Estimation of Directional Wave Spectra from an Autonomous Underwater Vehicle (AUV)

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Abstract

Directional wave energy and wave spectra play an important role in the physical processes associated with the ocean environment. Determining these directional wave characteristics is a fairly arduous task. An Autonomous Underwater Vehicle (AUV) provides a suitable platform for making in-situ measurements in the ocean environment. Small AUVs can typically cover a 3-5 mile square region at a speed of 3-4 knots, allowing for the survey of the water column over a substantial range. Further, an AUV is much more versatile than the traditional bottom-mounted ADCPs, suspended current meters or drifting buoys in so much as the sensor (the AUV) is mobile and can be programmed to collect data in any desired area vice having to locate the sensors, recover them, transit and re-deploy.

In this paper we examine the potential of an AUV to be used as a platform for making directional wave spectrum measurements, with particular reference to the Naval Postgraduate School's PHOENIX AUV. We show that by using relative velocity measurements from a SONTEK Acoustic Doppler Velocimeter (ADV), ground-referenced velocity measurements from a RDI Navigator Doppler Velocity Log (DVL), vehicle motion data from a Systron-Donner Motion Pak and employing a Maximum Entropy Method (MEM) the wave directional spectra can be found. The methodology used to determine the spectra, the corrections that are required to account for the Doppler shift due to the moving vehicle and the results obtained from data collected during AUVFEST '98 and experimental missions in Monterey Bay will be discussed and analyzed.

Introduction

Measurements of directional wave spectra have interests that range from fundamental physics, to accurate forecasting, to determination of tactical information for military operations. There are many techniques for collecting spectral information, each of which has advantages and disadvantages. These include deployable or in-situ instruments such as directional wave buoys [Grosskopf 1983], as well as arrays of current meters and pressure sensors [Allender 1989]. Remote sensing techniques involving microwave radar systems [Tyler 1974], aircraft [McLeish 1980], or satellites [Monaldo 1984] are also commonly used. The issue associated with these measuring approaches is that they are too expensive for many users and are not suitable for rapid deployment without significant pre-planning.

Currently many operational AUVs carry sensor suites, which allow the vehicles to record data that many be used to obtain directional spectra estimates. This paper will outline the underlying principles used in identification of wave directions from standard wave following buoys. It will present the mathematical formulas used in determining the wave direction as a function of frequency. Extension of these algorithms to subsurface velocity sensors will be made, where, through the use of the Doppler equation, a moving AUV can be used to determine wave directions. Lastly, it will be shown how a control command can be obtained from the frequency dependent wave direction estimates.

The information in this paper is not new, only the application to which this method is applied. For more detailed descriptions of the mathematical formulations presented in this paper, the reader is referred to [O'Reilly 1996] and the references therein.

Wave Spectra and Directional Estimates

The elevation of the sea surface $\eta(t)$ can be described as the superposition of an infinite number of sinusoids of the form:

$$\eta(t) = \sum_{n=1}^{\infty} a_n \cos(k_n x - \omega_n t + \phi_n) = \sum_{n=1}^{\infty} \eta_n \; . \qquad (1)$$

Wave components with different frequencies are usually assumed to be statistically independent because they are generated by random wind forces at different locations. From the central limit theorem it follows that the probability distribution of the surface elevation, $\eta(t)$, is approximately Gaussian, consistent with many observations, [Soong 1993].

The procedure presently employed by many of the operational data buoys in based on Fourier analysis. In Fourier analysis it is convenient to work with complex exponentials rather than sine and cosine functions, there fore using the relation

$$\cos(\omega t+\phi)=\frac{\exp(i(\omega t+\phi))+\exp(-i(\omega t+\phi))}{2} \quad (2)$$

the expression for the surface elevation can be written as

$$\eta(t)=\sum_{\omega} A_{\omega}\exp(i\omega t)\ , \quad (3)$$

where

$$A_{\omega}=\frac{1}{2}a_{\omega}\exp(i\phi_{\omega}) \quad (4)$$

and the summation is over both positive and negative frequencies.

