

# On the inversion of sub-mesoscale information to correct mesoscale velocity

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*May 4, 2011*



# Prerequisites

Altimetry

Data Assimilation

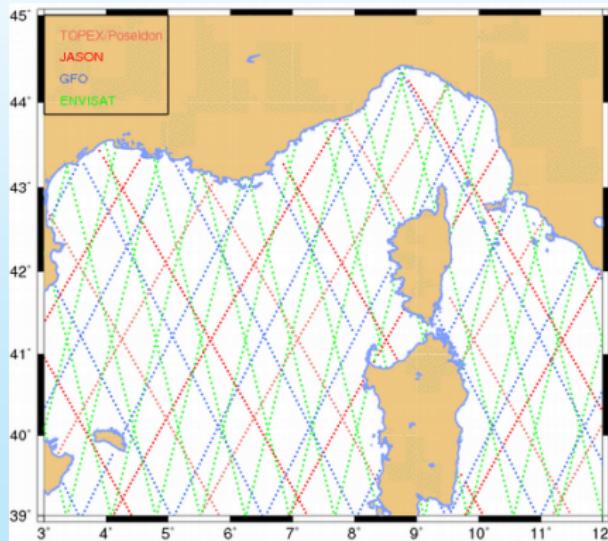
Sub-mesoscale

Tracer

# Prerequisites

## Altimetry

- Altimetry :
- Measure of sea surface height along track
  - Geostrophic velocity derived from ssh gradients
  - Data: e.g. AVISO (interpolated maps, velocity, ssh)



# Prerequisites

## Altimetry

## Data Assimilation

Data Assimilation aims at finding an optimal compromise between information of different natures, space and time sampling.

The sources are generally some observations (satellite, in-situ) and a numerical model.

## Sub-mesoscale

## Tracer

# Prerequisites

Altimetry

Data Assimilation

Sub-mesoscale

Sub-mesoscale: intermediate scale between Mesoscale and dissipative scales.  
Energetic and Dynamic importance have been recently brought to light  
(Capet & al, 2008, Thomas & al, 2008, Klein & al, 2008)

Tracer

# Prerequisites

Altimetry

Data Assimilation

Sub-mesoscale

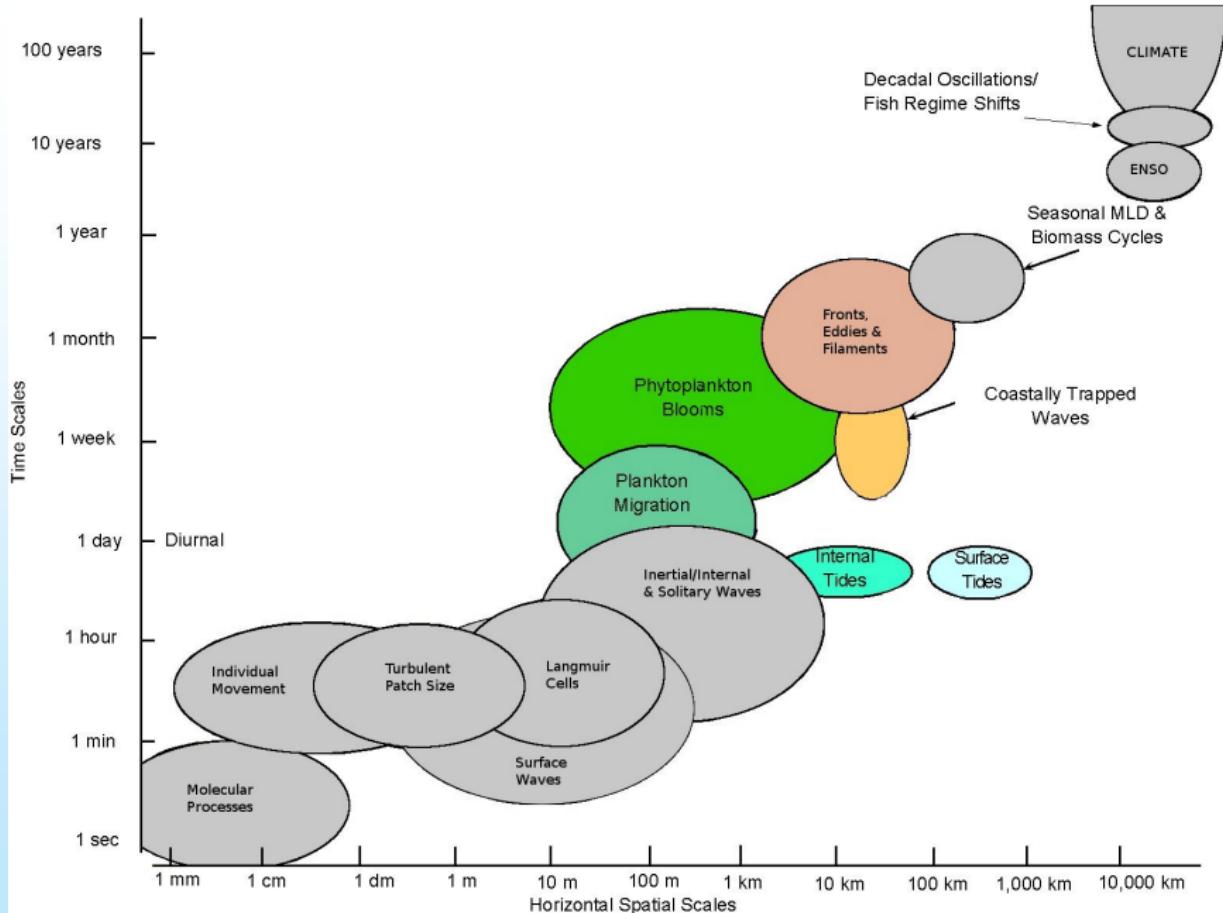
Tracer

We are interested in tracers visible from space:

