FisyMat
Máster en Matemática Aplicada

UNIVERSIDAD DE GRANADA

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8. Schematic stellar evolution

Evolution of the stellar center

The center of a star is

the point with highest P,$\boldsymbol{\rho}$,T

the most evolved part of the star

What sets the pace of evolution

It is characterized by P_c,$\boldsymbol{\rho}_c$,T_c, and $\boldsymbol{\mu}$, which are related by EoS

The evolution of a star can be represented with an evolutionary track in the (P_c,$\boldsymbol{\rho}_c$) or (T_c,$\boldsymbol{\rho}_c$) diagrams

HE and the P_c-ρ_c relation

From the homologous relations, a star in HE,

$$P_c = C \cdot GM^{2/3}\rho_c^{4/3} \qquad\qquad P_c \propto \boldsymbol{\rho}_c^{4/3}$$

C: constant that depends on the density distribution (e.g. polytropic n)

which is independent of EoS, so it is an Universal relation

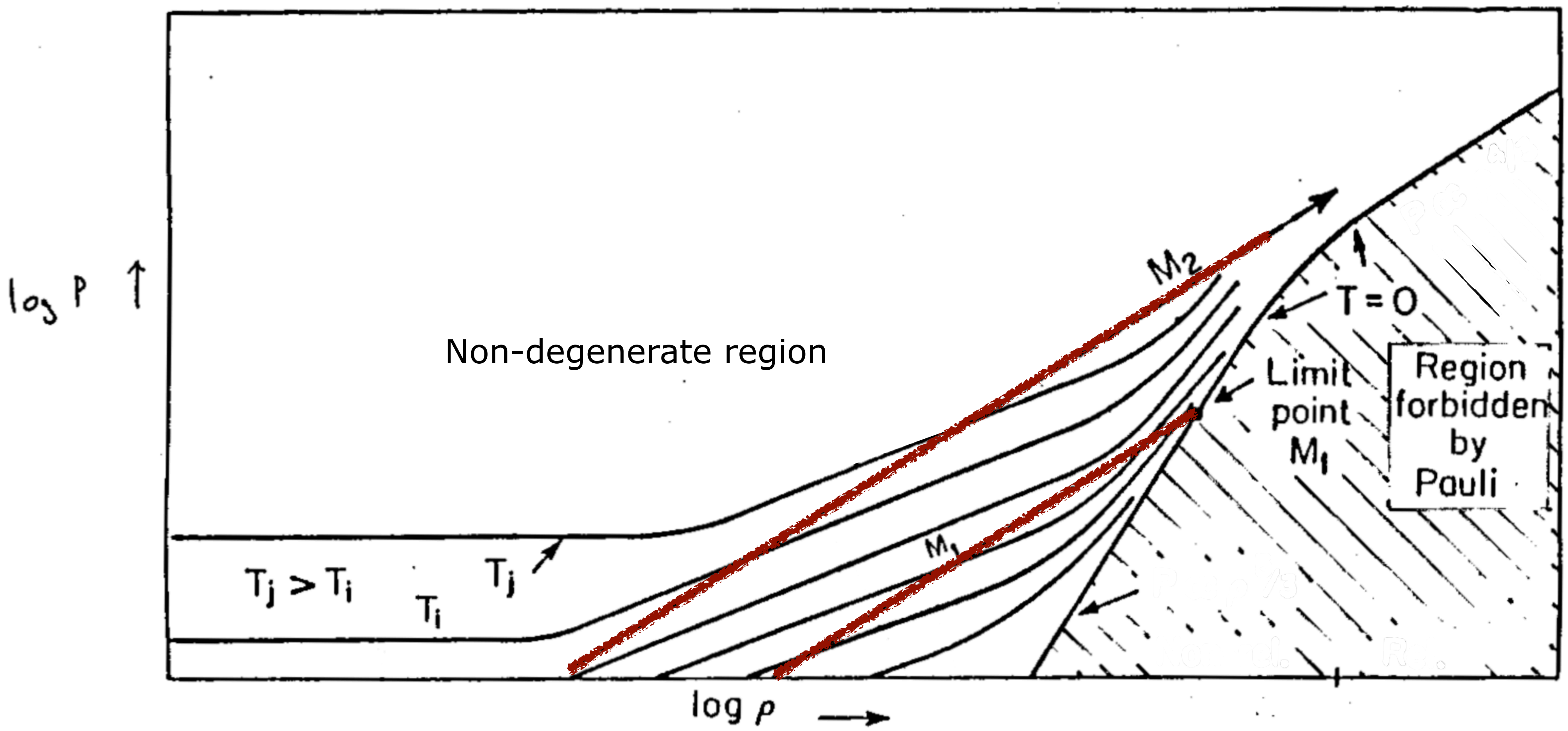
Now, considering EoS, we can derive the evolution of T_c