The energy spectrum $E(\omega)$, is defined as

$$E(\omega)=\frac{\left\langle \left|A_{\omega}\right|^{2}\right\rangle}{\Delta\omega} \quad (5)$$

where $\langle\ \rangle$ indicates an average over many data records and $\Delta\omega$ is the spacing of the frequency bands. The spectrum is closely related to the energy of the waves, and represents the distribution of wave energy as a function of frequency.

To describe the spatial and temporal characteristics of the sea surface linear superposition of wave components is used. In exponential terms this can be represented as

$$\eta(x,y,t)=\sum_{\omega}\sum_{\theta} A_{\omega,\theta}\exp(i(k(x\cos\theta+y\sin\theta)-wt)) \quad (6)$$

where x, y are the horizontal spatial coordinates, and ω and k obey the dispersion relation. The frequency directional wave spectrum is defined as

$$E(\omega)=\frac{\left\langle \left|A_{\omega,\theta}\right|^{2}\right\rangle}{\Delta\omega\,\Delta\theta} \quad (7)$$

and describes the distribution of energy as a function of frequency and direction.

Directional Estimates from Wave Buoys

The most commonly used instrument for measuring waves in deep water is the "heave, pitch and roll buoy" that measures the surface height and slope in two orthogonal directions. The newer Datawell® Directional Waverider measures 3-component accelerations of the buoy, which are integrated to yield the horizontal and vertical displacements of the buoy. The hull and mooring, of this buoy, were designed to give the buoy good wave following characteristics, thereby allowing the buoy displacements to approximate the displacements of an actual water particle at the sea surface.

For a wave train propagating in the positive x-direction, the fluid particle motion is given by

$$u=a\frac{gk}{\omega}\frac{\cosh(k(z+H))}{\cosh(kH)}\cos(kx-\omega t)\ . \quad (8)$$

However, for the more general case of a wave train propagating at some angle relative to the x-axis, it can be shown that the flow field is given by

$$\begin{aligned}
u(x,y,t)&=a\omega\cos\theta\exp(kZ)\cos(k(x\cos\theta+y\sin\theta)-\omega t)\\
v(x,y,t)&=a\omega\sin\theta\exp(kZ)\cos(k(x\cos\theta+y\sin\theta)-\omega t)\\
w(x,y,t)&=a\omega\exp(kZ)\sin(k(x\cos\theta+y\sin\theta)-\omega t)
\end{aligned} \quad (9)$$

Let the average position of the wave buoy be given by $x=y=z=0$. For small amplitude waves, the expected buoy displacements are small compared to the surface wavelength, therefore the buoy motion can be approximated by the fluid velocity at $x=y=z=0$.

For a full spectrum of waves, the buoy displacements can be expressed using complex notation as

$$\begin{aligned}
X(t)&=\sum_{\omega}\sum_{\theta}-iA_{\omega,\theta}\cos\theta\exp(iwt)\\
Y(t)&=\sum_{\omega}\sum_{\theta}-iA_{\omega,\theta}\sin\theta\exp(iwt)\\
Z(t)&=\sum_{\omega}\sum_{\theta}-A_{\omega,\theta}\exp(iwt)
\end{aligned} \quad (10)$$

where the $-i$ is due to the 90° phase difference between the vertical and horizontal displacements. The expressions in (10) can be written using Fourier transforms as

$$\begin{aligned}
X(t)&=\sum_{\omega}X(\omega)\exp(iwt)\\
Y(t)&=\sum_{\omega}Y(\omega)\exp(iwt)\ ,\\
Z(t)&=\sum_{\omega}Z(\omega)\exp(iwt)
\end{aligned} \quad (11)$$

where the Fourier transforms are given by

$$\begin{aligned}
X(\omega)&=\sum_{\theta}-iA_{\omega,\theta}\cos\theta\\
Y(\omega)&=\sum_{\theta}-iA_{\omega,\theta}\sin\theta\ .\\
Z(\omega)&=\sum_{\theta}A_{\omega,\theta}
\end{aligned} \quad (12)$$