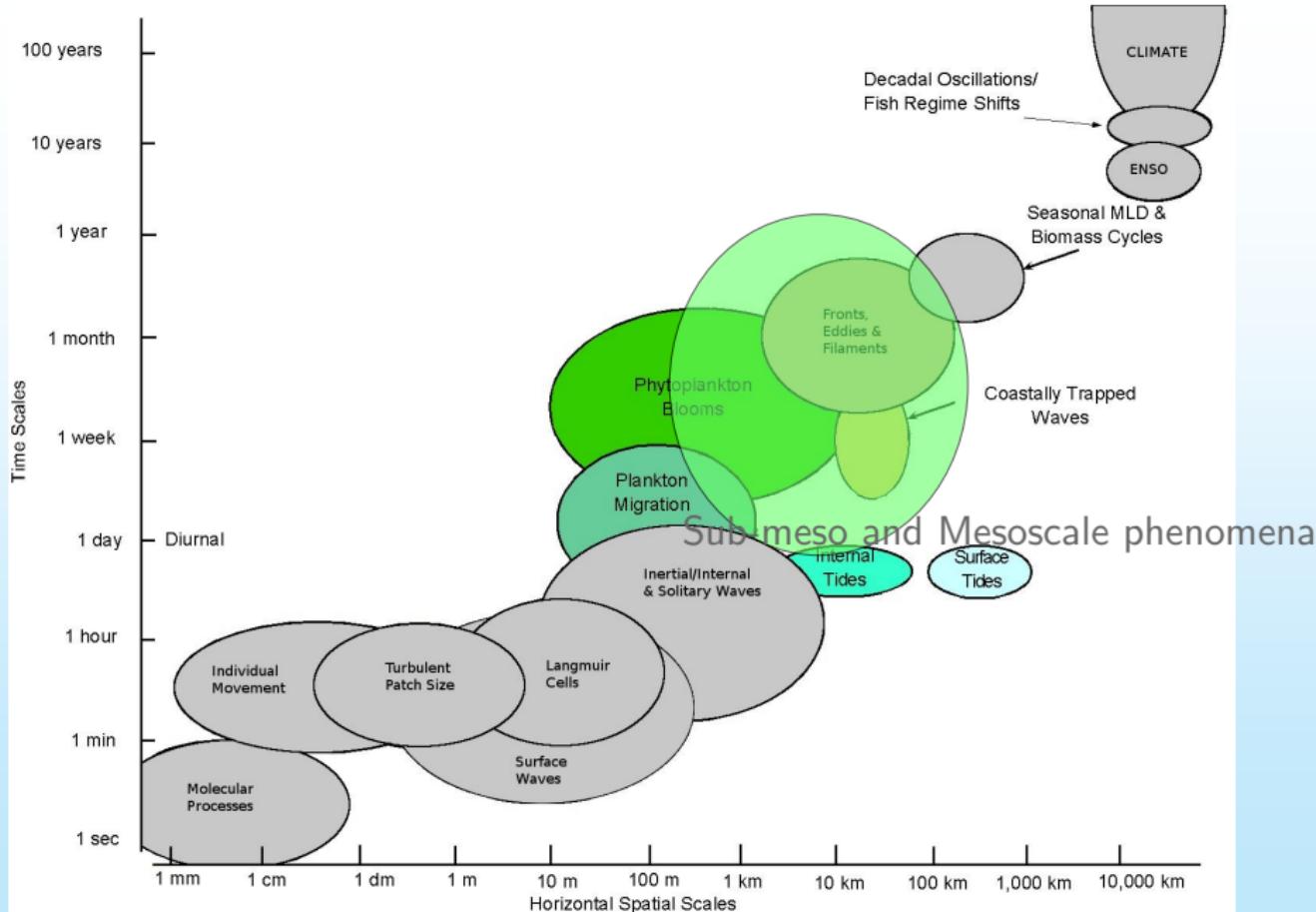
Sea Surface Temperature

Ocean Color: Chlorophyll

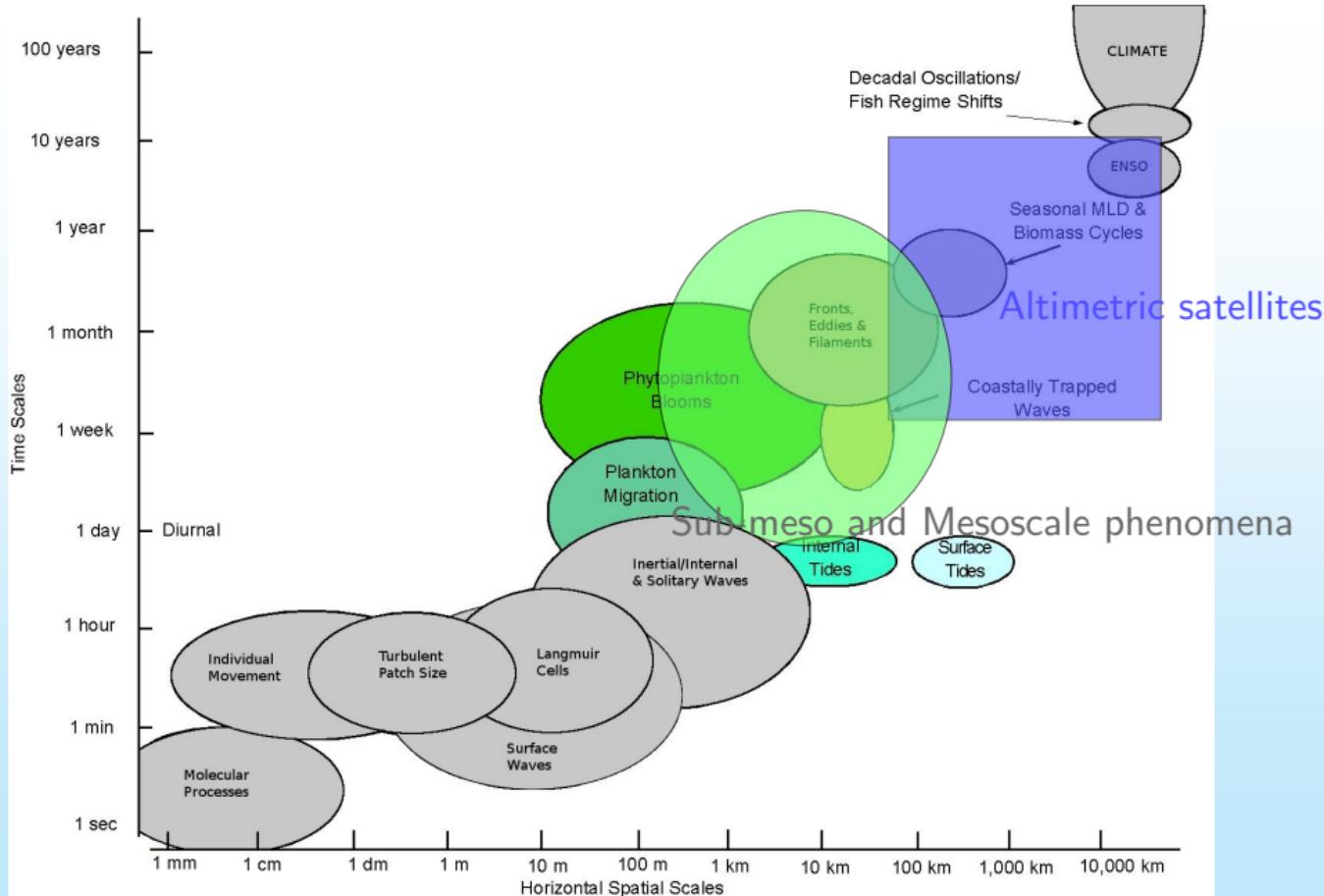
# Mesoscale and sub-mesoscale dynamics



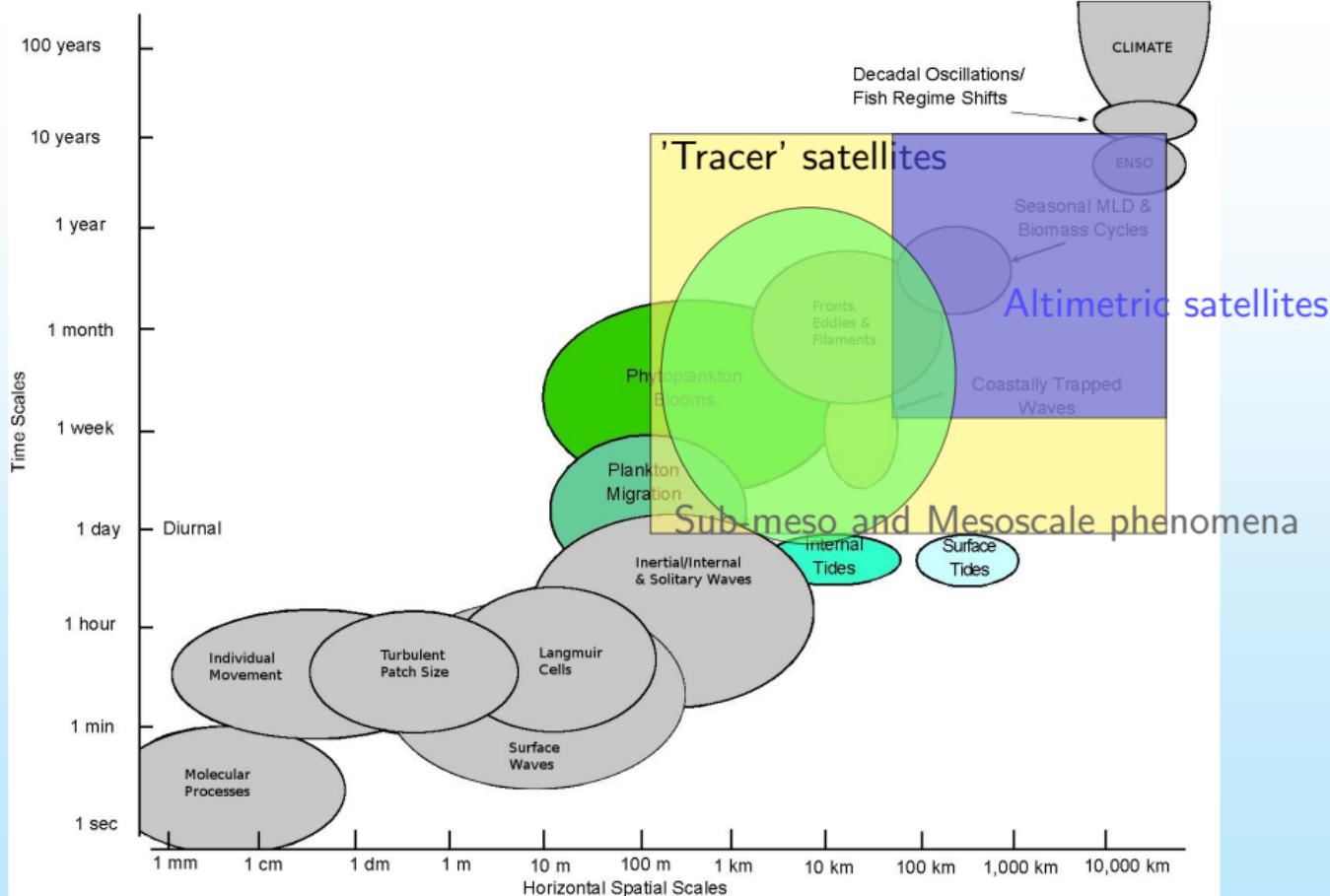
# Mesoscale and sub-mesoscale dynamics



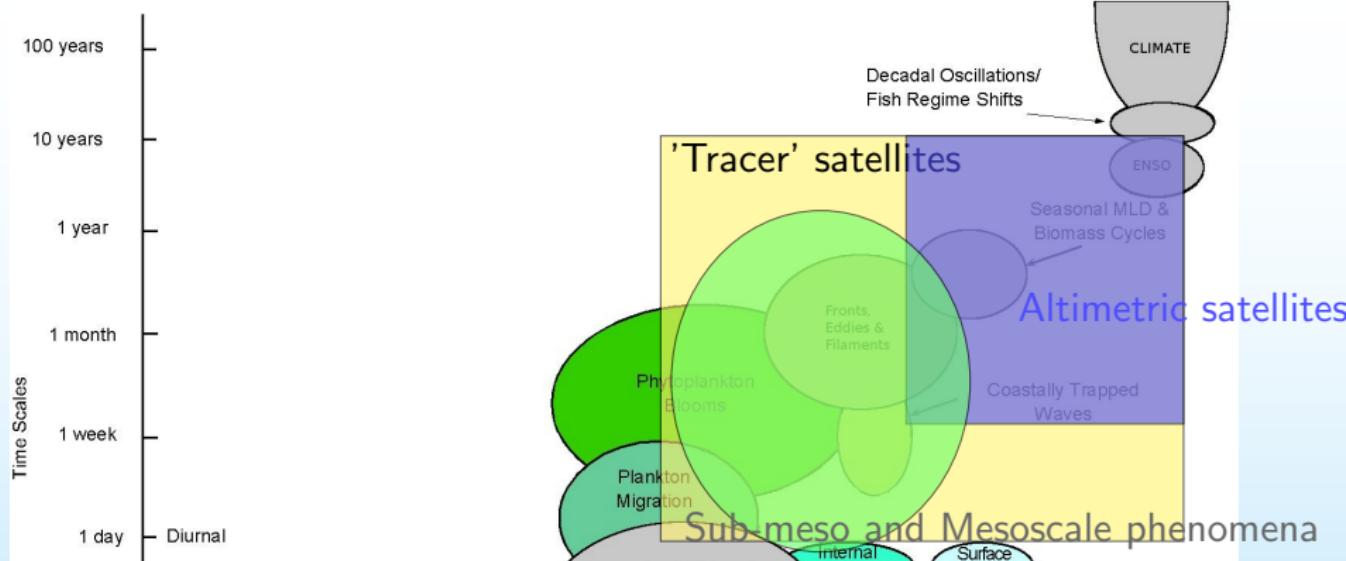
# Mesoscale and sub-mesoscale dynamics



# Mesoscale and sub-mesoscale dynamics

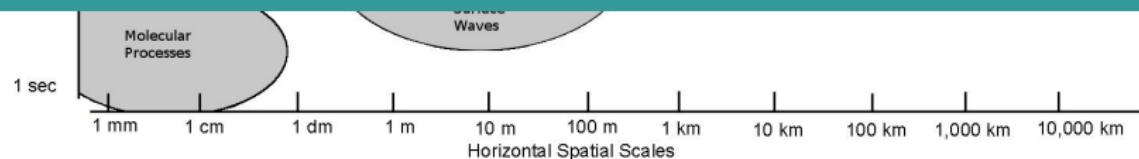


