FisyMat
Máster en Matemática Aplicada

UNIVERSIDAD DE GRANADA

Física Estelar

Lluís Galbany, Ed. Mecenas (#16)

Inma Domínguez, Ed. Mecenas (#16)

Antonio García, Ed. Mecenas (#16)

5. Energy transport in stellar interiors

Energy Transport

We have seen that in stellar interiors:

LTE is a good approach

Thermal equilibrium

Internal energy sources, nuclear reactions, compensate energy loss from the surface,

$$L = L_{\rm nuc} = -\frac{dE_{\rm nuc}}{dt}$$

The total energy is conserved, and the viral theorem states that the E_{int} and E_{pot} are conserved as well,

$$\dot{E}_{\rm tot} = \dot{E}_{\rm int} = \dot{E}_{\rm pot} = 0$$

This stationary state is known as **Thermal Equilibrium (TE)**

Energy is radiated at the surface at the same rate at which it is produced by nuclear reactions in the interior.

Energy Transport

We have seen that in stellar interiors:

LTE is a good approach

Mean free path is extremely small (l<<R)

The time radiation takes to escape from the center of the Sun by the random walk process is the K-H timescale.

Thermal (Kelvin-Helmholtz) timescale, (changes in the thermal structure; *from V. Th.*)

$$\tau_{KH} = \frac{E_{\rm int}}{L} \simeq \frac{|E_{pot}|}{2L} \simeq \frac{GM^2}{2RL} \simeq 1.5 \times 10^7 \left(\frac{M}{M_\odot}\right)^2 \frac{R_\odot}{R} \frac{L_\odot}{L} yr$$

describes how fast changes in thermal structure of a star can occur

Changes in the Sun luminosity would occur after millions of years, on the timescale for radiative energy transport: K-H timescale for thermal readjustment.

Energy Transport

In this situation, energy can be transported (hot to cold):

DIFFUSION: random thermal motion of particles.

- *Radiative diffusion* in the case of photons
- *Heat conduction* for gas particles

CONVECTION: collective ordered motion of gas particles

Very efficient & rapid mixing

This leads to 2 new equations for stellar structure

Local energy conservation

Virial Th, regulates the global energy budget.
(conservation)

In local scale the internal energy can be changed by two forms,

$$\delta u = \underbrace{\delta q}_{\text{Heat}} + \underbrace{\frac{P}{\rho^2}\delta\rho}_{\text{Work}}$$

Changes in the heat content can occur due to:

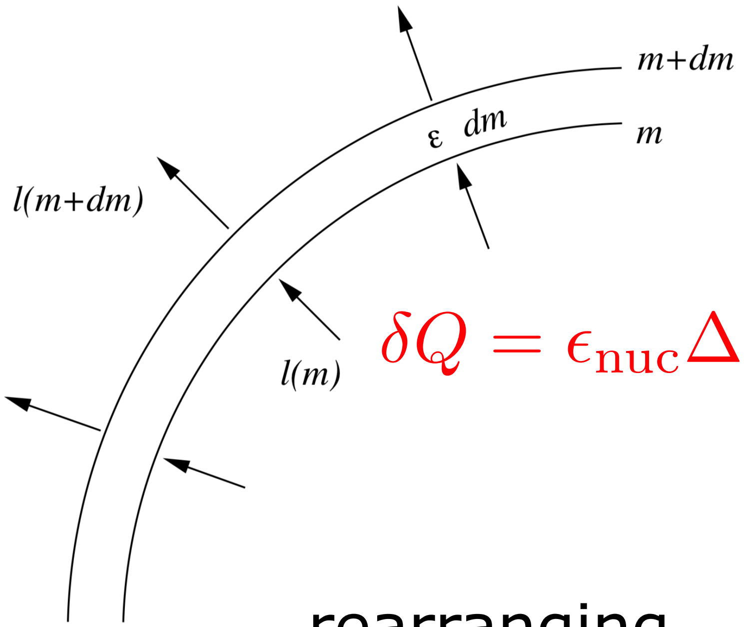
- Heat is added by the release of nuclear energy ($\epsilon_{\rm nuc}$)

- Heat is removed by the release of neutrinos ($\epsilon_{\rm nuc}$, ϵ_{ν})

- Heat is absorbed/emitted according to the balance of heat fluxes (*local luminosity*; $l = 4\pi r^2 F$)
where $l(0) = 0$ and $l(R) = L$

Local energy conservation

Consider a spherical shell. The heat content of the shell is:



$$\delta Q = \delta q \Delta m$$

so,

$$\delta Q = \epsilon_{\rm nuc}\Delta m\delta t - \epsilon_\nu \Delta m \delta t + l(m)\delta t - \underline{l(m+\Delta m)\delta t}$$

$$l(m+\Delta m)\delta t = l(m) + \frac{\partial l}{\partial m}\Delta m$$

rearranging,

$$\delta q = \left(\epsilon_{\rm nuc} - \epsilon_\nu - \frac{\partial l}{\partial m}\right)\delta t$$

And combined with the change of internal energy eq.,

$$\boxed{\frac{\partial l}{\partial m} = \epsilon_{\rm nuc} - \epsilon_\nu - \frac{\partial u}{\partial t} + \frac{P}{\rho^2}\frac{\partial \rho}{\partial t}}$$

LEC eq.

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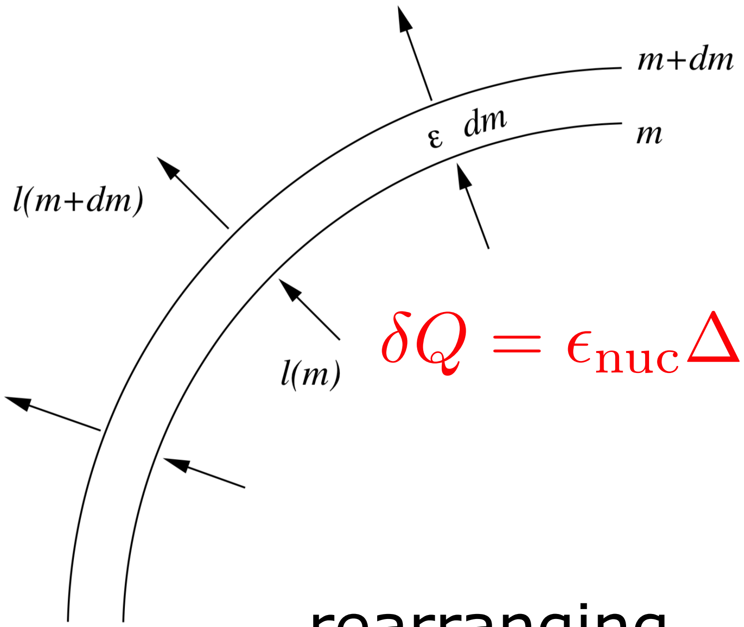

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$$\delta q = \left(\epsilon_{\rm nuc} - \epsilon_\nu - \frac{\partial l}{\partial m}\right)\delta t$$

And combined with the change of internal energy eq.,

$$\boxed{\frac{\partial l}{\partial m} = \epsilon_{\rm nuc} - \epsilon_\nu \boxed{- \frac{\partial u}{\partial t} + \frac{P}{\rho^2}\frac{\partial \rho}{\partial t}}}$$ LEC eq.

$$= -T\frac{\partial s}{\partial t} = \epsilon_{\rm gr}$$

Local energy conservation

$$\frac{\partial l}{\partial m} = \epsilon_{\rm nuc} - \epsilon_\nu + \epsilon_{\rm gr}$$

$\epsilon_{\rm gr} > 0$ Energy released (contraction)

$\epsilon_{\rm gr} < 0$ Energy absorbed (expansion)

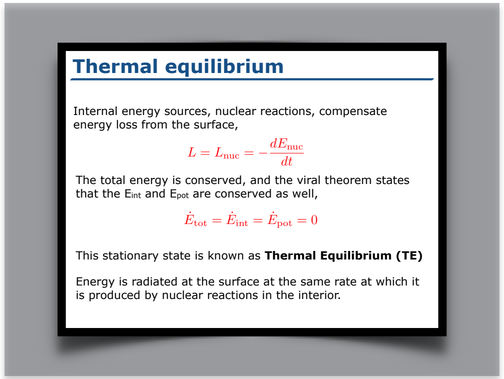
Thermal equilibrium achieved when $\epsilon_{\rm gr} = 0$, so

$$\frac{\partial l}{\partial m} = \epsilon_{\rm nuc} - \epsilon_\nu$$

And integrating over the mass (Lagr. coord.)

$$L = \int_0^M \epsilon_{\rm nuc} dm - \int_0^M \epsilon_\nu dm \equiv L_{\rm nuc} - L_\nu$$

Neglecting $L_\nu \rightarrow$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Energy is radiated at the surface at the same rate at which it is produced by nuclear reactions in the interior.

Equation of radiative transport

opacity matters in the interior layers of a star: impacts radiative transfer, energy transport!

If the source is surrounded by gas with opacity κ, then in traveling a distance *ds* the fraction of radiation absorbed is:

$$dI_\lambda = -\kappa_\lambda \rho I_\lambda ds + j_\lambda \rho I_\lambda ds$$

Change in I of a ray as it travels through a gas

mass absorption coefficient

mass emissivity coefficient

distance ray travels

Where K_λ = *opacity* ($cm^2\ g^{-1}$), and can therefore be interpreted as being the **fraction** of radiation absorbed by unit column density of gas.

Or the cross-section for absorbing photons at wavelength λ per g of star stuff

And the mean free path can be defined as the distance over which the intensity decreases by a factor of *e*

$$l_{\rm ph} = \frac{1}{\kappa_\nu \rho}$$

Diffusion

When gradient of particles, diffusion is given by Fick's diffusion law:
Flux equals a constant times the rate it changes in space

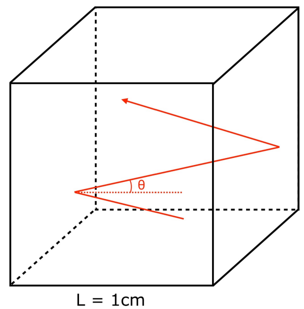
Mean free path

$$J = -D\nabla n \quad \text{where} \quad D = \frac{1}{3}\bar{v}l$$

Diffusion coefficient

average velocity

From Ch. 3:



$$\int_0^{\pi/2}\cos^2\theta\sin\theta d\theta$$

= 1/3

Consider particles crossing a unit surface area

The number of particles crossing a surface $\frac{dN}{dt} = \frac{1}{2}\left(\frac{1}{3}\bar{v}\right)n$

half in each direction

if there is a gradient in the particle density, $\frac{\partial n}{\partial z}$, particles moving up have density $n_{(z-l)}$, and particles moving down $n_{(z+l)}$. *So,*

$$J = \frac{1}{6}\bar{v}n_{(z-l)} - \frac{1}{6}\bar{v}n_{(z+l)} = \frac{1}{6}\bar{v}\left[n_{(z-l)} - n_{(z+l)}\right] = \frac{1}{6}\bar{v}\left[n - \frac{dn}{dz}l - \left(n + \frac{dn}{dz}l\right)\right]$$

$$= \frac{1}{6}\bar{v}\left(-2l\frac{dn}{dz}\right) = -\frac{1}{3}\bar{v}l\frac{dn}{dz} = -D\frac{dn}{dz}$$

Diffusion

The equivalent, when gradient of energy is present,

$$F = -D\nabla U \qquad \text{with} \qquad D = \frac{1}{3}\bar{v}l \qquad \nabla U = C_V \nabla T$$

$$F = -\frac{1}{3}\bar{v}lC_V\nabla T$$

For photons, $\bar{v} = c \qquad U = aT^4 \qquad C_V = \frac{dU}{dT} = 4aT^3$

so, $F_{\mathrm{rad}} = -\underbrace{\boxed{\frac{4}{3}\frac{acT^3}{\kappa\rho}}}_{K_{\mathrm{rad}}}\nabla T \underset{\text{in spherical Symmetry}}{\sim} \frac{l}{4\pi r^2}$ ← local luminosity

Radiative conductivity

Radiative diffusion (photons)

rearranging, $$\frac{\partial T}{\partial r} = -\frac{3\kappa\rho}{16\pi a c T^3}\frac{l}{r^2}$$

or combined with mass continuity eq. $$\frac{\partial T}{\partial m} = -\frac{3}{64\pi^2 a c}\frac{\kappa l}{r^4 T^3}$$

which describes the T gradient in stellar interiors when E is transported by radiation

Only valid when $l_{\rm ph} \ll R$, when LTE holds (e.g. in stellar surface $l_{\rm ph} \geq R$)

In HE, we can combine this with eq. Motion $\frac{dP}{dm} = -\frac{Gm}{4\pi r^4}$

$$\frac{dT}{dm} = \frac{dP}{dm}\frac{dT}{dP} = -\frac{Gm}{4\pi r^4}\frac{T}{P}\frac{d\log T}{d\log P}$$

so then, we can define the radiative T gradient

$$\nabla_{\rm rad} = \left(\frac{d\log T}{d\log P}\right)_{\rm rad} = \frac{3}{16\pi a c G}\frac{\kappa l P}{m T^4}$$

Heat conduction (gas particles)

Collisions between gas particles (e- or ions) can also transport heat

However, their cross sections are typically ~10[-19] cm² and $l_{\rm gas} \ll l_{\rm ph}$

Also $\bar{v} \ll c$, so in general this contribution can be ignored

Only important when particles (e.g. e-) are degenerate

- v increase (momenta approach the Fermi momentum)
- Mean free path $l_{\rm e} \gg l_{\rm ph}$, and e- conduction becomes more efficient

Similarly, $$F_{\rm cd} = -K_{\rm cd} \nabla T = -\frac{4acT^3}{3\kappa_{\rm cd}\rho} \nabla T$$

And in general,

$$F = F_{\rm rad} + F_{\rm cd} = -\left(K_{\rm rad} + K_{\rm cd}\right) \nabla T = -\frac{4acT^3}{3\kappa\rho} \nabla T$$

where $\frac{1}{\kappa} = \frac{1}{\kappa_{\rm rad}} + \frac{1}{\kappa_{\rm cd}}$

Rosseland mean opacity

So far, we have assumed radiative diffusion is independent of frequency

However, in practice, opacity depends on frequency

$$F_\nu = -D_\nu \nabla U_\nu = -D_\nu \frac{\partial U_\nu}{\partial T} \nabla T$$

where $$D_\nu = \frac{1}{3} c l_\nu = \frac{c}{3\kappa_\nu \rho}$$ and $$U_\nu = \frac{8\pi h}{c^3} \frac{\nu^3}{e^{\frac{h\nu}{kT}} - 1}$$