To derive the relationships between the measured time series and the unknown frequency-directional wave spectrum the cross spectrum must be considered. In general, the cross spectrum between two time series *X(t)* and *Y(t)* with Fourier transforms $X(\omega)$ and $Y(\omega)$ is defined as

$$C_{XY}(\omega)=\frac{\left\langle X(\omega)Y^{*}(\omega)\right\rangle}{\Delta\omega} \tag{13}$$

where * indicates the complex conjugate, [Soong 1993]. Substitution of (12) into (13) yields

$$C_{XY}(\omega)=\sum_{\theta}\cos\theta\sin\theta E(\omega,\theta) \tag{14}$$

where it is assumed that the wave components propagating in different directions are statistically independent. The cross spectrum C_{XY} can be expressed in continuous form as

$$C_{XY}(\omega)=\int_{0}^{2\pi}\cos\theta\sin\theta E(\omega,\theta)d\theta\ . \tag{15}$$

Cross-spectra of the other time series pairs can be obtained in a similar manner. The full set of relations for buoy cross-spectra can be found in [Dean 1984].

It is convenient to define a normalized directional distribution of energy at a frequency ω as

$$S(\theta;\omega)=\frac{E(\omega,\theta)}{E(\omega)} \tag{16}$$

with unit integral

$$\int_{0}^{2\pi}S(\theta;\omega)d\theta=\frac{\int_{0}^{2\pi}E(\omega,\theta)d\theta}{E(\omega)}=\frac{E(\omega)}{E(\omega)}=1\ . \tag{17}$$

With this definition, (15) and the other referenced spectral relations can be combined and expressed in terms of $S(\theta;\omega)$. Dropping the frequency dependence these relations can be expresses as

$$\begin{aligned}
\frac{\mathrm{Im}(C_{XZ})}{\left((C_{XX}+C_{YY})C_{ZZ}\right)^{1/2}} &= \int_{0}^{2\pi}\cos\theta S(\theta)d\theta\equiv a_1\\
\frac{\mathrm{Im}(C_{YZ})}{\left((C_{XX}+C_{YY})C_{ZZ}\right)^{1/2}} &= \int_{0}^{2\pi}\sin\theta S(\theta)d\theta\equiv b_1\\
\frac{C_{XX}-C_{YY}}{C_{XX}+C_{YY}} &= \int_{0}^{2\pi}\cos2\theta S(\theta)d\theta\equiv a_2\\
\frac{2\,\mathrm{Re}(C_{XY})}{C_{XX}+C_{YY}} &= \int_{0}^{2\pi}\sin2\theta S(\theta)d\theta\equiv b_2
\end{aligned} \tag{18}$$

These four relations between the cross-spectra of the buoy measurements and the directional distribution of wave energy, derived by [Long 1980], form the basis for most of the buoy analysis techniques currently used.

Extension to Subsurface Sensors

As discussed in the previous section, the motion of a wave buoy is directly related to the fluid velocity, therefore, the cross-spectra of a tri-directional current meter yields the same low-resolution directional wave information obtained from buoy measurements. Substituting the normalized spectra of the vertical (*Z*) velocity component w, and the horizontal (*X*, *Y*) velocity components u and v into (18), the lowest four Fourier moments of the directional distribution of wave energy can be obtained and are given by

$$a_1(\omega)=\frac{\mathrm{Im}(C_{wu}(\omega))}{\left[C_{ww}(\omega)\left[C_{uu}(\omega)+C_{vv}(\omega)\right]\right]^{1/2}}\ , \tag{19}$$

$$b_1(\omega)=\frac{\mathrm{Im}(C_{wv}(\omega))}{\left[C_{ww}(\omega)\left[C_{uu}(\omega)+C_{vv}(\omega)\right]\right]^{1/2}}\ , \tag{20}$$

$$a_2(\omega)=\frac{C_{uu}(\omega)-C_{vv}(\omega)}{C_{uu}(\omega)+C_{vv}(\omega)}\ , \tag{21}$$

$$b_2(\omega)=\frac{2\,\mathrm{Re}(C_{uv}(\omega))}{C_{uu}(\omega)+C_{vv}(\omega)}\ , \tag{22}$$

where $\boldsymbol{C}(\omega)$ is the spectral matrix of the velocity components u, v, w. Since the wave direction, θ, is referenced to the navigation frame (N-E-D), vehicle borne sensor measurements must be transformed prior to use. It is interesting to note that the estimates of these directional moments are insensitive to errors, so long as the errors are the same on all measurement axes of the sensors, which is typical with oceanographic sensors installed on AUVs.