# Mesoscale and sub-mesoscale dynamics



## Sub-sampling of altimetry: use of Biogeochemistry data

SWOT, Altika/SARAL project: High resolution altimetric satellites, a need to plan the use of this huge amount of data



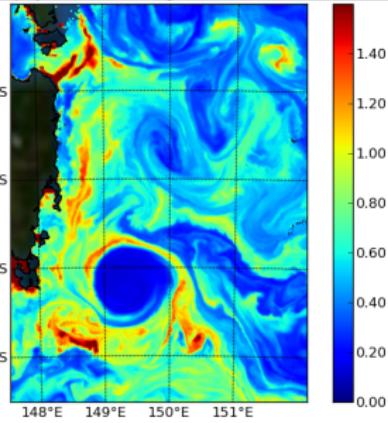
# Outline

- 1 Philosophy of the study
- 2 Methodology of the inversion
- 3 Test Case
- 4 Conclusion

# Philosophy of the study

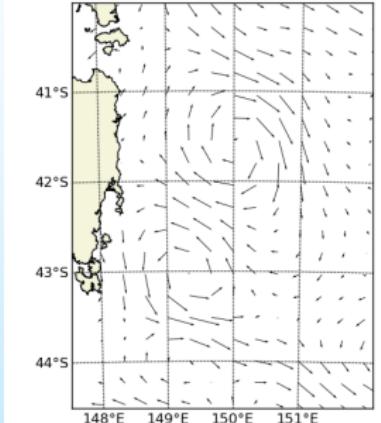
## Sub-mesoscale tracer image

Chlorophyll, Tasmania region, 24 December 2004