Planck function for BB radiation (Ch. 3)

the total flux is integrated over all frequencies

$$F = -\left[\frac{c}{3\rho} \int_0^\infty \frac{1}{\kappa_\nu} \frac{\partial U}{\partial T} d\nu\right] \nabla T = -\boxed{K_{\mathrm{rad}}} \nabla T$$

From both expressions of conductivity,

$$\frac{1}{\kappa} = \frac{1}{4aT^3} \int_0^\infty \frac{1}{\kappa_\nu} \frac{\partial U}{\partial T} d\nu$$

is the Rosseland mean opacity

(the average transparency of the star)

Opacity

The opacity coefficient κ determines the flux that can be transported by radiation for a certain temperature gradient

(how large the temperature gradient must be in order to carry a given luminosity *l* by radiation)

physical processes that contribute to opacity:

Electron scattering

Free electrons scatter photons. $\mu_e = 2/(1+X)$

Deep inside stars, gas fully ionized, this is dominant

Basically found dividing cross-section by the unit mass

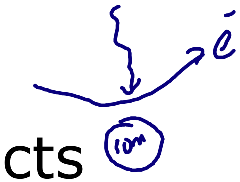
Thomson cross-section

$$\kappa_{\rm es} = \frac{\overline{\sigma_e}}{\underbrace{\mu_e m_u}_{\rho/n_e}} = \frac{8\pi}{3}\left(\frac{e^2}{m_e c^2}\right)^2 \frac{1}{\mu_e m_u} = 0.20(1+X)\ \mathrm{cm^2 g^{-1}}$$

High T, not-so-high densities

Opacity

Free-free absorption



Oposite of bremsstrahlung: γ absorbed by e- when interacts with ion

$$\kappa_{\rm ff} \sim Z_i^2 n_i n_e \sim 7.5\times 10^{22}\left(\frac{1+X}{2}\right)\left\langle\frac{Z_i^2}{A_i}\right\rangle \rho T^{-7/2}\ {\rm cm^2 g^{-1}}$$

(also called Kramer's opacity)

The steep T dependence implies that it is strongest at low T

bound-free absorption

absorption of a photon with sufficient energy by an atom not fully ionized, kicking a bound electron

$$\kappa_{\rm bf} \sim 4.3\times 10^{25}(1+X)Z\rho T^{-7/2}\ {\rm cm^2 g^{-1}}$$

Not applicable at low T<10^4 K (γ not energetic enough)

Not applicable at high T (ions fully ionized)

Opacity

Bound-bound absorption $\kappa_{\rm bb}$

If gas is not fully ionized, it can absorb γ, resulting in e-transitions from one bound state to another.

Mainly important at $T<10^6$ K

Negative H ion

Important in relatively cool stars (T ~ kK)

Bound-free absorption of H that has been previously ionized with free electrons from metals (Na, K, Ca, or Al)

$$\kappa_{\rm H-} \simeq 2.5 \times 10^{-31} \left(\frac{Z}{0.02}\right) \rho^{1/2} T^9 \ {\rm cm^2 g^{-1}}$$

Molecules and dust

In cool stars (T< 4000 K) opacity sources from molecules and dust (even lower T~1500 K; dust grains formation) become important

Opacity

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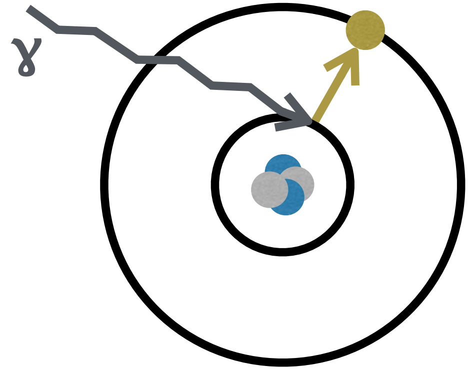

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Opacity

+ Conductive opacities

Energy transport by heat conduction can also be described by means of a conductive opacity $\kappa_{\rm cd}$

In general, $\kappa_{\rm cd} \gg \kappa_{\rm rad}$

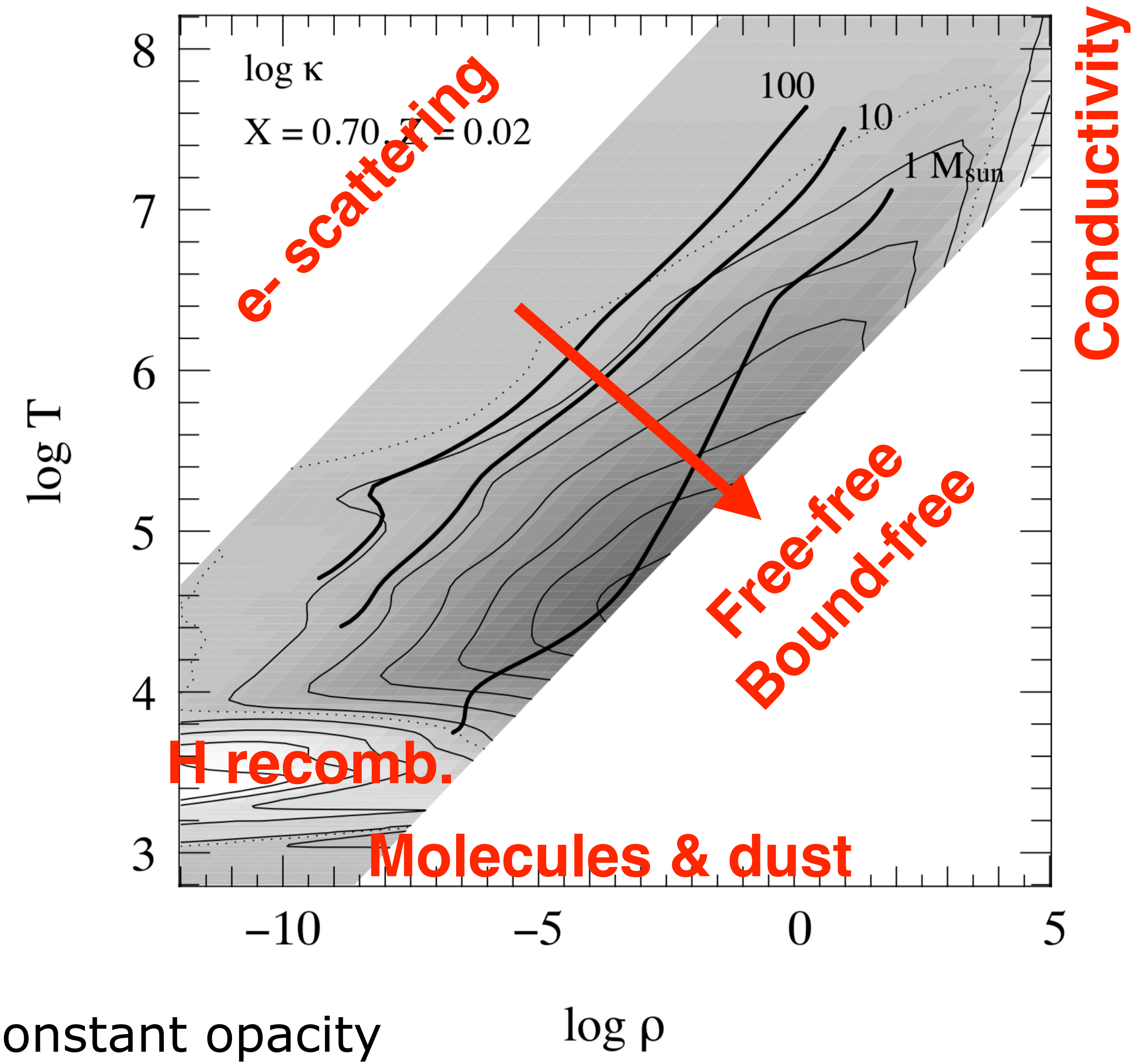
But for a degenerate gas,

$$\kappa_{\rm cd} \simeq 4.4 \times 10^{-3} \frac{\sum_i Z_i^{5/3} X_i/A_i}{(1+X)^2} \frac{(T/10^7 K)^2}{\rho/10^5 \text{ g/cm}^3)^2} \text{ cm}^2\text{g}^{-1}$$

at high ρ and low T, $\kappa_{\rm cd}$ becomes very small (large l_e)

In general, $\kappa = \kappa(\rho, T, X_i)$ is a complicated function (because it is not additive). But it has been calculated in a few cases…

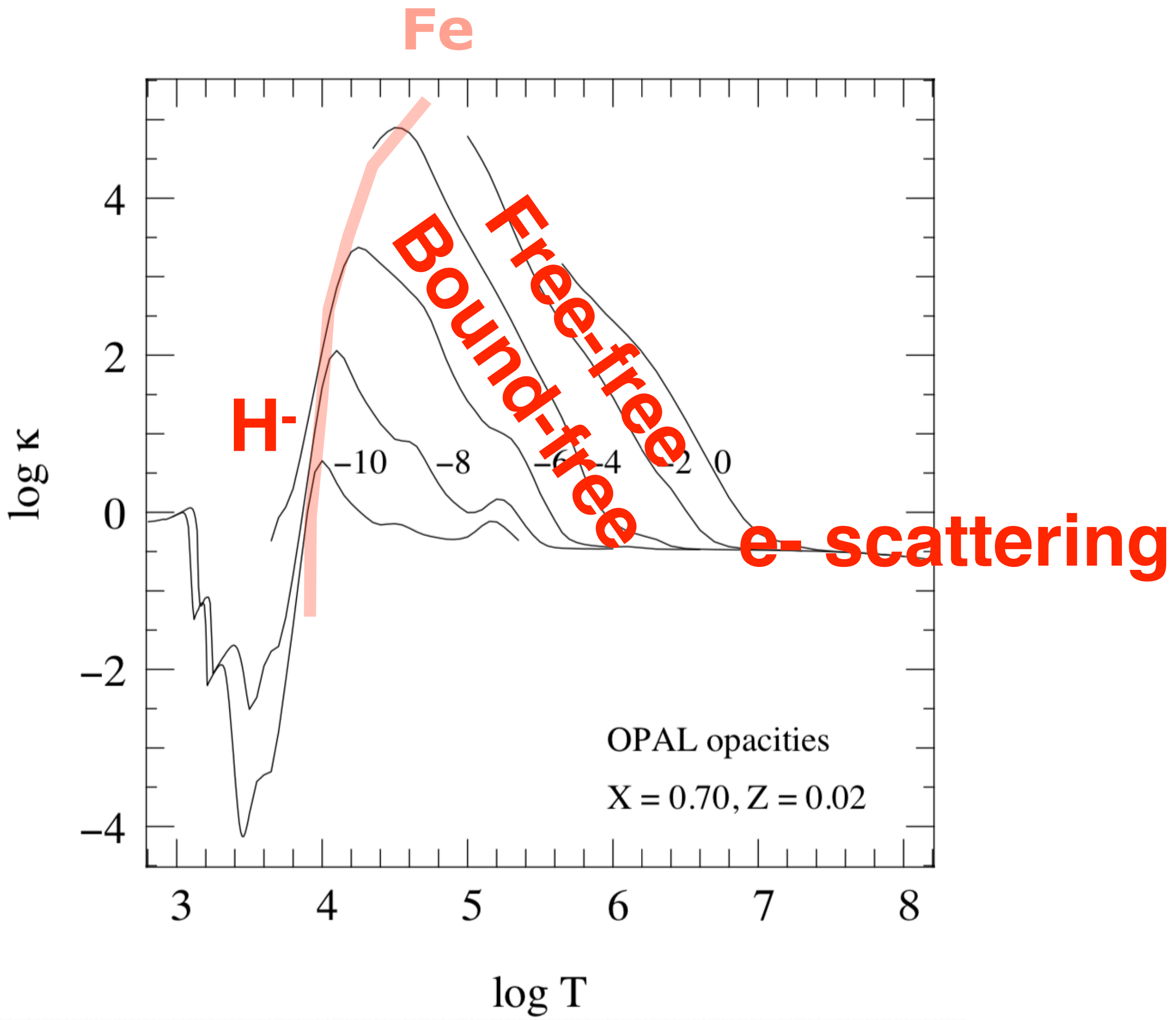
Opacity



lines of constant opacity

Opacity

- At high T and low ρ, all matter is ionized, $\kappa=\sigma_e$
- As T decreases, the bound–free and free–free absorption coefficient increases as $\kappa \sim \rho T^{-7/2}$
- At intermediate T (depending on ρ) the gas is partly ionized. This results in many more possible e- transitions and huge κ.
- This produces the so-called Fe-opacity peak around 10^5 K for densities 10^{-6} to 10^{-4} g cm^{-3}
- The peak shifts to higher T as the ρ increases.
- The peak at $10^4 < T < 10^5$ K at very low ρ (~10^{-10} to 10^{-8} g cm^{-3}) is due to H.
- At very low T < 10^4 K, κ due to H^- decreases steeply toward lower T as $\kappa(H^-) \sim T^9$.



