

# Física Estelar

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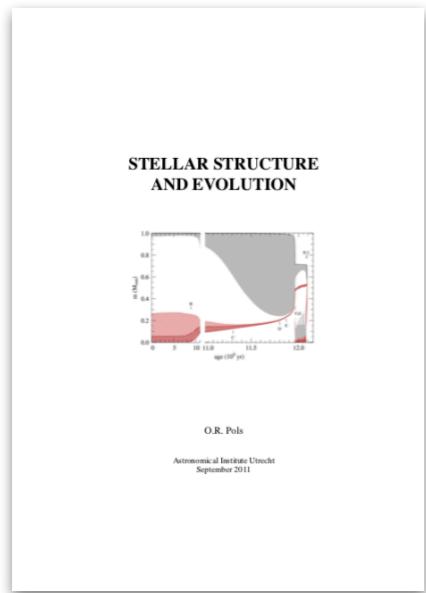
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**Curso 2020-2021**

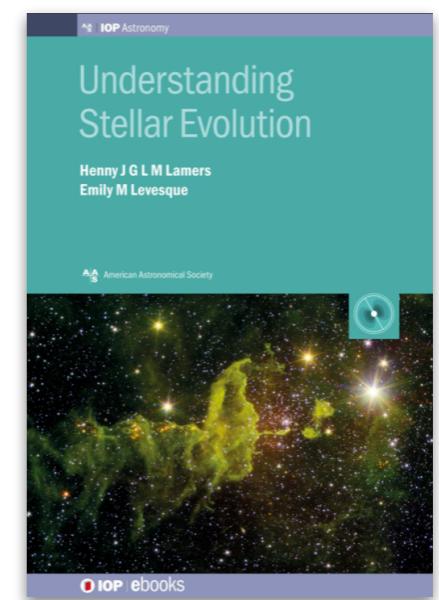
# Syllabus

**Aim:** to understand the structure and evolution of stars, and their observational properties, using known laws of physics.



Stellar structure and evolution  
(O. R. Pols)

Understanding Stellar Evolution  
(Henny J. G. L. M. Lamers & Emily M. Levesque)



# Goals

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In this course we will:

- understand the global properties of stars: energetics and timescales
- study the micro-physics relevant for stars: the equation of state, nuclear reactions, energy transport and opacity
- derive the equations necessary to model the internal structure of stars
- examine (quantitatively) the properties of simplified stellar models
- survey (mostly qualitatively) how stars of different masses evolve, and the endpoints of stellar evolution (white dwarfs, neutron stars)
- discuss a few ongoing research areas in stellar evolution

# Syllabus

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## Tema 1: Estructura estelar

**Introducción a la evolución estelar. Parámetros observacionales. Diagrama Hertzsprung-Russell. Principios de conservación de equilibrio Mecánico y Térmico. Ecuaciones de estructura estelar. Teorema del Virial. Ecuación de estado. Procesos adiabáticos. Modelos estelares polítropos.**

## Tema 2: Fuentes y transporte de energía estelar

**Transporte de energía: Radiación, convección y conducción. Opacidad. Luminosidad de Eddington. Reacciones termonucleares y ritmos de reacción. Principales cadenas y ciclos de combustión nuclear. Otros procesos nucleares de interés astrofísico. Relaciones homologas y criterios de estabilidad.**

## Tema 3: Evolución Estelar

Formación estelar y pre-secuencia principal. Límites de masa estelar: enanas marrones y planetas. Edad cero y secuencia principal. Estimación de edades de cúmulos estelares. Evolución en la rama de las gigantes: estrellas RGB y AGB. Formación de enanas blancas. Estrellas masivas y supernovas de colapso gravitatorio.

## Tema 4: Evolución Estelar en Sistemas Binarios.

Supernovas termonucleares: Aplicaciones cosmológicas. Binarias cataclísmicas. Novas. Erupciones de rayos X. Estrellas gigantes binarias y anomalías químicas.

## Tema 5: Objetos compactos

Estructura y evolución de enanas blancas. Masa límite de Chandrasekhar. Estrellas de neutrones: estructura y evolución. Ecuación de estado: ecuación de Tolman-Volkov-Openheimer. Púlsares: diagrama P-Pdot y sistemas binarios. Agujeros negros: dinámica. Métrica de Schwarzschild y de Kerr. Ondas gravitacionales.

## Tema 6: Pulsaciones Estelares.

Astrosismología: relación periodo-luminosidad. Análisis de señal: prewhitening. Pulsaciones esféricas adiabáticas y no adiabáticas. Oscilaciones radiales y no radiales. Mecanismos de pulsación. Identificación modal: regímenes asintóticos. Pulsaciones en alta rotación.

# Syllabus

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**Tema 1:** Introducción a la evolución estelar. Parámetros observacionales. Diagrama Hertzsprung-Russell.

**Tema 2:** Principios de conservación de equilibrio Mecánico y Térmico.

**Tema 3:** Ecuaciones de estructura estelar. Teorema del Virial. Ecuación de estado. Procesos adiabáticos.

**Tema 4:** Modelos estelares polítropos.

**Tema 5:** Transporte de energía: Radiación, convección y conducción. Opacidad. Luminosidad de Eddington.

**Tema 6:** Reacciones termonucleares y ritmos de reacción. Principales cadenas y ciclos de combustión nuclear.

**Tema 7:** Modelo estelares y criterios de estabilidad.

**Tema 8:** Evolución estelar.

# **1. Introduction**

# **What is a star?**

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**A star is...**

# What is a star?

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**A star is** an object that:

- (1) radiates energy from an internal source; and
- (2) is bound by its own gravity.

Stars must evolve: have *life* and *death*.



Stars can have only a limited range of masses:  $0.1 \lesssim M_{\odot} \lesssim 100$



# Fundamental properties of stars

The **Sun**, our nearest neighbor

Standard parameters of the Sun:

$$\boxed{\begin{aligned} M_{\odot} &\sim 1.99 \times 10^{33} \text{ g} \\ R_{\odot} &\sim 6.96 \times 10^{10} \text{ cm} \\ L_{\odot} &\sim 3.84 \times 10^{33} \text{ ergs}^{-1} \end{aligned}}$$

$$L = 4\pi R^2 \sigma T_{\text{eff}}^4 \longrightarrow T_{\odot, \text{eff}} \sim 5777 \text{ K}$$

Chemical composition:  $X + Y + Z = 1$       (...also  $v_{\text{rotation}}$ )  
*Mass fraction*

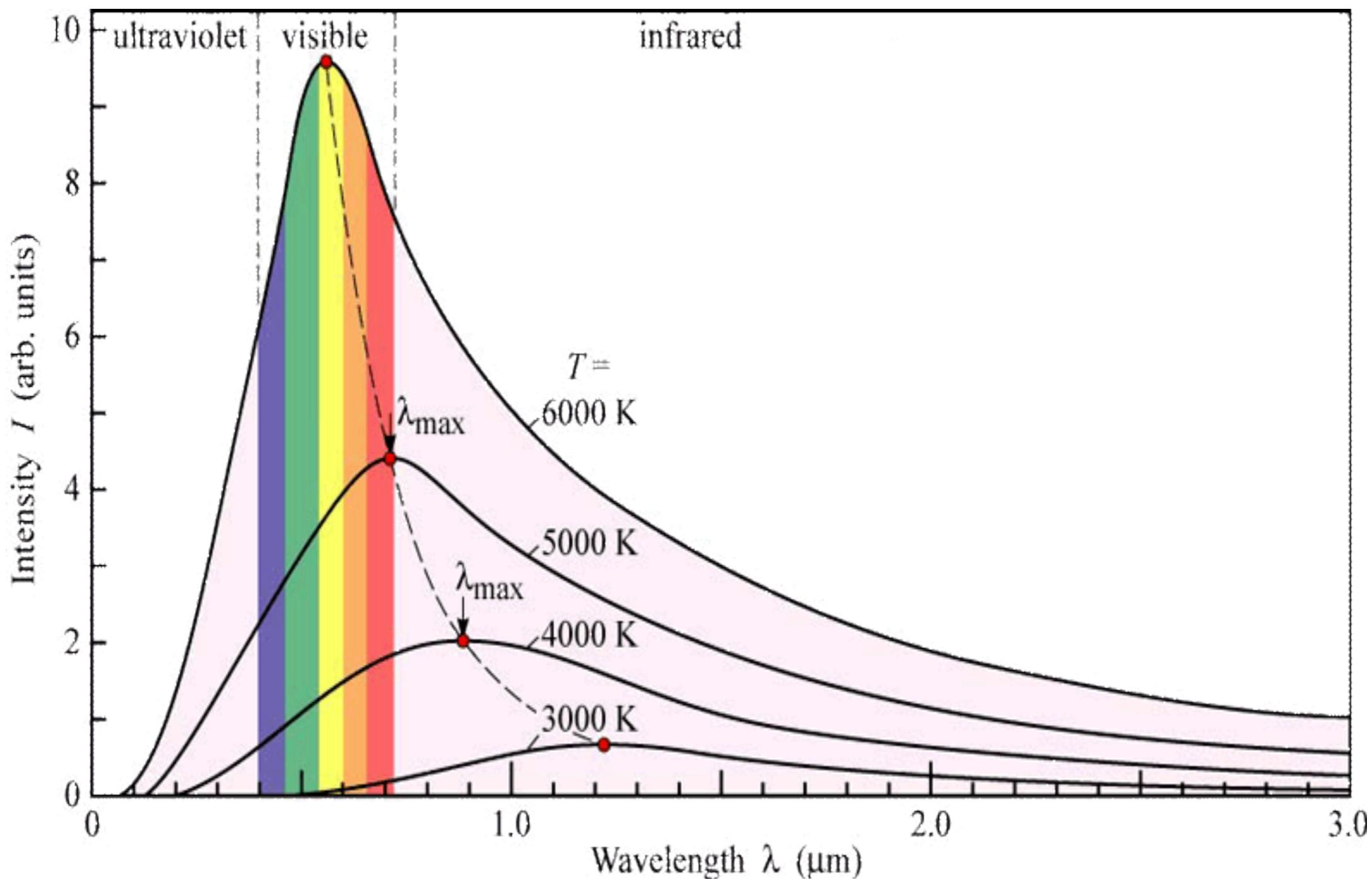
**Solar abundance:**

| Nr | Element | Z  | m (AMU) | Abund | Nr | Element | Z  | m (AMU) | Abund |
|----|---------|----|---------|-------|----|---------|----|---------|-------|
| 1  | H       | 1  | 1.0079  | 0.00  | 6  | N       | 7  | 14.007  | -3.95 |
| 2  | He      | 2  | 4.0026  | -1.01 | 7  | Mg      | 12 | 24.305  | -4.42 |
| 3  | O       | 8  | 15.999  | -3.07 | 8  | Si      | 14 | 28.086  | -4.45 |
| 4  | C       | 6  | 12.011  | -3.44 | 9  | S       | 16 | 32.066  | -4.79 |
| 5  | Ne      | 10 | 20.180  | -3.91 | 10 | Fe      | 26 | 55.847  | -4.46 |

$$H : He : C + O + Ne : \text{rest} = 0.70 : 0.28 : 0.016 : 0.003$$

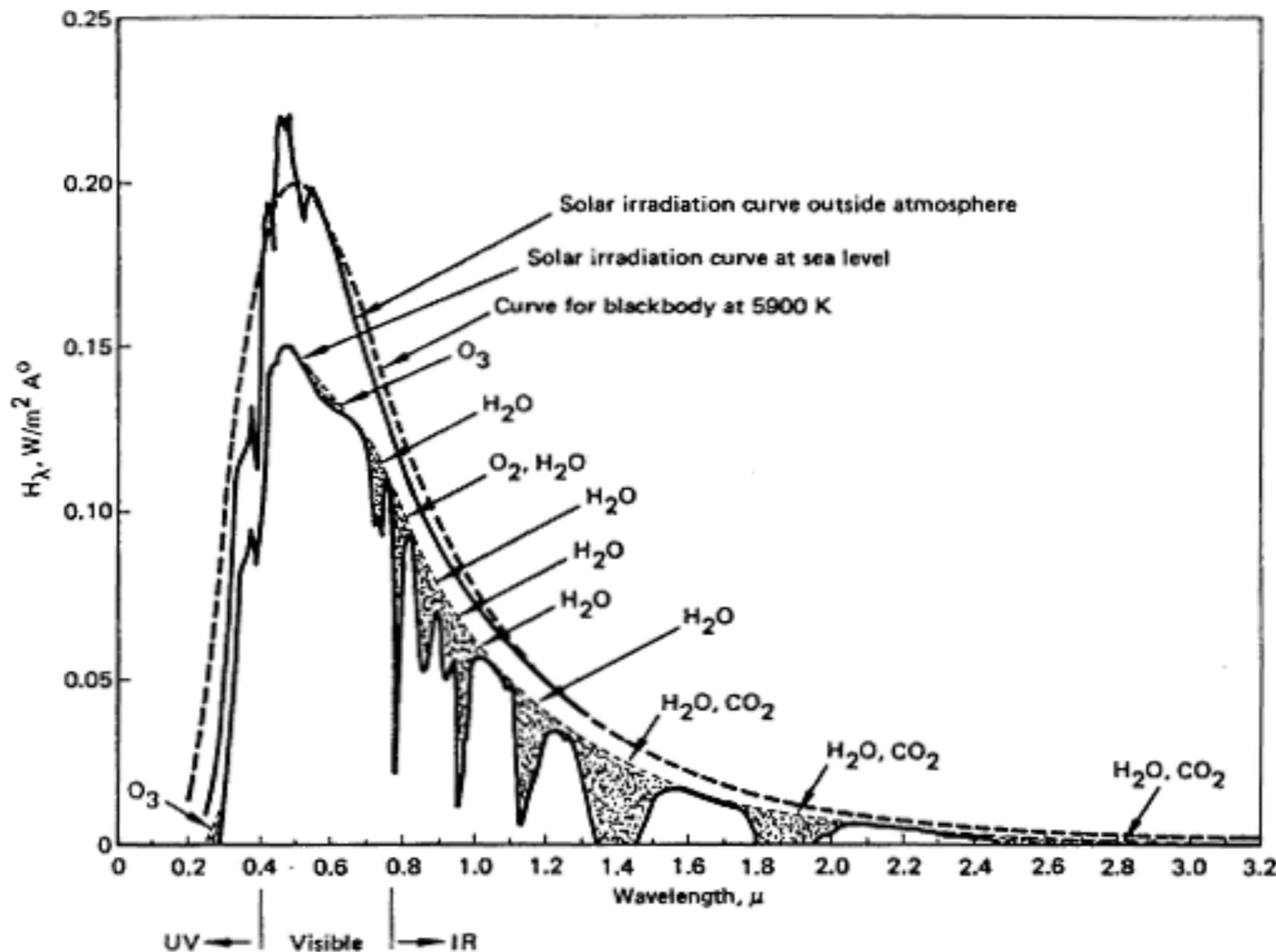
# Effective temperature

Radiation curves of Blackbodies:  $F = \sigma T^4 = \sigma(T_{\text{eff}}^4)$

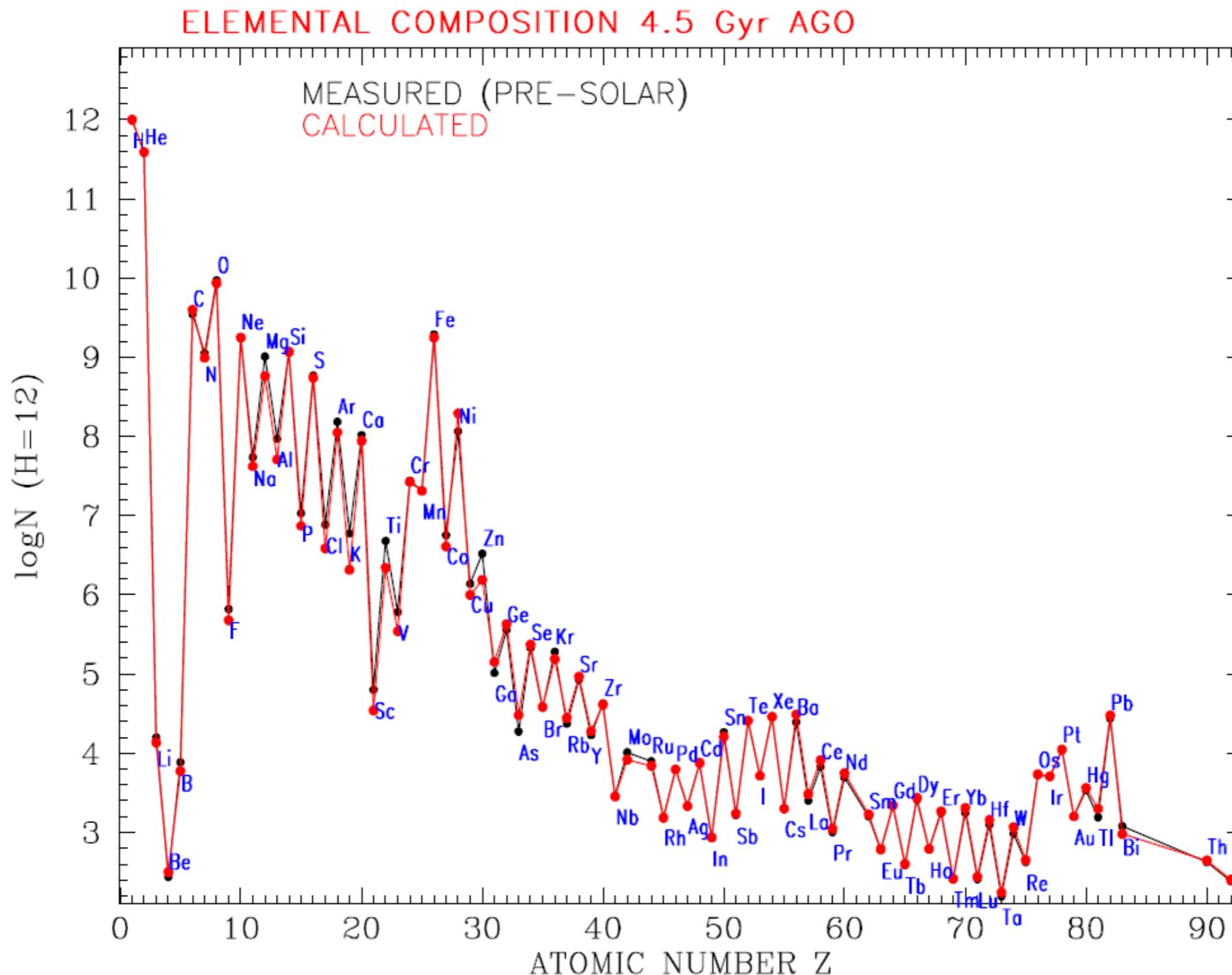


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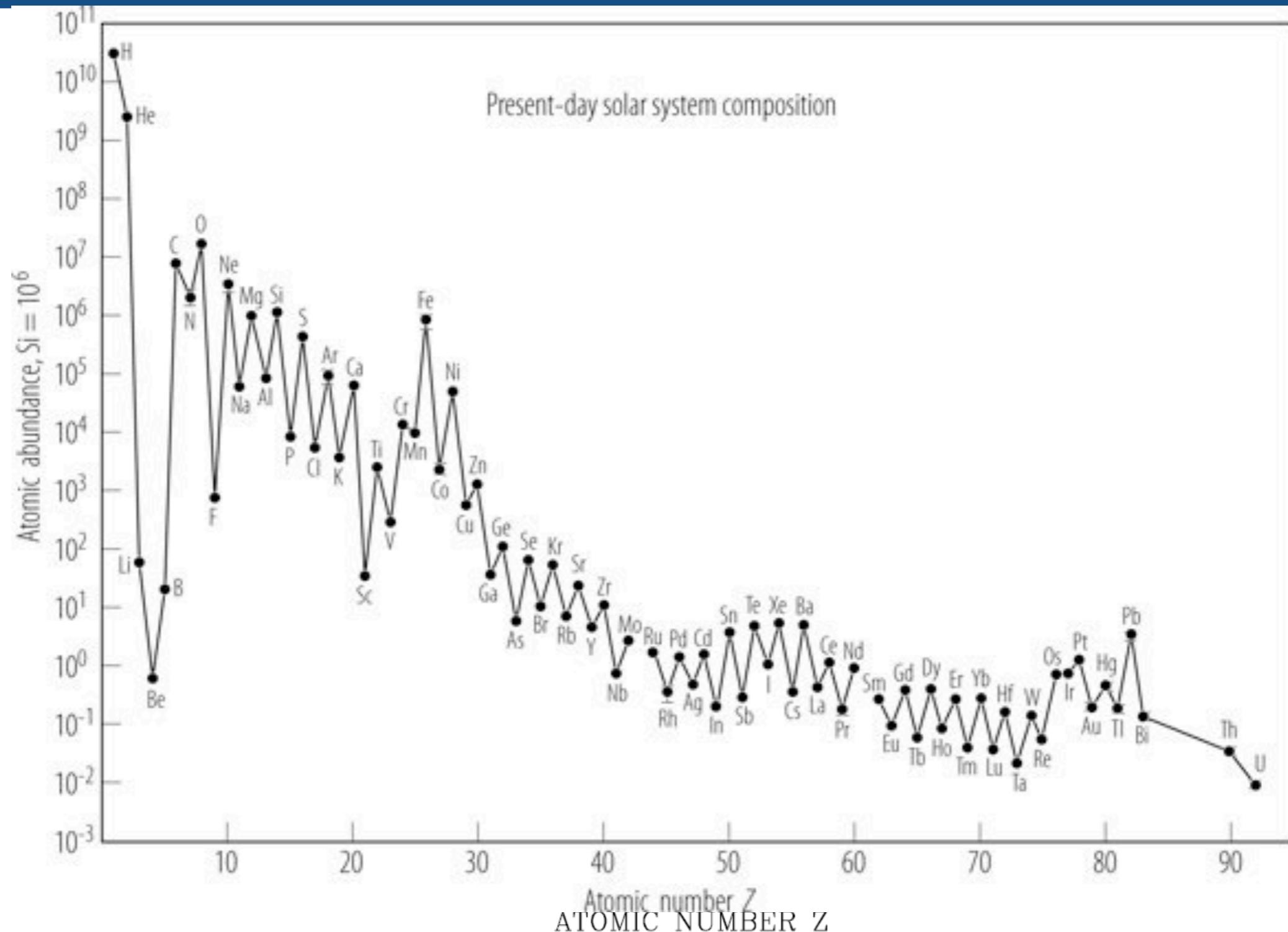