## Mesoscale field

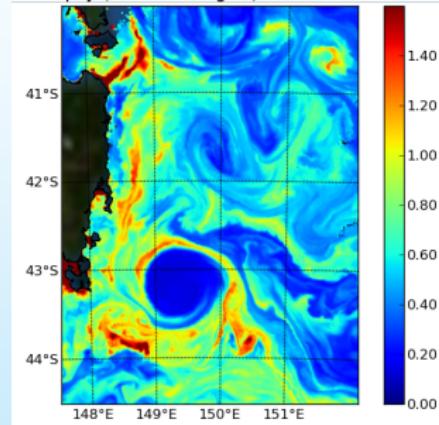
Velocity map in tasmania on day 20079



# Philosophy of the study

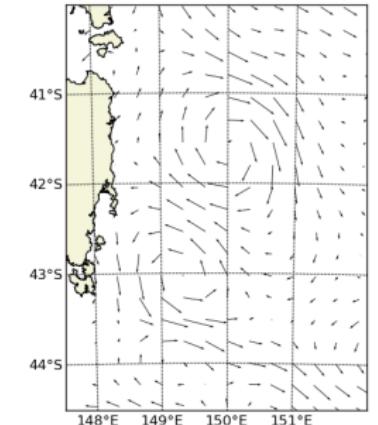
## Sub-mesoscale tracer image

Chlorophyll, Tasmania region, 24 December 2004



## Mesoscale field

Velocity map in tasmania on day 20079



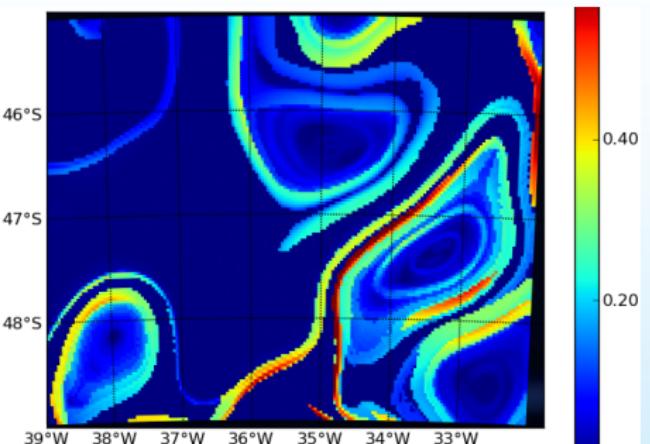
The inversion of sub-mesoscale tracer information to correct mesoscale velocity has never been done before