The Eddington luminosity

Radiative transport requires a T gradient

$$\frac{\partial T}{\partial r} = -\frac{3\kappa\rho}{16\pi a c T^3}\frac{l}{r^2}$$

Since $P_{\rm rad} = \frac{1}{3}aT^4$, this also implies a Pressure gradient

$$\frac{dP_{\rm rad}}{dr} = \frac{4}{3}aT^3\frac{dT}{dr} = -\frac{\kappa\rho}{4\pi c}\frac{l}{r^2}$$

this is an outward force that goes against gravitation in HE

$$\left(\frac{dP_{\rm rad}}{dr}\right) < \left(\frac{dP}{dr}\right)_{\rm HE} = -\frac{Gm\rho}{r^2} \quad \rightarrow \quad \frac{\kappa\rho}{4\pi c}\frac{l}{r^2} < \frac{Gm\rho}{r^2} \quad \rightarrow \quad l < \frac{4\pi c G m}{\kappa} \equiv l_{\rm Edd}$$

Which is the maximum luminosity that can be carried out by radiation

We can find $l > l_{\rm Edd}$, which may result from intense nuclear burning or large κ (e.g. at very low T)

At that point energy cannot be transported by radiation, and we need another mechanism: *convection*

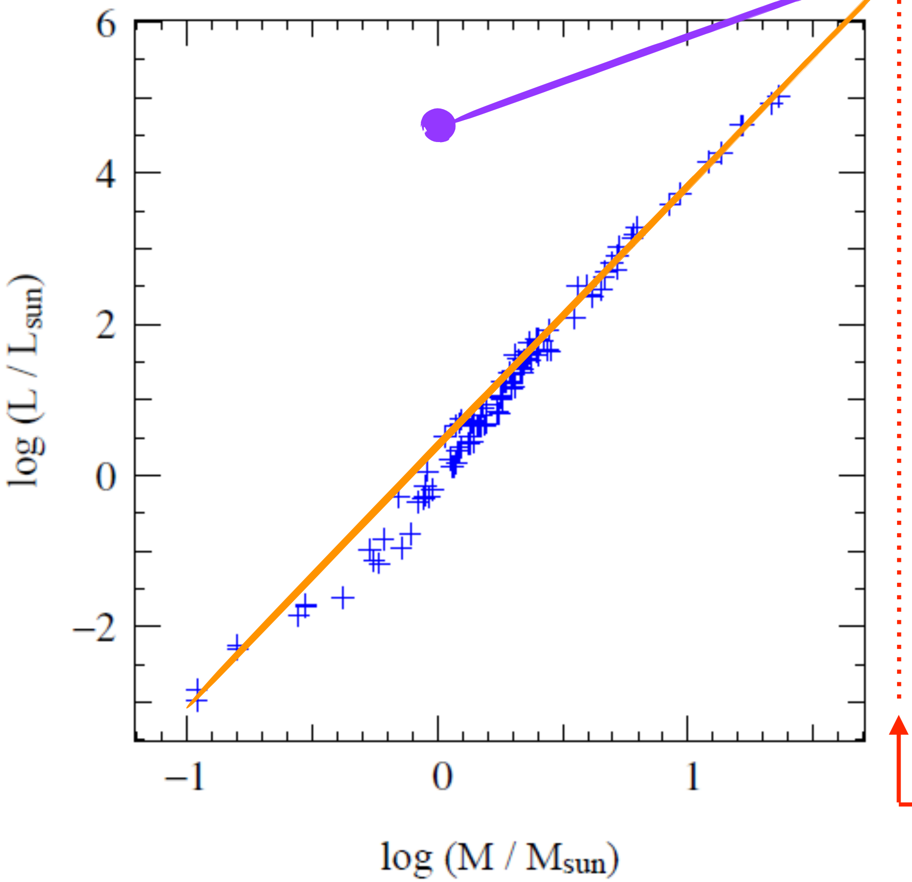
(collective motion of gas bubbles that carry heat)

The Eddington luminosity

At the surface (m -> M) energy scapes, so the surface layer is always radiative. Assuming κ constant,

e- scatter opacity for X=0.7 ~ solar

$$L_{\rm Edd} = \frac{4\pi c G M}{\kappa} = 3.8\times10^4 \left(\frac{M}{M_\odot}\right)\left(\frac{0.34\ {\rm cm\ g^{-1}}}{\kappa}\right) L_\odot$$



For main sequence stars, only e- scatter contributes to κ

We saw that L was proportional to M^α with α~3.8

so, L at some point will exceed the Edd. limit

$$\log_{10}(3.8\times10^4) \sim 4.5$$

$$M_{\rm max} \sim 100 M_\odot$$

Convective energy transfer

For diffusion to happen, a certain T gradient is needed

The larger the L, the larger the T gradient needed

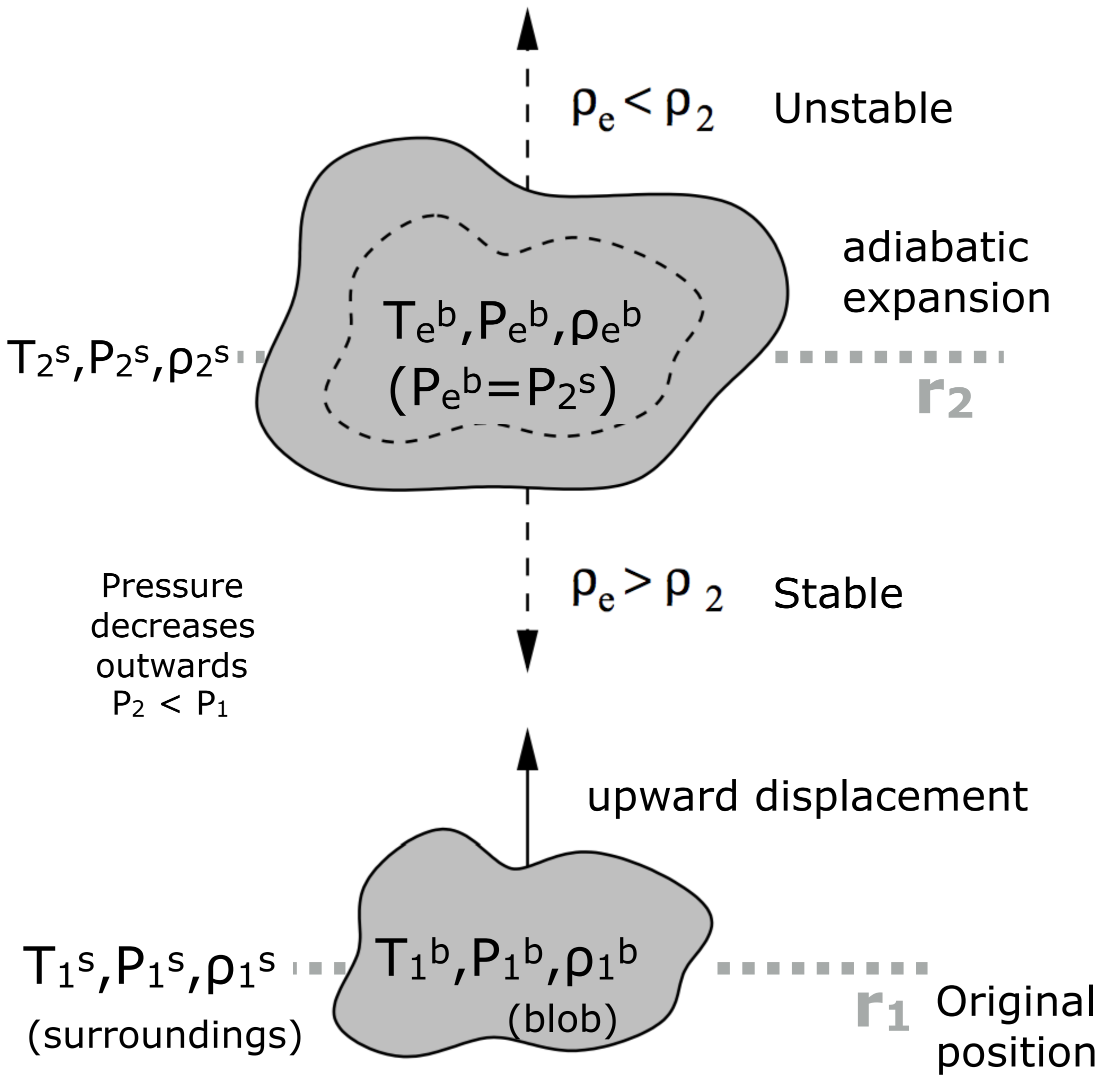
If the limit T gradient is exceeded, it leads to an instability in the gas, that produces cyclical motions of the gas

Convection is a dynamical instability that do not break HE.
(no disruptive consequences)

Convection affects the structure of a star only as an efficient means of ***heat transport*** and as an efficient ***mixing mechanism***.

Criteria for stability against convection

Consider a blob of gas in a star that due to small perturbations starts moving upwards. (by thermal motions of gas particles)



At r_2, and at lower pressure P_2, the blob will expand: $P_e = P_2$, but not necessarily $\rho_e = \rho_2$

If $\rho_e > \rho_2$, we have a stable situation. It will go back down

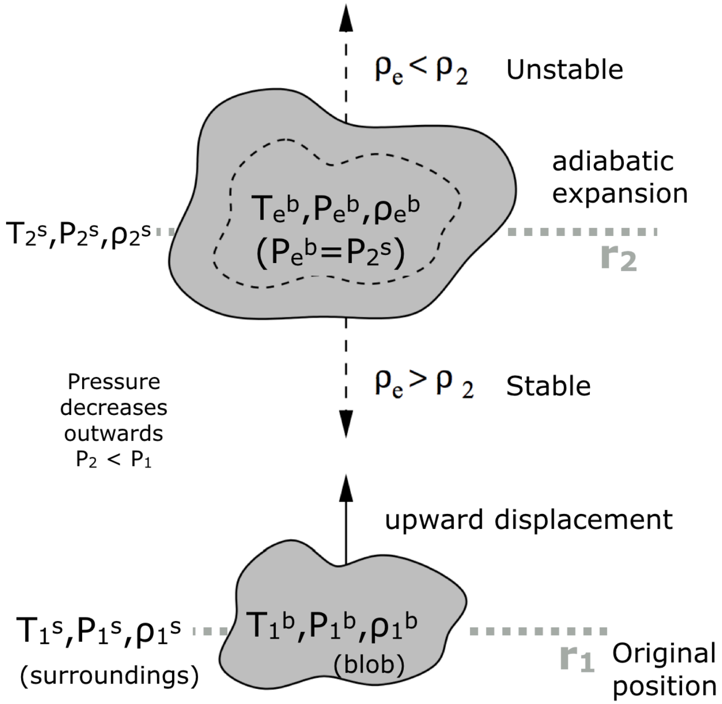
If $\rho_e < \rho_2$, it will keep going up. We have an unstable situation that leads to convection.

The gas expansion occurs at τ_{LD}(local dynamical t.)$<<\tau_{KH}$(heat exchange), so it is an adiabatic process

$$\gamma_{ad} = \frac{d \log P}{d \log \rho} \quad \longrightarrow \quad \frac{\delta P_e}{P_e} = \gamma_{ad} \frac{\delta \rho_e}{\rho_e}$$

Criteria for stability against convection

where the difference in pressure is described by the pressure gradient



$$P_e = P_2$$

$$\delta P_e = P_2 - P_1 = \frac{dP}{dr}\Delta r$$

$$\delta \rho_e = \frac{\rho_e}{P_e}\frac{1}{\gamma_{\rm ad}}\frac{dP}{dr}\Delta r$$

writing $\quad \rho_e = \rho_1 + \delta\rho_e \qquad \rho_2 = \rho_1 + (d\rho/dr)\Delta R$

we can express the stability criterion

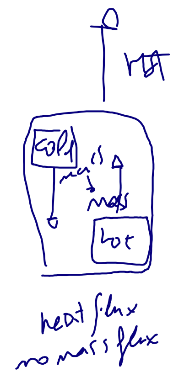
$$\rho_e > \rho_2 \qquad \text{as,} \qquad \delta\rho_e > \frac{d\rho}{dr}\Delta r$$

which combined with above expression yields a limit to the density gradient for stability

$$\frac{1}{\rho}\frac{d\rho}{dr} < \frac{1}{P}\frac{dP}{dr}\frac{1}{\gamma_{\rm ad}} \;\rightarrow\; \frac{d\log\rho}{d\log P} = \frac{1}{\gamma_{\rm ad}}$$

The general criterion for stability against convection

If violated, convection motions will develop $\rightarrow$



Criteria for stability against convection

(not part of the stellar structure equations)

The stability criterion involves the calculation of a density gradient

(Because it is also in the eq radiative transfer)

We can rewrite the criterion using EOS to get a T gradient

$$\frac{dP}{P} = \chi_T \frac{dT}{T} + \chi_\rho \frac{d\rho}{\rho} + \chi_\mu \frac{d\mu}{\mu} \qquad \text{where} \qquad \chi_i = \left(\frac{\partial \log P}{\partial \log i}\right)_{j,k}$$

In general, we can rearrange

$$\frac{\partial \log \rho}{\partial \log P} = \frac{1}{\chi_\rho}\left(1 - \chi_T \frac{\partial \log T}{\partial \log P} - \chi_\mu \frac{\partial \log \mu}{\partial \log P}\right) = \frac{1}{\chi_\rho}(1 - \chi_T \nabla - \chi_\mu \nabla_\mu)$$

In the displaced gas element, composition does not change, so from adiabatic derivatives,

$$\nabla_{\rm ad} = \frac{\gamma_{\rm ad} - \chi_\rho}{\gamma_{\rm ad}\chi_T} \qquad \rightarrow \qquad \frac{1}{\gamma_{\rm ad}} = \frac{1}{\chi_\rho}(1 - \chi_T \nabla_{\rm ad})$$

And the stability criterion $\frac{d\log\rho}{d\log P} = \frac{1}{\gamma_{\rm ad}}$ becomes

$$\nabla < \nabla_{\rm ad} - \frac{\chi_\mu}{\chi_T}\nabla_\mu$$

Criteria for stability against convection

If all energy is transported by radiation $\nabla = \nabla_{\rm rad}$

$$\nabla_{\rm rad} < \nabla_{\rm ad} - \frac{\chi_\mu}{\chi_T}\nabla_\mu$$

Ledoux criterion
(a layer is stable against convection if...)

