A transition loss theory for water waves reflected and transmitted by an overwashed body

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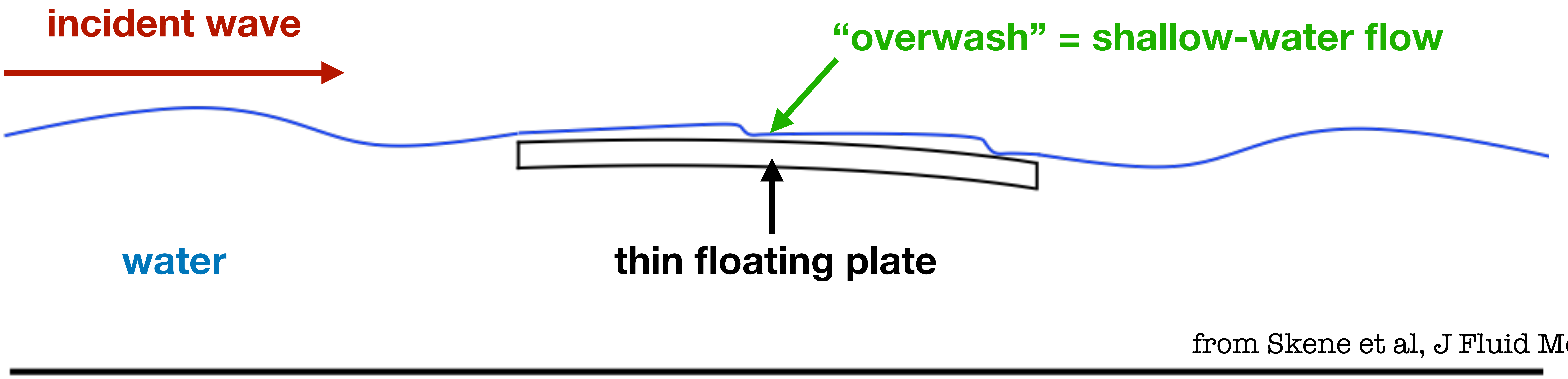
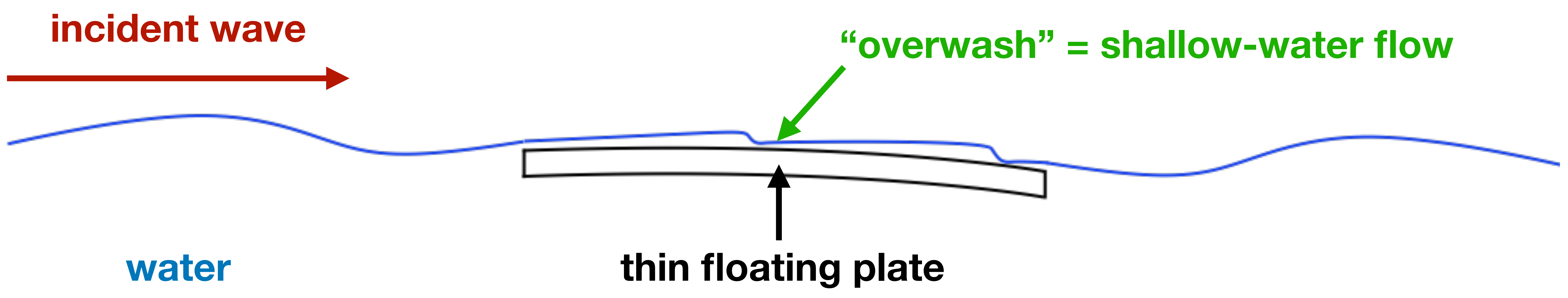
Australian Antarctic Division



from Skene et al, J Fluid Mech, 2015



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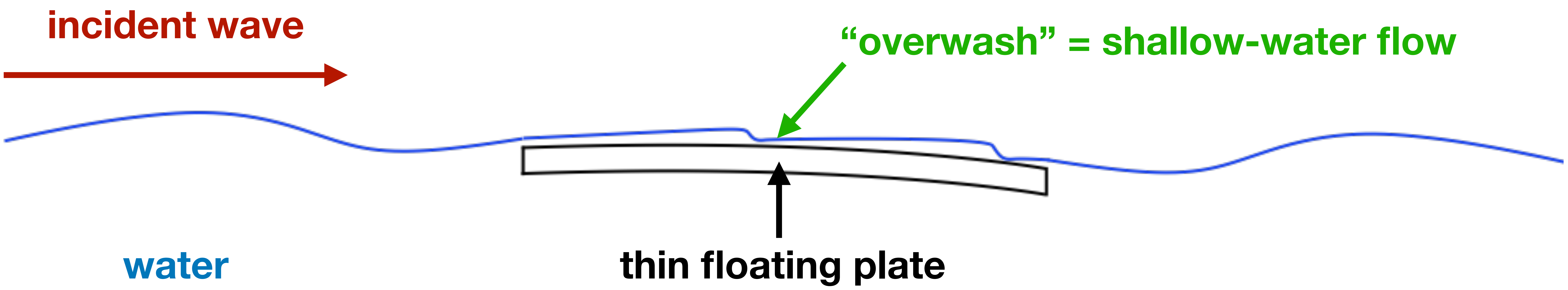
Overwash model: Nonlinear shallow-water equations

- Set boundary conditions using linear (hydroelastic) theory.
- No back-coupling from overwash to surrouding water.



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Overwash model: Nonlinear shallow-water equations

