FisyMat
Máster en Matemática Aplicada

UNIVERSIDAD DE GRANADA

Física Estelar

Lluís Galbany, Ed. Mecenas (#16)

Inma Domínguez, Ed. Mecenas (#17)

Antonio García, Ed. Mecenas (#16)

7. Stellar models and stability

Equations of Stellar Structure

So far, we have seen the most important physicals processes taking place in stellar interiors

By putting all together (+ boundary and initial conditions) we can construct models of spherically symmetric stars

Assumptions

1) Star is spherically symmetric: physical quantities vary only with *r*: P(r), ρ(r), T(r)

2) Star is in hydrostatic equilibrium

3) Energy sources are: grav. energy, thermonuclear reactions, internal (thermal) energy (important in WD)

4) Energy transport mechanisms are: radiation, convection, conduction (WD)

5) Chemical composition:
- Newly formed stars have homogeneous composition
- Assume initial composition (X+Y+Z=1 from surface spectrum)
- Composition changes during the star's evolution

Equations of Stellar Structure

1 Mass continuity *(Ch. 2)*

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho}$$

2 Hydrostatic equilibrium *(Ch. 2)*

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} - \frac{1}{4\pi r^2}\frac{\partial^2 r}{\partial t^2}$$

3 Energy generation *(Ch. 5)*

$$\frac{\partial l}{\partial m} = \epsilon_{\rm nuc} - \epsilon_\nu - T\frac{\partial s}{\partial t}$$

4 Energy transport *(Ch. 5)*

$$\frac{\partial T}{\partial m} = -\frac{Gm}{4\pi r^4}\frac{T}{P}\nabla \qquad \nabla = \begin{cases} \nabla_{\rm rad} = \dfrac{3\kappa}{16\pi acG}\dfrac{lP}{mT^4} & \text{if } \nabla_{\rm rad} \le \nabla_{\rm ad} \\ \nabla_{\rm ad} + \underline{\Delta\nabla} & \text{if } \nabla_{\rm rad} > \nabla_{\rm ad} \end{cases}$$

N Composition *(Ch. 6)*

$$\frac{\partial X_i}{\partial t} = \frac{A_i m_u}{\rho}\left(-\sum_j (1+\delta_{ij}) r_{ij} + \sum_{k,l} r_{kl,j}\right) \quad [+\text{ mixing terms}] \qquad i = 1 \ldots N$$

4+N *equations with* ***4+N*** *unknowns (r, ρ, T, l, X_i) - all function of m and t*

Equations of Stellar Structure

1 Mass continuity *(Ch. 2)*

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho}$$

2 Hydrostatic equilibrium *(Ch. 2)*

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} - \frac{1}{4\pi r^2} \underbrace{\cancel{\frac{\partial^2 r}{\partial t^2}}}_{\text{=0 in HE}}$$

=0 in HE

3 Energy generation *(Ch. 5)*

$$\frac{\partial l}{\partial m} = \epsilon_{\rm nuc} - \epsilon_\nu \underbrace{- T\frac{\partial s}{\partial t}}_{\epsilon_{\rm gr}}$$

$$\epsilon_{\rm gr} = -\frac{\partial u}{\partial t} + \frac{P}{\rho^2}\frac{\partial P}{\partial t}$$

<0 expansion
>0 contraction

4 Energy transport *(Ch. 5)*

$$\frac{\partial T}{\partial m} = -\frac{Gm}{4\pi r^4}\frac{T}{P}\nabla \qquad \nabla = \begin{cases} \nabla_{\rm rad} = \dfrac{3\kappa}{16\pi acG}\dfrac{lP}{mT^4} & \text{Radiactive if } \nabla_{\rm rad} \le \nabla_{\rm ad} \\ \nabla_{\rm ad} + \underbrace{\Delta\nabla}_{\text{Superadiabaticity}} & \text{Convective if } \nabla_{\rm rad} > \nabla_{\rm ad} \end{cases}$$