# Sun abundance



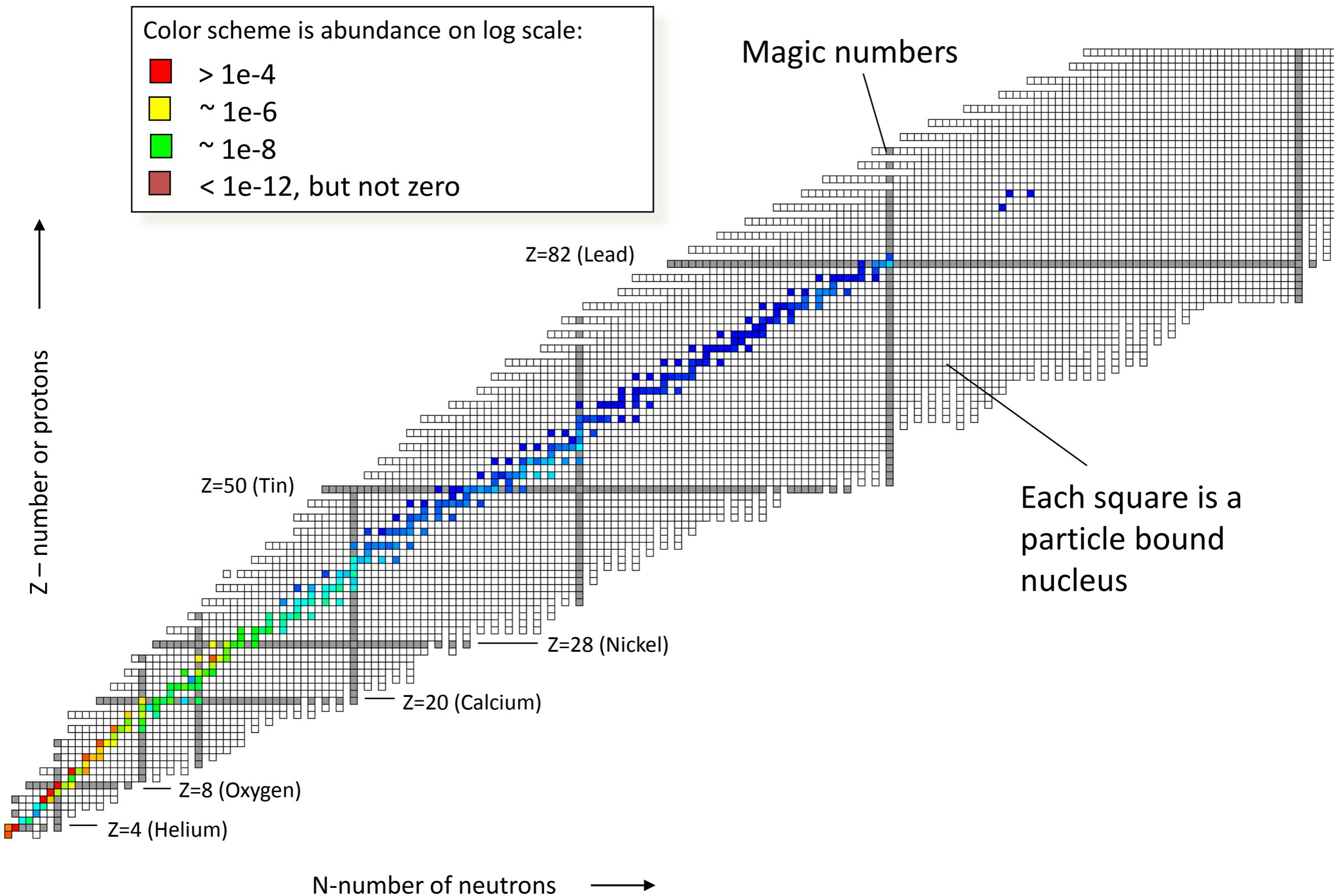
$A = \log(n_x/n_H) + 12$  (log of number of atoms per  $10^{12}$  H atoms)  
also used number of atoms per  $10^6$  Si atoms

# Sun abundance



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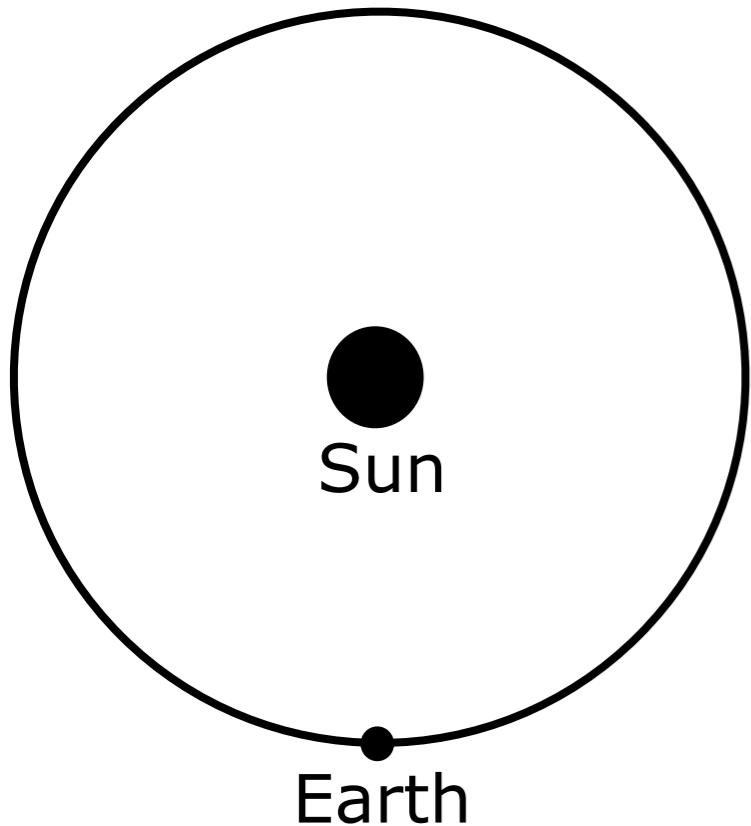
# Abundances of nuclei on the chart of nuclides:



# Observations of stellar parameters

Astronomical observations can yield information about stellar fundamental quantities:

## 1. Distance - parallax method (*Hipparcos/Gaia*)



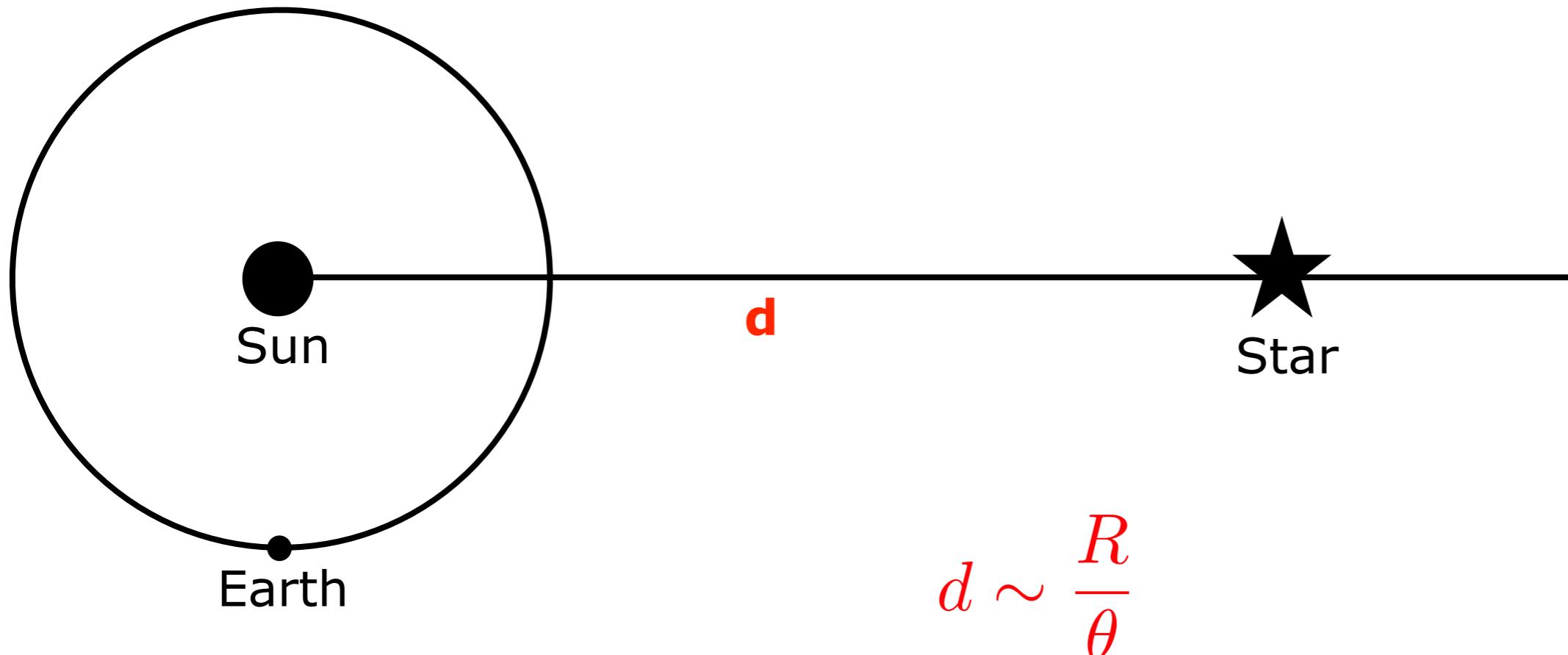
A diagram showing a single star labeled "Star" with a black five-pointed star symbol above it. In the background, there is a cluster of smaller, gray five-pointed star symbols arranged in a roughly triangular shape.

$$d \sim \frac{R}{\theta}$$

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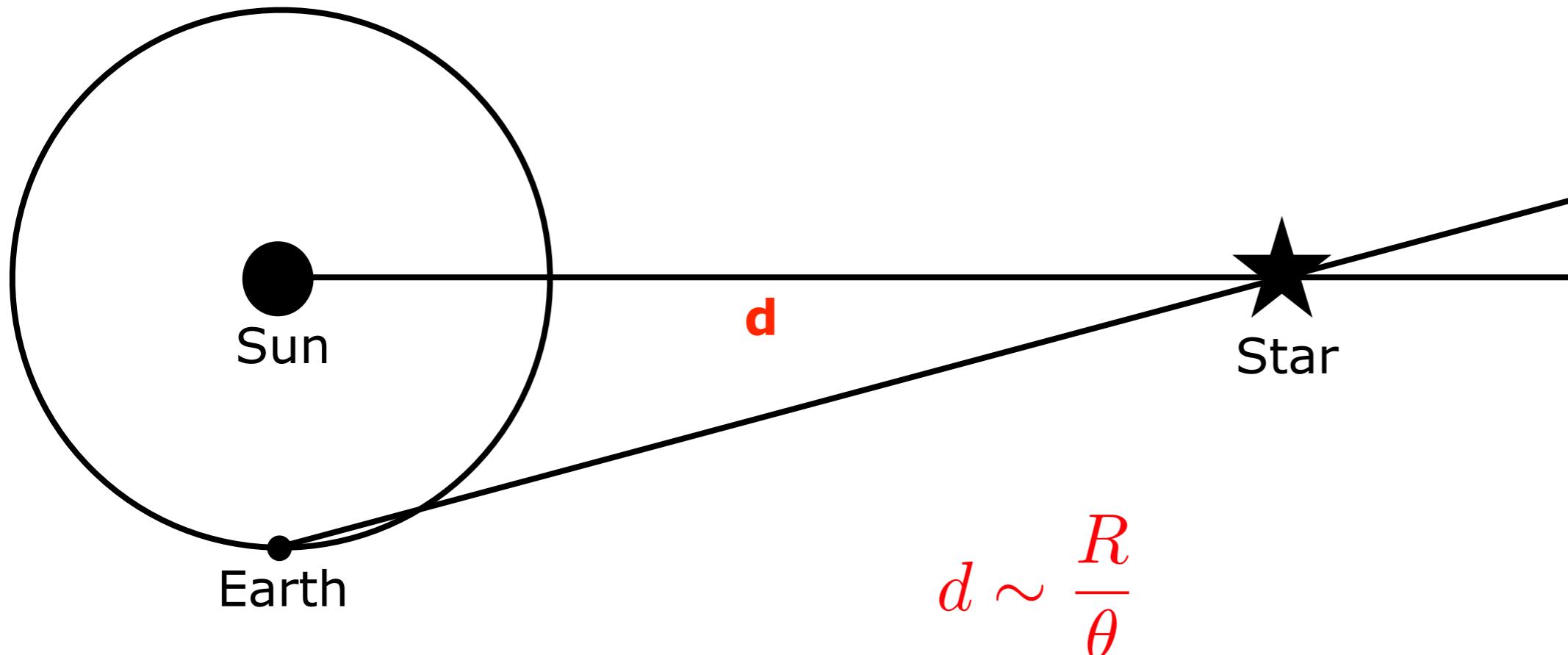
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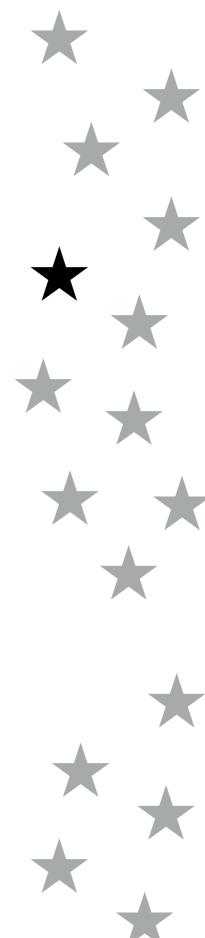
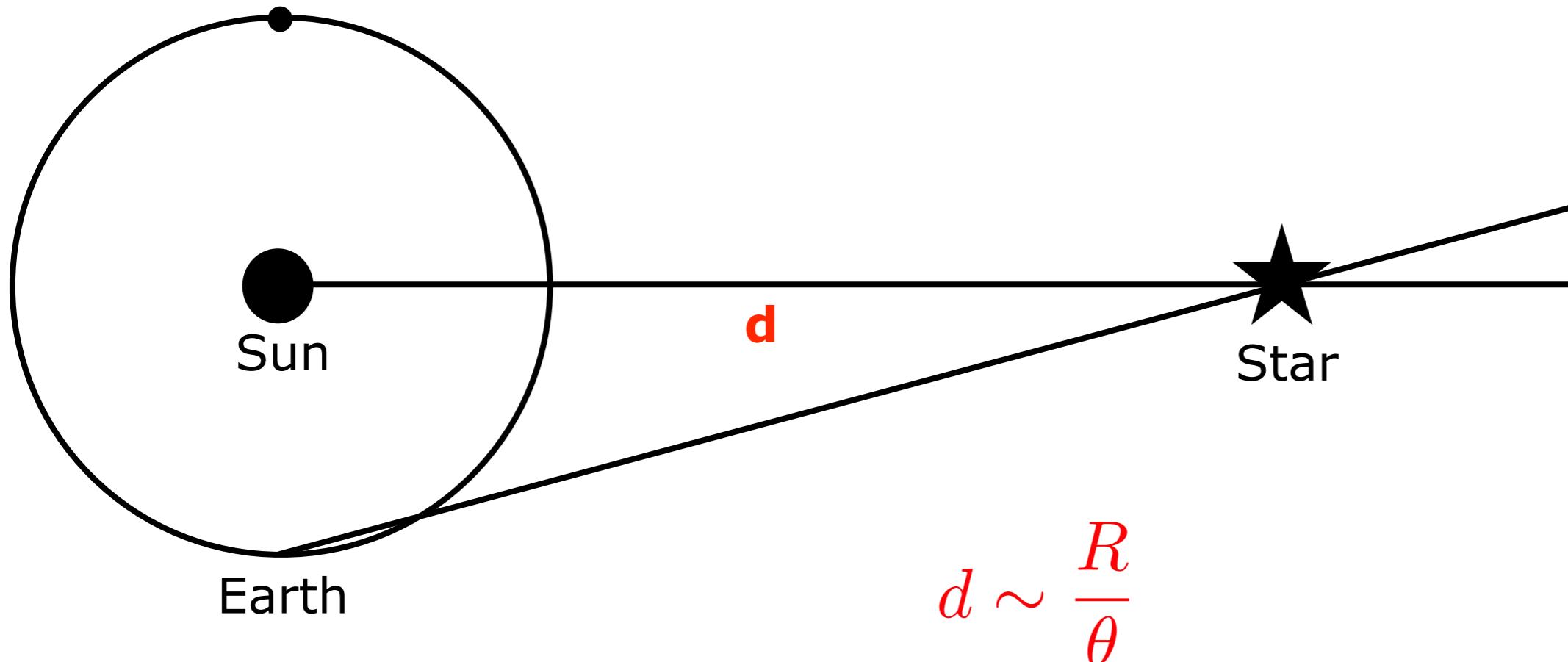
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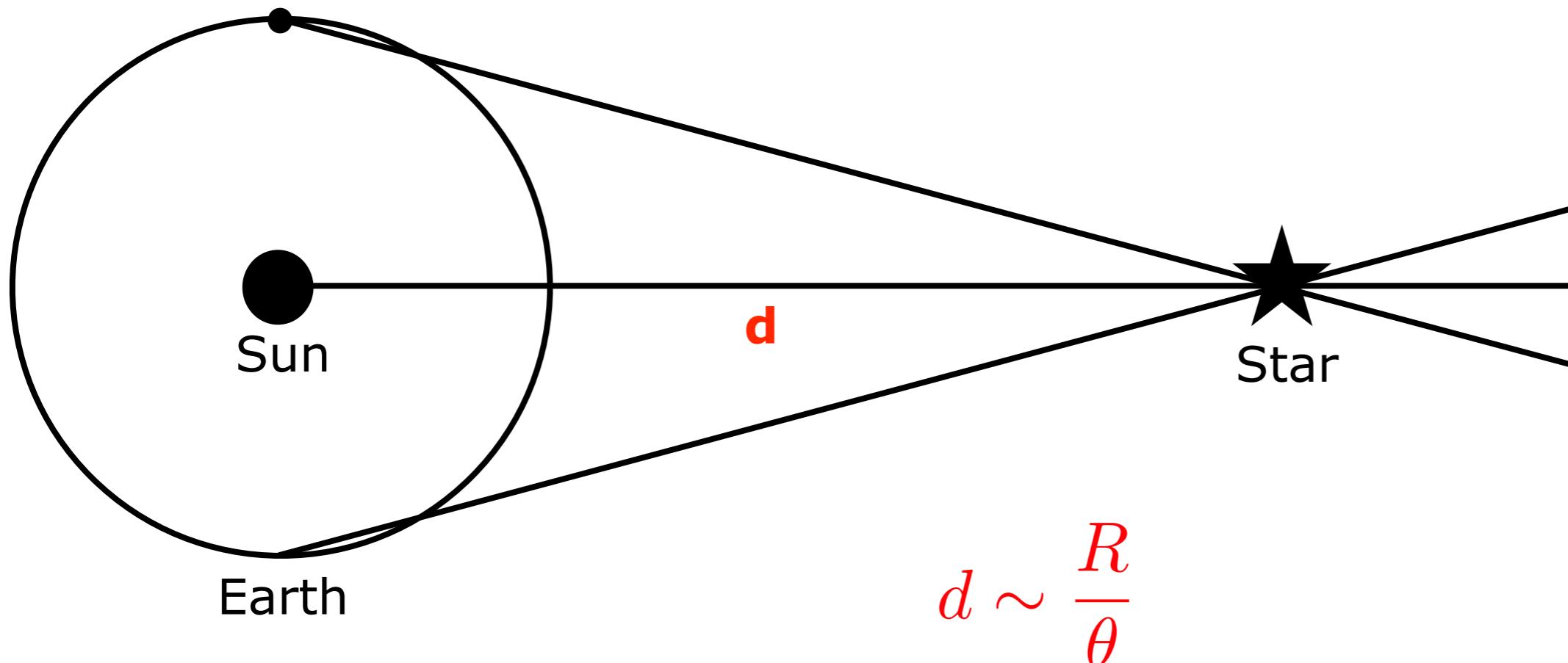
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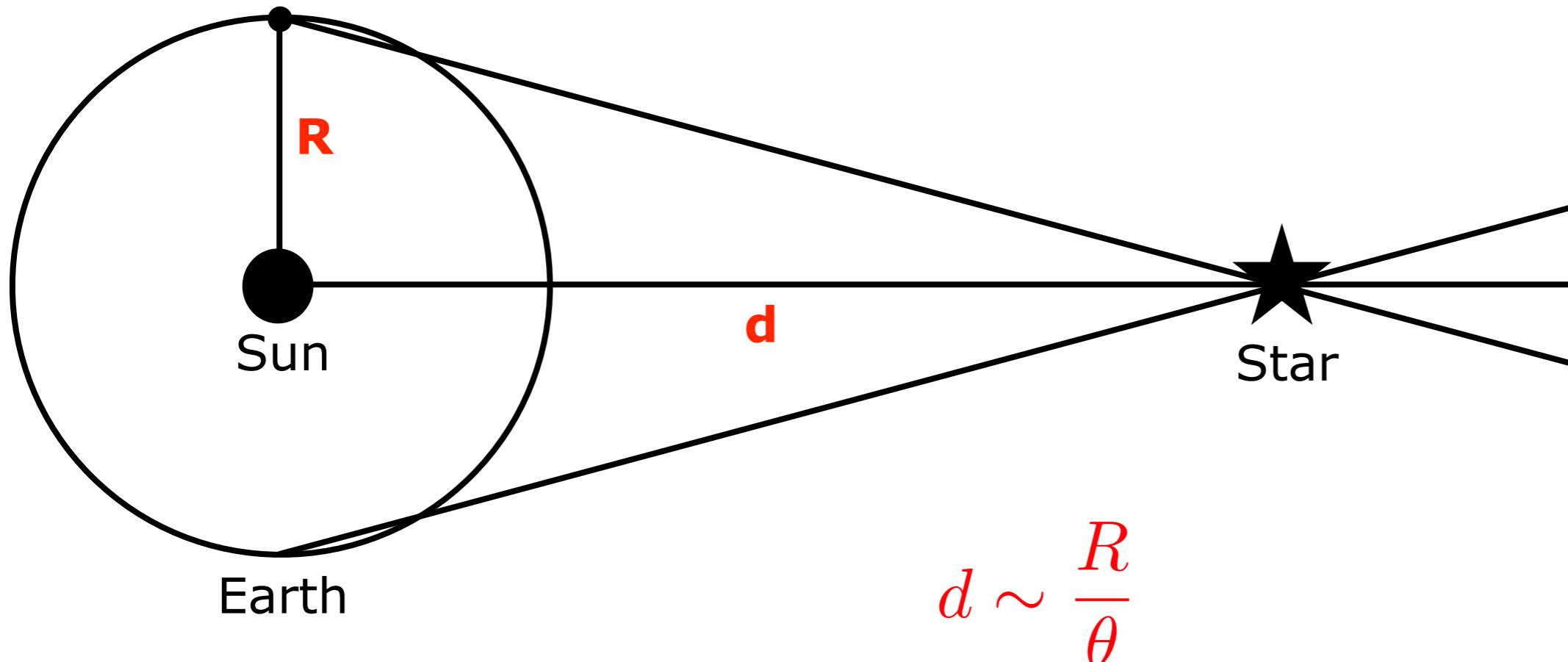
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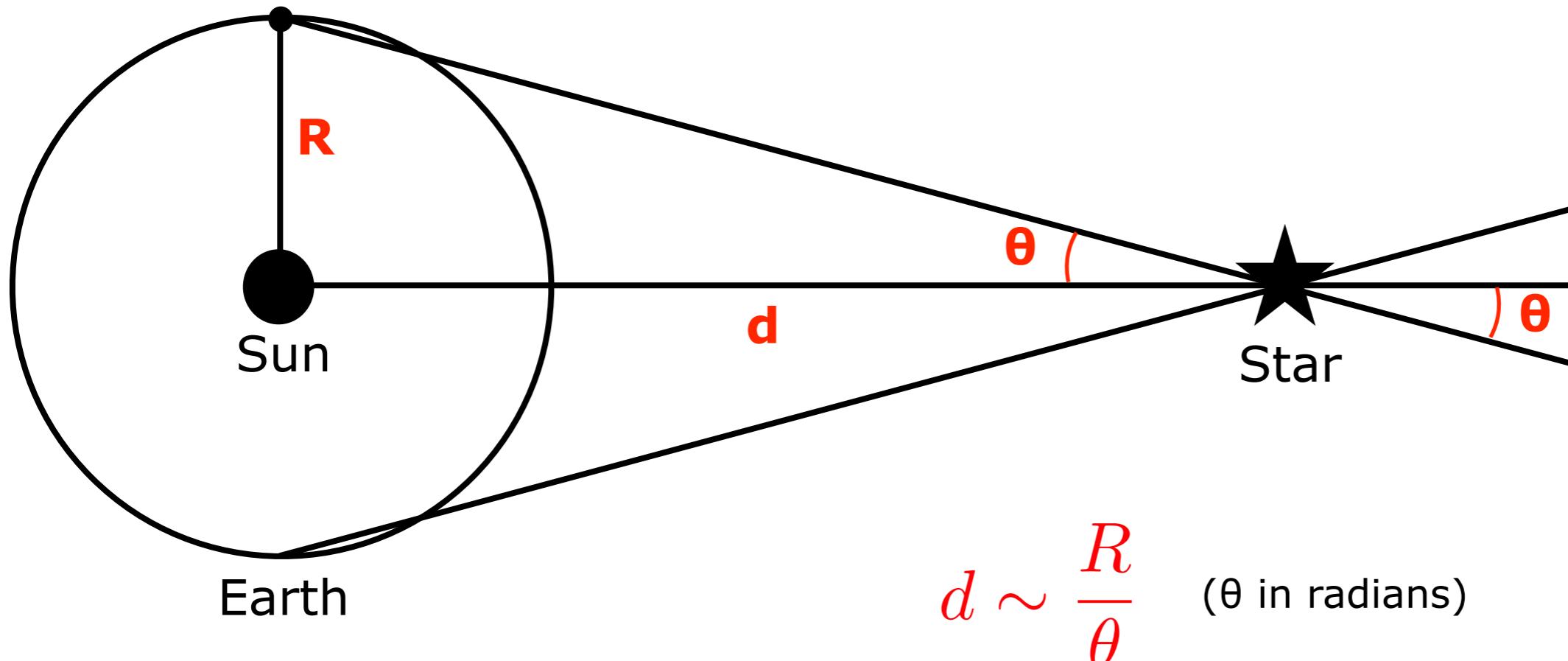
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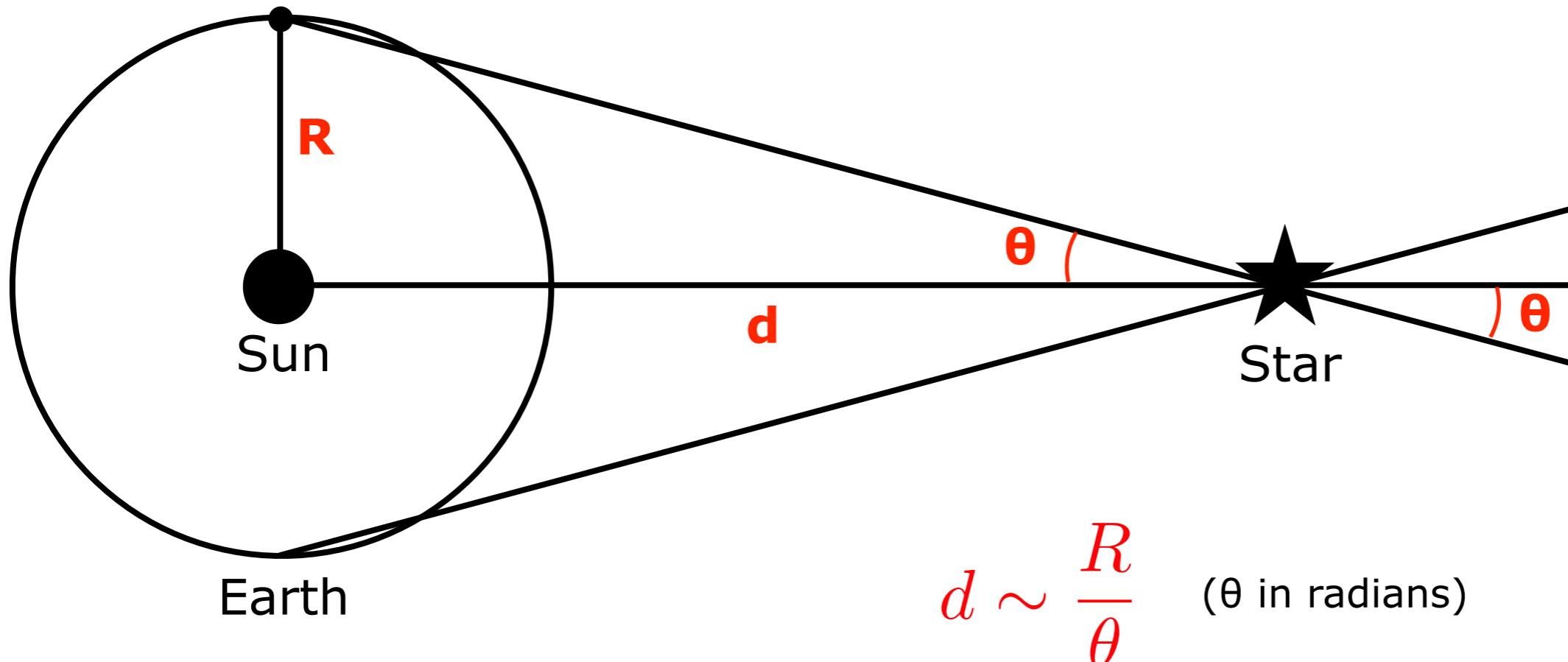
## 1. Distance - parallax method (*Hipparcos/Gaia*)