- Set boundary conditions using linear (hydroelastic) theory.
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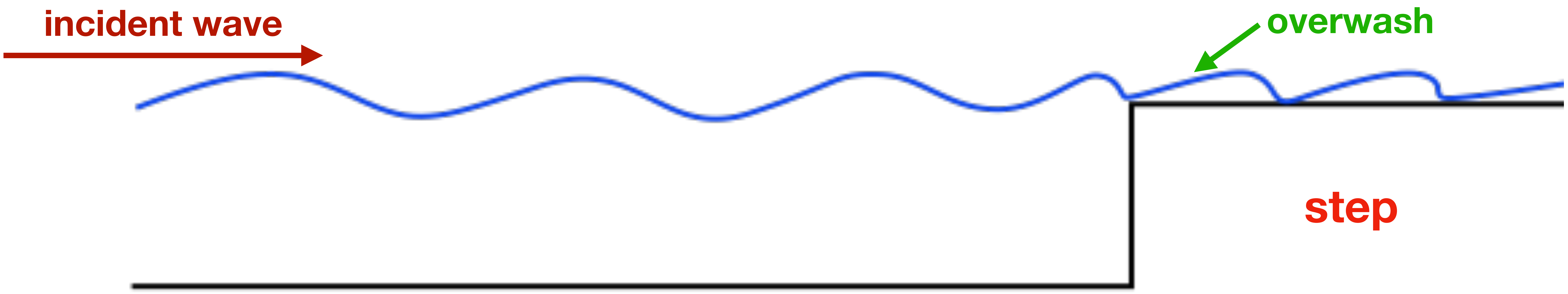
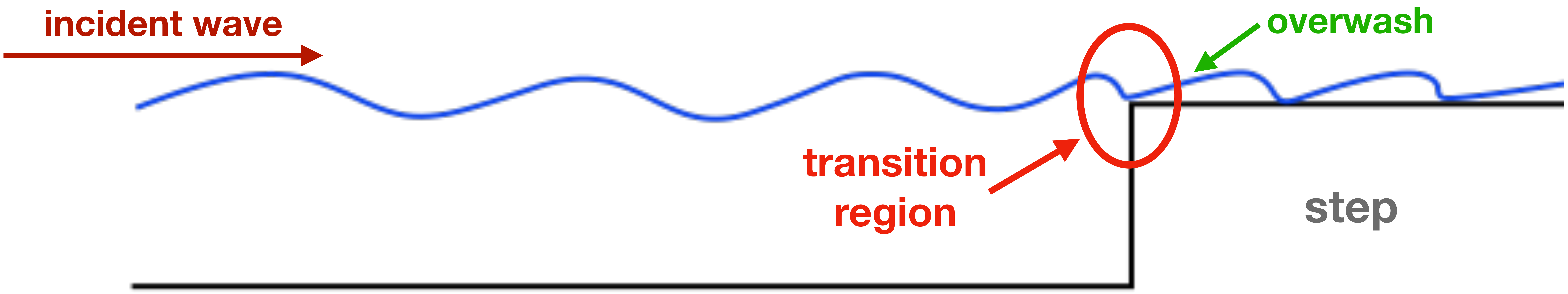
Assessment vs laboratory experiments

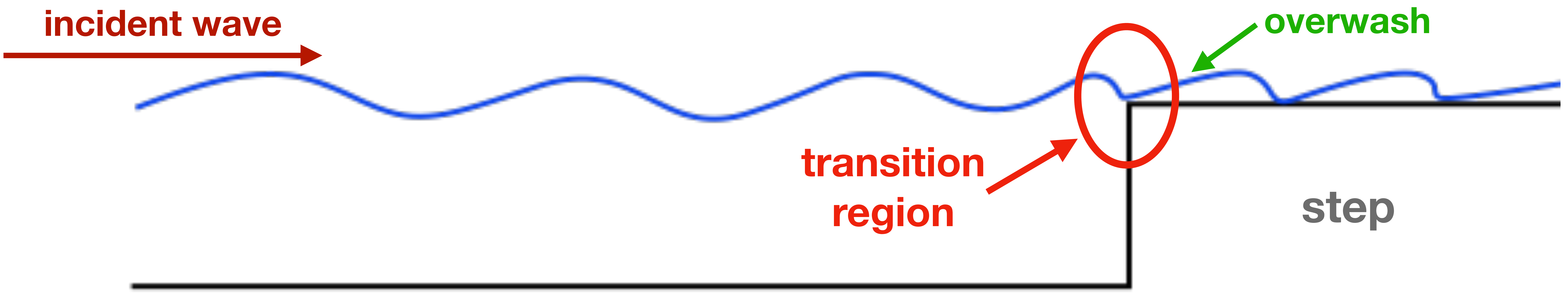
- Predicts overwash depths and bores accurately for small incident steepness.
- Inaccurate for large incident steepness.


incident wave
overwash
step



Step problem. . . a simpler problem

- Only one transition $\longrightarrow$ no bore collisions.
- No floating body motions



Step problem. . . a simpler problem

- Only one transition $\longrightarrow$ no bore collisions.
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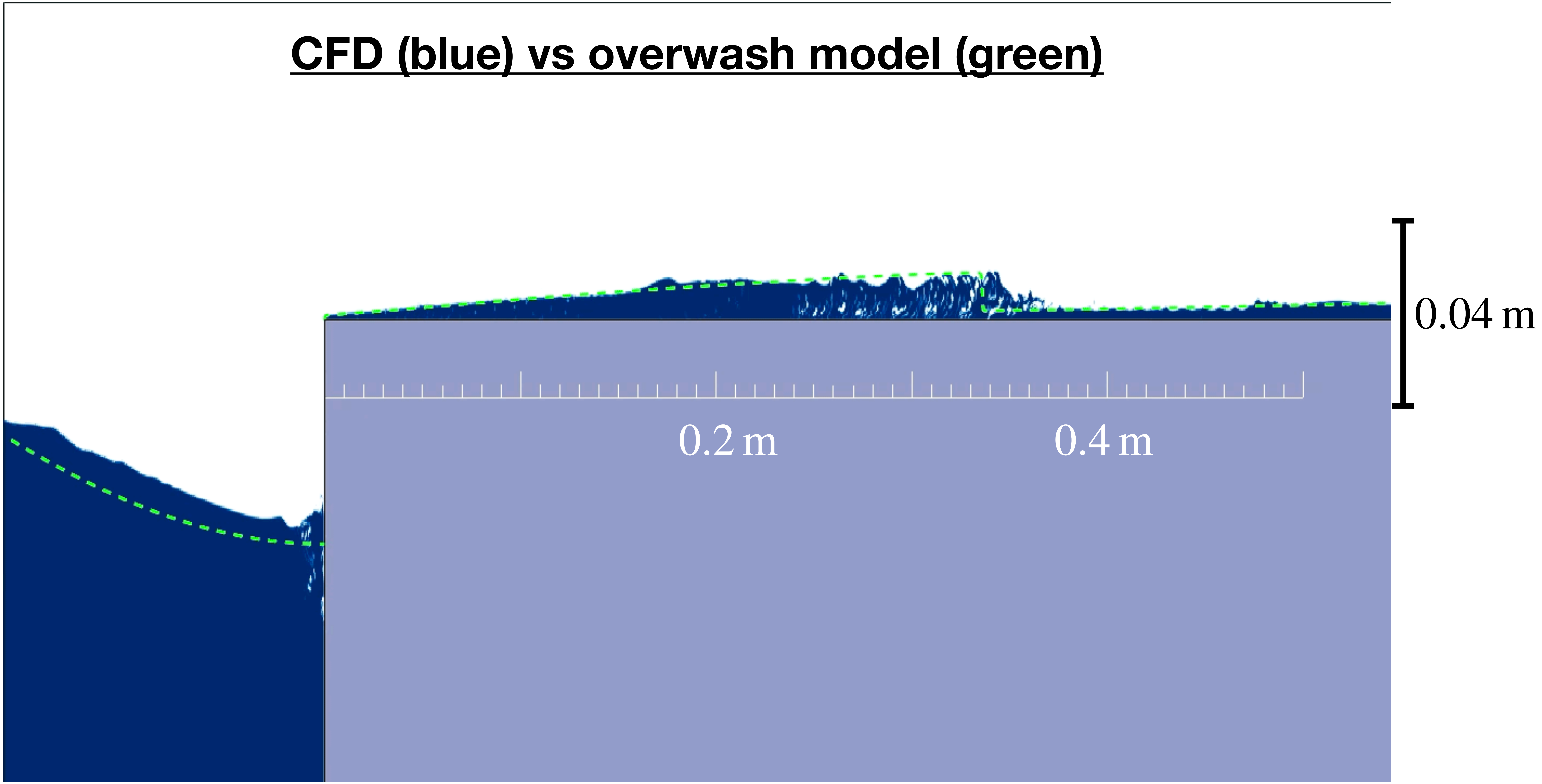
Assessment vs CFD (two-phase Navier–Stokes)