Radiactive if $\nabla_{\rm rad} \le \nabla_{\rm ad}$

Convective if $\nabla_{\rm rad} > \nabla_{\rm ad}$

Superadiabaticity

N Composition *(Ch. 6)*

$$\frac{\partial X_i}{\partial t} = \frac{A_i m_u}{\rho}\left(-\sum_j (1+\delta_{ij}) r_{ij} + \sum_{k,l} r_{kl,j}\right) \quad [+ \text{ mixing terms}] \quad i = 1 \ldots N$$

Due to convection

***4+N** equations with **4+N** unknowns (r, **ρ**, T, l, X_i) - all function of m and t*

Equations of Stellar Structure

1 Mass continuity
(Ch. 2)

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho}$$

2 Hydrostatic equilibrium
(Ch. 2)

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} - \frac{1}{4\pi r^2}\cancel{\frac{\partial^2 r}{\partial t^2}}$$

=0 in HE

Although perturbations may appear on a dynamical timescale τ_{dyn}

3 Energy generation
(Ch. 5)

$$\frac{\partial l}{\partial m} = \epsilon_{\rm nuc} - \epsilon_\nu \underbrace{- T\frac{\partial s}{\partial t}}_{}$$

on a KH timescale τ_{KH} otherwise ϵ_{gr}=0

$$\epsilon_{\rm gr} = -\frac{\partial u}{\partial t} + \frac{P}{\rho^2}\frac{\partial P}{\partial t}$$

<0 expansion
>0 contraction

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$$\frac{\partial X_i}{\partial t} = \frac{A_i m_u}{\rho}\left(-\sum_j (1+\delta_{ij}) r_{ij} + \sum_{k,l} r_{kl,j}\right) \quad [+ \text{ mixing terms}] \quad i = 1 \ldots N$$

Due to convection

on a nuclear timescale τ_{nuc}, which is $\gg \tau_{KH} \gg \tau_{dyn}$, and it can be decoupled from the other 4

***4+N** equations with **4+N** unknowns (r, ρ, T, l, X_i) - all function of m and t*

Equations of Stellar Structure

Stellar structure equations

Mechanical structure

Thermal and E structure

1 Mass continuity *(Ch. 2)*

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho}$$

2 Hydrostatic equilibrium *(Ch. 2)*

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} - \frac{1}{4\pi r^2}\cancel{\frac{\partial^2 r}{\partial t^2}}$$

=0 in HE

Although perturbations may appear on a dynamical timescale τ_{dyn}

3 Energy generation *(Ch. 5)*

$$\frac{\partial l}{\partial m} = \epsilon_{\rm nuc} - \epsilon_\nu \underbrace{- T\frac{\partial s}{\partial t}}_{\epsilon_{\rm gr}}$$

on a KH timescale τ_{KH} otherwise ϵ_{gr}=0

$$\epsilon_{\rm gr} = -\frac{\partial u}{\partial t} + \frac{P}{\rho^2}\frac{\partial P}{\partial t}$$

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N Composition *(Ch. 6)*

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Due to convection

on a nuclear timescale τ_{nuc}, which is $\gg \tau_{KH} \gg \tau_{dyn}$, and it can be decoupled from the other 4

4+N *equations with* ***4+N*** *unknowns (r, ρ, T, l, X_i) - all function of m and t*

Initial conditions

In HE and TE, the first 4 equations become independent of time, and we just need to specify the initial chemical composition profiles $X_i(m,t_0)$ as *initial conditions*:

Zero-age main sequence star models

In HE but not *TE*, then $\epsilon_{gr} \not\approx 0$, which has a time derivative, so the specific entropy profile $s(m,t_0)$ needs to be added as *initial conditions*.