# Observations of stellar parameters

Astronomical observations can yield information about stellar fundamental quantities:

## 1. Distance - parallax method (*Hipparcos/Gaia*)



$$R = 1.5 \times 10^{13} \text{ cm}, 1'' = 4.85 \times 10^{-6} \text{ radians}$$

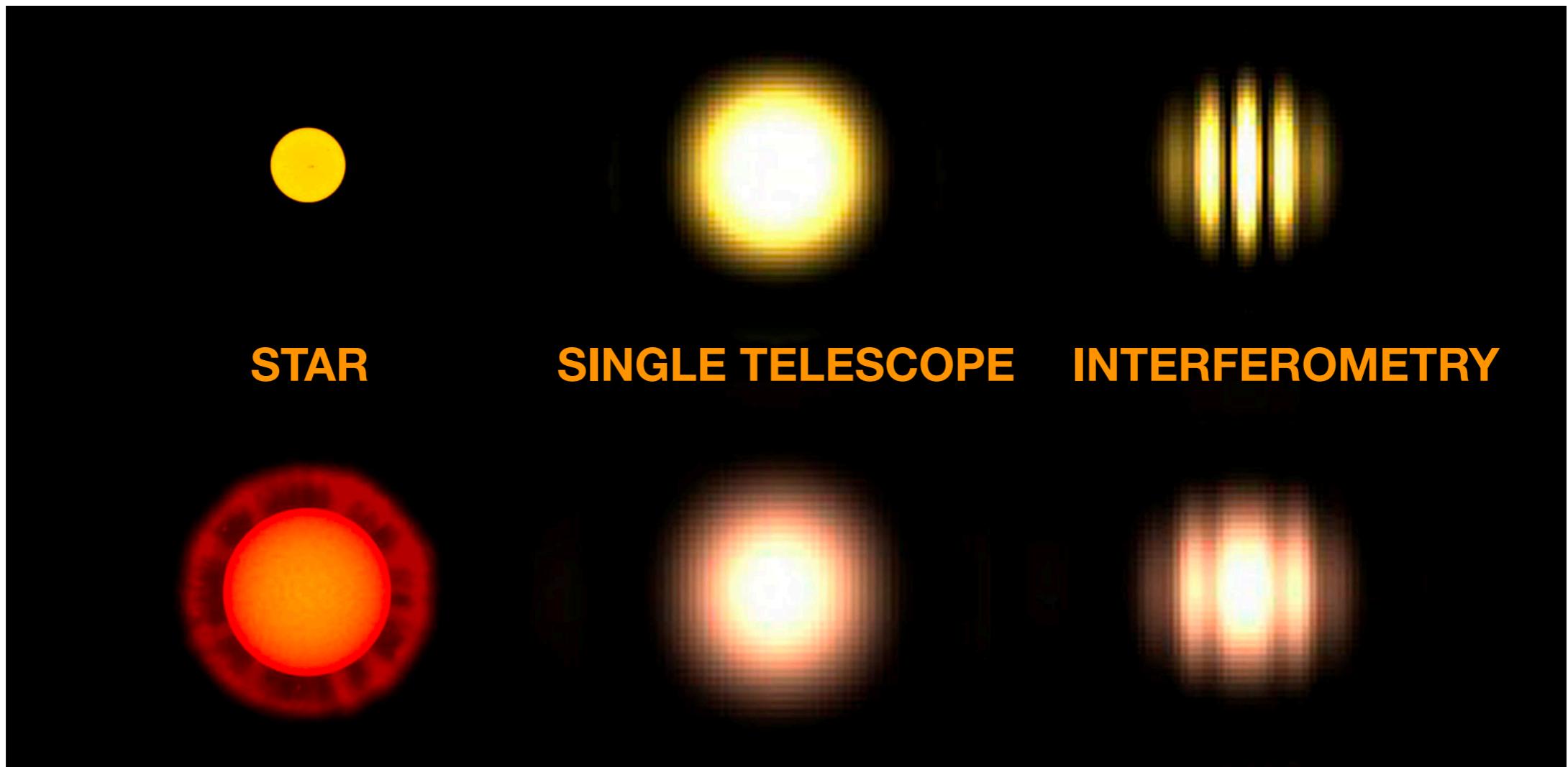
so a parallax of  $1''$  = distance of  $3.09 \times 10^{18} \text{ cm} = 1 \text{ parsec}$

# Observations of stellar parameters

## 2. Angular radius

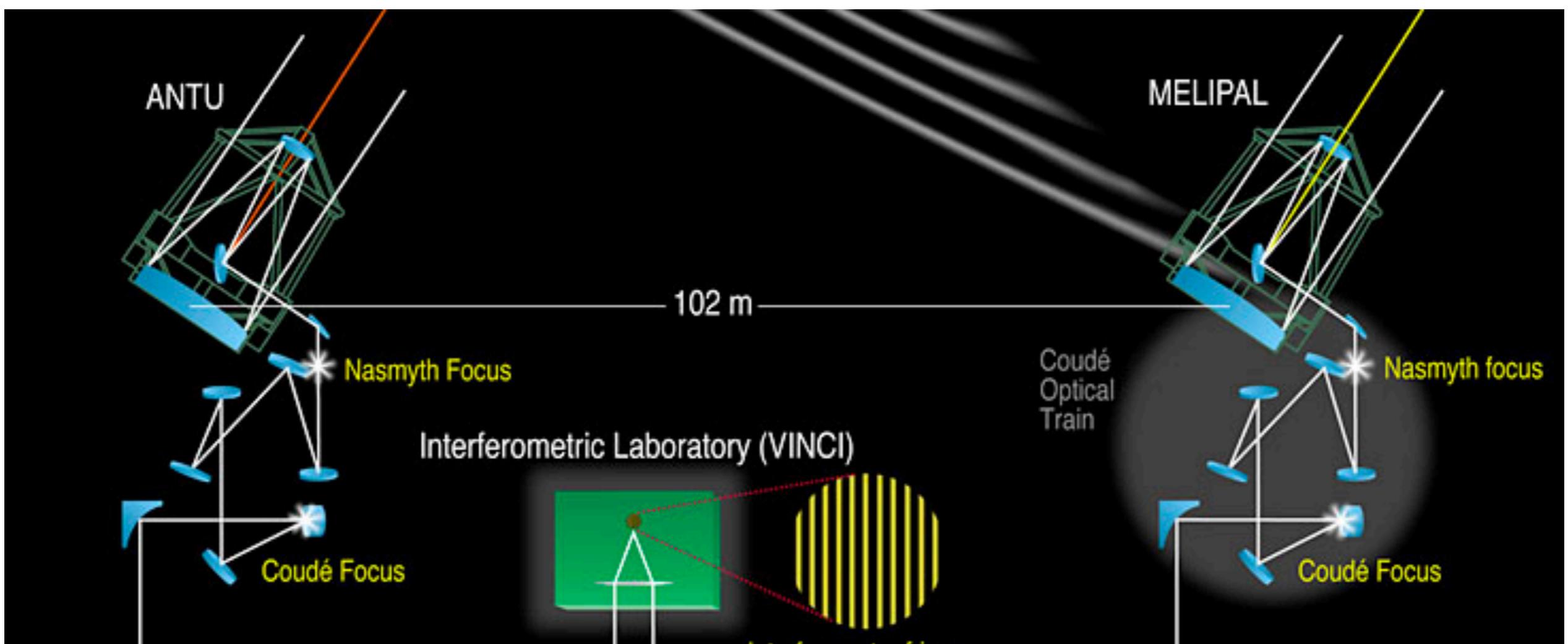
With interferometry, for stars sufficiently extended in the sky

$$\theta = \frac{R}{d} \quad \xrightarrow{\text{Ex: } L = 4\pi R^2 \sigma T_{\text{eff}}^4} \quad \sigma T_{\text{eff}}^4 = \frac{f_{\text{bol}}}{\theta^2}$$



# Observations of stellar parameters

VLT interferometer, Cerro Paranal Observatory, Chile



<https://www.eso.org/public/images/eso0111d/>

[https://www.eso.org/sci/facilities/paranal/telescopes/vlti/tuto/tutorial\\_introduction\\_to\\_stellar\\_interf.pdf](https://www.eso.org/sci/facilities/paranal/telescopes/vlti/tuto/tutorial_introduction_to_stellar_interf.pdf)

# Observations of stellar parameters

## 3. Mass

The most accurate mass measurements for stars come from binaries.

(Visual and eclipsing)

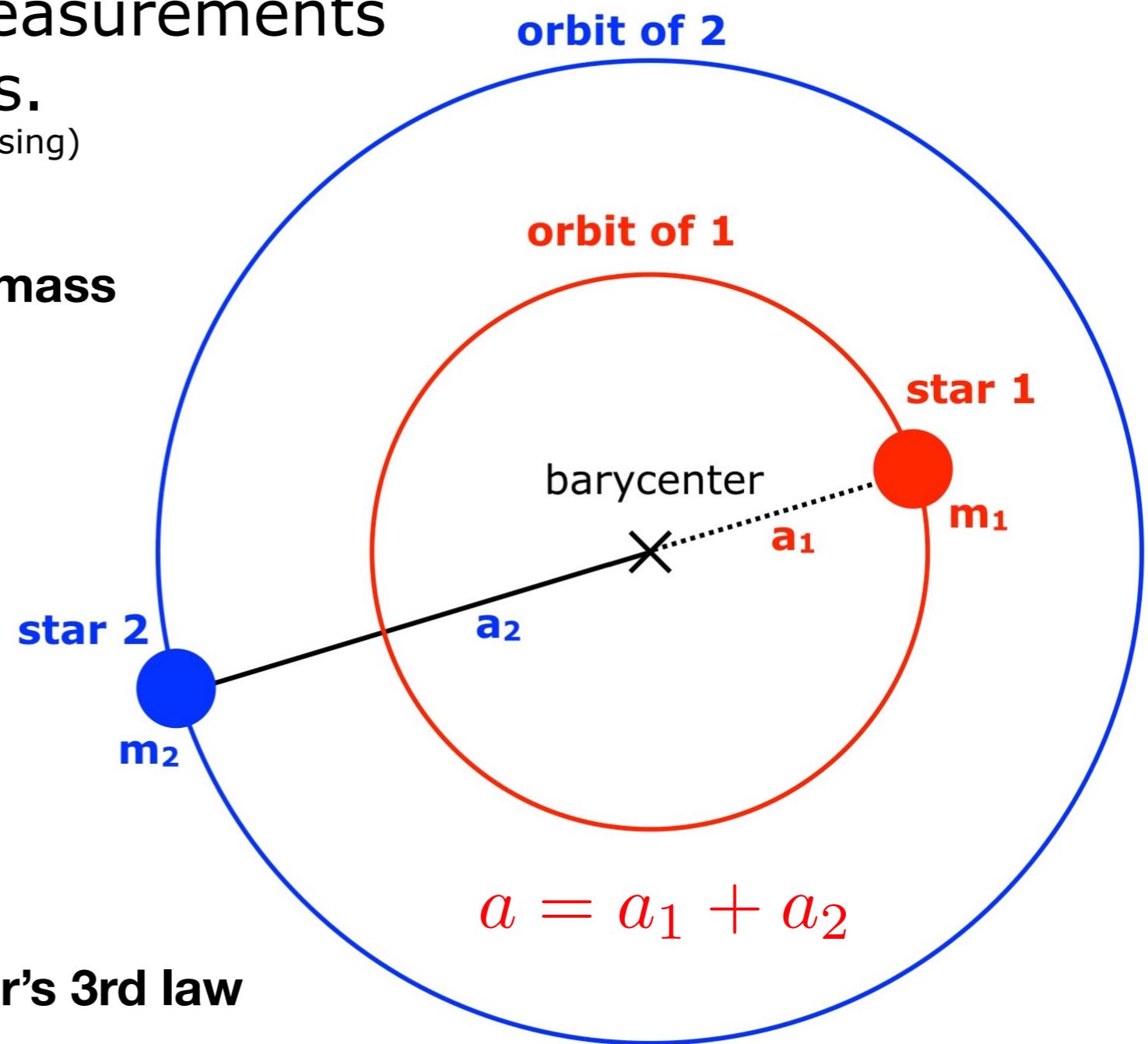
$$M_1 a_1 = M_2 a_2$$

and

$$\left(\frac{2\pi}{P}\right)^2 = G \frac{(M_1 + M_2)}{a^3}$$

$$= G \frac{(M_1 + M_2)}{(a_1 + a_2)^3}$$

**Kepler's 3rd law**



(requires orbits and radial velocities for both stars...)

