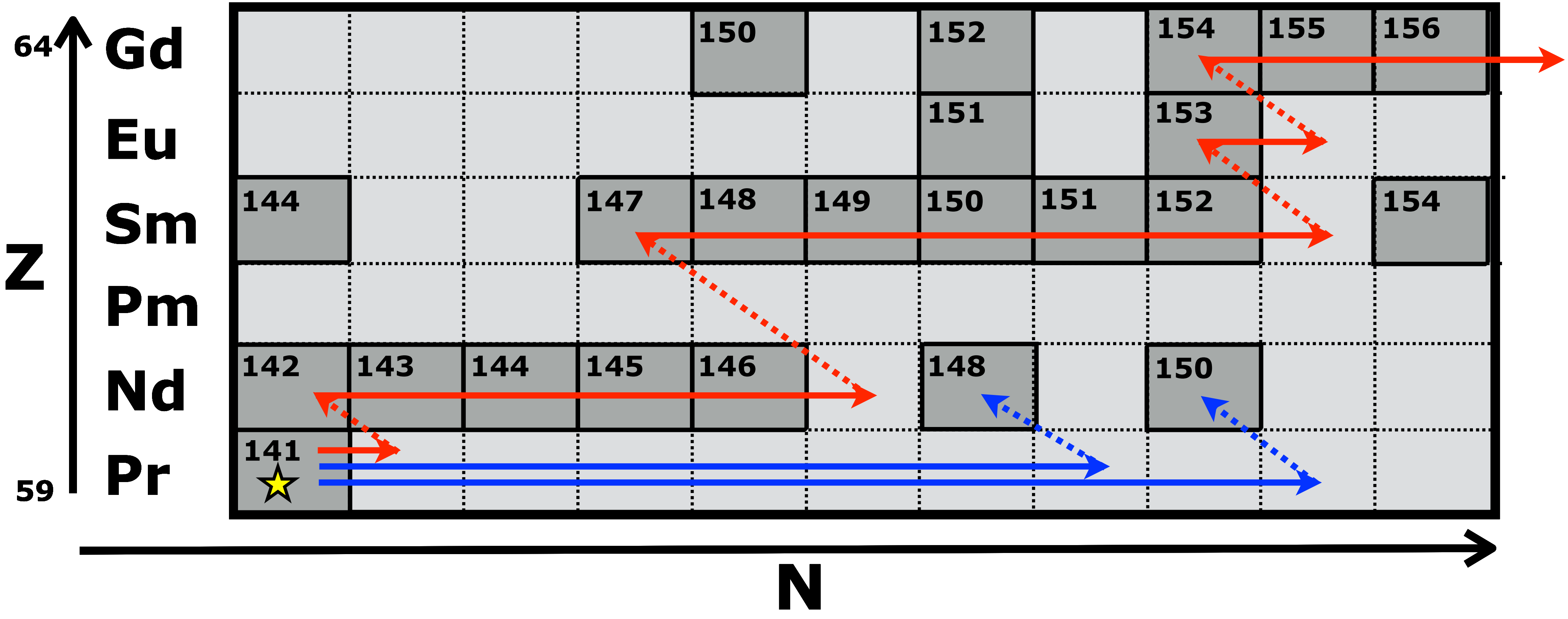
- s-process



Formation of heavy elements

s-process and r-process

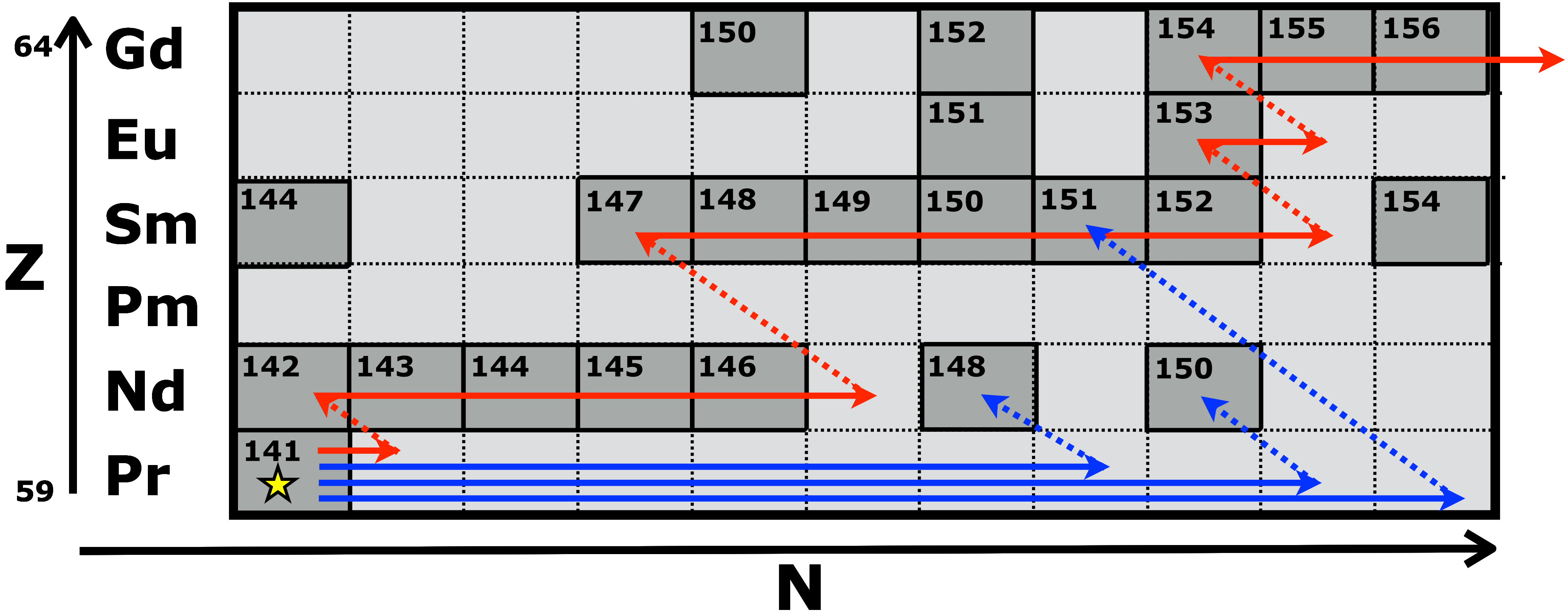
- stable isotopes