Pre-main sequence star models

Boundary conditions

Central boundary conditions (m=0)

At the centre, **ρ** and energy generation must remain finite,

$$r(m=0)=0$$

$$l(m=0)=0$$

No energy point sources

Surface boundary conditions (m=M or r=R)

$$P(m=M)=0$$

$$T(m=M)=0$$

However they never become zero
because of the stellar atmosphere

so, they can be better defined at the photosphere τ=2/3 (visible surface)

$$P(m=M_{\rm ph})=\frac{2}{3}\frac{GM}{\kappa_{\rm ph}R^2}$$

$$L(m=M_{\rm ph})=4\pi R^2\sigma T^4$$

or even better coupled with a detailed stellar atmosphere model (from τ>2/3)

Homology relations

Scaling relations between stellar models with different M and R, but with the same ***ρ*** distribution *(useful for ZAMS!)*

Consider 2 stars with M_1,R_1 and M_2,R_2, and the relative mass coord.

$$x = \frac{m_1}{M_1} = \frac{m_2}{M_2} \qquad \forall x$$

The two stars are homologous if all mass shells are located at the same relative radii r/R:

$$\frac{r_1(x)}{R_1} = \frac{r_2(x)}{R_2} \quad \rightarrow \quad \frac{r_1(x)}{r_2(x)} = \frac{R_1}{R_2} \qquad \forall x$$

so then two homologous stars have *the same density distribution*

Homology structure equations

The mass continuity equations,

$$\frac{dr_i}{dx} = \frac{M_i}{4\pi r_i^2 \rho_i} \longrightarrow \frac{dr_1}{dx} = \frac{R_1}{R_2}\frac{dr_2}{dx} = \frac{M_1}{4\pi r_2^2 \left(\frac{R_1}{R_2}\right)^2 \rho_1}$$

with $r1 = r2\frac{R_1}{R_2}$

or

$$\frac{dr_2}{dx} = \frac{M_2}{4\pi r_2^2 \rho_2}\left[\frac{\rho_2}{\rho_1}\frac{M_1}{M_2}\left(\frac{R_2}{R_1}\right)^3\right]$$

so, $$\frac{\rho_2(x)}{\rho_1(x)} = \frac{M_2}{M_1}\left(\frac{R_2}{R_1}\right)^{-3}$$

And $\boldsymbol{\rho}$ at any homologous shell

$$\boxed{\rho(x) \propto \rho_c \propto \bar{\rho}}$$

Homology structure equations

The mass continuity equations,

$$\frac{dr_i}{dx} = \frac{M_i}{4\pi r_i^2 \rho_i} \longrightarrow \frac{dr_1}{dx} = \frac{R_1}{R_2}\frac{dr_2}{dx} = \frac{M_1}{4\pi r_2^2 \left(\frac{R_1}{R_2}\right)^2 \rho_1}$$

with $r1 = r2\dfrac{R_1}{R_2}$

or

$$\boxed{\frac{dr_2}{dx} = \frac{M_2}{4\pi r_2^2 \rho_2}} \left[\frac{\rho_2}{\rho_1}\frac{M_1}{M_2}\left(\frac{R_2}{R_1}\right)^3\right]$$

Is the mass continuity eq for 2

so, $$\frac{\rho_2(x)}{\rho_1(x)} = \frac{M_2}{M_1}\left(\frac{R_2}{R_1}\right)^{-3}$$

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Homology structure equations

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$$\frac{dr_i}{dx} = \frac{M_i}{4\pi r_i^2 \rho_i} \longrightarrow \frac{dr_1}{dx} = \frac{R_1}{R_2}\frac{dr_2}{dx} = \frac{M_1}{4\pi r_2^2 \left(\frac{R_1}{R_2}\right)^2 \rho_1}$$

with $r1 = r2\frac{R_1}{R_2}$

or

$$\boxed{\frac{dr_2}{dx} = \frac{M_2}{4\pi r_2^2 \rho_2}} \underbrace{\left[\frac{\rho_2}{\rho_1}\frac{M_1}{M_2}\left(\frac{R_2}{R_1}\right)^3\right]}_{=1}$$