- Does **not** capture complex dynamics at interface.
- Predicts depths and velocities accurately short distance onto step.
- Predicts mass and energy fluxes accurately at all points along step.



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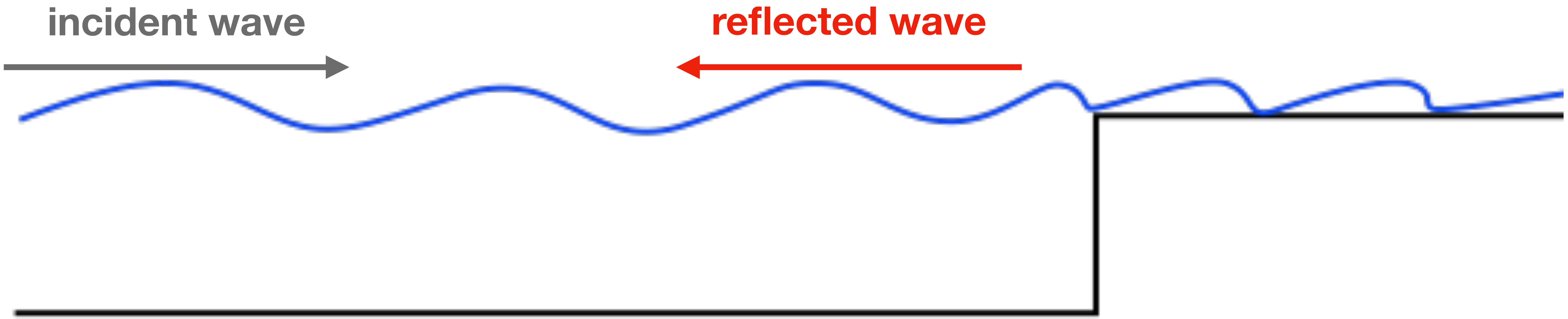
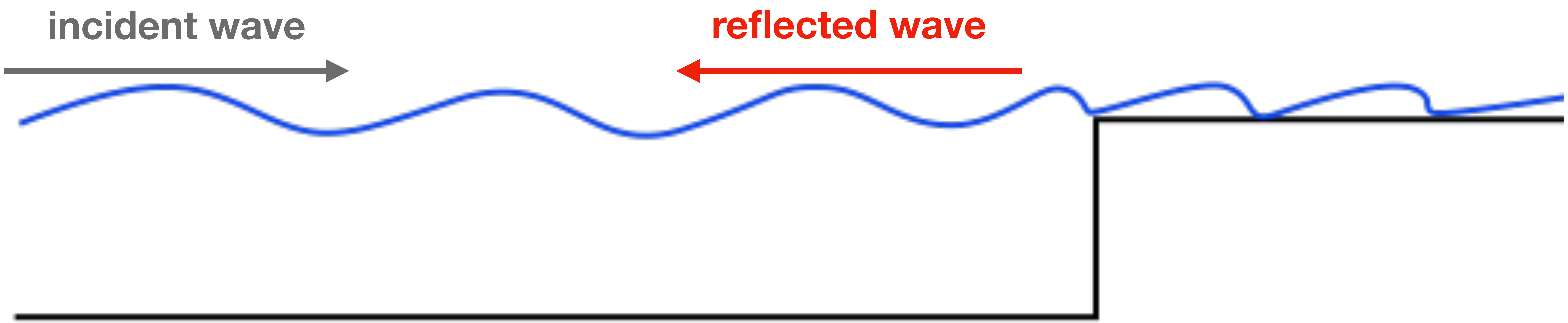


Reflection by a step

- Linear theory (no overwash) predicts all wave energy reflected by step.



Reflection by a step

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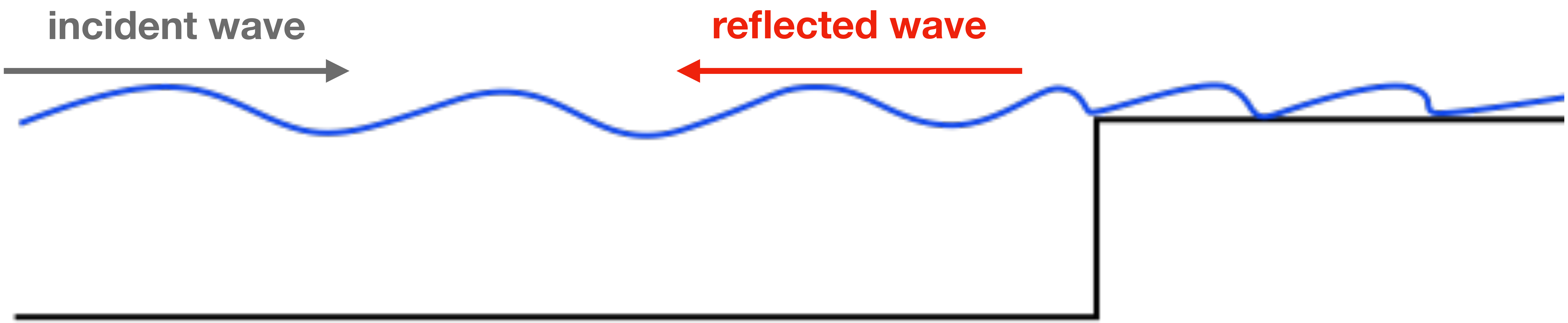
- CFD shows overwash reduces reflection.

