

4.6

- 6) There is no greatest neg real number
 There is a greatest neg real number
 Let a be the greatest neg real number
 $a < 0$ $a \geq x$ $0 < \frac{1}{2} < 1$ $0 > \frac{a}{2} > a$

- 4) Prove by contradiction

The product of any nonzero rational number and any irrational number is irrational

\exists a rational number r , with $r \neq 0$ and an irrational number S such that rs is rational

Proof: assume statement is false and the negation is true. Assume r is a rational number $r \neq 0$ & S is an irrational number

& rs is rational

$r = \frac{a}{b}$ and $rs = \frac{c}{d}$ then $\left(\frac{a}{b}\right)S = \frac{c}{d}$ then $\left(\frac{b}{a}\right)\left(\frac{a}{b}\right)S = \left(\frac{b}{a}\right)\frac{c}{d}$
 $S = \frac{bc}{ad}$ S has been written as a quotient of
 $ad \in \mathbb{Z}$ and is rational

- 28) For all \mathbb{Z} m and n , if mn is even then m is even and n is even

$$m = 2r + 1$$

$$n = 2s + 1$$

$$mn = (2r + 1)(2s + 1)$$

$$mn = 4rs + 2r + 2s + 1$$

$$mn = 2(2rs + r + s) + 1$$

mn is odd