

# **Session 7: Portfolio Theory III**

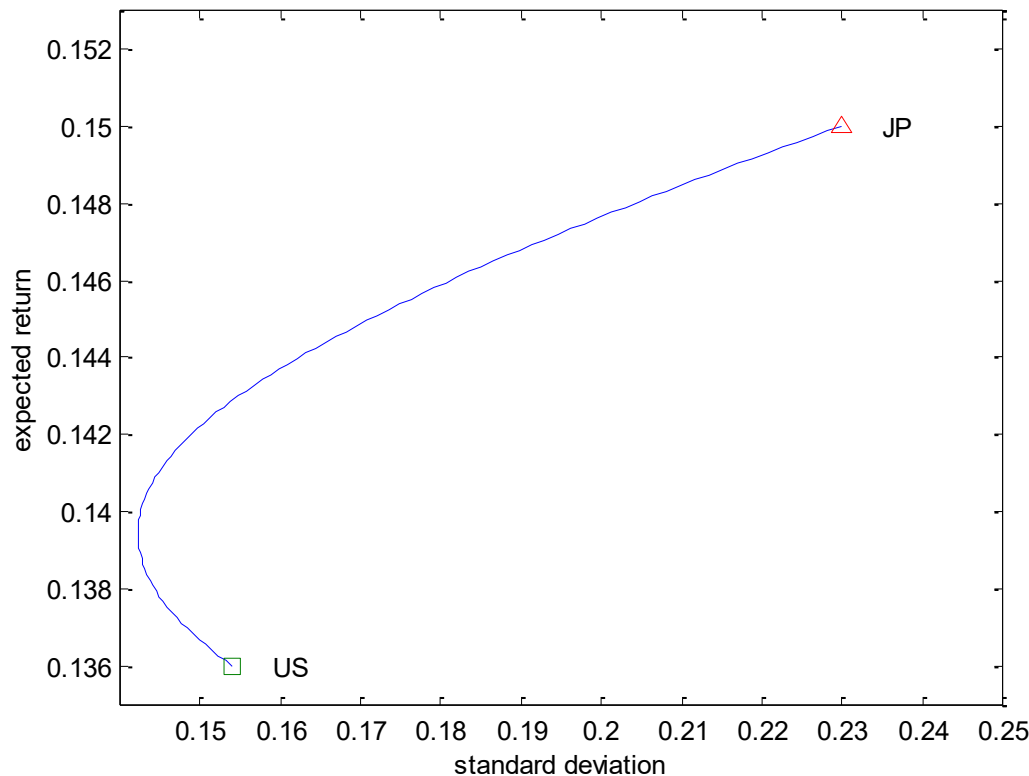
Fall 2025

# Outline

- Risk-return tradeoff
- Indifference curves
- Optimal portfolio choice



# Optimal Portfolio Choice

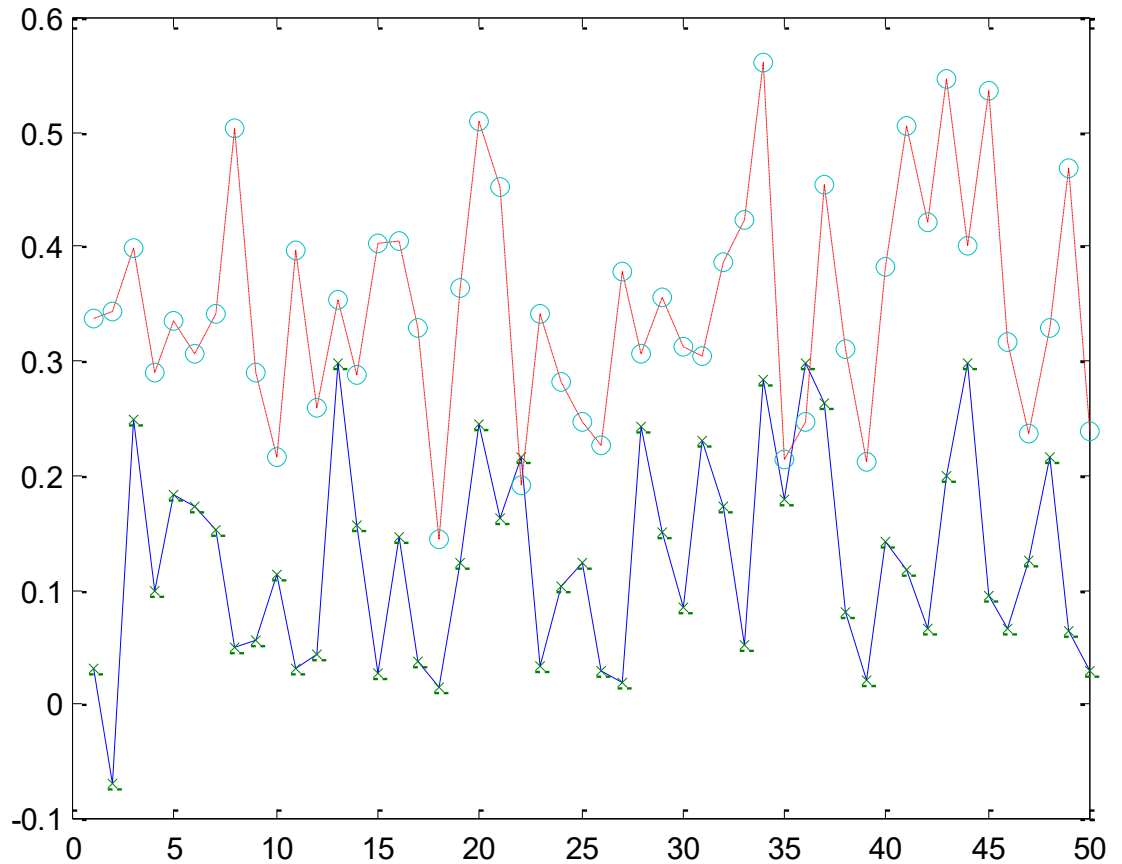