# Observations of stellar parameters

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## 4. Luminosity

The luminosity ( $L$ ) of a star can be derived from its distance and its radiative flux measured at Earth using the relations:

$$L = 4\pi d^2 f_{\text{bol}} = 4\pi R^2 \sigma T_{\text{eff}}^4 \quad \text{or} \quad \frac{L}{L_{\odot}} = \left( \frac{R}{R_{\odot}} \right)^2 \left( \frac{T_{\text{eff}}}{5777\text{K}} \right)^4$$

where

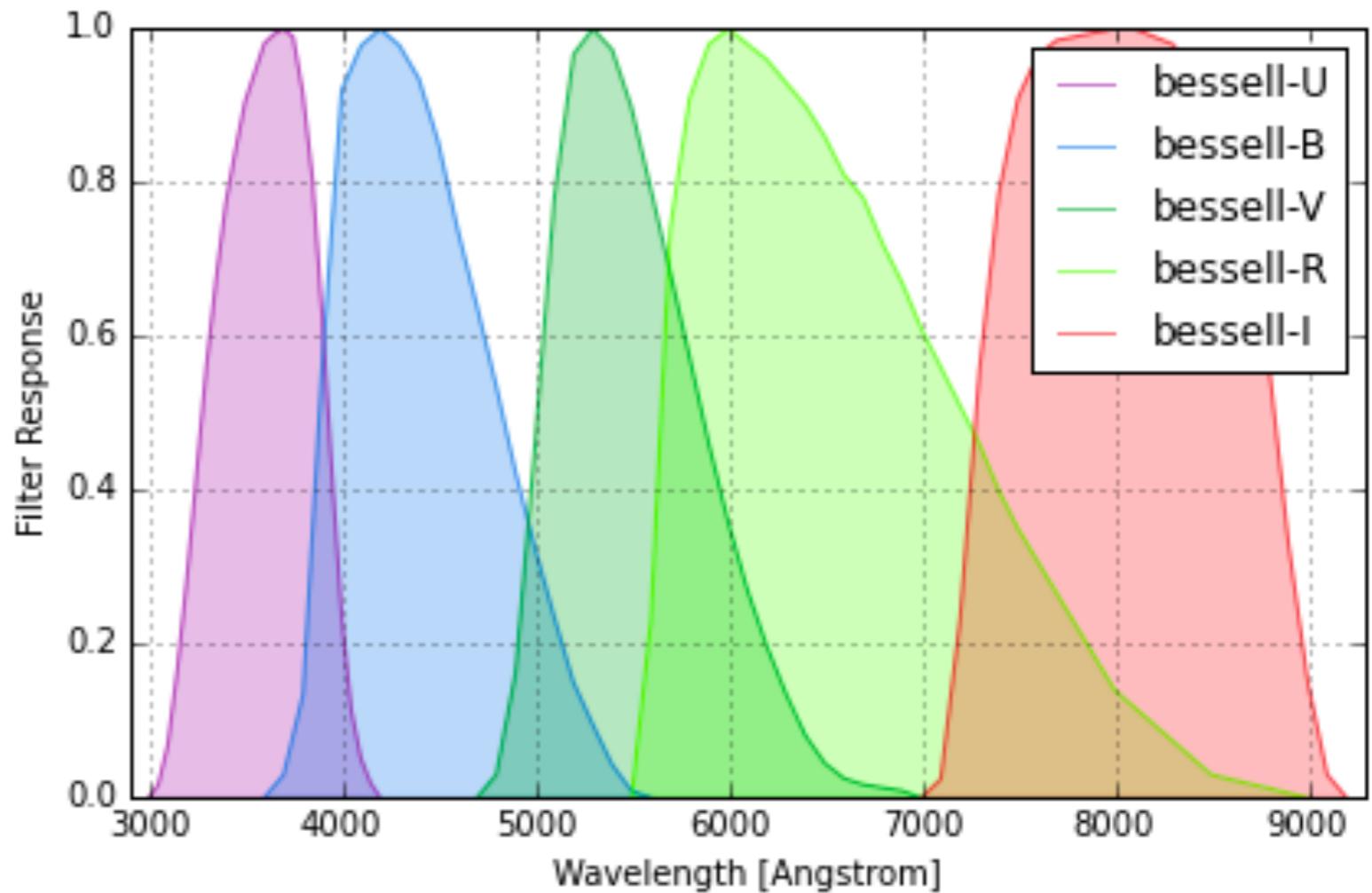
$f_{\text{bol}}$  = flux integrated over all wavelengths and corrected for ISM and atmospheric attenuation

$T_{\text{eff}}$  = approximation of the gas temperature in the photosphere of the star at optical depth  $\tau \approx 1$

# Observations of stellar parameters

## 5. Magnitude and color

**Magnitudes** reflect the energy flux received on Earth in different wavelength bands.



**Color indices** are defined as the ratio of the flux in two wavelength bands, and often expressed as the difference between two apparent magnitudes ( $m_\lambda$ ) in different filters

# Observations of stellar parameters

$$m_\lambda = -2.5 \times \log \left( \frac{f_\lambda}{f_{0,\lambda}} \right)$$

**Note:** a *lower* value of  $m_\lambda$  corresponds to a *larger* flux

where  $f_\lambda$  is in  $\text{erg cm}^{-2} \text{ s}^{-1}$  and  $f_{0,\lambda}$  is the reference flux at that  $\lambda$

Magnitude systems (ref. fluxes) typically used are :

**Vega**, star

$$f_0(\text{U|B|V}) = 4.2|6.4|3.8 \times 10^{-8} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ nm}^{-1}$$

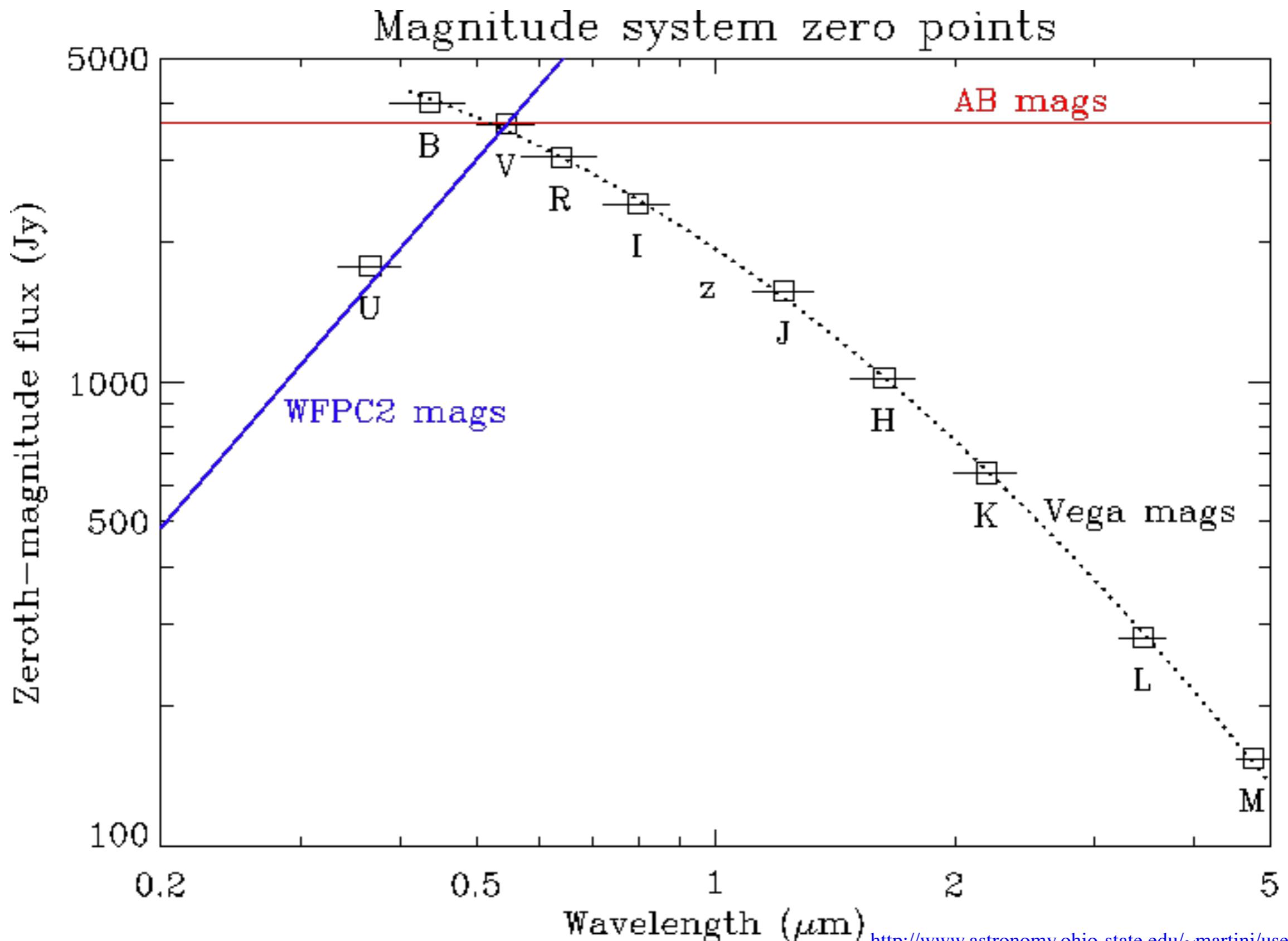
$$\textbf{AB}, \text{ flat } 3.63 \times 10^{-20} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ Hz}^{-1} = 3631 \text{ Jy}$$

$$AB_{\text{mag}} = -2.5 \times \log F_\nu - 48.6$$

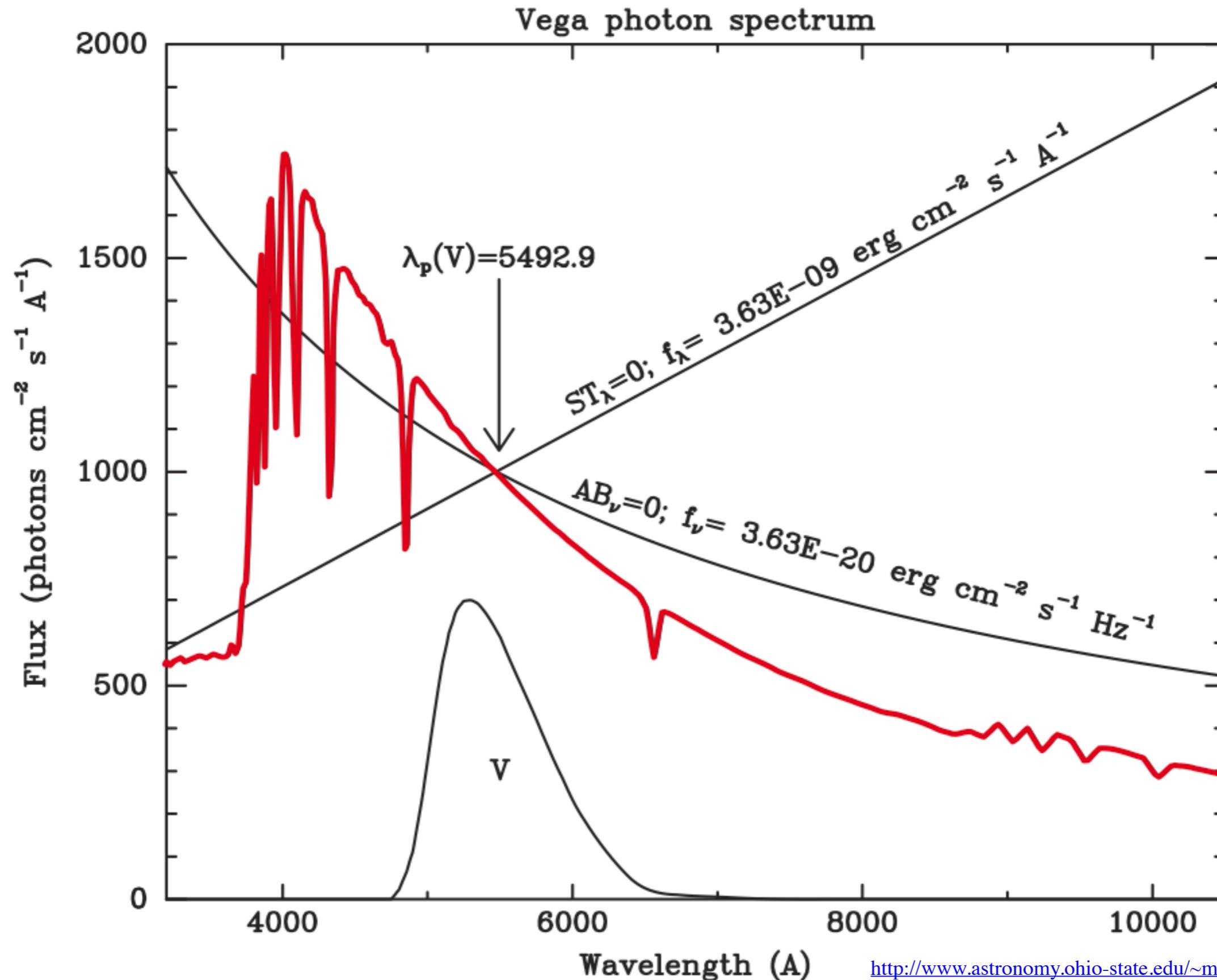
$$\textbf{ST}, \text{ flat } 3.63 \times 10^{-9} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ \AA}^{-1}$$

$$ST_{\text{mag}} = -2.5 \times \log F_\lambda - 21.1$$

# Observations of stellar parameters



# Observations of stellar parameters



# Observations of stellar parameters

Adopting the Johnson UBVRI filters,  $m_V = V = -2.5 \times \log \left( \frac{f_V}{f_{0,V}} \right)$

B-V color index is  $B - V = -2.5 \times \log \left( \frac{f_B}{f_V} \right)$  [ + difference in  $f_{0,\lambda}$ ]  
Ex. Calculate term  
for Vega and AB.

Colors are defined as BLUE-RED mags. Following BB radiation, a **larger** (redder) value for color corresponds to a **lower** (cooler)  $T_{\text{eff}}$

If a star's distance is known we can calculate its **absolute magnitude**  $M_\lambda$  (the  $m_\lambda$  the star would have at a distance of 10pc):

$$M_\lambda = m_\lambda + 5 - 5 \times \log(d)$$

The **absolute bolometric magnitude**  $M_{\text{bol}}$  describes the flux of the star *integrated over all wavelengths* at a distance of 10pc:

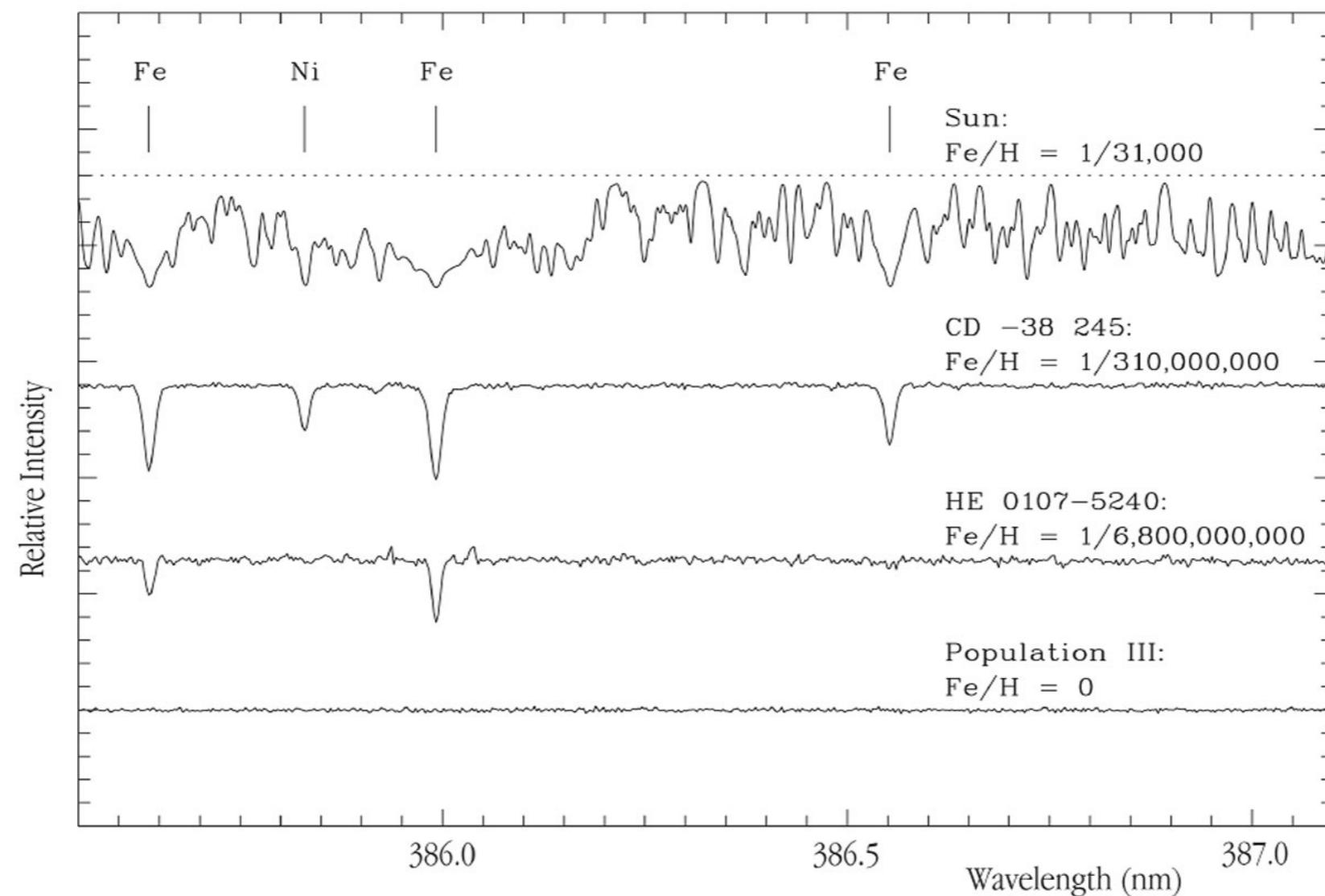
$$M_{\text{bol}} = -2.5 \times \log \left( \frac{L}{L_\odot} \right) + 4.74$$

and  $M_{\text{bol}} - M_\lambda = BC_\lambda$ , **the bolometric correction**

# Observations of stellar parameters

## 6... Other **spectral** parameters

gravity, chemical abundances, Teff, rotational velocity...



Spectra of Stars with Different Metal Content

# Observations of stellar parameters

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So far, we listed ***surface*** stellar parameters

We need a theory of stellar structure to derive the internal properties of a star.

However, direct windows on the ***interior*** of a star exist:

**Neutrinos** that escape without interaction. Only detected from the Sun

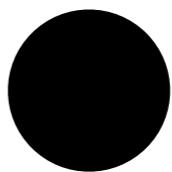
**Seismology**, the frequency of different stellar oscillations contains information about the speed of sound waves inside the star, so do density and temperature.

# The Structure of Stars

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Stars become highly structured. These are the main regions:

- a) **core** - site of current/former fusion

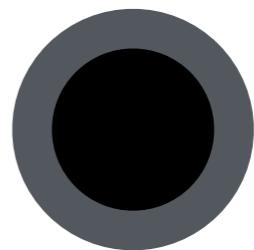


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- b) **shell zone** - fusion occurring  
in shell(s) around core