- s-process
- r-process



Formation of heavy elements

s-process and r-process

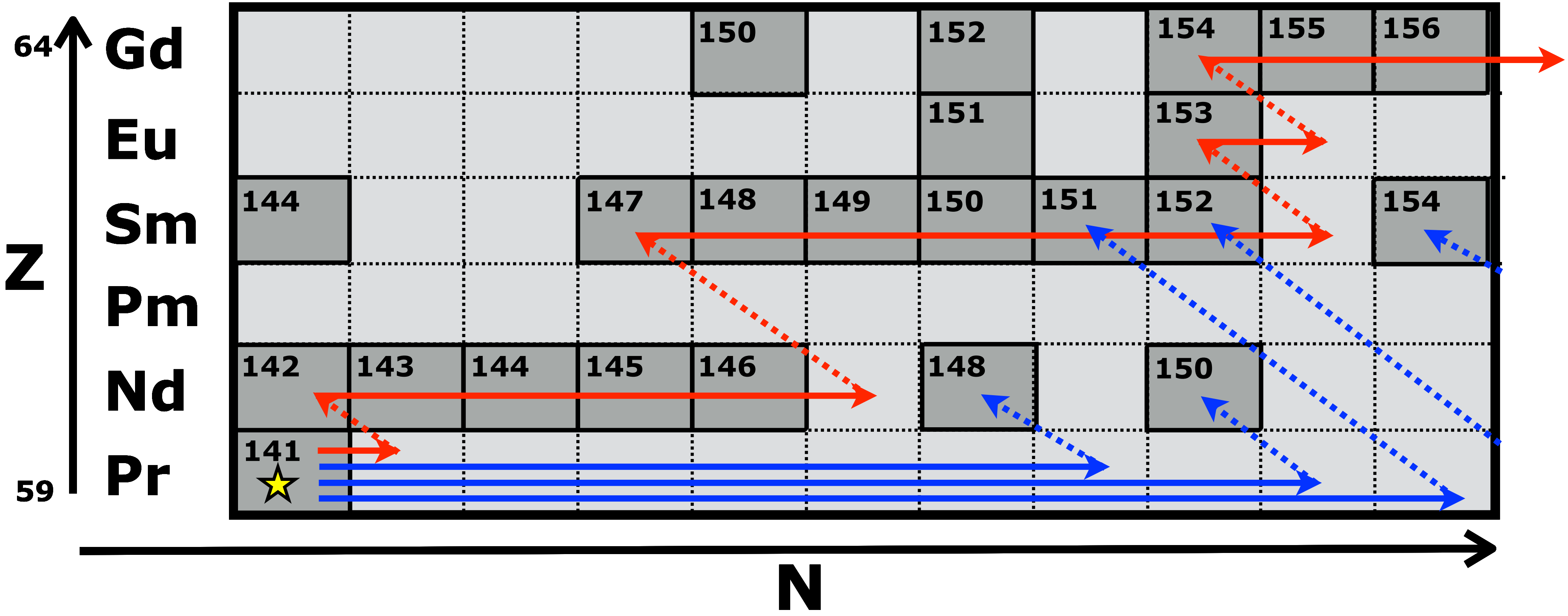
- stable isotopes
- s-process
- r-process