- Any (mean-variance) investor should choose an *efficient* portfolio to benefit from diversification
- Which efficient portfolio is *optimal* depends on the investor's preferences, in particular her risk aversion



# Risk-Return Tradeoff

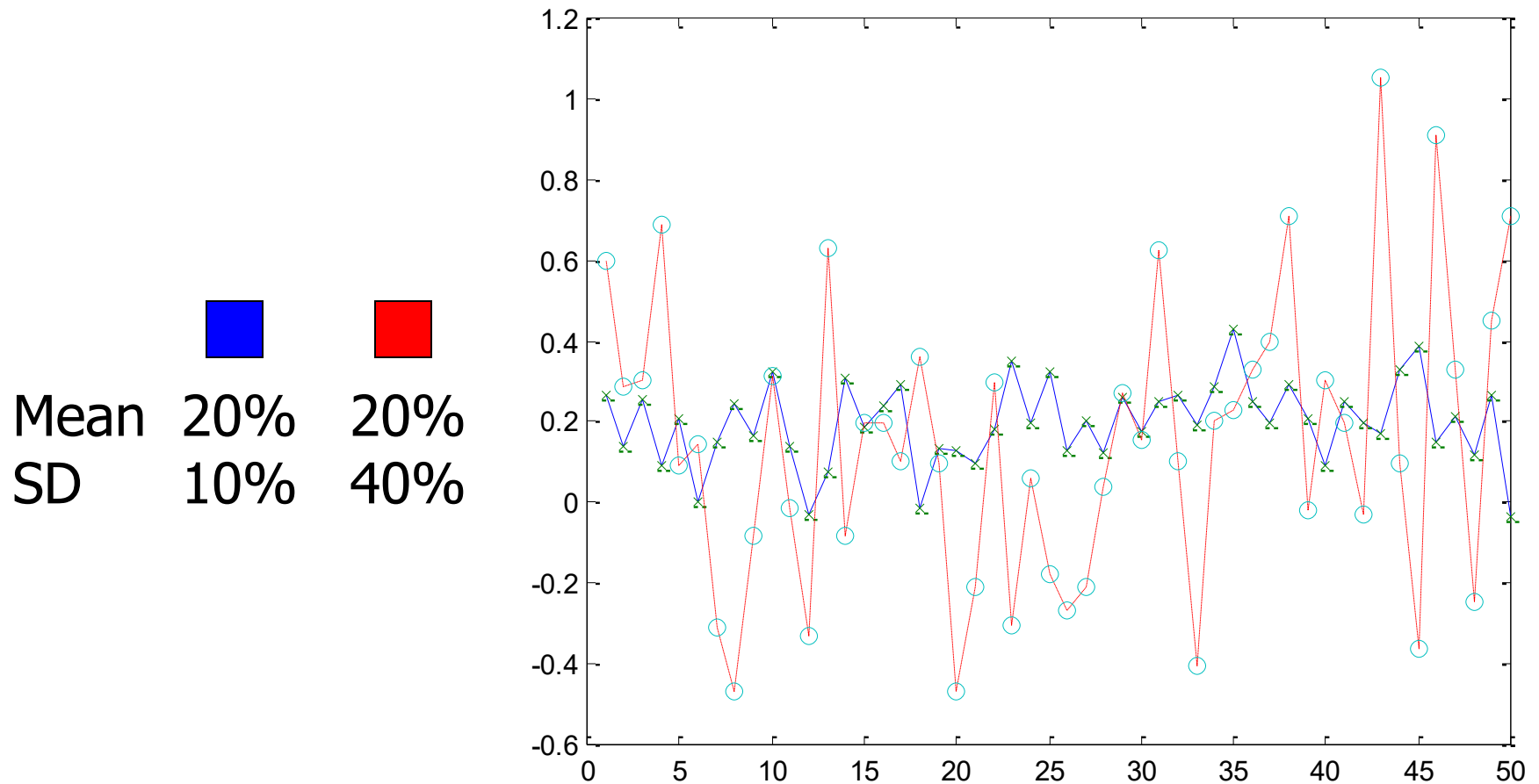
Which do you prefer?

