Demostración PCA. Para compuesta.

$$\min_{w} \mathcal{E}_x \{ \|x_n - \hat{x}_n\|_2^2 = \mathcal{E}_x \{ \langle x_n - z_n w^T, x_n - z_n w^T \rangle \}$$

$$= \mathcal{E}_x \{ x_n x_n^T - 2x_n (z_n w^T) + z_n w^T (z_n w^T)^T \}; \quad x_n \in \mathbb{R}^{1\times p}, \; w \in \mathbb{R}^{p\times 1}$$

$$= \mathcal{E}_x \{ x_n x_n^T - 2x_n w w^T x_n^T + x_n w w^T w w^T x_n^T \}; \quad \underline{w^T w = 1}$$

$$= \mathcal{E}_x \{ x_n x_n^T - 2x_n w w^T x_n^T + x_n w w^T x_n^T \}$$

$$= \mathcal{E} \{ x_n x_n^T - x_n w w^T x_n^T \}$$

$$\min_{w} \mathcal{E}_x \{ \|x_n - \hat{x}_n\|_2^2 \} = \min_{w} - \mathcal{E}_x \{ x_n w w^T x_n^T \}$$

restricción $\swarrow$ s.t. $w^T w = 1$

$z_n = x_n w$

$z_n \in \mathbb{R}$.

$$\min_{w} - \mathcal{E}_x \{ z_n z_n^T \} = - \mathcal{E}_x \{ z_n^T z_n \}$$

$$= - \mathcal{E}_x \{ w^T x_n^T x_n w \} = - w^T \mathcal{E} \{ x_n^T x_n \} w; \quad \mathcal{E}_x \{ x_n^T x_n \}$$

$$\min_{w} - w^T \Sigma_x w = \max_{w} w^T \Sigma_x w \qquad R_{xx} \in \mathbb{R}^{p\times p}$$

s.t. $w^T w = 1$ $\qquad$ s.t. $w^T w = 1$

$z_n = x_n w$.

$$\mathcal{L}(w, \lambda) = w^T \Sigma_x w - \lambda (w^T w - 1); \quad \frac{d\mathcal{L}}{dw} = 2 \Sigma_x w - 2\lambda w^T w = 0$$

$\boxed{\Sigma_x w = \lambda w} \rightarrow e{:}y$

$$\boxed{\begin{aligned} & w^T \Sigma_x w = \lambda w^T w = \lambda \\ & \mathcal{E} \{ w^T x_n^T x_n w \} = \mathcal{E} \{ z_n^T z_n \} = \sigma_z^2 = \lambda \end{aligned}}$$