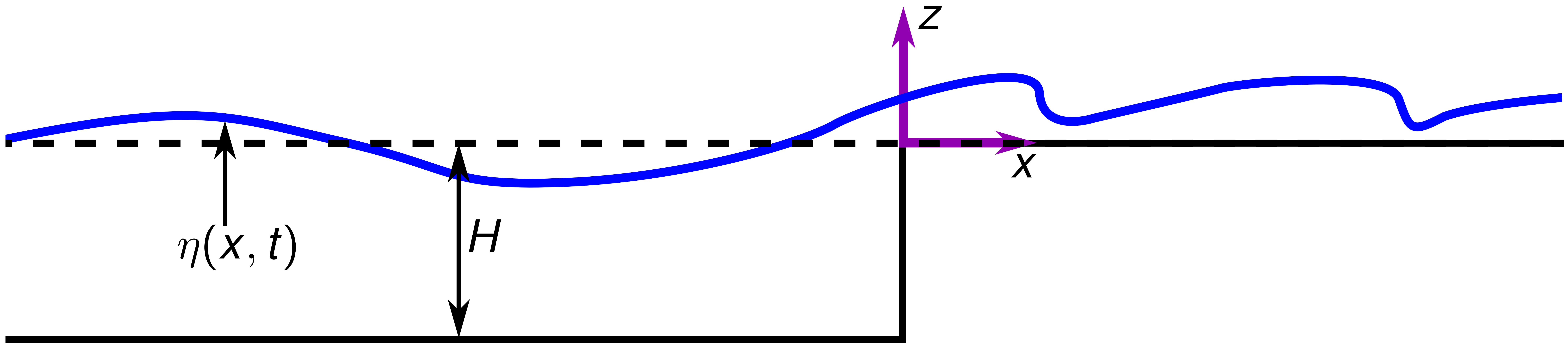


Reflection by a step

- Linear theory (no overwash) predicts all wave energy reflected by step.
- CFD shows reflection reduced by overwash.
- We made first attempt to capture reduced reflection:

linear theory $\longrightarrow$ overwash prediction $\longrightarrow$ correct reflection

Problem 1: Reflection by a step, Attempt #2

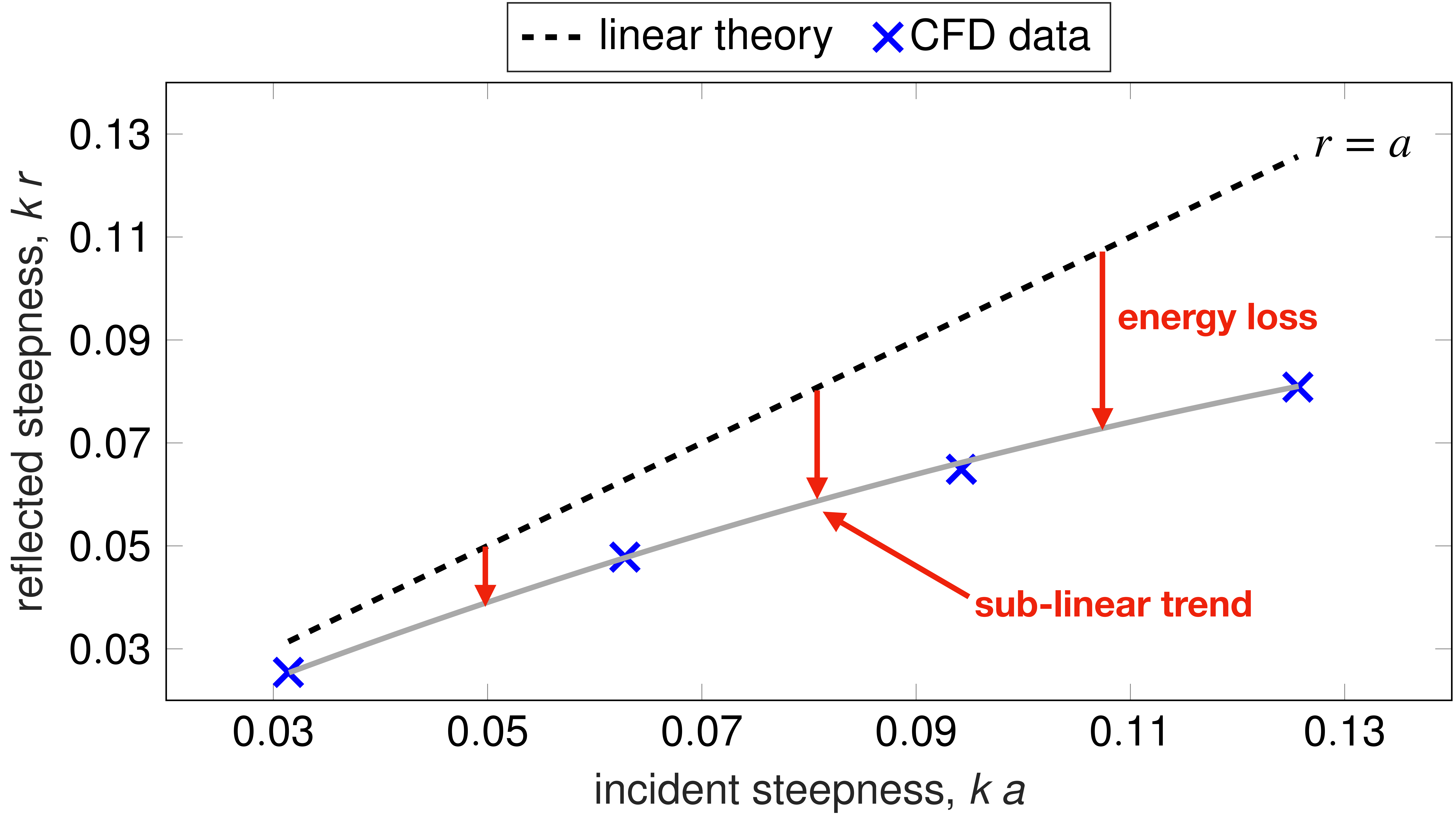


- Regular incident wave

$$\eta_{\text{inc}} = a\cos(k\,x - \omega\,t)$$

where a = amplitude, ω = angular frequency, $k(\omega)$ = wavenumber.