Formation of heavy elements

s-process and r-process

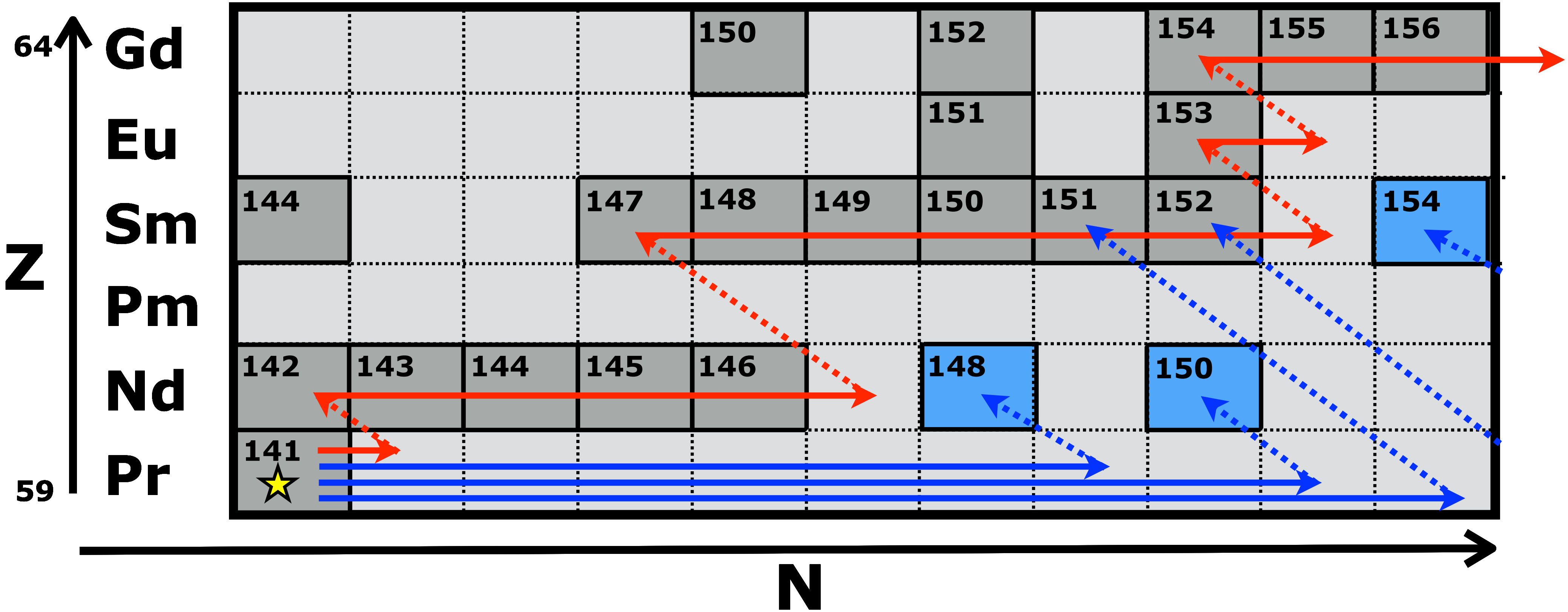
- stable isotopes
- s-process
- r-process



