Fis y Mat
Máster en Matemática Aplicada

UNIVERSIDAD DE GRANADA

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5. Energy transport in stellar interiors

Energy Transport

We have seen that in stellar interiors:

LTE is a good approach

Thermal equilibrium

Internal energy sources, nuclear reactions, compensate energy loss from the surface,

$$L = L_{\rm nuc} = -\frac{dE_{\rm nuc}}{dt}$$

The total energy is conserved, and the viral theorem states that the E_{int} and E_{pot} are conserved as well,

$$\dot{E}_{\rm tot} = \dot{E}_{\rm int} = \dot{E}_{\rm pot} = 0$$

This stationary state is known as **Thermal Equilibrium (TE)**

Energy is radiated at the surface at the same rate at which it is produced by nuclear reactions in the interior.

Energy Transport

We have seen that in stellar interiors:

LTE is a good approach

Mean free path is extremely small (l<<R)

The time radiation takes to escape from the center of the Sun by the random walk process is the K-H timescale.

Thermal (Kelvin-Helmholtz) timescale, (changes in the thermal structure; *from V. Th.*)

$$\tau_{KH} = \frac{E_{\rm int}}{L} \simeq \frac{|E_{pot}|}{2L} \simeq \frac{GM^2}{2RL} \simeq 1.5 \times 10^7 \left(\frac{M}{M_\odot}\right)^2 \frac{R_\odot}{R}\frac{L_\odot}{L} yr$$

describes how fast changes in thermal structure of a star can occur

Changes in the Sun luminosity would occur after millions of years, on the timescale for radiative energy transport: K-H timescale for thermal readjustment.

Energy Transport

In this situation, energy can be transported (hot to cold):

DIFFUSION: random thermal motion of particles.

- *Radiative diffusion* in the case of photons
- *Heat conduction* for gas particles

CONVECTION: collective ordered motion of gas particles

Very efficient & rapid mixing

This leads to 2 new equations for stellar structure

Local energy conservation

Virial Th, regulates the global energy budget.
(conservation)

In local scale the internal energy can be changed by two forms,

$$\delta u = \underbrace{\delta q}_{\text{Heat}} + \underbrace{\frac{P}{\rho^2}\delta\rho}_{\text{Work}}$$

Changes in the heat content can occur due to:

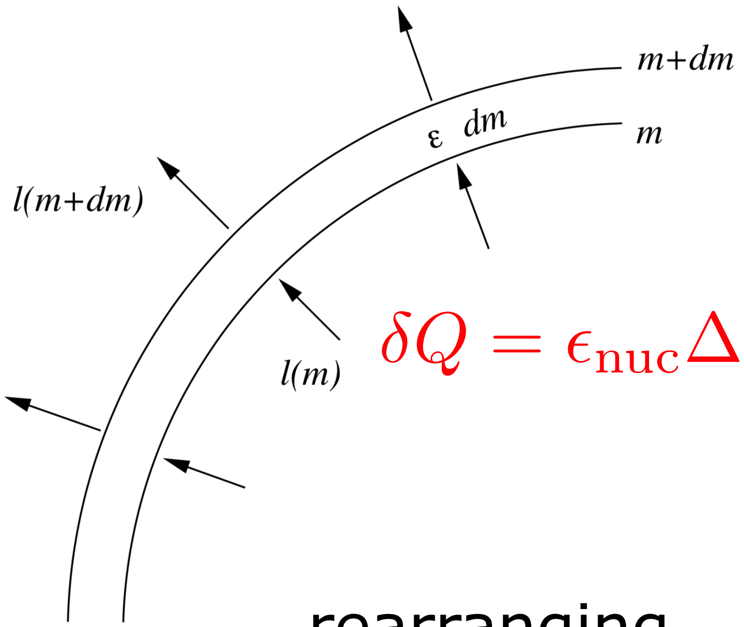
- Heat is added by the release of nuclear energy ($\epsilon_{\rm nuc}$)

- Heat is removed by the release of neutrinos ($\epsilon_{\rm nuc}$, ϵ_{ν})

- Heat is absorbed/emitted according to the balance of heat fluxes (*local luminosity*; $l = 4\pi r^2 F$)
where $l(0) = 0$ and $l(R) = L$