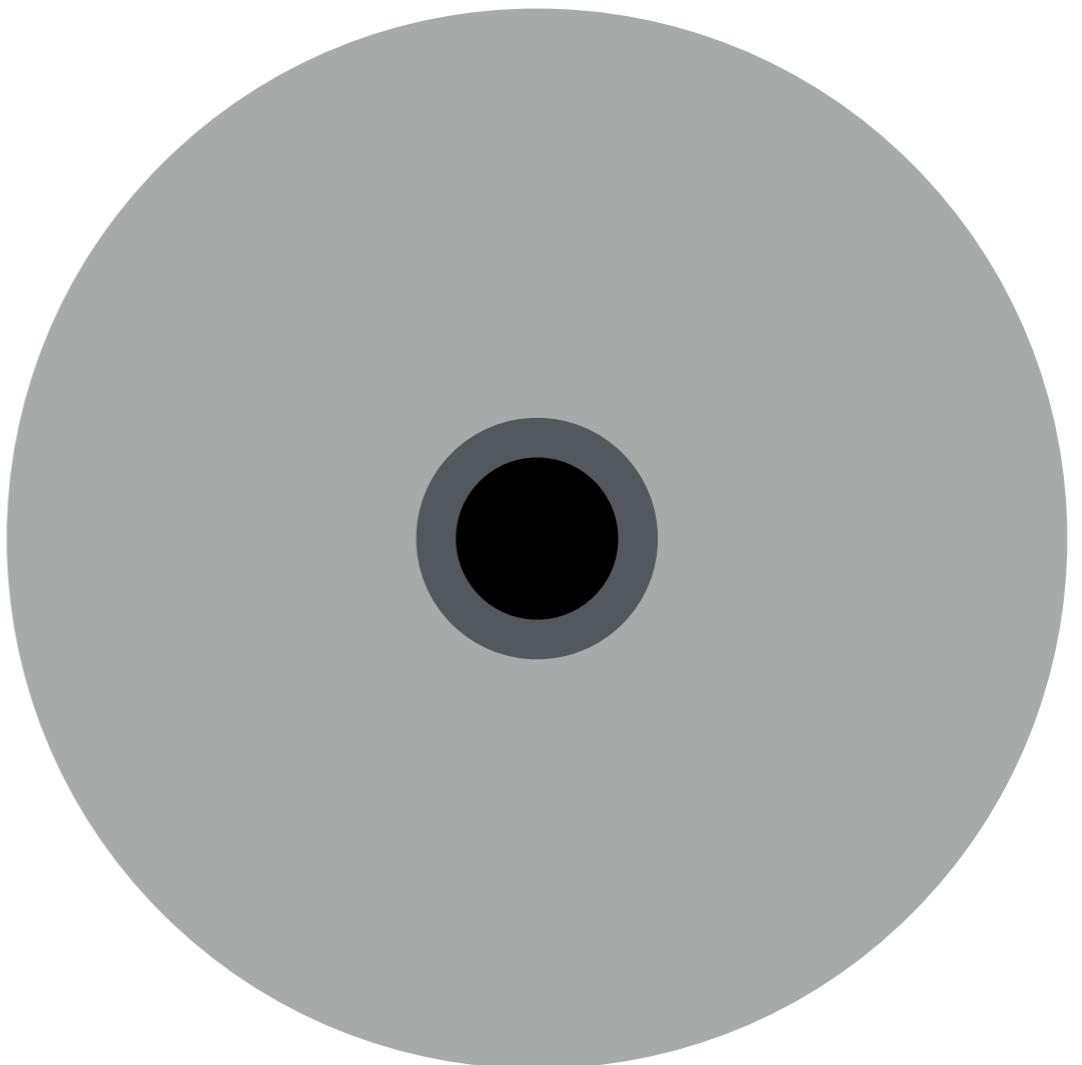


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shell and atmosphere

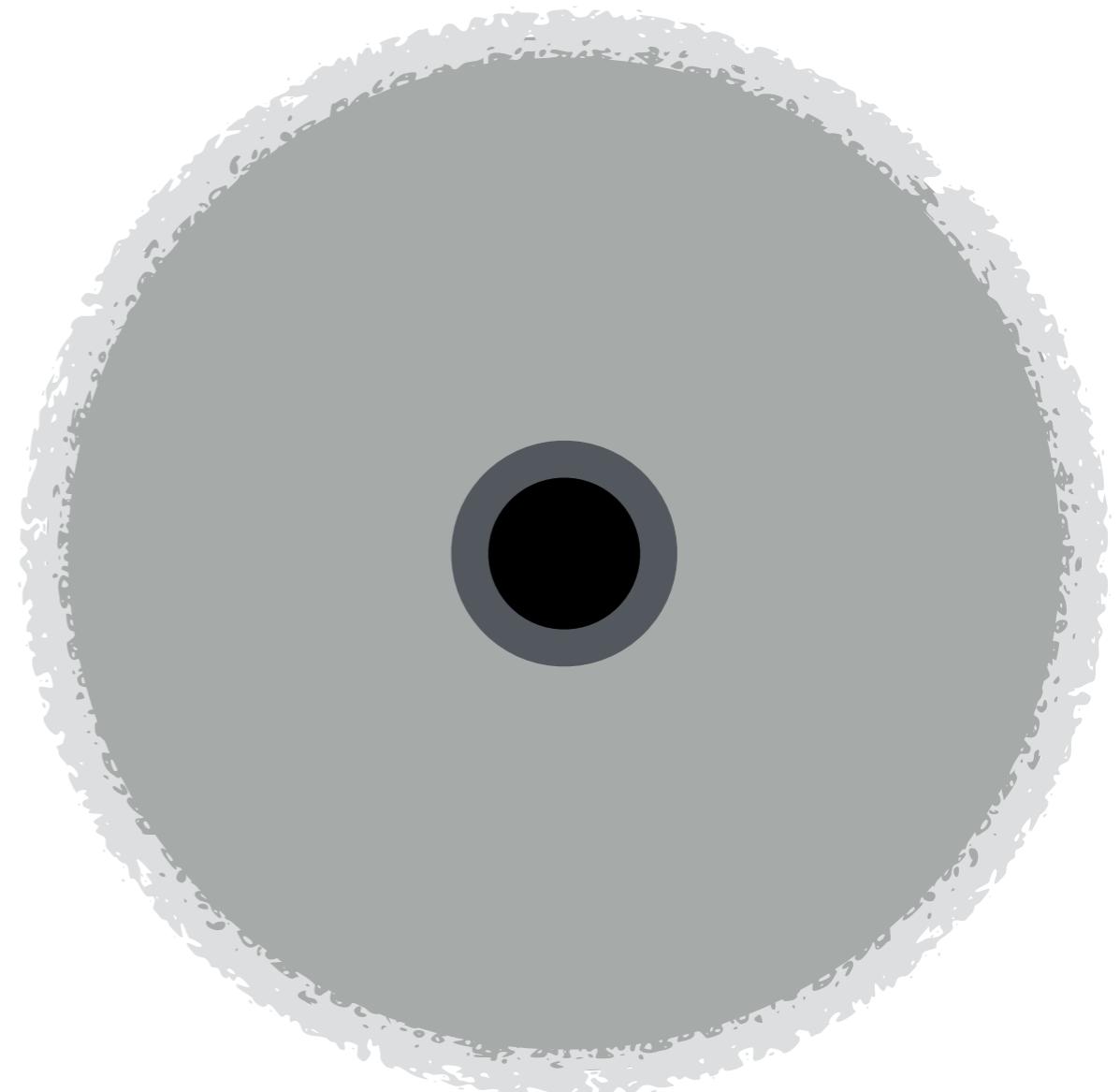


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radiation can escape

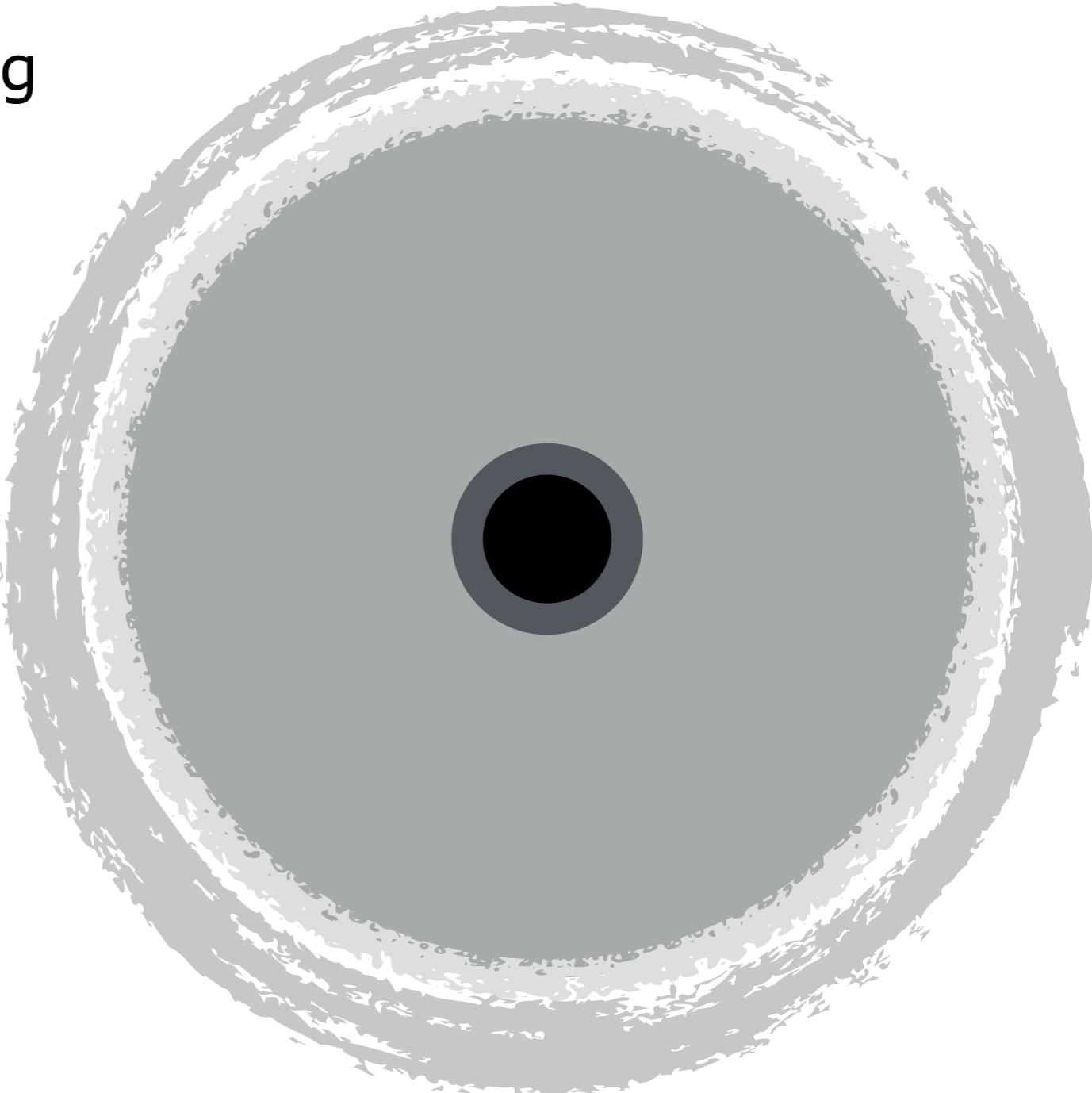


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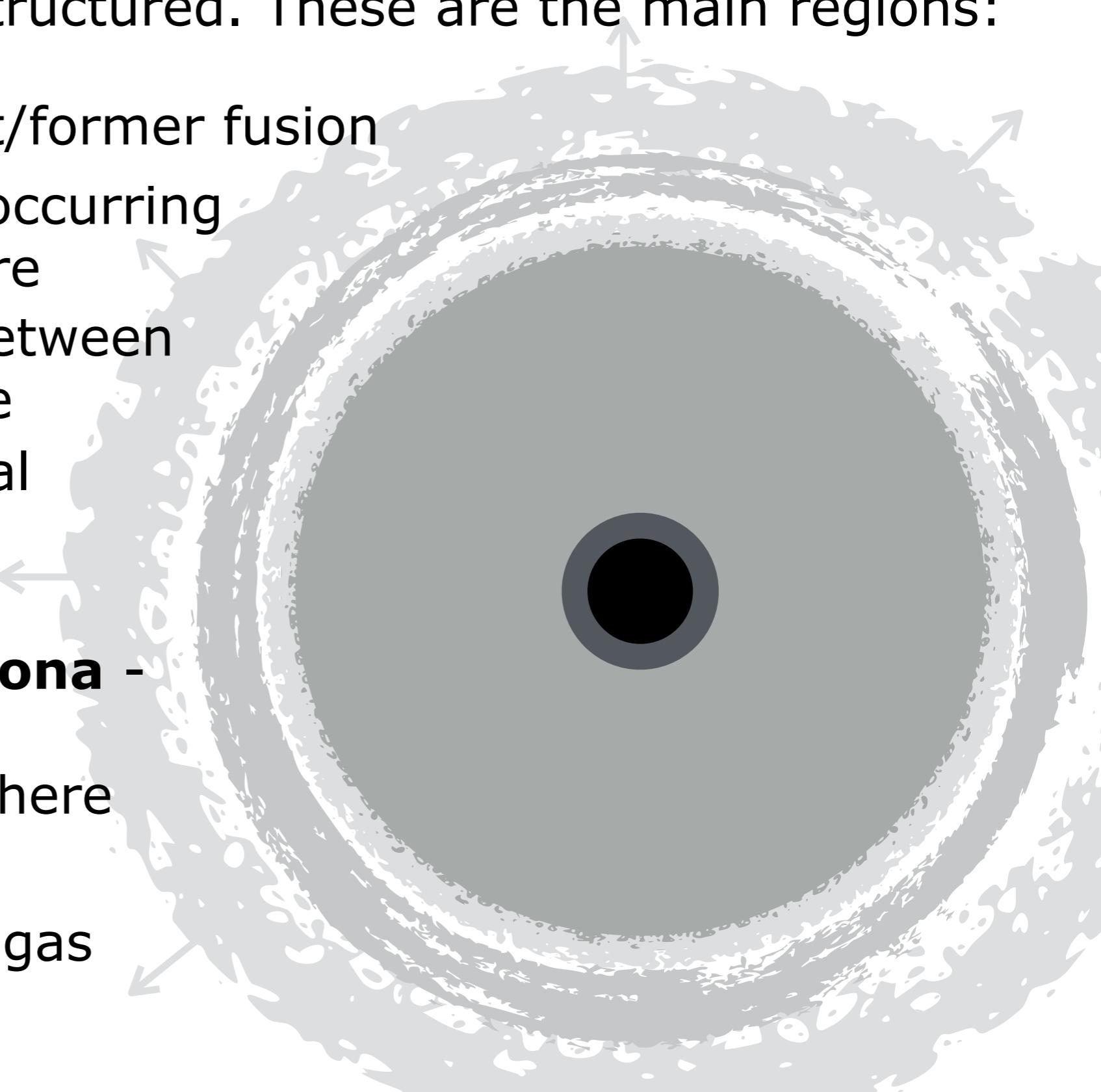
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- e) **chromosphere/corona** - region in cool star above atmosphere where T rises due to shocks



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radiation can escape
- e) **chromosphere/corona** - region in cool star above atmosphere where T rises due to shocks
- f) **wind** - region where gas escapes at  $>> V_{esc}$



# Stellar Evolution in a Nutshell

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Stars are spheres of gas held by **gravity**

Stable for a long time. **Gravity** balanced by **pressure** + **radiation**

Strong T and  $\rho$  gradient. Energy flows outward **radiated** into space

Stars replenish this energy loss with **nuclear fusion** in the center

If **fusion** stops the star contracts and T raises starting the next **fusion reaction**

(this cycle continues until **fusion** fuel is exhausted )

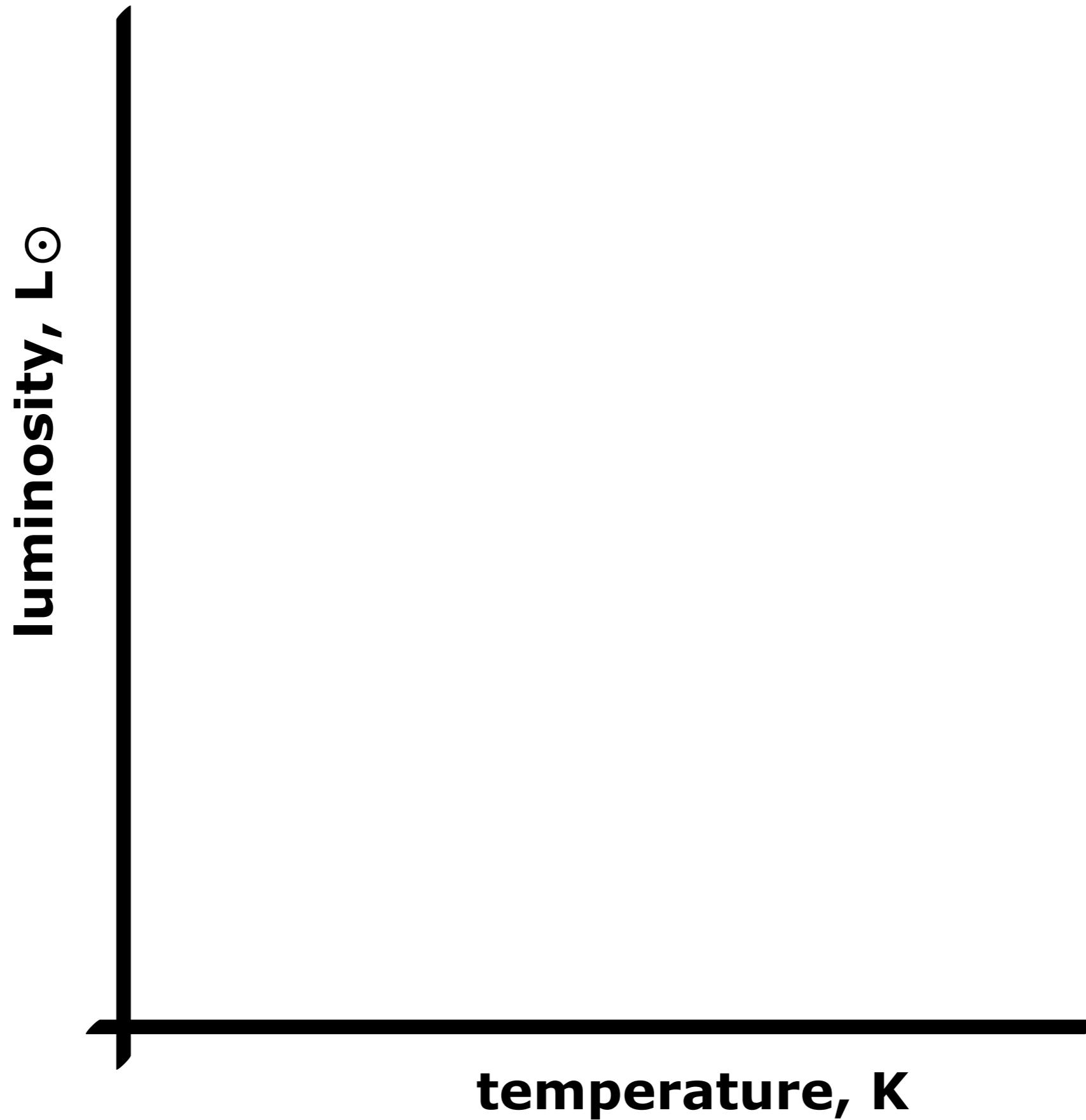
...then **gravity** wins.

In **low-mass stars** the core compresses and becomes a white dwarf.

In **high-mass stars** the core collapses, producing a neutron star or black hole, and ejects the outer layers, producing a supernova.

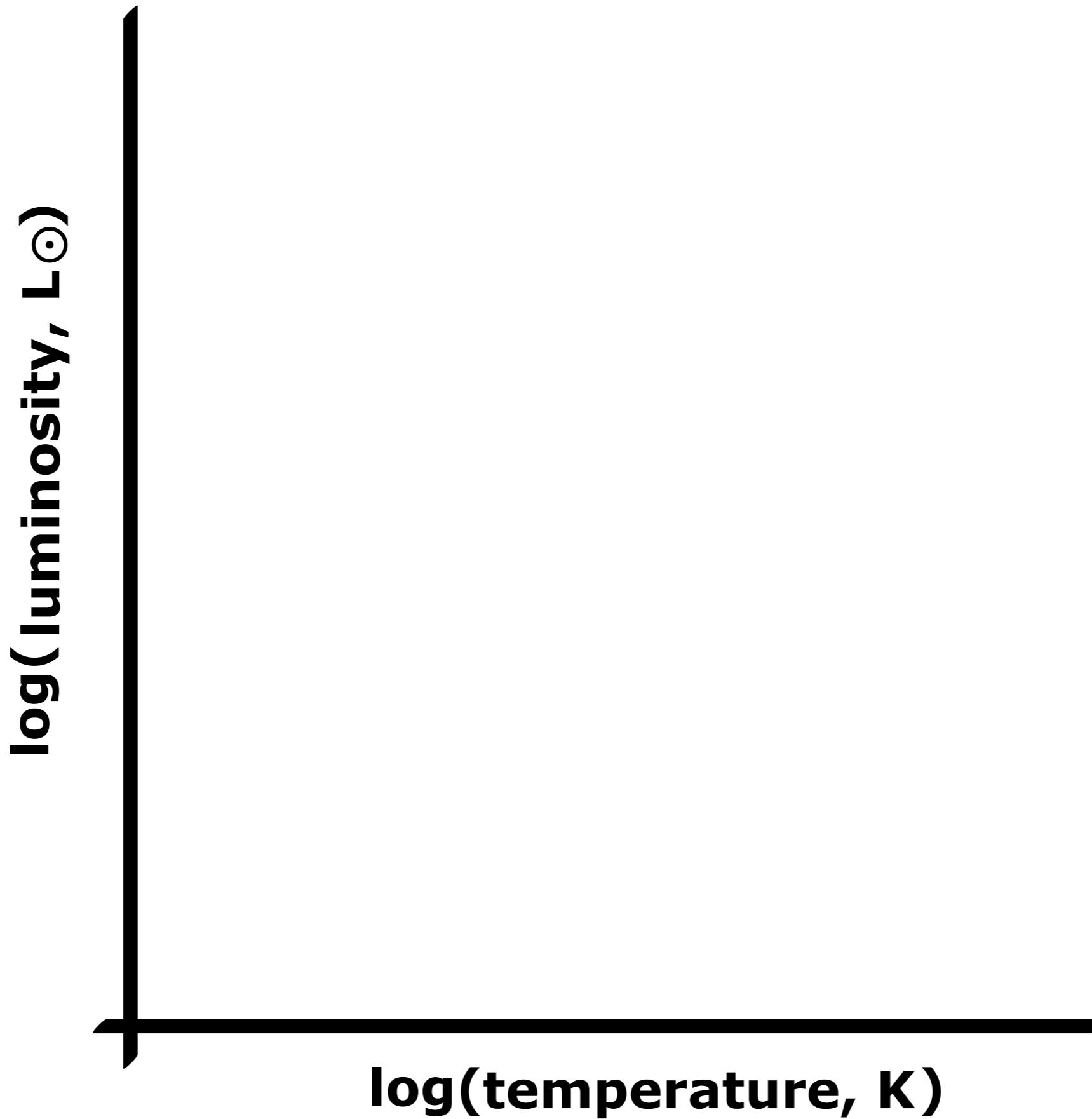
# The Hertzsprung-Russell Diagram

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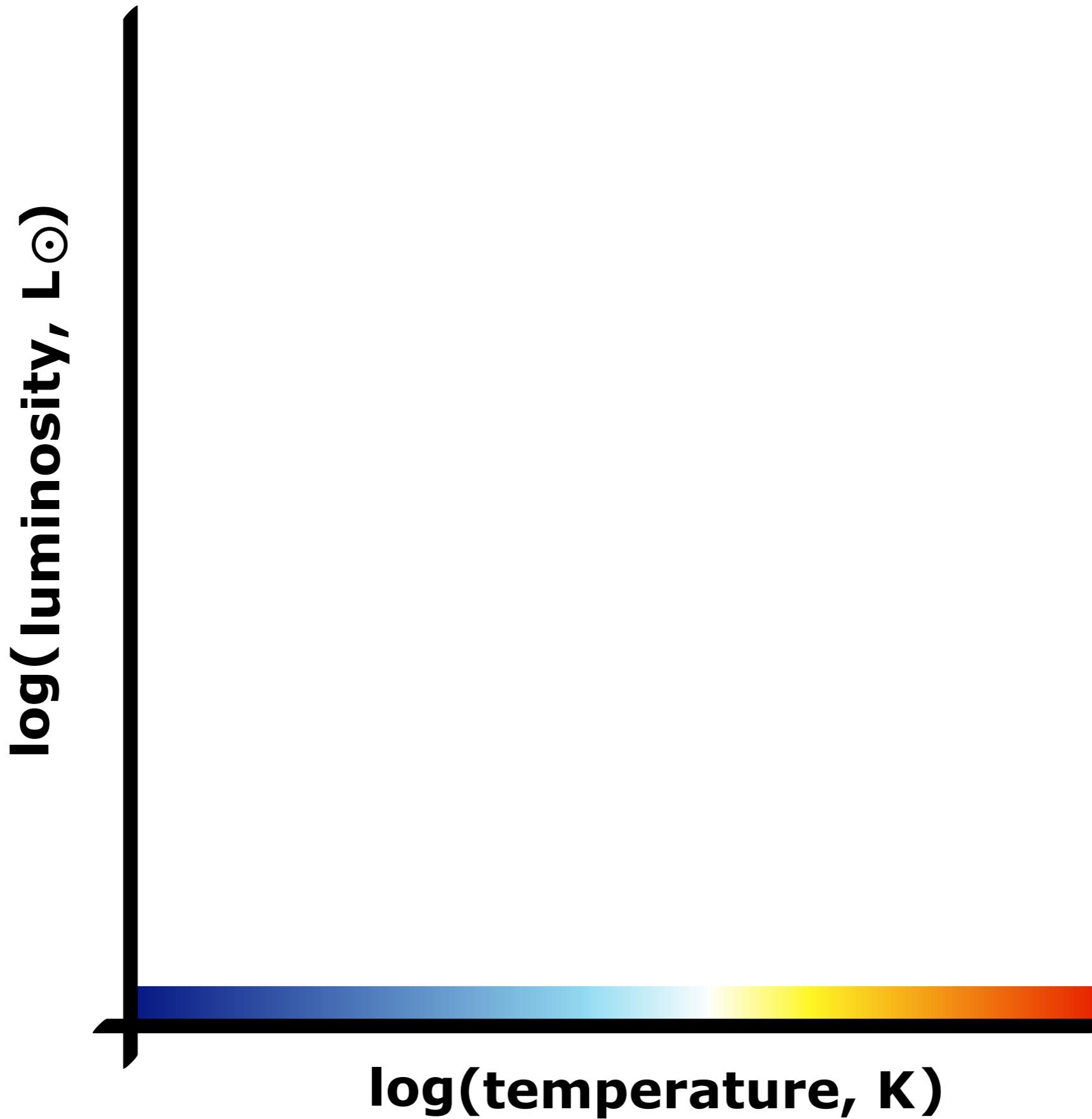
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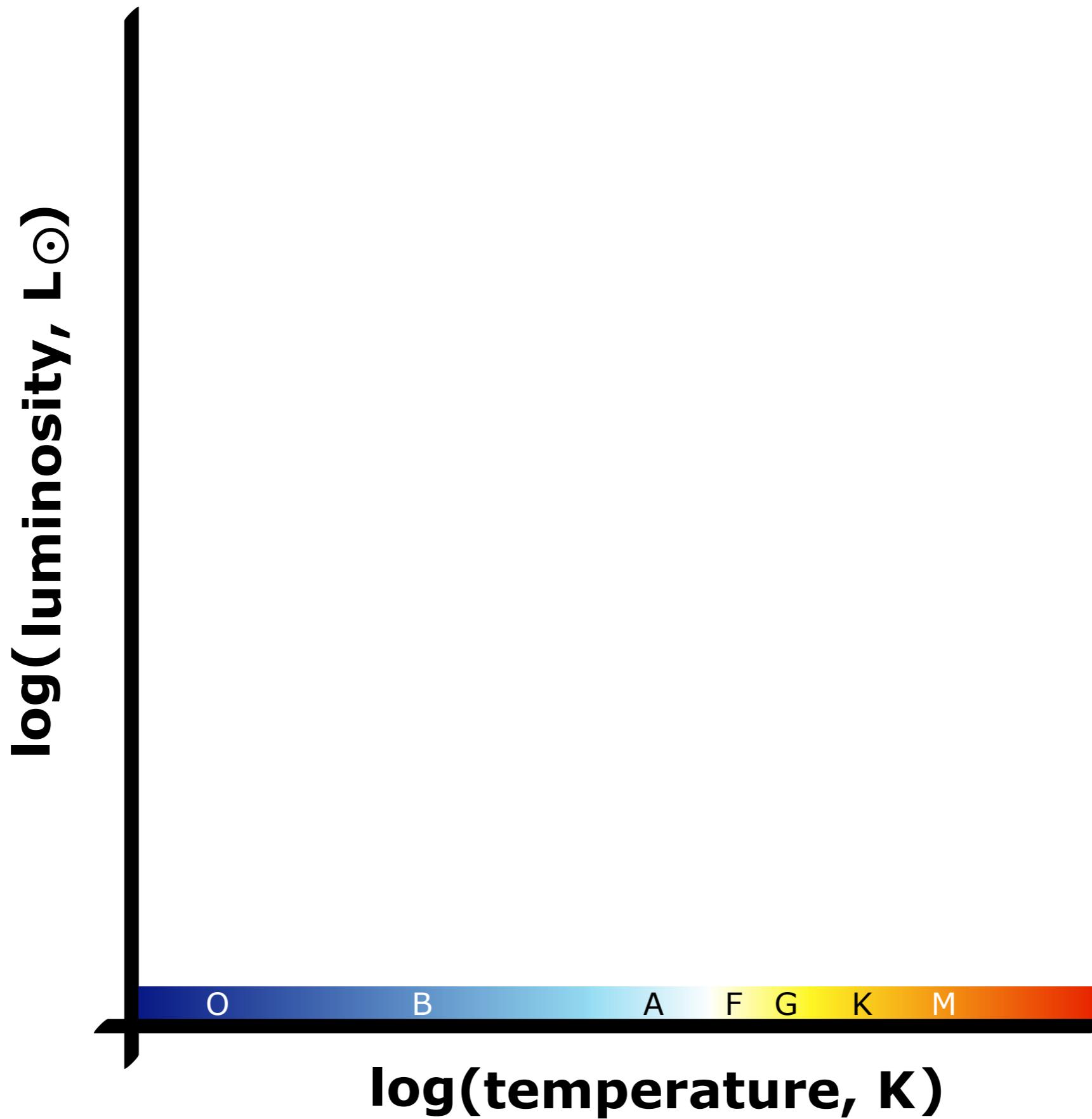