Reflection by a step: Wave period $T \equiv 2\pi/\omega = 0.8$ s



“Naive” correction theory

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- The (density scaled) energy flux in the overwash is

$$E_{ow}(x,t) = \frac{1}{2}\, h\, \hat{u}^3 + g\, h^2\, \hat{u} \quad \text{for} \quad x > 0,$$

where $h(x,t)$ = overwash depth, $\hat{u}(x,t)$ = depth averaged horizontal velocity.

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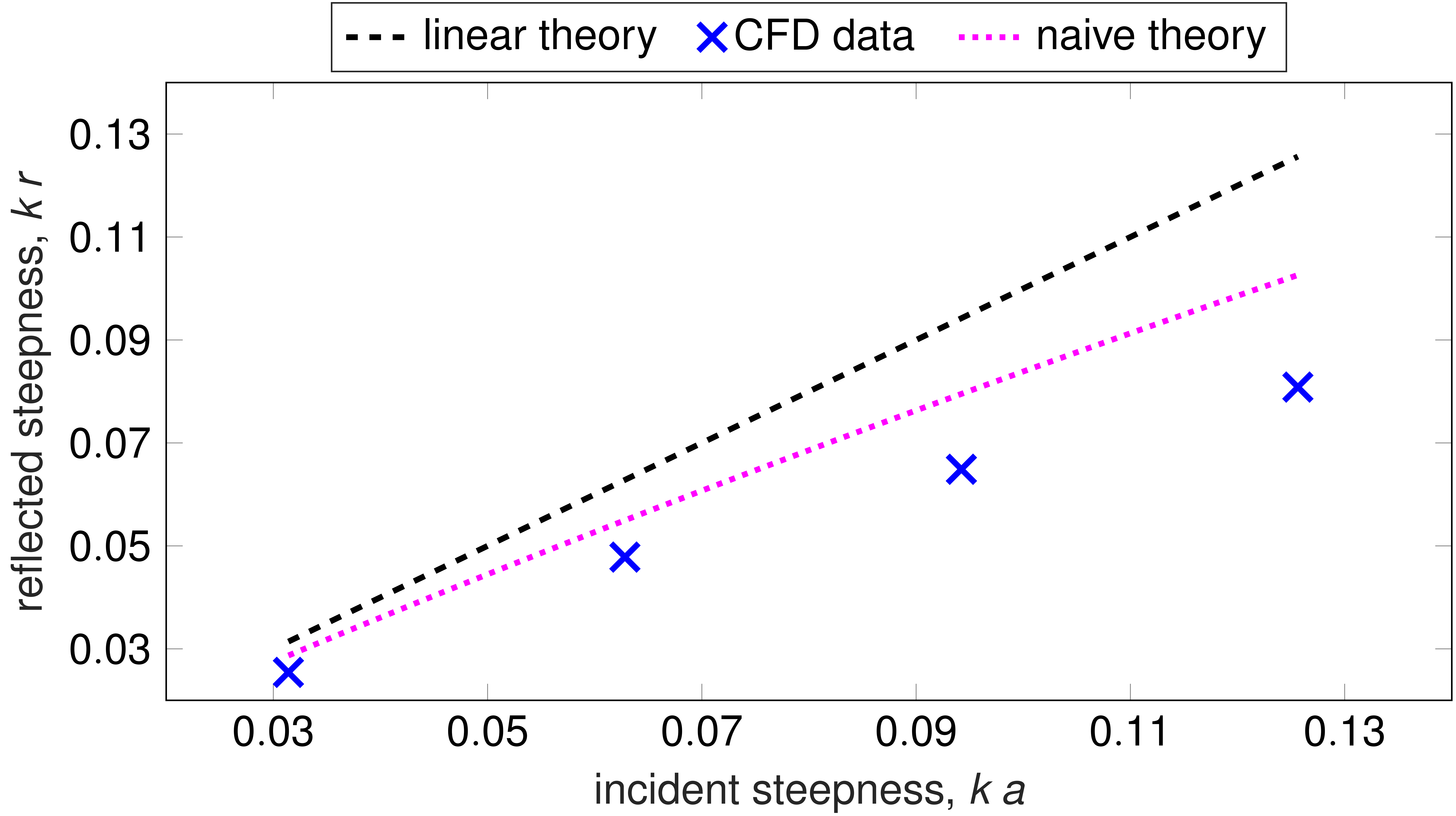
  where $h(x,t) =$ overwash depth, $\hat{u}(x,t) =$ depth averaged horizontal velocity.
- Derive expression for reflected amplitude

$$\frac{g^2}{4\,\omega}\,(a^2 - r^2) = \overline{E_{ow}} \quad \Rightarrow \quad r = \sqrt{a^2 - \frac{4\,\omega}{g^2}\,\overline{E_{ow}(0^+,t)}}$$