		
Mean	15%	35%
SD	10%	10%





# Risk-Return Tradeoff

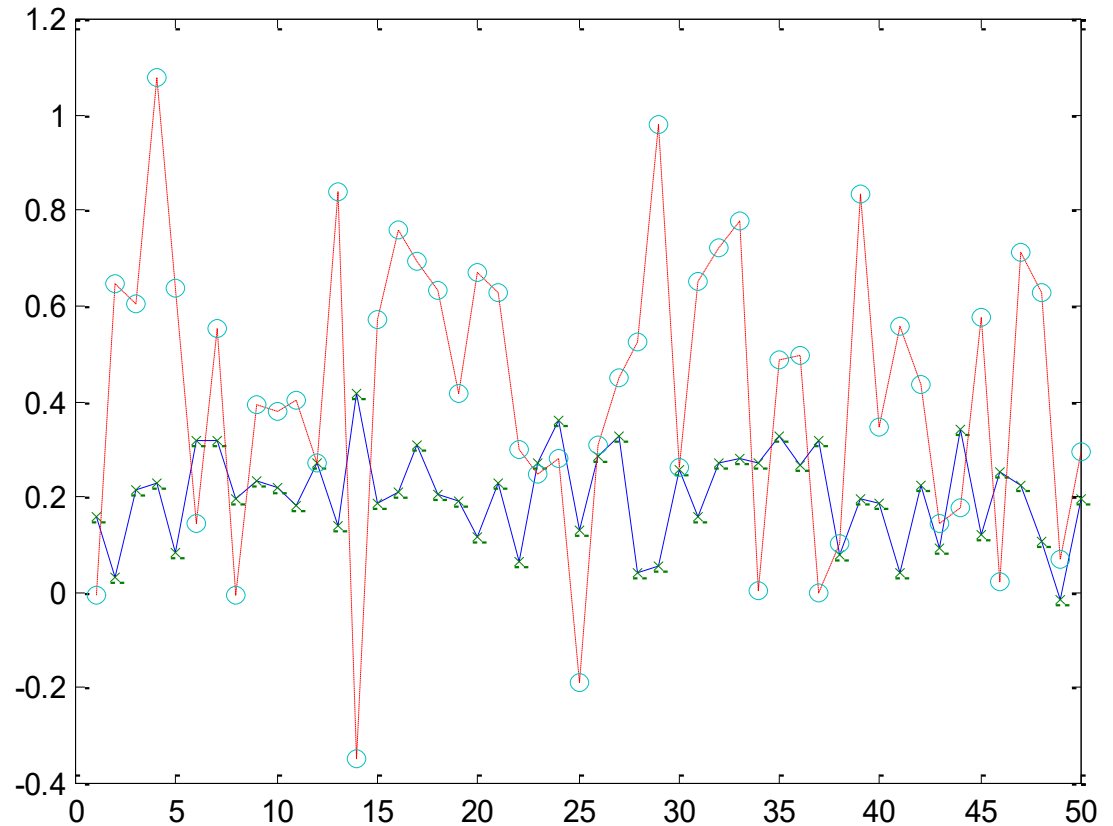
Which do you prefer?