Formation of heavy elements

s-process and r-process

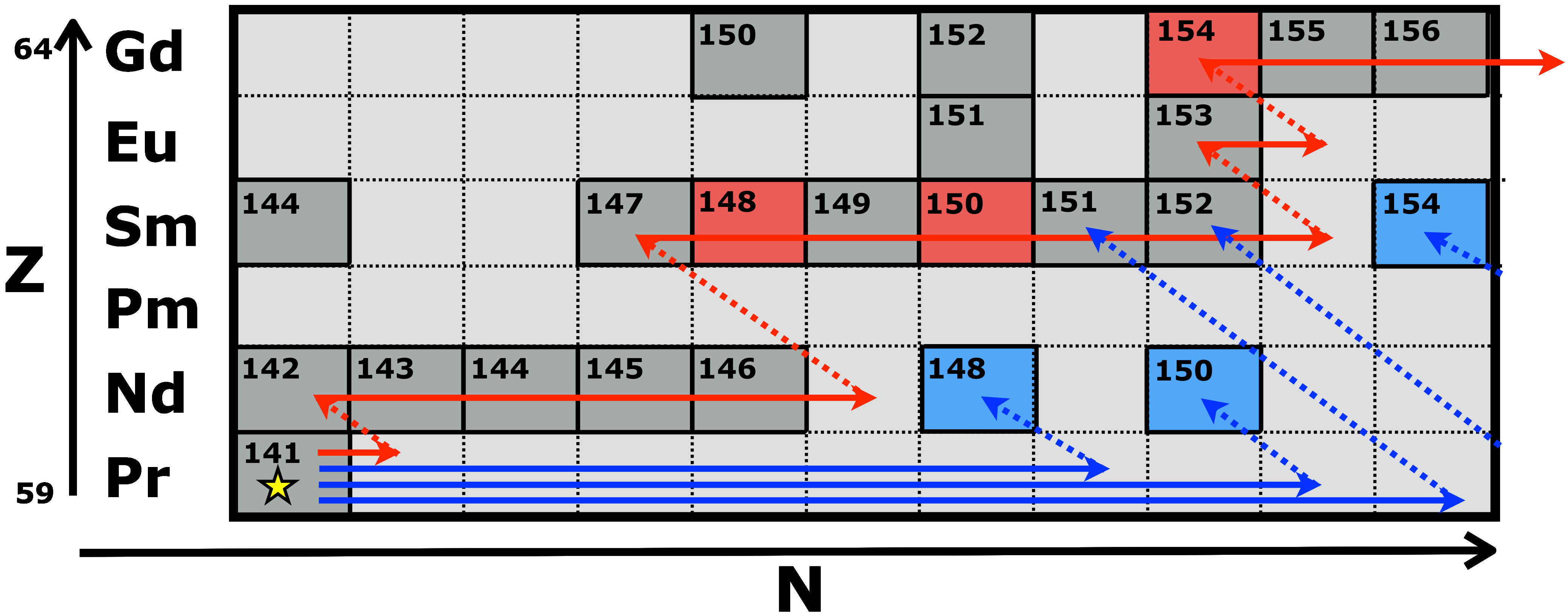
- stable isotopes
- r-process only
- s-process
- r-process