Find the correction of this background the most compatible with tracer information

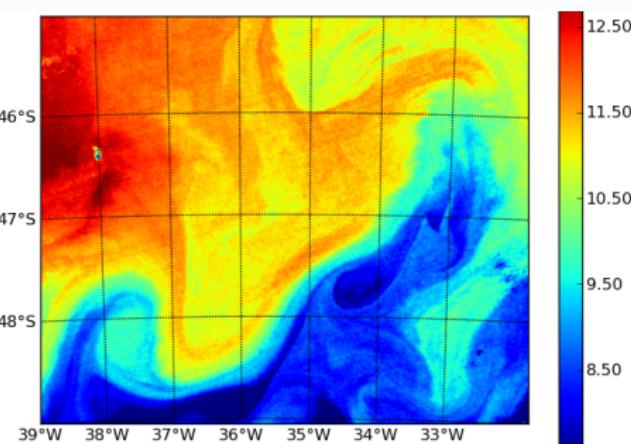
- The direct measure of the distance between  $\vec{u}$  and **Tracer** is not possible
- Need to find a go-between variable
- Use of Finite-Size Lyapunov Exponents as a proxy (FSLE)

# Are Lyapunov exponents a reliable proxy/image?



*FSLE, South Atlantic region,*

*December 27, 2006*



*Tracer (SST), South Atlantic region,*

*December 27, 2006*

Lyapunov measures stirring in a fluid

→ Link between sub-mesoscale dynamics and biologic stirring.  
(Lehahn & al, 2008, d'Ovidio & al, 2004)

# Methodology

- Cost function:

$$J(u) = \|\lambda(u) - \lambda_{obs}\| + \text{background term}$$

The cost function is strongly non linear, with many local minima.

- Explore sub-space of errors to find the velocity that minimizes the cost function.

Velocity panel using Principal Component Analysis with all velocity fields available:

$$\mathbf{u}_k = \bar{\mathbf{u}} + \sum_{i=0}^n \underbrace{a_k^i}_{\text{Eigenvalue}} \underbrace{\mathbf{u}^i}_{\text{EOF}}$$

The number of degrees of freedom is reduced, using only 100 or less EOFs.

# An exploratory study

- **Step 1:** Is FSLE the right proxy for this study?

Inversion of synthetic sub-mesoscale images to larger scale ocean circulation (twin experiment approach)

- **Step 2:** Link real information with sub-mesoscale proxy

Inversion of sub-mesoscale tracer to larger scale ocean circulation

# Choice of a Study area

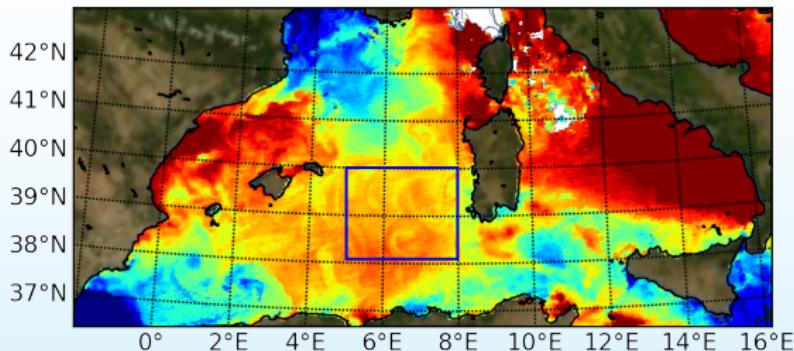
## Required by a tracer

- Low cloud cover: Visible and Near IR wavelengths do not go through clouds
- Strong filament signature

## Needed by FSLE

- Being far from any coast : Computing problems with particules advection in the presence of land
- Being far from any upwelling or downwelling

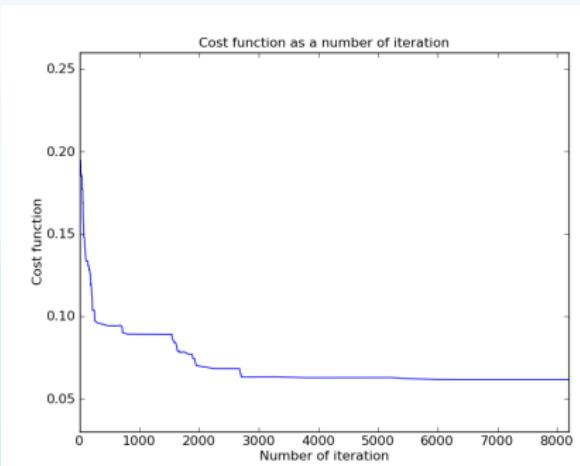
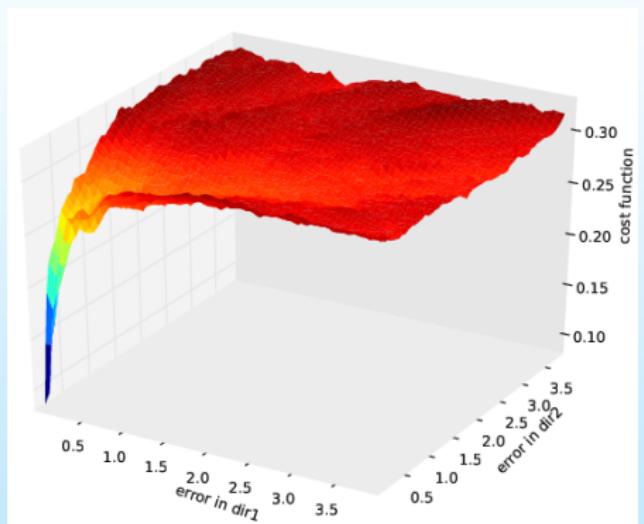
# Test case : Mediterranean Sea