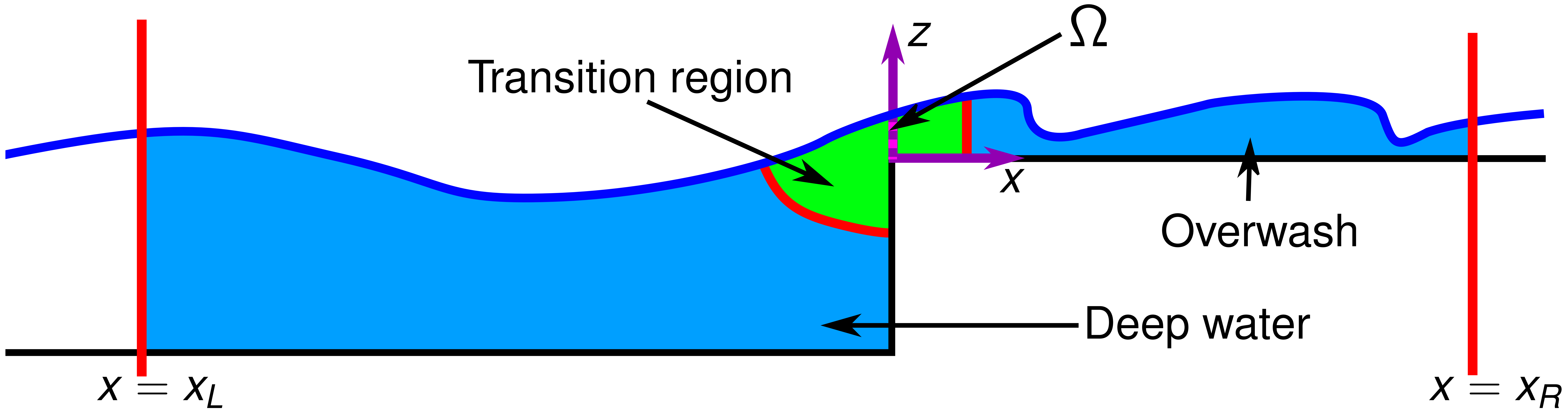
  where overbars denote time averages over wave period.

“Naive” correction theory: Better but still inaccurate



Transition loss theory

Let C be water domain between $x = x_L$ and $x = x_R$.



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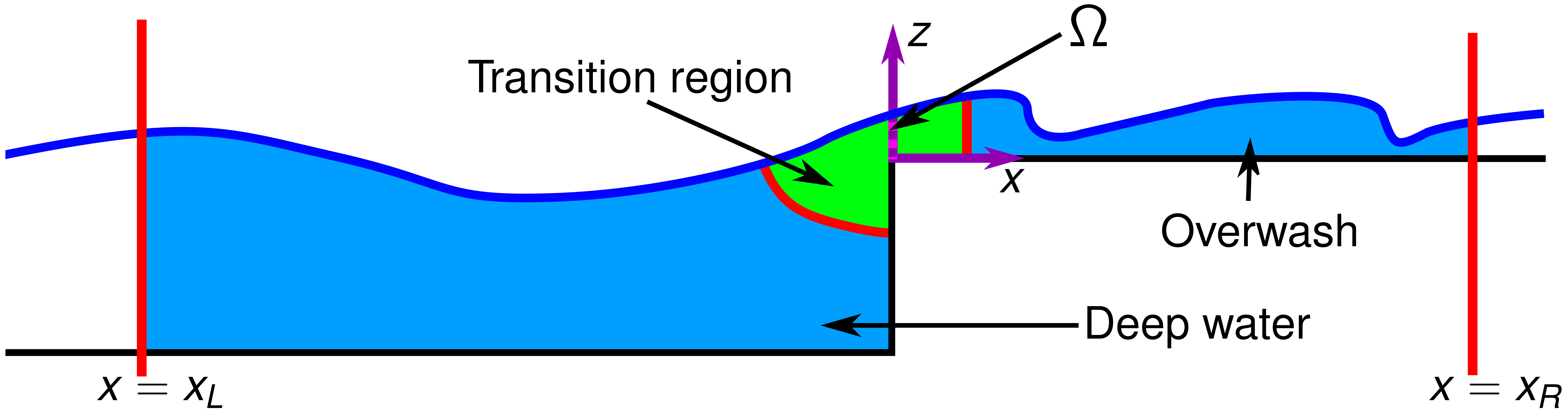

- Idea: Allow for energy loss due to hydraulic jump in transition region.

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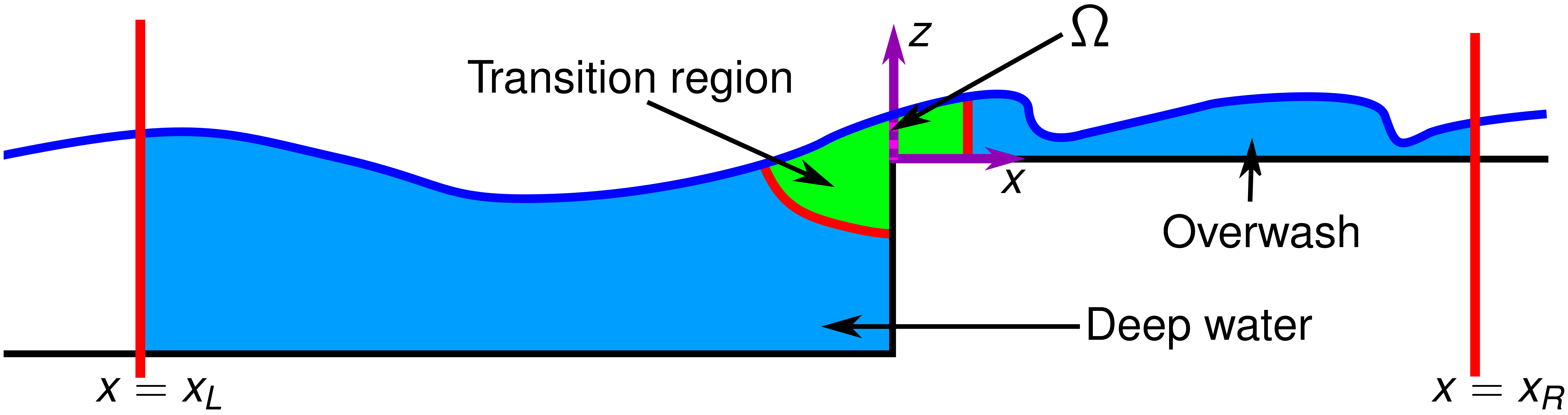

- Idea: Allow for energy loss due to hydraulic jump in transition region.
- Idea: Collapse transition region to vertical branch Ω.

Transition loss theory

- Energy “conservation"

$$\int_C \frac{\partial E}{\partial t}\,\mathrm{d}V + \int_{\partial C\setminus\Omega} (E+p)\mathbf{u}\cdot\mathbf{n}\,\mathrm{d}S + \boxed{\left[\int_\Omega (E+p)\mathbf{u}\cdot\mathbf{n}\,\mathrm{d}S\right]_{x=0^-}^{x=0^+}} \cancel{-L_b} = 0$$

where $\mathbf{n}$ = normal, E =energy, p =pressure, $\mathbf{u} = (u, w)$= velocity.