The objective of the data analysis is to infer the directional distribution $S(\theta)$, from the four measured moments a_1, b_1, a_2, and b_2. The most widely used techniques are described below.

The Cosine Power Distribution

[Longuet-Higgins 1963] introduced a simple cosine-power distribution,

$$S(\theta)=A\cos^{2s}\left(\frac{\theta-\theta_{mean}}{2}\right) \tag{23}$$

with θ_{mean} the mean propagation direction, s a parameter that controls the width of the distribution and A, a normalization coefficient. The parameters θ_{mean} and s are determined by fitting (23) to the relations given in Equations (19)-(22).

The main drawback to this simple method is that (23), with only two free parameters can describe only unimodal distributions, and thus fails in situations with a bimodal sea state (e.g., multi-directional seas during a wind veering event or swell arriving from two different sources).

The Maximum Entropy Method

[Lygre and Krogstad 1986] introduced the maximum entropy or MEM estimate of $S(\theta)$. Unlike Equation (23), this approach can describe both unimodal and bimodal distributions and exactly fits the relations given in (19)-(22). This directional spectrum is given by

$$S(\theta)=\frac{1}{2\pi}\frac{1-\phi_1 c_1^*-\phi_2 c_2^*}{\left(1-\phi_1 e^{-i\theta}-\phi_2 e^{-2i\theta}\right)^2} \tag{24}$$

with

$$\begin{aligned} c_1 &\equiv a_1+ib_1\\ c_2 &\equiv a_2+ib_2\\ \phi_1 &\equiv \frac{c_1-c_2c_1^*}{1-\left|c_1\right|^2}\\ \phi_2 &\equiv c_2-c_1\phi_1 \end{aligned} \tag{25}$$

Still, the directional distribution is poorly constrained by only four moments and the estimates require careful interpretation, [Krogstad 1991].

Mean Direction and Directional Spread

An alternative approach that avoids the pitfalls of $S(\theta)$ estimation, is to describe the directionality of waves by a few simple parameters. For narrow $S(\theta)$, a mean propagation direction θ_m and a root-mean-square measure of the directional spreading energy σ_θ can be defined in terms of the first-order and second-order Fourier moments a_1, b_1, a_2 and b_2 [Kuik 1988], given by,

$$\theta_m=\tan^{-1}\left(\frac{b_1}{a_1}\right) \tag{26}$$

$$\sigma_\theta^2=2\left[1-\left[a_1\cos(\theta_m)+b_1\sin(\theta_m)\right]\right] \tag{27}$$

$$\theta_m=\frac{1}{2}\tan^{-1}\left(\frac{b_2}{a_2}\right) \tag{28}$$

$$\sigma_\theta^2=\frac{1}{2}\left[1-\left[a_2\cos(2\theta_m)+b_2\sin(2\theta_m)\right]\right]. \tag{29}$$

Again, this method fails to identify bimodal distributions but it is useful to determine a base direction so that a control command could be determined. More on this approach will be discussed later in this chapter.

Correction For A Moving Platform (The Doppler Equation)

The equations for the wave directionality estimations presented in the previous section are valid for a non-moving sensor. However, when information is obtained from an AUV, corrections must be made to account for the vehicle motion. The wave frequency, which the vehicle encounters while moving through the wave field, has been shifted. This shift can be determined by using the well-known Doppler equation,

$$\omega_e\approx\omega-\frac{\omega^2}{g}U\cos\beta\ , \tag{30}$$

where the spectrum that the vehicle encounters is a function of the vehicle's forward speed U and its heading angle relative to the propagation direction of the sea waves, β. Using the techniques outlined in the previous section will give the wave directional distribution as a function of vehicle encounter frequency. If these estimations are used in conjunction with the Doppler equation in a recursive manner, the estimation of $S(\theta)$ can be found as a function of true frequency.

The method by which this is performed is outlined below;

1. Determine the three components of the fluid velocity in vehicle body fixed coordinates.
2. Transform the fluid velocities into the global navigation frame.
3. Compute the auto- and cross-spectra of the fluid velocity components.
4. Determine the Fourier moments using (19)-(22).
5. Convert the Fourier moments into Krogstad notation and use the MEM to determine the directional distribution $S(\theta)$.
6. Using the vehicles mean velocity, and the mean direction obtained from (26) or (28), apply the Doppler equation to determine the frequency shift.
7. Return to 3 and complete the process until frequencies converge.