Is the mass continuity eq for 2

so, $$\frac{\rho_2(x)}{\rho_1(x)} = \frac{M_2}{M_1}\left(\frac{R_2}{R_1}\right)^{-3}$$

And $\boldsymbol{\rho}$ at any homologous shell

$$\boxed{\rho(x) \propto \rho_c \propto \bar{\rho}}$$

Homology structure equations

The hydrostatic equilibrium equations,

$$\frac{dP_i}{dx} = -\frac{GM_i^2 x}{4\pi r_i^4} \xrightarrow{(\ldots)} \frac{P_2(x)}{P_1(x)} = \left(\frac{M_2}{M_1}\right)^2 \left(\frac{R_2}{R_1}\right)^{-4}$$

And P at any homologous shell

$$\boxed{P(x) \propto P_c \propto \frac{M^2}{R^4}}$$

And combining both eq. We obtain an expression for the mechanical structure

$$\frac{P_2(x)}{P_1(x)} = \left(\frac{M_2}{M_1}\right)^{2/3} \left(\frac{\rho_2}{\rho_1}\right)^{4/3}$$

Which describes the relation between P and $\boldsymbol{\rho}$ two homologous stars must obey

$$\boxed{P(x) \propto M^{2/3} \rho(x)^{4/3}}$$

Homology (radiative stars of ideal gas)

Assuming an ideal gas EoS ($P = \frac{\Re}{\mu}\rho T$), and an homogeneous composition ($\mu \sim \mathrm{ct}$), we can rearrange

$$\frac{T_2(x)}{T_1(x)} = \frac{\mu_2}{\mu_1}\frac{M_2}{M_1}\left(\frac{R_2}{R_1}\right)^{-1}$$

And T at any homologous shell

$$\boxed{T(x) \propto T_c \propto \mu \frac{M}{R}}$$

In addition, if we also assume radiative equilibrium, the eq. of energy transport

$$\frac{\partial T}{\partial m} = -\frac{Gm}{4\pi r^4}\frac{T}{P}\nabla_{\mathrm{rad}} \longrightarrow \frac{d(T^4)}{dx} = -\frac{3M}{16\pi^2 ac}\frac{\kappa l}{r^4}$$

Keeping opacities constant (although $\boldsymbol{\kappa}_1$ does not need to be the same than $\boldsymbol{\kappa}_2$)

$$\left(\frac{T_2(x)}{T_1(x)}\right)^4 = \frac{l_2(x)}{l_1(x)}\frac{M_2}{M_1}\frac{\kappa_2}{\kappa_1}\left(\frac{R_2}{R_1}\right)^{-4} \quad ; \quad \frac{l_2(x)}{l_1(x)} = \left(\frac{\mu_2}{\mu_1}\right)^4\left(\frac{M_2}{M_1}\right)^3\left(\frac{\kappa_2}{\kappa_1}\right)^{-1}$$

that holds for all x, including at the surface

$$\boxed{l(x=1) \propto L \propto \frac{1}{\kappa}\mu^4 M^3}$$

Mass-luminosity relation

or with a Kramers $\kappa = \kappa_0 \rho^a T^b$

$$L \propto \mu^{7.5} M^{5.5} R^{-0.5}$$

Homology (TE)

For stars in TE we can use the E generation eq. to derive more homology relations. Assuming,

$$\epsilon_{\rm nuc} = \epsilon_0 \rho T^\nu$$

Mass-radius relation

$$\boxed{R \propto \mu^{(\nu-4)/(\nu+3)} M^{(\nu-1)/(\nu+3)}}$$

so that $$\frac{dl}{dx} = \epsilon_0 M \rho T^\nu$$

which combined with previous relations

$$\rho_c \propto \mu^{3(4-\nu)/(\nu+3)} M^{2(3-\nu)/(\nu+3)}$$

$$T_c \propto \mu^{7/(\nu+3)} M^{4/(\nu+3)}$$

All 3 depend on **ν**

pp-chain	$\nu \approx 4$	$R \propto M^{0.43}$	$T_c \propto \mu M^{0.57}$	$\rho_c \propto M^{-0.3}$
CNO cycle	$\nu \approx 18$	$R \propto \mu^{2/3} M^{0.81}$	$T_c \propto \mu^{1/3} M^{0.19}$	$\rho_c \propto \mu^{-2} M^{-1.4}$