- **Time Range:** from 1998 to June 2009, 595 velocity maps
- **Velocity fields:** AVISO, altimetric data
- **Resolution:**  $1/8^\circ$ , grid points: 26\*17
- **FSLE Resolution:**  $1/48^\circ$ , grid points: 119\*86
- **Tracer field:** SST data from MODIS captor, L2 product
- **Resolution needed to detect filament:**  $1/100^\circ$

# Study of the cost function: Inversion of FSLE

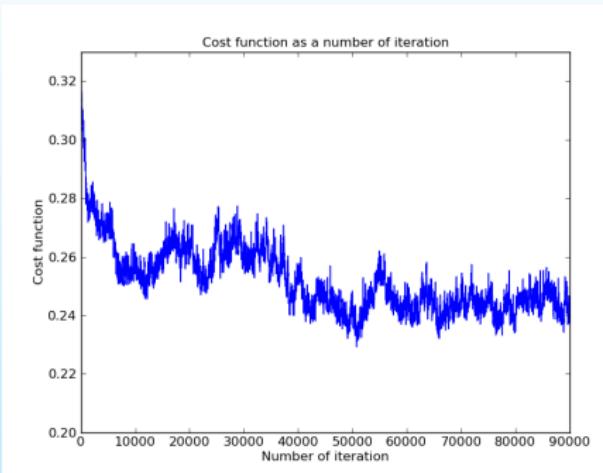
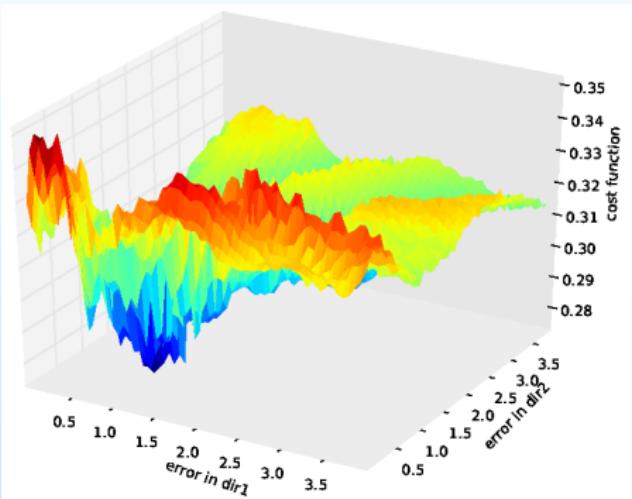
## STEP 1



Cost function:  $J(u) = \|\lambda(u) - \lambda_{obs}\| + \text{background term}$

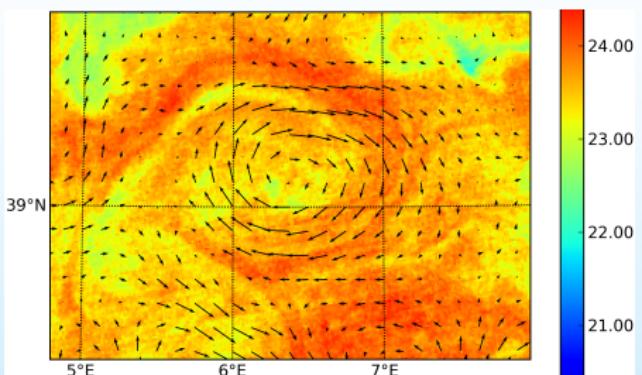
# Study of the cost function: Full inversion

## STEP 2

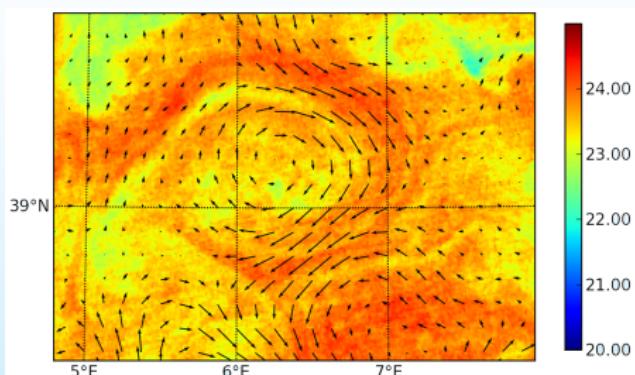


Cost function:  $J(u) = \|\lambda(u) - \lambda_{obs}\| + \text{background term}$

# Results: correction on velocity



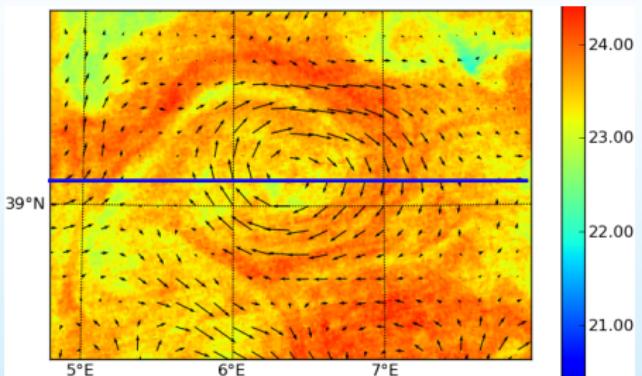
*Aviso velocity and Tracer (SST), cost  
function: 0.33*



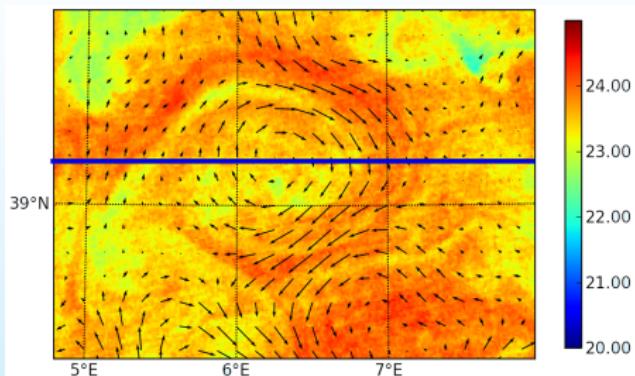
*Corrected velocity and Tracer (SST), cost  
function: 0.23*