# Risk-Return Tradeoff

Which do you prefer?

		
Mean	20%	40%
SD	10%	40%

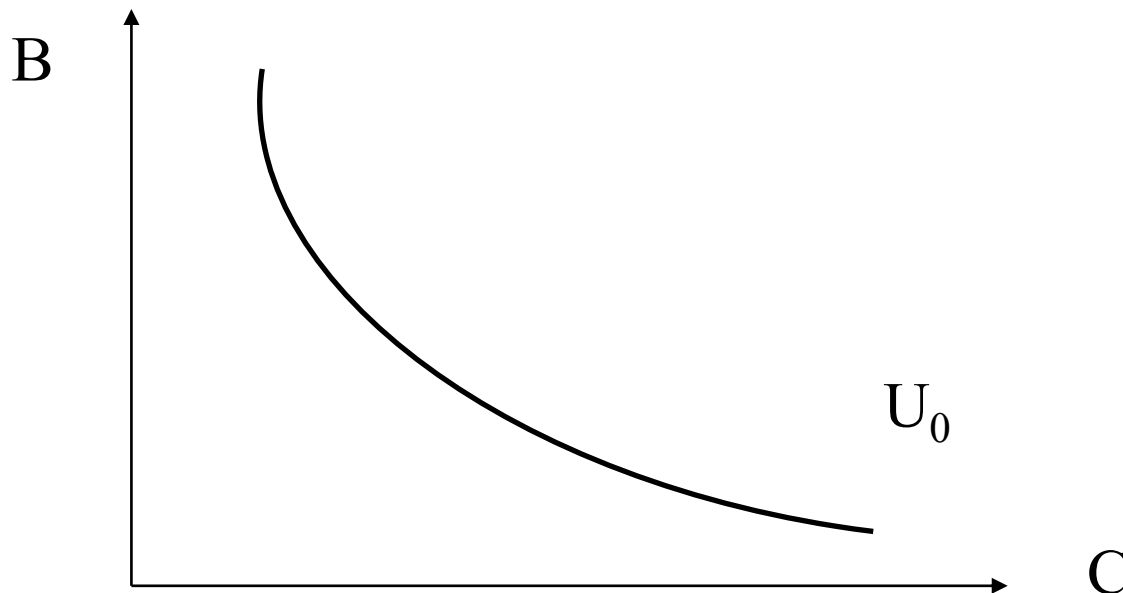


# Risk-Return Tradeoff

- Recall two of the finance axioms:
  - Investors prefer more to less
  - Investors are risk averse
- This means that investors prefer an investment
  - With a higher expected return,  $E(R_i)$
  - With a lower variance or standard deviation,  $\sigma_i$
- Investors optimally **trade off** risk and return in order to maximize their expected utility