For an ideal gas χ_μ = -1 and χ_T = 1, so $\nabla_{\rm rad} < \nabla_{\rm ad} + \nabla_\mu$

$\nabla_\mu > 0$, so it contributes to stability

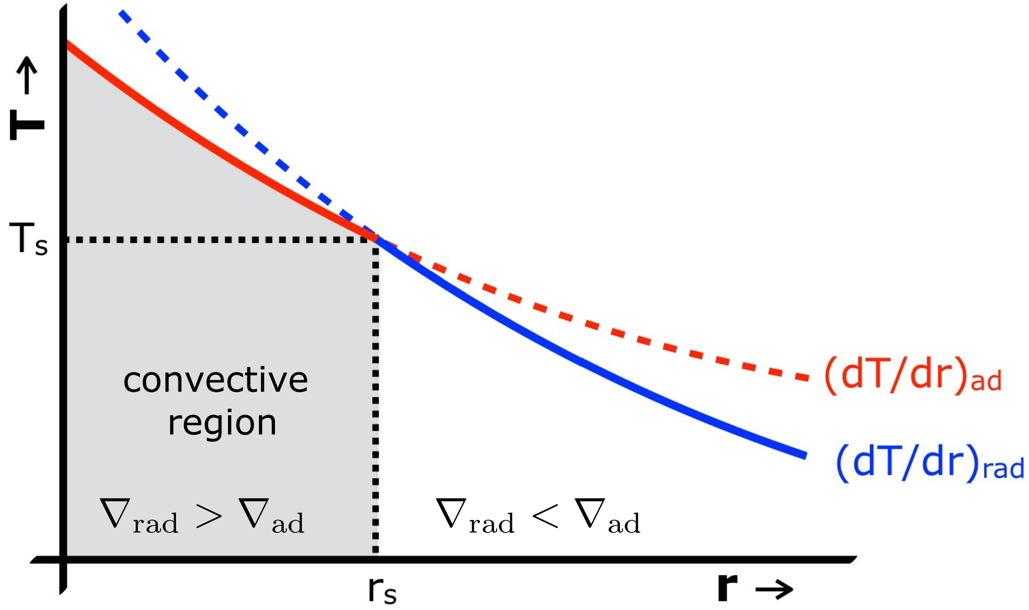
In chemically homogenous layers $\nabla_\mu = 0$

$$\nabla_{\rm rad} < \nabla_{\rm ad}$$

Schwarzschild criterion

So we expect convection to occur when

$$\nabla_{\rm rad} = \frac{3}{16\pi acG}\frac{P}{T^4}\frac{\kappa l}{m} > \nabla_{\rm ad}$$



Schwarzschild criterion

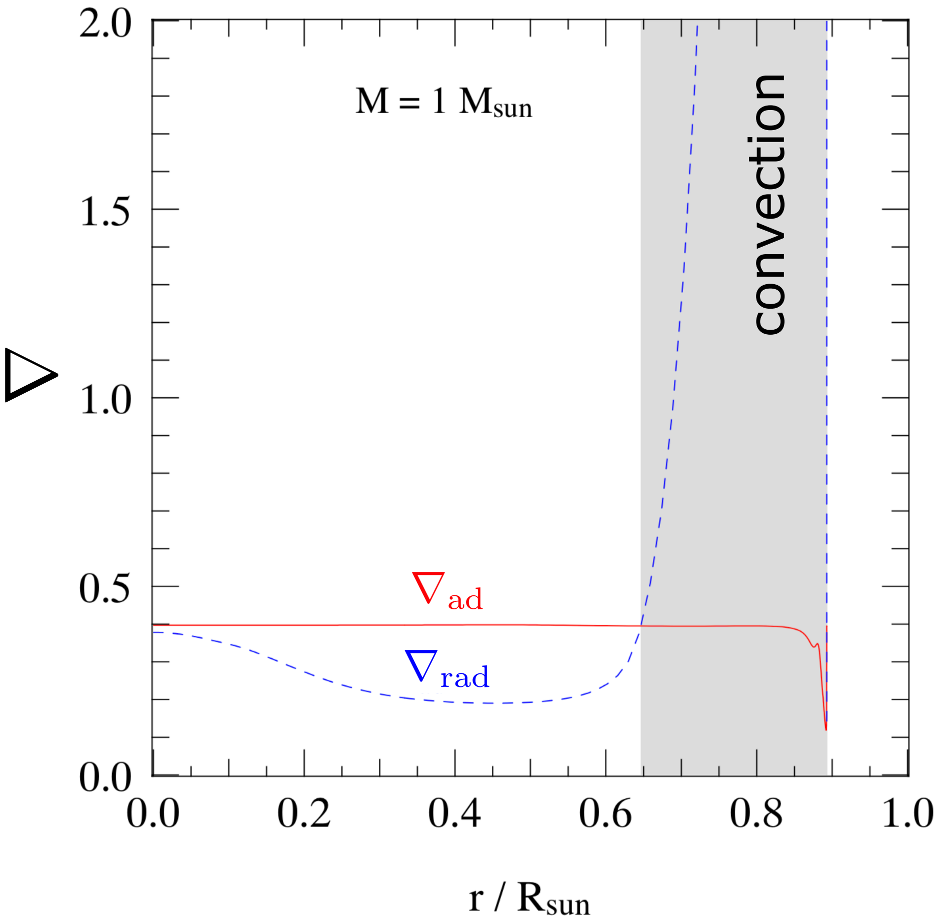
convection requires:

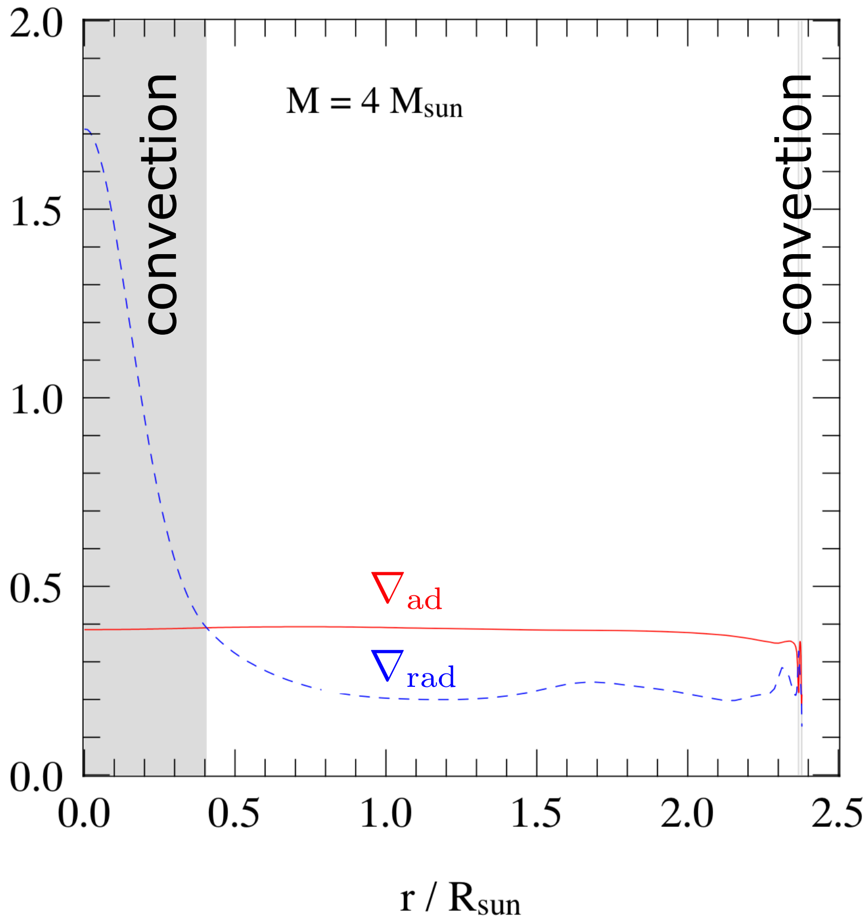
$$\nabla_{\rm rad} = \frac{3}{16\pi acG}\frac{P}{T^4}\frac{\kappa l}{m} > \nabla_{\rm ad}$$

Large κ opacity increases with decreasing T.
low-mass stars to have convective envelopes

Large $\frac{l}{m}$ towards the centre of a star $l/m \approx \epsilon_{\rm nuc}$
massive stars will have convective cores.

Small $\nabla_{\rm ad}$ occurs in not fully ionized zones at low T.
Even if **κ** is not large, surface layers may be unstable to convection.



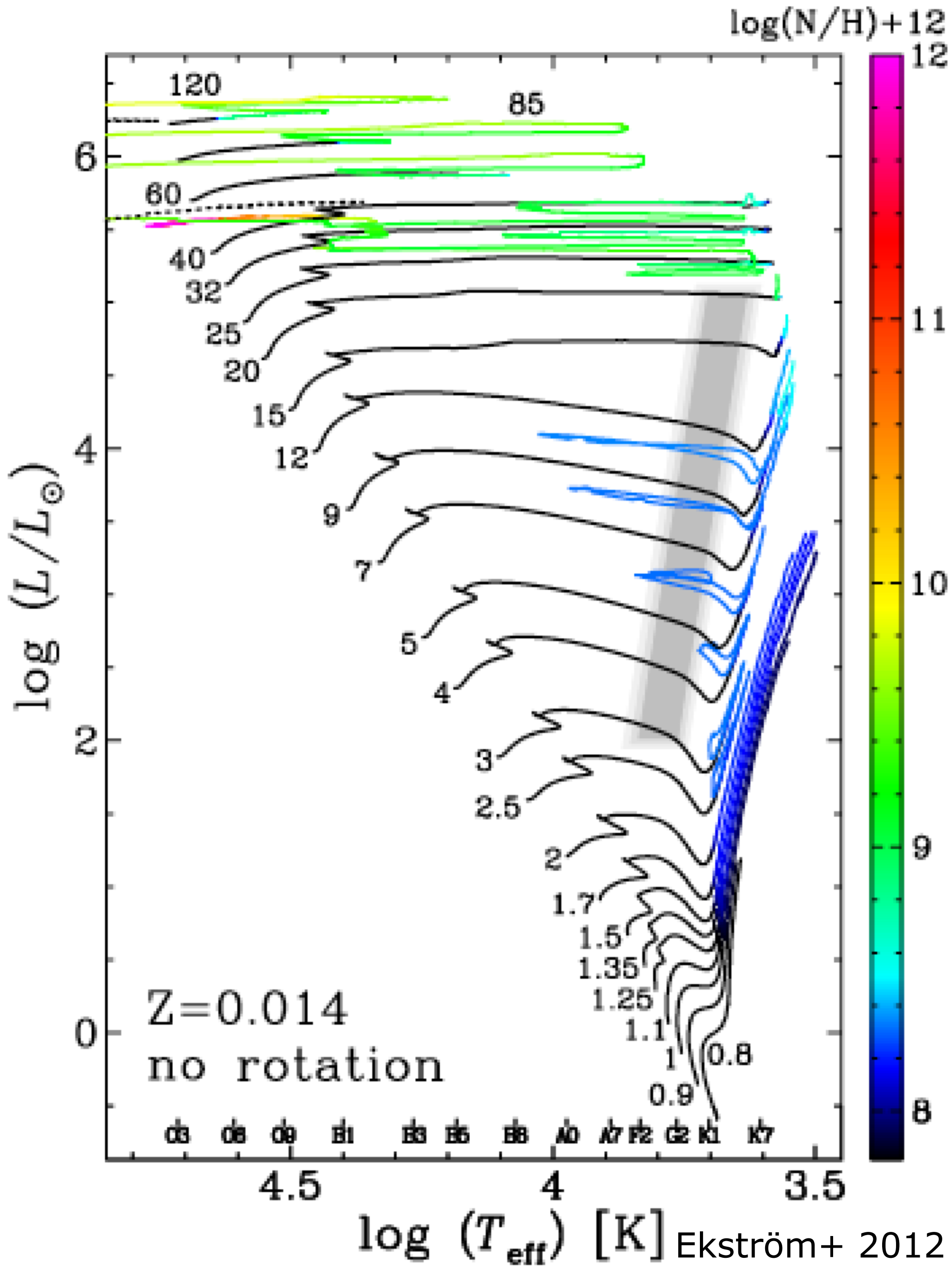


Convection in Stars

Where does convection actually happen?

$$\nabla_{\rm rad} = \frac{3}{16\pi acG}\frac{P}{T^4}\frac{\kappa l}{m} > \nabla_{\rm ad}$$

1. Large κ and low T



Ekström+ 2012

Convection in Stars

<u>Where does convection actually happen?</u>

$$\nabla_{\rm rad} = \frac{3}{16\pi acG}\frac{P}{T^4}\frac{\kappa l}{m} > \nabla_{\rm ad}$$

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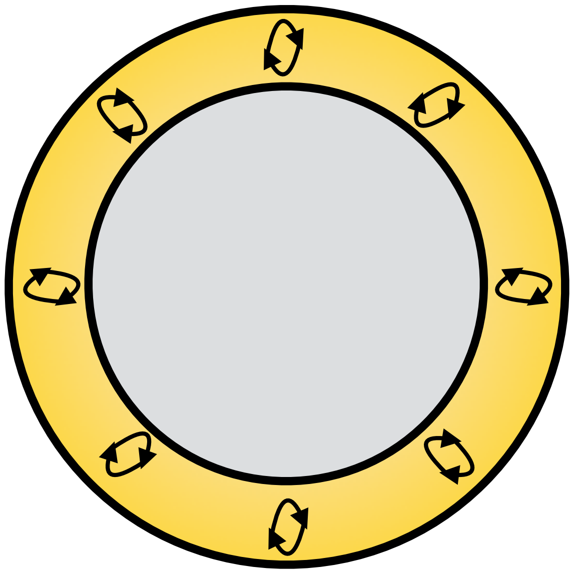