For main sequence stars

Homology (contraction)

A star without internal E must contract. If this happens homologously, and according to the definition,

$$\frac{r_1(x)}{r_2(x)} = \frac{R_1}{R_2} \xrightarrow{\dot{r} = \frac{\partial r}{\partial t}} \frac{\dot{r}(m)}{r(m)} = \frac{\dot{R}}{R}$$

each mass shell maintains its relative radius

The density eq. becomes

$$\frac{\dot{\rho}(m)}{\rho(m)} = -3\frac{\dot{R}}{R}$$

the pressure

$$\frac{\dot{P}(m)}{P(m)} = -4\frac{\dot{R}}{R} = \frac{4}{3}\frac{\dot{\rho}(m)}{\rho(m)}$$

and using the general EoS ($\frac{dP}{P} = \chi_T \frac{dT}{T} + \chi_\rho \frac{d\rho}{\rho}$), the temperature

$$\frac{\dot{T}}{T} = \frac{1}{\chi_T}\left(\frac{4}{3} - \chi_\rho\right)\frac{\dot{\rho}}{\rho} = \frac{1}{\chi_T}\left(3\chi_\rho - 4\right)\frac{\dot{R}}{R}$$

T increases as a result of contraction if $\chi_T < 4/3$

For an ideal gas $\chi_T=1$, so T will increase,
but for a degenerate gas, $\chi_T=5/3$, it will decrease (upon contraction)!

Stellar stability

We have so far considered stars in HE and TE.

We will now find out under what circumstances stars may become unstable:

- *dynamical stability*: what happens when HE is perturbed?

- *thermal (secular) stability:* what happens when the TE situation is perturbed?

. . . .

Dynamical stability

This relates to the response to a perturbation of the balance of forces (HE)

We have seen that the response to a local dynamical perturbation of HE, produces a change in the way E is transferred giving rise to *convection*

Here we will treat the case of global *radial* perturbations of HE that would produce expansion or contraction of the star.

Dynamical stability

Suppose that a star in HE is compressed on a short $\tau \ll \tau_{\mathrm{KH}}$, so that the expansion happens adiabatically, and homologously from R to R'

the density becomes $\rho \quad \rightarrow \quad \rho' = \left(\frac{R'}{R}\right)^{-3}$ (we just seen this)

And since it is adiabatic $\frac{P'}{P} = \left(\frac{\rho'}{\rho}\right)^{\gamma_{\mathrm{ad}}} = \left(\frac{R'}{R}\right)^{-3\gamma_{\mathrm{ad}}}$

which compared to the homologous relation in HE $\left(\frac{P'}{P}\right)_{\mathrm{HE}} = \left(\frac{\rho'}{\rho}\right)^{4/3} = \left(\frac{R'}{R}\right)^{-4}$

so, if

$\gamma_{\mathrm{ad}} > 4/3$ *DYNAMICALY STABLE*

$P' > P_{\mathrm{HE}}$ produces an expansion back to HE

Criterion for dynamical stability

$\gamma_{\mathrm{ad}} = 4/3$ *DYNAMICALY NEUTRAL*

$\gamma_{\mathrm{ad}} < 4/3$ *DYNAMICALY UNSTABLE*

$P' < P_{\mathrm{HE}}$ Not enough to go back to HE

What was the γ_{ad} of an ideal gas? P_{e-} NR? Radiation dominated? P_{e-} R?

Dynamical stability

Stars dominated by an **ideal gas** or by **non-relativistic degenerate e-** have $\gamma_{ad}=5/3$ and are therefore *dynamically stable*.