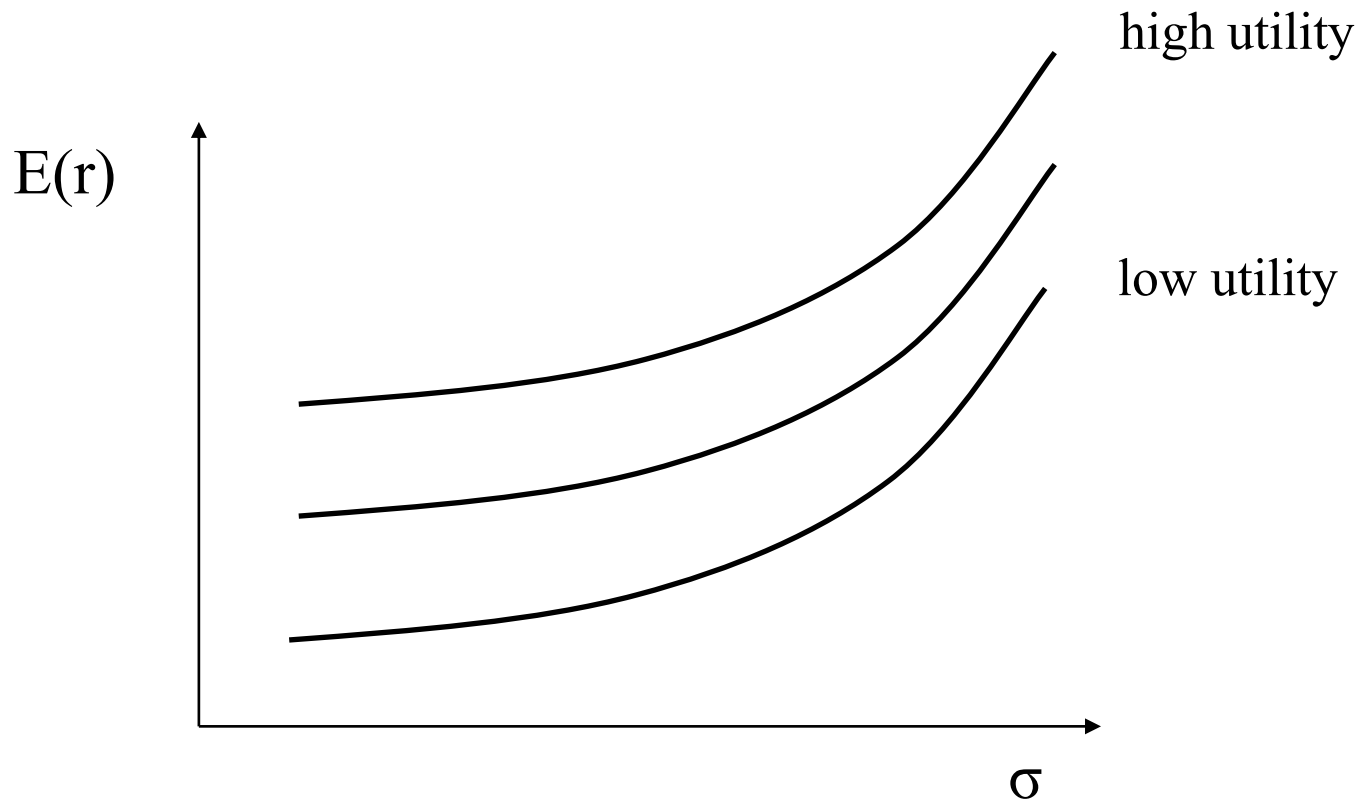
# Indifference Curves: Review

- A person consumes 2 goods: beer (B) and chocolate (C)
- An indifference curve gives all the combinations of B and C that give the same utility level  $U_0 = U(B,C)$
- People like to be on the highest possible indifference curve



# Indifference Curves in Finance

Indifference curve: A set of  $[E(r_p), \sigma_p]$  combinations that gives an investor the same expected utility



# Utility Functions

- One utility function that is easy to work with is **mean-variance** utility:

$$U(r_p) = E(r_p) - 0.5A \text{var}(r_p)$$

- A portfolio with higher utility score  $U$  has a more attractive risk-return profile
- Parameter  $A > 0$  measures **risk aversion**
- If  $A = 0$ , investor is risk-neutral (does not care about risk, only about expected return)

# Which is the “Better” Portfolio?

- Investor 1 has  $A=2$ , investor 2 has  $A=5$
- Consider the following 2 investments:

Asset	Expected return	Standard Deviation
Low risk stock L	7%	5%
High risk stock H	13%	20%

- Calculate the utility scores:

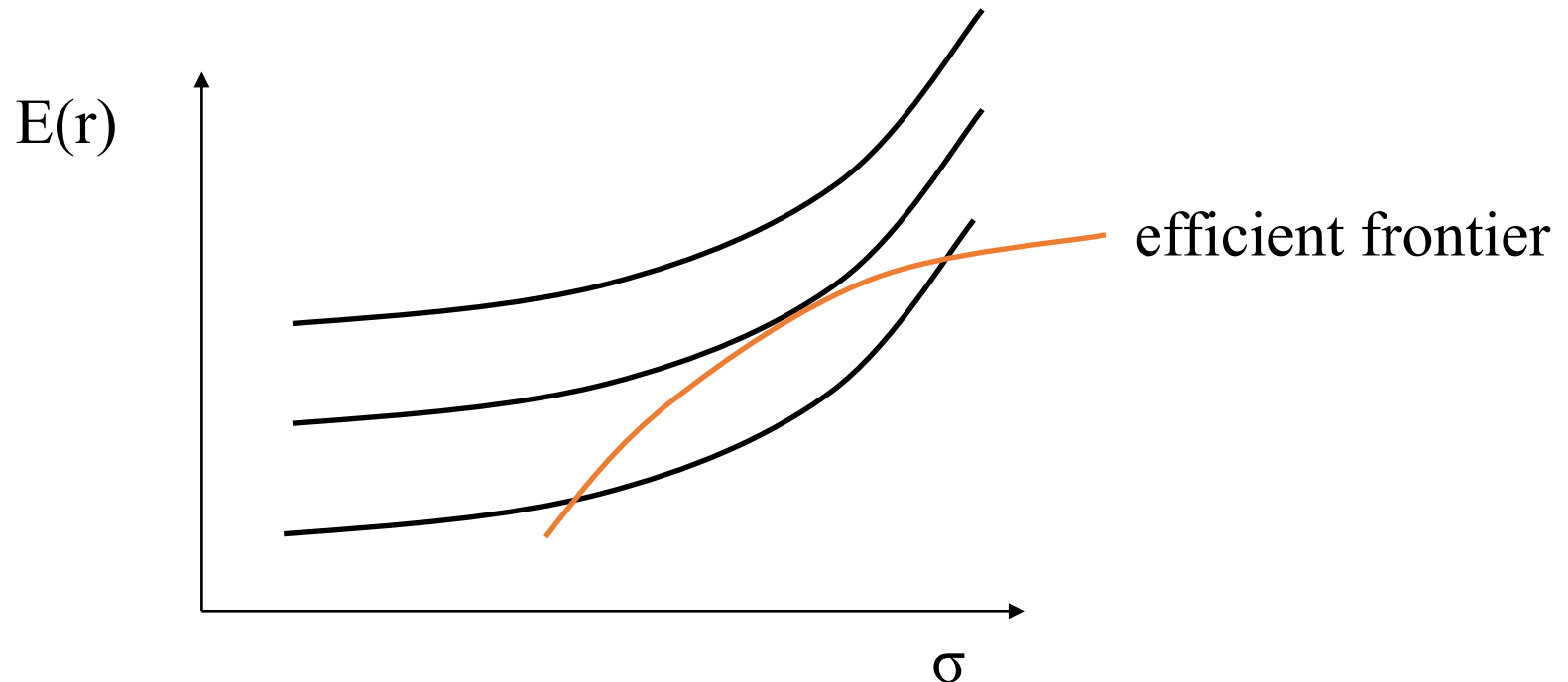
Investor	Investment L	Investment H
Investor 1 ( $A=2$ )	$U =$	$U =$
Investor 2 ( $A=5$ )	$U =$	$U =$