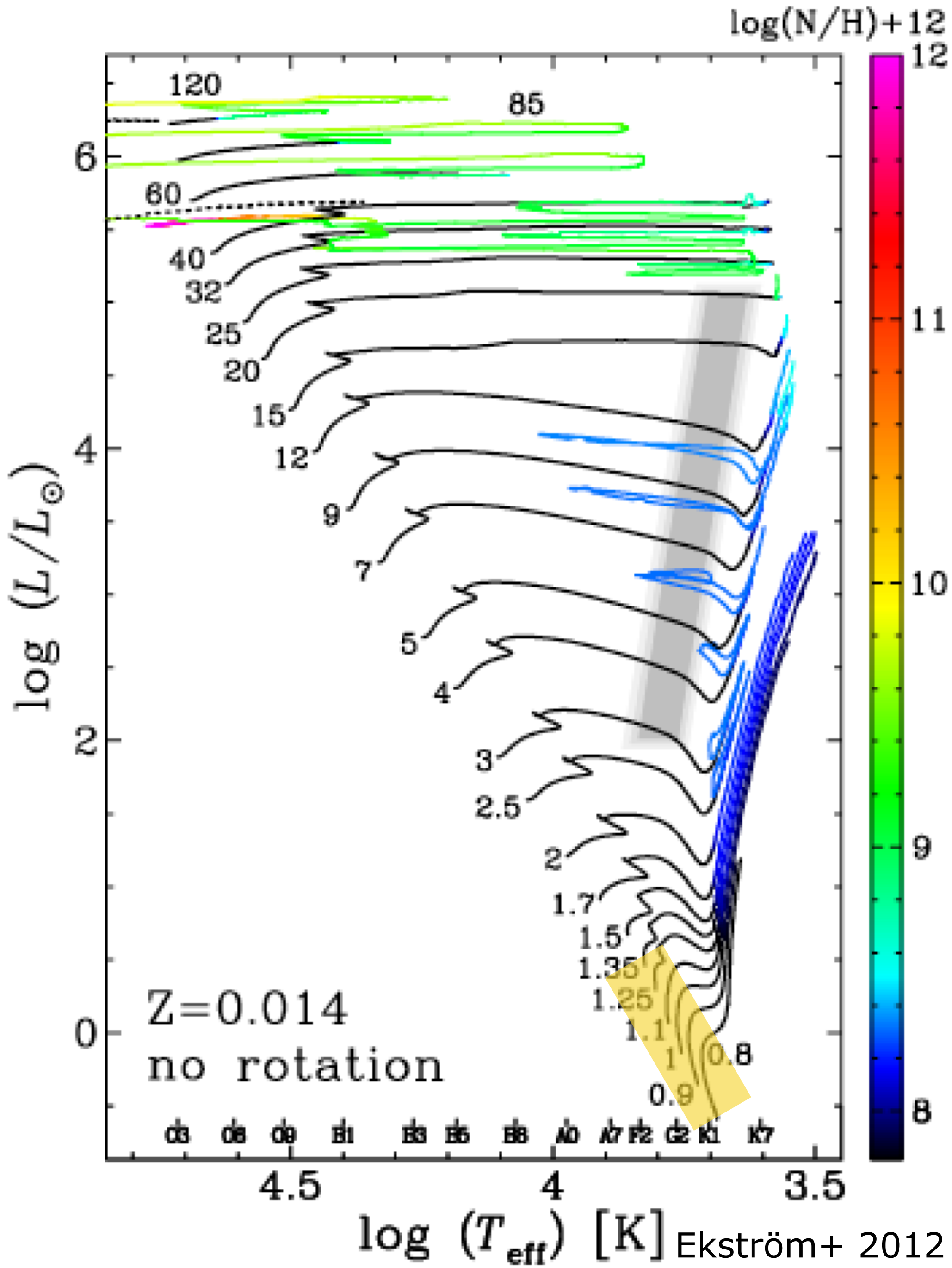

Ekström+ 2012

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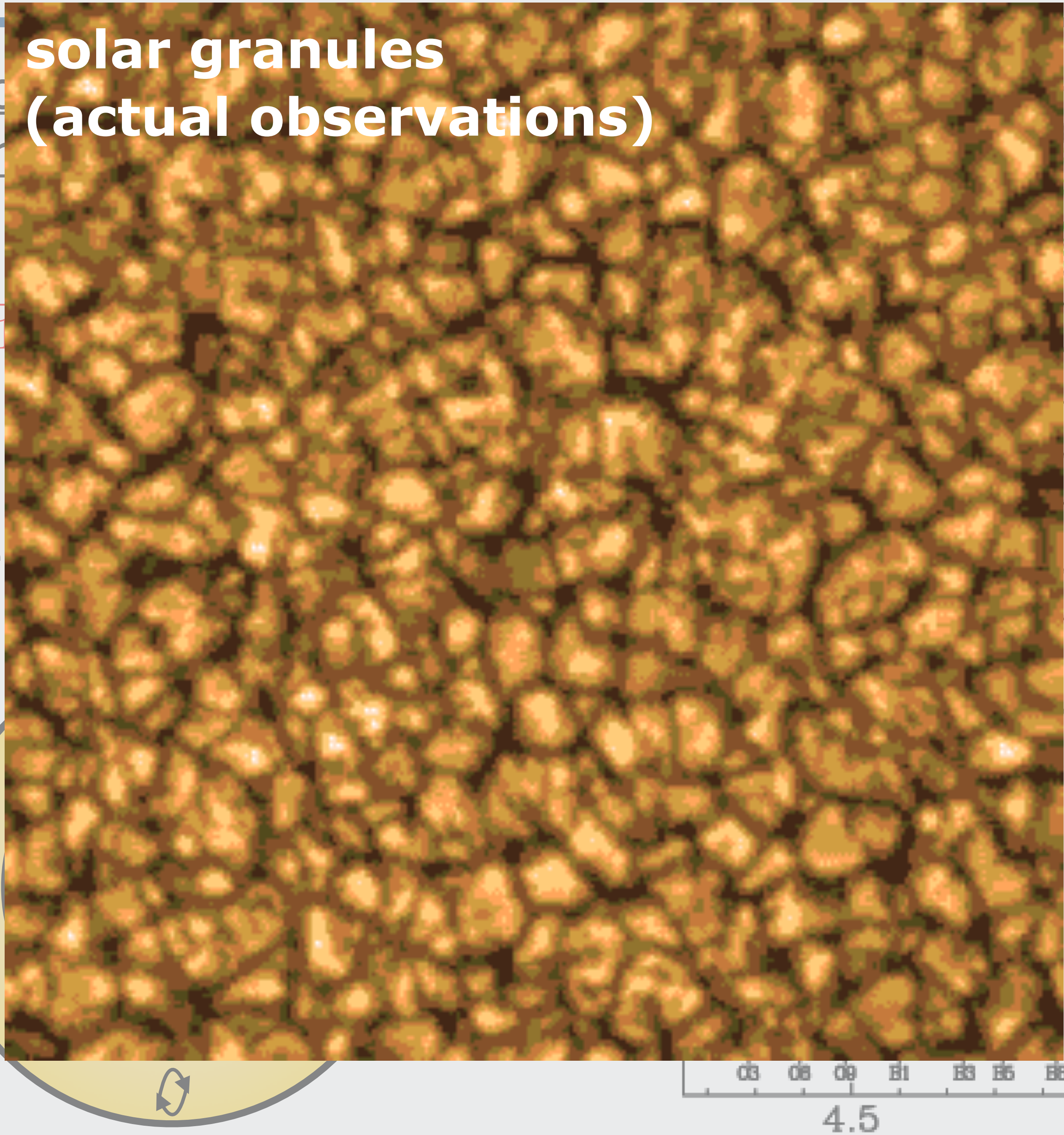

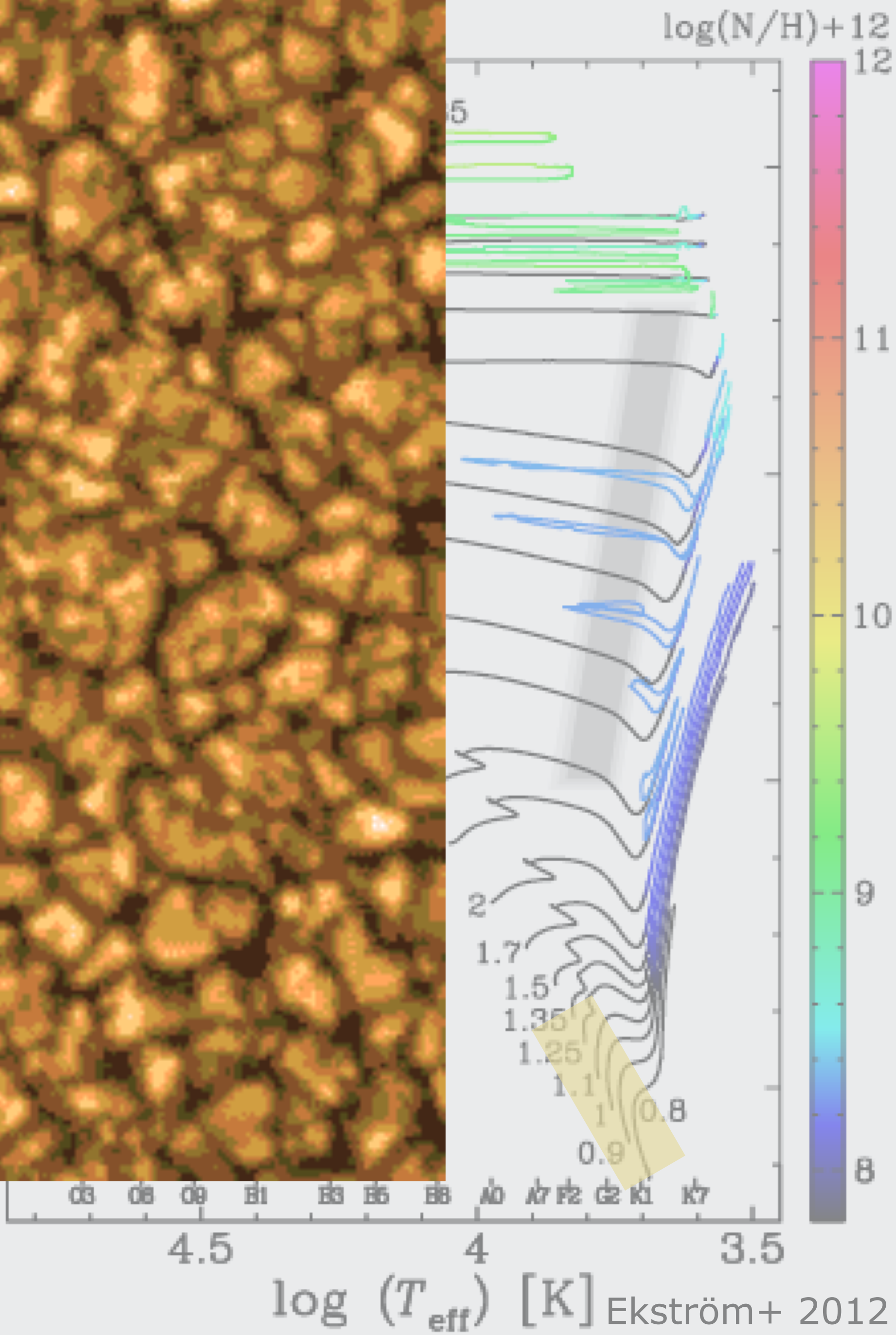

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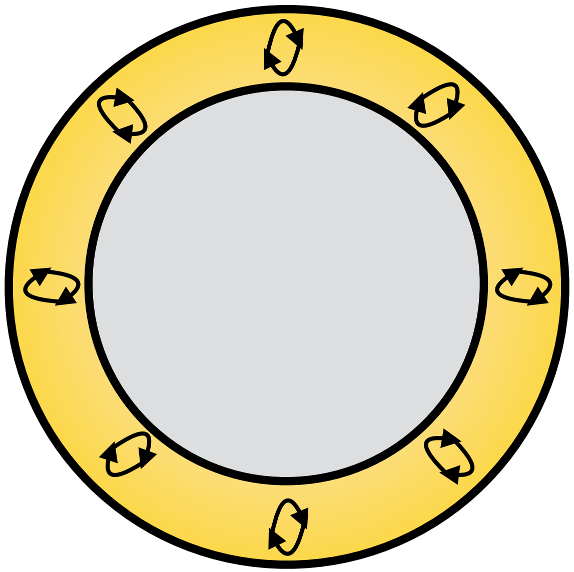

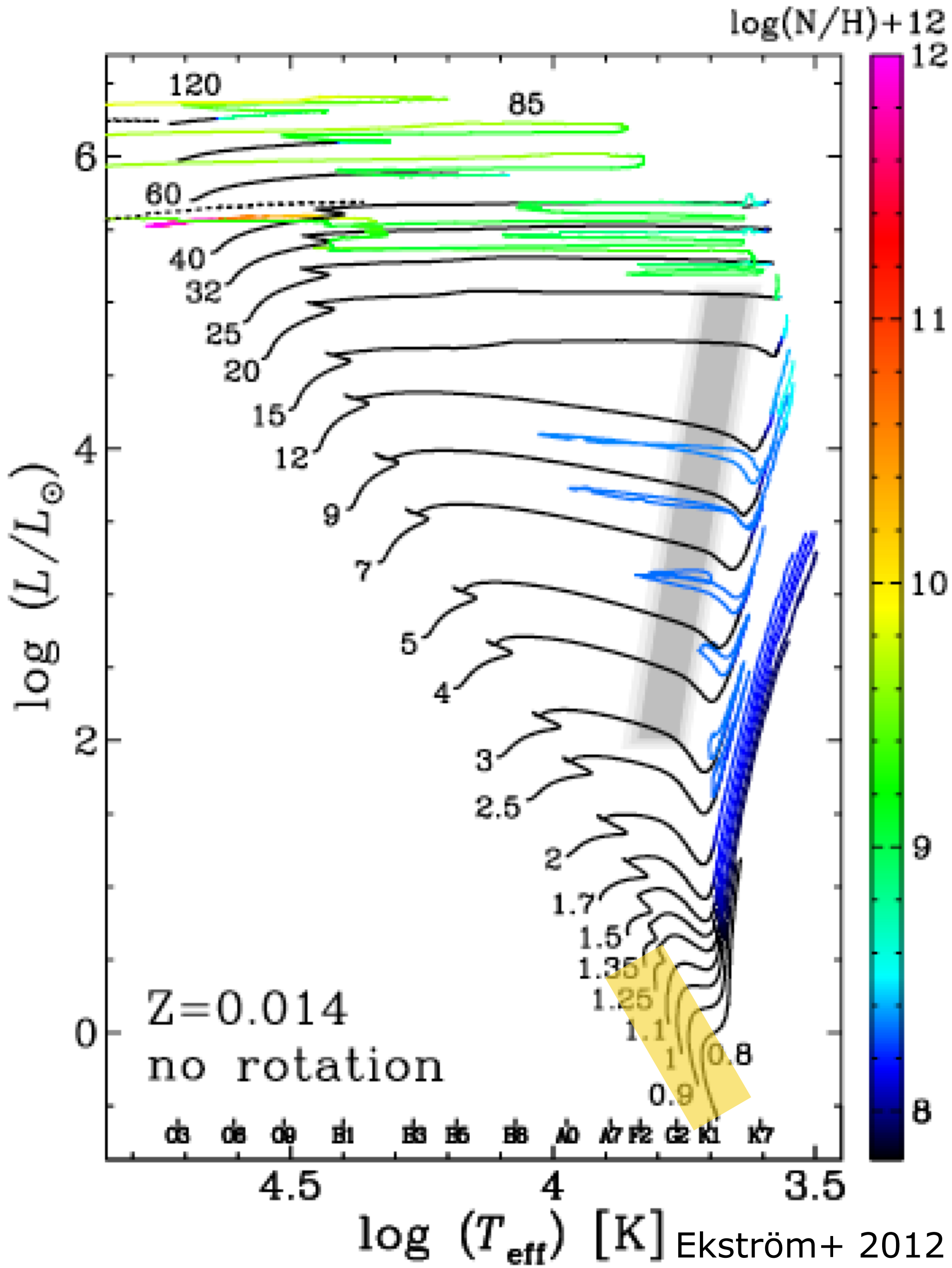

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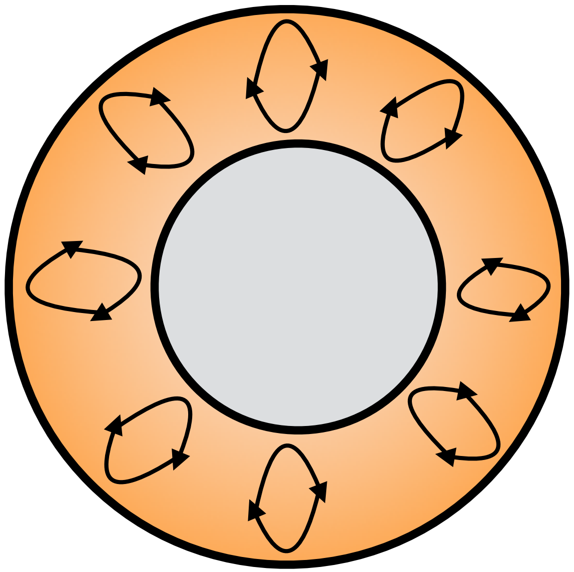