Evolution in the P_c-ρ_c plane

Let's start by considering isotherms in the (P,$\boldsymbol{\rho}$) plane

Two evolutionary tracks of stars with masses $M_2 > M_1$

$P_c \propto \boldsymbol{\rho}_c^{4/3}$



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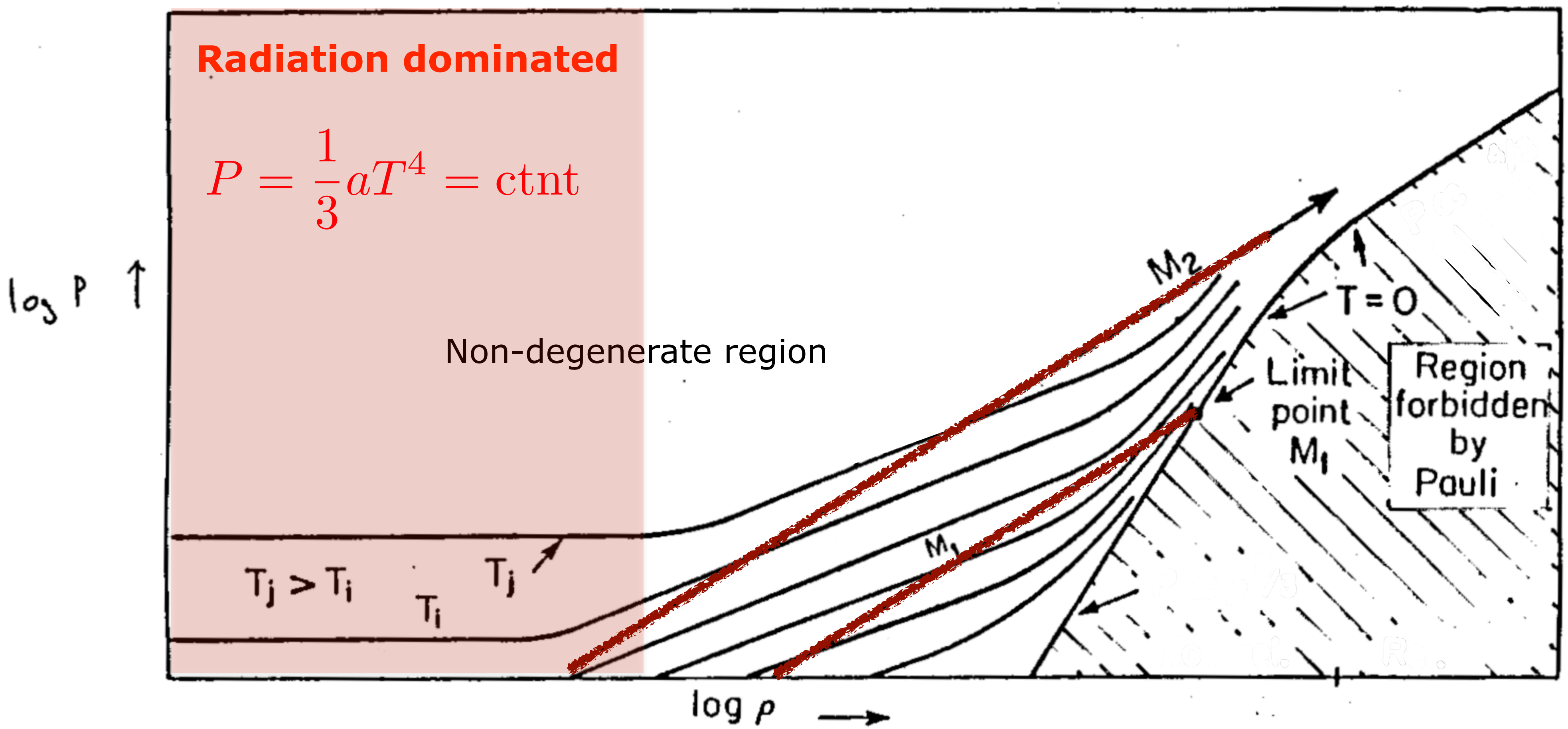

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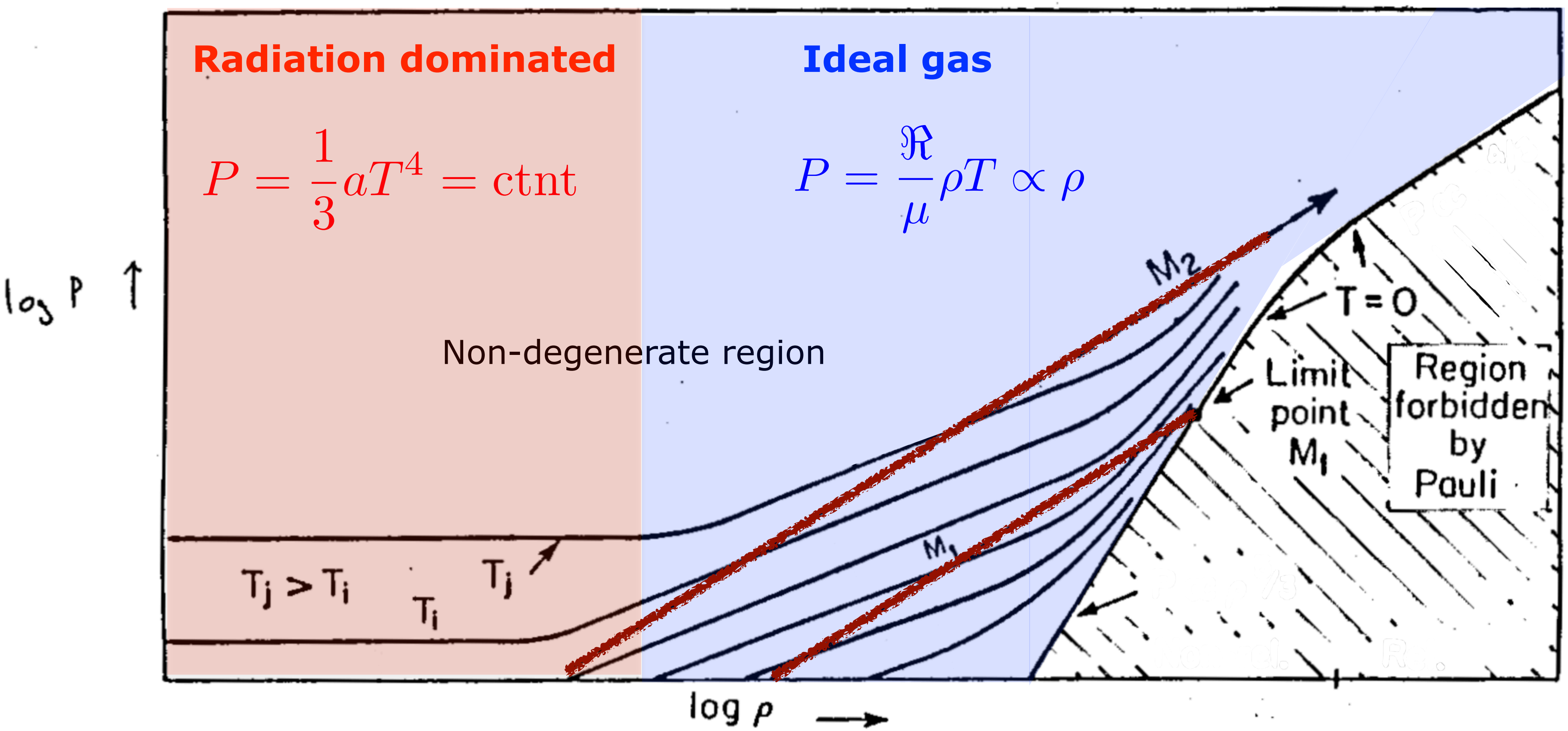