The corrections to the estimation of $S(\theta)$ using the Doppler equation are quite small for slow moving vehicles. Considering, for example, the NPS Phoenix AUV which has a maximum forward velocity of 1.5 m/s, the error associated with not using the Doppler equation are approximately ± 1 sec in identification of wave periods between 4 and 40 seconds. Similarly, the error in direction estimation is approximately 5-7 degrees. When an AUV goes into a station-keeping mode, where vehicle velocities are significantly reduced, the modifications required due to the Doppler shift are negligible.

Experimental Results Of Wave Directional Estimation Using The NPS Phoenix AUV

During November 1998, the NPS Center for AUV Research, under the direction of Professor Anthony Healey, participated in the ONR sponsored AUVFEST '98. This AUV technology demonstrator was held in the Gulf of Mexico, south of Gulfport, MS. A complete description of the exercise can be found in [Bunce 1998].

The Phoenix AUV was used during this exercise as a mobile sensor to gather oceanographic data. Using the concepts presented in the previous section, the vehicle conducted 27 short-term sampling missions. The products that were obtained included directional energy plots, directional spectrum plots and mean current estimations.

The key to the ability of Phoenix to obtain data capable of producing this information is the combined ADV/RDI/MotionPak/Directional Gyro sensor suite. The physical layout of the NPS Phoenix AUV is shown in Figure 1. Detailed description of the vehicle can be found in [Marco 1996] and [Brutzman 1997]. In addition an online description can be found on the NPS center for AUV Research web site at http://www.cs.nps.navy.mil/research/auv/auvstats. With these sensors, accurate three-dimensional fluid velocities, expressed in global quantities, were capable of being obtained in post-processing. Since the RDI/ADV sensors are collocated, little vehicle induced motion remains after processing the data.

The data obtained validated the concept of obtaining tactical oceanographic data from an underwater vehicle. During the collection of the data, remnants of Hurricane Mitch were still present in the Gulf, providing an excellent source of ground swell. In addition, there was a significant wind generated wave component in a different direction than the swell component, resulting in a multi-modal spectrum, Figure 2.

The results presented in Figures 3-5 provide a sample of the oceanographic data obtained during this offshore exercise. As seen in Figure 3, the bi-modal properties of the seaway are captured, as well as an estimate of the significant wave height. The ability to estimate the dual directions is due to the use of the MEM algorithm employed. Figure 4 presents the associated spectrum plots for this energy estimate. The ability of an AUV to estimate currents is shown in Figure 5. Using a triangular, time based run; the current can be determined using set and drift calculations from the error in final position as well as the heading error on each leg. This information can be feed directly into the vehicle's navigation process to account for the offset due to current thereby increasing navigation accuracy. Short-term averages from each of the three legs are in general agreement with the overall average determined from the navigational drift.

During experimental validation of a new Disturbance Compensation Controller [Riedel 1999], wave direction estimates were obtained in Monterey Bay, see Figure 6. Again the Phoenix was able to record data which can be used to determine directionality spectra, Figure 7.

Conclusions

This paper has presented the techniques currently employed for the determination of wave directional estimations from standard wave following buoys. It has extended this analysis for use in determining directional estimates from data gathered by an Autonomous Underwater Vehicle. Using this data gathered, an approach was presented which will allow the deployed vehicle to obtain information about the directionality of its working environment thereby allowing the vehicle to have information available to make decisions regarding mission execution. Actual experimental results using the NPS Phoenix AUV have been presented. It was shown that tactical oceanographic data is obtainable from a moving AUV. The vehicle in this manner becomes an intelligent, mobile off-board sensor, thus presenting the argument for AUV deployment with operational fleet units.