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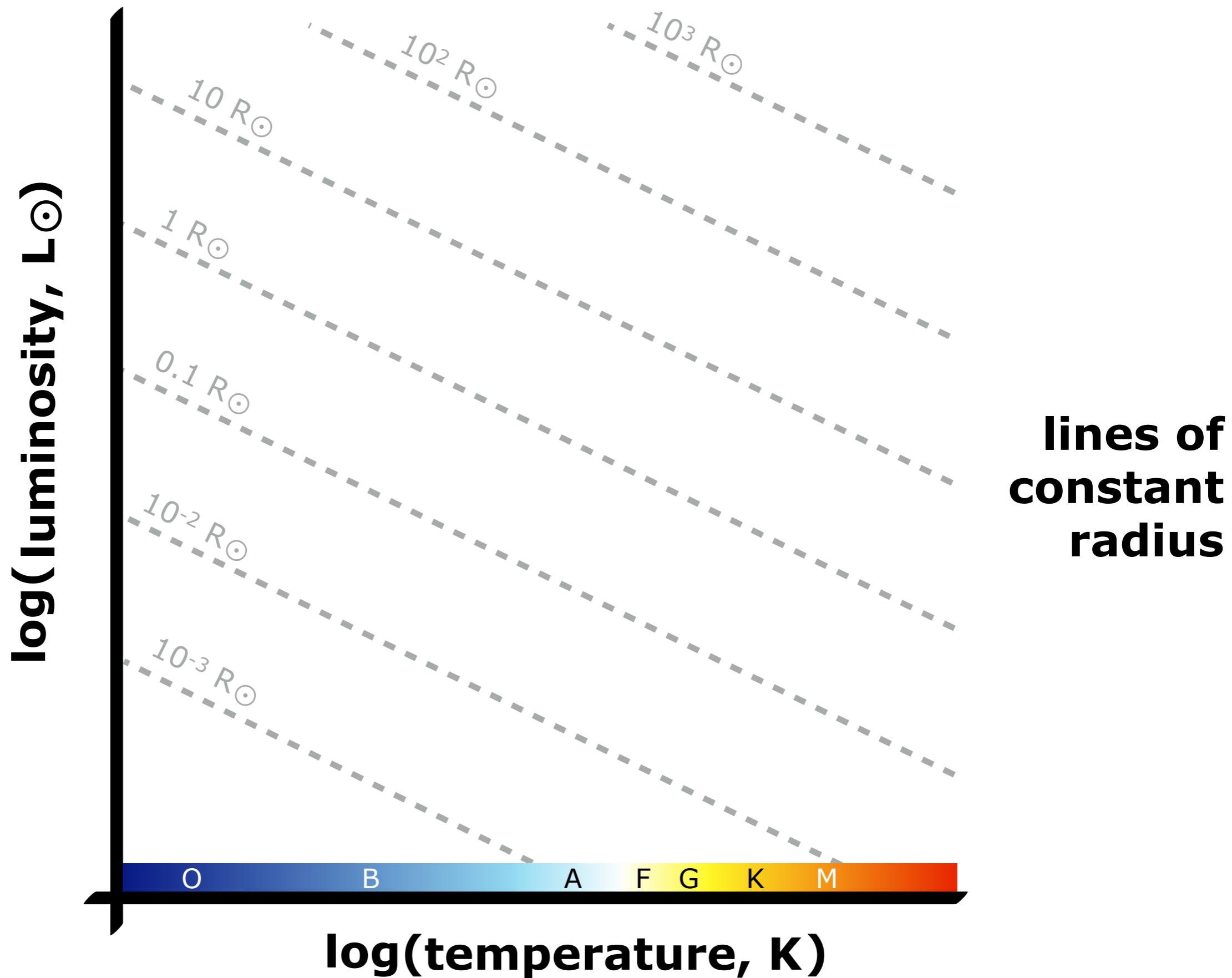
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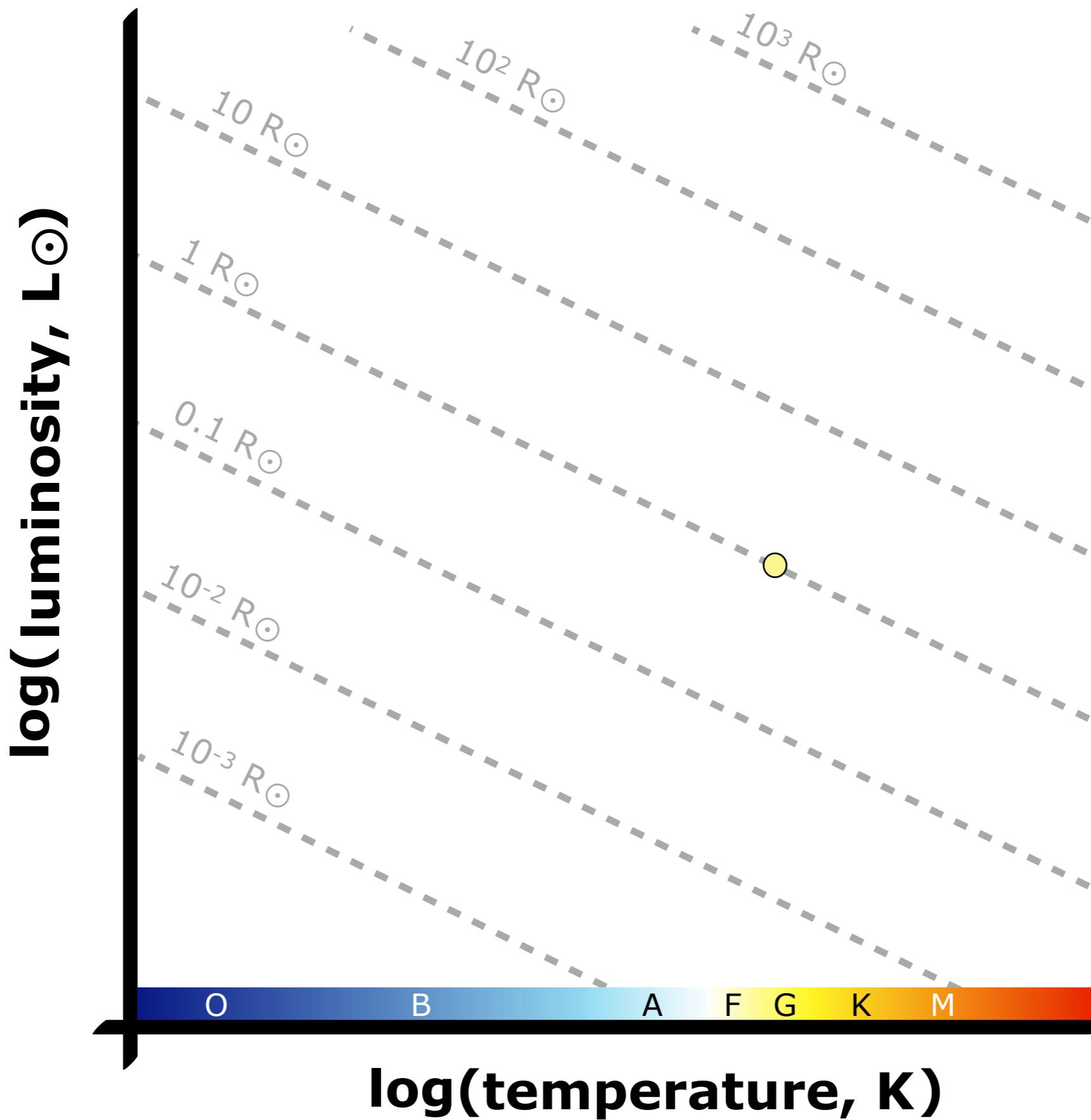
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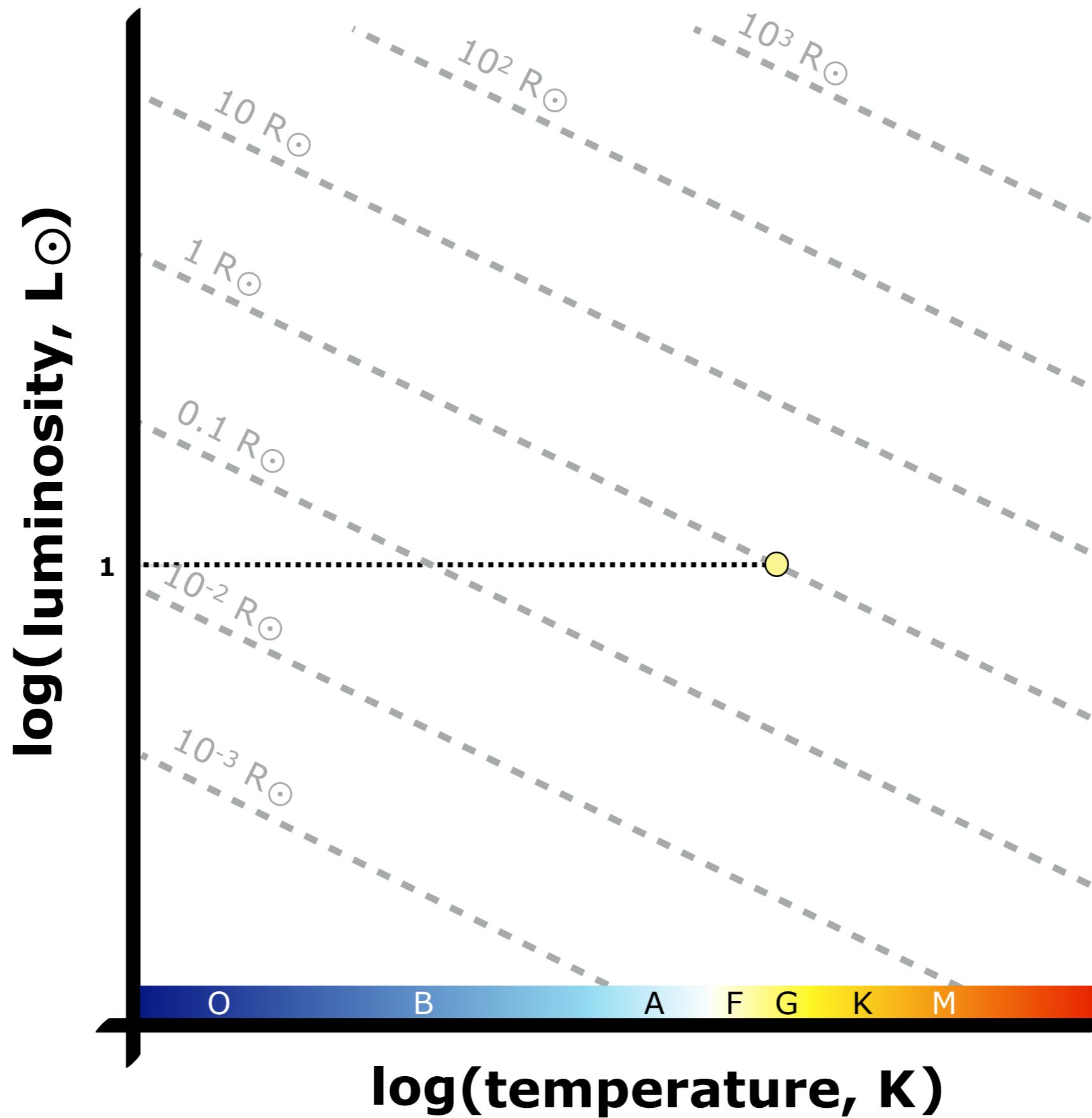
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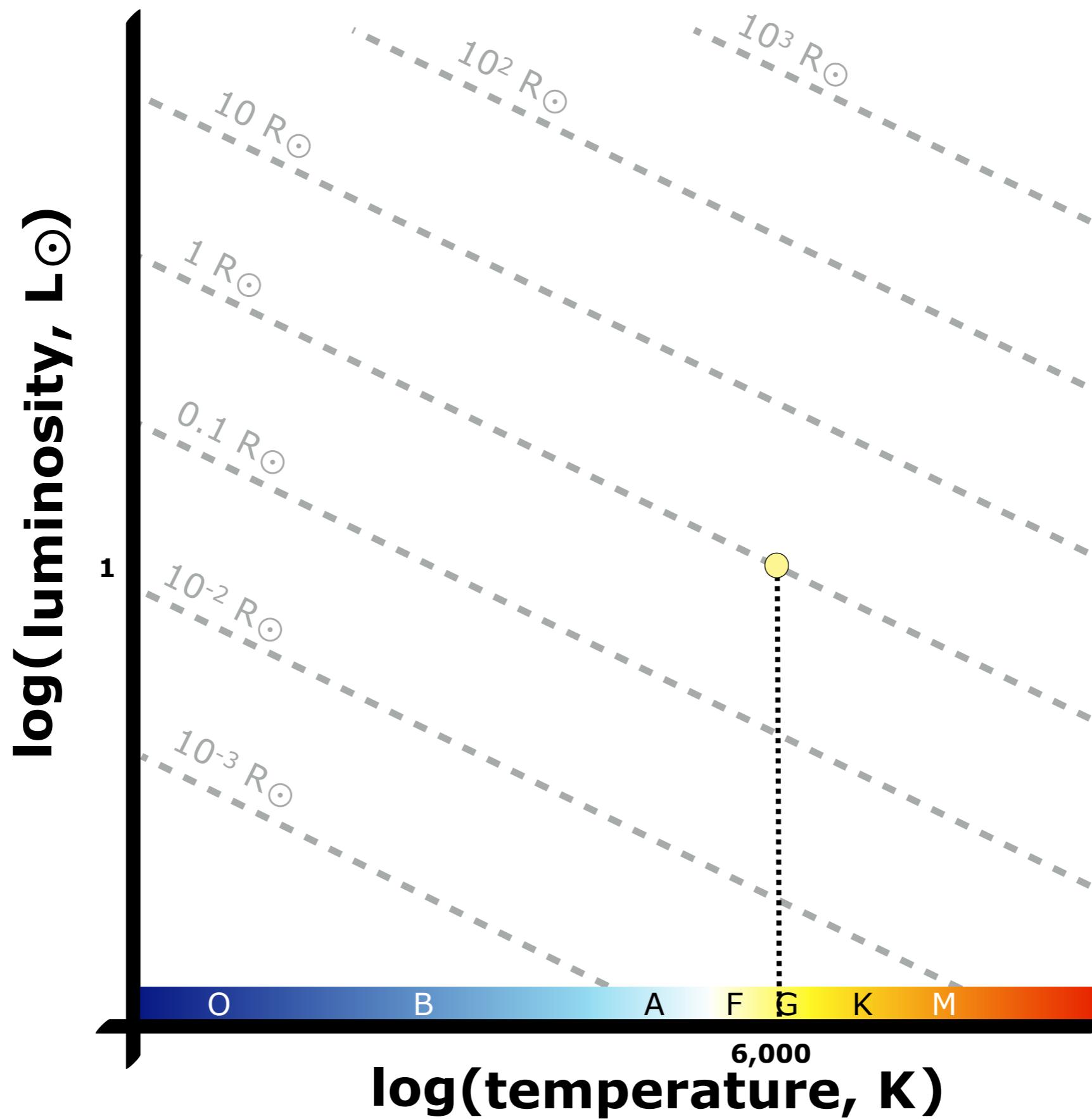
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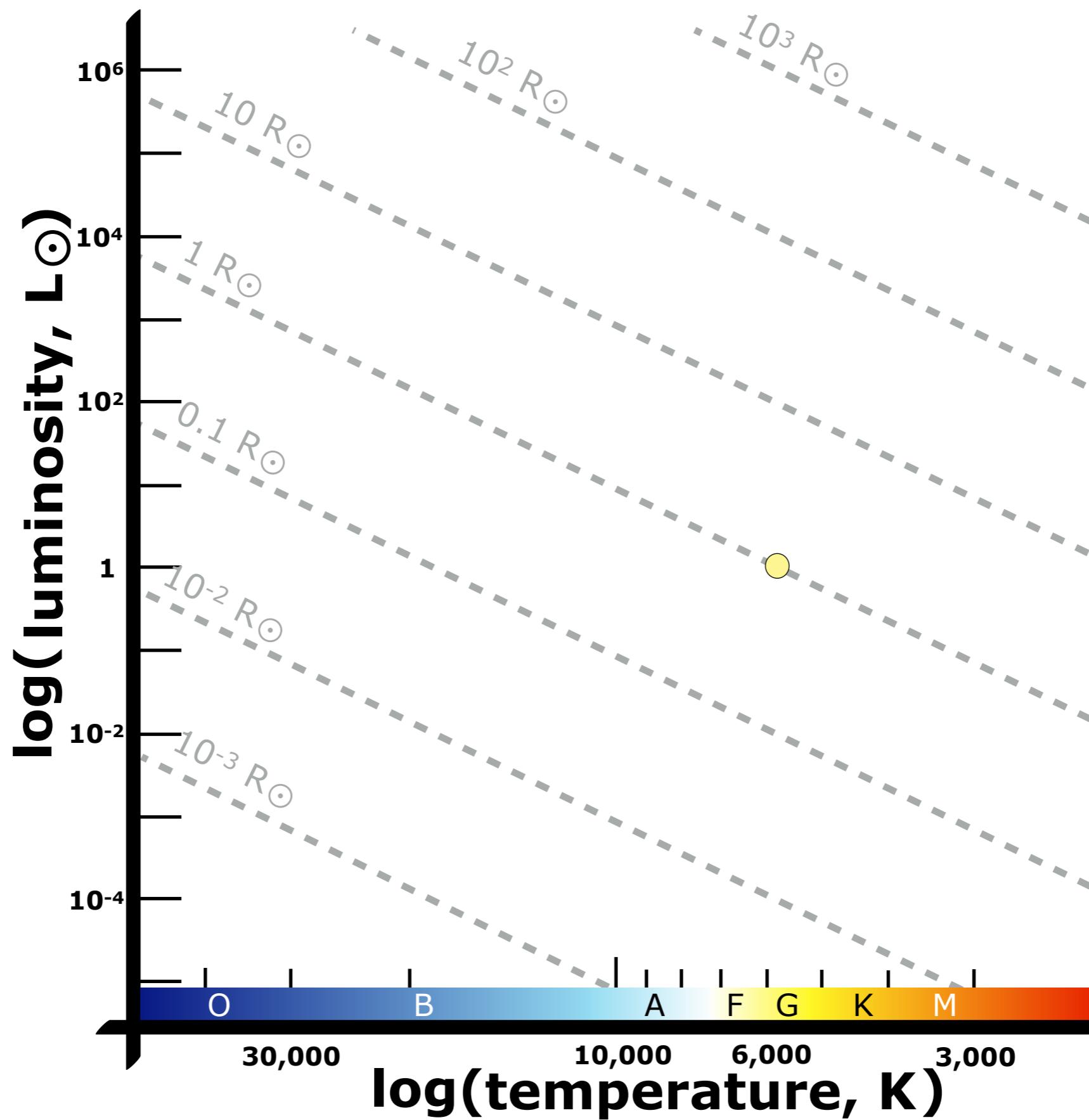
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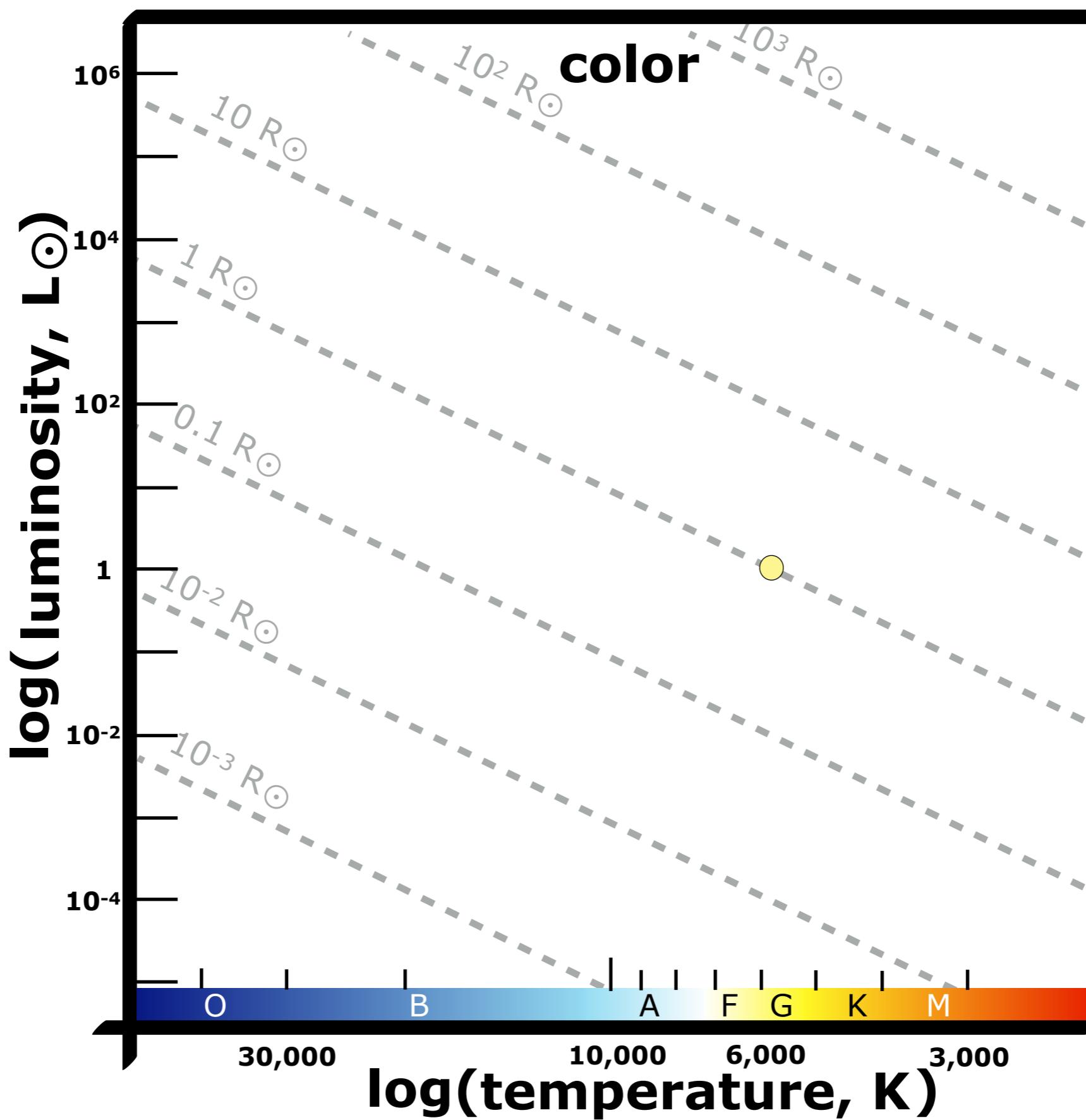
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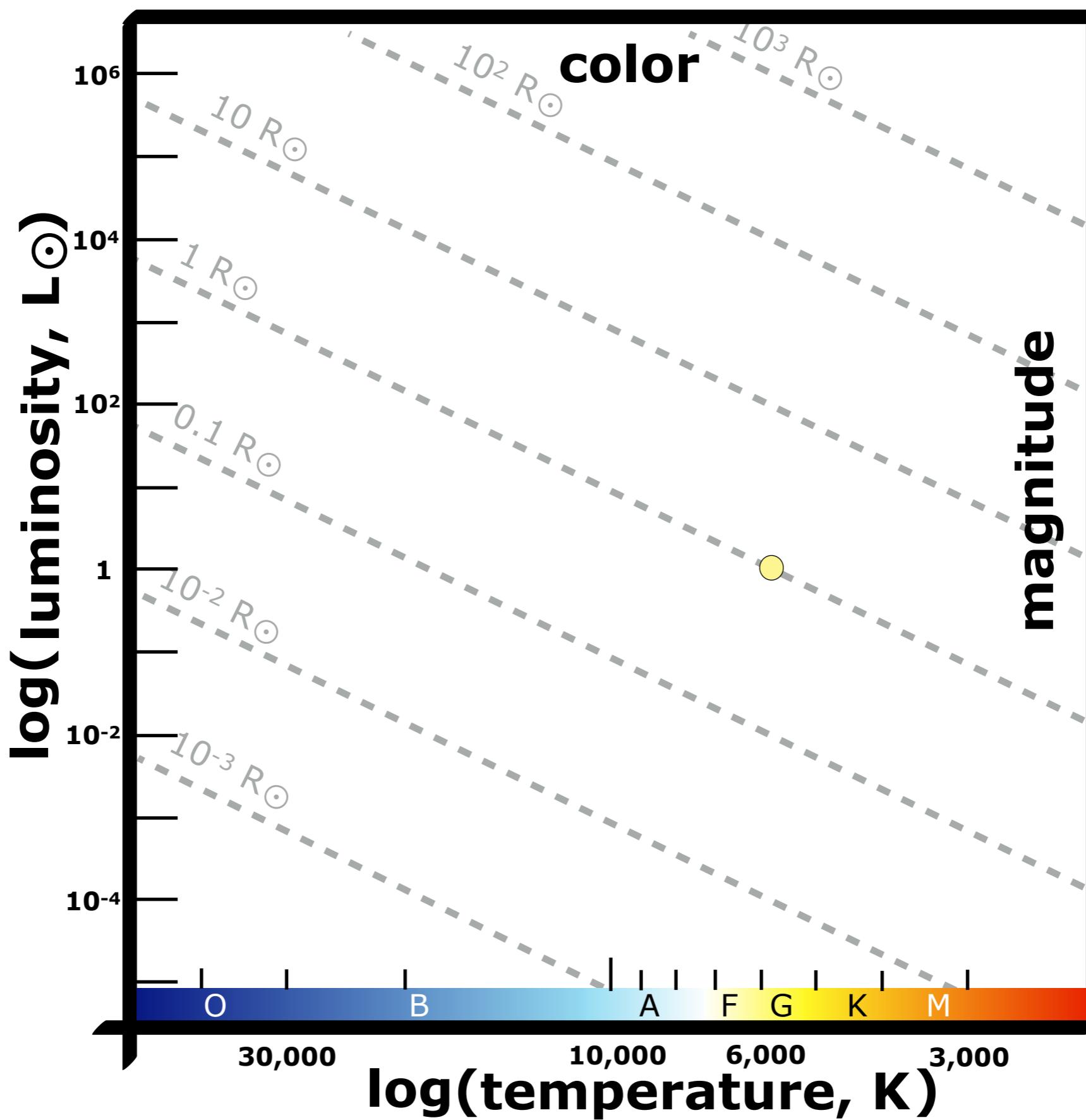
# The Hertzsprung-Russell Diagram