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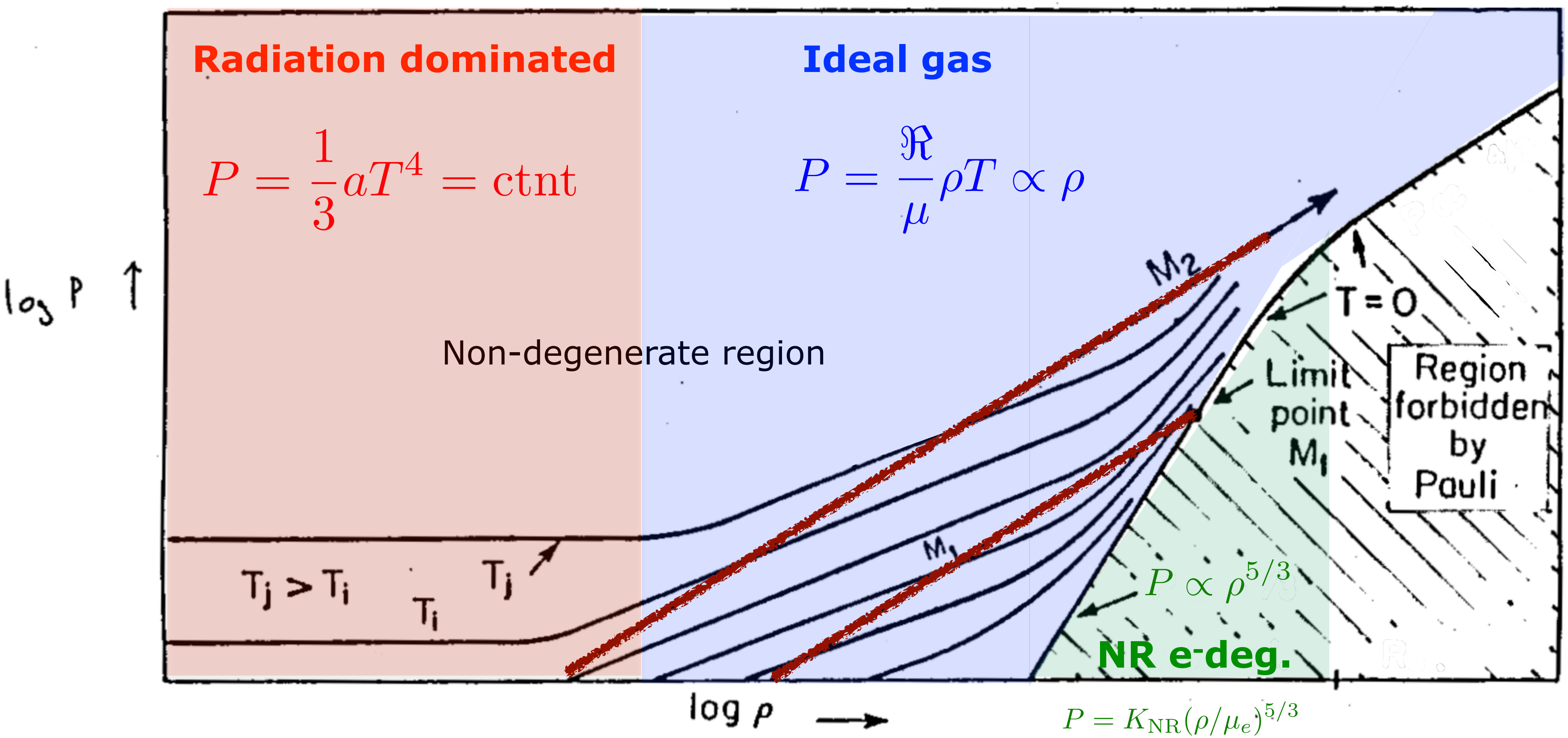

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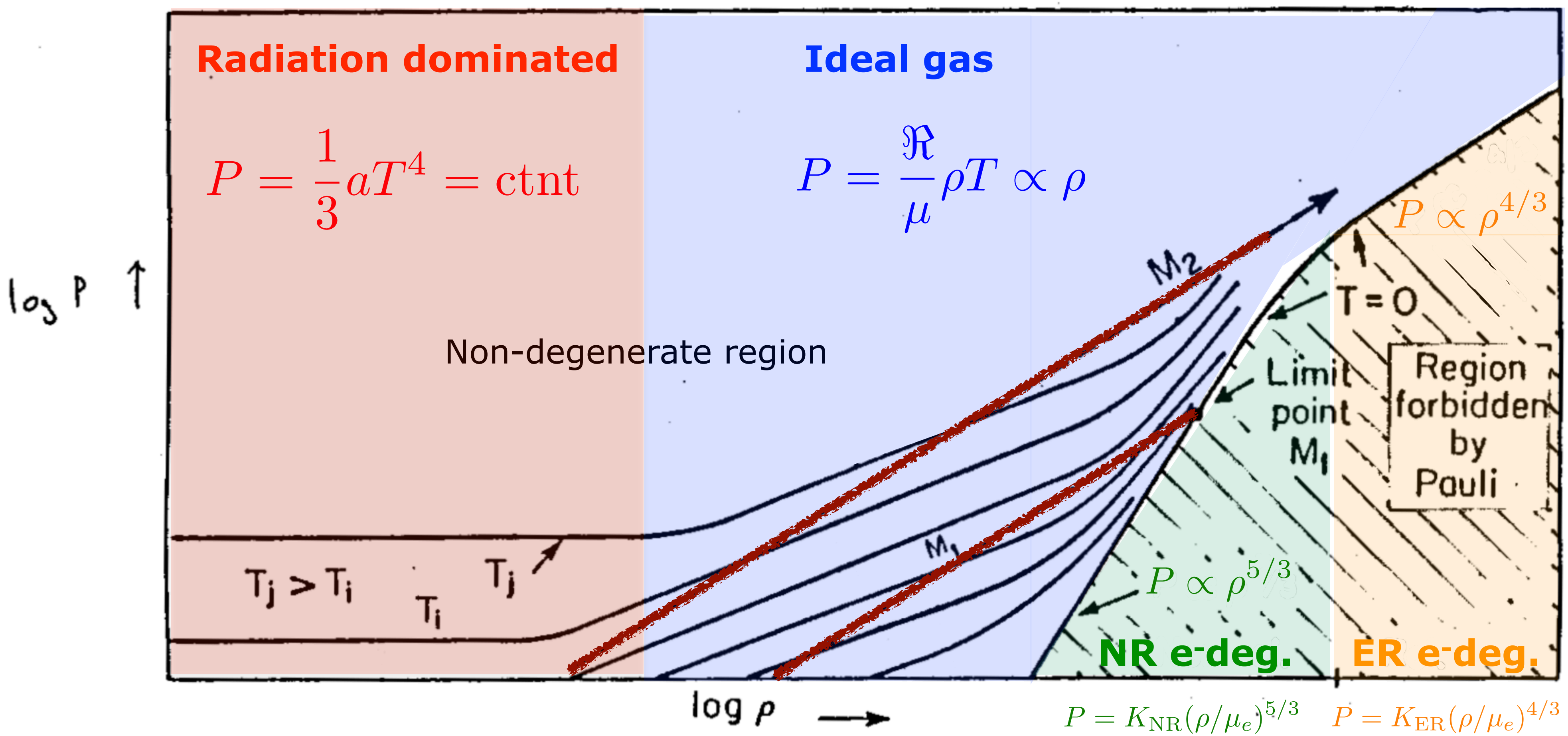