Local energy conservation

Consider a spherical shell. The heat content of the shell is:



$$\delta Q = \delta q \Delta m$$

so,

$$\delta Q = \epsilon_{\rm nuc} \Delta m \delta t - \epsilon_\nu \Delta m \delta t + l(m)\delta t - \underline{l(m+\Delta m)\delta t}$$

$$l(m+\Delta m)\delta t = l(m) + \frac{\partial l}{\partial m}\Delta m$$

rearranging,

$$\delta q = \left(\epsilon_{\rm nuc} - \epsilon_\nu - \frac{\partial l}{\partial m}\right)\delta t$$

And combined with the change of internal energy eq.,

$$\boxed{\frac{\partial l}{\partial m} = \epsilon_{\rm nuc} - \epsilon_\nu - \frac{\partial u}{\partial t} + \frac{P}{\rho^2}\frac{\partial \rho}{\partial t}}$$

LEC eq.

Local energy conservation

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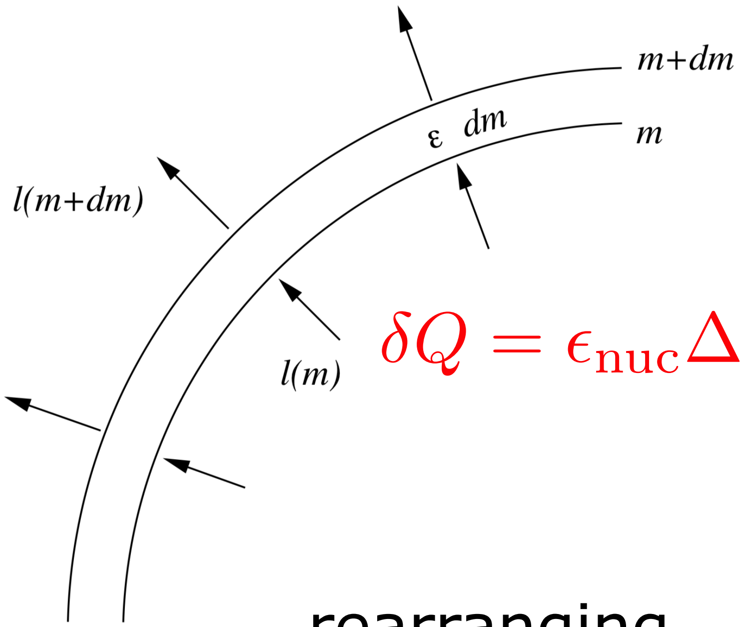

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And combined with the change of internal energy eq.,

$$\boxed{\frac{\partial l}{\partial m} = \epsilon_{\rm nuc} - \epsilon_\nu \boxed{- \frac{\partial u}{\partial t} + \frac{P}{\rho^2}\frac{\partial \rho}{\partial t}}}$$ LEC eq.

$$= -T\frac{\partial s}{\partial t} = \epsilon_{\rm gr}$$

Local energy conservation

$$\frac{\partial l}{\partial m} = \epsilon_{\mathrm{nuc}} - \epsilon_{\nu} + \epsilon_{\mathrm{gr}}$$

$\epsilon_{\mathrm{gr}} > 0$ Energy released (contraction)

$\epsilon_{\mathrm{gr}} < 0$ Energy absorbed (expansion)

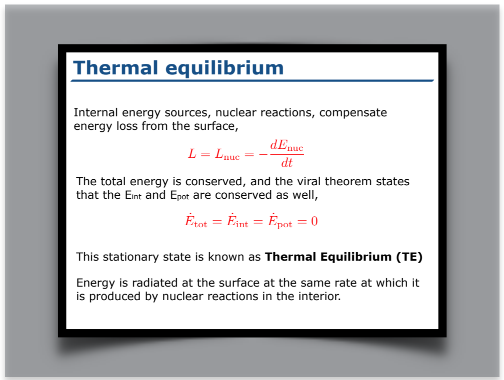
Thermal equilibrium achieved when $\epsilon_{\mathrm{gr}} = 0$, so

$$\frac{\partial l}{\partial m} = \epsilon_{\mathrm{nuc}} - \epsilon_{\nu}$$

Andd integratinng over the mass (Lagr. coord.)

$$L = \int_0^M \epsilon_{\mathrm{nuc}} dm - \int_0^M \epsilon_{\nu} dm \equiv L_{\mathrm{nuc}} - L_{\nu}$$

Neglecting $L_{\nu} \rightarrow$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Diffusion