# Putting It All Together

An **optimal** portfolio reconciles what is  
**desirable** – described by the *indifference* or utility *curves*

with what is

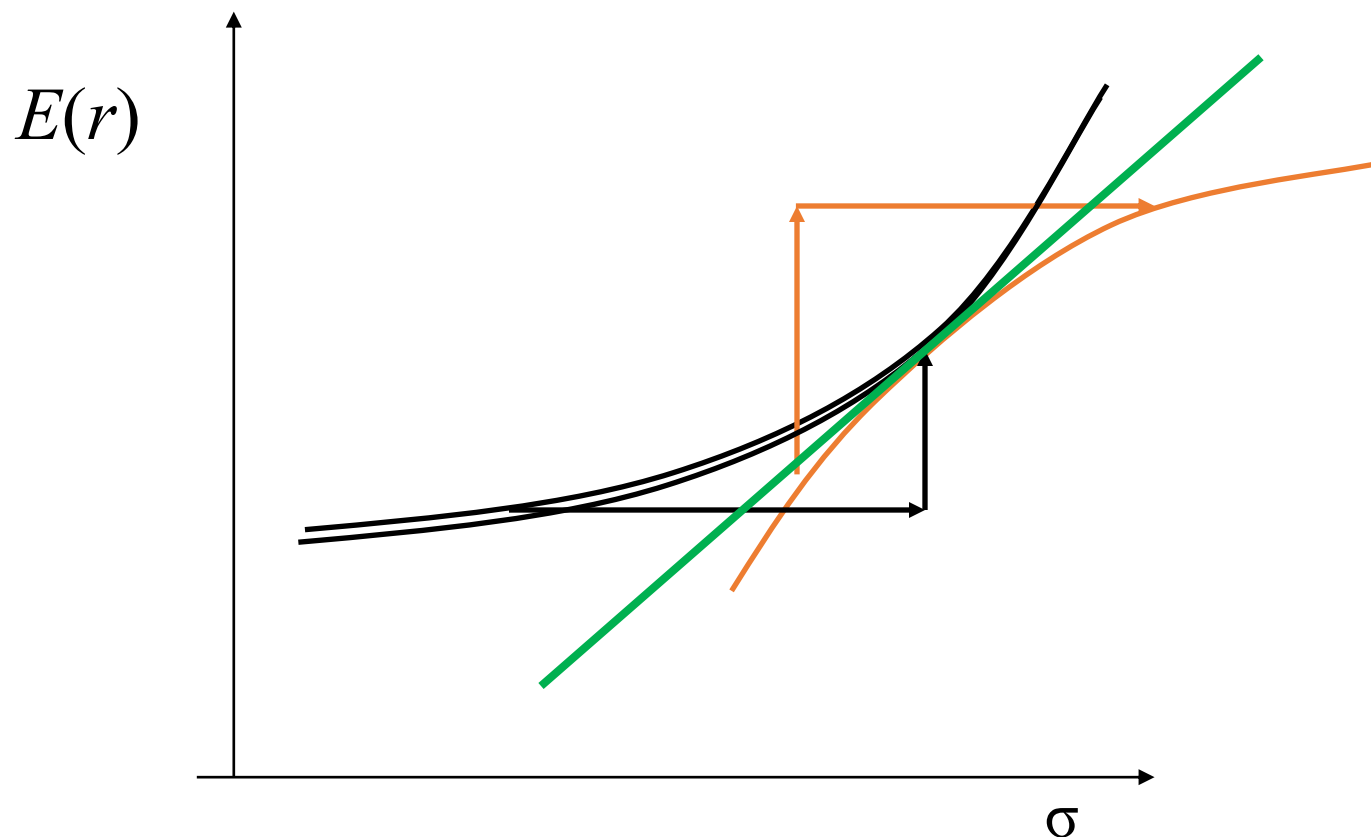
**feasible** – described by the upper portion of the investment  
opportunity set, the *efficient frontier*



# Optimal Portfolio

The **marginal rate of substitution** is how much investors need to be compensated in terms of higher utility for taking on more risk.

The **marginal rate of transformation** is how much more return the economy can generate when more risk is undertaken.



# Conclusion

- Everyone should hold a portfolio on the efficient frontier
- But the location depends on risk aversion

# Assignments

## ➤ Reading

➤ BKM: Chapters 5.4-5.6, 6.3

➤ Problems: 5.12-5.14, 5.17, 6.3-6.4, 6.14