However, stars dominated by **relativistic particles** $\gamma_{ad} \rightarrow 4/3$ tend towards a *neutrally stable state*. A small disturbance of such a star could either lead to a collapse or an explosion. This is the case if $\mathbf{P_{rad}}$ dominates (at high *T* and low ρ), or the pressure of relativistically degenerate e- (at very high ρ).

A process that can lead to $\gamma_{ad} < 4/3$ is **partial ionization** (e.g. $H \leftrightarrow H^+ + e^-$).

Since this normally occurs in the very outer layers, where P/ρ is small, it does not lead to overall dynamical instability of the star, but it is connected to driving *oscillations* in some kinds of star.

At very high T two other processes can occur in massive stars that also lead to $\gamma_{ad} < 4/3$ but now in the core of the star: **pair creation** ($\gamma + \gamma \leftrightarrow e^+ + e^-$)

and **photo-disintegration** of nuclei (e.g. γ + Fe $\leftrightarrow \alpha$). These processes can

lead to a *stellar explosion* or *collapse*.

Secular stability

Thermal stability is linked to the Virial Th. $E_{\rm tot} = -E_{\rm int} = \frac{1}{2} E_{\rm gr} < 0$

(bound ideal gas star)

The rate of change of E is $\dot{E} = L_{\rm nuc} - L$

In TE, $L_{\rm nuc} = L \quad \rightarrow \quad E_{\rm tot} = {\rm ct}$

Considering a small perturbation,

$$L_{\rm nuc} > L \quad \rightarrow \quad \delta E_{\rm tot} > 0 \quad \rightarrow \quad \delta E_{\rm gr} > 0 \quad \rightarrow \quad \delta E_{\rm int} < 0 \quad \xrightarrow{\epsilon_{\rm nuc} \propto \rho T^{\nu}} \quad \delta L_{\rm nuc} < 0$$

ETOT less negative

$\delta \rho < 0$

$\delta T < 0$

perturbation will be quenched and TE will be restored

STABILITY

More generally, a mixture of ideal gas and radiation,

$$\frac{P}{\rho} = \frac{P_{\rm gas}}{\rho} + \frac{P_{\rm rad}}{\rho} = \frac{2}{3} u_{\rm gas} + \frac{1}{3} u_{\rm rad} \quad \xrightarrow{\text{V Th}} \quad E_{\rm gr} = -3 \int_0^M \frac{P}{\rho} dm = -2 E_{\rm int,gas} - E_{\rm int,rad}$$

and $E_{\rm tot} = -E_{\rm int,gas} = \frac{1}{2} \left(E_{\rm gr} + E_{\rm int,rad} \right)$

Radiation reduces effective Potential E

It can be parametrized as $\beta = \frac{P_{\rm gas}}{P_{\rm tot}}$, so $E_{\rm tot} = \frac{1}{2} \beta E_{\rm gr}$

which is negative as long as $\beta > 0$ ($\beta = 1$ for an ideal gas)

Secular stability

For degenerate e- gas, P and u are independent of T

A perturbation $L_{\rm nuc} > L$ will have no effect on P: no expansion and no cooling