# Results: correction on velocity



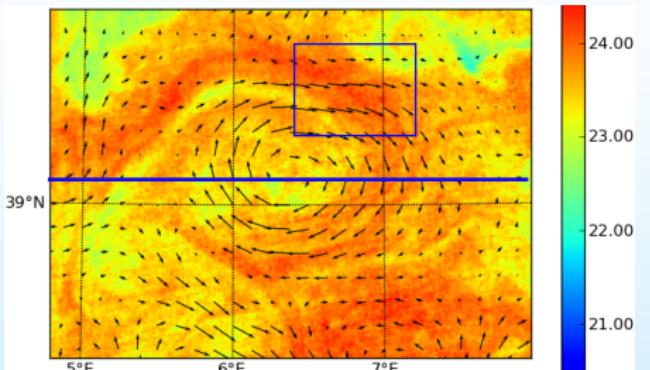
*Aviso velocity and Tracer (SST), cost function: 0.33*



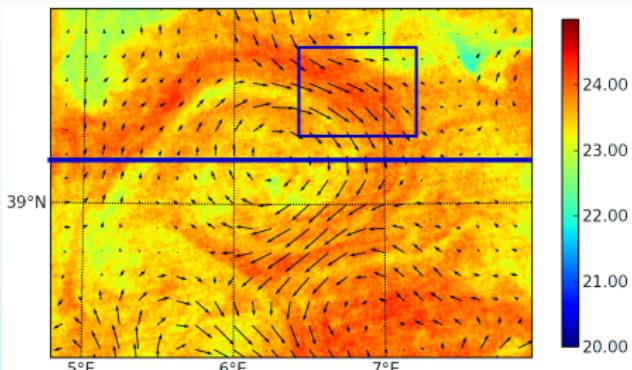
*Corrected velocity and Tracer (SST), cost function: 0.23*

- Gyre moved upward
-

# Results: correction on velocity



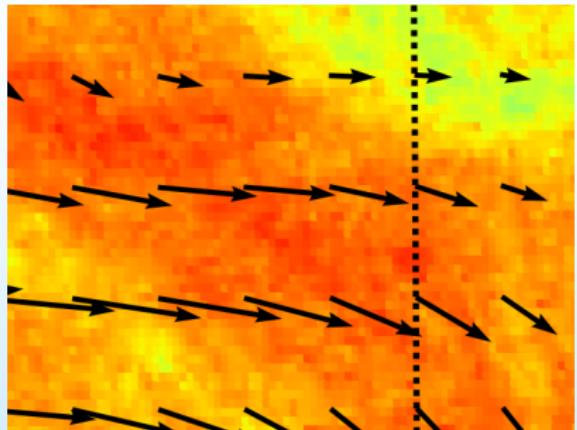
*Aviso velocity and SST,  
cost function: 0.33*



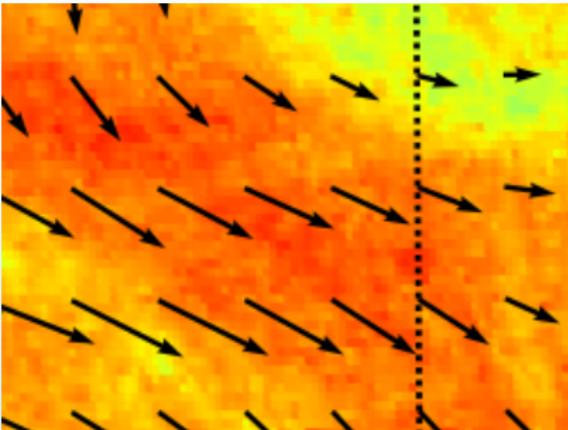
*Corrected velocity and SST,  
cost function: 0.23*

- Gyre moved upward
- Velocity strengthen in the south East of the picture
-

# Results: correction on velocity



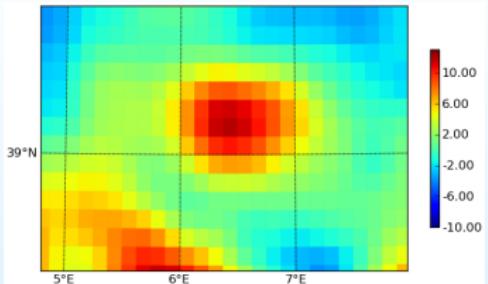
Aviso velocity and SST,  
cost function: 0.33



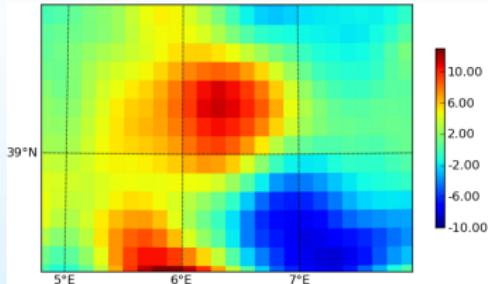
Corrected velocity and SST,  
cost function: 0.23

- Gyre moved upward
- Velocity strengthen in the south East of the picture
- Velocity does not cross frontal structure anymore

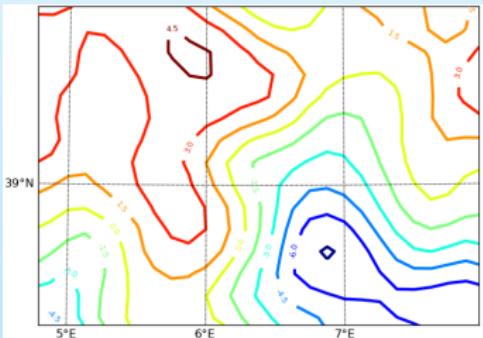
# Correction to the ssh



Aviso SSH

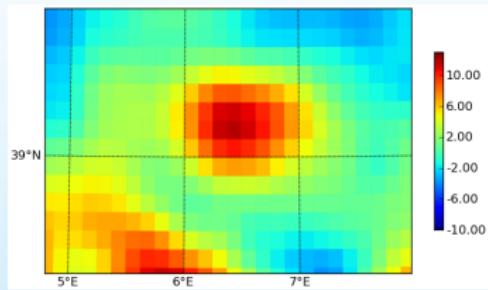


Corrected SSH

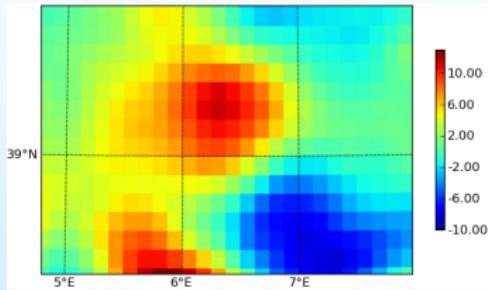


Resulting correction

# Correction to the ssh



*Aviso SSH*



*Corrected SSH*

High resolution tracer data makes the improvement of AVISO products possible.