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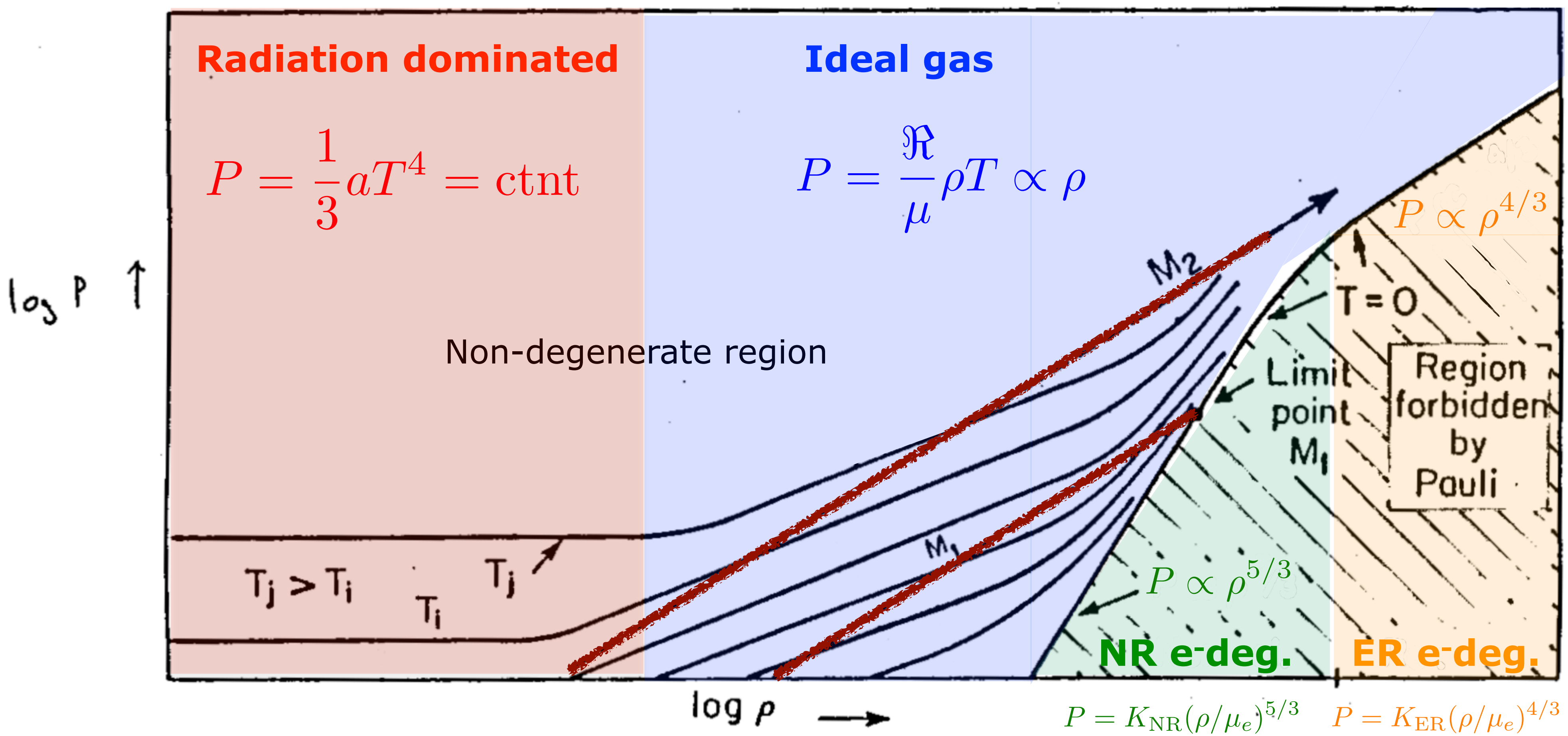


Flat isotherms

Slope 1

P ind. of T
Crosses iso T=0

Evolution in the P_c-ρ_c plane

- As long as the gas is ideal, contraction (⇧ρ_c) leads to ⇧T_c

VTh: E_{in} =- 1/2 E_{gr}. Contraction (⇩E_{gr}, i.e. ⇧-E_{gr}) leads to ⇧E_{in}, (heating of the stellar gas)

- Tracks of $M<M_{crit}$ eventually run into the line for complete e^- deg

Hence for stars with $M<M_{crit}$ there exist a $\rho_{c,max}$, $P_{c,max}$, that define the endpoint of their evolution.

They also reach a $T_{c,max}$, at the point where degeneracy starts to dominate P, after which further contraction leads to decreasing T_c.