Formation of heavy elements

s-process and r-process

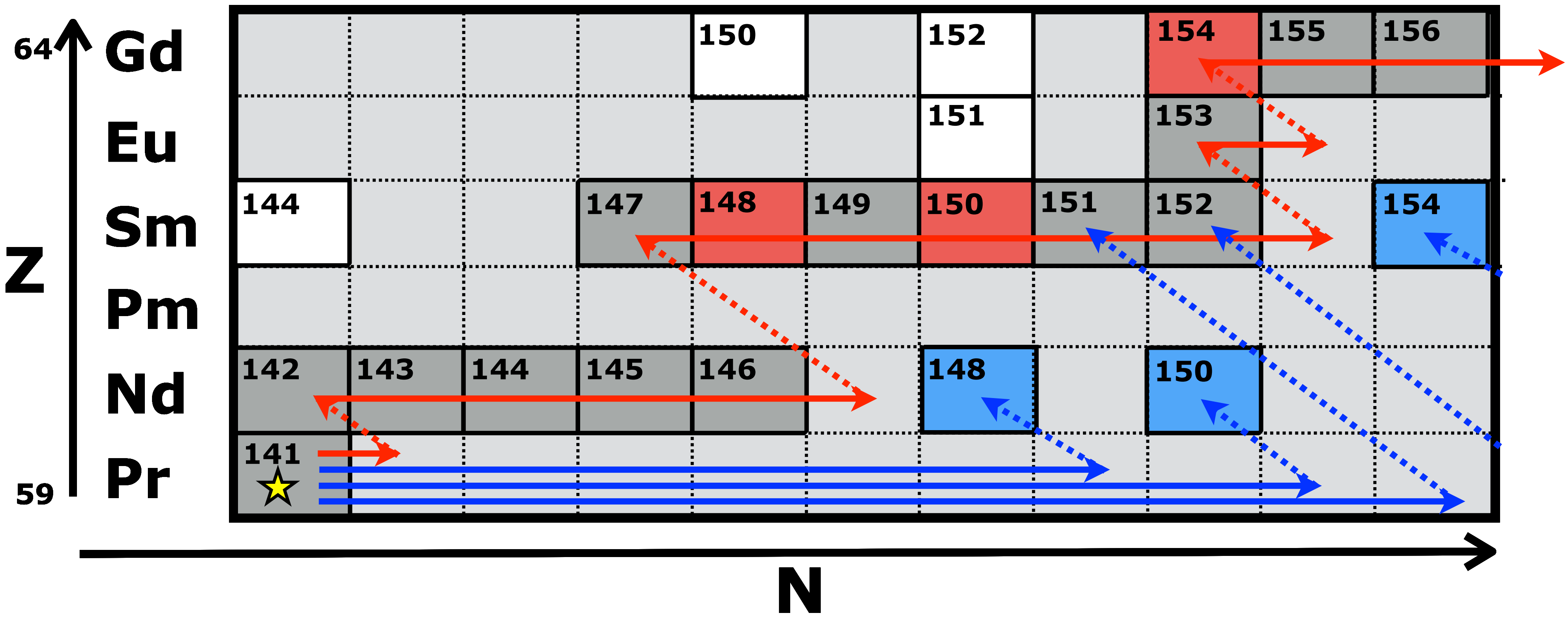
- stable isotopes
- s-process
- r-process
- r-process only
- s-process only



Formation of heavy elements

s-process and r-process

- stable isotopes
- s-process
- r-process
- r-process only
- s-process only
- neither process



Exercises (1)

6.1 Conceptual questions: Gamow peak

N.B. Discuss your answers to this question with your fellow students or with the assistant.

In the lecture (see eq. 6.22) you saw that the reaction rate is proportional to

$$\langle\sigma v\rangle = \left(\frac{8}{m\pi}\right)^{1/2} \frac{S(E_0)}{(kT)^{3/2}} \int_0^\infty e^{-E/kT} e^{-b/E^{1/2}} dE,$$

where the factor $b = \pi(2m)^{1/2} Z_1 Z_2 e^2/\hbar$, and $m = m_1 m_2/(m_1 + m_2)$ is the reduced mass.

(a) Explain in general terms the meaning of the terms $e^{-E/kT}$ and $e^{-b/E^{1/2}}$.

(b) Sketch both terms as function of E. Also sketch the product of both terms.

(c) The reaction rate is proportional to the area under the product of the two terms. Draw a similar sketch as in question (b) but now for a higher temperature. Explain why and how the reaction rate depends on the temperature.

(d) Explain why hydrogen burning can take place at lower temperatures than helium burning.

(e) Elements more massive than iron, can be produced by neutron captures. Neutron captures can take place at low temperatures (even at terrestrial temperatures). Can you explain why?

6.2 Hydrogen burning

(a) Calculate the energy released per reaction in MeV (the Q-value) for the three reactions in the pp1 chain. (Hint: first calculate the equivalent of $m_u c^2$ in MeV.)

(b) What is the total effective Q-value for the conversion of four ^{1}H nuclei into ^{4}He by the pp1 chain? Note that in the first reaction ($^1\text{H} + {}^1\text{H} \rightarrow {}^2\text{H} + \text{e}^+ + \nu$) a neutrino is released with (on average) an energy of 0.263 MeV.

(c) Calculate the energy released by the pp1 chain in erg/g.

(d) Will the answer you get in (c) be different for the pp2 chain, the pp3 chain or the CNO cycle? If so, why? If not, why not?

Exercises (2)

6.3 Relative abundances for CN equilibrium

Estimate the relative abundances of the nuclei CN-equilibrium if their lifetimes against proton capture at $T = 2 \times 10^7$ K are: $\tau_p(^{15}N) = 30$ yr, $\tau_p(^{13}C) = 1600$ yr, $\tau_p(^{12}C) = 6600$ yr and $\tau_p(^{14}N) = 6 \times 10^5$ yr.