# Conclusion

Sub-mesoscale information are invertible to control larger scales dynamics

- Altimetry and tracer observation are complementary.
- Tracer information can compensate for the lack of SSH resolution in time and space.
- High resolution Sea Surface Temperature or Ocean Color data are useful to control ocean physics.

# Conclusion

## Next

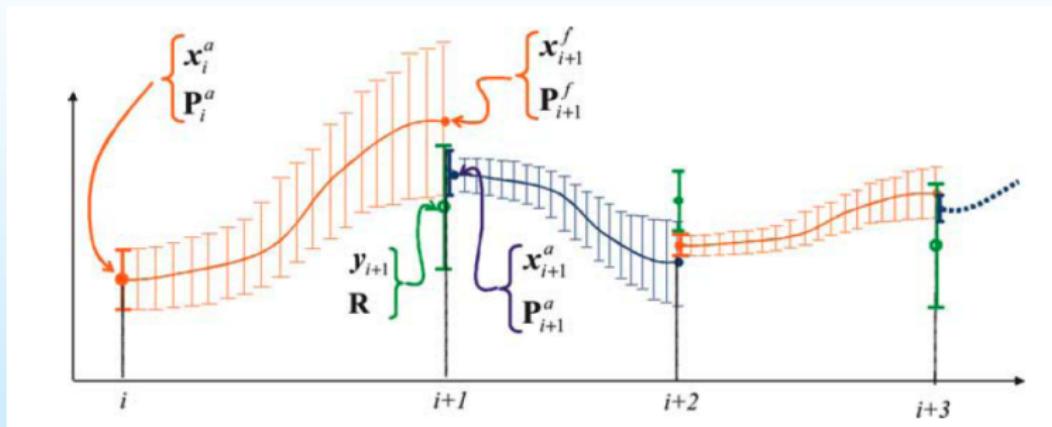
- Quantify the error made on the estimated velocity.
- Inversion of image in a coupled physico-biogeochemical model.

## Prospects

- Data Assimilation of image in a coupled physico-biogeochemical model.

Thank you for your attention

# Data Assimilation



*Conceptual representation of filtering with sequential assimilation, Brasseur, 2006*

# Sub-mesoscale

Sub-mesoscales are scales defined by a Rossby number of order one

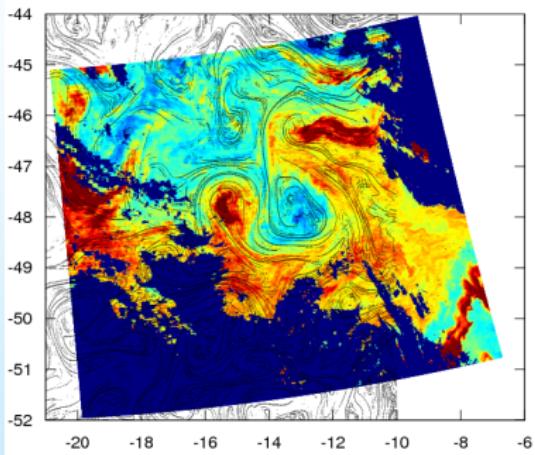
$$R_o = \frac{\text{inertial force}}{\text{Coriolis force}} = \frac{U}{fL}$$

It is characterized by ageostrophic circulation: strain dominates over rotation.

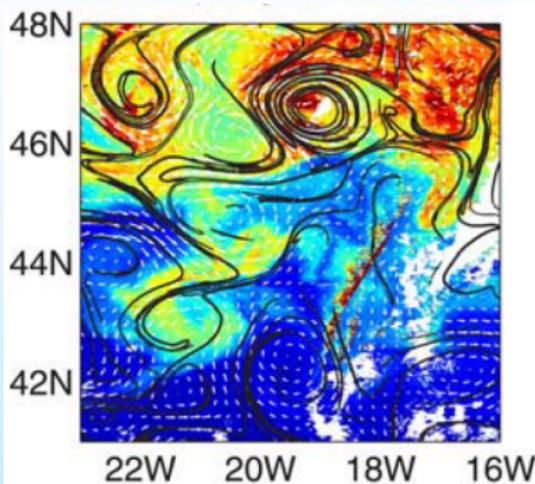
Three major ingredients:

- frontogenesis
- straining by the mesoscale turbulent field
- sub-mesoscale baroclinic instability.

# Connection between FSLE and tracer filaments



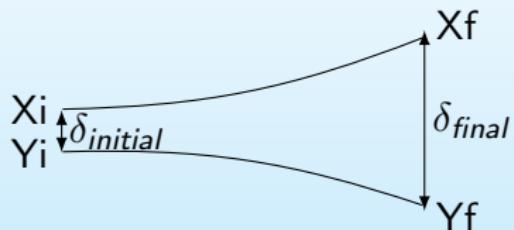
*Chlorophyll, South Atlantic, d'Ovidio & al,  
2004*



*Chlorophyll, Pomme area, Lehahn & al,  
2008*

# Physical meaning of Lyapunov Exponents

Lyapunov exponents are defined as the exponential rate of separation, averaged over time



FSLE

$$\lambda = \frac{1}{T} \times \log\left(\frac{\delta_{final}}{\delta_{initial}}\right)$$

Lyapunov exponents constitute Lagrangian transport barriers between different regions (Lehahn & al (2007)).