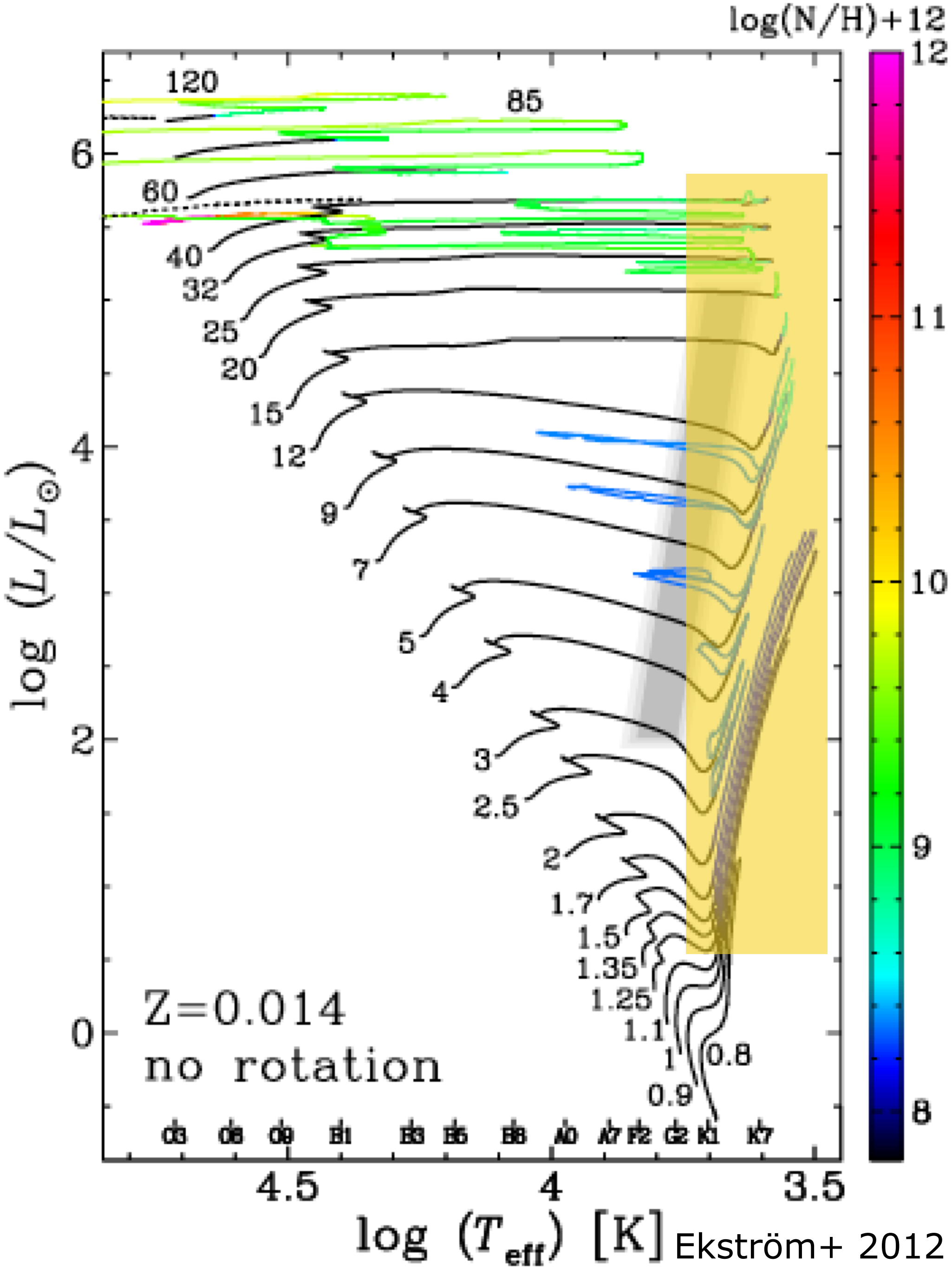

Ekström+ 2012

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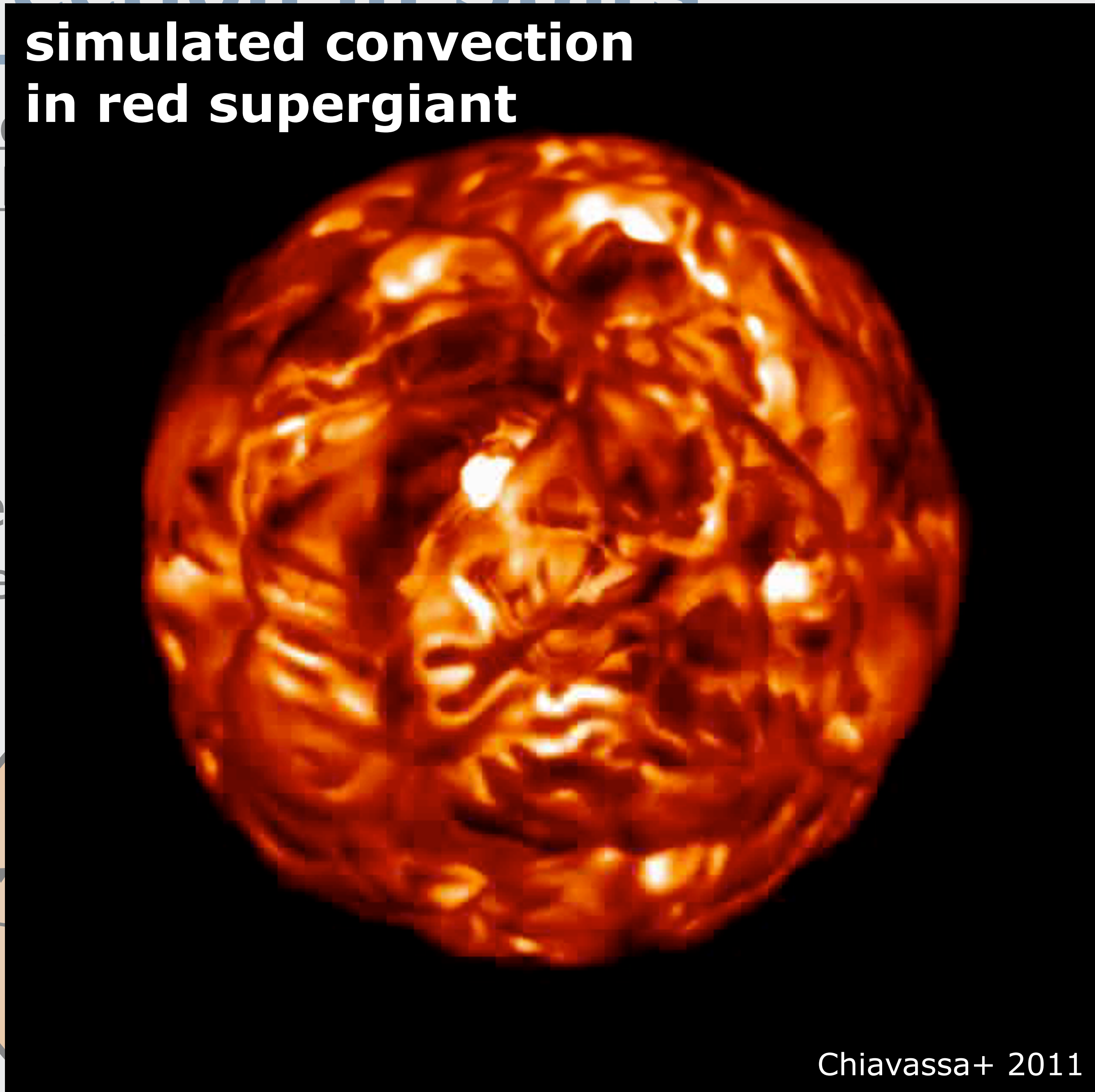

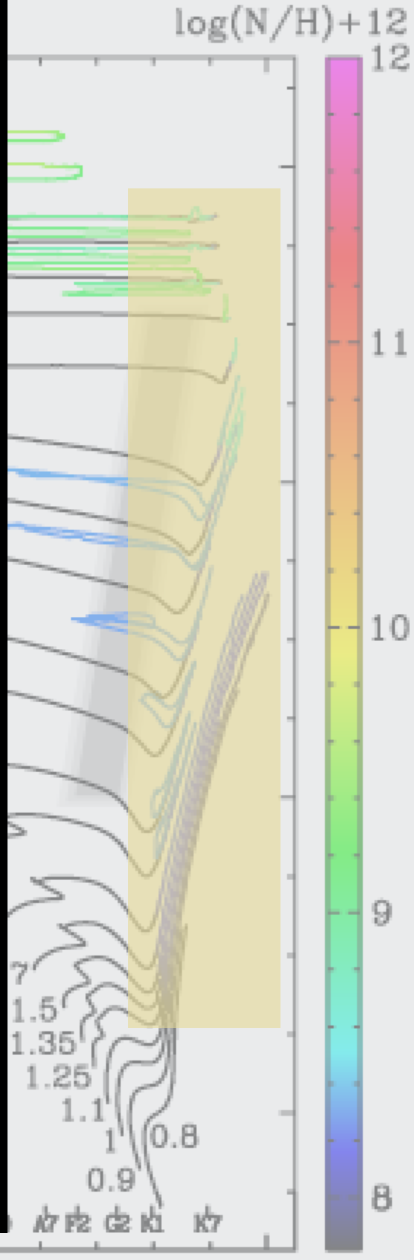

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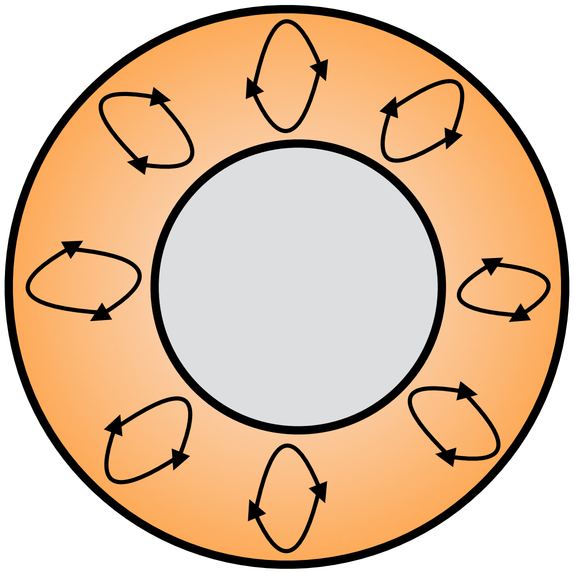