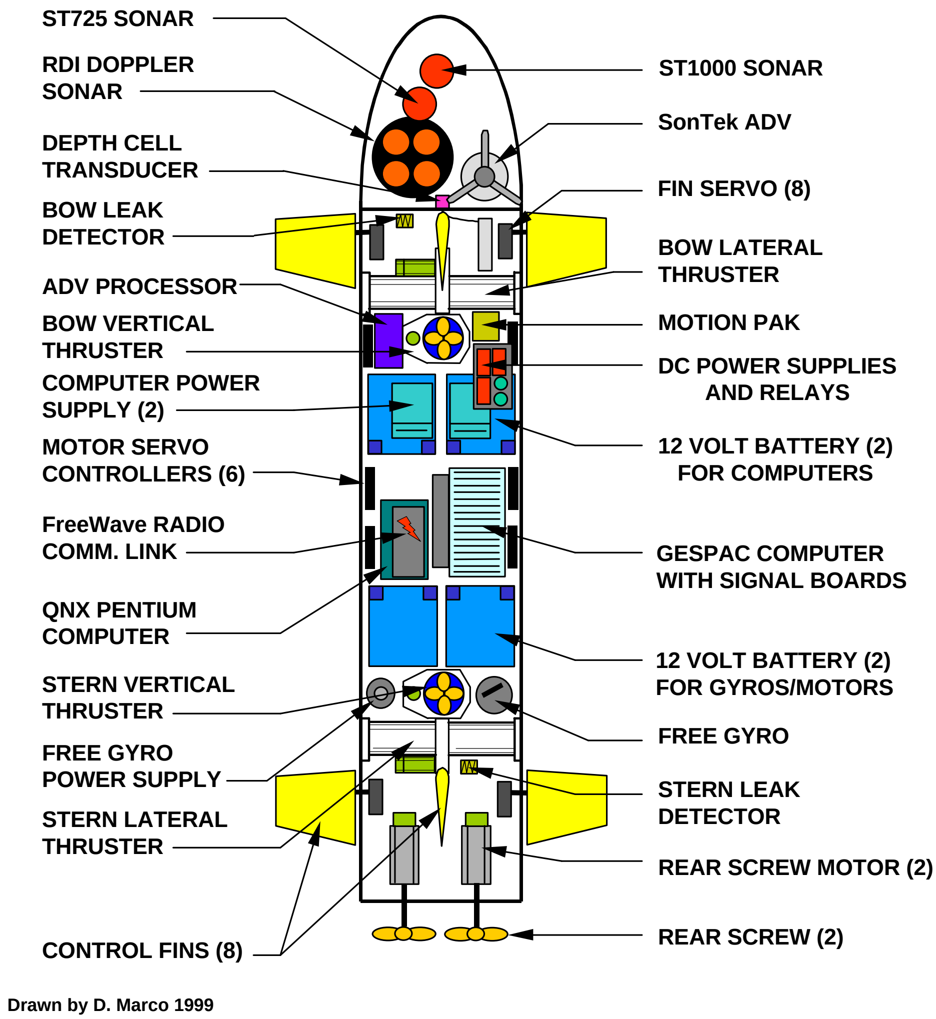

Figure 1 Physical Layout of Phoenix AUV

Figure 2 Phoenix Being Launched from the R/V Gyre

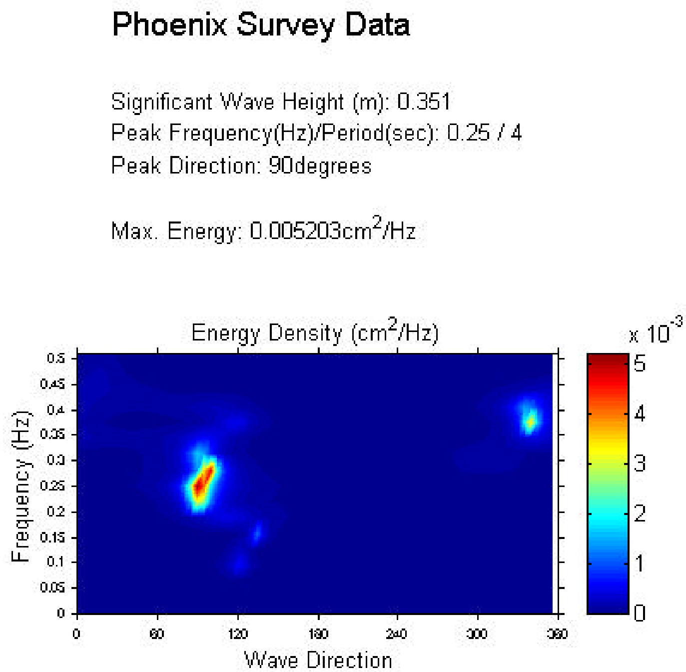

Figure 3 Sample Direction Energy Plot From Phoenix Data, Nov. 4, 1998 (Run# 9), Gulf of Mexico, AUVFEST '98