When gradient of energy, diffusion is given by Fick's diffusion law:

$$F = -D\nabla U$$

where

$$D = \frac{1}{3}\bar{v}l$$ Diffusion coefficient

$$\nabla U = C_V \nabla T$$

$$F = -\frac{1}{3}\bar{v}lC_V\nabla T$$

For photons, $\bar{v} = c$ $\quad U = aT^4 \quad C_V = \frac{dU}{dT} = 4aT^3$

And l can be taken from the eq. of radiative transfer

$$\frac{dI_\nu}{ds} = -\overbrace{\kappa_\nu}^{\text{Opacity}}\rho I_\nu \rightarrow l_{\text{ph}} = \frac{1}{\kappa_\nu \rho}$$

so, $F_{\text{rad}} = -\underbrace{\boxed{\frac{4}{3}\frac{acT^3}{\kappa\rho}}}_{K_{\text{rad}}}\nabla T \sim \frac{l}{4\pi r^2}$

in spherical Symmetry

Radiative conductivity

Radiative diffusion (photons)

rearranging, $$\frac{\partial T}{\partial r} = -\frac{3\kappa\rho}{16\pi ac T^3}\frac{1}{r^2}$$

or combined with mass continuity eq. $$\frac{\partial T}{\partial m} = -\frac{3}{64\pi^2 ac}\frac{\kappa l}{r^4 T^3}$$

which describes the T gradient in stellar interiors when E is transported by radiation

Only valid when $l_{\rm ph} \ll R$, when LTE holds (e.g. in stellar surface $l_{\rm ph} \geq R$)

In HE, we can combine this with eq. Motion $$\frac{dP}{dm} = -\frac{Gm}{4\pi r^4}$$

$$\frac{dT}{dm} = \frac{dP}{dm}\frac{dT}{dP} = -\frac{Gm}{4\pi r^4}\frac{T}{P}\frac{d\log T}{d\log P}$$

so then, we can define the radiative T gradient

$$\nabla_{\rm rad} = \left(\frac{d\log T}{d\log P}\right)_{\rm rad} = \frac{3}{16\pi acG}\frac{\kappa l P}{mT^4}$$

Rosseland mean opacity

So far, we have assumed radiative diffusion is independent of frequency

However, in practice, opacity depends on frequency

$$F_\nu = -D_\nu \nabla U_\nu = -D_\nu \frac{\partial U_\nu}{\partial T} \nabla T$$

where $D_\nu = \frac{1}{3} c l_\nu = \frac{c}{3\kappa_\nu \rho}$ and $U_\nu = \frac{8\pi h}{c^3} \frac{\nu^3}{e^{\frac{h\nu}{kT}} - 1}$

Planck function for BB radiation (Ch. 3)

the total flux is integrated over all frequencies

$$F = -\left[\frac{c}{3\rho} \int_0^\infty \frac{1}{\kappa_\nu} \frac{\partial U}{\partial T} d\nu\right] \nabla T = -\boxed{K_{\mathrm{rad}}} \nabla T$$

From both expressions of conductivity,

$$\frac{1}{\kappa} = \frac{1}{4aT^3} \int_0^\infty \frac{1}{\kappa_\nu} \frac{\partial U}{\partial T} d\nu$$

is the Rosseland mean opacity

(the average transparency of the star)

Heat conduction (gas particles)

Collisions between gas particles (e- or ions) can also transport heat

However, their cross sections are typically ~10^{-19} cm^2 and $l_{\rm gas} \ll l_{\rm ph}$

Also $\bar{v} \ll c$, so in general this contribution can be ignored

Only important when particles (e.g. e-) are degenerate

- v increase (momenta approach the Fermi momentum)
- Mean free path $l_{\rm e} \gg l_{\rm ph}$, and e- conduction becomes more efficient

Similarly, $$F_{\rm cd} = -K_{\rm cd} \nabla T = -\frac{4acT^3}{3\kappa_{\rm cd}\rho} \nabla T$$

And in general,

$$F = F_{\rm rad} + F_{\rm cd} = -\left(K_{\rm rad} + K_{\rm cd}\right) \nabla T = -\frac{4acT^3}{3\kappa\rho} \nabla T$$

where $\dfrac{1}{\kappa} = \dfrac{1}{\kappa_{\rm rad}} + \dfrac{1}{\kappa_{\rm cd}}$


LOONEY TUNES
That's all Folks!