$\rho_{c,max}$, $P_{c,max}$ and $T_{c,max}$ all depend on M and increase with M.

$M_{crit} \sim M_{Ch}$

- Tracks for $M>M_{crit}$, miss the completely e- deg region of the EOS

a star with $M>M_{crit}$ must keep on contracting and getting hotter indefinitely

(up to the point where the assumptions we have made break down, e.g. when ρ becomes so high that p inside the nuclei capture free e- and a n gas is formed, which can also become degenerate)

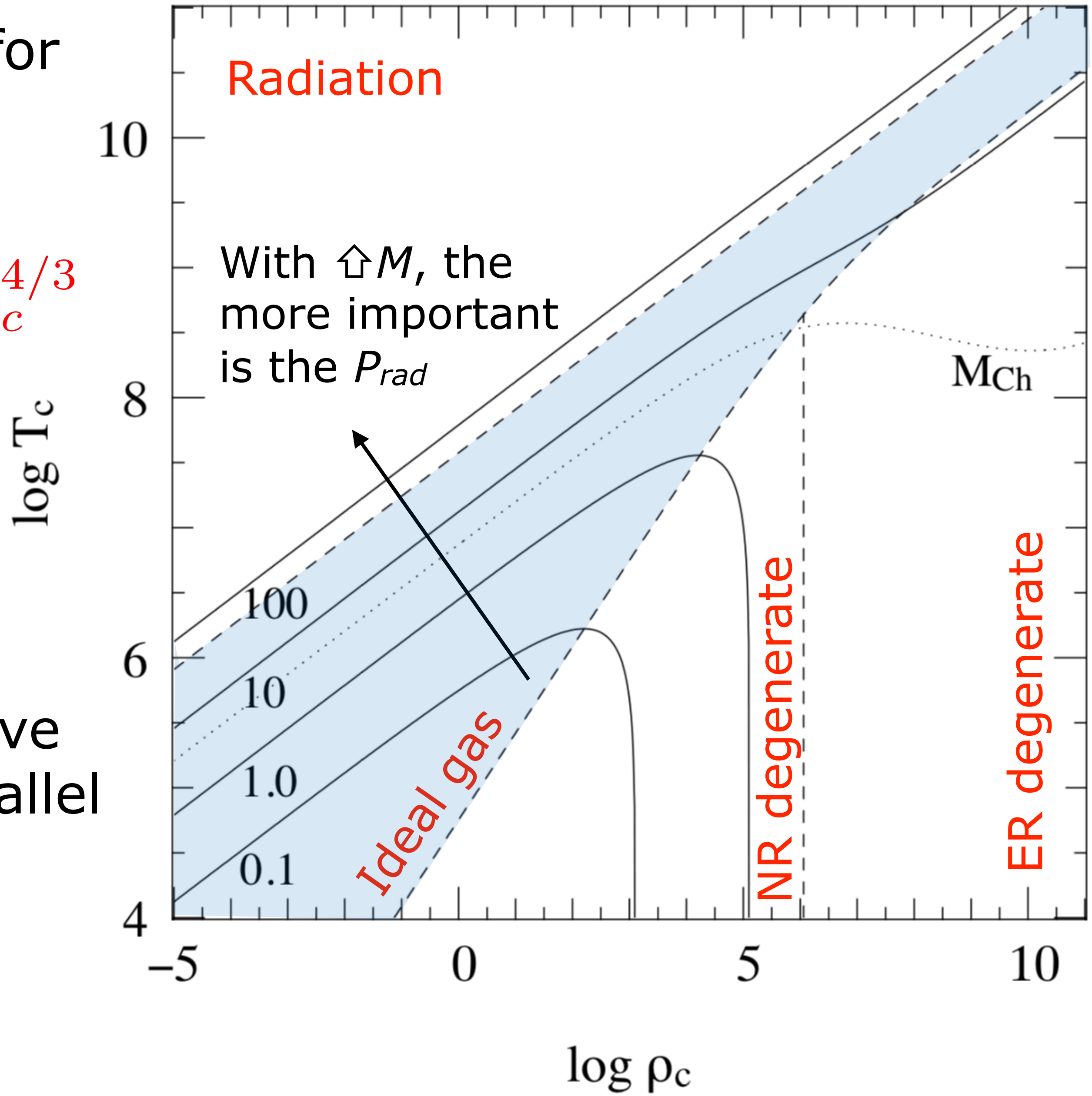
Evolution in the T_c-ρ_c plane

From the P-$\boldsymbol{\rho}$ relation for an ideal gas,

$$\frac{\Re}{\mu} T_c \rho_c = C \cdot G M^{2/3} \rho_c^{4/3}$$

$$\boxed{T_c = \frac{CG}{\Re} \mu M^{2/3} \rho_c^{1/3}}$$

Stars of different M, have evolutionary tracks parallel to each other



Evolution in the T_c-ρ_c plane

Stars with $M<M_{crit}$ will cross the NR deg. region

$$K_{\rm NR}\left(\frac{\rho_c}{\mu_e}\right)^{5/3} = CGM^{2/3}\rho_c^{4/3}$$

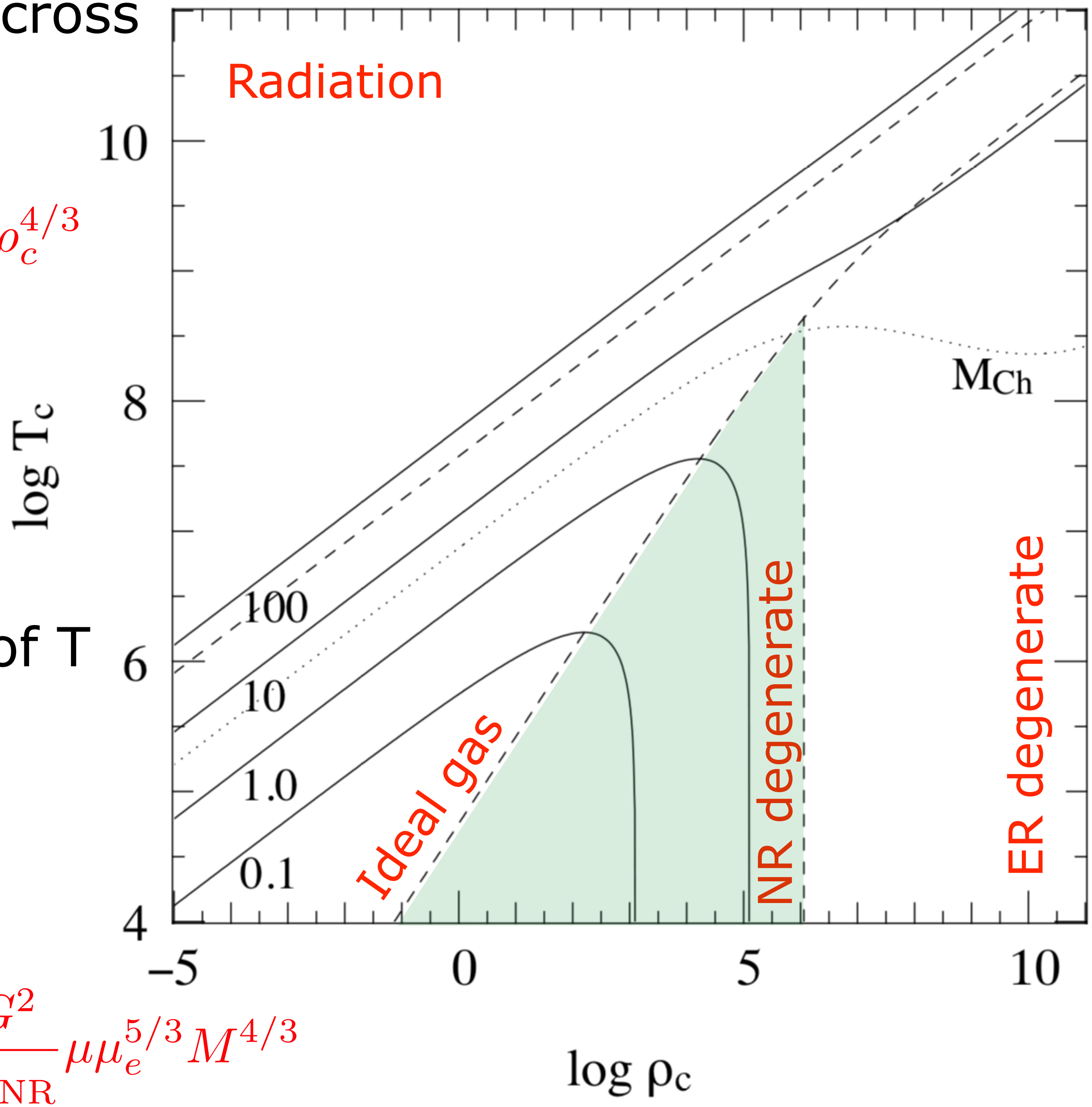
$$\boxed{\rho_c = \left(\frac{CG}{K_{\rm NR}}\right)^3 \mu_e^5 M^2}$$

which is ctnt and ind. of T

the star cools down at fixed density

$T_{c,max}$ is reached when $P_{ig} = P_{deg}$

$$T_{\rm c,max} = \frac{C^2G^2}{4\Re K_{\rm NR}}\mu\mu_e^{5/3}M^{4/3}$$



Nuclear burning regions

We just found out that stars with $M<M_{crit}$ reach a $T_{c,max} \propto M$

Only stars of certain M will reach sufficient T for nuclear burning

The nuclear energy generation rate,

$$\epsilon_{\rm nuc} = \epsilon_0 \rho^\lambda T^\nu$$

$\boldsymbol{\lambda}$: depends on number of nuclei
=1 for 2 parti., =2 for 3 part.
$\boldsymbol{\nu}$: depends on masses and charges
=4 for p-p and =18 for CNO

As a consequence of this dependence,

- each nuclear reaction takes place at a particular, nearly constant T
- nuclear burning cycles of subsequent heavier elements are well separated in T

Limits to stellar mass