Instead it will produce a $\delta T > 0$ while $\delta\rho \approx 0$

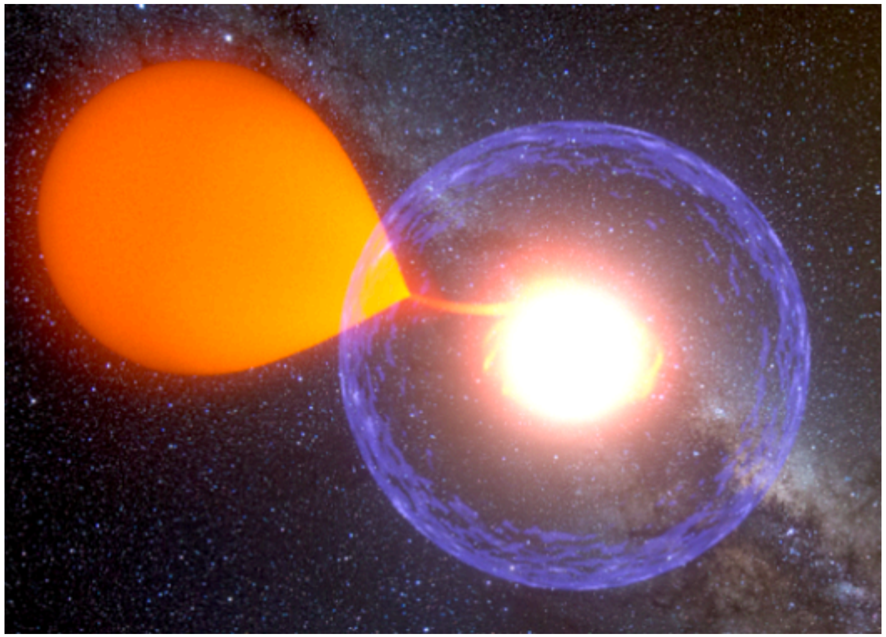
And this increase of T will lead to $\delta L_{\rm nuc} > 0$ and the star will become *UNSTABLE*

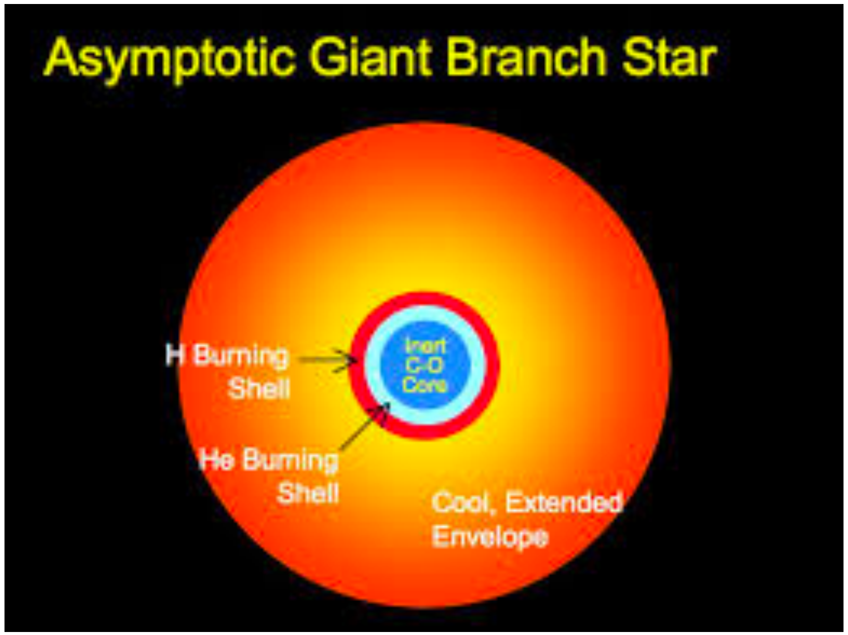
thermonuclear runaway

For instance, H ignition on WD surfaces (*nova outburst*)

He ignition in stars with $M \lesssim 2M_\odot$ (*He flash*)

Nuclear burning in a shell around an inert star core of $M \lesssim 8M_\odot$ (*asymptotic giant branch*)





Exercises (1)

7.1 General understanding of the stellar evolution equations

The differential equations (7.1–7.5) describe, for a certain location in the star at mass coordinate m, the behaviour of and relations between radius coordinate r, the pressure P, the temperature T, the luminosity l and the mass fractions X_i of the various elements i.

(a) Which of these equations describe the mechanical structure, which describe the thermal-energetic structure and which describe the composition?

(b) What does ∇ represent? Which two cases do we distinguish?

(c) How does the set of equations simplify when we assume hydrostatic equilibrium (HE)? If we assume HE, which equation introduces a time dependence? Which physical effect does this time dependence represent?