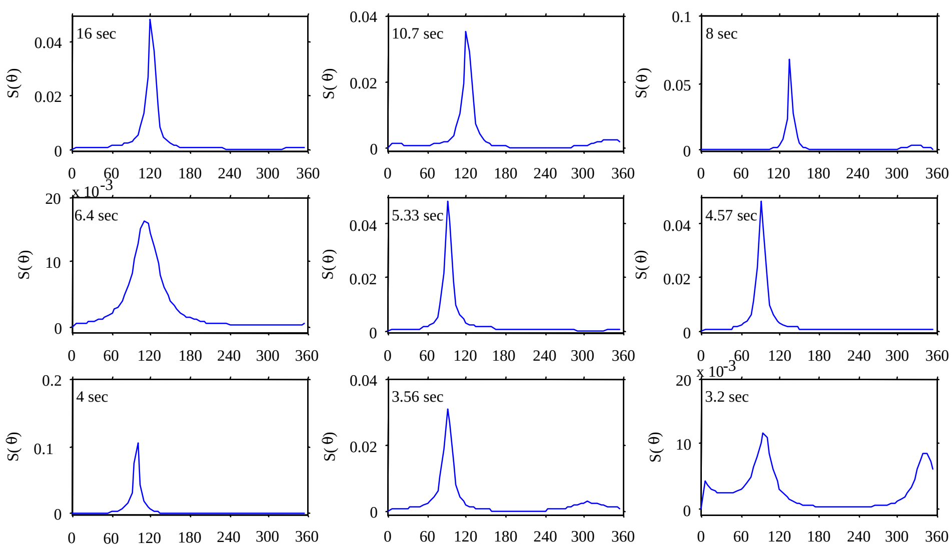

Figure 4 Sample Direction Spectrum Plots From Phoenix Data, Nov. 4, 1998 (Run# 9), Gulf of Mexico, AUVFEST '98

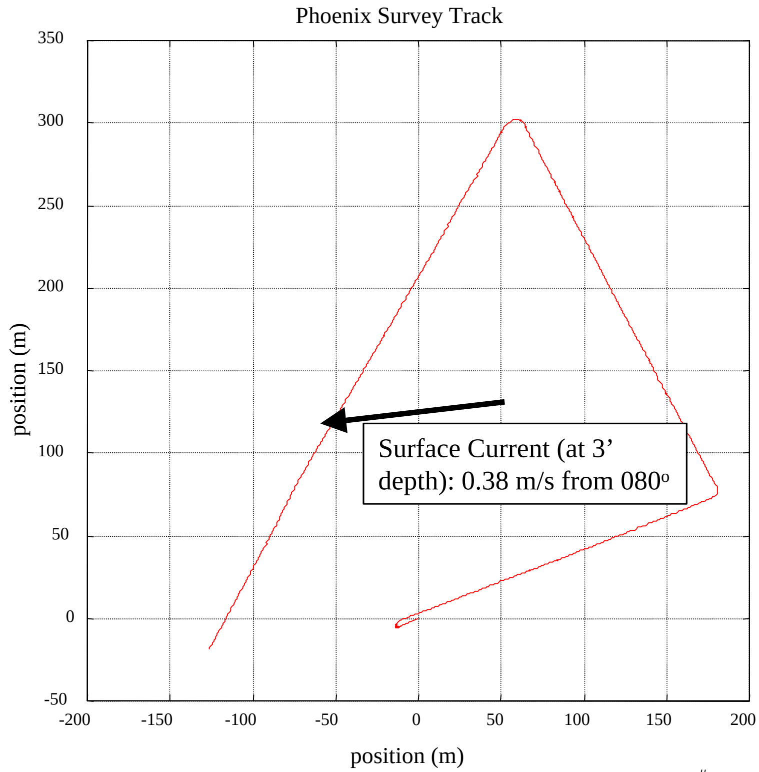

Figure 5 Short-term Current Estimation from Phoenix, November 8, 1999, (Run# 2), Gulf of Mexico, AUVFEST '98