- We can estimate the minimum mass of a star to reach the T needed to burn H (at $T_c \sim 10^7$ K)

Using homology relations, one can see that this mass is $\sim 0.08\ M_\odot$

Less massive objects become partially degenerate before reaching T, and continue to contract and cool without ever burning H: *brown dwarfs*.

- On the other hand, $M \gtrsim 100\ M_\odot$ are dominated by P_{rad}, which has a $\gamma_{ad} = 4/3$, and HE becomes unstable

Schematic stellar evolution

A star contracts until it reaches HE, and T_C and ρ_C remain ~ctnt, at the values needed for H burning.

When H is exhausted in the core – which how consists of He and has a mass ~10% of M – this He core resumes its contraction, while the layers around it expand.

This constitutes a large deviation from homology, however the core itself still contracts more or less homologously until reaching HE, while the weight of the envelope decreases as a result of its expansion.

Therefore, homologous relations remain approximately valid for the *core* of the star: replacing M by the core mass M_C, which now determines the further evolution.

Similar arguments lead to the existence a minimum M_C for He-ignition of ~0.3 $M_\odot$.

Schematic stellar evolution

In stars with a He $M_c < 0.3\ M_\odot$ the core becomes degenerate before reaching $T_c = 10^8$ K, and in the absence of a surrounding envelope it would cool to become a *He White Dwarf*
(In practice, however, H-burning in a shell around the core keeps the core hot and when Mc has grown to $\approx 0.5\ M_\odot$ He ignites in a degenerate flash.)

After the exhaustion of He in the core, the core again resumes its contraction until the next fuel can be ignited.

Following a similar line of reasoning the minimum (core) mass for C-burning, which requires $T \approx 5\times10^8$ K, is $\approx 1.1\ M_\odot$. Less M_c are destined to never ignite C but to become degenerate and cool as *CO White Dwarfs*.