6.4 Helium burning

(a) Calculate the energy released per gram for He burning by the 3α reaction and the $^{12}C + \alpha$ reaction, if the final result is a mixture of 50% carbon and 50% oxygen (by mass fraction).

(b) Compare the answer to that for H-burning. How is this related to the duration of the He-burning phase, compared to the main-sequence phase?

6.5 Comparing radiative and convective cores

Consider a H-burning star of mass $M = 3M_\odot$, with a luminosity L of $80L_\odot$, and an initial composition $X = 0.7$ and $Z = 0.02$. The nuclear energy is generated only in the central 10% of the mass, and the energy generation rate per unit mass, ϵ_{nuc}, depends on the mass coordinate as

$$\epsilon_{nuc} = \epsilon_c \left(1 - \frac{m}{0.1M}\right)$$

(a) Calculate and draw the luminosity profile, l, as a function of the mass, m. Express ϵ_c in terms of the known quantities for the star.

(b) Assume that all the energy is transported by radiation. Calculate the H-abundance as a function of mass and time, $X = X(m, t)$. What is the central value for X after 100 Myr? Draw X as a function of m. (Hint: the energy generation per unit mass is $Q = 6.3 \times 10^{18}$ erg g^{-1}).

(c) In reality, ϵ_{nuc} is so high that the inner 20% of the mass is unstable to convection. Now, answer the same question as in (b) and draw the new X profile as a function of m. By how much is the central H-burning lifetime extended as a result of convection?

Exercises (3)

6.6 (a) Calculate the mass defect fractions of the following fusion reactions

$4\ {}^{1}H \rightarrow {}^{4}He$

$3\ {}^{4}He \rightarrow {}^{12}C$

${}^{12}C + {}^{4}He \rightarrow {}^{16}O$

$2\ {}^{16}O \rightarrow {}^{28}Si + {}^{4}He$

$2\ {}^{28}Si \rightarrow {}^{56}Fe$

(b) Describe the trend and discuss what this trend implies for stellar evolution.

6.7 The efficiency of He-fusion has a much steeper T-dependence than H-fusion.

(a) What does it imply for the mass of the He-fusing core compared to that of the H-fusing core of a star with a given initial mass?

(b) What is the consequence for the chemical distribution of the star at the end of the He-fusion phase.

6.8 The lifetimes of the major isotopes involved in the CNO cycle are given in Equation (8.16).

(a) Calculate the equilibrium ratios of these isotopes in the core of the star at the start and at the end of the H-fusion phase.

(b) Sketch the variation of the logarithmic isotope number ratios ${}^{4}He/{}^{1}H$, ${}^{12}C/{}^{1}H$, ${}^{14}N/{}^{1}H$, ${}^{12}C/{}^{4}He$, ${}^{14}N/{}^{4}He$, ${}^{13}C/{}^{12}C$, and ${}^{15}N/{}^{14}N$ as a function of time in a single diagram, for a star of about $2M_{\odot}$ fusing H $\rightarrow$ He at $T = 2 \times 10^{7}$ K. Start with the cosmic logarithmic abundance ratios by number of (${}^{1}H$, ${}^{4}He$, ${}^{12}C$, ${}^{13}C$, ${}^{14}N$, ${}^{15}N$) = (0.0, −1.0, −3.4, −5.4, −4.0, −6.4). Discuss the trends.

6.9 Suppose that the He-fusion requires a minimum core mass of about $0.3M_{\odot}$. What would be the minimum core mass for the next fusion phases?

Questions

Q (6.1) Explain why the cosmic abundance of Li and Be is low

Q (6.2) Why is $\varepsilon_{pp} \sim \rho^1$ and why is $\varepsilon_{pp} \sim X^2$?

Q (6.3) What is the net reaction of these four cycles?

Q (6.4) Energy production by the CNO cycle has a much steeper dependence on T than that of the pp chain. What does this imply for the mass of the region where H-fusion occurs in massive stars?

Q (6.5) Approximately how long does it take for the CN cycle to reach equilibrium?

Q (6.6) Which one of these two changes occurs faster?

Q (6.7) Why is $\varepsilon_{3a} \sim \rho^2$? Why is $\varepsilon_{3a} \sim Y^3$? What would the dependence be if He+He→Be was not an equilibrium reaction?

Q (6.8) Would you exist if the ^{12}C nucleus did not have a resonant energy level around 8 MeV? Explain.