Appendix 3 :
GODA's method explanations

Waves generated by a wave maker in a basin are reflected by the beach.

The reflection coefficient is in general less than 10 % for short waves (reflection coefficients less than 5 % are unusual). For long waves, the reflection coefficient can reach as high values as 30% or 40 %.

In such cases, it is necessary to resolve the incoming wave generated by the wave maker into incident and reflected waves.

The method of GODA is commonly used at OCEANIDE.

We explain this method in the case of regular waves.

Let the wave measured at position x be $\eta(x,t)$ and let A be the complex amplitude of the fundamental simple-harmonic component (obtained through a Fourier analysis). Ignoring the other harmonic terms, we can write:

$$\eta(x,t) = A\, e^{i(kx+\omega t)}$$

But this measured wave profile is the sum of the incident and reflected wave, so that:

$$A\, e^{i(kx+\omega t)} = A_I\, e^{i(kx+\omega t)} + A_R\, e^{i(-kx+\omega t)}$$

If the wave profiles are recorded by 2 probes at position x_1 and x_2 :

$$A_1\, e^{i(kx_1+\omega t)} = A_I\, e^{i(kx_1+\omega t)} + A_R\, e^{i(-kx_1+\omega t)}$$

$$A_2\, e^{i(kx_2+\omega t)} = A_I\, e^{i(kx_2+\omega t)} + A_R\, e^{i(-kx_2+\omega t)}$$

We have two equations for the two unknown A_I and A_R. The resolution of these equations yields:

$$A_I\, e^{ikx_1} = \frac{A_2 - A_1 e^{-ik\Delta l}}{2i\sin k\Delta l}$$

$$A_R\, e^{-ikx_1} = -\frac{A_2 - A_1 e^{ik\Delta l}}{2i\sin k\Delta l}$$

The complex reflection coefficient is expressed as:

$$K_R(\omega) = \frac{A_R}{A_I} = C_R(\omega)\, e^{i\varphi} = -\left(\frac{A_2 - A_1 e^{ik\Delta l}}{A_2 - A_1 e^{-ik\Delta l}} \right) e^{2ikx_1}$$

with module $C_R(\omega)$ and argument φ.

Practical aspects:

When the distance between the two probes is equal or near to n.λ /2 with n = 0, 1, 2,…the estimates of incident and reflected wave are expressed as a quotient with a divisor nearly equal to zero. These estimates diverge because numerical errors and errors due to noise are greatly amplified.

The technique adopted by OCEANIDE makes use of a linear array of 5 wave probes. By taking the average of the results of pairs of wave probes giving a non-diverging estimate, the resolution is accurate on a large range of wavelengths. This technique is also well adapted for the spectral resolution of irregular waves.