7) $\begin{array}{r} 4\,4 \\ 9\overline{)70} \\ \underline{63} \\ 7 \end{array}$ $q = 7$ $r = 7$

7) A) 43 div 9 = 4

$\begin{array}{r} 4 \\ 9\overline{)43} \\ \underline{36} \\ 7 \end{array}$

B) 43 mod 9 = 7

20) a is a INT
If a mod 7 = 4 what is 5a mod 7

given a mod 7 = 4 $a = 7q + 4$ by def of mod
then $5a = 5(7q + 4)$
$5a = 35q + 20$
So 5a mod 7 = $(35q + 20) \bmod 7 = 6$

$\begin{array}{r} 5q + 2 \\ 7\overline{)35q + 20} \\ -35q \\ \hline +20 \\ -14 \\ \hline 6 \end{array}$

23) Prove that for all INT n, if n mod 5 = 3 then n^2 mod 5 = 4 Let a be particular but arbitrary INT
Then $a = 5q + 3 \Rightarrow a^2 = (5q+3)^2 \Rightarrow a^2 = (5q+3)(5q+3) \Rightarrow$
$a^2 = 25q^2 + 30q + 9 \Rightarrow a^2 = 25q^2 + 30q + 5 + 4$
$(5q^2 + 6q + 1$ is INT since the sum & product of INT are INT
$\therefore a^2 \bmod 5 = 4$