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where $\mathbf{n}=$ normal, $E=$energy, $p=$pressure, $\mathbf{u}=(u,w)$= velocity.

- In overwash ($x>0$), shallow-water theory gives

$$\mathbf{u}=(\hat{u},0), \quad p=\rho\,g\,(h-z), \quad E=\frac{\rho\,\hat{u}^2}{2}+\rho\,g\,z$$

Transition loss theory

- In deep water ($x < 0$), potential-flow theory gives

$$p = -\rho \left(\frac{\partial \phi}{\partial t} + \frac{1}{2}(w^2 + u^2) + g\,z \right), \qquad E = \frac{\rho}{2}(w^2 + u^2) + \rho\,g\,z$$

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- $\mathbf{u} = \nabla\phi$, where $\phi =$ velocity potential

$$\phi(x, z, t) = \frac{g}{\omega}\{a\cos(k\,x - \omega\,t) + r\cos(k\,x + \omega\,t)\}\,\frac{\cosh\{k\,(z + H)\}}{\cosh(k\,H)}$$

where $r =$ reflected amplitude is unknown.

Transition loss theory

- Average over wave period; let $x_R \to 0_+$; evaluate integrals; neglect terms $\mathrm{O}(k^4\, a^4)$, ...

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$$\frac{g^2}{4\,\omega}(a^2 - r^2) = \boxed{\left(\overline{E_{ow}(0^+, t)} - \frac{2\,g^2\,k}{3\,\pi\,\omega}(a - r)(a + r)^2\right)} + \overline{E_{ow}(0^+, t)}.$$

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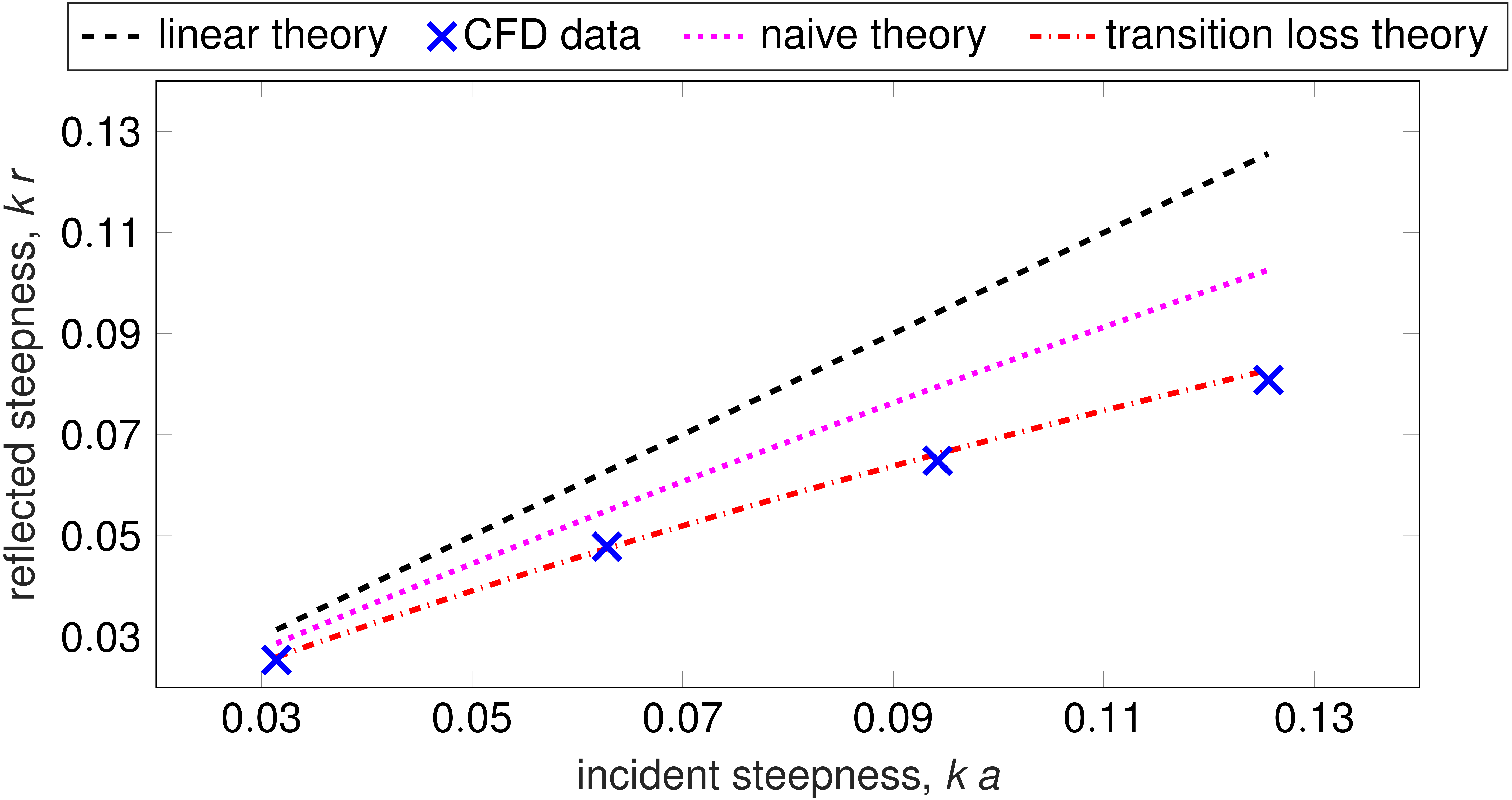
- A cubic to be solved for r such that $0 < r < a$.

Transition loss theory: Nailed it!