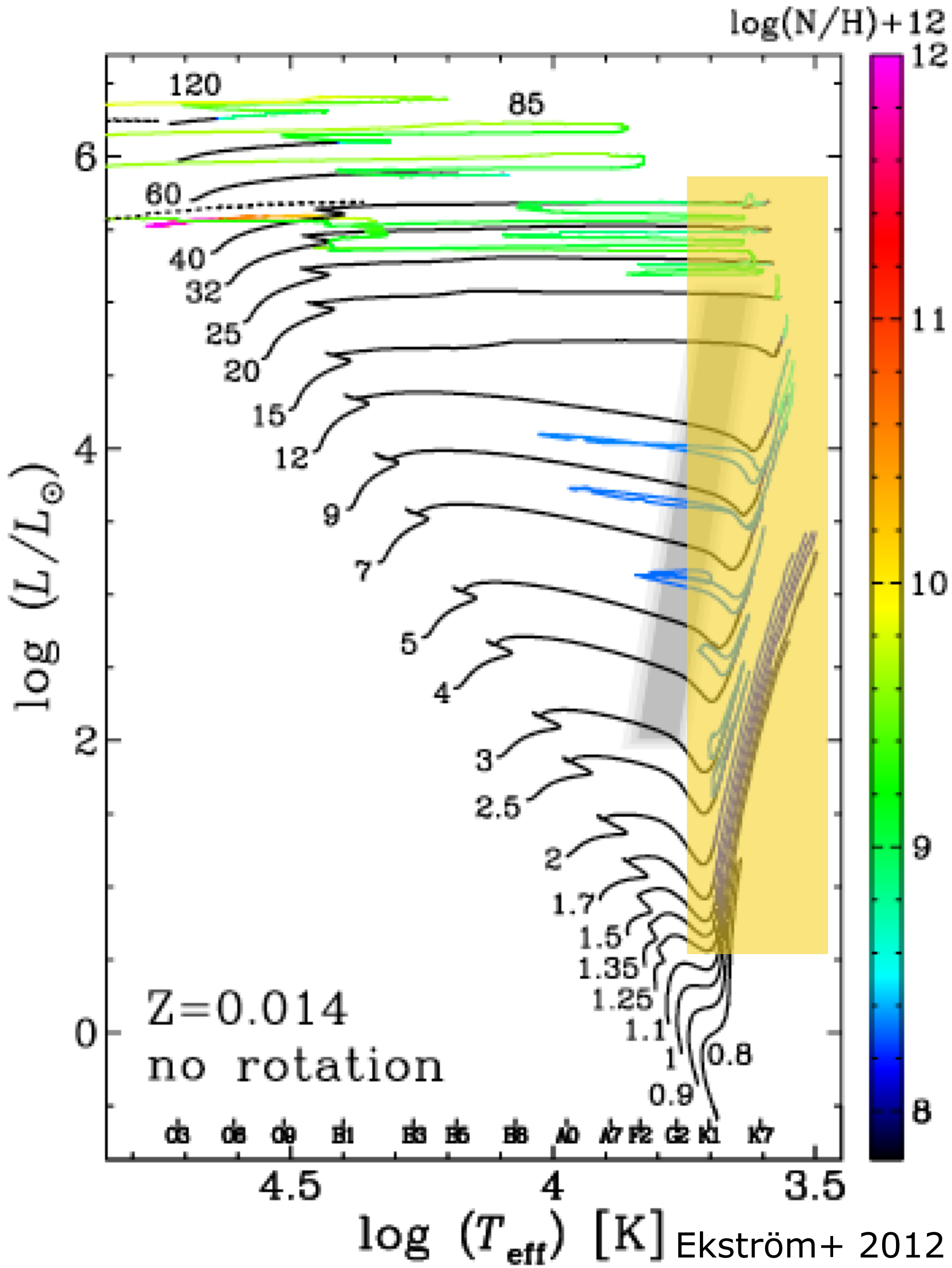

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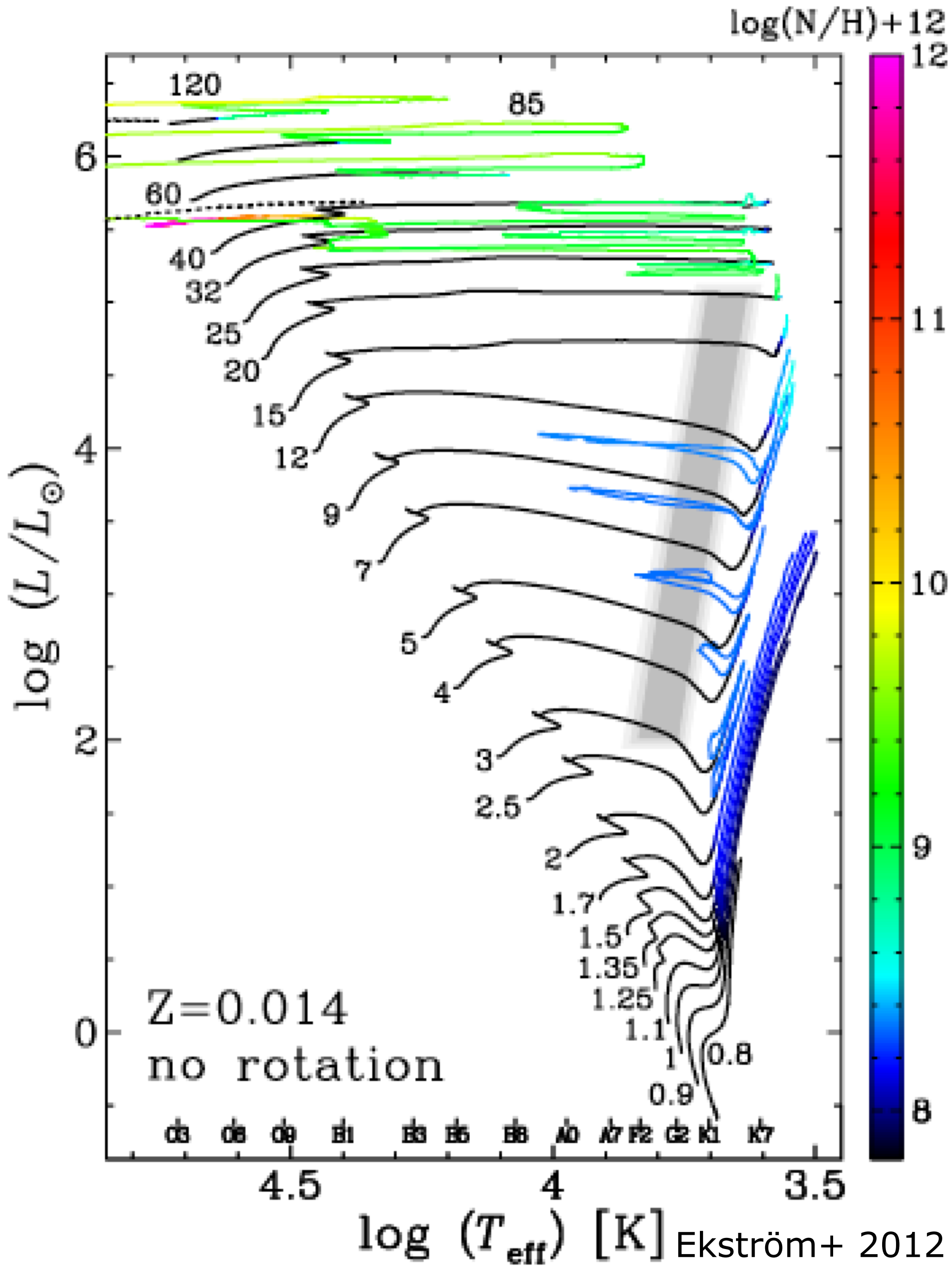

Ekström+ 2012

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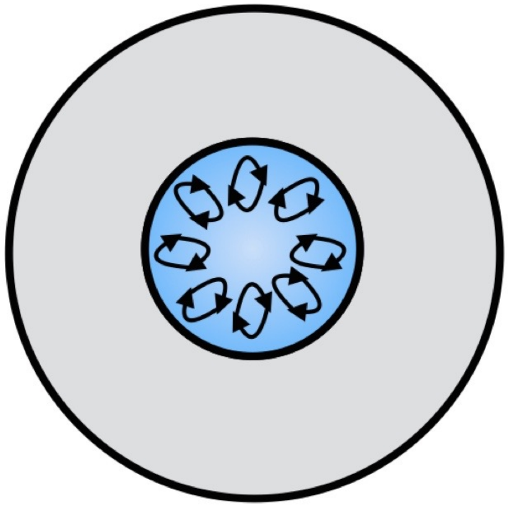
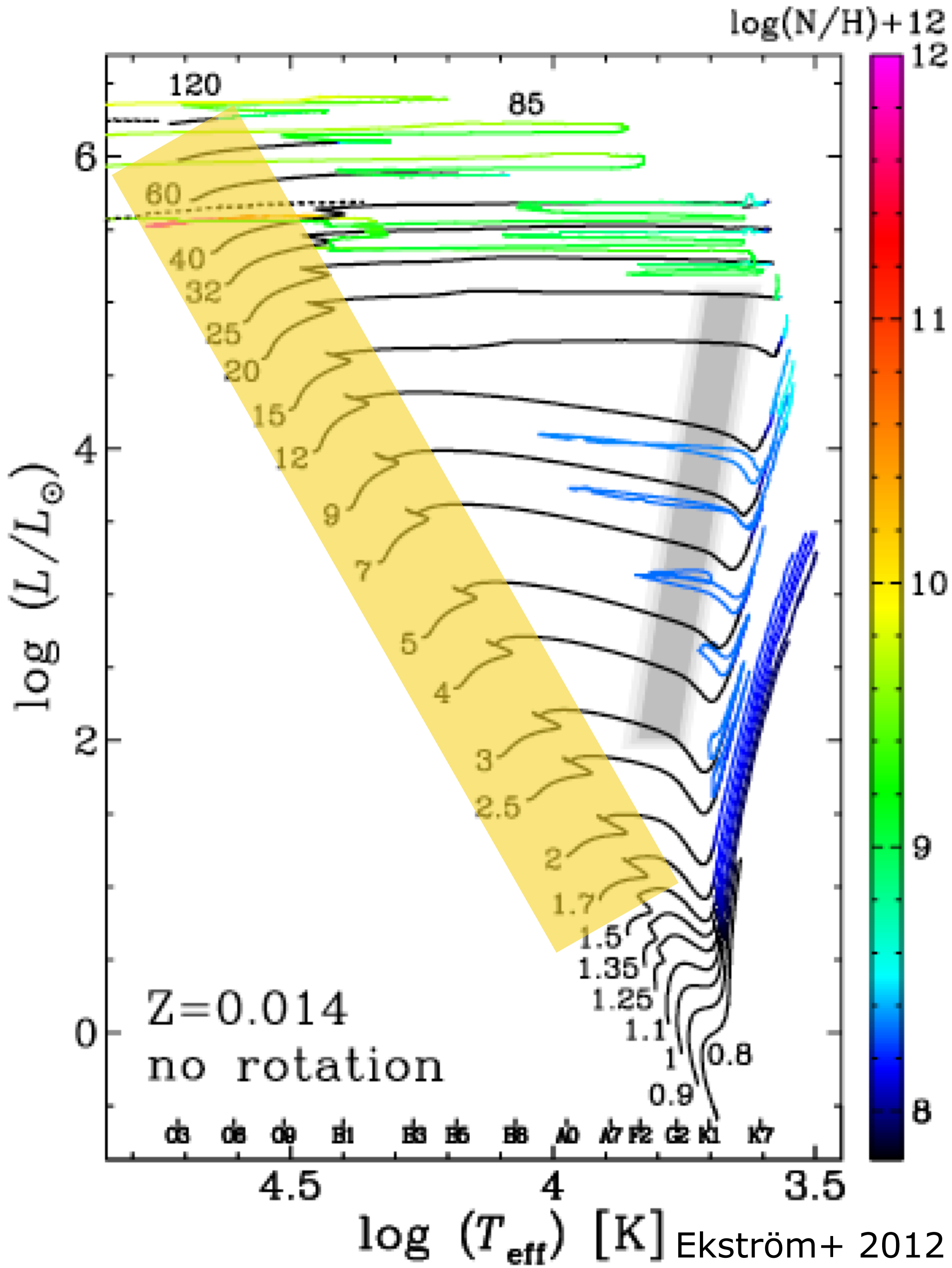
   —cores of massive stars





Ekström+ 2012

Convection in Stars

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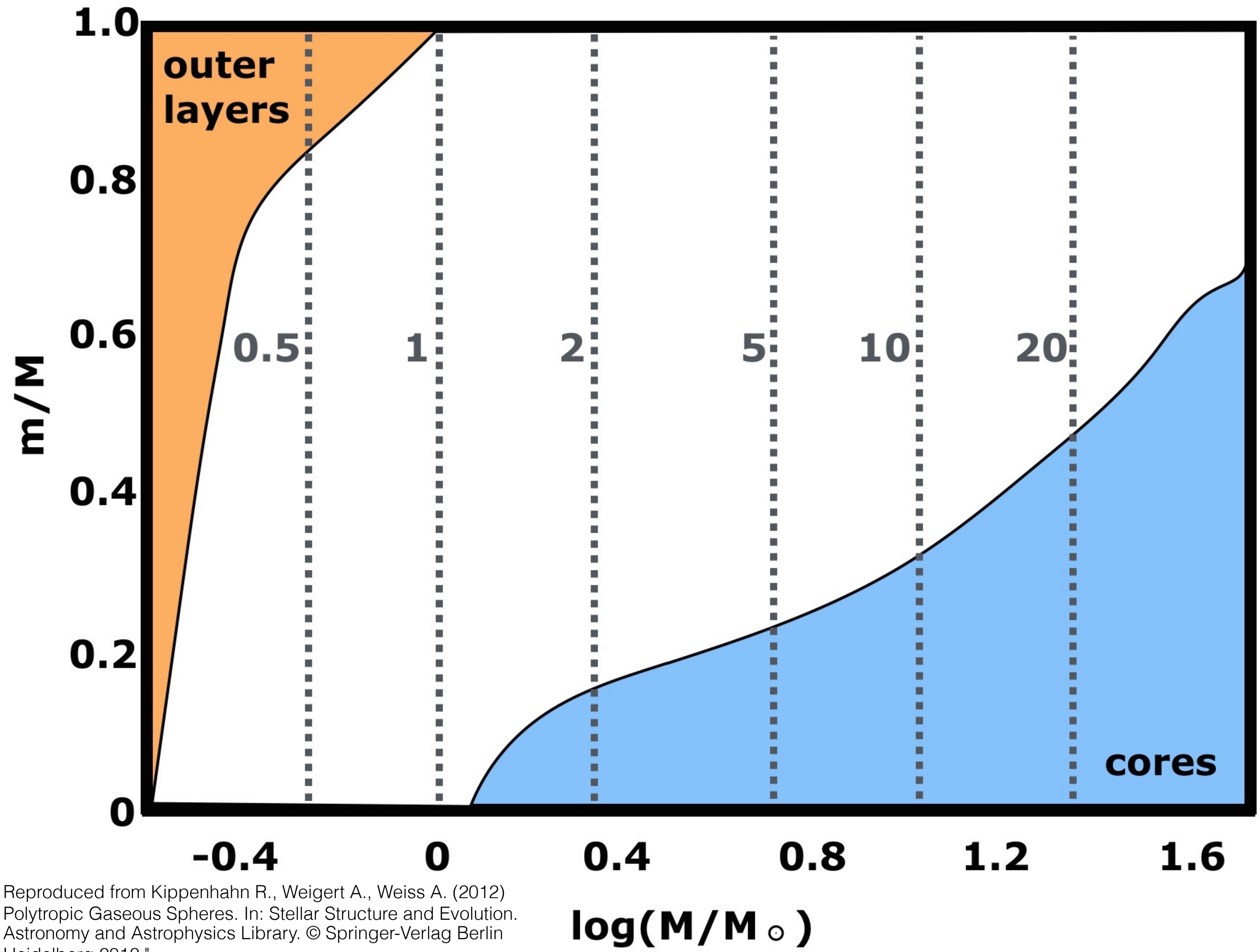

Reproduced from Kippenhahn R., Weigert A., Weiss A. (2012) Polytropic Gaseous Spheres. In: Stellar Structure and Evolution. Astronomy and Astrophysics Library. © Springer-Verlag Berlin Heidelberg 2012."