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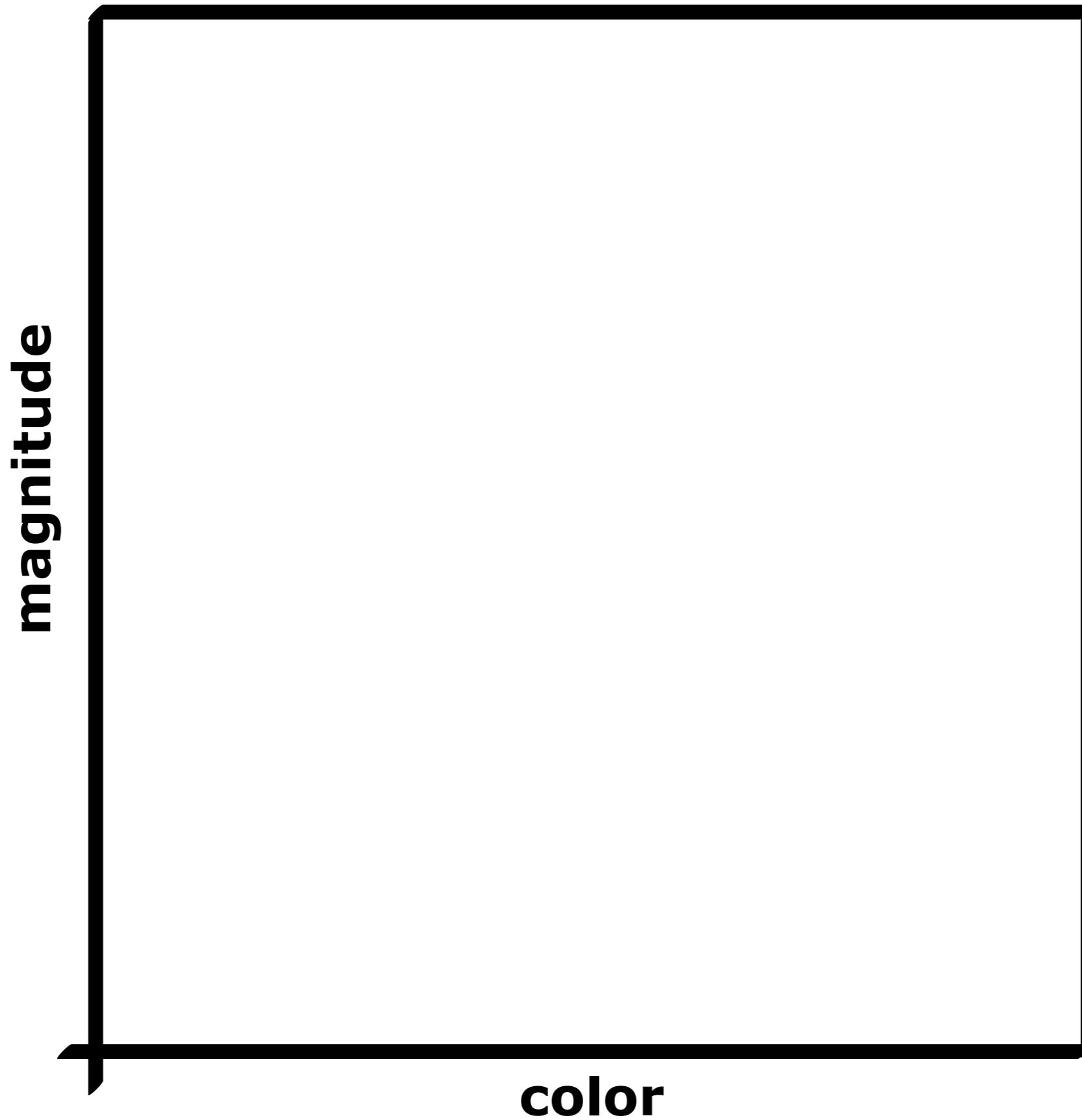


# The Hertzsprung-Russell Diagram



# The Color-Magnitude Diagram

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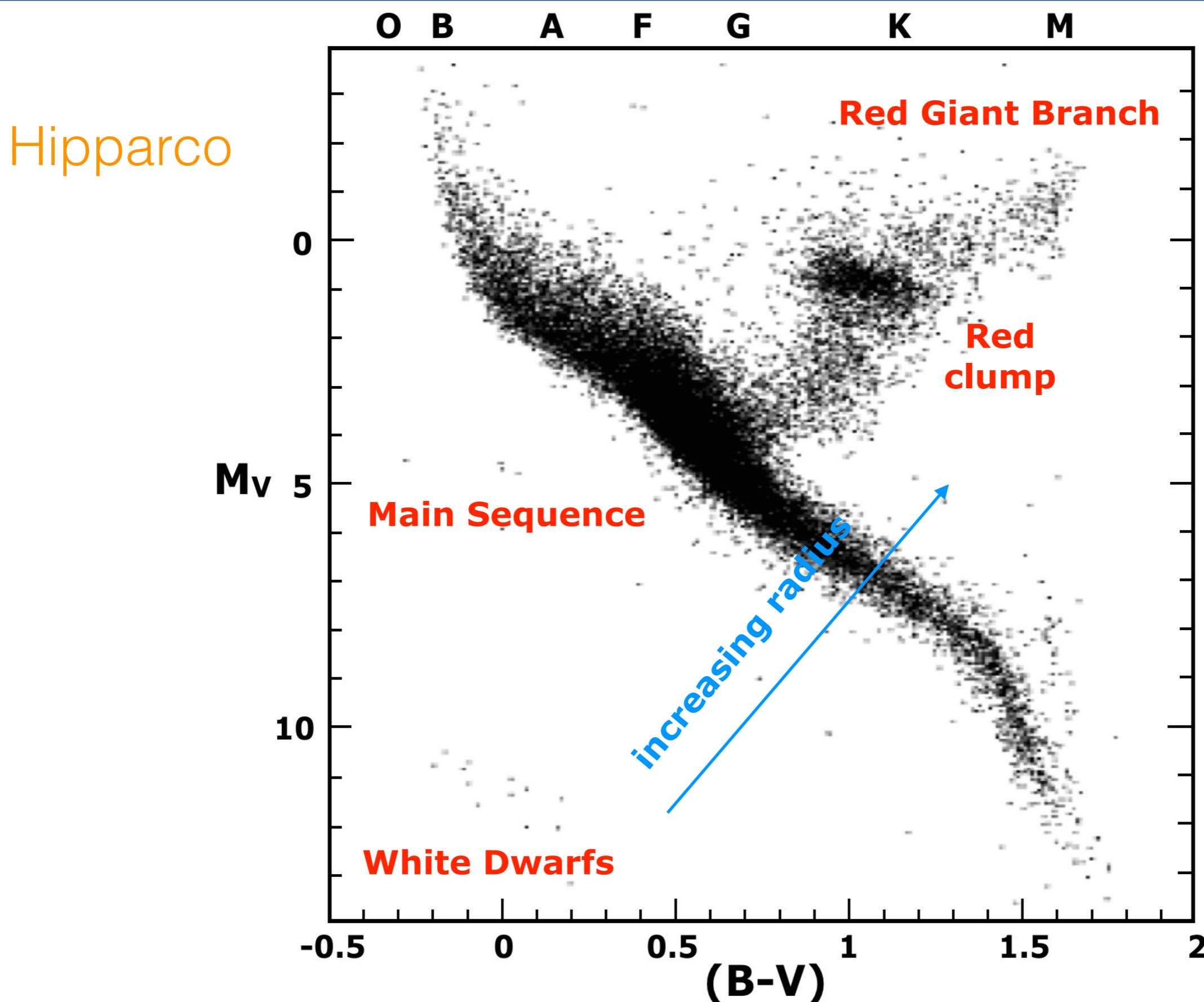
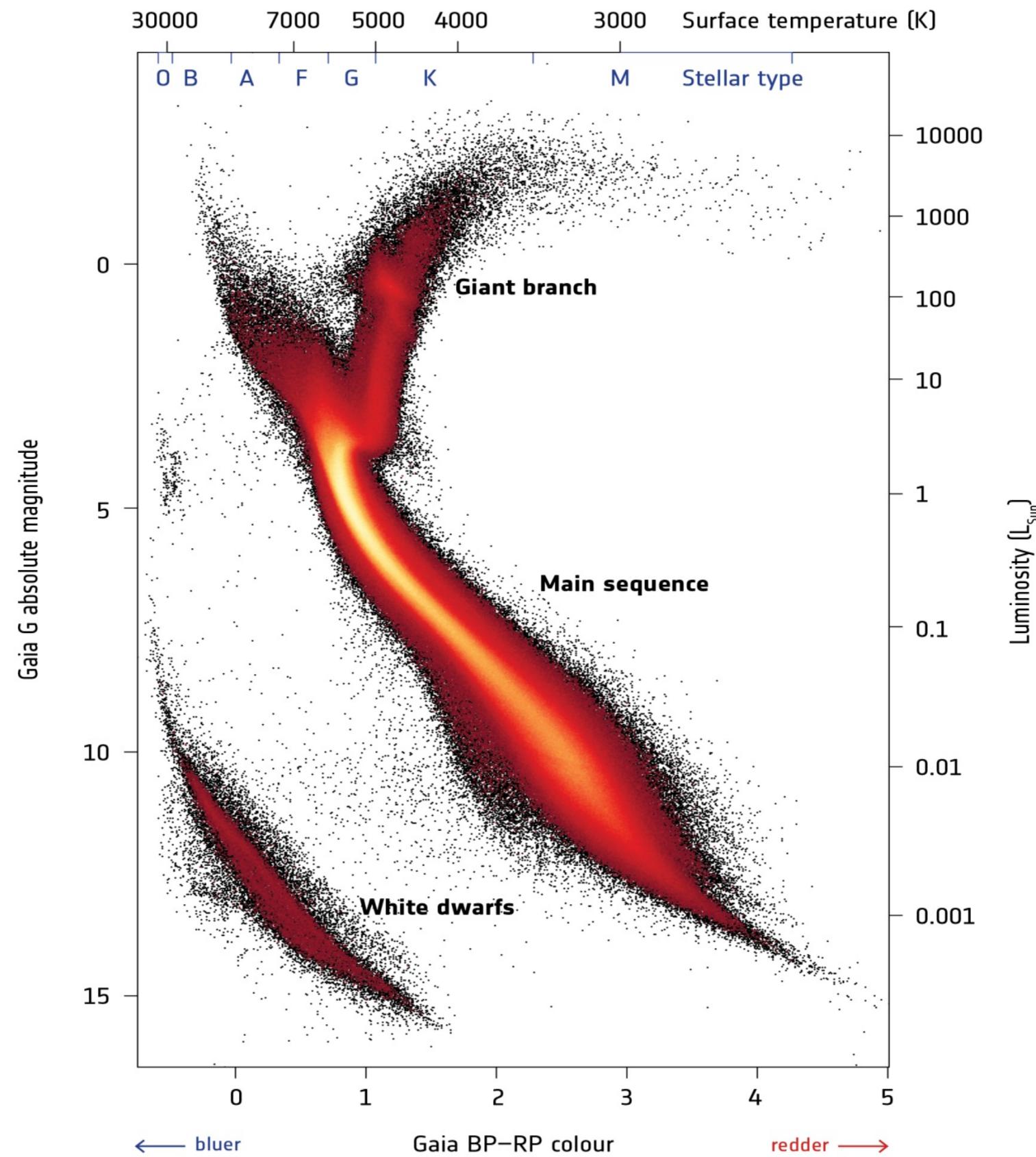


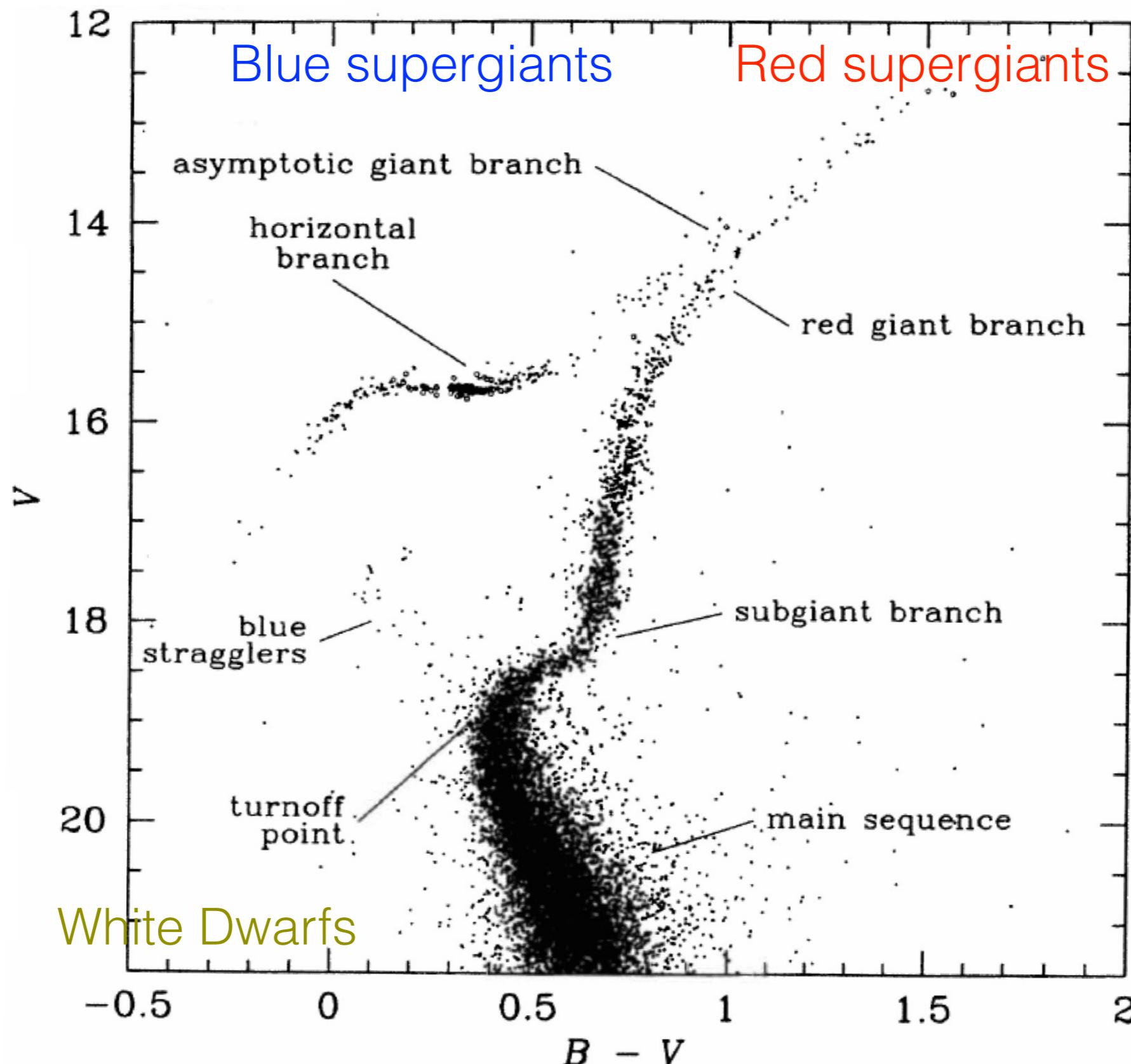
Figure courtesy of ESA.