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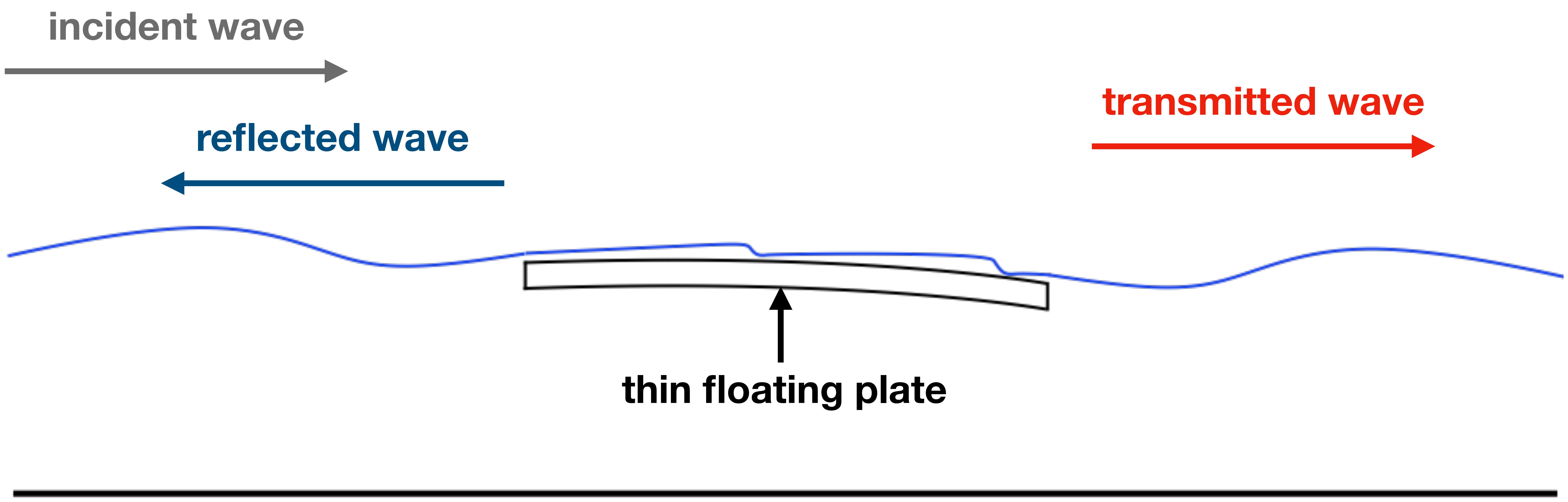
Problem 2: Transmission by a plate

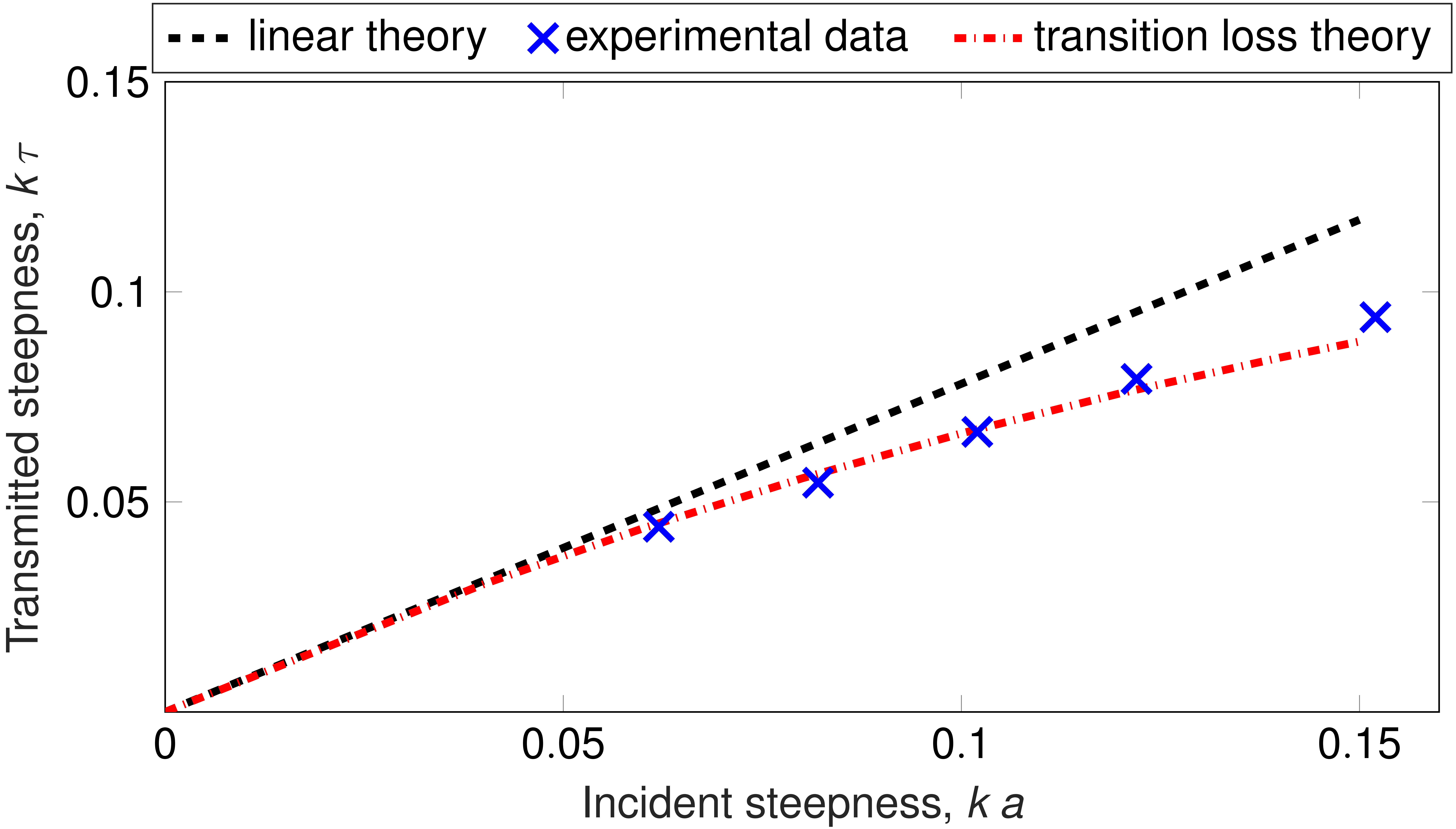


Transition loss theory

Assumptions

- Reflection coefficient is given by linear theory.
    - Based on experimental evidence.
- Local “evanescent" waves are ignored.
    - Consistent with overwash prediction theory.
- Horizontal velocity ≈ 0 above plate ends.
    - Based on no-flow condition at plate ends.
- There is an energy sink in the middle of the plate.
    - So overwash does not return to surrounding water.

Transmission by a plate: Nailed it again!



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Final thoughts

Limits of theory

- Up to what steepness does the theory hold?
- What if the plate is allowed to drift?

Extensions

- Multiple plates.
- 3D problem, e.g. a floating disk.
- Irregular waves.