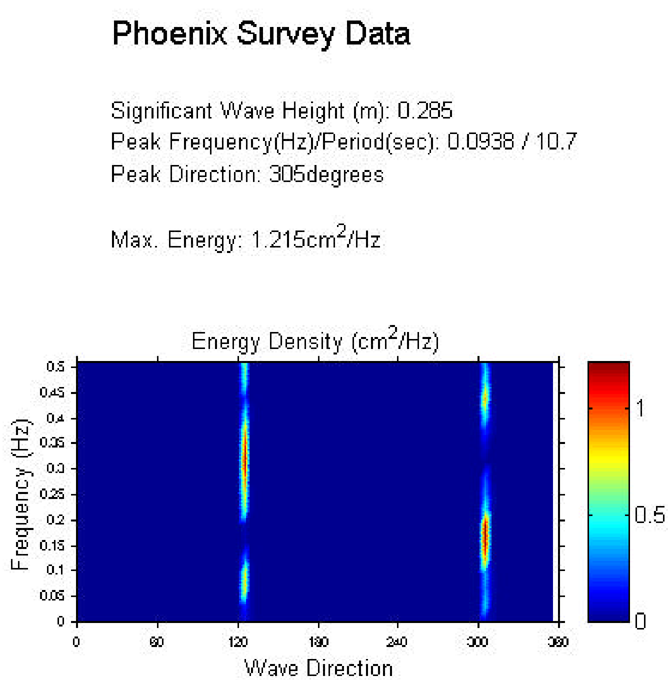

Figure 6 Sample Direction Energy Plot From Phoenix Data, June 15, 1999 (Run# 4), Monterey Bay

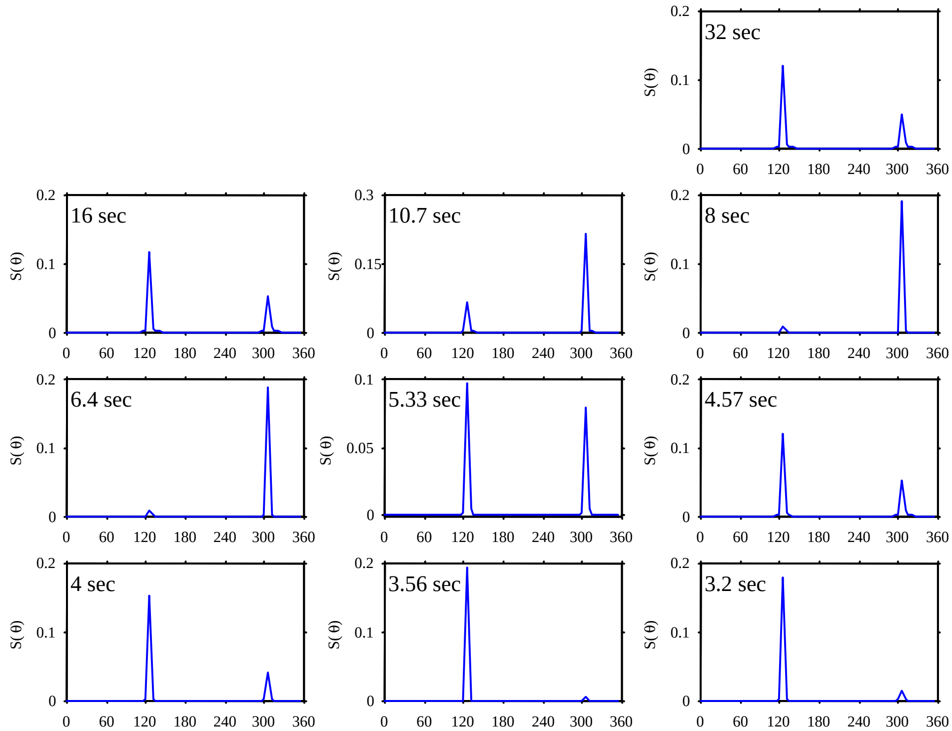

Figure 7 Sample Direction Spectrum Plots From Phoenix Data, June 15,1999 (Run# 4), Monterey Bay

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