log (T_{eff}) [K] Ekström+ 2012

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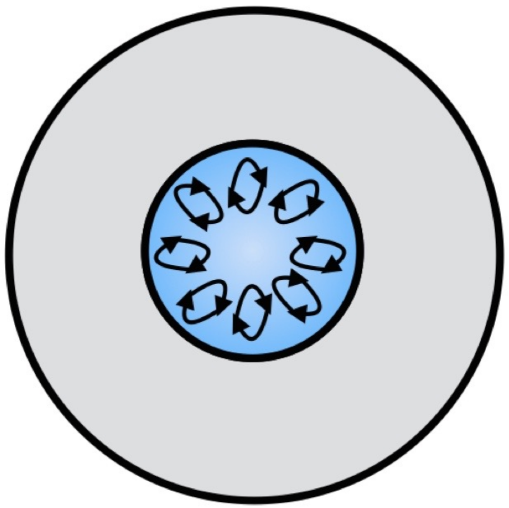
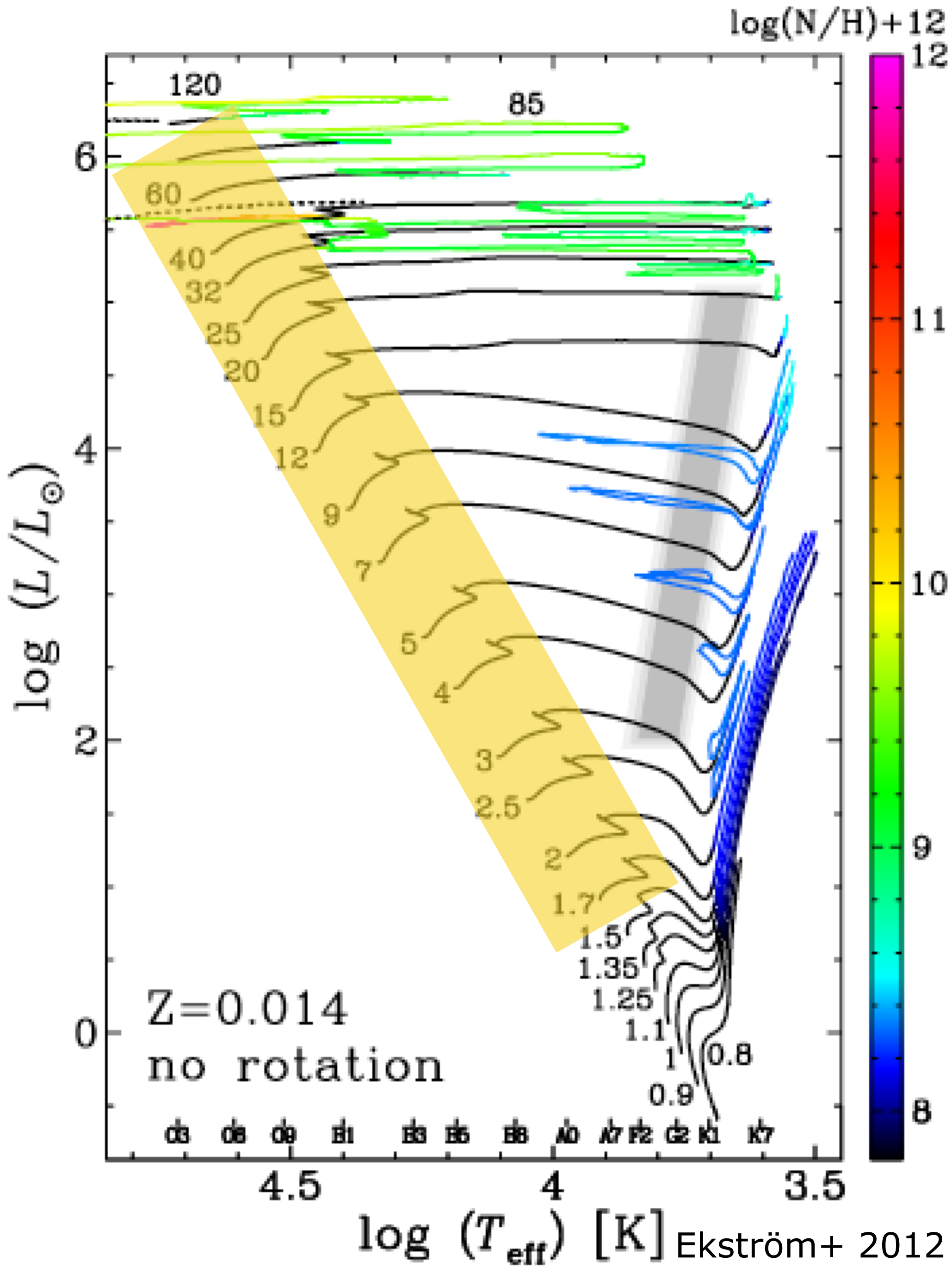




Ekström+ 2012

Convective Energy Transport

(how much can be transferred, T gradient needed,...)

To address in detail how convective energy transport works we need detailed and complicated 3D models

However, we can get an approximate results with the 1D *mixing length theory (MLT)*

In *MLT* convective motions are approximated to blobs that move a distance l_m, before they dissolve and transfer heat

l_m can be approx. to $$l_m \sim H_P = \left| \frac{dr}{d\ln P} \right| = \frac{P}{\rho g}$$

(local P scale height: radial distance over which P changes by an *e*-folding factor)

After moving l_m, the T difference between blob and media is

$$\Delta T = T_e - T_s = \left[\left(\frac{dT}{dr} \right)_e - \frac{dT}{dr} \right] l_m$$

Convective Energy Flux

$$\Delta T = T_e - T_s = \left[\left(\frac{dT}{dr} \right)_e - \frac{dT}{dr} \right] l_m$$

We can write ΔT in terms of ∇ and ∇_{ad}

$$\left(\frac{dT}{dr} \right)_e = -\frac{T}{H_P} \nabla_{\mathrm{ad}}$$

$$\frac{dT}{dr} = T \frac{d \ln T}{dr} = T \overbrace{\frac{d \ln T}{d \ln P}}^{\nabla} \overbrace{\frac{d \ln P}{dr}}^{-1/H_P} = -\frac{T}{H_P} \nabla$$

$$\Delta T = T \frac{l_m}{H_P} (\nabla - \nabla_{\mathrm{ad}})$$

And the energy flux carried by the convection gas

$$F_{\mathrm{conv}} = v_c \rho \Delta u = v_c \rho c_P \Delta T$$

we therefore need a way to calculate the convective velocity

buoyancy force by a $\Delta\rho$

$$a = -g \frac{\Delta \rho}{\rho} \sim g \frac{\Delta T}{T}$$

$$P \propto \rho T$$

$$\Delta P = 0$$

for an ideal gas

Accelerated over a distance $l_m = \frac{1}{2} a t^2$

$$v_c \approx l_m / t = \sqrt{\frac{1}{2} l_m a} \approx \sqrt{\frac{1}{2} l_m g \frac{\Delta T}{T}} \approx \sqrt{\frac{l_m^2 g}{2 H_P} (\nabla - \nabla_{\mathrm{ad}})}$$

Convective Energy Flux

Combined with the previous expression

$$F_{\text{conv}} = \rho c_P T \left(\frac{l_m}{H_P}\right)^2 \sqrt{\frac{1}{2} g H_P} (\nabla - \nabla_{\text{ad}})^{3/2}$$

Where is superadiabaticity

The superadiabaticity needed to carry the whole Flux by convection

$F_{\text{conv}} = \frac{l}{4\pi r^2} \sim \frac{L}{R^2}$ for an ideal gas $\quad \rho \approx \bar{\rho} = \frac{3M}{4\pi R^3} \quad T \approx \bar{T} \sim \frac{\mu}{\Re}\frac{GM}{R} \quad c_P = \frac{5}{2}\frac{\Re}{\mu} \quad \sqrt{gH_P} = \sqrt{\frac{P}{\rho}} = \sqrt{\frac{\Re}{\mu}T} \sim \sqrt{\frac{GM}{R}}$

$$F_{\text{conv}} \sim \frac{M}{R^3}\left(\frac{GM}{R}\right)^{3/2} (\nabla - \nabla_{\text{ad}})^{3/2} \quad \rightarrow \quad \nabla - \nabla_{\text{ad}} \sim \left(\frac{LR}{M}\right)^{2/3} \frac{R}{GM}$$

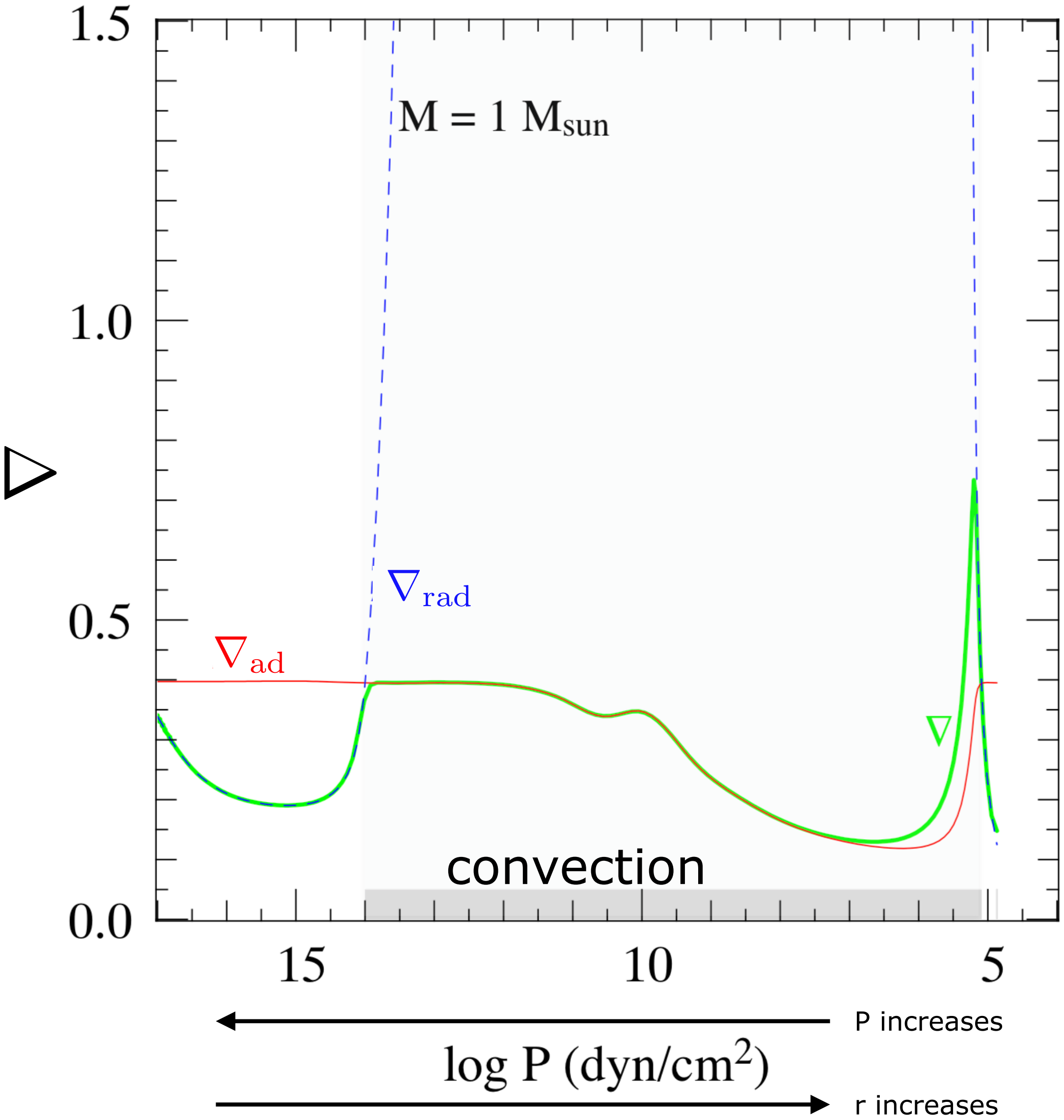
For the Sun, $\nabla - \nabla_{\text{ad}} \sim 10^{-8}$, a small difference translates in a huge amount of energy transfer. *Convection is highly efficient.*

$F_{\text{conv}} \gg F_{\text{rad}}$ in convective regions

Convective Energy Flux

In a convective zone $\nabla \sim \nabla_{\rm ad}$ therefore $\frac{dT}{dm} = -\frac{Gm}{4\pi r^4}\frac{T}{P}\nabla$

eq. of structure in convection



However, at the surface

$$\rho \ll \bar{\rho} \qquad T \ll \bar{T}$$

And $\nabla > \nabla_{\rm ad}$, so convection becomes inefficient, so

$$F_{\rm conv} \ll F{\rm rad}$$

And $\nabla \approx \nabla_{\rm rad}$

Convection mixing

Convection is also an efficient mixing mechanism

From the expression of v_c and approximating

$$v_c \approx \sqrt{\frac{l_m^2 g}{2H_P}(\nabla - \nabla_{\rm ad})} \approx v_s \sqrt{(\nabla - \nabla_{\rm ad})}$$

$$\sqrt{gH_P} \approx v_s$$
$$l_m \sim H_P$$

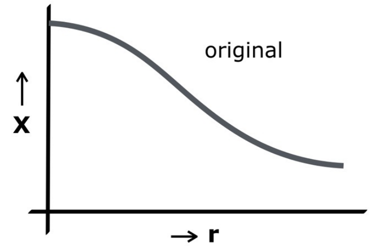
So in general, $v_c \ll v_s$

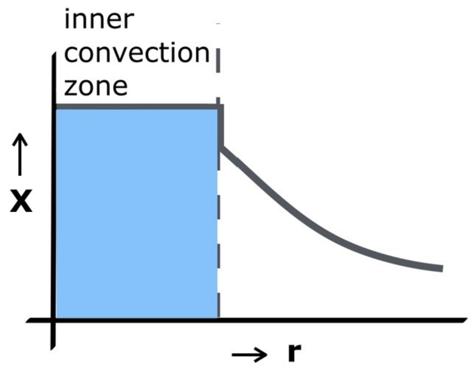
(e.g. for the Sun $v_c \sim 5\ 10^3$ cm/s)

but high enough to mix different layers in a timescale

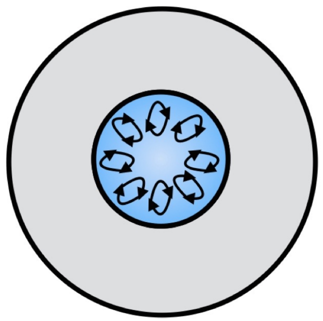
$$\tau_{\rm mix} \ll \tau_{\rm KH} \ll \tau_{\rm nuc} \ll \tau_{\rm life}$$

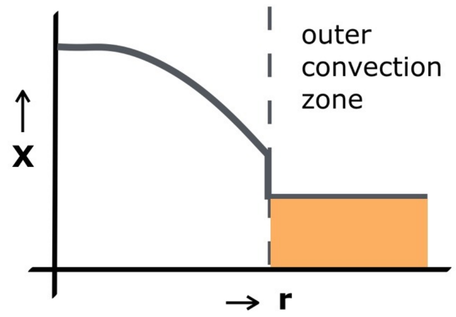
consequences:



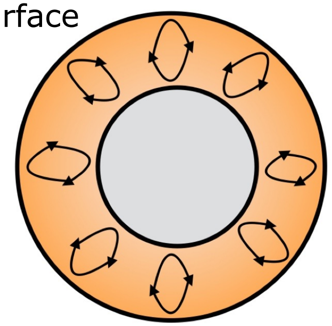


Mixing in massive star cores extends their MS lifetimes by transporting ashes upwards and fuel downwards





Mixing in stars with large convective *outer* layers causes "dredge-up" transporting ashes to the surface



Convective overshooting

At the boundary between radiative and convective zones (Schwarzschild criterion, $\nabla_{\rm rad} = \nabla_{\rm ad}$) the acceleration due to buoyancy vanishes.

However, the blob has certain inercia and will *overshoot* by some distance

In principle this distance is $l_{ov} << H_P$, so Schwarzschild criterion would still stand

But as it also carry heat and mix, it makes $|\nabla - \nabla_{\rm ad}|$ to decrease, and creates a positive feedback loop

As a consequence, overshooting introduces a large uncertainty in the extent of mixed regions

Chapter 9

Usually it is parametrized as $d_{\rm ov} = \alpha_{\rm ov} H_P$
where $\alpha_{\rm ov}$ is calibrated against observations