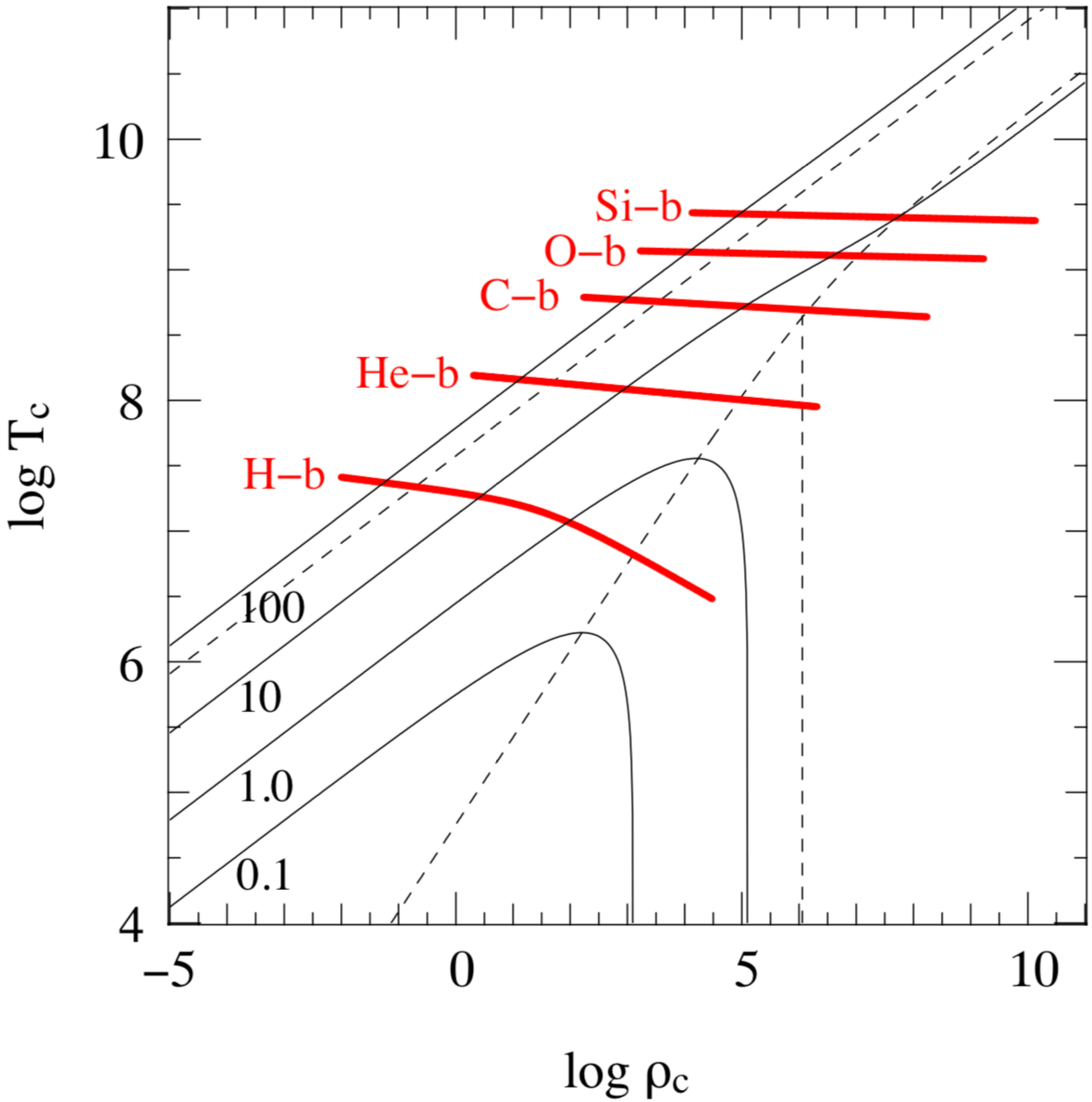
The $M_{c,min}$ required for the next stage, Ne-burning, turns out to be $\approx M_{Ch}$. Stars with $M_C > M_{Ch}$ will undergo all subsequent nuclear burning stages (Ne-, O- and Si- burning) and never become degenerate.

Eventually they develop a Fe core, from which no further nuclear energy can be squeezed. The Fe core must collapse in a *cataclysmic event* (a SN or a GRB) and become a NS or BH.