# The Color-Magnitude Diagram

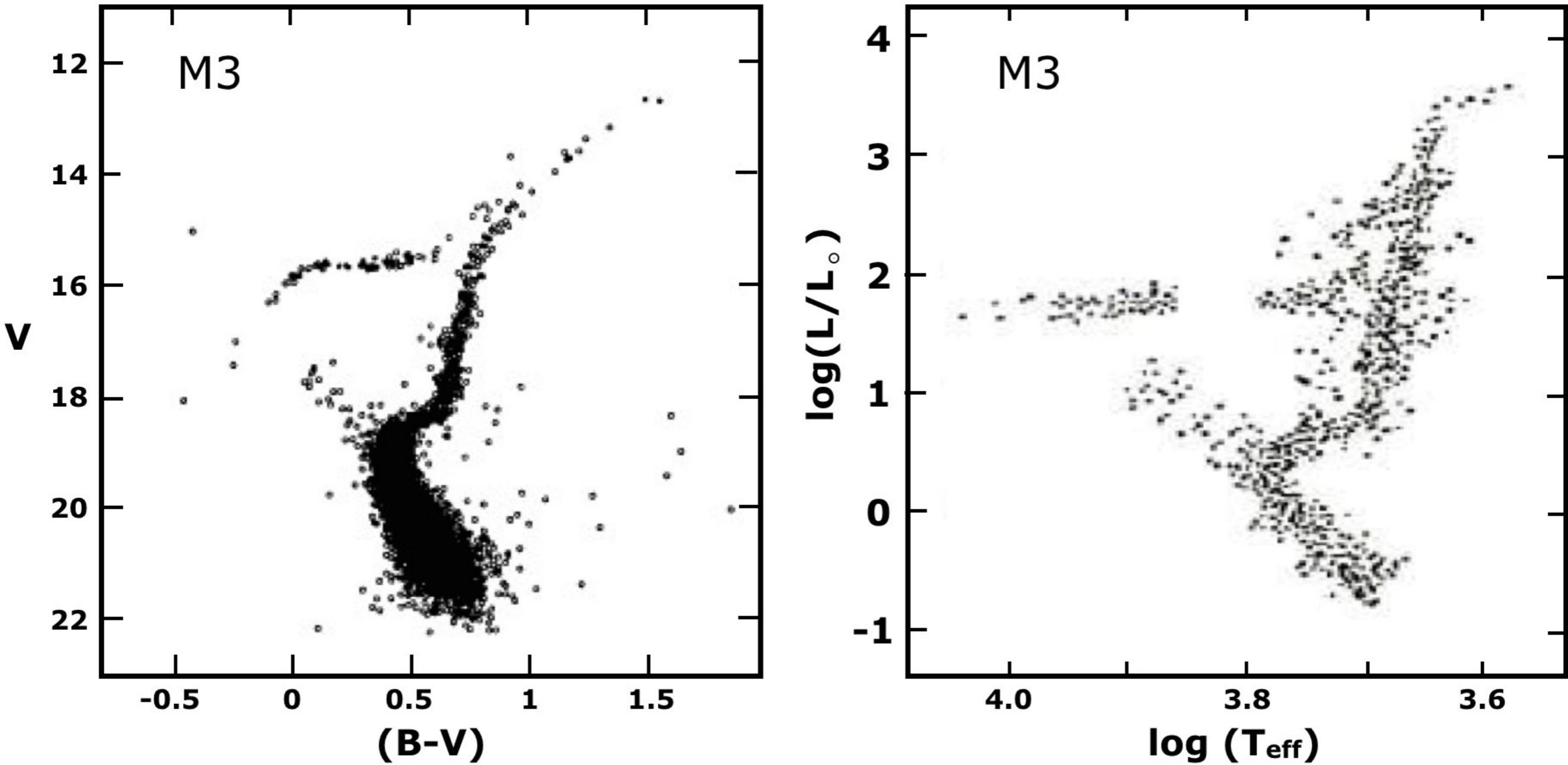
Gaia



# The Color-Magnitude Diagram



# CMD and HRD

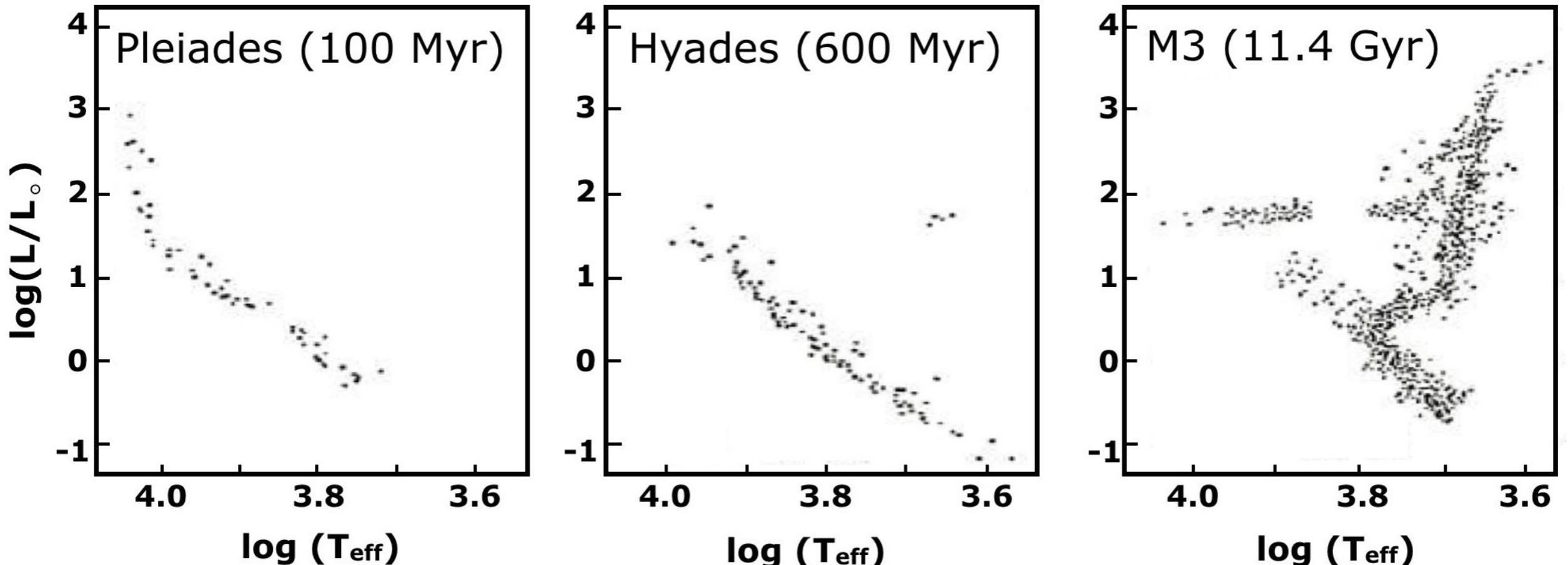


1. on the blue/hot side stars cover a small (B-V), large  $T_{\text{eff}}$  range
2. on the red/cool side, stars cover a small  $T_{\text{eff}}$ , large (B-V) range
3. horizontal branch curves down strongly in the CMD

Left: Reproduced from Buonanno, R et al, A&A, Vol. 290, p.69-103 (1994), reproduced with permission. © ESO.

Right: Reproduced from Johnson, H. L. and Sandage, A. R., 'Three-Color Photometry in the Globular Cluster M3', Astrophysical Journal, vol. 124, p.379, 1956.

# The Color-Magnitude Diagram



As the clusters increase in age:

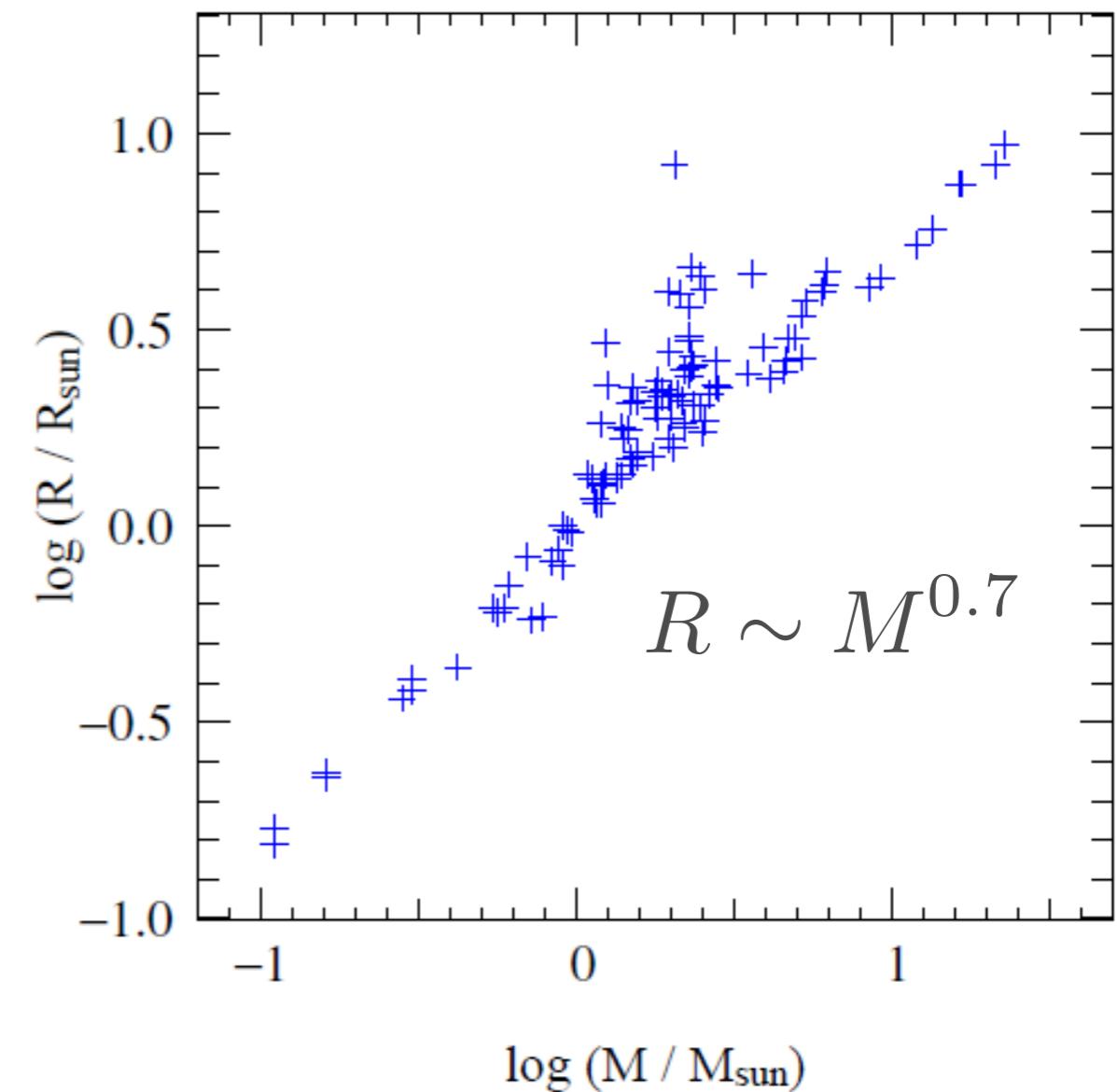
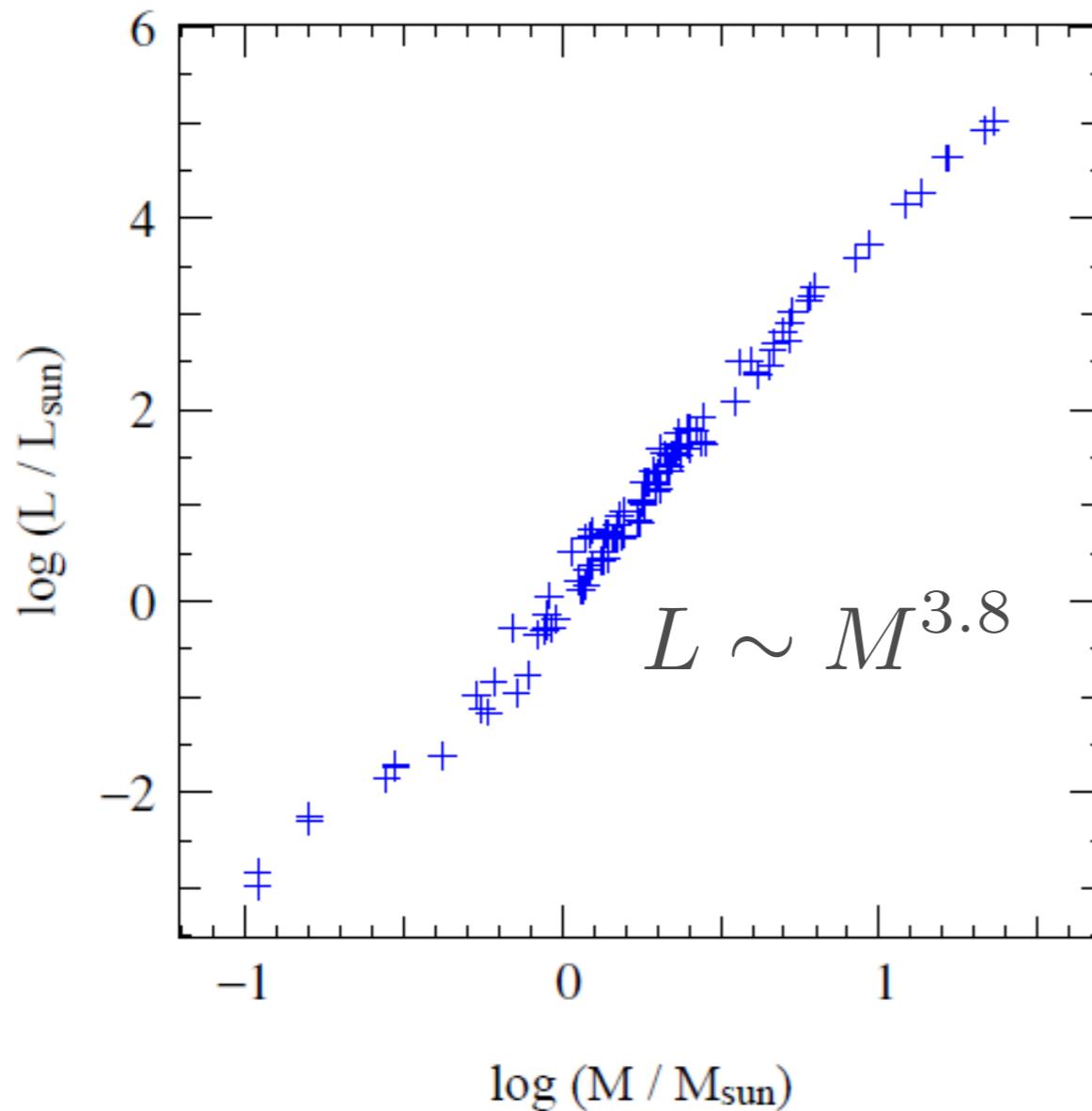
- the MS gets shorter
- the ratio of red giants to MS stars increases

# Stellar properties relations

## Mass-luminosity & Mass-radius relations

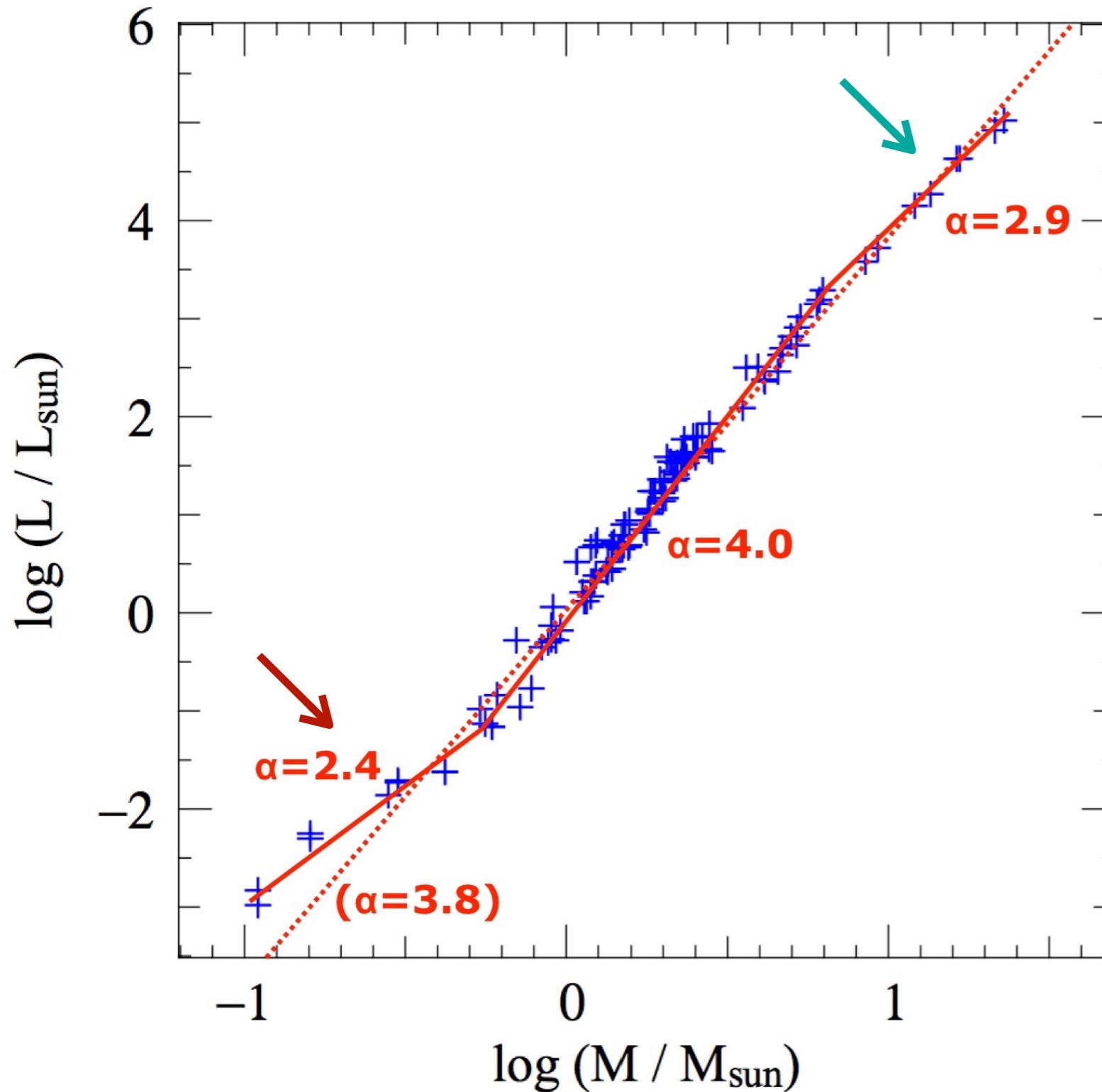
All 3 parameters are needed, so only possible for *main sequence stars\** from double-lined eclipsing binaries

\*stars that fuse H in their center



# Stellar Evolution in a Nutshell

## Mass-luminosity relation



Relation is only valid for stars during core H fusion (red giants/supergiants, degenerate stars, etc., do not follow the relation)

For  $M > 6M_{\odot}$ , slope flattens due to increasing role of radiation pressure

For  $M < 0.6M_{\odot}$ , slope flattens due to increasing role of convection throughout the star and changes in opacity

# Basic assumptions: Th. Stell. Evo.

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## 1. Isolated in space

- Their structure and evolution depend only on intrinsic properties (mass and composition)
- For binaries and dense clusters, evolution can be influenced by interaction

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## 3. Have spherical symmetry

- Promoted by self-gravity.
- Possible deviation from non-central forces: No rotation, no magnetic fields

# Exercises

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- 1.1** Stars are assumed to be *isolated in space*. The star closest to the sun, Proxima Centauri, is 4.3 light-years away. How many solar radii is that? How it affects the Sun gravitationally? How this force compares to the attractive gravitational force the Sun suffers from Jupiter?
- 1.2** Stars are assumed to form with a *uniform composition*. What elements is the Sun made of?
- 1.3** The Sun rotates around its axis every 27 days. Calculate the ratio of the centrifugal acceleration  $a$  over the gravitational acceleration  $g$  for a mass element on the surface of the Sun.
- 1.4** The masses of stars are approximately in the range  $0.08 M_{\odot} \leq M \leq 100 M_{\odot}$ . Why is there an upper limit? Why is there a lower limit?
- 1.5** Can you think of methods to measure (1) the mass, (2) the radius, and (3) the luminosity of a star?
- 1.6** Which stars live longer, high mass stars (which have more fuel) or low mass stars? Derive an expression for the lifetime of a star as a function of its mass.

# Exercises (2)

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- 1.7** (a) Calculate the radius of an M5 I supergiant with  $\log(L/L_\odot) = 5.50$  and  $T_{\text{eff}} = 2700$  K.  
(b) Assume a mass of approximately  $20M_\odot$  and calculate the mean density of the star.  
(c) Calculate the escape velocity.  
(d) Compare these values with those for the Sun.
- 1.8** (a) Calculate the luminosities of the horizontal branch stars with  $B-V \approx +0.30$  and  $BC = 0.11$  in the cluster M3 (NGC 5272). The distance to M3 is 10.4 kpc and its interstellar extinction is negligible.  
(b) What is the bolometric correction of the two stars at  $B-V \approx 0.25$ ?
- 1.9** The star  $\tau$  Sco has an apparent visual magnitude of  $V = +2.8$  and a spectral type of approximately B0V. Parallax measurements indicate a distance of 470 ly.  
(a) Calculate the absolute visual magnitude.  
(b) Adopt the bolometric correction from Table 2.1 and calculate the luminosity  $L$ .  
(c) Adopt the value of  $T_{\text{eff}}$  from Table 2.1 and calculate the radius.  
(d) Estimate the mass.  
(e) Calculate the acceleration of gravity at the stellar surface and the escape velocity.  
(f) Calculate the mean density of the star.  
(g) Compare these values with those of the Sun.
- 1.10** The Gaia satellite will measure parallaxes with an accuracy of  $2 \times 10^{-5}''$  for stars with  $V < 15$   
(a) What is the distance  $d$  of an M5 I supergiant with  $\log(L/L_\odot) = 5.50$  and  $BC = 3.70$  that has  $V = 12$ , if the effect of interstellar extinction is ignored?  
(b) What is the relative distance accuracy  $\sigma(d)/d$  of such a star?  
(c) Assuming that  $T_{\text{eff}}$  is known, what is the relative accuracy in radius  $\sigma(R)/R$ ?

# Questions

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**Q1:** Note that the B-V color hardly changes with Teff for temperatures above about 30,000 K. Explain this in physical terms, using the properties of Planck's formula for blackbody radiation.

**Q2:** Explain in physical terms why the red giant branch curves strongly to high values of B-V for a relatively small change in Teff. (Hint: use the properties of the Planck curve for blackbody radiation)

**Q3:** Explain why the distribution of stars in the CMD of M3 is completely different from the Milky Way.