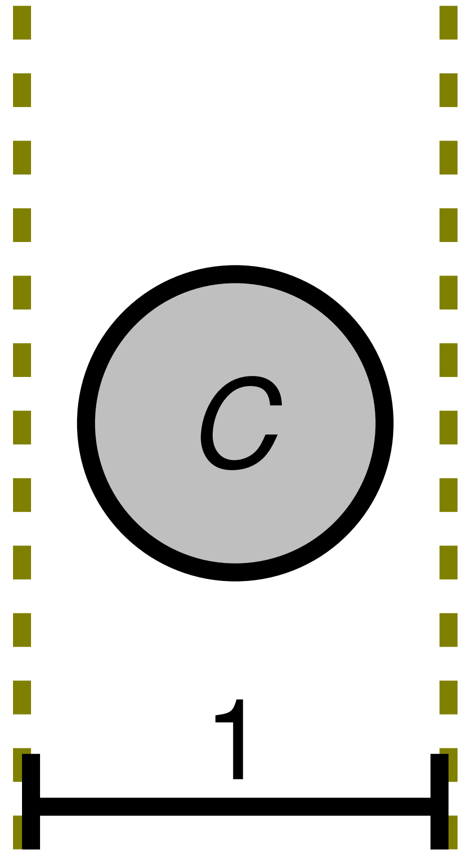
New approach: Transfer operators and directional spectrum

Generalised channel problem

$$\phi_-^{\rightarrow} = \int_{\Gamma_\rightarrow} a_-(\chi)\varphi(x+0.5, y:\chi)\,\mathrm{d}\chi \qquad \phi_+^{\rightarrow} = \int_{\Gamma_\rightarrow} b_+(\chi)\varphi(x-0.5, y:\chi)\,\mathrm{d}\chi$$



$$\phi_-^{\leftarrow} = \int_{\Gamma_\leftarrow} b_-(\chi)\varphi(x+0.5, y:\chi)\,\mathrm{d}\chi \qquad \phi_+^{\leftarrow} = \int_{\Gamma_\leftarrow} a_+(\chi)\varphi(x-0.5, y:\chi)\,\mathrm{d}\chi$$

- $\Gamma_\rightarrow = \{\gamma \in \mathbb{R} : -\pi/2 < \gamma < \pi/2\}$ + complex branches; $\Gamma_\leftarrow = \Gamma_\rightarrow + \pi$

- Reflection and transmission operators

$$b_- = \mathcal{R}\{a_-\} + \mathcal{T}\{a_+\} \quad \text{and} \quad b_+ = \mathcal{T}\{a_-\} + \mathcal{R}\{a_+\}$$

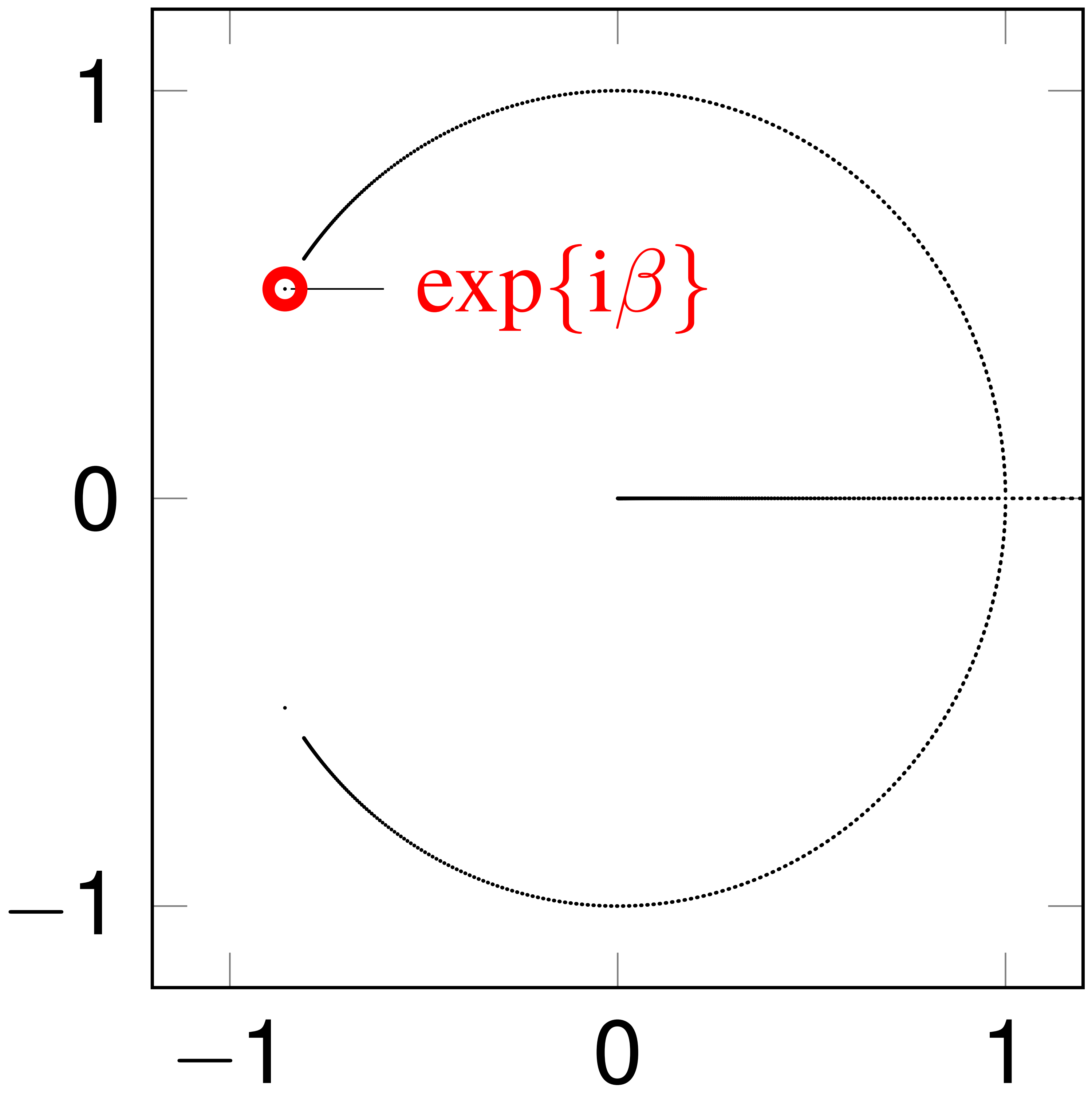
- Transfer operator (left to right map) is

$$\left\{ \begin{array}{c} b_+ \\ a_+ \end{array} \right\} = \mathcal{P} \left\{ \begin{array}{c} a_- \\ b_- \end{array} \right\} \quad \text{where} \quad \mathcal{P}(k, a) = \begin{pmatrix} \mathcal{T} - \mathcal{R}\mathcal{T}^{-1}\mathcal{R} & \mathcal{R}\mathcal{T}^{-1} \\ -\mathcal{T}^{-1}\mathcal{R} & \mathcal{T}^{-1} \end{pmatrix}$$

Transfer operator: Spectrum below cut-off

Example: $a = 0.25$; $k = 0.8\pi$

Eigenvalues $\in \mathbb{C}$



Eigenfunction