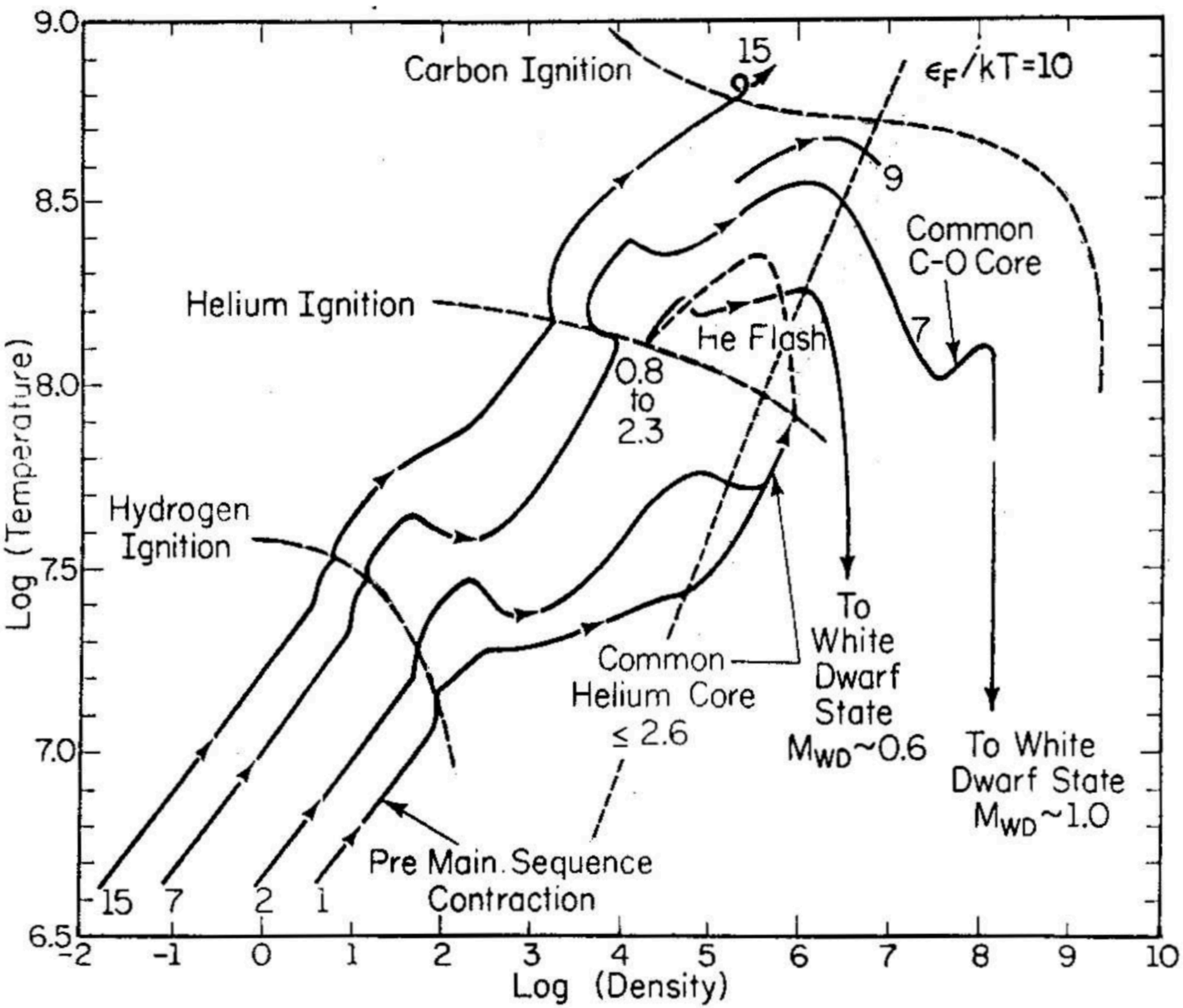
Schematic stellar evolution

phase	T (10^6 K)	total E_{gr}/n	main reactions	total E_{nuc}/n	M_{min}	γ (%)	ν (%)
grav.	$0 \to 10$	~ 1 keV/n				100	
nucl.	$10 \to 30$		$^{1}H \to {}^{4}He$	6.7 MeV/n	$0.08\,M_\odot$	~95	~5
grav.	$30 \to 100$	~ 10 keV/n				100	
nucl.	$100 \to 300$		$^{4}He \to {}^{12}C, {}^{16}O$	≈ 7.4 MeV/n	$0.3\,M_\odot$	~100	~0
grav.	$300 \to 700$	~ 100 keV/n				~50	~50
nucl.	$700 \to 1000$		$^{12}C \to Mg, Ne$	≈ 7.7 MeV/n	$1.1\,M_\odot$	~0	~100
grav.	$1000 \to 1500$	~ 150 keV/n					~100
nucl.	$1500 \to 2000$		$^{16}O \to S, Si$	≈ 8.0 MeV/n	$1.4\,M_\odot$		~100
grav.	$2000 \to 5000$	~ 400 keV/n	$Si \to \ldots \to Fe$	≈ 8.4 MeV/n			~100

Schematic stellar evolution



Schematic stellar evolution



Schematic stellar evolution

We have obtained the following picture:

Nuclear burning cycles can be seen as long-lived but temporary interruptions of the inexorable contraction of a star (or at least its core) under the influence of gravity.

This contraction is dictated by the *virial theorem*, and a result of the fact that stars are hot and lose energy by radiation.

If the core mass is less than the M_{Ch}, the contraction can eventually be stopped (after one or more nuclear cycles) when e- degeneracy supplies the pressure needed to withstand gravity.

However if the core mass exceeds the M_{Ch}, degeneracy pressure is not enough and contraction, interrupted by nuclear burning cycles, must continue at least until nuclear densities are reached.

What is a star?

A star is...

Una masa de gas que se encuentra en equilibrio entre la presión hidrostática y la de radiación

Cuerpo gaseoso que emite radiación

Grandes bolas de plasma que emiten energía y permiten la vida en la Tierra

Reactor nuclear que eventualmente explota o se apaga

Cuerpo celeste alrededor del cual orbitan planetas

Causante de las quemaduras solares

Objeto al que se le piden deseos

Exercices (1)

8.1 Homologous contraction (1)

(a) Explain in your own words what *homologous contraction* means.

(b)A real star does not evolve homologously. Can you give a specific example? *[Think of core versus envelope]*

(c) Fig.8.3 shows T_c vs. $\boldsymbol{\rho}_c$ for schematic evolution tracks assuming homologous contraction. Explain qualitatively what we can learn form this figure (nuclear burning cycles, difference between a $1M_\odot$ and a $10M_\odot$ star, ...)

(d) Fig. 8.4 shows the same diagram with evolution tracks from detailed (i.e. more realistic) models. Which aspects were already present in the schematic evolution tracks? When and where do they differ?

8.2 Homologous contraction (2)

In this question you will derive the equations that are plotted in Figure 8.2b.

(a) Use the homology relations for P and ϱ to derive the equation, $P_c = CGM^{2/3}\rho_c^{4/3}$

To see what happens qualitatively to a contracting star of given mass M, the total gas pressure can be approximated roughly by:

$$P \approx P_{id} + P_{deg} = \frac{\Re}{\mu}\rho T + K\left(\frac{\rho}{\mu_e}\right)^\gamma$$

where γ varies between 5/3 (non-relativistic) and 4/3 (extremely relativistic).

(b) Combine this equation, for the case of NR degeneracy, with the central pressure of a contracting star in hydrostatic equilibrium (eq. 8.1, assuming $C \approx 0.5$) in order to find how T_C depends on ϱ_C.

(c) Derive an expression for the maximum central temperature reached by a star of mass M.

Exercices (2)

8.3 Application: minimum core mass for helium burning

Consider a star that consists completely of helium. Compute an estimate for the minimum mass for which such a star can ignite helium, as follows.

- Assume that helium ignites at $T_c = 10^8$ K.
- Assume that the critical mass can be determined by the condition that the ideal gas pressure and the electron degeneracy pressure are equally important in the star at the moment of ignition.
- Use the homology relations for the pressure and the density. Assume that $P_{c,\odot} = 10^{17}$ g cm^{-1} s^{-2} and $\varrho_{c,\odot} = 60$ g cm^{-3}.