(d) What do the terms $\epsilon_{\rm nuc}$ and $T\partial s/\partial t$ represent?

(e) How does the set of equations simplify if we also assume thermal equilibrium (TE)? Which equation introduces a time dependence in TE?

(f) Equation (7.5) describes the changes in the composition. In principle we need one equation for every possible isotope. In most stellar evolution codes, the nuclear network is simplified. This reduces the number of differential equations and therefore increases speed of stellar evolution codes. The STARS code behind *Window to the Stars* only takes into account seven isotopes. Which do you think are most important?

Exercises (2)

7.2 Dynamical Stability

(a) Show that for a star in hydrostatic equilibrium ($\mathrm{d}P/\mathrm{d}m = -Gm/(4\pi r^4)$) the pressure scales with density as $P \propto \rho^{4/3}$.

(b) If $\gamma_{\mathrm{ad}} < 4/3$ a star becomes dynamically unstable. Explain why.

(c) In what type of stars $\gamma_{\mathrm{ad}} \approx 4/3$?

(d) What is the effect of partial ionization (for example $\mathrm{H} \leftrightarrows \mathrm{H}^+ + \mathrm{e}^-$) on γ_{ad}? So what is the effect of ionization on the stability of a star?

(e) *Pair creation* and *photo-disintegration* of Fe have a similar effect on γ_{ad}. In what type of stars (and in what phase of their evolution) do these processes play a role?

7.3 Mass radius relation for degenerate stars

(a) Derive how the radius scales with mass for stars composed of a *non-relativistic completely degenerate* electron gas. Assume that the central density $\rho_c = a\bar{\rho}$ and that the central pressure $P_c = bGM^2/R^4$, where $\bar{\rho}$ is the mean density, and a and b depend only on the density distribution inside the star.

(b) Do the same for an *extremely relativistic degenerate* electron gas.

(c) The electrons in a not too massive white dwarf behave like a completely degenerate non-relativistic gas. Many of these white dwarfs are found in binary systems. Describe qualitatively what happens if the white dwarf accretes material from the companion star.

Exercises (3)

7.4 Main-sequence homology relations

We speak of two *homologous stars* when they have the same density distribution. To some extent main sequence stars can be considered as stars with a similar density distribution.

(a) You already derived some scaling relations for main sequence stars from observations in the first set of exercises: the mass-luminosity relation and the mass-radius relation. Over which mass range were these simple relations valid?

(b) During the practicum you plotted the density distribution of main sequence stars of different masses. For which mass ranges did you find that that the stars had approximately the same density distribution.

(c) Compare the L-M relation derived form observational data with the L-M relation derived from homology, eq. (7.32). What could cause the difference? (Which assumptions may not be valid?)

(d) Show that, if we replace the assumption of a constant opacity with a Kramers opacity law, the mass-luminosity-radius relation becomes eq. (7.33),

$$L \propto \frac{\mu^{7.5} M^{5.5}}{R^{0.5}}.$$

(e) Substitute a suitable mass-radius relation and compare the result of (d) with the observational data in Fig. 1.3. For which stars is the Kramers-based L-M relation the best approximation? Can you explain why? What happens at lower and higher masses, respectively?

7.5 Central behaviour of the stellar structure equations

(a) Rewrite the four structure equations in terms of $\mathrm{d}/\mathrm{d}r$.

(b) Find how the following quantities behave in the neighbourhood of the stellar center:
- the mass $m(r)$,
- the luminosity $l(r)$,
- the pressure $P(r)$,
- the temperature $T(r)$.

Exercises (4) + Questions

7.6 How do we know which one of the two energy transport equations to use?

7.7 How can we calculate the entropy term due to expansion or contraction if the evolution is calculated by a series of models in hydrostatic equilibrium?

(Q 1) Why is it more practical to use m as the free parameter rather than r?

(Q 2) Which time steps would you use for calculating stellar evolution?